2 QDQH6XSSOHPHQWV

$ QXPEHU RI VXSSOHPHQWV DUH DYDLODEOH WR VWXGHQWV DQG RU LQWUXFWRUV DVVRFLDWHG WLWZHEVL
ZZZ PKKHFRP ZKLWH 6WXGHQWKDYHDFFHVVWRD6WXGHQW6WXG\ * XLGHGHYHORSHGE\ -HUU ' XQQ
7H[DV $ 0 8QLYHUVLW\ 7KH\ DUH DOR DEOH WR XWLOL]H (QJLQHHULQJ (TXDWRQ 6RIWHU ((6 IO
PHFKDQLFVYLGHRVGHYHORSHGE\ * DU 6HWWHVRI 3HQQV\OYDQLD6WDWH8QLYHUVLW DQG&) ' LPDJHVDO
DQLPDWRQVSUHSDUHGE\ ) OXHQWJQF $OR DYDLODEOHWR VWXGHQWVDUH) XQGDPHQWDOVRI (QJLQHHULQJ)(
([DP TXL]]HV SUHSDUHGE\ (GZDUG$QGHUVRQRI 7H[DV7HFK8QLYHUVLW
,QWUXFWRUVPD\ REWDLQDVHUHVRI 3RZHU3RLQWVOLGHVDQGLPDJHV SOXV WKHIXOO6RXUFH0DQXDO
LQ3') IRUPDW7KH6RXUFHV0DQXDOSURYLGHVFRPSOHWHDQG GHWDLOHG VROXWLRQV LQFOXGLQJ SUREOH
VWDWHPHQW DQG DUWZRUN WR WKH HQG RI FKDSWU SUREOPV ,WPD\ EH SKRWRFRSLHG IRU SRVWQJ R
SUHSDULQJ WDQVPDUHQFLHVIRUWKHFODVVURRP ,QWUXFWRUVFDQDOR REWDLQDFFHVVWR&2 6 0 2 6 IRUW
VHYHQWK HGLWLRQ &2 6 0 2 6 LV D &RPSOHWH 2QOLQH 6RXUFH 0DQXDO 2UJDQL]DWLRQ 6\ WV
LQWUXFWRUV FDQ XVH WR FUHDWH H[DPV DQG DVLJQPHQW FUHDWH FXWRP FRQWHQW DQG HGLWXSSOHP
SUREOPVDQG VROXWLRQV

(OHFWURQLF 7H[WERRN 2SWLRQV

(ERRNVDUHDQLQGRYDLYHZD\ IRUVWXGHQWVWR VDYHPRQH\ DQG FUHDWDJUHQHUHQYLURQPHQWDWWK
VDPHWPH $QHERRNFDQVDYHVWXGHQWDERXWKDOWKHFRVWRI DWDGLWLRQDOWH[WERRNDQG RIIHUVXQLTX
IHDWXUHV OLNH D SRZHUJUOVHDUFK HQJLQH KLJKOJKWQJ DQG WKH DELOLW WR VKDUH QRWHV ZLWK FODVVPDW
XVLQJ HERRNV
0 F* UDZ +LOO RIIHUV WKLV WH[WDV DQ HERRN 7R VDOO DERXWKH HERRN RSWLRQV FRQWDFW RX
0 F* UDZ +LOOVDOHVUHSRUYLVWKHVLWHZZZ FRXUVHVPDUWRP WR OHDUQPRUH

$FNQRZOHGJPHQWV

$VXXDOVRPDQ SHRSOHKDYHKHOSHGPHWKDW PD\ IDLOWROWWKHP DOO6KHOGRQ* UHHQRI VK
8QLYHUVLWRI %UDGWK&ROXPELD * RUGRQ+RORZD\ RI WKH8QLYHUVLWRI 1HZ %UXQZLFN 6XNDQD.
' DVKRI 7KH,QGLDQ,QWHUQDWLRI 7HFKQRORJ\ DW KDUDJSXUDQG3HKPDQ6KLYDQDQRI)RUG0RWRU&
JDYHPDQ KHOSIXOVXJJHVWLRQV +XEHUW&KDQRQ WKH8QLYHUVLWRI 4XHHQVDQG)UDQN- &XQKDI
3UDWW : KLQH 6DPXHO6FKZHJKDUWRI 7HUUDXJLD ,QF 0DUN6SHDU RI WKH : RRGV +R
2FHDQRJUDSKLF ,QVWLWXWRQ . HLW +DQQD RI $1 6< 6 ,QF (OHQD 0HJD RI WKH -HW3URSXOL
/DERUDWRU\ $QH6WDFNRI 6N\6DLO ,QF DQG (OHQ(PHUVRQ: KLWH SURYLGHG JUHDWQHZ LPDJH
6DPXHO6 6LK RI : DQD: DQD &ROHJH 7LPRWK 6LQJOHURI 681< %LQJKDPWRQ 6DHG0RYHL
0LQQHVRWD6WDWH8QLYHUVLW DQG-RKQ%RJ RI 0DUTXHWW8QLYHUVLW ZHUHHVSHFLDOO KHOSIXOZ LWK WK
\RXUQRV/PDQXDO

7KH IROORZLQJ SUHUHYLHZHUV JDYH PDQ H[FHOOHQWVXJJHVWLRQV IRU LPSURYLQJ WKH PDQXVFULS
5RDQGR %UDYR RI 6RXWKHUQ,OOLQRLV8QLYHUVLW -RVKXD % . RQDWRI 3HQQ6WDWH8QLYHUVLW ' DQ
0 D\QHV RI %ULJKDP <RXQJ 8QLYHUVLW -RVHSK 6FKDHUHU RI ,RZD 6WDWH8QLYHUVLW DQG ; LDQJFK
; XDQRI &OHPVRQ8QLYHUVLW

,QSUHSDUDWRQ WKHZULWUJRWXWXFNRQ&KDSWU EXWZDVUHVFXHGE\ WKHIROORZLQJ UHYLHZHUV
6HUK\ <DUXVHY\FK RI WKH8QLYHUVLW RI : DWHUORR + 3LURX] . DYHKSRXDQG-HI (OUHGJH RI WK
8QLYHUVLW RI &DOLRUQLD / RV$QJHOHV 5D\KDQH$NKDYDQ RI WKH8QLYHUVLW RI 0LFKLJDQ 6R\RX
6WHYH&KD RI WKH8QLYHUVLW RI ,OOLQRLV &KLFDJR * HRUJLD 5LFKDUGVRQ RI WKH8QLYHUVLW RI $OEDP
. ULVKDQ%KDWD RI 5RZDQ8QLYHUVLW +XJK &ROHPDQ RI WKH8QLYHUVLW RI $OEDPD+XQWVYLOOH ' :
2VWHQGRUI RI WKH8QLYHUVLW RI 0DVVDFKXVHWW DQG' RQDO0 H\HURI WKH8QLYHUVLW RI 5KRGH,VODQ
7KHZULWUFRQWULEXWRUVWR EH ILQDQFLDOVXSSRUWHG E\ WKH1DWLRQDO6FLHQFH)RXQGDWLRQ
\HDUVDQGHGLWRQV/

0DQ RWKHUUHYLHZHUVDQG FRUUHVSRQGHQWJDYH JRRG VXJJHVWLRQV HQFRXUDJHPHQWFRUUHFWRQ
DQGPDWULDOV (OL]DEHWK- . HQRQRI 0DWK RUNV -XDQ5 &UX] RI 1$6$ / DQJOH\ 5HVHDUFK&HQ
/L DL/ LRI 8QLYHUVLW RI 6FLHQFH DQG7HFKQRORJ\ RI &KLQD 7RP 5REELQV RI 1DWLRQDO,QVWUXPHQ
7DSDQ . 6HQJXSWD RI WKH,QGLDQ,QVWLWXWH RI 7HFKQRORJ\ DW. DQSXU 3DXOR 9DVDYXN RI 8QLFDP
8QLYHUVLW $QGUV 6NDWWHER RI 6FDQG SRZHU$ 6 -HIUH\ 6 $OHQ RI 0LFKLJDQ 7HFKQRORJLFD
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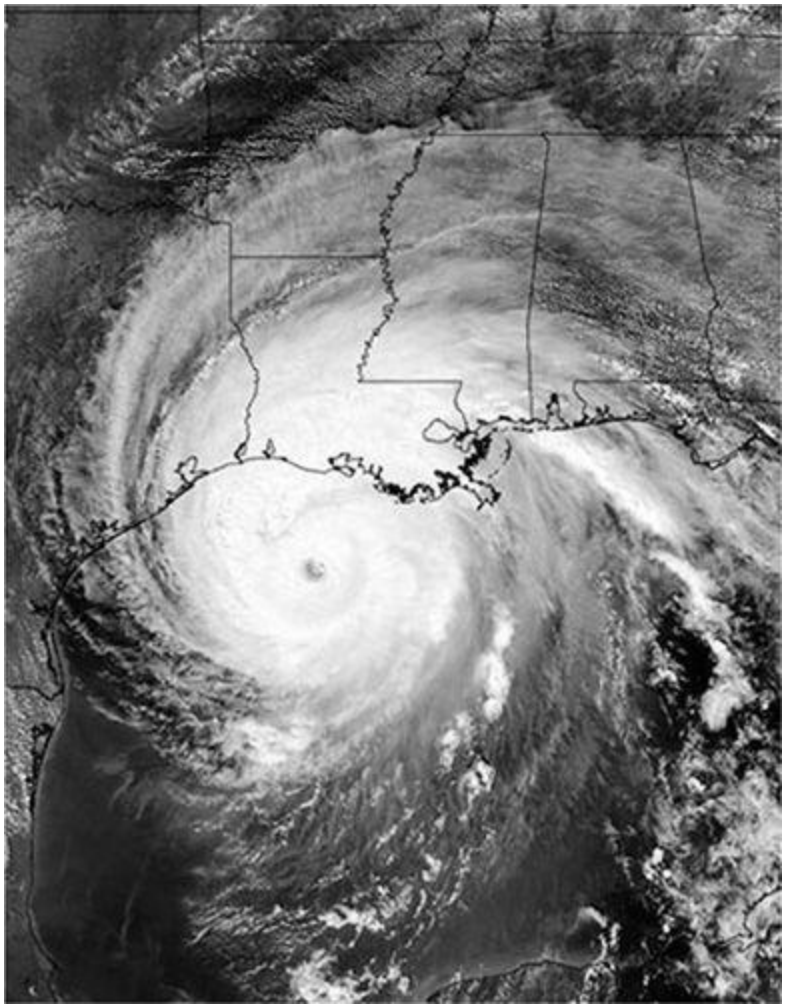
: HVWPLQ0LFKLJDQ8QLYHUVLW 0LFKDHO. ' DLGH RI * HQHUDO0 RWRUV &UXLVHGL/ $UFKHURI 6DQR
8QLYHUVLW 3DXO) -DFREV RI 7HFKQRORJ\ ' HYHORSPHQW$VVRFLDWH 5HEHFFD &XOOHQ: HEE RI WK
8QLYHUVLWRI &RORUDGR DW&RORUDGR 6SULQJV ' HEHQGUD. ' DVRI WKH8QLYHUVLWRI $OEXTXHUI
. HYLQ2¶6XOOLYDQ DQG 0DWWKHZ / XWWRI WKH$VVRFLDWG 3UHVV / HQQDUW •WW DQG 1LQD. RQKD
5(SRZHU6\VWHPV$* +DPEXUJ * HUPDQ &KL<DQJ &KHQJ RI $1 6< 6 ,QF ' HEDEUDW' DVJXSWD
7KH,QGLDQ,QVWLWXWH RI 7HFKQRORJ\ DW KDUDJSXU)DELDQ$QHOHPHWRI WKH,QVWLWXWH 5HFKHUFKHVXU
3KHQRPHQHV + RUV (TXLOLEUH 0DWHULDOV ' DYLG &KDQG]H 5LFKDUG / HVPDQ ' RQDO0 H\HU $U
6KXNOD 3HWHU/ DWHQ DQG0 DURP 6SDXOGLQJ RI WKH8QLYHUVLW RI 5KRGH,VODQG &UDJ 6Z DQVRQ
$SSOHG6FLHQFH$VVRFLDWH,QF -LP 6PD\ RI 2NODKRPD6WDWH8QLYHUVLW ' HERUDK3HQFHRI 2UHJ
6WDWH8QLYHUVLW (UF%UDVFKRWRI 3KLODGHOSKLD 3$ DQG' DOH+ DWWRI / RXLVLDQD7HFK8QLYHUVLW

7KH 0F* UDZ +LOO VWDII Z DV DV XVXDO HQRUPRXVO KHOSIXO 0 DQ WKDQNV DUH GH VR %L
6WDQTXLVW/ RUD. DE1H\HQV &XUWV H\QROGV -RKQ/ HDQG -DQH0 RKU%UHQGD5RZH VZHOODVL
WKRVHZKR ZRUNHGRQSUHYLRXVHGLWLRQV/

) LQDOO\ WKH FRQWLQXLQJ VXSSRUW DQG HQFRXUDJHPHQW RI P\ ZLIH DQG IDPLO\ DUH DV DOZD\V PXFK DSSUHFLDWHG 6SHFLDO WKDQNV DUH GXH WR RXU GRJ 6DGLH DQG RXU FDW &RFKDQG. HUV

) OXLG0 HFKDQLFV



+ XUULFDQH 5LWD LQ WKH * XOI RI 0 H[LFR 6HSW 5LWD P DGH ODQGIDOODW WKH 7H[D
/ RXLVLDQD ERUGHU DQG FDXVHG EORRQV RI GRODUV LQ ZLQG DQG IORRGLQJ GDP DJH 7 KRXJK P RU
GUDP DWF WKDQ WSLFDOODSSOFDWRQVLQWKLVWH[W5LWDLVDWWXHIOLGIORZ VWRQJ0 LQOH HQFHGE\ WK
HDUWK¶VURWDWRQDQGWKHFRFHDQWHP SHUDWXUH ***Photo courtesy of NASA***

3UHGLQDU\ 5HPDUNV

) OXLGPHFKDQLFVLVWKHVWXG\ RI IOXLGVHLWKHULQPRWRQ IOXLGGQDPLFV RUDWWHWWIOXLGVWDWFV
%RWKJDVHVDQGGTXLGVDUHFODVVLILHGDVIOXLGV DQGWKHQPEHURI IOXLGHQJLQHHULQJ DSSOFDWRQVL
HQRUPRXV EUHDWKLQJ EORRG IORZ VZLPPLQJ SXPSV IDQV WXUELQHV DLSODQHV VKLSV UIYHU
ZLQGPLOOV SLSHV PLVWLOHV LFHEHUJV HQJLQHV ILOWHUV NHWV DQG VSULQNOHUV WR QDPH DIHZ : KHQ\R
WKLQNDERXWDWDOPRWWHYHUWKLQJ RQWKLVSODQHWLWKHULVDIXLGRUPRYHVZLWKLQRUQHDUIXLG
7KH HWHQFH RI WKH VXEMHFWRI IOXLG IORZ LV D MXGLFLRXV FRPSURPLVH EHWZHHQ WKRU DQ

H[SHULPHQW6LQFH IOXLG IORZ LV D EUDQFK RI PHFKDQLFV LWV DQDOLHV D VHWRI ZHOOGRFXPHQWHG EDVL
OZ V DQG VKXV D JUHDWGHDO RI VKRUWFDOVHDWHQWLV DYDLODEOH +RZHYHU VKH VKRUV LV RIWH
IUXVWUDWQJ EHFDXVH LWDSSOLHV PDLQO VR LGHDOL]HG VLWXDWRQV ZKFK PD\ EH LQYDOG LQ SUDFWFH
SUREOPV 7KH VWR FKLH REVWDFOV VR D ZRUNDEOH VKRU DUH JHRPHWU DQG YLVFRVLW 7KH EDVL
HTXDWRQV RI IOXLG PRWRQ &KDS DUH VRGLIILFXOWR HQDEOH VKH DQDOVWRDWWFN DUELWUDU JHRPHWU
FRQLJXUDWRQV 7KXV PRVWVWX[VERRNV FRQFHQWUDWH IODWSODWH FLUFXODU SLSHV DQG RWKHU HDV
JHRPHWLHV ,WLV SRVLEOH VR DSSO QXPHULFDO FRPSXWHFKQLTXHV VR FRPSOH[JHRPHWLHV DQ
VSHFLDO]HG VW[VERRNV DUH QRZ DYDLODEOH VR H[SODLQ VKH QHZ FRPSXWDWRQDO IOXLG G QDPLFV &) '
DSSUR[LPDWRQV DQG PHWKRGV> ± @ 7KLV ERRN ZLOO SUHVHQW PDQ VKRUHWFDOUHVXOW ZKLFK NHSLQ
VKHLUOPLWDWRQV LQPLQG
7KH VHFRQG REVWDFOH VR D ZRUNDEOH VKRU LV VKH DFWRQ RI YLVFRVLW ZKFK FDQEH QHJOHFWG RQ
LQFHUWLQ LGHDO]HG IORZ V &KDS) LUVWYLVFRVLW LQFUHDVHV VKH GLIILFXOW RI VKH EDVLF HTXDWRQ
DQVRXJK VKH ERXQGDU CDVHUDSSUR[LPDWRQ IRXQG E\ / XGZLG 3UDQGWOLQ &KDS KDV JUHDW
VLPSOLILHG YLVFRXV IORZ DQDOVHV 6HFRQG YLVFRVLW KDV D GHVWDELOL]LQJ HIFWRQ DOOIOXLGV JLYLQ
ULVH DWXUXWDWQJO VPDOOYHORFLWHV VR D GLVRUGHU UDQGRP SKHQRPHQRQ FDOOHG WXUEXOHQFH 7K
VKHRU RI WXUEXOHQWORZ LV FUXGH DQG KDYLO EDFNHGXS E\ H[SHULPHQW&KDS \HWWFDQEH TXL
VHULFDEOH DV DQ HQJLQHHULQJ HVWLP DW 7KLV WR[VERRN LQWRGXFHV VKH VWDQGDUG H[SHULPHQW
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VLPXODWRQ ' 1 6 RI IOXFWXDWQJ WXUEXOHQFH> @
7KXV VKHUHV VKH RV DYLODEOH IRU IOXLG IORZ SUREOPV EXWQDOOFDVHV LWV KRXGEHEDFNHGXS E\
H[SHULPHQW2IWQ VKH H[SHULPHQWDOGDW SURYLGHV KHPDLQ VRXUFH RI LQIRUPDWRQ DERXW SHFLLF IORZ \
VXFK DV WKH GUDJ DQG OLWRI LPPHUVHGERGLHV &KDS) RUXQDWH IOXLG PHFKDQLFV LV DKJKO YLVXD
VXEMHFWZ LWKJRRG LQWXLWP HQDWRQ> ± @DQG WKHXVH RI GLPHQVLRQDOQDOVLV DQG PRGHOQJ FRQFHSW
&KDS LVZ LGHVSUHDG 7KXV H[SHULPHQWDWRQ SURYLGHV DQ XUDODQGHX FRPSOPHQWWR WKH WKRU
<RX VKRXG NHHS LQPLQG WKDW WKH RU DQG H[SHULPHQWWKRXG JR KDQG LQ KDQG LQ DOOWXGLHV RI IOXLG
PHFKDQLFV

+ LVWRU\ DQG 6FRSH RI) OXLG 0 HFKDQLFV

/ LNH PRVW VFLHQWLF GLVFLSOQHV IOXLG PHFKDQLFV KDV D KLVWRU RI HUDWFDOO RFFXUULQ HDU
DFKLHYPHQW VKQ DQ LQWUP HGLDW HUD RI VWDG IXQGDPHQWDO GLVFRYHULHV LQ WKH HJKWHQWK DQ
QLQHWHQWK FHQWXULHV OHDGLQJ WR WKH VWHQW ILUWFHQWXU HUD RI ³PRGHQ SUFWFH´ DV ZH VHO
FHQWUHG WUP RXUQP LWV GEXVS WR GDWNQRZ OHGJH $QFLHQWFLYLQ DWRQV KDG HQRXJK NQRZ OHGJH W
VROYH FHUWLQ IORZ SUREOPV 6DQQQ VKLSV ZLWK RDU DQG LUULJDWRQ V\VWP V ZHUH ERWK NQRZ Q L
SUHKLWRULF WP HV 7KH* UHNV SURGXFHG TXDQWWDWLY HLQRUP DWRQ $UFKLP HGHV DQG+ HUR RI $O[DQGU
ERWK SRVXODWG WKH SDUDGR[UP OZ IRU DGGLWRQ RI YHFWRU V LQ WKH WKLUG FHQWXU %& $UFKLP HGH
± %& IRUPXODWG WKH OZ VRI EXR\DQF\ DQG DSSOHG WKP WR IORDWQJ DQG VXEP HUJHG ERGLHV
DFWDOO GHULYLQ D IRUP RI WKH GLIHUHQWDO FDOFXOXV DV SDUWRI WKH DQDOVLV 7KH 5RP DQ EXL
H[WHQVLYH DTXHGXFWV\VWP V LQ WKH IRXUWK FHQWXU %& EXWOHWQR UHFRUGV VKRZ LQJ DQ TXDQWWDWLY
NQRZ OHGJHRI GHVLJQSULQFLSOHV
) URP WKHELUWKRI &KULVWWR WKH 5HQLVVDQFH WKH UHZDVDVWDG LP SURYHPHQWQ WKH GHVLJQRI VXF
IORZ \VWP V DV VKLSV DQG FDQDOV DQG ZDWU FRQGXLW EXWQR UHFRUGHG HYLGHQFH RI IXQGDP HQW
LP SURYHPHQW LQ IORZ DQDOVLV 7KHQ / HRQDUGR GD 9LQFL ± VWDWG WKH HTXDWRQ F

FRQVHUYDWRQ RI PDW LQRQHGLPHQVLRQDO VWDG IORZ / HRQDUGR ZDV DQ H[FHOOHQW[SHULPHQWDO
DQG KLVWRULDQV FRQVLGHU DFFXUDWH GHVFULSWLRQV RI ZDYHV MWW K\GUDXOLF MXPSV HGG IRUPDWLRQ DQG ERI
ORZ GUDJ VWUHDPOLQHG DQG KLJK GUDJ SDUDFKXWH GHVLJQV $) UHQFKPDQ (GPH 0 DURWW :
EXLOWKHILUWZ LQGXVWULDOL]HGVWDWHV,VDDF1HZWRQ :
3URFHVVLQYROYLQJ WKHPRPHQWXP RI IOXLGVFRXOGILQDOO\EHDQDO]HGDWu,VDDF1HZWRQ :
SRWXDDG KLV ODZV RI PRWLRQ DQG WKH ODZ RI YLVFRVLW\ WKH OLQHDU IOXLGV QRZ FDOOH
QHZWRQLDQ 7KH WKHRU ILUVW\LHOGHG WR WKH DVXPSWLRQ RI D ³SHUIHFW´ RU IULFWLRQOHVV IOXLG DQ
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/DJUDQJH DQG 3LHUUH6LPRQ /DSODFH SURGXFHG PDQ EHDXWLXO VROXWLRQV RI IULFWLRQOHVV IORZ
SUREOHPV (XOHU)LJ GHYHORSHG ERWK WKH GLIIHUHQWLDOHTXDWLRQV RI PRWLRQ DQG WKHLUL[WUDM
IRUP QRZ FDOOHG WKH%HUQRXOOLHTXDWLRQ ' ¶$OPEHUWXVHG WKHP WR VKRZ KLVIDPRXVSDUDGR[WKDW
ERG\ LPPHUVHG LQ DIULFWLRQOHVV IOXLG KDV]HUR GUDJ 7KHVHEHDXWXOUHVXOWVDPRXQWHG WR RYHUWKO
VLQFH SHUFHLYHG DVXPSWLRQV KDYH YHU\ OPLWHG DSSOLFDWLRQLQSUDFWLFHDQG PRVWHQJLQHHULQJ IORZ
DUH GRPLQDWHG E\ WKH HIHFW RI YLVFRVLW\ (QJLQHHUVEHJDQ WR UHMHFWZKDWWKH UHJDUGHG DV DWRDO
XQUHDOLVWLF WKHRU DQG GHYHORSHG WKH VFLHQFH RI K\GUDXOLFV UHOLQJ DPRVWHQWLUHO RQ H[SHULPHQ
6XFK H[SHULPHQWDOWV DV &Kp]\ 3LWRW%RUGD : HEHU) UDQFLV + DJHQ 3RLVHXLOOH ' DUF\ 0 DQQLQ
%D]LQ DQG: HLVEDFK SURGXFHG GDWRQDYDULHWR RI IORZ VVXFK DVRSHQFKDQQHOV VKLS UHLVWDQFH SLSI
IORZ V ZDYHV DQG VKUELQHV $OOWRR RIWQ WKH GDWZHUH XVHG LQ UDZ IRUP ZLWKRXWUHJDUG WR WK
IXQGDPHQWDOSK\VLFVRI IORZ

) LJ / HRQKDUG (XOHU ± ZDV WKH JUHDWWXP DWKHP DWLFDQ RI WKH HJKWHQW
FHQWXU\ DQG XVHG 1HZWRQ¶V FDOFXOXV WR GHYHORS DQG VROYH WKH HTXDWLRQV RI PRWLRQ RI LQYLVFL
IORZ +H SXEOLVKHG RYHU ERRNV DQG SDSHUV *Courtesy of the School of Mathematics and
Statistics, University of St Andrew, Scotland. @

$WWKHHQGRI WKHLQHWHHQWK FHQWXU\ XQLILFDWLRQEHJ7HHQ PDWKHPDWLFV DQG WKRUHWLFDO
K\GURG\QDPLFV ILQDOO\ EHJDQ : LOOLDP) URXGH ± DQG KLV VRQ 5REHUW ±
GHYHORSHG DZ VRI PRGHOWAWLQJ / RUG 5D\OHLJK ± SURSRVHG WKHWFKQLTXHRI GLPHQVLRQ
DQDOVLV DQG 2VERUQH5H\QROGV ± SXEOLVKHG WKHFODVVLF SLSHH[SHULPHQWLQ Z KLF

VKRZ HG WKH LPSRUWDQFH RI WKH GLPHQVLRQDWV 5HQROGVQXPEHUQDPHGDIWHU 0HDQ ZKLOH YLVFRXV
IORZ WKHRU Z DV DYDLODEOH EXW[SORLWHG VLQFH 1DYLHU ± DQG 6WRNHV ± KD
VXFFHVVIXOO DGGHG QHZ WRQDQ YLVFRXV WHUPV WR WKH HTXDWRQV RI PRWRQ 7KHUHXQRQ 1DYLHU6WRN
HTXDWRQV ZHUH WKRURXJKO\ DQDO]HG IRU DUELWUDU IORZ V 7KHQ LQ D* HUPDQHQJLQHU / XGZL
3UDQGWO ±)LJ SXEOLVKHG SHUKDSV WKH PRVW LPSRUWDQW SDSHU HYHU ZULWWQ RQ IXLG
PHFKDQLFV 3UDQGWO SRLQWHG RXW WKDW IXLG IORZ V ZLWK VPDOO YLVFRVLW VXFK DV Z DWHU IORZ V DQ
DLU IORZ V FDQ EH GLYLGHG LQWR D WKLQ YLVFRXV OD\HU RUERXQGDU OD\HU QHDU VROG VXUIDFHV DQ
LQWUDFHV SDWKHGRQV DQG LQYLVFLG IXLG OD\HU ZKHUHWH(XOHUDQG %HUQRXOOL HTXDWRQV DSSO
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. iUPiQ ± DQG 6LU* HRIUH , 7D\ORU ± 0DQ RI WKH UHVXOW VNHWFK HG KL
IURP DKLVWRULFDOSRLQWR YLHZ ZLOO RI FRXUVH EHGLVFXVVHG LQWKLV WR[WERRN 0RUHKLVWRULFDOGHWDLO
FDQEHIRXQGLQ5HV VR

)LJ / XGZLJ 3UDQGWO ± RI WKQ FDOHG WKH ³IDWKHU RI PRGHUQ IXLG PHFKDQLFV
> @GHYHORSHG ERXQGDU OD\HU WKHRU DQG P DQ RWKHULQQRYDWLYH DQDO VHV + HDQG KLV WXGHQV
ZHUH SLRQHHUV LQ IORZ YLVXDOL]DWLRQ WHFKQLTXHV ***Aufnahme von Fr. Struckmeyer, Gottingen,***

courtesy AIP Emilio Segre Visual Archives, Lande Collection. @

7KH VHFRQG KDOI RI WKH WZHQWLHWK FHQWXU LQWURGXFHG DQHZ WRRO &RPSXWDWLRQDO) OXLG ' \QDPLF &)' 7KH HDUOLHVW SDSHUV RQ WKH VXEMHFW QRZ QRW WKLV ZULWHU ZDV E\ \$ 7KRP LQ > @ ODERUDWRU EXWDFFXUDWH KDQG FDOFXODWRQ RI IORZ SDWWD F\QDPLF DWWRZ 5H\QROGV QXPEHU &RPPHUFLDOGLJLWDOFRPSXWHUV EHFDPH DYDLODEOH LQ WKH V DQG SHUVRQDOFRPSXWHUV LQ WKH \ EULQJLQJ &)' LQWR GXOKRRG \$ GHJHQGDU ILUVWWA[WRRRN ZDV E\ 3DWDQDU > @ 3UHVHQW ZLV LQFUHDVHV LQ FRPSXWHU SHHG DQG PHPRU DOPRVWDQ ODPLQDU IORZ FDQ EH PRGHOHG DFFXUDWH 7XUEXOHQW IORZ LV VWLOOFFDOFXODWG ZLWK HPSLULFDOPRGHOV EXW LUHFW1XPHULFDO6LPXODWRQ> @L SRVVLEOHIRUIORZ 5H\QROGV QXPEHUV \$QRWKHUILYHRUGHUVRI PDJQLWXGHILQFRPSXWVSHG DUH0H EHRUHJHQHUDOWXUEXOHQWIORZ VFDQEHFDOFXODWG 7KDWPD\ QRWEHSRVVLEOH GXHWRPL]HOPLWVRI QDQR DQG SLFR HOPHQW %XWLJHQHUDO' 1 6 GHYHORSV * DGHO+DN> @DLVHVWKHSURVSHFWRI DVKRFNLQ IXXUH DOORI ICXLG PHFKDQLFV UHGXFHG WR DEODFNER[ZLWK QR UHDOQHG IRU WKDFKHUV UHVHDUFKHUV ZULWHUV RUICXLGVHQJLQHUV

6LQFHWKHDUWKLV SHUFHQWRYHUHG ZLWK ZDWUDQG SHUFHQWRYHUHG ZLWKDLUWKHVFRSHR ICXLG PHFKDQLFV LV YDVWDQG WRXFKHV QHDUO HYHU KXPDQHQGHDYRU 7KH VFLHQFHV RI PHWRURORJ\ SK\VLFDORFHDQRJUDSK\ DQG K\GURORJ\ DUH FRQFHQHG ZLWK QDWXUDO RFFXUULQJ ICXLG IORZ V DV DU PHGLFDOWXGLHVRI EUHDWKLQJ DQG EORRG FLUFXODWRQ \$OWKRXJKSRUDWRQSURECOPVLQYROYHICXLG PRWRQ ZLWKZHOGHYHORSHG VSHFLDOWLHVLQDHURG\QDPLFVRI DLUFUDWDQG URFNHWV DQG URFNHWSURSXOVLRQ ICXLG PHFKDQLFV UHVHDUFKHUV DOVRXHOHFWULF HQHJ\ LVGHYHORSHGHLWKHUIURP ZDWUICXLG RUIURP VWDWLF SURECOPVLQYROYHICXLG PRWRQDQG FRQWRO ZDWUICXLG IORRG FRQWRO SURECOPV RI LUULJDWRQ ICRRG FRQWRO ZDWUVSSO\ VHZDJHGLVSRVDO SURWHFWHPRWRQ DQG RLODQG JDVSLSHOLQHV 7KHDLP RI WKLVERRN LV WR SUHVHQWHQXJK IXQGDPHQWDOFRQFHSWV DQG SUDFWLFDO DSSOLFDWRQV LQICXLG PHFKDQLFV WR HQDEOH \RX WR VROH WKH ZKROH VSHFWUXP RI DWJQPHQW ZLWK RU ZLWRXWA \$QZHUVLQWKH\$SSHQGL[+HUHDUHWKHUHFRPPHQGHGWAVIRUSURECOPVROWRQ

5HDGWKHSURECOPDQGUHWDWLWZLWK\RXUVXPPDU RI WKHUHVXOWGHVLUHG

) URP WDEOHVRUFKDUW JDWKHUWKHQHHGHGSURSHUWGDWD GHQVLW YLVFRVLW HW

0 DNHVXUH\RXXQGHUWDQGZKDWDVNHG6WDUWDWHDSWRQZHUWKHZURQJ TXHVWRQ IR H[DPSOH SUHVXUH LQWADG RI SUHVXUH JUDGLHQWOWRUFHLQWADG RI GUDJ IRUFH RUPDVIORZ LQWADG RI YROXPHIORZ 5HDGWKHSURECOPFDUHXOO

0 DNHDGHWDLOHG OEHOHG VNHWFK RI WKHVXWAP RUFRQWROYROXPHQHGHG

7KLQNFDUHXOO DQG OWVRXUDWXPSWRQV <RX PXWGHFLGHLI WKHICXLG LVWADG RUXQWADG FRPSUHVVLEOHRULQFRPSUHVVLEOH YLVFRXV RULQYLVFLG DQG ZKHWKHUDFRQWROYROXPHRUSDWW GLIHUHQWDOHTXDWRQV/DUHQHGHG

) LQG DQDOJEUDLF VROXWRQLI SRVVLEOH 7KHQ LI DQXPHULFDOYDOXHLVQHGG XVHHLWKHUWKH6

RU%* XQLWX VWAPV VRE-HUHYLHZ-HGLQ6HF

5HSRUW RXU VROXWLRQ DEHDG ZLWK WKH SURSHUXQLWDQG WKH SURSHUQPEHUR VLJQLILFDQ

ILJXUHV XVXDOO VZRRUWKHH WKDWWKHGDWXQFHUWDLQWDOOZV

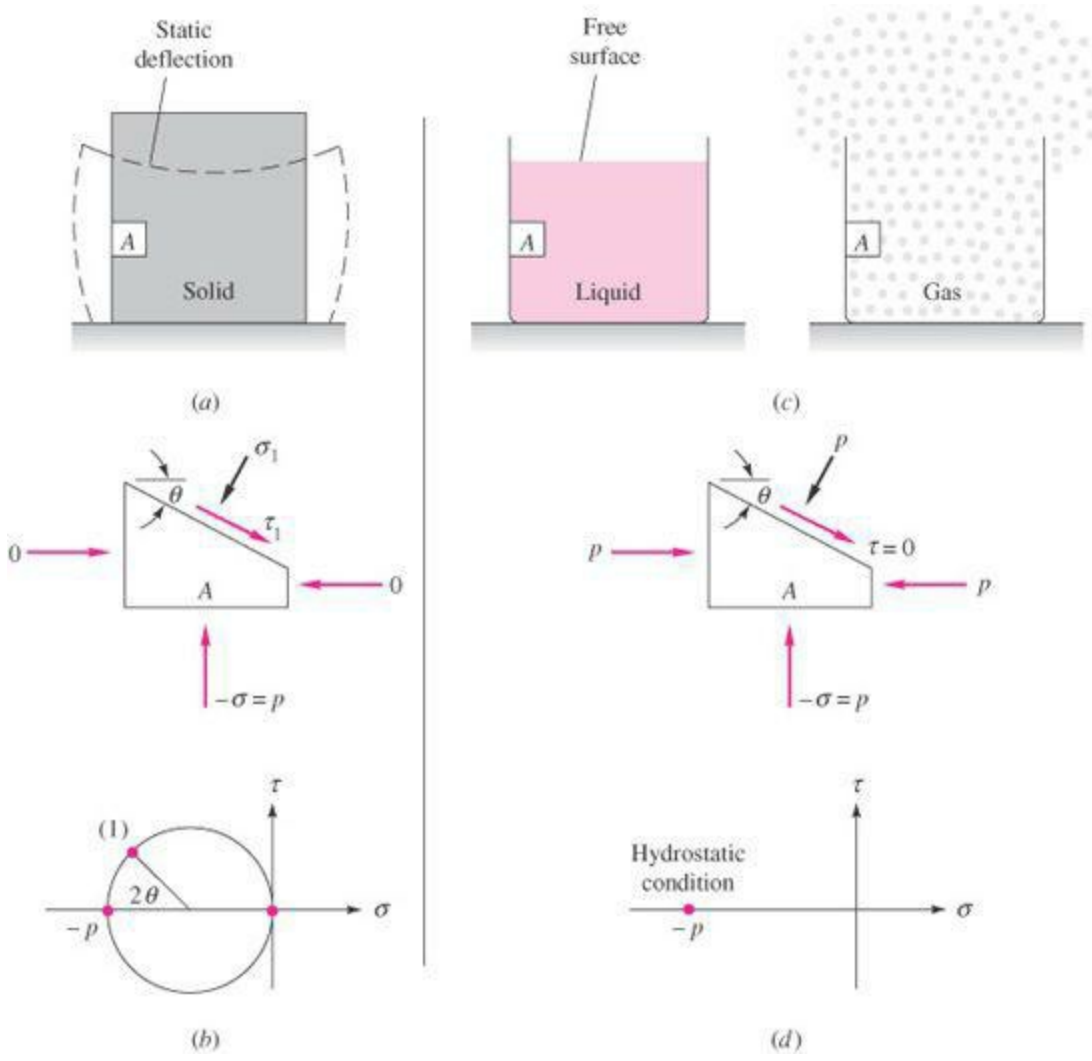
: HVKDOOIROORZ WKHVHVWHSV ZKHUHDSSURSULDWH LQRXU[DPSOHSURECOPV

7KH&RQFHSWRI D) OXLG

) URP WKHSRLQWR YLHZ RI IXLGPHFKDQLFV DOOP DWWUFRQLWRI RQO VZR VWDWV IXLG DQG VROG 7KHGLIHUHQFHEHZHHQWKHVZRLVSHUFW REYLRXVWRWKHDSHUVRQ DQGLWVDQLQWHUHVWLQJ H[HUFLV WRDVNDOOSHUVRQWRSXWKLVGLIHUHQFHLQWZRUGV 7KHWFKQLFDOGLVWQFWLRQ RWKHUWRQDHVZLWKWKHUHDFWRQ VKHVZR WR DQ DSSOLHG VKDUURUWQJHQWDO VWUHVV \$ VROG FDQ UHLVWD VKDUWUHVV E\ D VWDWH GHOHFWRQ DIXLG FDQQRW\$Q VKDUWUHVV DSSOHG WR DIXLG QR PDWWUKRZ VPDOO ZLOOUHVXOW PRWQRI WKDWVXLG 7KHIXLG PRYHVDQG GHRUPVFRQWLQXXVO DVORQJ DV WKHVKDUWUHVV LVDSSOHG \$ VD FRURODU ZHFDQ VD\ WKDWD IXLG DWUHWPXWEHLQDWDWRI]HUR VKDUWUHVV DWDWRI WHFDOHG WKHK\GURVWDWFVWUHVVFRQGLWQQLQWKFKXDODQDOVLV ,QWKLVFRQGLWQ 0RKU¶VFLUFOHIRUVWUHVV UHGXFHVWRDSRLQWDQGWKHUHVQRVKHDUVWUHVVRQ SDQHFXWKURXJKWKHHOPHQWXQGHUVWUHVV

* LYHQWKLVGHILQLWQRI DIXLG HYHU\ OD\HUQDOOR NQRZ VWKDWWKHUHDUHZR FODVVHVRI IXLGV OTXLGVDQGJDVHV \$JDLQWKHGLVWQFWLRQLVDWFKQLFDORQHFRQFHUQLQJ WKHHIHFWRI FRKHVLYHIRUFHV \$ OTXLG EHLQJ FRPSRVHGRI UHDLWYHO FORVHSDFNHGPROHFXOHVZLWKVWRQJ FRKHVLYHIRUFHV WQGVWR UHWDLQLWVYROXPHDQGZLOOIRUP DIUHHVXUDFHLQDJUDYLWDWQDOILHOGLI XQFRQILQHGIURP DERYH) UH-VXUDFHIORZV DUHGRPLQDWHG E\ JUDYLWDWQDOHIHFWDQG DUHVXGLHGLQ&KDSV DQG 6LQFHJD PROHFXOHV DUH ZLGHO VSDFHG ZLWK QHJOLJLEOH FRKHVLYH IRUFHV D JDV LV IUHH WR H[SDQG XQWLOLW HQFRXQWHUVFRQILQLQJ ZDOOV \$ JDVKDVQR GHILQLWVKDSHDQG ZKHQOHIWWRLWVHOZLWKRXWFRQILQHPHQ DJDV IRUPV DQ DPRSKHUH WKDWLV HVVHQWLDOO K\GURVWDWF 7KH K\GURVWDWF EHKDYLRUR OTXLGV DQ JDVHV LV VWNHQ XS LQ&KDS * DVHV FDQQRWRUP D IUHH VXUDFH DQG WKXV JDV IORZV DUH UDUHO FRQFHQHGZLWKJUDYLWDWQDOHIHFWRWKHUWKDQEXR\DQF\

) LJXUH LOOXVWDWVDVRQGEORFNUHVWQJ RQDULJLGSODQHDQGVWUHVVHGE\ LWVRZQZHLJKW7K VRQGVDJVLQRDVWDWF GHOHFWRQ VKRZQDVKJKO H[DJJHUDWHGDVKHGQQH UHLVWQJ VKHDZLWKRX IORZ \$ IUHEERG\ GLDJUDP RI HOPHQ\$RQWKHVLGHRI WKEORFNVKRZVWKDWWKHUHVVHDULQWKEORF DORQJ DSODQHFXWDWDQDQJOH WKURXJK\$ 6LQFHWKEORFNVLGHVDUHXQXSSRUWHG HOPHQW\$ KDV]HU VWUHVVRQWKHOHWDQGULJKWVLGHVDQGFRPSUHVLRQVWUHVV1 í SRQWKHVRS DQGERWWRP 0RKU¶VFLUFO GRHVQRWUHGXFHWRDSRLQWDQGWKHUHVQRJHURVHDUVWUHVVLQWKEORFN



) LJ $ VRGDWUHWWFDQUHVWWKHDU *a* 6WDWF GHOHFWRQ RI WKH VRGG *b* HTXLOEUXP DQ 0 RKU¶V FLUFOH IRU VRGG HOHP HQW $ IXLG FDQQRWUHVWWKHDU *c* &RQWLQXLQJ ZDOY DUHQHGG *d* HTXLOEUXP DQG0 RKU¶V FLUFOH IRU IXLG HOHP HQW

%\ FRQWDWWKH OTXLG DQG JDV DWUHWWLQ) LJ UHTXLUH WKH VXSSRUWQJ ZDOY LQ RUGHU V HOP LQDWWKH DUWWHW 7KHZDOVH[HUWDFRP SUHWLRQWWHWRí SDQGUHGXFH0 RKU¶V FLUFOHWRDSRLQ ZLWK]HUR VKDUHVWUZKHUH WKDWLV WKH K\GURWDWF FRQGLWRQ 7KH OTXLG UHDLQV LWV YROXP H DQ IRUP VDIUHVXUDFHLQWKHFRQWLQHU ,I WKHZDOY DUHUPRYHG VKDUDGHYHOSVLQWKHOTXLGDQGDEL VSODVKUHVXOW ,I WKHFRQWLQHULVWWDG VKDUDJDLQGHYHOSV ZDVHIRUP DQGWKHIUHVXUDFHVHN DKRU]RQWDOFRQLJXUDWRQ SRXULQJ RXWRYHUWKHOSLI QHFHVDU 0 HDQZKLOH WKHJDVLVXQHWWDLQH DQGH[SDQGVRXWWKHOSLI QHFHVDU DQGH[HUWDFRP SUHWLRQWWHWí SRQWKHZDOY

,QWKHSUHYLRXVGLVFXWLRQ FOHDUGHFLVLRQVFRXGEHPDGHDERXWRQGV OTXLGV DQGJDVHV 0 RV HQJLQHULQJ OXLGPHFKDQFVPREOPVGHDOZLWKWKHVHFOHDUFDVH² WKDWLV WKHFRP PRQOTXLGV VXFH DVZDWU RLO PHUFXU JDVRQQH DQGDORKRO DQGWKHFRP PRQJDVHV VXFK DVLU KHOXP K\GURJHQ DQGVWDP LQWKHUFRP PRQVWP SHUDXUH DQGSUHVXUHUDQJHV 7KHUH DUH PDQ ERUGHUOLQH FDVHV KRZHYHU RI ZKLFK \RX VKRXOGEHDZDUH 6RPHDSSDUHQW³VRQG VXEVWDQFHV VXFK DVDSKDOWDQ OHDG UHVLVWWKDU VWUHVV IRU VKRUWSHULRGV EXWDFWDOO GHRUP VRZ0 DQG H[KLELWGHIQLWH IXLG EHKDYLRURYHUQRJ SHULRGV 2WKHUVXEVWDQFHV QRWDEO FRQFRLG DQG VXLU PL[WXHV UHLWWP DOOVKD VWUHWHVEXW\LHOG DWDJHVWUHVDQGEHJLQVIORZ DVIXLGVGR 6PHFLDOLHG WUHDWPHQWVDUHGHYRWHG WRWKLV WXG RI PRUHJHQHUDOGHRUP DWRQ DQGIORZ DILHGFDOHG UKHRJ\ > @$OR OTXLGV DQ

JDVHV FDQ FRH[LVW LQ VW R SKDVH P[WXUHV VXFK DV VXJDP±Z DWHU P[WXUHV RU Z DWHU Z LWK HQJLQHHULQJ DL
EXEEOHV 6SHFLDO[HG WH[WERRNV SUHVHQW WKH DQDO VLV RI VXFK PXOWLSKDVH IORZV > @) LQDO LQ VRP H
VLWXDWLRQV WKH GLVWLQFWLRQ EHWZHHQ D GLTXLG DQG D JDV EXUV 7KLV LV WKH FDVH DW WHP SHUDWXUHV DQ
SUHVVXUHV DERYH WKH VR FDOOHG FULWLFDO SRLQW RI D VXEVWDQFH Z KHUH RQO D VLQJOH SKDVH H[LVW
SUP DULO UHVHP EOQJ DJDV $ V SUHVXUH LQFUHDVHV IDUDERYH WKH FULWLFDOSRLQWWKH JDVQN HVXEVWDQF
EHFRP HV VR GHQVH WKDWWKHUH LV VRP H UHVHP EOQJ WR D LQTXLG DQG WKH XVXDO WHUP RG QDP L
DSSUR[LP DWRQV QN HWKH SHUFHWJDVQZ EHFRP HLQFFXUDW 7KH FULWLFDO P SHUDWXUH DQG SUHVVXUH R
Z DWUDUH7 F . DQG S F DW DWRVSKHUH VR WKDWWSLFDOSUREOP VLQYROYLQJ Z DWUDQ
WHP DUH EHOZ WKH FULWLFDOSRLQW$ LU EHLQJ DP[WXUH RI JDVHV KDVQR GLVWQFWLVWLQFWLRQ EXWW
SULQFLSDOFRP SRQHQWQLWURJHQ KDV7 F . DQG S F DW 7KXVWSLFDOSUREOP VLQYROYLQJ DL
DUHLQWKHUDQJHRI KJK WP SHUDWXUHDQG GRZ SUHVVXUHZ KHUHDLUVGLVWQFW DQG GHILQHG DJDV 7KI
W[WZLOOEHFRQFHQHG VROHO Z LWKFOHDU LGHQWLWLDEOHOTXLGVDQG JDVHV DQG WKHERUGHUDQHFDVHVXV
GLVFXVHGZLOOEHEH\RQGRXUSFRSH

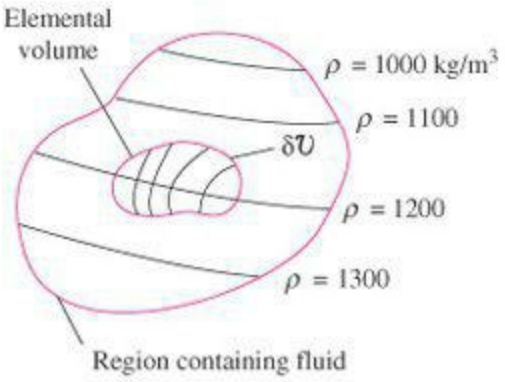
7KH) OXLG DVD &RQWLQXXP

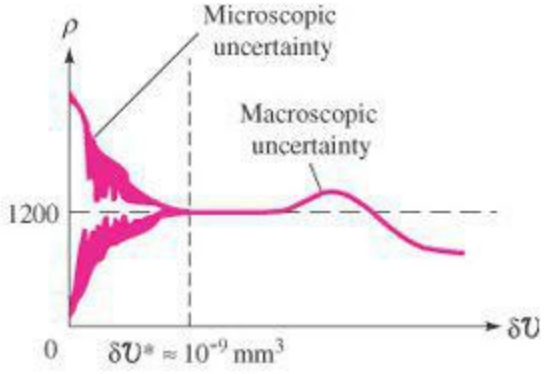
: H KDYH DOUHDG\ XVHG WKFKQLFDO WHUP V VXFK DV IOXLG SUHVXUH DQG GHQVLW Z LWKRXW D ULJRURX
GLVFXVLRQ RI WKHLU GHILQLWLRQ $ V IDU DV Z H NQRZ IOXLGV DUH DJJUHJDWLRQV RI PROHFXOHV Z LGHO
VSDFHG IRUD JDV FORVHO VSDFHG IRUD OTXLG 7KHGLVWDQFHEHWZHHQP ROHFXOHV LVYHU ODUJHFRP SDUH
Z LWKWKHP ROHFXODUGLP HWU 7KHP ROHFXOHV DUHQRWI[HGLQD OWWLFHEXWP RYH DERXWUHO UHODWLYH
HDFK RWKHU 7KXV IOXLG GHQVLW RUP DV SHUXQLWYROXP H KDVQR SUHFLVHP HDQLQJ EHFDXVHWKHQXP EH
RI P ROHFXOHV RFFXS\LQJ DJLYHQYROXP H FRQWQXDOO FKDQJHV 7KLV HIHFWEHFRP HV XQP SRUWDQWL WK
XQLWYROXP H LV ODUJH FRP SDUHG Z LW VD\ WKH FXEH R WKH P ROHFXOU SDFLQJ Z KHQ WKH QXP EHUR
P ROHFXOHV Z LWLQ WKH YROXP H Z LOOU P DLQ QHDU FRQVDQWL SLW R WKH HQUP RXV LQWUFKDQJH R
SDUWFOHV DFURVV WKH ERXQGDUL ,I KRZ HVHU WKH FKRVHQ XQLWYROXP H LV VR OODUJH WKHUH FRXOG EH I
QRWWDEOH YDUDWRQ LQ WKH EXON DJJUHJDWRQ R WKH SDUWFOHV 7KLV VLWXDWRQ LV LOOXWUDWHG LQ) LJ
Z KHUHWKH³GHQVLW´ DVFDOFXODWHG IURP P ROHFXODUP DW// P Z LWKLQDJLYHQYROXP H δ9 LVSORWWHGYHUVX
WKHVL]HR WKH XQLWYROXP H 7KHUHLV DOP LWQJ YROXP H δ9 EHORZ Z KLFK P ROHFXODUYDUDWRQV PD\ EH
LP SRUWDQWDQG DERYH Z KLFK DJJUHJDWHYDUDWRQV PD\ EH LP SRUWDQW7KHGHQVLW! RI DIOXLG LV EH
GHILQHGDV

$$\rho = \lim_{\delta V \to \delta V^*} \frac{\delta m}{\delta V} \qquad (1.1)$$

7KH OP LWQJ YROXP H/ LV DERXW í PP IRUDOOOTXLGV DQG IRUJDVHV DWDWP RVSKHULF SUHVVXU
) RU H[DP SOH í PP RI DLU DWWDQGDUG FRQGLWRQV FRQWLQV DSSUR[LP DWHO î P ROHFXOH
Z KLFK LV VXIILFLHQWWR GHILQH D QHDU FRQWLQXXP GHQVLW DFFRUGLQJ WR(T 0 RWHQGLQHUL
SUREOP V DUH FRQFHQHG Z LW SK\VLFDO GLP HQVLRQV P XFK ODUJHU WKDQ WKLV OP LWQJ YROXP H VR WKD
GHQVLW LV HVHQWLDOO DSRLQWXQFWLRQ DQG IOXLG SURSHUWLHV FDQ EH WKRXJKWRI DV YDU LQJ FRQWLQXDOO L
VSDFH DV VNHWFKHG LQ) LJ D 6XFK D IOXLG LV FDOOHG DFRQWLQXXP Z KLFK VLP SO P HDQV WKDW LV
YDULDWRQLQSURSHUWLHV LV VR VP RRWK WKDW GLIIHUHQWLDOFDOFXOXV FDQEH XVHGWR DQDO]H WKHVXEVWDQFH : I
WKDOO DVXP H WKDWFRQWLQXXP FDOFXOXV LV YDOG IRU DOOWKH DQDO\VHV LQ WKLV ERRN $JDLQ WKH DU
ERUGHUDQH FDVHV IRU JDVHV DWWXFK ORZ SUHVXUHV WKDWP ROHFXODU VSDFLQJ DQG P HDQ IUHH SDW DU
FRP SDUDEOH WR RU ODUJHU WKDQ WKH SK\VLFDO VL]H R WKH V\VWP 7KLV UHTXLUHV WKDWWKH FRQWLQXXP

DSSUR[LPDWLRQEHFRPHVURSSHGLQIDYRUURP DPRGHFXODUWKHRU RI UDUHILHGJDVHVZ > @,QSULQFLSOH DO
IOXLGPHFKDQLFVSUREOPVFDQEHDWWDFNHGIURP WKHPRGHFXODUYLHZSRLQWEXWQR VXFK DWWHPSWZ LOOEH
PDGHKHUH 1RWWKDWWKHXVHRI FRQWLQXXP FDOFXOXVGRHVQRWSUHFOXGHWKHSRVVLELOLWLHV GLVFRQWLQXRX
MXPSV LQ IOXLG SURSHUWLHV DFURVV D IUHH VXUIDFH RU IOXLG LQWHUIDFH RU DFURVV D VKRFN Z DYH LQ I
FRQ SUHVVLEOH IOXLG &KDS 2 XUFDOFXOXVLQDQDO]LQJ IOXLGIORZ PXVWEHIDPLOLDUHQRXJK WKDQGG
GLVFRQWLQXRXVERXQGDU\ FRQGLWLRQV





) LJ 7KH GHPLVWGHILQLWLRQ RI FRQWLQXXP IOXLG GHQVLW *a* DQ HDPHQWDOYROXPH LQ D IOXL
UHJLRQ RI YDULDEOH FRQWLQXXP GHQVLW *b* FDOFXODWHG GHQVLW YHUXVV[HRI WKHHDPHQWDOYROXPH

' LPHQVLRQVDQG8QLW

$ GLPHQVLRQLV WKHPHDVXUHE\ Z KLFK DSK\VLFDOYDULDEOHLVH[SUHVVHGTXDQWLWDWLYHO $ XQLWLV
SDUWFXODU Z D\ RI DWWDFKLQJ D QXPEHU WR WKH TXDQWLWDWLYH GLPHQVLRQ 7KXV OHQJWK LV D GLPHQVLR
DVRFLDWHG Z LWK VXFK YDULDEOHV DV GLVWDQFH GLVSODFHPHQW Z LGWK GHOHFWLRQ DQG KHLJKW Z KLC
FHQWLPHWHUV DQG LQFKHV DUH ERWK QXPHULFDO XQLWV IRU H[SUHVVLQJ OHQJWK ' LPHQVLRQ LV D SRZHUX
FRQFHSWDERXW Z KLFK D VSHFLGLG VRFDOHGGLPHQVLRQDODQDOVLV KDV EHQGHYHORSHG &KDS
Z KLOHXQLWDUHWKHQXPHULFDOTXDQWLWVWKDWWKHFXWMPHUZDQWDVWKHILQDODQZHU

,Q DQ LQWHUQDWLRQDOPHHWLQJ LQ) UDQFH SURPRVHG D WHDW FDOOHG WKH 0 HWUF &RQYHQWLR
Z KLFK Z DVVLJQHGLQ E\ FRXQWLHVLQFOXGLQJ WKH8QLWHG6WDWHV ,WZ DV DQLPSRYHPHQWRY
%ULWLVK VVWHPV EHFDXVHLWV XVHRI EDVH LV WKHIRXQGDWLRQ RI RXUQXPEHUVVWHP OHDGLQGIUF
FKLOGKRRGE\ DOO 3UREOPVVWDUWHPDLQHGEHFDXVHHYHQWKHPHWUF FRXQWLHVGLIIHUHGLQWKHUXVHF
NLORSRQGVLQWRDG GHVRUQHZWRQ NLORJUDPVLQWRDG JUDPV RUFDORULHVLQWRDG NRXOHV 7F
WDQGDUG]HWKHPHWUF VVWHP D* HQHUDO&RQHUHQFH RI : HLJKWDQG0 HDVXUHV DWWKQGHGLQ E
FRXQWLHV SURSRVHG WKH,QWHUQDWLRQDO6VWHP RI 8QLWV 6, : HDUHQRZ XQGHUJRLQJ D SDLQX
SHULRGRI WDQVLWLRQWR6, DQDGXVWHQWKDWPD\ WDNHPDQ PRUH\HDUVWRFRPSOHW 7KHSURHVWRQ
VRFLHWLHVKDYHHGGWKHZD\ 6LQFH-XO 6, XQLWVKDYHEHHQUHTXLUHGE\ DOOSDSHUVSXEOVKHGE\
WKH$PHULFDQ6RFLHWRI 0 HFKDQLFDO(QJLQHHUV DQG WKHUHLV DWHQGHQF WRZDUGV[SDQGLQJ WKH6, >
SUHVHQWWZ LGHUXVH6, XQLWVWRJHWKHUZ LWK%ULWLVK JUDYLWDWLRQDO %* XQLWV

3ULP DU\ ' LPHQVLRQV

,QIOXLGPHFKDQLFVWKHUHDUHRQO IRXUSULPDU GLPHQVLRQVIURP ZKLFK DOOWKHUGLPHQVLRQVFD EHGHULYHG PDW/ CHQWK WLPH DQGWKPSHUDWXUH 7KHVHGLPHQVLRQVDQGWKHLUXQLWVLQERWKVWVWHPVDU JLYHQLQ7DEOH 1RWWKDWWKHN-OYLQXQLWXVHVQRGHJUHHV\PERO7KHEUDFHVDURXQGDV\PERODN ^0 ` PHDQ³WKHGLPHQVLRQ RI PDW \$ OORWKHUYDULDEOHVLQIXLGPHFKDQLFVFDQEHH[SUHVVHGLQWHUP RI ^0 ` ^/ ` ^7` DQG^, ` ) RUH[DPSOH DFFHOHUDWRQKDVWKHGLPHQVLRQV^/ 7í ` 7KHPRWWFUFLD RI WKHVH VHFRQGDU GLPHQVLRQV LV IRUFH ZKLFK LV GLUHFWO UHODWHG WR PDW CHQWK DQG WPH E\ 1HZWRQ¶V VHFRQGODZ) RUFHHTXDOVWKHWLPHUDWH RFKDQJHRI PRPHQWXP RU IRUFHRXDOVPDW/

$$\mathbf{F} = m\mathbf{a} \tag{1.2}$$

) URP WKLVZHVHHWKDWGLPHQVLRQDOO ^) ` ^0 / 7í `

7DEOH 3ULPDU\ ' LPHQVLRQVLQ6, DQG%* 6\VWHPV

Primary dimension	SI unit	BG unit	Conversion factor
Mass {*M*}	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length {*L*}	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time {*T*}	Second (s)	Second (s)	1 s = 1 s
Temperature {Θ}	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

7KH,QWHUQDWLRQDO6\VWHP 6,

7KHXVHRI DFRQVLVWHQW SURSRUWLRQDOLW\ 1HZWRQ¶V (T LV DYRLGHGE\ GHILQLQJ WKH IRUFHXQLW[DFWO LQWHUP VRI WKHRWKHUEDVLF XQLW ,QWKH6, VWHP WKHEDVLF XQLWDUHQHZWRQV^) ` NLORJUDPV^0 ` PHWHUV^/ ` DQGVHFRQGV^7` : HGHILQH

QHZWRQRI IRUFH 1 NJ P V

7KHQHZWRQLV DUHODWLYHO VPDOOIRUFH DERXWWKHZHLJKWRI DQDSSOH GE ,QDGGLWLRQ WK EDVLF XQLWRI WPSHUDWXUH^, ` LQWKH6, VWHP LVWKHGHJUHH. HOYLQ . 8VHRI WKHVH6, XQLW 1 NJ P V . ZLOOUHTXLUHQR FRQYHUVLRQIDFWRUVLQRXUHTXDWLRQV

7KH%ULWLVK* UDYLWDWLRQDO%* 6\VWHP

,Q WKH%* VWVWHP DOVR DFRQVLVWHQW SURSRUWLRQDOLW\(T LV DYRLGHGE\ GHILQLQJ WKHIRUF XQLW[DFWO LQWHUP VRI WKHRWKHUEDVLF XQLW ,QWKH%* VWVWHP WKHEDVLF XQLWDUH SRXQG IRUFH^) ` VOXJV^0 ` IHHW/ ` DQGVHFRQGV^7` : HGHILQH

SRXQGRI IRUFH GE VOXJ IWV/

2QHGE § 1 DQGDSSUR[LPDWWKHZHLJKWRI RXUDSSOHV : HZLOOXVHWKHDEEUHYLDWRQGE IRUSRXQG IRUFH DQGGBP IRUSRXQG PDW 7KHVOXJ LVDUDWKHUKHWPDW HTXDOWR GBP 7K EDVLF XQLWRI WPSHUDWXUH^, ` LQWKH%* VWVWHP LVWKHGHJUHH5DQNLQH *5 5HFDOOWKDWDWPSHUDWX GLIIHUHQFH . *5 8VHRI WKHVH%* XQLW GE VOXJ IWV *5 ZLOOUHTXLUHQR FRQYHUVLRQIDFWR

LQRXUHTXDWRQV

2 WKHU8QLW6\VWHPV

7KHUHDUHRWKHUXQLWVYVWHPVVWLOOLQXVH $VWHDWRQHQHGVQR SURSRUWRQDOWFRQVWDQWWKH&* (
VVWHP GQH JUDP FP V . +RZHYHU &* 6 XQLWVDUHWRVPDOOIRUPRVWDSSOLFDWRQV GQH
í 1 DQGZLOOQRWEHXVHGKHUH
,QWH86$ VRPHVWLOOXVHWKH(QJOLVK(QJLQHHULQJ VVWHP GEI GEP IWV *5 ZKHUHWKHEDV
PDWXQLWVWKHSRXQGRI PDW 1HZWRQ¶VDZ PXWEHUHZULWWHQ

$$\mathbf{F} = \frac{m\mathbf{a}}{g_c}, \quad \text{where} \quad g_c = 32.174\,\frac{\text{ft}\cdot\text{lbm}}{\text{lbf}\cdot\text{s}^2} \tag{1.3}$$

7KHFRQVWDQWRI SURSRUWRQDOLW J$ KDVERWKGLPHQVLRQVDQGDQXPHULFDOYDOXHRIWHQDOWR
7KHSUHVHQWWZRXVHVRQO WKH6, DQG %* VVWHPVDQGZLOOQRWWROVHSUREOHPVRUH[DPSOHVLQWK
(QJOLVK(QJLQHHULQJ VVWHPV %HFDXVH$PHULFDQVWXGHQWVWKHP DIHZ SUREOHPVLQWKHWH[WZLOOE
WDWHGLQWKO DZNZDUGXQLW DFUHV JDOORQV RXQFHV RUPLOHV <RXDWJQPHQWZLOOEHWRFRQVH
WKHVHDQGVRQYHLQWKH6, RU%* VVWHPV

7KH3ULQFLSOHRI ' LPHQVLRQDO+RPRJHQHLW

,QHQJLQHHULQJ DQG VFLHQFH DOO HTXDWRQV PXWEHGLPHQVLRQDOO KRPRJHQHRXV WKDWLV HDF
DGGLWLYH WHUP LQ DQ HTXDWRQ PXWKDYH WKH VDPH GLPHQVLRQV)RU H[DPSOH WDNH %HUQRXOOL
LQFRPSUHVLEOHHTXDWRQ WR EHWUDQVIRUPHGDQGXVHGWKURXJKRXWWKLVWH[W

$$p + \frac{1}{2}\rho V^2 + \rho g Z = \text{constant}$$

(DFK DQG HYHU WHUP LQ WKLV HTXDWRQPXWKDYH GLPHQVLRQV RI SUHVVXUH ^0 / í 7í ` : HZLO
H[DPLQHWKHGLPHQVLRQDOKRPRJHQHLW RI WKLVHTXDWRQLQGHWDLOLQ([
$ OWHUQDWLYH VRPHLPSRUWDQWFRQGXFWYDULDEOHVLQIOXLG PHFKDQLFV ZLWKGLPHQVLRQVGHULYHG D
FRPELQDWLRQV RI WKH IRXU SULPDU GLPHQVLRQV LV JLYHQ LQ7DEOH $ PRUH FRPSOHWH OLWWR
FRQYHUVLRQIDFWRUVLVJLYHQLQ$SS &

7DEOH 6HFRQGDU ' LPHQVLRQVLQ) OXLG0 HFKDQLFV

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	1 m^2 = 10.764 ft^2
Volume $\{L^3\}$	m^3	ft^3	1 m^3 = 35.315 ft^3
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	1 ft/s^2 = 0.3048 m/s^2
Pressure or stress $\{ML^{-1}T^{-2}\}$	Pa = N/m^2	lbf/ft^2	1 lbf/ft^2 = 47.88 Pa
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	1 s^{-1} = 1 s^{-1}
Energy, heat, work $\{ML^2T^{-2}\}$	J = N · m	ft · lbf	1 ft · lbf = 1.3558 J
Power $\{ML^2T^{-3}\}$	W = J/s	ft · lbf/s	1 ft · lbf/s = 1.3558 W
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	1 $slug/ft^3$ = 515.4 kg/m^3
Viscosity $\{ML^{-1}T^{-1}\}$	kg/(m · s)	slugs/(ft · s)	1 slug/(ft · s) = 47.88 kg/(m · s)
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot °R)$	1 $m^2/(s^2 \cdot K)$ = 5.980 $ft^2/(s^2 \cdot °R)$

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$ EORFN ZHLJKV GHI ZKHQH[SRVHGWRDVXDQGDUGHDUWKJUDYLWJ IWV D : KDWLVLV PDVVLQNJ" E : KDWZLOOWKHZHLJKWRI WKLVEORFN EHLQ1 LI LWLV H[SRVHGWRWKHPRRQ V VXDQGDUG DFFHOHUDWRQJ PRRQ P V" F +RZ IDVWZLOOWKHEORFN DFFHOHUDWH LI DQHWRUFH RI GHI L DSSOLHGWRLWRQWKHPRRQRUQWKHHDUWK"

6RXWLRQ

: HQHHGWRILQGWKH D PDVV E ZHLJKWRQWKHPRRQ DQG F DFFHOHUDWRQ RI WKLVEORFN 7KLVLVI IDLUO VLPSOHH[DPSOHRI FRQYHUWLRQ IDFWRUVIRUGLIIHUHQW XQLWVWVWHPV 1R SURSHUWGDWDLVQHHGHG 7KHH[DPSOHLVWRR OHYHOIRUDVNHWK

3DUWD

1HZWRQ¶V ODZ KRGVZLWKNQRZQ QZHLJKWDQGJUDYLWDWRQDODFFHOHUDWRQ 6RYHIRUP

$$F = W = 1000\ \text{lbf} = mg = (m)(32.174\ \text{ft/s}^2), \quad or \quad m = \frac{1000\ \text{lbf}}{32.174\ \text{ft/s}^2} = 31.08\ \text{slugs}$$

&RQYHUWWKLVWRNLORJUDPV

$$m = 31.08\ \text{slugs} = (31.08\ \text{slugs})(14.5939\ \text{kg/slug}) = 454\ \text{kg} \qquad Ans.\ (a)$$

3DUWE

7KHPDVVRI WKHEORFN UHPDLQV NJ UHJDUGOHVVRI LWVORFDWRQ (TXDWRQ DSSOLHVZLWKDQHZ JUDYLWDWRQDODFFHOHUDWRQDQGKHQFHDQHZ ZHLJKW

$$F = W_{\text{moon}} = mg_{\text{moon}} = (454\ \text{kg})(1.62\ \text{m/s}^2) = 735\ \text{N} \qquad Ans.\ (b)$$

3DUWF

7KLVSDUWGRHVQRWOQYROYHZHLJKWRUJUDYLWRUOORFDWRQ ,WV VLPSOH DQDSSOLFDWRQRI 1HZWRQ¶ ODZ ZLWKDNQRZQPDVVDQGNQRZQIRUFH

) GHI PD VOXJV D

6RYHIRU

$$a = \frac{400\ \text{lbf}}{31.08\ \text{slugs}} = 12.87\frac{\text{ft}}{\text{s}^2}\left(0.3048\frac{\text{m}}{\text{ft}}\right) = 3.92\frac{\text{m}}{\text{s}^2} \qquad Ans.\ (c)$$

&RPPHQWF 7KLVDFFHOHUDWRQZRXOGEHWKHVDPHRQWKHHDUWKRUPRRQRUDQ\ZKHUH

0DQ GDWDLQWKHODWDUXHDUHUHSRUWHGLQLQFRQYHQLHQWRUDUFDQHXQLWVXVLQJEHHRQO VRVRPH LQGXVWU RUVSHFLDOVWRUFRXQW 7KHHQJLQHHUVKRXOGFRQYHUWKHVHGDWDWRWKH6, RU%* VXVWHP EHRU XVLQJ WKHP 7KLV UHTXLUHV WKH VXVWHP DWF DSSOLFDWLRQ RI FRQYHUVLRQ IDFWRUV DV LQ WKH IROOZLQ H[DPSOH

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,QGXVWULHVLQYROYHGLQYLVFRVLWPHDVXUHPHQW> @FRQVLGHUWXVHWKH &* 6 VXVWHP RI XQLV VLQFHFHQWPHWHUVDQGJUDPVVHFRQGHQRWHGDVP DQ IOLGV 7KHEVRXWYLVFRVLW XQ LVWKHSRLVH QDPHGDWU- / 0 3RLVHXLOOH D)UHQFKSK\VLFLDQZKRLQ SHUIRUPHGSLRQHHU H[SHULPHQWVRQZDWUIORZ LQSLSHV SRLVH J FP V 7KHNLQHPDWF YLVFRVLW XQLWLV WKH VWRNHV QDPHGDWU* * 6WRNHV D%ULWLVKSK\VLFLWWZKRLQ KHSHG GHYHORS WKHEDVLF SDUWD GLIIHUHQWDOHTXDWRQVRI IOXLGPRPHQWXP VWRNHV FP V: DWUDW *& KDV SRLVHDQGDOV VWRNHV ([SUHVVWKHVHUHVXOWLQ D 6, DQG E %* XQLWV

6RXWLRQ

3DUWD

- $SSURDFK 6\VWHPDWFDO FKDQJHJUDPVWRNJRUVOXJVDQGFKDQJHFHQWPHWHUVWRPHWHUVR IHHW
- 3URSHUWYDOXHV * LYHQ J FP V DQG FP V
- 6RXWLRQVWHSV D)RUFRQYHUVLRQWR6, XQLWV

$$\mu = 0.01\,\frac{\text{g}}{\text{cm}\cdot\text{s}} = 0.01\,\frac{\text{g(1 kg/1000g)}}{\text{cm(0.01 m/cm)s}} = 0.001\,\frac{\text{kg}}{\text{m}\cdot\text{s}}$$

$$\nu = 0.01\,\frac{\text{cm}^2}{\text{s}} = 0.01\,\frac{\text{cm}^2(0.01\text{ m/cm})^2}{\text{s}} = 0.000001\,\frac{\text{m}^2}{\text{s}} \qquad \textit{Ans. (a)}$$

3DUWE

-) RUFRQYHUVLRQWR%* XQLWV

$$\mu = 0.01\,\frac{\text{g}}{\text{cm}\cdot\text{s}} = 0.01\,\frac{\text{g(1 kg/1000 g)(1 slug/14.5939 kg)}}{\text{(0.01 m/cm)(1 ft/0.3048 m)s}} = 0.0000209\,\frac{\text{slug}}{\text{ft}\cdot\text{s}}$$

$$\nu = 0.01\,\frac{\text{cm}^2}{\text{s}} = 0.01\,\frac{\text{cm}^2(0.01\text{ m/cm})^2(1\text{ ft}/0.3048\text{ m})^2}{\text{s}} = 0.0000108\,\frac{\text{ft}^2}{\text{s}} \qquad \textit{Ans. (b)}$$

- &RPPHQWV 7KLV ZDV D ODERUDWRXV FRQYHUVLRQ WKDWFRXOG KDYH EHHQ GRQH E\ XVLQJ WK GLUHFWYLVFRVLWFRQYHUVLRQ IDFWRUVLQ$SS &) RUH[DPSOH %* 6,

: HUHSHDWRXUDGYLFH) DFHGZ LWKGDWDLQXQXVXDOXQLW FRQYHUWKH LPPHGLDWH VRHLWHU6, R
%* XQLWEHFDXVH LWLV PRUH SURHVLRQDODQG WKHRUHWLFDOHTXDWRQV LQ IXLG PHFKDQFV DU
GLPHQVLRQDO FRQVLVWHQWGUHTXLHQR IXUWHUFRQYHUVLRQIDFWRUV ZKHQWHVWRIXQGDPHQWDOXQ
V\VWHP VDUHXVHG DVWKHIROORZLQJ H[DPSOHVKRZV

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$ XVHXOWHRUHWLFDOHTXDWRQIRUFRPSXWLQJ WKHUHDWRQEHWZHHQSUHVXUH YHORFLW DQG DOWLWXG
LQDVWDGG IORZ RI DQHDO LQYLVFLG QHDO LQFRPSUHVLEOH IXLG ZLWK QHJOLJLEOH KHDWWDQVHUDQ
VKDWZ RUN LVWKH%HUQRXOOLUHDWRQ QDPHG DWWU' DQHO%HUQRXOO ZKR SXEOLVKHG DK\GURG\QDPLF
WK[WWRRNLQ

$$p_0 = p + \tfrac{1}{2}\rho V^2 + \rho g Z \qquad (1)$$

ZKHUHS WRWDQDWRQSUHVXUH
S SUHVXUHLQPRYLQJ IOXLG
9 YHORFLW
! GHQVLW
= DOWLWXGH
J JUDYLWDWRQDODFFHOHUDWRQ
D 6KRZ WKDW(T VDWLVILHV WKH SULQFLSOH RI GLPHQVLRQDO KRPRJHQHLW ZKLFK VWDWHV WKDWDQ
DGGLWLYH WUP VLQD SK\VLFDOHTXDWRQ PXWKDYH WKH VDPH GLPHQVLRQV E 6KRZ WKDWFRQVLVWHQQ\XQL
UHVXOWZLWKRXWDGGLWLRQDOFRQYHUVLRQIDFWRUVLQ6, XQLWV F 5HSHDWE IRU%* XQLWV

6RXWLRQ

3DUWD

: HFDQH[SUHVV(T GLPHQVLRQDO XVLQJ EUDFHV E\ HQFORVLQJ WKHGLPHQVLRQV RI HDFK WHUP IURP
7DEOH

$$\{ML^{-1}T^{-2}\} = \{ML^{-1}T^{-2}\} + \{ML^{-3}\}\{L^2T^{-2}\} + \{ML^{-3}\}\{LT^{-2}\}\{L\}$$
$$= \{ML^{-1}T^{-2}\} \text{ for all terms} \qquad \textit{Ans. (a)}$$

3DUWE

(QHUWKH6, XQLWVIRUHDFKTXDQWLWIURP 7DEOH

$$\{\mathrm{N/m^2}\} = \{\mathrm{N/m^2}\} + \{\mathrm{kg/m^3}\}\{\mathrm{m^2/s^2}\} + \{\mathrm{kg/m^3}\}\{\mathrm{m/s^2}\}\{\mathrm{m}\}$$
$$= \{\mathrm{N/m^2}\} + \{\mathrm{kg/(m \cdot s^2)}\}$$

7KHULJKWKDQGVLGHORRNVEDGXQLWZHUHPHPEHUIURP (T WKDW NJ 1 V P

$$\{\text{kg}/(\text{m}\cdot\text{s}^2)\} = \frac{\{\text{N}\cdot\text{s}^2/\text{m}\}}{\{\text{m}\cdot\text{s}^2\}} = \{\text{N}/\text{m}^2\} \qquad \textit{Ans. (b)}$$

7KXV DOOWUP VLQ%HUQRXOOL¶V HTXDWLRQ ZLOONDYH XQLWV RI SDVFDOV RUQHZWRQV SHUVTXDUH P HWH ZKHQ6, XQLWVDUHXVHG 1RFRQYHUVLRQIDFWRUVDUHQHHGHG ZKLFKLVWKHRI DOOWKHRUHWFDOHTXDWLRQVL IOXLGPHFKDQLFV

3DUWF

,QWRGXFLQJ %* XQLWVIRUHDFKWUP ZHKDYH

$$\{\text{lbf}/\text{ft}^2\} = \{\text{lbf}/\text{ft}^2\} + \{\text{slugs}/\text{ft}^3\}\{\text{ft}^2/\text{s}^2\} + \{\text{slugs}/\text{ft}^3\}\{\text{ft}/\text{s}^2\}\{\text{ft}\}$$
$$= \{\text{lbf}/\text{ft}^2\} + \{\text{slugs}/(\text{ft}\cdot\text{s}^2)\}$$

%XWURP (T VOXJ GEI V IWVRWKDW

$$\{\text{slugs}/(\text{ft}\cdot\text{s}^2)\} = \frac{\{\text{lbf}\cdot\text{s}^2/\text{ft}\}}{\{\text{ft}\cdot\text{s}^2\}} = \{\text{lbf}/\text{ft}^2\} \qquad \textit{Ans. (c)}$$

$ OOWUP VKDYHWKHXQLWVRI SRXQGVIRUFHSHUVTXDUHIRRW1RFRQYHUVLRQIDFWRUVDUHQHGHGLQWK %* VVWUP HLWKHU

7KHUHLVVWUDLQHQHUJ LQ(QJOLVK VSHDNLQJ FRXQWULHVWRXVHSRXQGIRUFHSHUVTXDUHLQFK DV SUHVVXUHXQLWVEHFDXVHWKHQPEHUVDUHPRUHPDQDJHDEOH)RU[DPSOH VDQGDUGDWPRVSKHUF SUHVVXU LV GEI LQ GEI IW 3D 7KHSDFLDVPDOXQLWVEHFDXVHWKHQHZWRQLVOHVVWKD ó GEI DQGDVTXDUHPHWULDYHU ODUJHUHD

&RQVLVWHQWXQLWV

1RWWKDWQRWRQO PXVWDOO IOXLG PHFKDQLFVHTXDWLRQVEHGLPHQVLRQDOO KRPRJHQHRXV RQHPX\ DOVR XVHFRQVLVWHQWXQLWV WKDWLV HDFK DGGLWLYH WUP PXWKDYHWKH VDPH XQLW 7KHUHLVQR WURXEO GRLQJ WKLV ZLWK WKH 6, DQG %* VVWUPV DV LQ[DPSOH EXWZRH XQRI WKRVH ZKR WU WR PL FRQRTXDO(QJOLVK XQLW)RU[DPSOH LQ&KDS ZHRIWQXVHWKHDXXPSWRQ RI VWDG DGLDEDW FRPSUHVVLEOHJDVIORZ

$$h + \tfrac{1}{2}V^2 = \text{constant}$$

ZKHUHK LV WKH IOXLG HQWKDS\ DQG9 LV LWV NLQHWF HQHUJ\ SHU XQLWPDV &RQRTXD WKHUPRG\QDPLFVDEOHVPLJKWOLVWK LQXQLWVRI %WXWKHUPDOXQLWSHUSRXQGPDV %WX GEI ZKHUHV LVOHNHO XVHGLQIW ,WV FRPSHWHO HUURQHRXVWRDGG %WX GEI WR IW 7KHSURSHUXQLWUXK LQWKLV FDVHLV IW GEI VOXJ ZKLFK LVGHQWFDOWR IWV 7KHFRQYHUVLRQIDFWRULV %WX GE§ IWV IW GEI VOXJ

+ RP RJHQHRXVYHUVXV' LP HQVLRQDOO ,QFRQVLVWHQW(TXDWLRQV

$ OO WKH RUHILFH HTXDWLRQV LQ PHFKDQLFV DQG LQ RWKHU SK\VLFDO VFLHQFHV DUH GLPHQVLRQDOO
KRP RJHQHRXV WKDWV HDFK DGGLWLYH WHUP LQ WKH HTXDWLRQ KDV WKH VDP H GLPHQVLRQV + RZ HYHU WK
UHDGHUVKRXOG EH ZDUQHG WKDW P DQ HPSLULFDO IRUPXODV LQWKHHQJLQHHULQJ OLWHUDWXUH DUH VLQJ SUP DUL
IURP FRUUHODWLRQV RI GDWD DUH GLPHQVLRQDOO LQFRQVLVWHQW7KHUXOHWFDQQRWEH UHFRQFLOHG VLPSO
DQG VRPH WHUPV PD\ FRQWDLQ KLGGHQ YDULDEOHV $Q H[DPSOH LV WKH IRUPXOD WKDWSLSH YDOY
PDQXIDFWXUHUVFLWHIRUOLTXLG YROXPHIORZ UDWH4 P V WKURXJK DSDUWDOO RSHQYDOYH

$$Q = C_V\left(\frac{\Delta p}{SG}\right)^{1/2}$$

ZKHUH ûS LV WKH SUHVVXUH GURS DFURVV WKH YDOYH DQG 6* LV WKH VSHFLILF JUDYLWY RI WKH OLTXLG WK
UDWR RI LWV GHQVLW WR WKDW RI ZDWU 7KH TXDQWLW&9 LV WKH YDOYH ICRZ FRHILFLHQW ZKLFK
PDQXIDFWXUHUV VDEXODWH IQ WKHUYDOYH EURFKXUHV 6LQFH 6* LV GLPHQVLRQOHVV ^ ` ZH VHH WKDWWKH
IRUPXOD LV WRWDOO LQFRQVLVWHQW ZLWK RQH VLGH EHLQJ DICRZ UDW^/ 7` DQG WKH RWKHU EHLQJ WKH VTXU
URRWRI DSUHVVXUHGURS ^0 / 7` ,WRICRZ VWKDW&9 PXWWKDYH GLPHQVLRQV DQG UDWKHURGGRQHVD
WKDW^/ 0 ` 1 RULV WKH UHVROXWRQ RI WKLV GLVFUHSDQF\ FOHDU DOKRXJK RQH KLQWV WKDWWKH YDOXHV R
&9 LQ WKH OLWHUDWXUH LQFUHDVH QHDU DV WKH VTXDUH RI WKH VL]H RI WKH YDOYH 7KH SUHVHQWDWRQ R
H[SHULPHQWDOGDWD IQKRPRJHQHRXV IRUP LV WKH VXEVHFWRI GLPHQVLRQDOODQDOVLV &KDS 7KHUH ZI
VKDOOHDUQWKDWDKRPRJHQHRXV IRUP IRUWKHYDOYHICRZ UHDWRQV

$$Q = C_d A_{\text{opening}}\left(\frac{\Delta p}{\rho}\right)^{1/2}$$

ZKHUH! LV WKH OLTXLG GHQVLW DQG$ WKH DUHD RI WKH YDOYH RSHQLQJ 7KHGLVFKDUJHFRHILFLHQW&G LV
GLPHQVLRQOHVV DQG FKDQJHV RQO PRGHUDWO ZLWK YDOYH VL]H 3OHDVH EHOHYH? XQOZ H HVDEOVK WK
IDFWQ&KDS ² WKDWWKLVDWWULVDPXFK EHWWUIRUP XODWRQ WKHGDW

0 HDQZKLOH ZH FRQFOXGH WKDWGLPHQVLRQDOO LQFRQVLVWHQWHTXDWRQV WKRXJK WKH RFFXU L
HQJLQHHULQJ SUDFWH DUH PLVOHDGLQJ DQG YDJXH DQG HYHQGDQJHURXV LQ WKH VHQVH WKDWWKH DUH RWM
PLVXVHGRXWLGHWKHUUDQJHRI DSSOFDELOW

&RQYHQLHQW3UHL[HVIRU3RZHUVRI

(QJLQHHULQJ UHVXOWVRI WKQDUH WRR VPDOO RU WRR ODUJHIRU WKHFRP PRQXQLW ZLWK WRR PDQ]HURVRQ
ZD\ RU WKHRWKHU) RUH[DPSOH WR ZULWS 3DVFRQ DQG DZ NZ DUG 8VQJ WKHSUHL[
³0´ WR PHDQ ZH FRQYHUWWKLV WR D FRQFLVHS 0 3D PHJDSDVFDO 6LPLODUO W
VLVDSURRUHDGHU¶VQJKWDUHFRPSDUHGWRWKHHTXLYDOHQW QV QDQRVHFRQGV 6XF
SUHL[HVDUHFRP PRQDQG FRQYHQLHQWLQERWKWKH6, DQG%* V\VWHPV $ FRPSOHWHWDWLVJLYHQLQ7DEO

7DEOH &RQYHQLHQW3UHL[HVIRU(QJLQHHULQJ 8QLW

Multiplicative factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

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,Q 5REHUW0 DQQLQJ DQ ,UVK HQJLQHHU SURSRVHG WKH IRUPXOD IRU WK
DYHUDJH YHORFLW9 LQ XQIRUP IORZ GXHWR JUDYLWGRZ QDQRSHQFKDQQHO %* XQLW

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \qquad (1)$$

ZKHUH5 K\GUDXOLF UDGLXV RI FKDQQHO &KDSV DQG
6 FKDQQHOVORSH VDQJHQWR DQJOH WKDWHRWWP PDNHVZLWKKRUL]RQWDO
Q 0DQQLQJ¶VURXJKQHVVIDFWRU &KDS

DQGQLVDFRQVWDQWRUDJLYHQVXUFHFRQGLWRQIRUWKHZDOOVDQGERWWRP RI WKHFKDQQHO D ,
0DQQLQJ¶VIRUPXODGLPHQVLRQDO FRQVLVWHQW E (TXDWRQ LVFRPPRQO VDNHQWREHYDOGLQ%*
XQLWZLWKQVDNHQDVGLPHQVLRQOHVV 5HZULWLWQ6, IRUP

6RXWLRQ

- $VXPSWLRQ 7KHFKDQQHOVORSH6 LVWKHVDQJHQWR DQDQJOHDQGLVWKXVGLPHQVLRQOHVVUDW
ZLWKWKHGLPHQVLRQDOQRWDWLRQ^ `² WKDWLV QRWFRQVLGHUHG0 / RU7
- $SSURDFK D 5HZULWHWKHGLPHQVLRQVRI HDFKWHUP LQ0DQQLQJ¶VHTXDWLRQ XVLQJ EUDFNHW
^`

$$\{V\} = \left\{\frac{1.49}{n}\right\}\{R^{2/3}\}\{S^{1/2}\} \quad \text{or} \quad \left\{\frac{L}{T}\right\} = \left\{\frac{1.49}{n}\right\}\{L^{2/3}\}\{1\}$$

7KLVIRUPXOD LV LQFRPSDWEOH XQGHUWV^ Q` ^/ 7` ,I Q LV GLPHQVLRQOHVV DQG LWL
QHYHUDLWWRGZLWKXQLWVLQW[WERRNV WKHQXPEHU PXWFDUU WKHGLPHQVIRQVRI ^/ 7` $QV
D

- &RPPHQWD)RUPXODV ZKRVH QXPHULFDO FRHIILFLHQW KDYH XQLWV FDQ EH GLVDWWRXV IR
HQJLQHHUVZRUNLQDGLIIHUHQWXVWP RUDQWHUIOXG 0DQQLQJ¶VIRUPXOD WKRXJK SRSXODU L

LQFRQVLVWHQWEHWKH GLPHQVLRQDOO DQG SKYVLFDOO DQG LV YDOLG RQO IRU ZDWHU ZLWK FHUWDL
ZDOOURXJKQHVVHV 7KH HIHFWV RI ZDWHU YLVFRVLWLHV DQG GHQVLWLHV KLGGHQ LQ WKH QXPHULFDOYDOX

- $SSURDFK E 3DUW D VKRZHG WKDW KDVGLPHQVLRQV ,I WKH IRUPXOD LV YDOLG LQ%* XQLW
WKHQLWP XWHTXDO IW V %\ XVLQJ WKH6, FRQYHUVLRQ IRUGHQWK ZHEVLQ

IW V P IW P V

7KHUHIRUH0 DQDOLQJ¶VLQFRQVLVWHQWRUPXODFKDQJHVIRUP ZKHQFRQYHUWHG WR WKH6, VYVWHP

SI units: $V = \frac{1.0}{n} R^{2/3} S^{1/2}$ *Ans. (b)*

ZLWK5 LQPHWHUV DQG9 LQPHWHUVSHUVHFRQG

- &RPPHQWE $FWXDOO ZH PLVVHG \RX 7KLV LV WKH ZD\ 0 DQDLQJ D PHWLF XVHU ILUV
SURSRVHG WKH IRUPXOD ,WZDV ODWHU FRQYHUWHG WR %* XQLW 6XFK GLPHQVLRQDOO LQFRQVLVWHV
IRUPXODV DUH GDQJHURXV DQG VKRXOG HLWKHU EH UHQDPHG RU WDNH DV KDYLQJ YHUV GPLVV
DSSOLFDWLRQ

3URSHUWLHV RI WKH9 HORFLWV) LHOG

,QDJLYHQIORZ VLWXDWLRQ WKH GHWUPLQDWLRQ E\ H[SHULPHQWRUV RI WKHSURSHUWLHV RI WKHIOXLG
DVDIXQFWLRQ SRVLWLRQDQG WLPHVFRQVLGHUHGWREHWKHVROXWLRQ WR WKHSUREOP ,QDOPRVWDOFDVH
WKHP SKDVLVLVRQ WKHVSFH±WLPHGLWXEXWLRQ RI WKHIOXLG SURSHUWLHV 2QHUDUH NHHSVWDFN RI WK
DFWXDOIDWRI WKHVSHFLILF IOXLG SDUWFOH- 7KLVWHDWHQWRI SURSHUWLHV DVFRQVQXP ILHOG IXQFWLRQ
GLVWQJXLVKHVIOXLG PHFKDQFVIURP VROGPHFKDQFV ZKHUHZHDUHPRUHOLNHO WREHLQWHUHVWHG LQWKH
WDMFWRUHVRI LQGLYLGXDOSDUWFOHVRUX\VWPV

(XOHULDQ DQG/ DJUDQJLDQ' HVFULSWLRQV

7KHUH DUH WZR GLIHUHQWSRLQWV RI YLHZ LQ DQDO]LQJ SUREOPV LQ PHFKDQFV 7KH ILUVWYLHZ
DSSURSULDWH WR IOXLG PHFKDQFV LVFRQFHUQHG ZLWK WKHILHOG RI IORZ DQG LVFDOOHG WKHHXOHULDQ PHWRG
RI GHVFULSWLRQ ,Q WKH HXOHULDQ PHWRG ZHFRPSXWH WKH SUHVXUHILHOGS [\ ] WR WKHIOXLG SDWWLQ
QRWWKHSUHVXUHFKDQJHVS WWKDWDSDUWFOH[SHULHQFHVDVLWPRYHVWKURXJK WKHILHOG
7KHVHFRQG PHWKRG ZKLFKIRQORZV DQLQGLYLGXDOSDUWFOHPRYLQJ WKURXJK WKHIOXLG LVFDOOHG WK
DJUDQJLDQGHVFULSWLRQ 7KHDJUDQJLDQDSSURDFK ZKLFKLVPRUHDSSURSULDWHVROGPHFKDQFV ZLQ
QRWEHWWDQGLQWKLVERRN + RZHYHUFRQVLGHUDWLRQ RI VKDSSO ERXQGHGIOXLGIOZV VXFH
DV WKH PRWRQ RI LVRODWHG IOXLG GURSOHWV DUH YHU FRQYHQLHQW FRPSXWDWLRQDO DJUDQJLDQ FRRUGLQDW
> @

) OXLG G\QDPLF PHDVXUHPHQWV DUH DOVR XVLWG WR WKH HXOHULDQ V\VWWP) RU H[DPSOH ZKHQ
SUHVXUH SUREHV LQWRGXFHG LQWR D ODERUDWRU IOZV IL[HG DVD VSHFLILF SRVLWLRQ [\ ] ,V
RXSXWXXV FRQWLEXWV WR WKH GHVFULSWLRQ RI WKH HXOHULDQ SUHVXUH IHLGS [\ ] W 7R VLPXODWH
DJUDQJLDQPHDVXUHPHQWWKH SUREHZRXOG KDYHWR PRYHGRZQWUHDP WKHIOXLG SDWWFOHVHHGV WKV

LV VRPHWLPHV GRQH LQ RQH GRJUDSKLF PHDVXUHPHQW ZKHUHIRUH PHWKRGRORJLFDO WDOHQW FXUHQW

7KH VZRGLIIHUHQW GHVFULSWRQV FDQ EH FRQQHFWHG LQ WKH DQDO\VLV RI WDILF IORZ DORQJ DIUHZD\ \$ FHUWDLQ QHJWKR IUHZD\ PD\ EH VHOHFWHG IRU VWXG\ DQG FDOOHG WKH ILHOG RI IORZ 2EYLRXVO DV WLPH SDVVHV YDULRXV FDUV ZLOO HQWHU DQG OHDYH WKH ILHOG DQG WKH LGHQWLWLHV RI WKH VSHFLILF FDUV ZLWKLQ WKH ILHOG ZLOO FRQWLQXH EH FKDQJLQJ 7KH WDILF HQJLQHHULQJ VSHFLILF FDUV DQG FRQFHQWUDWLRQ WKH DYHUDJH YHORFLW DV DIXQFWLRQ RI WLPH DQG SRVLWLRQ ZLWKLQ WKH ILHOG SOXV WKH IORZ UDWH RUQXPEHU FDUV SHU KRXU DFURVV D JLYHQ VHFWLRQ RI WKH IUHZD\ 7KLV HQJLQHHULQJ DQHXGLDQG HVFULSWRQ WKH WDILF IORZ 2WKHULQYHVWLJDWRUV VXFK DV WKH SRQFHRUVFLHQWVW PD\ EH LQWHUHVWHG LQ WKH SDWKRUVSHHG RUGHVWLQDWLRQ RI VSHFLILF FDUV LQ WKH ILHOG %\ IROORZLQJ D VSHFLILF FDUDV DIXQFWLRQ RI WLPH WKH DUHXVLQJ DGJUDQJLDQ GHVFULSWRQ RI WKH IORZ

7KH9HORFLW)LHOG

)RUHPRVWDPRQJ WKH SURSHUWLHV RI DIORZ LV WKH YHORFLWILHOG9 [\ ] W ,QIDFW GHWHUPLQLQJ WK YHORFLWLVRIWHQVDQDP RXQWRVROLQJ DIORZ SUREOP VLQFHRWKHUSURSHUWLHVIORZ GLUHFW IURP WK YHORFLWILHOG &KDSWU LVGHYRWHGWRWKHFDOFXODWLRQ RI WKHSUHVXUHILHOGIRU NQRZQ %RRNVRQKHDWWUDQVIHU IRUH[DPSOH 5H DUHGLUHG GHYRWHGWRILQGLQJ WKHWHPSHUDWU ILHOGIURP NQRZQYHORFLWILHOGV

,QJHQHUDO YHORFLWLVDYHFWRUIXQFWLRQRI SRVLWLRQDQG WLPHDQG WKXVKDVWKUHHFRPSRQHQWX DQGZ HDFKDVFDODUILHOGLQLWVHO

$$V(x, y, z, t) = \mathbf{i}u(x, y, z, t) + \mathbf{j}v(x, y, z, t) + \mathbf{k}w(x, y, z, t) \qquad (1.4)$$

7KHXVHRI X DQGZ LQWKDGRI WKHPRUHORJLFDOFRPSRQHQWQRWDWRQ9[9\ DQG9] LVWKHUHVX RI DQDPRWXQEUHDNEOHFXUP LQIOXGPHFKDQFV 0 XFKRI WKLVWH[WERRN HVSHFLDOO &KDSV DQG LVFRQHQHG ZLWKILQGLQJ WKHGLVWUEXWRQ RI WKHYHORFLW YHFWRU9 IRUDYDULHWRI SUDFWFD IORZV

7KH\$FFHOHUDWRQ)LHOG

7KHDFFHOHUDWRQYHFWRU D G9 GWRFFXUVLQ1HZWRQ¶V DZ IRUDIOXLGDQGWKXVLVYHU LPSRUDQ ,QRUGHUWR IORZ DSDUWFOHLQWKH (XOHUDQIUDPH RI UHHUHQFH WKHILQDOUHVXOWRUDFFHOHUDWRQ QRQOLQHDUDQGTXLWFRPSOFDWHG +HUHZ HQO JLYHWKHIRUPXOD

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \qquad (1.5)$$

ZKHUH X Y Z DUHWKHYHORFLWFRPSRQHQWIURP (T : HVKDOOWXG WKLVIRUPXODLQGHWDL LQ&KDS 7KH ODVWWKUHH WHUPV LQ(T DUH QRQOLQHDU SURGXFWV DQG JUHDW FRPSOFDW WKH DQDOVLVRI JHQHUDOIOXLGPRWRQ HSHFLDOO YLVFRXVIORZ V

7KHUP RG\QDP LF3URSHUWLHVRI D)OXLG

: KLGH WKH YHORFLW ILHOG9 LV WKH PRWLP SRUDQW OLG SURSHUW LWLQWUDFW FRVHO Z LWK WK
WKHUPRG\QDPLF SURSHUWLHV RI WKH IOXLG : HKDYH DOHDG\ LQWURGXFHG LQWR WKH GLVFXVVLRQ WKH WKUHP RV
FRPPRQVXFK SURSHUWLHV

3UHVVXUHS
' HQVLW!
7HPSHUDWXH7

7KHVH WKUHH DUH FRQVLGHUHG PSDUWLFXODU RI WKH YHORFLW YHFWRULQIRZ DQDOVHV) RXURWKHULQWQLY
WKHUPRG\QDPLF SURSHUWLHV EHFRPH LPSRUWDQWZKHQ ZRUN KHDW DQG HQHJ\ EDODQFHV DUH WUHDW
&KDSV DQG

,QWHUQDOHQHJ\ €
(QWKDOS\ K € S!
(QWURS\ V
6SHFLILF KHDWF$ DQGF9

,Q DGGLWLRQ IULFWLRQ DQG KHDWFRQGXFWLRQ HIHFWV DUH JRYHUQHG E\ WKH VZR VR FDOOHG WDQSRU
SURSHUWLHV

&RHILFLHQWRI YLVFRVLW
7KHUPDOFRQGXFWLYLW

$ OO QLQH RI WKHVH TXDQWLWLHV DUH WUXH WKHUPRG\QDPLF SURSHUWLHV WKDWDUH GHWHUPLQHG E\ WK
WKHUPRG\QDPLF FRQGLWLRQ RUVWDWH RI WKH IOXLG) RU H[DPSOH IRUD VLQJOH SKDVHVXEVWDQFH VXFK D
Z DWHURUR[\JHQ VZR EDVLF SURSHUWLHV VXFK DV SUHVVXUHDQG WHPSHUDWXHDUHVXIILFLHQWRIL[WKHYDOX
RI DOOWKHRWKHUV

! ! S7 K KS7 S7

DQG VR RQ IRU HYHU TXDQWLW LQ WKH OLW1RWH WKDWWKH VSHFLILF YROXPH VR LPSRUWDQWL
WKHUPRG\QDPLF DQDOVHV LVRPHWGKHUHQIDYRUR LWWLQYHUVH WKHGHQVLW!
5HFDOOWKDWWKHUPRG\QDPLF SURSHUWHV GHVFULEH WKH VWDWH RI DV\VWHP² WKDWLV D FROOHFWLRQ R
PDWWURI IL[HG LGHQWLW WKDWQRWHWZLWK LWV VXUURXQGLQJV ,QPRVWFDVHV KHUH WKH V\VWHP ZLOOEH I
VP DOOIOXLG HOHPHQWDQG DOOSURSHUWHVZLOOEH DVXPHGWREH FRQWLQXXP SURSHUWHVRI WKHIORZ ILHOG
! ! [\ ] W DQGVRRQ
5HFDOODOVR WKDWWKHUPRG\QDPLFVLVQRUP DOO FRQFHUQHG ZLWKWDWF V\VWHP V ZKHUHDVIOXLGVDU
XVXDOO LQYDULABEHPRWRQZLWKFRQWDQW FKDQJLQJ SURSHUWHV ' RWKHSURSHUWHVUHODWLQWKHUPHDQG
LQDIOXLGIORZ WKDWV\VWFKQFDOO QRWQHTXLOEULXP" 7KHDQZHULV\HV IURP DVWDWXWFDODJXPHQW,
JDVHVDWQRUP DOSUHVXUH DQGHYHQPRUVR IRUQTXLGV DQHQRUP RXQXPEHURI PROFXDUFRQLRQ
RFFXURYHUDYHU VKRUWGLVWDQFH RI WKHRUGHURI P VR WKDWDIOXLG VXEVHFWG WR VXGGHQFKDQJH
UDSLGO DGXVWVLWHOVWZDUGHTXLOEULXP : HWKHUHRUHDXPHWKDWDOOWKHWKHUPRG\QDPLF SURSHUWH
MXVWDVWG H[LWVDVSRLQWXQFWLRQVLQDIOXLGVDQG IRQOZ DOOWKHOZV DQG VWDWHUHDWRQVR
RUGLQDU HTXLOEULXP WKHUPRG\QDPLFV 7KHUH DUH FRXUH LPSRUWQQHTXLOEULXP HIHFWVXFK D
FKHPLFDOQG QXFOHDUHDFWRQVLQIOXLGV ZKLFK DUHQRWWDDGLQWKLV\W

3UHVVXUH

3UHVVXUH LV WKH FRPSUHVVLRQ VWUHVV DWD SRLQWLQ D VWDWF IOXLG) LJ 1H[WRQ YHORFLW WK
SUHVVXUHS LV WKH PRVWG\QDPLF YDULDEOH LQ IOXLG PHFKDQLFV ' LIIHUHQFHV RUJUDGLHQWV LQ SUHVVXU
RIWQGULYHDIOXLGIORZ HVSHFLDOO LQGXFWV ,QORZ VSHHGIORZ V WKHDFWDOP DJQLWXGHRI WKHSUHVVXU
LV RIWQ QRWLP SRUWDQWXQDVVLWGURSV VR ORZ DV WR FDXVH YDSRUEXEEOHV WR IRUP LQ D OTXLG) R
FRQYHQLHQFH ZHVHWP DQ VXFK SUREOHP DWLJQPHQWDWWKHOHYHORI DW GEI IW
3D + LJK VSHHG FRPSUHVVLEOH JDVIORZV &KDS KRZHYHU DUHLQGHHGVHQVLWLYHWRWKHPDJQLWXG
RI SUHVVXUH

7HP SHUDWXUH

7HP SHUDWXUH7 LV UHDDG WR WKHLQWHUQDOHQHUJ\ OHYHORI DIOXLG ,WP D\ YDU FRQVLGHUDEO GXLQ
KLJK VSHHGIORZ RI DJDV &KDS $OWRXJK HQJLQHHUV RIWQ XVH &HOLXV RU) DKUHQKHLWFDOHV IR
FRQYHQLHQFH PDQ DSSOLFDWLRQVLQWKLVWH[WXUHDEVROXW . HOYLQRU5DQNLQH WHP SHUDWXUHVFDOHV

*f*5 *f*)
. *f*&

,I WHP SHUDWXUHGLIIHUHQFHVDUHVXUQJ KHDWWDQVHUPD\ EHLPSRUWDQW> @EXWRXUFRQFHUQKHUH
PDLQO ZLWKG\QDPLF HIHFW

' HQVLW

7KHGHQVLWRI DIOXLG GHQRWGE\ ! GRZHUFDVH* UHHNUKR LVLWPDWSHUXQLWYROXPH ' HQVLWL
KLJKO YDULDEOHLQJDVHVDQGLQFUHDVHVQHDUO SURSRUWLRQDO WR WKHSUHVVXUHOYHD' HQVLWLQ QTXLG
LVQHDU FRQVDQWWKHGHQVLWRI ZDWU DERXW NJ P LQFUHDVHVRQO SHUFHQW WKHSUHVVXUHL
LQFUHDVHG E\ D IDFWRU RI 7KXV PRWWQTXLG IORZV DUH WUHDWG DQDO WFDO DV QHDU
³LQFRP SUHVVLEOH´
,QJHQHUDO QTXLGV DUH DERXWKUHHRUGHUV RI PDJQLWXGH PRUH GHQVH WKDQ JDVHV DWDWP RSKHUL
SUHVVXUH 7KH KHDYLHWFRP PRQ QTXLG LV PHUFXU DQG WKH QJKWWDV LV K\GURJHQ &RPSDUH WKHL
GHQVLWLHVDW *f& DQG DW

0 HUFXU ! NJ P +\GURJHQ ! NJ P

7KH\ GLIHUE\ DIDFWRURI 7KXV WKHSK\VFDOSDUDPHWHUV LQYDULRXV QTXLG DQG JDV IORZ
PLJKWYDU FRQVLGHUDEO 7KH GLIHUHQFHV DUH RIWQ UHVROYHG E\ WKH XVH RI GLPHQVLRQDO DQDO VL
&KDS 2WKHUIOXLG GHQVLWLHVDUHDLWWGLQ7DEOHV$ DQG$ LQ$SS $ DQGLQ5H

6SHFLILF: HLJKW

7KHVSHFLILF ZHLJKWRI DIOXLG GHQRWGE\ GRZHUFDVH* UHHNJDPPD LV LWZHLJKWSHUXQ
YROXPH -XWDV D PDWKDV D ZHLJKW PJ GHQVLW DQG VSHFLILF ZHLJKWDUH VLPSO UHODWG E\

JUDYLW

$$\gamma = \rho g \qquad (1.6)$$

7KHXQLWVRI DUHZHLJKWSHUXQLWYROXPH LQGEI IWRU1 P ,QVWDQGDUGHDUWKJUDYLW J IWV P V 7KXV IRUH[DPSOH WKHVSHFLILF ZHLJKWRI DLUDQG ZDWHUDW *& DQG DW DU DSSUR[LPDWHO

DLU	NJ P	P V	1 P	GEI IW
ZDWHU	NJ P	P V	1 P	GEI IW

6SHFLILF ZHLJKWLV YHU XVHXOLQ WKH KYGURVWDWF SUHVVXUH DSSOLFDWLRQV RI &KDS 6SHFLILF ZHLJKWRI RWKHUIOXLGVDUHJLYHQLQ7DEOHV$ DQG$

6SHFLILF* UDYLWV

6SHFLILF JUDYLW GHQRWHG E\ 6* LV WKH UDWLR RI D IOXLG GHQVLW WR D VWDQGDUG UHIHUHQFH IOXLG XVXDOO ZDWHUDW *& IRUOLTXLGV DQG DLU IRUJDVHV

$$\mathrm{SG}_{\mathrm{gas}} = \frac{\rho_{\mathrm{gas}}}{\rho_{\mathrm{air}}} = \frac{\rho_{\mathrm{gas}}}{1.205\ \mathrm{kg/m^3}} \qquad (1.7)$$

$$\mathrm{SG}_{\mathrm{liquid}} = \frac{\rho_{\mathrm{liquid}}}{\rho_{\mathrm{water}}} = \frac{\rho_{\mathrm{liquid}}}{1000\ \mathrm{kg/m^3}}$$

) RUH[DPSOH WKHVSHFLILF JUDYLWRI PHUFXU +J LV6*$+J §(QJLQHHUVILQ WKHVHGLPHQVLRQOHVVUDWRVHDVLHUWRUHPHPEHUWKDQWKHDFWXDOQXPHULFDOYDOXHVRI GHQVLWRI DYDULHW RI IOXLGV

3RVLWLRQLQDOODQG. LQHWLF(QHUJLHV

,QWKHUPRG\QDPLFVWKHRQO\ LQD\XEVWDQFHLV WKDWWRUHGLQD\\WHP E\ PRGHFXODUDFWLYLWDQ PRGHFXODUERQGLQJ IRUFHV 7KLV LV FRPPRQO GHORWHG DVLQWHUQDO HQHUJ\ € $ FRPPRQ DFFHSWHG DGYXWVHQWWR WKLV WDWF VLXDWRQ IRUIOXLG IORZ LV WR DGG WZR PRUH HQHUJ\ WHUPV WKDWDULVH IURF QHZ WRQDQPHFKDQLFV SRWHQWLDOHQHUJ\ DQGNLQHWLFHQHUJ\

7KHSRWHQWLDOHQHUJ\ HTXDOV WKHZRUNUHTXLUHGWRPRYHWKH\VWHP RI PDVV IURP WKHRUJLQWR SRVLWRQ YHFWRUU [M N] DJDLQVWD JUDYLWLILHOGJ ,WYDOXH L\½mV² or ½V² SHUXQLWPDV 7K NLQHWLF HQHUJ\ HTXDOV WKHZRUNUHTXLUHGWRFKDQJHWKHVSHHGRI WKHPDVIURP]HR WRYHORFLW9 ,V YDOXHLVò P9 RUò 9 SHUXQLWPDV 7KHQE\ FRPPRQFRQYHQWLRQ WKHWRWDOWRUHGHQHUJ\ HSHUXQ PDVLQIOXLGPHFKDQLFVLVWKHVXP RI WKUHHWHUPV

$OVR WKURXJKRXWWKLVERRNZHVKDOOGHILQH] DVXSZDUG VRWKDWJ í JN DQGJ U í J] 7KHQ(T EHFRPHV

7KH PRGHFXODUOLQHXQDOHQHUJ\ € LV D IXQFWLRQ RI 7 DQGS IRUWKH VLQJOHSKDVH SXUH VXEVWDQF ZKHUHDVWKHSRWHQWLDOHQHUJLHVDUHNLQHPDWFTXDQWLWLHV

6WDWH5HODWLRQVIRU* DVHV

7KHUPRG\QDPLF SURSHUWLHV DUHIRXQGERWKWKHRUHWFDOO DQGH[SHULPHQWDOO VR EHUHDG WR HDF RWKHUE\ VWDWUHDWLRQVKDWGLIIHUIRUHDFK VXEVWDQFH $VPHQWLRQHG ZHVKDOOFRQLQHRXUHOYHVKH VRVLQJOHSKDVHSXUHVXEVWDQFHV VXFKDVZDWULQLWWOTXLGSKDVH 7KHVHFRQGPRWWRP PRQIOXLG DLLVDPL[WXUHR JDVHV EXWWLQFHWKHPL[WXUHDWRVUHPDLQQHDO FRQWDQWEHWZHHQ DQG . LQ WKLVWXP SHUDXUHDQJHDUFDQEHFRQVLGHUHGWREHDSXUHVXEVWDQFH

$OOJDVHV DWKLJK WP SHUDXUHV DQG ORZ SUHVVXUHV UHDLYH WR WKHLUFULWFDOSRLQWDUH LQJRRGDJUHPHQWZLWKWKHSHUHFWJDVODZ

$$p = \rho RT \qquad R = c_p - c_v = \text{gas constant} \tag{1.10}$$

ZKHUHWKHVSHFLLF KHDWF$ DQGF9 DUHGHILQHGLQ(TV DQG

6LQFH(T LV GLPHQVLRQDOO FRQVLVWHQW5 KDV WKH VDPH GLPHQVLRQV DV SHFLLF KHD ^/ 7í ,í ` RUYHORFLW VTXDUHG SHUWP SHUDXUH XQLWNHOYLQ RUGHUH5 DQNLQH (DFK JDV KDV LV RZQFRQVWDQW5 HTXDOWR DXQLYHUVDOFRQVWDQW GLYLGHGE\ WKHPROHFXODUZHJKW

$$R_{\text{gas}} = \frac{\Lambda}{M_{\text{gas}}} \tag{1.11}$$

ZKHUH IWGEI VXJPRO @f5 - NPRO @ 0 RWDSSOLFDWRQVLQWKLVERRNDO IRUDLU ZKRVHPROHFXODUZHJKWLV0 PRO

$$R_{air} = \frac{49{,}700\ \text{ft}\cdot\text{lbf}/(\text{slugmol}\cdot{}^\circ\text{R})}{28.97/\text{mol}} = 1716\frac{\text{ft}\cdot\text{lbf}}{\text{slug}\cdot{}^\circ\text{R}} = 1716\frac{\text{ft}^2}{\text{s}^2{}^\circ\text{R}} = 287\frac{\text{m}^2}{\text{s}^2\cdot\text{K}} \quad (1.12)$$

6\DQGDUG DWPRVSKHULF SUHVVXUH LV OE IW VOXJ IW V DQG VWDQGDUG WHPSHUDWXUH LV
f *f*5 7KXV VWDQGDUG DLU GHQVLW\ LV

$$\rho_{air} = \frac{2116\ \text{slug}/(\text{ft}\cdot\text{s}^2)}{[1716\ \text{ft}^2/(\text{s}^2\cdot{}^\circ\text{R})](520{}^\circ\text{R})} = 0.00237\ \text{slug}/\text{ft}^3 = 1.22\ \text{kg}/\text{m}^3 \quad (1.13)$$

7KLV LV DQRPLQDOO\ YDOXH VXLWDEOH IRU SURE\OHPV) RUR WKHUJDVHV VHH7DEOH$
2QH SURSHUW\ LQ WKHUPRG\QDPLFV WKDW UHTXLUHV WKDW WKH LQWHUQDO RQH FXDUHQHJ\ € RI I
SHUIHFW JDV YDU RQO\ Z LWK WHPSHUDWXUH € € 7 7KHUHRUH WKH VSHFLILF KHDW\ DOR YDULHV RQO\ Z LW
WHPSHUDWXUH

$$c_v = \left(\frac{\partial \hat{u}}{\partial T}\right)_\rho = \frac{d\hat{u}}{dT} = c_v(T)$$
$$d\hat{u} = c_v(T)dT \quad (1.14)$$

RU
,QQHPDQQHUK DQG F$ RI DSHUHFW JDV DOR YDU RQO Z LWK WHPSHUDWXUH

$$h = \hat{u} + \frac{p}{\rho} = \hat{u} + RT = h(T)$$
$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = \frac{dh}{dT} = c_p(T) \quad (1.15)$$
$$dh = c_p(T)dT$$

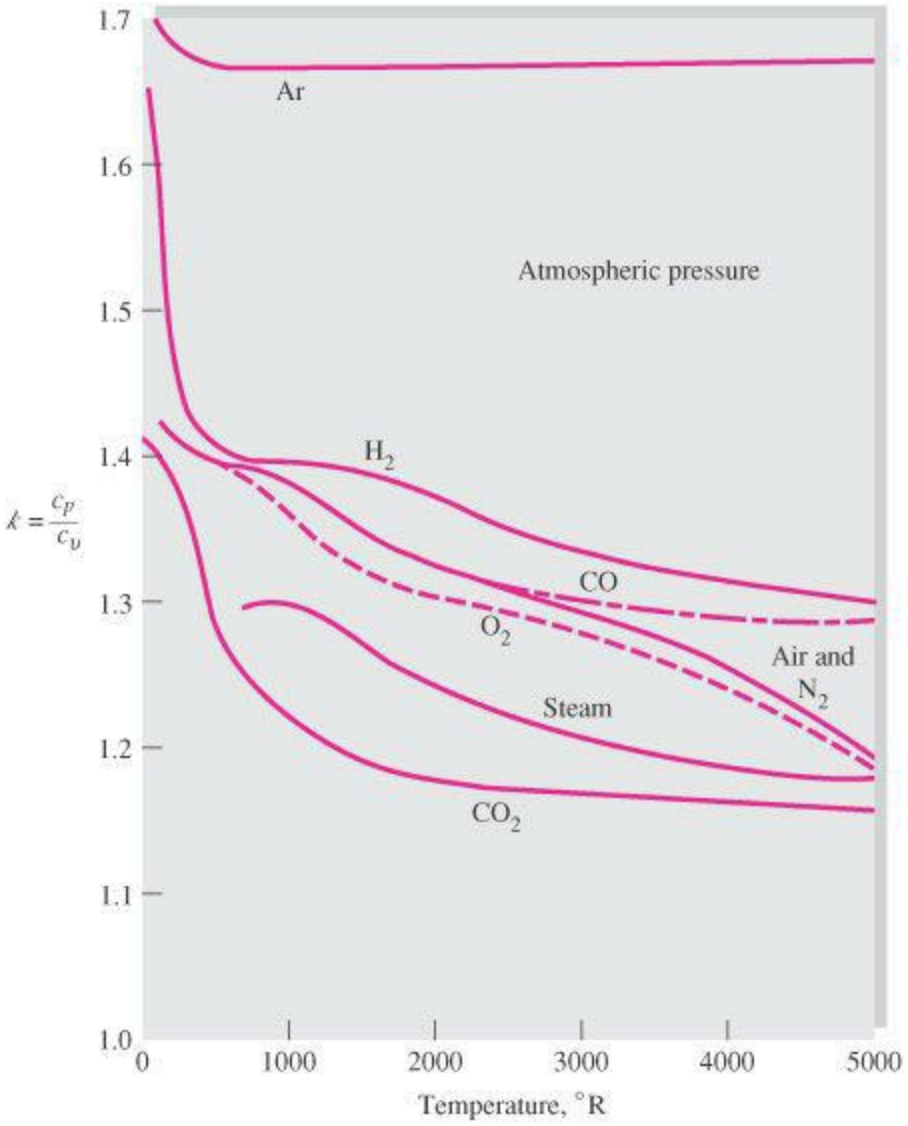
7KH UDWLR RI VSHFLILF KHDWV RI D SHUHFW JDV LV DQ LPSRUWDQW GLPHQVLRQOHVV SDUDPHWHU L
FRPSUHVVLEOH IORZ DQDO VLV &KDS

$$k = \frac{c_p}{c_v} = k(T) \geq 1 \quad (1.16)$$

$ VDILUVW SSUR[LPDWLRQ LQ DLUFRZ DQDO VLV Z HFRPPRQO WDNHF$ F$ DQG NWR EHFRQVWDQW

$$k_{air} \approx 1.4$$
$$c_v = \frac{R}{k-1} \approx 4293\ \text{ft}^2/(\text{s}^2\cdot{}^\circ\text{R}) = 718\ \text{m}^2/(\text{s}^2\cdot\text{K}) \quad (1.17)$$
$$c_p = \frac{kR}{k-1} \approx 6010\ \text{ft}^2/(\text{s}^2\cdot{}^\circ\text{R}) = 1005\ \text{m}^2/(\text{s}^2\cdot\text{K})$$

$ FWXDOO IRUDOOJDVHV F$ DQG F% LQFUHDVH JUDGXDOO Z LWK WHPSHUDWXUH DQG NGHFUHDVHV JUDGXDOO
([SHULPHQWDO YDOXHV RI WKH VSHFLILF KHDWDWDR IRUHJKWPRP PRQJDVHVDUHVKRZ QLQ) LJ



) LJ 6SHFLILF KHDWUDWLR RI HJKWFRPPRQJDVHVDV DIXQFWLRQRI WHPSHUDWXUH*(Data from Ref. 22.)*

0DQ\ IORZ SUREOHPV LQYROYH WHWP 7\SLFDOWHWP RSHUDWLQJ FRQGLWLRQV DUH UHODWLYHO\ FORVH WKH FULWLFDOSRLQWVR WKDWKH SHUIHFWJDVSSURPLDWRQLVLQDFFXUDWH6LQFHQRVLPSOHIRUPXODVSSO\ DFFXUDWH WHWP SURSHUWLHV DUH DYDLODEOHHERWK LQ((6 VHH6HF DQGRQD&' 520 > @DQ HYHQRQWKH,QWHUQHWDV D0 DWK3DG&RUS DSSOHW> @0 HDQZKLOH WKHHURURI XVLQJ WKH SHUHFWJDO FDQEHPRGHUDWHDVWKHIROORZLQJ H[DPSOHVKRZV

(; $0 3/ (

(WHWP DW! DQGF$ RI WHWP DW OEI LQ DQG *f) LQ(QJDWK XQLW D E\ WKH SHUHFWJD DSSURPLDWRQDQG E E\ WKH$60 (6WHP 7DEOHV> @RUE\ ((6

6RXWRQ

- $SSURDFK D² WKH SHUHFWJDV ODZ $ QRRXJK WHWP LV QRWDQ LGHDOJDV ZHFDQHWWP DW WKHVH SURSHUWLHV ZLWK PRGHUDWH DFFXUDF\ IURP (TV DQG) LUWFRQYHUWSUHWXU IURP OEI LQ WR OEI IW DQG XVHDEVROXWWHWP SHUHWXU *f) *f5 7KH

ZHQHGWKHJDVFRQVWDQWRUVWDP LQ(QJQLK XQLW) URP 7DEOH$ WKHPROHFXODUZHJKWR
+ 2 LV ZKHQH

$$R_{steam} = \frac{\Lambda_{English}}{M_{H_2O}} = \frac{49{,}700 \text{ ft} \cdot \text{lbf/(slugmol } ^\circ\text{R)}}{18.02\text{/mol}} = 2758 \frac{\text{ft} \cdot \text{lbf}}{\text{slug } ^\circ\text{R}}$$

7KHQWKHGHQVLWHVWDPDWIRQBRZ VIURP WKHSHUHFWJDVODZ (T

$$\rho \approx \frac{p}{RT} = \frac{14{,}400 \text{ lbf/ft}^2}{[2758 \text{ ft} \cdot \text{lbf/(slug} \cdot ^\circ\text{R)}](860 ^\circ\text{R})} \approx 0.00607 \frac{\text{slug}}{\text{ft}^3} \qquad \textit{Ans. (a)}$$

$W f5 IURP) LJ NWDP FS FY § 7KHQ IURP (T

$$c_p \approx \frac{kR}{k-1} = \frac{(1.3)(2758 \text{ ft} \cdot \text{lbf/(slug } ^\circ\text{R))}}{(1.3-1)} \approx 12{,}000 \frac{\text{ft} \cdot \text{lbf}}{\text{slug } ^\circ\text{R}} \qquad \textit{Ans. (a)}$$

- $SSURDFK E² VDEOHVRUVRVZDUH 2QHFDQHLWKHUUHDGWKHVWDP VDEOHVRUSURJUDP DIHZ
QQHVLQ((6 ,QHLWKHUFDVH WKH(QJQLK XQLW SVL %WX OEP DUHZ NZ DUGZ KHQDSSOLHGWRIOX
PHFKDQLFVIRUPXODV (YHQVR ZKHQXVLQJ ((6 PDNHVXUHWKDWWKH9DULDEOH,QRUPDWRQ PHQ
VSHFLILHV(QJQLK XQLW SVLDDQG f) ((6 VDWWHPSWVIRUHYDOXDWLQJ GHQVLWDQG SHFLILF KHDW
VWDP DUH IRUWKHVHFRQGLWRQV

```
Rho = DENSITY(steam, P = 100, T = 400)
Cp = SPECHEAT(steam, P = 100, T = 400)
```

1RWWKDWWKHVRWZDUHLVVHWXSIRUSVLDDQG f) ZLWRXWFRQYHUWLQJ ((6 UHWXUQVWKHFXUHI
YDOXHV

$$\rho \approx 0.2027 \text{ lbm/ft}^3 \text{ ; } c_p \approx 0.5289 \text{ Btu/(lbm-F)}$$

$VMXWWXDWG %WX DQG OEP DUHH[WPHO XQZLHOG ZKHQDSSOLHGWRPDW PRPHQWXP DQ
HQHUJ\ SUREOPVLQIOLGPHFKDQLFV 7KHUHUH HLWKHUFRQYHUWRIWOEE DQGVOXJVXLQJ \RXURZ
UHVRXUFHV RUXVHWKH³&RQYHUWIXQFWLRQLQ((6 SDFLQJ WKHRGDQGQHZ XQLWVLQVLQJOHTXR
PDUNV

```
Rho2 = Rho*CONVERT('lbm/ft^3','slug/ft^3')
Cp2 = Cp*CONVERT('Btu/lbm-F','ft^2/s^2-R')
```

1RWWKDW \RXPXOWLSO WKHRG5KRDQG&SE\ WKH&219(57IXQFWLRQ DQG XQLWVWRWK
ULJKWRI WKHGLYLVRQVLJQ³ ´ LQ&219(57 DUHDVXPHGWREHLQWKHGHQRPLQDWRU((6 UHWXO
WKHVHUHVXOWV

$$\text{Rho2} = 0.00630 \text{ slug/ft}^3 \quad \text{Cp2} = 13{,}200 \text{ ft}^2/(\text{s}^2\text{-R}) \qquad \textit{Ans. (b)}$$

- &RPPHQWV 7KH VWDP VDEOHV ZRXOG \LHOG UHVXOWV TXLWH FORVH WR ((6 7KH SHUHFWJD
HVWDP DWIRP ! LV SHUHFWQRZ DQG WKHHVWDP DWIRP FS LV SHUHFWQRZ 7KHFKLH UHDVRQIRUWK
GLVFUHSDQF\ LV WKDWWKLV VWP SHUDXUH DQG SUHVXUH DUH UDWKHU FORVH WR WKH FULWFDO SRLQWDQ

VDXUDWRQ QQHR VWDP $WKJKUWHSHUDWXUHV DQG RZ HUSUHVVXUHV VD f) DQG GEL LQ WKHSHUHFWJDVODZ \LHOGVSURSHUWHVZLWKDQDFFXUDF\ RI DERXW SHUFHQW

2QH DJDLQ HWXV ZDUQ WKDW(QJOLK XQLW SVLD GEP %WX DUH DZ NZ DUG DQG PXWE FRQYHUWG LQ PRWWOLG PHFKDQFV IRUPXODV ((6 KDQGDV 6, XQLWV QFHO ZLWK QR FRQYHUVLR IDFWRUV QHGHG

6WDWH5HODWLRQVIRU/ LTXLGV

7KH ZULWUNQRZ VRI QR³SHUHFWOTXLG OZ´ FRP SDUDEOHWR WKDWRUJDVHV / LTXLGV DUH QHDU LQFRP SUHVVLEOHDQG KDYH DVLQJOH UHDRQDEO FRQVDQWVSHFLILF KHDW7KXV DQLGHQ]HG VDWUHDWR IRUDQTXLGV

$$\rho \approx \text{const} \qquad c_p \approx c_v \approx \text{const} \qquad dh \approx c_p\, dT \tag{1.18}$$

0 RVWRI WKHIORZ SUREOPV LQWKLV ERRN FDQEHDWDFNHG ZLWK WKHVHVLPSOHDVXPSWRQV : DWUL QRUPDO VDNQWR KDYHDGHQLWV RI NJ P DQG D VSHFLILF KHDW$S P V . 7KHVWDF VDEOHV PD\ EHXVHGLI PRUHDFFXUDF\ LVUHTXLUHG

7KHGHQLWV RI D QTXLG XVXDOO GHFUHDVHV VOJKWO ZLWK WHPSHUDWXUH DQG LQFUHDVHV PRGHUDWO ZLWKSUHVVXUH ,I ZHQHJOHFWWKHWHPSHUDWXUHHIHFWDQHPSLULFDOSUHVVXUHGHQLWUHDWRQIRUDQTXLGV

$$\frac{p}{p_a} \approx (B + 1)\left(\frac{\rho}{\rho_a}\right)^n - B \tag{1.19}$$

ZKHUH%DQG QDUHGLPHQVLRQOHVVSDUDPHWHUV WKDWYDU VOJKWO ZLWK WHPSHUDWXUHDQG S$ DQG! $ DU VWDQGDUGDWPRVSKHULF YDOXHV : DWUFDQEHILWWGDSSUR[LPDWHO WRWKHYDOXHV%§ DQG Q§

6HDZDWHU LV D YDULDEOH PL[WXUH RI ZDWHU DQG VDOWVDQG WKXV UHTXLUHV WKUHH WHUPRG\QDPLF SURSHUWLHV WR GHILQH LWV VWDWH 7KHVHDUH QRUPDO WHPSHUDWXUH DQG WKHVDOLQLW Å GHILQHGDV WKHZHLJKW RI WKHGLVROYHGVDOWGLYLGHGE\ WKHZHLJKW RI WKHPL[WXUH 7KHDYHUDJHVDOLQL RI VHZDWHULV XVXDOO ZULWWQDV SDUWSHU RU 7KHDYHUDJHGHQLWLRI VHZDWHU LV VXJJVIW§ NJ P 6WUFWO VSHDNLQJ VHZDWHUKDV WKUHHVSHFLILF KHDW DOODSSUR[LPDWHO HTXDOWR WKHYDOXHIRUSXUHZDWHURI IW V *5 P V .

(; $0 3/ (

7KHSUHVWXUHDWWKHGHHSHVWSDUWRI WKHRFHQLV DSSUR[LPDWHO DW (VWP DWWKHGHQLWLRI VHZDWHULQ\OXJ IWDWKLVSUHVWXUH

6RXWLRQ

(TXDWRQ KRGVIRUHLWHUZDWURUVHZDWU 7KHUDWRS S$ LVJLYHQDV

7KH YLVFRVLWLHV RI QHZWRQLDQ IOXLGV LV DWWKH WKHUPRG\QDPLF SURSHUWLHV DQG YDULHV ZLWK WHPSHUDWXUH DQG SUHVVXUH $WDJLYHQ WHPS 7 WKHUH LV D YDOXH RI YDOXH DP RQJ WKH FRPPRQ IOXLGV 7DEOH GLVWLQFW YLVFRVLWLHV HJKWOXLGV DWWDQGDUG SUHVVXUH DQG WHPSHUDWXUH 7KHUH LV DYDULDWLRQ RI VL RUGHUV RI PDJQLWXGH IURP K\GURJHQ XS WR JOFHULQ 7KXV WKHUH ZLOOEH ZLGHGLIIHUHQFHV EHWZHHQ IOXLGV VXEMHFWHG WR WKHVDP HDSSOLHG WUHVV

* HQHUDOO VSHDNLQJ WKH YLVFRVLWLHV DIOXLGLQFUHDVHV RQO ZHDNO ZLWK SUHVXUH) RUH[DPSOH LQFUHDVLQJ S IURP WR DW ZLOOLQFUHDVH RI DURQG SHUFHQW7HP SHUDWXUH KRZHYHU KDV D VWURQJ HIHFWZLWK LQFUHDVLQJ ZLWK7 IRUJDVHV DQG GHFUHDVLQJ IRUOLTXLGV) LJXUH$ LQ$SS $ VKRZV VWKLV WHPSHUDWXUHYDULDWLRQIRUYDULRXVFRPPRQIOXLGV ,WVFXUWRP DU LQPRVWHQJLQHHULQJ ZRUN WKHSUHVVXUHYDULDWLRQ

7KH YDULDWLRQ S 7 IRUD WSLFDOIOXLG LV QLFHO VKRZQ E\ ) LJ IURP 5H ZKLFK QRUPDOL]HV WKH GDWD ZLWK WKHFULWLFDOSRLQWVXVDM F S= 7F 7KLV EHKDYLRU FDQEH WKHSULQFLSOHR FRUUHVSRQGLQJ VWDWHV LV FKDUDFWHULVWLF RI DOOIOXLGV EXWWKHDFWXDOQXPHULFDOYDOXHV DUH XQFHUWDLQ “ SHUFHQWRUDQ JLYHQIOXLG) RUH[DPSOH YDOXHV RI 7 IRUDLUDW DW IURP 7DEOH$ IDC DERXW SHUFHQWBZ FRPSDUHG WR WKH³GRZ GHQVLWOPLWLQ) LJ

1 RWH LQ) LJ WKDWFKDQJHV ZLWK WHPSHUDWXUH RFFXUYHU UDSLGO QHDU WKH FULWLFDOSRLQW,(JHQHUDO FULWLFDOSRLQWP HDXUHP HQWDUH[WPHO GLIILFXOWQGXQHUWDLQ

7KH5H\QROGV1XPEHU

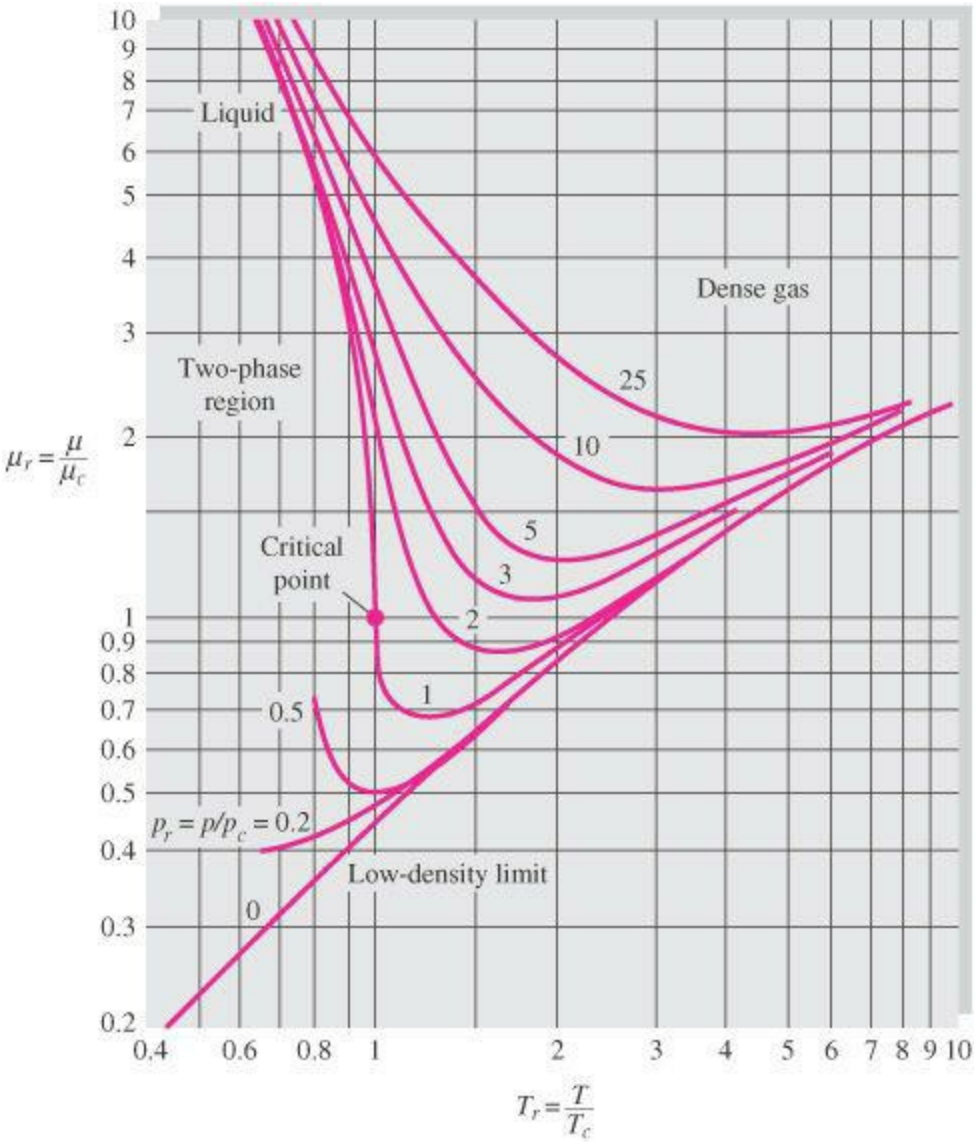
7KH SULPDU SDUDPHWHU FRUUHODWLQJ WKH YLVFRXV EHKDYLRU RI DOO QHZWRQLDQ IOXLGV LV WK GLPHQVLRQOHVV5H\QROGVQXPEHU

$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{V L}{\nu} \qquad (1.24)$$

ZKHUH9 DQG/ DUH FKDUDFWHULVWLF YHORFLW DQG OHQJWK VFDOHV RI WKH IORZ 7KH VHFRQG IRUP RI 5H LOOXVWUDWHV WKDWWKH UDWLR RI WR! KDVLWVRZQQDPH WKHNLQHPDWLF YLVFRVLW

$$\nu = \frac{\mu}{\rho} \qquad (1.25)$$

,WVFDOOHGNLQHPDWLF EHFDXVHWKHPDVXQLWVFDQFHOLQJ RQO WKHGLPHQVLRQ^/ 7`



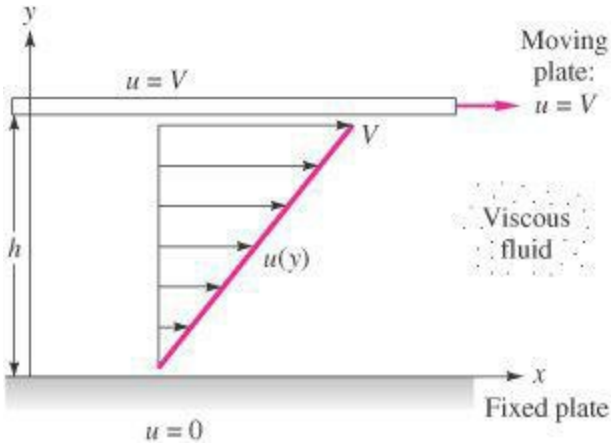
) LJ) OXLG YLVFRVLW QRQGLPHQVLRQDO[HG E\ FULWLFDOSRLQWSURSHUWLHV 7KLV JHQHUDO[H FKDUWLVFKDUDFWHULVWLF RI DOO OXLGVEXWLV DFFXUDWH RQO VR“ SHUFHQW*(From Ref. 25.)*

* HQHUDO VWHILWWKLQJ DIOXLGVHQJLQHHUVKRXOG GR LVHWLPDWWKH 5H\QROGVQXPEHUDQJH R VWHIGRZ XQGHUVWXG 9HUI GRZ 5H LQGLFDWV YLVFRXVFUHSLQJ PRWRQ ZKHUH LQHUWLD HIHFW DU QHJOJLEOH 0 RGHUDW 5H LPSOHV D VPRRWKO YDULQJOPLQDU IGRZ +JK 5H SUREDEO VSHO VXUEXOHQWGRZ ZKFK LV VGRZ 0 YDULQJ LQWKHWP H PHQEXWDVXSHUPSRVHG WURQ UDQGRP KJK IUHTXHQF\ IOXFWXDWRQV ([SDFLWQPHUFDOYDOXHV IRU GRZ PRGHUDW DQG KJK 5H\QROGV QXPEHV FDQQRWEHVWDWGKHUH 7K\ GHSHQGRQIGRZ JHRPHWW DQGZLOOEHGLVFXVHGLQ&KDSV WKURJK 7DEOH DOR OLWW YDOXHV RI IRU WKH VPH HJKWIOXLGV 7KH SHFNLQ RUGHU FKDQJH FRQVLGHUDEO DQG PHUFXU WKH KDYLHWWKDV WKH VPDOOHWYLVFRVLW UHDLYH WR LWV RZ QZHJKW$ JDVHV KDYH HKJK UHDLYH WR WKQ OTXLGV VXFK DV JDVROLQH ZDWU DQG DOR2LODQG JOFHULQ WW KDYHWKHKJKHWW EXWKHUDWR LV VPDOOHU)RUJLYHQYDOXHVRI 9DQG/ LQDIGRZ WKHVHIOXLGVH[KLE DVSUHDGRI IRXURUGHUVRI PDJQLWXGHLQWKH5H\QROGVQXPEHU

) GRZ EHWZHHQ3ODWHV

$ FODVLF SUREOP LV WKHIGRZ LQGXFHG EHWZHHQDIL[HG GRZ HUSODWDQG DQXSSHUSODWPRYLQ WWDGLO DWYHORFLW DV VKRZQLQ) LJ 7KH FHDUDQFH EHWZHQSODWV LVK DQG WKHIOXLG L

QHZ WRQDQDQG GRHVQRWOS DWHLWKHUSDW ,I WKHSDWV DUHDJH WKLVWADG VKHDLQJ PRWRQZ LOOV-
XS DYHORFLWGLWXEXWRQX \ DVVKRZ Q Z LWK Z 7KHIXLDFFHOHUDWRQLV]HURHYHUZKHUH



) LJ 9 LVFRXV IORZ LQGXFHG E\ UHDWLYH P RWRQ EHWZHHQ WZR SDUDOOHO SODWHV

: LWK]HUR DFFHOHUDWRQ DQG DVVXPLQJ QR SUHVVXUH YDULDWRQ LQ WKH IORZ GLUHFWRQ \RX VKRXO VKRZ VKDW IRUFHEDODQFH RQ D VP DOO IXLG HOP HQWGHG WR WKH HXOWKDW WKH VKHDUVWUHVV LV FRQVWDQ WKURXJKRXW WKH IXLG 7KHQ(T EHFRPHV

$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{const}$$

Z KLFK Z HFDQ LQWJUDWH WR REWDLQ

X D E\

7KH YHORFLW GLWXEXWRQ LV OQHDU DV VKRZ Q LQ) LJ DQG WKH FRQVWDQWV D DQGE FDQ EH HYDOXDWHG IURP WKH QR VOLS FRQGLWRQ DW WKH XSSHU DQG ORZ HUZ DOOV

$$u = \begin{cases} 0 = a + b(0) & \text{at } y = 0 \\ V = a + b(h) & \text{at } y = h \end{cases}$$

+ HQFH D DQGE 9 K 7KHQ WKH YHORFLW SURILOH EHZHHQ WKH SODWHV LVJLYHQ E\

$$u = V\frac{y}{h} \qquad (1.26)$$

DVLQGLFDWHG LQ) LJ 7XUEXOHQW ORZ &KDS GRHVQRW KDYH WKLV VKDSH
$ OWKRXJK YLVFRVLW KDV D SURIRXQG HIHFWRQ IXLG PRWRQ WKH DFWXDOYLVFRXV VWUHVVHV DUH TXL VP DOOLQ PDJQLWGHHYHQIRUOV DVVKRZQLQWKHIROORZLQJ H[DPSOH

(; $0 3/ (

6XSSRVHWKDWWKHIXLGEHLQJ VKHDUHGLQ) LJ LV6$(RLODW f& &RPSXWWKH VKHDUVWUHVV LQ WKHRLOLI 9 P VDQGK FP

6RXWLRQ

- 6\VWHP VNHWFK 7KLVLVVKRZQHDUDHULQ) LJ
- $VXPSWLRQV /LQHDU YHORFLW SURILOH PDLQWDLQHGQRQDQ IOXLG QR VOLS DWHLWKHU SODWH VXUIDFH
- $SSURDFK 7KHDQDOVLVRI) LJ GHDGVWR(T IRUDPLQDUIORZ
- 3URSHUWLYDOXHV) URP 7DEOH IRU6$(RLOWHRLOYLVFRVLW NJ P V
- 6RXWLRQVWHSV ,Q(T WKHRQO XQNQRZQLVWKHIULGVWUHVV

$$\tau = \mu\frac{V}{h} = \left(0.29\,\frac{\text{kg}}{\text{m}\cdot\text{s}}\right)\frac{(3\text{ m/s})}{(0.02\text{ m})} = 43.5\,\frac{\text{kg}\cdot\text{m/s}^2}{\text{m}^2} = 43.5\,\frac{\text{N}}{\text{m}^2} \approx 44\text{ Pa} \qquad \textit{Ans.}$$

- &RPPHQWV 1RWHWKHXQLWGHQLWHV NJ P V A 1 DQG 1 P A 3D $OWKRXJKRLOLVYHU YLVFRXV WKLVWKHDUWHWVLPRGHWDERXW WPHVOHWVWKDQDWRSKHULF SUHVVXUH 9LVFRX VWUHWHVLQJDVHVDQGWKLQ ZDWHU OTXLGVDUHHYHQVPDOHU

9DULDWLRQRI9LVFRVLW ZLWK7HPSHUDWXUH

7HPSHUDWXUHKDV DVWURQJ HIHFWDQG SUHVXUH DPRGHUDWHHIHFWRQ YLVFRVLW 7KH YLVFRVLW R JDVHVDQGPRVWOTXLGVLQFUHDVHVZLWK SUHVXUH : DWULVDQPDRXVLQVKRZLQJ DYHU VOJK GHFUHDVHEHORZ *f& 6LQFHWKHFKDQJHLQYLVFRVLW LVRQO DIHZ SHUFHQWSRWDW ZHVKO QHJOHFWSUHVXUHHIHFWLQWKLVERRN

* DVYLVFRVLW LQFUHDVHVZLWKWPSHUDWXUH 7Z RFRPPRQDSSUR[LPDWRQVDUHWKHSRZHUODZ DQ WKH6XWKHUODQGODZ

$$\frac{\mu}{\mu_0} \approx \begin{cases} \left(\dfrac{T}{T_0}\right)^n & \text{power law} \\ \dfrac{(T/T_0)^{3/2}(T_0+S)}{T+S} & \text{Sutherland law} \end{cases} \qquad (1.27)$$

ZKHUH LVDNQRZQYLVFRVLWDWDNQRZQDEVROXWWPSHUDWXUH7 XVXDOO . 7KHFRQVDQ QDQG6 DUHILWWRWKHGDWD DQGERWKIRUPXODVDUHDGHTXDWHRYHUDZLGHUDQJHRI WPSHUDWXUHV) RUDLU Q§ DQG6 § . *f5 2WKHUYDOXHVDUHJLYHQLQ5H

/LTXLGYLVFRVLWGHFUHDVHVZLWKWPSHUDWXUHDQGLVURXJKO H[SRQHQWLDO §DÍE7 EXWDEHWWH ILWLVWKHHPSLULFDOHXDWLRQ LVTXDGUDWLF 7 ZKHUH7 LVDEVROXWWPSHUDWXUH

$$\ln\frac{\mu}{\mu_0} \approx a + b\left(\frac{T_0}{T}\right) + c\left(\frac{T_0}{T}\right)^2 \qquad (1.28)$$

) RUZDWU ZLWK7 . NJ P V VXJJHVWWG YDOXHVDUHD í E í DQGF ZLWKDFFXUDF\ DERXW SHUFHQW7KH YLVFRVLW RI ZDWULVDEXQGDWG LQ7DEO $ &XUYH ILWVYLVFRVLW IRUPXODVIRU RUJDQLF OTXLGVDUHJLYHQE\ <DZ VHWDO > @) RUIXUWKH YLVFRVLWGDWD VHH5HV DQG

1RQQHZWRQLDQ)OLGV

)OLGVWKDWGRQRWRFFXU ZKHQHYHUDZ R(T DUHFDOHGQRQQHZWRQLDQDGDUHWHDMGL
ERRNVRQUKHORRJ\> @)LJXUH DFRPSDUHVVRPHH[DPSOHVWRDQHZWRQLDQIOLG)RUWKHQRQDQH
FXUYHV WKHVORSHDWZQ SRLQWVFDOHGWKHDSSDUHQWYLVFRVLW

' LDWDQVV

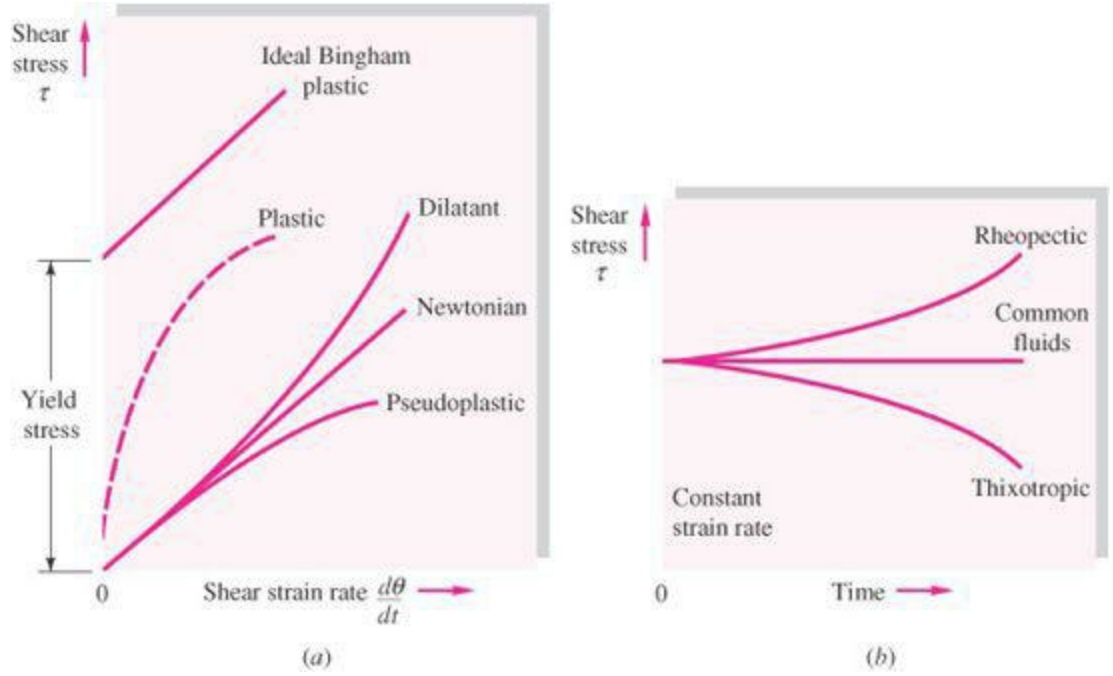
7KLVIOLGLVVKHDUWKLFNHQLQJ LQFUHDVLQJ LWVUHVLVWDQFHZLWKLQFUHDVLQJ VWUDLQUDWH ([DPSOHVDU
VXSHULRUVR FRQVWDUFK RUVDQGLQZDWU 7KHFDWLF FDHLVTXLFNVDQG ZKLFK VWUHQVXS LI RQ
WKUDKHVDERXW

3VHXGRSODVWLF

$ VKHDUWKLQQLQJ IOLG LV OHVV UHVLVWDQWDWKJKHUVWUDLQUDWHV $ YHU VWURQJ WKLQQLQJ LV FDOOH
SODVWLF 6RPHRI WKH PDQ H[DPSOHVDUHSROPHUVROXWLRQV FRORLGDOVXVSHQVLRQV SDSHUSXOS L
ZDWU ODWH[SDLQWEORRGSODVPD \UXS DQGPRODVVHV 7KHFDWLF FDHLVSLQWZKLFK LVWKLFNZKH
SRXUHGEXWWKLQZKHQEUXVKHGDWDKJKVWUDLQUDWH

%LQJKDP SODVWLF

7KHOPLWQJ FDHRI DSODVWLF VXEVWDQFHLVRQHWKDWHTXLHVDILQLWH\LHOGVWUHVEHORZLWEHJLQV\
IORZ)LJXUH DVKRZV\LHOGLQJ IROORZHG E\ OQHDUEHKDYLRU EXWQRQQHDUIORZ FDQDOR RFFXU
6RPHH[DPSOHVDUHFOD\ VXVSHQVLRQV GULOOLQJ PXG VRRWSDWW PDYRQOLVH FKRFRODW DQGPXWWRQ
7KHFDWLF FDHLVFDWXS ZKFK ZLOOQRWFRP HRXWR WKHERWWRPXQOO\RX WUHVVLVE\ VKDNLQJ
$IXUWKHUFRP SOFDWRQ QRQQHZWRQLDQEHKDYLRULV WKHWDQLHQWHIFWKRZ QLQ)LJ E 6RPI
IOLGV UHTXLUH D JUDGXDOO LQFUHDVLQJ VKHDUWWHV WR PDLQWDLQ D FRQVWDQWVWUDLQ UDWH FDOOH
UKHRSHFWF 7KHRSSRVLWH FDHRI DIOLG WKDWKLQV RXWZLWK WP H DQG UHTXLUHV GHFUHDVLQJ VWHV LV
WUHGWKL[RWRSLF : HQHJOHFWQRQQHZWRQLDQHIFWLQWKLVERRN VHH5H IRUIXUWKHUWXG\



) LJ 5 KH RBJLFDOEHKDYLRUR YDULRXV YLVFRXV PDWHULDOV a VWHVV YHUVXV VWUDLQ UDWH b HIHFWV WLPH RQ SSOLHG VWHVV

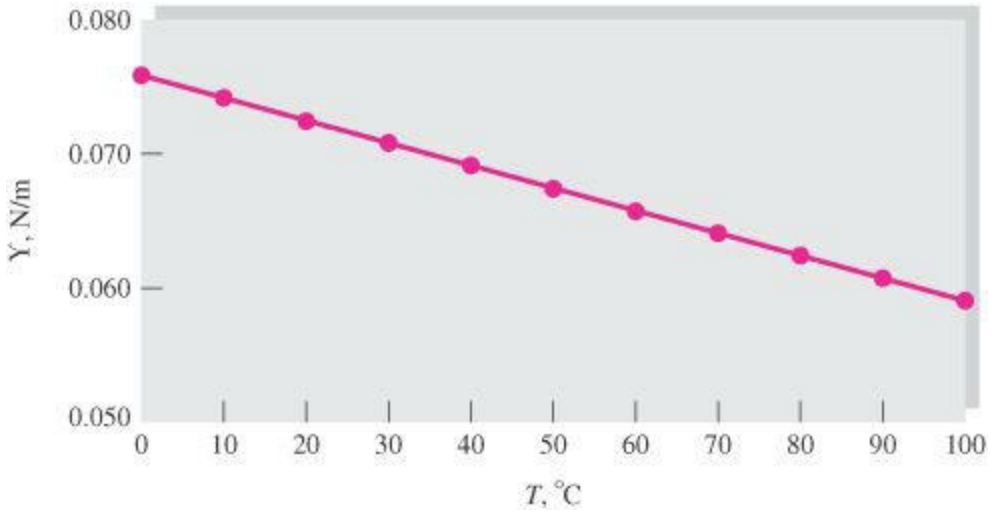
6XUIDFH7HQVLRQ

$ QXPEHU XQDEOH WR H[SODLQ IUHQ ZLOO IRUP DQLQWHUFHZLWK DVHFRQG QXPEHU RUJDV 7K SK\VLFDOFKHPLVWU RI VXFK LQWHUIDFLDOVXUFHV LV TXLWH FRP SOH[DQG ZKROH WH[WERRNV DUH GHYRWHG V WKLV VSHFLDOW> @0 ROHFXOHV GHSZLWKLQ WKH QXPEHU UHSHOHDFK RWKHUEHFDXVH RI WKHLUFRVHSDFNLQ 0 ROHFXOHV DWWKH VXUIDFH DUH GHW GHQVH DQG DWWDFWHDFK RWKHU 6LQFH KDO RI WKHLU QHLJKERUV DU PLVLQJ WKHPHFKDQLFDOHIHFWV WKDWKH VXUIDFH LV LQWQVLRQ : HFDQDFFRXQWDGHTXDWO IRUVXUIDH HIHFWLQIXLGPHFKDQFVZLWKWKH VXUIDFH HTXDOODQGRSSRVLWHIRUFHVRI PDJQLWXGH< G DUH H[SVHGQRUPDOWR WKHFXWDQG SDUDOOHOWR WKH VXUIDFH ZKHUH< LV FDOHG WKHFRHILFLHQWR VXUIDFHWQVLRQ 7KHGLPHQVLRQVRI < DUH^) / ` ZLWK6, XQLWVRI QHZRQVSHUPHWUDQG%* XQLWVF SRXQGVIRUFHSHUIRRW$QDOWHUQDWLQDWFRQHSWV WRSHQXS WKHFXWRDQDUHDG$ WKLVUHTXLUHVZRUNW EHGRQHRI DPRXQW< G$ 7KXV WKHFRHILFLHQW< FDQDOR EHUHJDUGHG DV WKH VXUIDFHHQHUJ\ SHUXQ DUHDRI WKHLQWHUIDFH LQ1 P P RUIW GE IW

7KHVZRPRWWFRPPRQLQWHUIDFHV DUHZDWHU±DLUDQG PHUFXU±DLU)RUDFHDQVXUIDFHDW *& f) WKHPHDVXUHGVXUIDFHWQVLRQV

$$\gamma = \begin{cases} 0.0050 \text{ lbf/ft} = 0.073 \text{ N/m} & \text{air–water} \\ 0.033 \text{ lbf/ft} = 0.48 \text{ N/m} & \text{air–mercury} \end{cases} \qquad (1.29)$$

7KHVH DUH GHVLJQ YDOXHV DQG FDQ FKDQJH FRQVLGHUDEO LI WKH VXUIDFH FRQWDLQV FRQWDP LQDQWV GHVWUJHQWV RU VOXEN V * HQHUDOO < GHFUHDVHV ZLWK LQFUHDVH GHW DQG LV]HR DWWKH FUWLFDOSRLQ 9DOXHV RI < IRUZDWHU DUH JLYHQLQ) LJ DQG7DEOH$



) LJ 6XUIDFHWHQVLRQRI DFHWDQDLLHZDWHUDWLIIHUHQW 7 DEOH\$

,I WKHLQWHUIDFHLVFXUYHG DPHFKDQLFDOEDODQFHVKRZVWKDWWKHUHLV DSUHVVXUHGLIIHUHQFHDFURVV WKHLQWHUIDFH WKHSUHVVXUHEHLQJ KLJKHURQWKHFRQFDYHVLGH DVLOOXVWUDWHGLQ) LJ ,Q) LJ D WKHSUHVVXUHLQFUHDVHLQWKHLQWHUURU DOTXLGFA\OLQGHULVEDODQFHGE\ WZR VXUIDFHWHQVLRQIRUFHV

$$2RL\,\Delta p = 2\Upsilon L$$

or

$$\Delta p = \frac{\Upsilon}{R} \qquad (1.30)$$

: HDUH QRWFRQVLGHULQJ WKH ZHJKWRI WKH OTXLG LQ WKLV FDOFXODWRQ ,Q) LJ E WKH SUHVVXU LQFUHDVHLQWKHLQWHUURU DVSKHULFDOGURSOHWEDODQFHVDULQJ RI VXUIDFHWHQVLRQIRUFHV

$$\pi R^2\,\Delta p = 2\pi R\Upsilon$$

or

$$\Delta p = \frac{2\Upsilon}{R} \qquad (1.31)$$

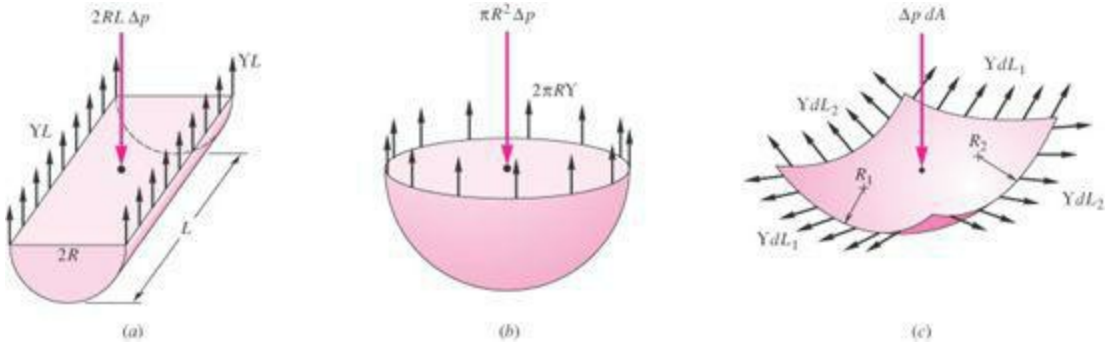
: HFDQ XVH WKLV UHVXOWWR SUHGLFWWKH SUHVVXUH LQFUHDVH LQVLGH D VRDS EXEEOH ZKLFK KDVWZF LQWHUIDFHVZ LWKDLU DQLQQHUDQGRXWHUVXUIDFHRI QHDUO WKHVDPHUDGLXV5

$$\Delta p_{\text{bubble}} \approx 2\,\Delta p_{\text{droplet}} = \frac{4\Upsilon}{R} \qquad (1.32)$$

) LJXUH F VKRZV WKHJHQHUDOFDVHRI DQDUELWUDULO FXUYHGLQWHUIDFHZKRVH SULQFLSDOUDGLL R FXUYDWXUH DUH5 DQG5 \$ IRUFHEDODQFHQRUPDOWR WKHVXUIDFHZLOOVKRZ WKDWWKH SUHVVXUHLQFUHDVHRQWKHFRQFDYHVLGHLV

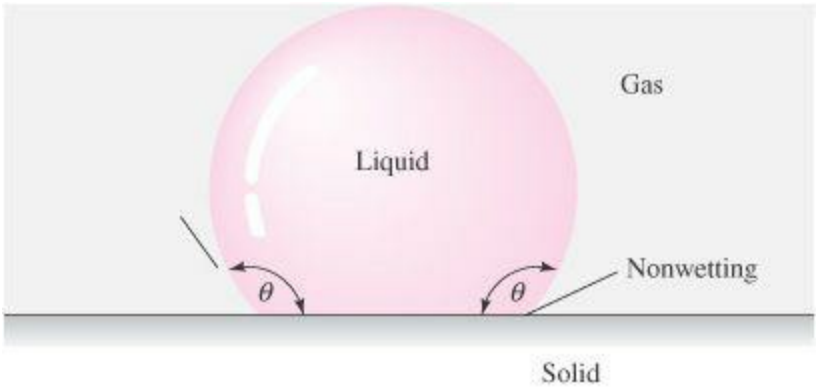
$$\boxed{\Delta p = \Upsilon(R_1^{-1} + R_2^{-1})} \qquad (1.33)$$

(TXDWLRQV WR FDQ DOOEH GHULYHG IURP WKLV JHQHUDOUHODWLRQ IRUH[DPSOH LQ(T 5 5 DQG5 ⊁



) LJ 3 UHVVXUH FKDQJH DFURVV D FXUYHG LQWHUIDFH GXH WR VXUIDFH WHQVLRQ *a* LQWHUIDFH RI D GURSOHW *b* LQWHUIDFH RI D VSKHULFDO EXEEOH *c* JHQHUDO FXUYHG LQWHUIDFH

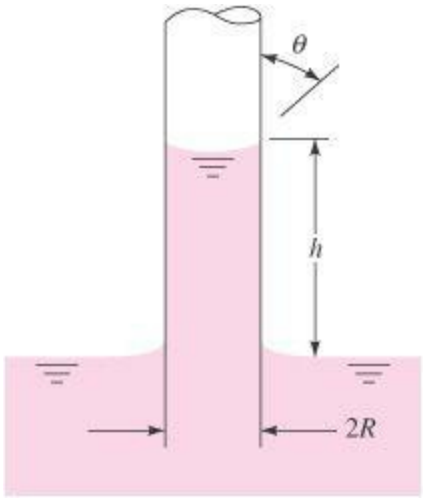
$ VHFRQG LPSRUWDQW XUIDFH HIIHFW LV WKH FRQWDFW DQJOH Z KLFK DSSHDUV Z KHQ D OLTXLG LQWHUIDFH LQWHUVHFWV Z LWK D VROLG VXUIDFH DV LQ) LJ 7KH IRUFH EDODQFH Z RXOG WKHQ LQYROYH ERWK < DQG , WKH FRQWDFW DQJOH LV GHWHUPLQHG WKDQ *f* WKH OTXLG LV VDLG WR Z HW WKH VROG LI ! *f* WKH OTXLG LV WHUP HG QRQZHWWLQJ) RU H[DPSOH Z DWHU Z HWV VRDS EXWGRHV QRW Z HW Z D[: DWHU LV H[WUHPHO Z HWWLQJ WR I FOHDQ JODVV VXUIDFH Z LWK § *f* / LNH< WKH FRQWDFW DQJOH LV VHQVLWLYH WR WKH DFWXDO SK\VLFRFKHPLFDO FRQGLWLRQV RI WKH VROG±OTXLG LQWHUIDFH) RU D FOHDQ PHUFXU\±DLU±JODVV LQWHUIDFH *f*



) LJ &RQWDFW DQJOH HIIHFW DW OTXLG±JDV±VROG LQWHUIDFH ,I *θ* *f* WKH OTXLG³ Z HWV WKH VROG LI *θ* ! *f* WKH OTXLG LV QRQZHWWLQJ

([DPSOH LOOXVWUDWHV KRZ VXUIDFH WHQVLRQ FDXVHV D OXLG LQWHUIDFH WR ULVH RU IDOO LQ D FDSLOODU\ WXEH

(



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' HULYH DQH[SUHVVLRQ IRU WKH FKDQJH LQ KHLJKW K LQ D FLUFXODU WXEH RI GLDPHWHU ' GXH WR VXUIDFH WHQVLRQ < DQG FRQWDFW DQJOH DVLQ) LJ (

6RXWLRQ

7KH YHUWLFDO FRPSRQHQW RI WKH IRUFH GXH WR VXUIDFH WHQVLRQ DFWLQJ RQ WKH OLQH RI FRQWDFW LQ WKH WXEH PXVW EDODQFH WKH ZHLJKW RI WKH FROXPQ RI OLTXLG K HLJKW K

$$\uparrow F_V = \downarrow W$$

6RONLQJ IRU K ZH KDYH WKH GHVLUHG UHVXOW

$$h = \frac{2\gamma \cos\theta}{\rho g R} \qquad \textit{Ans.}$$

7KXV WKH FDSLOODU\ KHLJKW LQFUHDVHV LQYHUVHO\ ZLWK WXEH UDGLXV 5 DQG LV SRVLWLYH ! f ZHWWLQ OLTXLG DQG QHJDWLYH FDSLOODU\ GHSUHVVLRQ LI ! f

6XSSRVH WKDW D PP 7KHQ WKH FDSLOODU\ ULVH IRU D ZDWHU±DLU±JODVV LQWHUIDFH § f < 1 P DQG! NJ P LV

$$h = \frac{2(0.073\ \text{N/m})(\cos 0^\circ)}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.001\ \text{m})} = 0.015\ (\text{N}\cdot\text{s}^2)/\text{kg} = 0.015\ \text{m} = 1.5\ \text{cm}$$

) RU D PHUFXU±DLU±JODVV LQWHUIDFH ZLWK f < 1 P DQG! NJ P WKH FDSLOODU\ ULVHLV

$$h = \frac{2(0.48)(\cos 130^\circ)}{13{,}600(9.81)(0.001)} = -0.0046\ \text{m} = -0.46\ \text{cm}$$

: KHQ D VPDOO GLDPHWHU WXEH LV XVHG WR PDNH SUHVXUH PHDVXUHPHQWV &KDS WKH HFDSLOODU\ HIHFW PXVW EH FRUUHFWHG IRU

9DSRUSUHVVXUH

9DSRUSUHVVXUHLVWKHSUHVVXUHDWZKLFKDOLTXLGEHOLYDQGLVLQHTXLOLEULXP ZLWKLWVRZQYDSRU) RUH[DPSOH WKHYDSRUSUHVVXUHRI ZDWHUDW f) LV GEI IW ZKLOHWKDWRI PHUFXU LVRQO GEI IW ,I WKHOLTXLGSUHVVXUHLVJUHDWHUWKDQWKHYDSRUSUHVVXUH WKHRQO H[FKDQJHEHWZHHQOLTXLGDQ YDSRULVHYDSRUDWRQDWWKHLQWHUIDFH ,I KRZHYHUWKHOLTXLGSUHVVXUHIDOOVEHORZ WKHYDSRUSUHVVXU YDSRUEXEEOHVEHJLQWRDSSHDULQWKHOLTXLG ,I ZDWHULVKHDWHGWR f) LWYDSRUSUHVVXUHULVHVW GEI IW DQGWKXV ZDWHUDWQRUPDODWPRVSKHULF SUHVVXUHZLOOERLO : KHQWKHOLTXLGSUHVVXUHL GURSSHGEHORZ WKHYDSRUSUHVVXUHGXHWRDIORZ SKHQRPHQRQ ZHFDOOWKHSURFHVVFDYLWDWLRQ , ZDWHULVDFFHOHUDWHGIURP UHVWWRDERXW IWV LWSUHVVXUHGURSVE\ DERXW GEI LQ RU DW 7KL FDQFDXVHFDYLWDWLRQ> @

7KHGLPHQVLRQOHVVSDUDPHWHUGHVFULELQJIORZ LQGXFHGERLOLQJ WKHFDYLWDWLRQQXPEHU

$$\mathrm{Ca} = \frac{p_a - p_v}{\frac{1}{2}\rho V^2} \qquad (1.34)$$

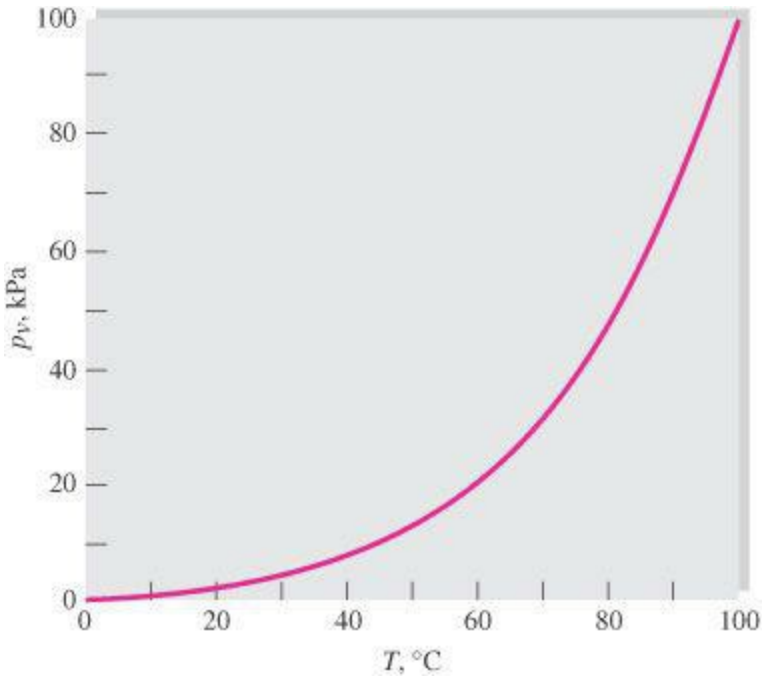
ZKHUHS$ DPELHQWSUHVVXUH

S9 YDSRUSUHVVXUH

9 FKDUDFWHULVWLFIORZ YHORFLW

! ILXLGGHQVLW

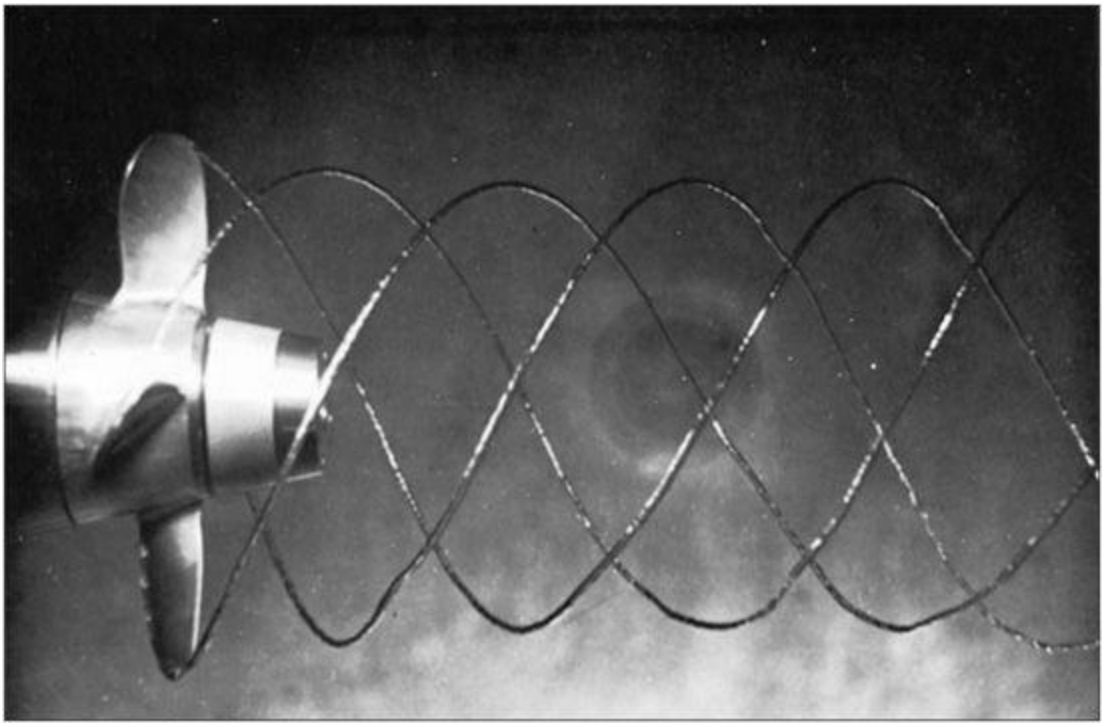
' HSHQGLQJ RQWKH JHRPHWU DJLYHQIORZ KDV DFULWFDOYDOXHRI &DEEORZ ZKLFK WKHIORZ ZL EHJLQWR FDYLWDWH 9DOXHVRI VXUFHWHQVLRQDQGYDSRUSUHVVXUHRI ZDWHUDUHJLYHQLQ7DEOH$ 7KH YDSRUSUHVVXUHRI ZDWHULVSORWWHGLQ)LJ

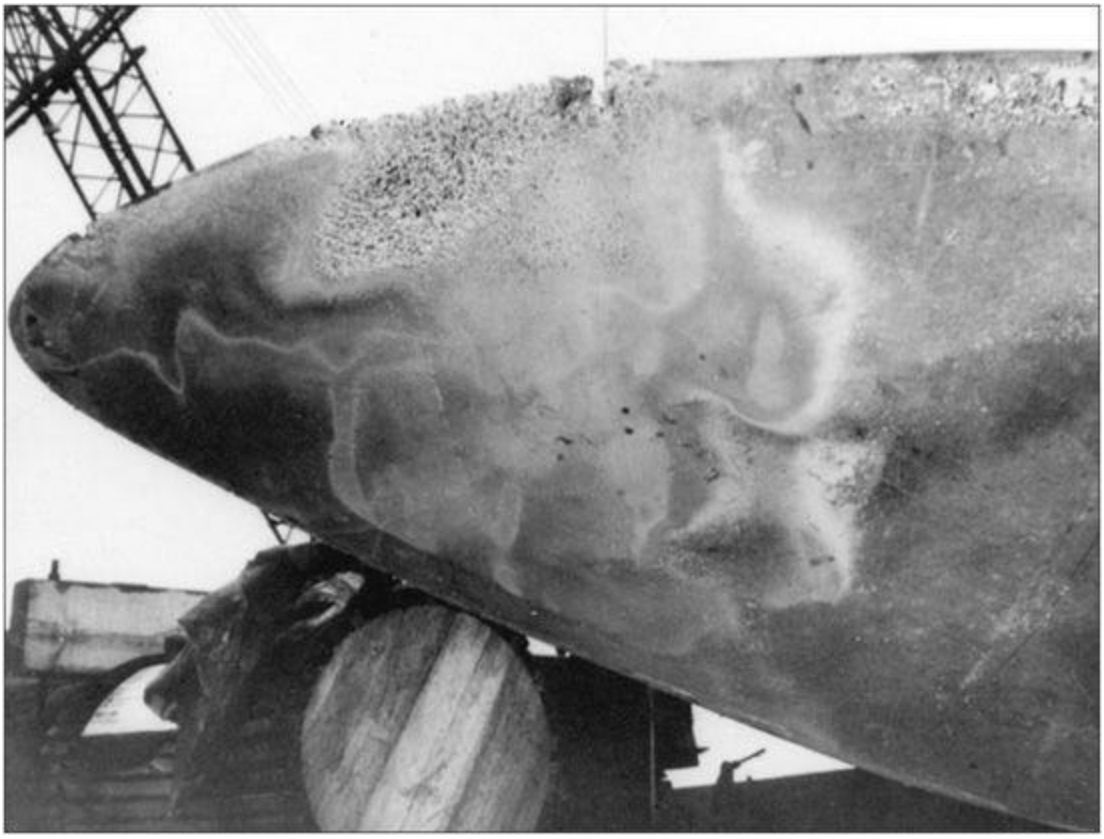


)LJ 9DSRUSUHVVXUHRI ZDWHU' DWDIURP 7DEOH$

)LJXUH DVKRZVFDYLWDWLRQEXEEOHVEHLQJIRUPHGRQWKHORZ SUHVVXUHVXUIDFHVRI DPDULQ SURSHOOHU : KHQ WKHVH EXEEOHV PRYH LQWR D KLJKHUSUHVVXUH UHJLRQ WKH FROODSVH LPSORVLYHO &DYLWDWLRQ FROODSVH FDQ UDSLGO VSDOODQG HURGH PHWDOF VXUIDFHV DQG HYHQWXDOO GHVWR\ WKHP D

VKRZ QLQ) LJ E





) LJ 7Z R DVSHFWV RI FDYLWDWLRQ EXEEOH IRUP DWLRQ LQ OTXLG IORZV *a* %HDXW VSLUDO EXEEOH VKHHW IRUP IURP WKH VXUIDFH RI D PDULQH SURSHOOHU(*courtesy of the Garfield Thomas Water Tunnel, Pennsylvania State University); b* XJOQHV FROODSVLQJ EXEEOHV HURGHD SURSHOOHU VXUIDFH(*courtesy of Thomas T. Huang, David Taylor Research Center)*

FDVHV VKHUHLV FOHDU QR VOLS DW WKH ZDOO ZKHUH WKH ZDWHU WRXFKHV WKH]HUR YHORFLW\ RI WKH IL[HG SODWH
7KH YHORFLW\ SURILOH LV PDGH YLVLEOH E\ WKH GLVFKDUJHG RI D GQH RI K\GURJHQ EXEEOHV IURP WKH ZLU
VKRZ QWXWHWKHG DFURVV WKH IORZ

) LJ 7KH QO VOLS FRQGLWLRQ LQ ZDWHU SDWWD WKLQ IL[HG SODWH 7KH XSSHU IORZ L
VXUEXOHQW WKH IORZ HU IORZ LV ODP LQDU 7KH YHORFLW\ SURILOH LV PDGH YLVLEOH E\ D QQH RI K\GURJH
EXEEOHV GLVFKDUJHG IURP WKH ZLUH DFURVV WKH IORZ *(National Committee for Fluid Mechanics*
Films, Education Development Center, Inc, © 1972)

7R GHFUHDVH WKH PDWK PDWFDOGLIILFXOW WKH QO VOLS FRQGLWLRQ LV SDUWLDOO UHDL[HG LQ WKH DQDO VL
RI LQYLVFLG IORZ &KDS 7KH IORZ LV DGDZHG WR ³VOLS´ SDVW WKH VXUIDFH EXWQRW WR SHUPHDWH WKURXJ
WKH VXUIDFH

$$V_{\text{normal}}(\text{fluid}) \equiv V_{\text{normal}}(\text{solid}) \qquad (1.36)$$

ZKLOH WKH WDQJHQWLDO YHORFLW9 $_W$ LV DGBRZ HG WR EH LQG HSHQG HQW RI WKH ZDO 7KH DQDO VLV LV PXF
VLP SOHU EXW WKH IORZ SDWWHUQ/DUHKJKO LGHDOL]HG
) RU KJK YLVFRVLW\ QHZ WRQLDQ IOXLGV WKH OQHDU YHORFLW DVXP SWLRQ DQG WKH QO VOLS FRQGLWLRQ
FDQ \LHOG VRPH VRSKLWWFDWG DSSUR[LP DWLQDO VHV IRU VZR DQG WKUHH GLP HQVLRQDO YLVFRXV IORZ V
7KH IROBRZ LQJ H[DP SOH IRUD WSHR URDWLQJ GLVNYLVFRP HWU ZLOLOXWDW

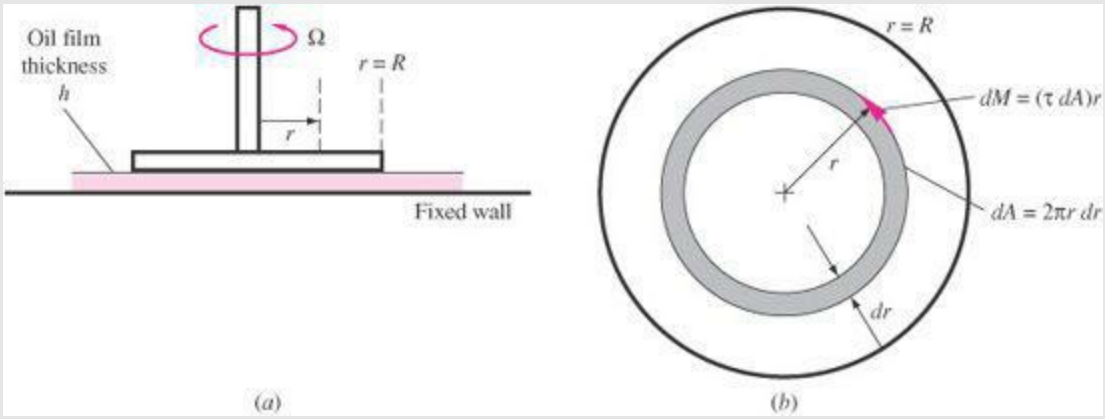
(; $0 3/ (

$ RLOILOP RI YLVFRVLW DQG WKLFNQHVWK ≪ 5 OHVEHWZHHQ D VROG ZDOO DQG DFLFXODUGLVN DV/L
) LJ (7KH GLVN LV URWDWG WKURXJH DGLO DWDQJXODU YHORFLW 9 1 RWQQ WKDWERWK YHORFLW DQG VKD

VWHHV YDU ZLWK UDGLXV U GHULYH DIRUPXOD IRU WKH WRUTXH0 UHTXLUHG WR URWDWH WKH GLVN 1 HJOHFW DQ GUDJ

6RXWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZ VDVLGH YLHZ D DQG DWRS YLHZ E RI WKH VXVWHP



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- $VVXPSWLRQV / LQHDU YHORFLW SURILOH OPLQDU IORZ QR VOS OLGH DOVK HDUVWHWV JLYHQ E\ (T
- $SSURDFK (VWDUW WKH VKHDUVWHVV RQ D FLUFXODU VWULS RI ZLGWK GU DQG DUHD G$ "UGUL) LJ (E WKH QILQG WKH PRPHQW G0 DERXW WKH RUJLQ FDXVHG E\ WKLV VKHDUVWHVV ,QWHJUDWH RYHU WKH HQWLUH GLVN WR ILQG WKH WRWDO PRPHQW0
- 3URSHUWLHV &RQVWDQW OLVFRVLW ,QWKLV VWHS IORZ RLOGHQVLW VLQRVWHOYDQW
- 6RXWRQVWHSV $VWDUW XVH WKH YHORFLW LQWHRLOVWDQGHQW DO YDULQJ IURP]HUR DW WKH ILL[H ZDOO QR VOS VRX 9UDWKHGLVN VXUIDFH DOR QR VOS 7KHVKHDUVWHVV DWKLVSRVLWRQ LV WKX

$$\tau = \mu \frac{du}{dy} \approx \mu \frac{\Omega r}{h}$$

7KLV VKHDU VWHVV LV HYHUZKHUH SHUSHQGLFXODU WR WKH UDGLXV IURP WKH RULJLQ VHH) LJ (E 7KHQ WKH HOOPRPHQWDERXWWKHGLVN RULJLQ FDXVHGE\ VKHDULQJ WKLV FLUFXODUVWULS FD EHIRXQGDQGLQWHJUDWHG

$$dM = (\tau)(dA)r = \left(\frac{\mu\Omega r}{h}\right)(2\pi r\, dr)r, \quad M = \int dM = \frac{2\pi\mu\Omega}{h}\int_0^R r^3 dr = \frac{\pi\mu\Omega R^4}{2h} \quad Ans.$$

- &RPPHQWV 7KLVLVDVLPSOLILHGHQJLQHHULQJ DQDOVLV ZKLFK QHJOHFWVSRVLEOHHGJHHIHFWV DUGUDJ RQWKHWRS RI WKHGLVN DQGWKHVXUEXOHQFHWKDWPLJKWHQWHU WKHGLVNURXJKVWRR IDVW

6OLS)ORZ LQ* DVHV

7KH³IUHHVOS´ERXQGDU FRQGLWRQ(T LVDQXQUHDOLVWF PDWKHPDWDOODUWLFHWRHQDEO

)LJ * HRPHWULF UHODWLRQVIRUGHILQLQJ DVWHDPOLQH

,I WKHYHORFLWLHV X Z DUHNQRZQIXQFWLRQVRI SRVLWLRQDQGWLPH (T FDQEHLQWHJUDWHGV
ILQGWKHVWUHDPOLQHSDWWLQJWKURXJKWKHLQLWLDOSRLQW[\ ] W 7KHPHWKRGLVVWUDLJKWIRZDUGIR
VWHDG\IORZV ([DPSOH EXWPD\EHGDQJHURXVIRUXQVWHDG\IORZ
7KHSDWKOLQH RUGLVSODFHPHQWRI DSDUWLFOHLVGHILQHGE\ LQWHJUDWLQRQ WKHYHORFLWLHVFRPSRQHQW

$$\text{Pathline:} \qquad x = \int u\,dt \qquad y = \int v\,dt \qquad z = \int w\,dt \tag{1.40}$$

* LYHQ X Z DVNQRZQIXQFWLRQVRI SRVLWLRQDQGWLPH WKHLQWHJUDWLQRQLVEHJXQDWD VSHFLILHG
LQLWLDOSRVLWLRQ [\ ] W $JDLQWKHLQWHJUDWLRQPD\EHGDQJHURXV
6WHDG\IORZV HDVLO JHQHUDWHG H[SHULPHQWDOO ZLWK VPRNH GH RU EXEEOHU HDVHV DUH YHU
GLIILFXOWRFRPSXWDQDOWLFDOO 6HH5H IRUPDWKHPDWLFDOGHWDLOV

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* LYHQWKHVWUHDG\ VZRGLPHQVLRQDOYHORFLWGLVWULEXWLRQ

$$u = Kx \qquad v = -Ky \qquad w = 0 \tag{1}$$

ZKHUH. LVDSRVLWLYHFRQVWDQWVFRPSXWHDQGSORWWKHVWUHDPOLQHVRI WKHIORZ LQFOXGLQJGLUHFWLRQ
DQGJLYHVRPHSRVLWLEHLQWLSUHDWLRQVRI WKHSDWWHUQ

6ROXWLRQ

6LQFH WLPH GRHV QRWDSSHDU H[SOLFLWO LQ(T WKH PRWLRQ LV VWHDG\ VR WKDWVWUHDPOLQHV

SDUWLDO GHULYDWLYHV DQG VXEVWLWXWLQJ LQWR WKH PRUH JHQHUDO HTXDWLRQ WKH H[SODQH 7KH VXEVWLWXWLRQ FDQ EH FRPSXWHG E\ VXEWUDFWLQJ WKH H[SUHVVLRQV IRU DQG LQWR(T

$$\frac{dx}{Kx} = -\frac{dy}{Ky}$$

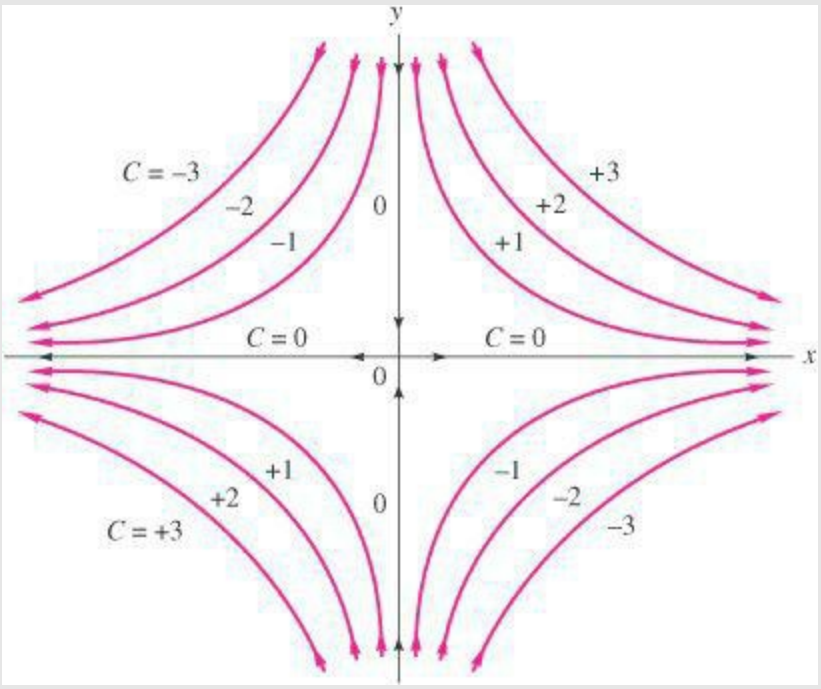
or

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

,QWHJUDWLQJ ZHREWDLQOQ[íOQ\ OQ& RU

$$xy = C \qquad \textit{Ans. (2)}$$

7KLV LV WKH JHQHUDO[SUHVVLRQ IRU WKH VWUHDPOLQHV ZKLFK DUH K\SHUERODV 7KH FRPSOHWH SDWWHUQ LV SORWWHG LQ) LJ (E\ DVVLJQLQJ YDULRXV YDOXHV WR WKH FRQVWDQW& 7KH DUURZ KHDGV FDQ EH GHWHUPLQHG RQO E\ UHDOLQJ WR(T WR DVFHUWDLQ WKH YHORFLW\ FRPSRQHQWV DW[PLQJ . L SRVLWLYH) RU[DP SOH LQ WKH XSSHU ULJKW TXDGUDQW[! \ ! XLV SRVLWLYH DQG LV QHJDWLYH KHQF WKH IORZ PRYHV GRZQ DQG WR WKH ULJKW WHUWXEDWLQJ WKH DUURZ KHDGV DV VKRZ Q



) LJ (6WUHDPOLQHV IRU WKH YHORFLW\ GLVWULEXWLRQ JLYHQ E\ (T IRU K !

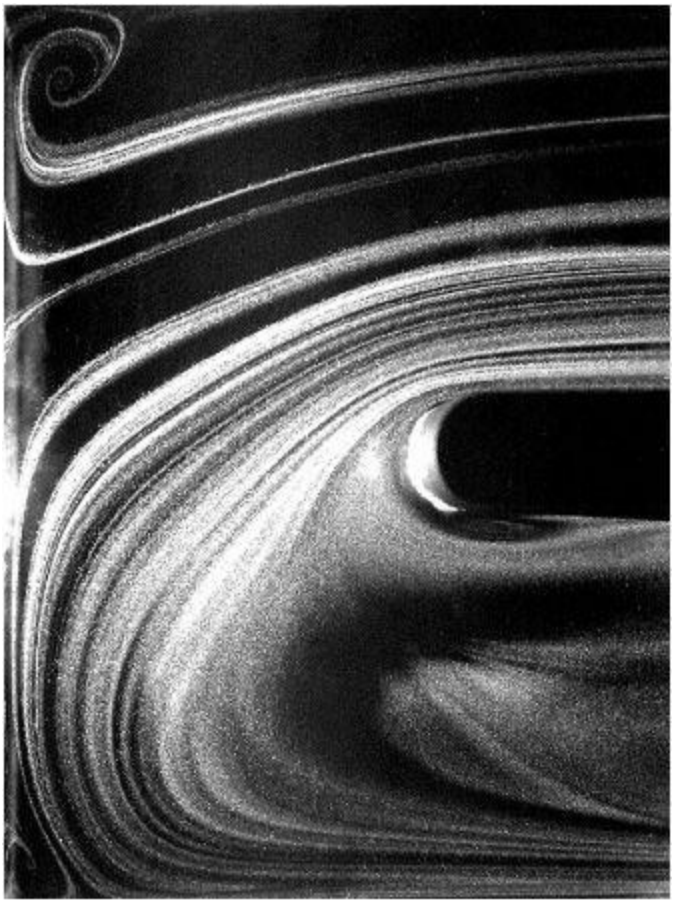
1 RWH WKDW WKH VWUHDPOLQH SDWWHUQ LV HQWLUHO LQGHSHQGHQW RI FRQVWDQW ,W FRXOG UHSUHVHQW WKH LPSLQJHPHQW RI VWR RSSRVLQJ VWUHDPV RU WKH XSSHU KDOI FRXOG VLPXODWH WKH IORZ RI D VLQJO GRZQ QZ DUG VWUHDP DJDLQVW D IODW ZDOO 7DNHQ LQ LVRODWLRQ WKH XSSHU ULJKW TXDGUDQW LV VLPLODU WR WK IORZ LQD *ORZ QQ[ƒ FRUQHU 7KLV LVGHILQLWHO DUHDOXWF IORZ SDWWHUQ DQG LV GLVFXVVHG DJDLQ LQ&KDS)LQDOO QRWH WKH SHFXOLDULW WKDW WKH VWUHDPOLQHV & KDYH RSSRVLWH GLUHFWLRQ DQ LQWHUVHFW 7KLV LV SRVVLEOH RQO DV D SRLQW ZKHUH Y Z ZKLFK RFFXUV DW WKH RULJLQ LQ WKLV FDVH 6XFK D SRLQW]HUR YHORFLW LV FDOOHG D VWDJQDWLRQ SRLQW

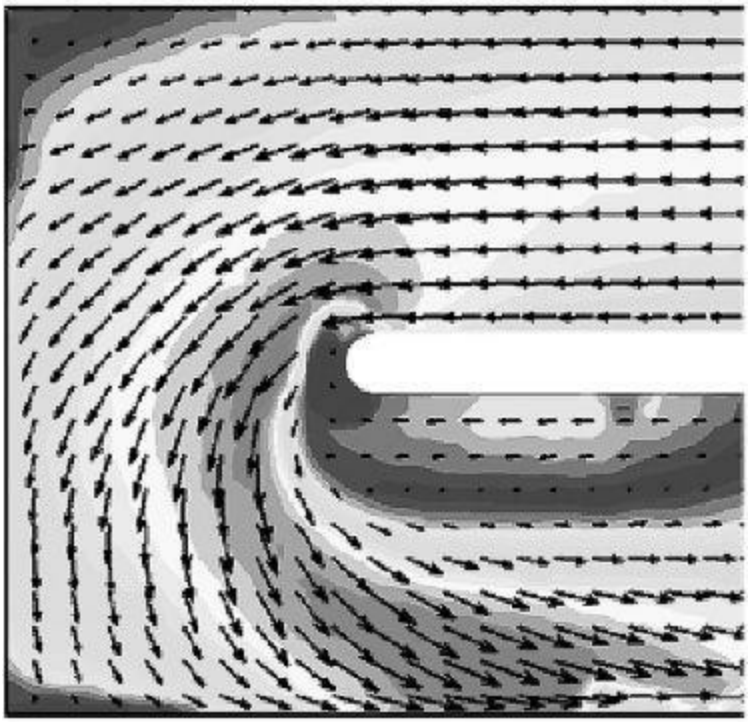
) ORZ 9 LVXDOL]DWLRQ

&OHYHU[SHULPHQWDWRQFDQSURGXFHUHYHDOQJ LPDJHVRI DIOXLGIORZ SDWWHUQ DV VKRZ QHDUDHUL
) LJV D DQG) RUH[DPSOH VWUHDNOLQHV DUH SURGXFHG E\ WKH FRQWRXV UHDVHR PDUNH
SDUWFOHV G\H VPRNH RUEXEEOHV IURP DJLYHQSRLQW,I WKHIORZ LV VWHDG\ WKH VWUHDNOLQHV ZLOOEH
LGHQWFDOWR WKH VWUHDPOLQHVDQG SDWKOLQHVRI WKHIORZ
6RPHPHWKRGVRI IORZ YLVXDOL]DWLRQLQFOXGHWKHIROORZLQJ > ± @

' \H VPRNH RUEXEEOHGLVFKDUJHV
6XUIDFHSRZ GHURUIODNHVRQOLTXLG IORZV
) ORDWLQJ RUQHXWUDOGHQVLW SDUWFOHV
2 SWFDOWHFKQLTXHV WKDWGHWFWGHQVLW FKDQJHV LQ JDV IORZV VKDGRZ JUDSK VFKOLHUHQ DQ
LQWHUIHURPHWU
7XIWRI \DUQDWDFKHGVRERXQGDU VXUIDFHV
(YDSRUDWLYHFRDWLQJVRQERXQGDU VXUIDFHV
/ XPLQHVFHQWOLGV DGGLWLYHV RUELRXPLQHVFHQFH
3DUWFOHLP DJHYHORFLP HW 3,9

) LJXUHV D DQG Z HUHERWKYLVXDOL]HG E\ EXEEOHUHDVHV $ QRWKHU[DPSOHLV WKHXVHR
SDUWFOHVLQ) LJ VRYLVXDOL]HDIORZ QHJRWDWLQJ D ƒ VXUQLQDVHSHQWHFKQLTXH> @





) LJ 7ZR YLVXDOL]DWLRQV RI IORZ PDNLQJ D *f* VWXQLQD VHSDUDWHG *a* SDUWLF
VWUHDNOLQHV DVD 5H\QROGV QXPEHU RI *b* WLPH PHDQ SDUWLFOH LP DJH YHORFLP HWU 3,9 DV
VXUEXOHQW 5H\QROGV QXPEHU RI ***From Ref. 42, by permission of the American Society of***
Mechanical Engineers

) LJXUH DLVDWDORZ ODPLQDU 5H\QROGV QXPEHU 7KH IORZ LV VWDG\ DQG WKH SDUWFO
IRUP VWUHDNOLQHV VKRZ LQJ WKDWWKH IORZ FDQQRW PDNH WKH VKDUS WXUQ ZLWKRXWSHUDWLQJ DZD\ IURP WK
EORWWR ZDOO

) LJXUH E LVDWDKJKHU WXUEXOHQW5H\QROGV QXPEHU 7KH IORZ LV XQVWDG\ DQG WK
VWUHDNOLQHV ZRXOG EH FKDRWF DQG VPHDUHG XQVXLWDEOH IRUYLVXDOL]DWLRQ 7KHIMDJH LV WKXV SURGXFH
E\ WKHQHZ WHFKQLTXH RI SDUWFOHIMDJHYHORFLP HWU > @,Q3,9 KXQGUHGV RI SDUWFOHV DUH WDJHG DQ
SKRWRJUDSKHG DWWZR FORVH OSDFHG WLP HV 3DUWFOHPRYHPHQWV WKXV LQGLFDWH YHORFLW YHFWRU
7KHVH KXQGUHGV RI YHFWRU V DUH WKHQ VPRRWKHG E\ UHSHDWHG FRPSXWHUSHUDWRQV XQWLOWKH WP HPHD
IORZ SDWWHUQ LQ) LJ E LV DFKLHYHG 0RGHUQ IORZ H[SHULPHQWV DQG QXPHULFDO PRGHOV XV
FRPSXWHUVH[WQVLYHO WR FUHDWWKHUYLVXDOL]DWRQV DVGHVFUEHGLQWKHWH[WE\ <DQJ > @

0 DWKHPDWFDOGHVLOVRI VWUHDPOQHVWUHDNOLQHSDWKOQHDQDOVLV DUH JLYHQLQ5H 5HUHQFH
± DUH EHDXWXO DEXPV RI IORZ SKRWRJUDSKV 5HUHQFHV ± DUH PRQRJUDSKV RQ IORZ
YLVXDOL]DWRQWFKQLTXHV

) OLGPHFKDQFVLVDP DUYHORXVVXEVFFWRUYLVXDOL]DWRQ QRWXWWRUVWRU VWDG\ SDWWHUQV EX
DOOR IRU PRYLQJ XQVWDG\ PRWRQ VXGLHV \$Q RXWDQGLQJ OLWWRI DYDLODEOH IORZ PRYLHV DQ
YLGHRVDSHVLVJLYHQE\ &UUDQG<RXQJ > @

7KH(QJLQHHULQJ (TXDWLRQ6ROYHU

0 RWWRI WKH H[DPSOHV DQG H[HUFLVHV LQ WKLV WH[WDUH DPHQDEOH WR GLUHFWFDOFXODWRQ Z LWKRX
JXHVVLQJ RULWHUDWRQRUORRSLQJ 8QIRUWXQDWHOQ RQO VXFK GLUHFWSUREOP DWLJQP HQW Z KHWKHU³SOXJ
DQG FKXJ´ RUPRUH VXEWOH Z HUH DSSURSULDWH IRU XQGHUJUDGXDWH HQJLQHHULQJ FRXUVHV +RZ HYHU WK

LQWRGXFWRQ RI FRPSXWHU VRIWZDUH VROYHUV PDNHV DOPRVWDQ VHWRI DJHEUDLF UHDWLRQV YLDEOH IR
DQDOVLV DQG VROXWLRQ 7KH VROYHU UHFRPPHQGHG KHUH LV WKH(QJLQHHULQJ (TXDWLRQ 6ROYHU ((6
GHYHORSHGE\ . OHLQDQG%HFNPDQ> @

$Q VRIWZDUH VROYHUVKRXOGKDQGOHDSXUH0 PDWHPDWLFDOVHWRI UHDWLRQV VXFKDVWKHRQHSRVH
LQ5H ; QQ ; < ; < 6XEPLWWHGDSDLUVRDQ FRPPHUFLDOVROYHUDQG\RXZLOOQRGRXE
UHFHLYHWKHDQZHU; < +RZHYHU IRUHQJLQHHUV LQWKHDXWRPRWLYHQ((6 L
VXSHURU VR PRWVROYHUV EHFDXVH HTXDWLRQV FDQ EH HQWHUHG LQ DQ RUGHU VFRUHV R
PDWHPDWLFDOIRUPXODVDUHEXLOWLQ VXFKDVWKH%HWWHOIXQFWLRQV DQG WKHUPRSK\VLFDOSURSHUWLHV
PDQ IOXLGVDUHEXLOWLQ VXFKDVWKHVWDQGDUGVDO%6,& RU)25775$1 VWWOH)RUH[DPSOH; < LVSHUFHQW VDWXUDWHGWKHUHLVRQHGURSOHW)RUH[DPSOH UHFRQVLGHU([DPSOH DVDQ((6 H[HUFLVH 2QHZRXOGILUVWHQWHUWKHUHHUHQF
SURSHUWLHVS DQG! SOXVWKHFXUYHILWFRQVWDQWV%DQGQ

$$Pz = 1.0$$

$$Rhoz = 2.0$$

$$B = 3000$$

$$n = 7$$

7KHQVSHFLI\ WKHJLYHQSUHVVXUHUDWLR DQGWKHFXUYHILWUHDWLRQ(T IRUWKHHTXDWLRQR
VWDWHRI ZDWHU

$$P = 1100.Pz$$

$$P/Pz = (B + 1)*(Rho/Rhoz)^n - B$$

,I \RX UHTXHVWDQ LQWHUSRODWLRQIURP WKH&+(&.)250$7 PHQX ((6 VWDWHVWKDWWKHUHDUHV
HTXDWLRQVLQVL[XQNQRZQVDQGWKHUHQR REYLRXVGLIILFXOWLHV 7KHQUHTXHVW62/9(IURP WKHPHQ
DQG((6 TXLFNO SURPSWRXW5KR WKHFRUUHFWDQZHUDVHHQDOHDG LQ([DPSOH ,WDOVR
SURPSWRXWYDOXHV RI WKHRWKHUILYH YDULDEOHV 2FFDVLRQDOO ((6 UHSRUW³XQDEOHWR FRQYHUJHí DQ
WDWHWZKDWZHQWZURQJ GLYLVLRQE\ ]HUR VTXDUHURRWRI DQHJDWLYHQXPEHU 2QHQHGVRQO WR
LPSURYHWKHJXHVVDQGUDQJHVRI WKHXQNQRZQVLQ9DULDEOH,QRUPDWRQWRDWLW((6 WRWKHVROXWLRQ
,QVXEVHTXHQWFKDSWHUVZHZLOOLQWURGXFHPHWKRGVWRVROYH H[DPSOHVE\ XVLQJ ((6 DQ
ZLOOGRDOORDWLJQVRPHDGYDQFHGSUREOP H[HUFLVHVIRUZKLFK((6LVDQLGHDOSSURDFK 7KHXVHRI D
HQJLQHHULQJ VROYHU QRWDEO ((6 LV UHFRPPHQGHG WR DOO HQJLQHHUV LQ WKLV HUD RI WKH SHUVRQ
FRPSXWHU,I ((6 LVQRWDYDLODEOH WKHZULWWUHFRPPHQGVXVLQJ DQ([FHOVSUHDGVKHHW

8QFHUWDLQWLHV LQ ([SHULPHQWDO DWD

8QFHUWDLQW\ LV D IDFWRU GXH LQ HQJLQHHULQJ : H UDUHO NQRZ DQ HQJLQHHULQJ SURSHUWLHV R YDULDEOHV VR DQH[WHPHGHUH RI DFFXUDF\ 7KHXQFHUWDLQWLHV RI GDWD LV QRUPDOO GHILQHG DV WKHEDQ ZLWKLQ ZKLFK RQH LV SHUFHQWFRQILGHQWWKDWWKH WUXH YDOXH OLHV 5HFDOO IURP) LJ WKDWWKH XQFHUWDLQW\ RI WKH UDLR F ZDV HVWLPDWHG DV" SHUFHQW7KHUH DUH ZKROH PRQRJUDSKV GHYRWHG WR WKH VXEMHFW RI H[SHULPHQWDO XQFHUWDLQWLHV> ± @VR ZH JLYH RQO DEULH VXPPDU KHUH

$OOH[SHULPHQWDOGDWD KDYH XQFHUWDLQW\ VHSDUDWHG LQWR WZR FDXVHV DV WKH LQWUXPHQWDO XQFHUWDLQW\ DQG WKH UDQGRP HUURUGXH WR VFDWWULQ UHSHDWHG UHDGLQJV : H PLQLPL]H WKH V\VWHPDWLF HUURU E\ FDUHIXOFDOLEUDWRQ DQG WKHQHXWUDOL WKH UDQGRP HUURU VDWXUDWLQJ 7KHMXGJPHQW WKH H[SHULPHQWDUL FUXFLDOLP SRUWDQFH

+HUH LV WKH DFFHSWHG PDWKHPDWLFDOHXUH 6XSSRVH D GHSHQGV XSRQ D VLQJO H[SHULPHQWDO YDULDEOH[,I [KDV DQ XQFHUWDLQW\/[WKHQ WKH XQFHUWDLQW\/3 LV HXDO IURP WK FDOFXOXV

$$\delta P \approx \frac{\partial P}{\partial x}\delta x$$

,I WKHUH DUH PXOWSOH YDULDEOHV 3 3 [[[« [1 WKHRYHUDOO XQFHUWDLQW\/3 LV FDOFXODWG D DURRWPHDQVTXDUH HXDWLRQ> @

$$\delta P = \left[\left(\frac{\partial P}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial P}{\partial x_2}\delta x_2\right)^2 + \cdots + \left(\frac{\partial P}{\partial x_N}\delta x_N\right)^2\right]^{1/2} \qquad (1.41)$$

7KLV FDOFXODWRQ LV VWDLJKWIRUZDUG PXFK PRUH SUREDEOH WKDQ VLPSO DGGLQJ OQHDUO WKH YDULRX XQFHUWDLQWLHV/[L WKHUE\ PDNLQJ WKH XQDN DVVXPSWRQ WKDWDOO YDULDEOHV VLPXOWDQHRXVO DWWDL PD[LPXP HUURU 1RWWKDWWKLV WKH UHVSRQVLELOLW RI WKH H[SHULPHQWUVR HXDEOLK DQG UHSRUWDFFXUDW HXDWRI DOOWKHUHOHYDQWXQFHUWDLQWLHV/[L

,I WKH TXDQWW3 LV D VLPSOH SRZHU H[SUHVVLRQ RI WKH RWKHU YDULDEOHV IRU H[DPSOH $x_1^{n_1}x_2^{n_2}x_3^{n_3}\ldots$, WKHQ HDFK GHULYDWLYH LQ(T LV SURSRUWRQDO WR3 DQG WKH UHOHYDQWSRZHUH[SRQHQWDQG LVIHUHQW SURSRUWRQDOWR WKDWYDULDEOH

,I$x_1^{n_1}x_2^{n_2}x_3^{n_3}\ldots$, WKHQ

$$\frac{\partial P}{\partial x_1} = \frac{n_1 P}{x_1}, \quad \frac{\partial P}{\partial x_2} = \frac{n_2 P}{x_2}, \quad \frac{\partial P}{\partial x_3} = \frac{n_3 P}{x_3}, \cdots$$

7KXV IURP (T

$$\frac{\delta P}{P} = \left[\left(n_1\frac{\delta x_1}{x_1}\right)^2 + \left(n_2\frac{\delta x_2}{x_2}\right)^2 + \left(n_3\frac{\delta x_3}{x_3}\right)^2 + \cdots\right]^{1/2} \qquad (1.42)$$

(YDOXDWRQ RI /3 LV WKHQD VWUDLJKWRUZ DUG SURFHGXUH DV LQWKHIRORZLQJ H[DPSOH

(; $0 3/ (

7KH VR FDOOHG GLPHQVLRQOHVV 0 RRG\ SLSH IULFWLRQ IDFWRU SORWWHG LQ) LJ LV FDOFXODWHG H[SHULPHQWDOO IURP WKH IROORZLQJ SLSH GLPHWHU' SUHVVXUH GURS ûS GHQVLW! YROXPHIORZ UDWH4 DQG SLSHOHQJWK/

$$f = \frac{\pi^2}{8}\frac{D^5\Delta p}{\rho Q^2 L}$$

0 HDVXUHPHQWXQFHUWDLQWLHVDUHJLYHQIRUDFHUWDLQH[SHULPHQW SHUFHQWûS SHUFHQ ! SHUFHQW4 SHUFHQWDQG/ SHUFHQW(VWLPDWHWKHRYHUDOOXQFHUWDLQWRI WKHIULFWLRQ IDFWRUI

6RXWLRQ

7KH FRHIILFLHQWE LVDVVXPHGWREHDSXUHWKHRUHWLFDOQXPEHU Z LWKQR XQFHUWDLQW 7KHRVWH YDULDEOHVPD\ EHFRQVLGHUHGXQLQ(TV DQG

$$U = \frac{\delta f}{f} = \left[\left(5\frac{\delta D}{D}\right)^2 + \left(1\frac{\delta \Delta p}{\Delta p}\right)^2 + \left(1\frac{\delta \rho}{\rho}\right)^2 + \left(2\frac{\delta Q}{Q}\right)^2 + \left(1\frac{\delta L}{L}\right)^2\right]^{1/2}$$
$$= [\{5(0.5\%)\}^2 + (2.0\%)^2 + (1.0\%)^2 + \{2(3.5\%)\}^2 + (0.4\%)^2]^{1/2} \approx 7.8\% \quad Ans.$$

%\ IDUWKHGRPLQDQWHIHFWLQ WKLVSDUWFXOUFDOFXODWRQLV WKH SHUFHQWHUURULQ4 ZKLFK LV DPSOLILHG E\ GRXEOLQJ GXH WR WKH SRZHUR RQ IGRZ UDWH 7KHGLDPHWU XQFHUWDLQW ZKLFK LV TXLQXSOHG ZRXOGKDYHFRQWULEXWHGPRUHKDG/' EHHQDJHUWKDQ SHUFHQW

7KH) XQGDPHQWDOVRI (QJLQHHULQJ) (([DPLQDWLRQ

7KH URDG WRZDUG D SURIHVVLRQDO HQJLQHHU\ OFHQVH KDV D ILUVWVWRS WKH) XQGDPHQWDOV R (QJLQHHULQJ ([DPLQDWLRQ NQRZ QDV WKH) (H[DP ,WZ DVIRUPHU0 NQRZ QDV WKH(QJLQHHULQ 7UDLQL (, 7 ([DPLQDWLRQ 7KLVHJKWKRXUQDWLRQDOWHVWZ LOOSUEDE0 VRQEHUHTXLUHGR DOOHQJLQHULQ JUDGXDWHV QRWWXWWRUOFHQXUH EXWDVDVWXGHQWDVVHVVPHQWWRO 7KH SUREOHP IRXUKRXUPRUQLQ VHVVLRQFRYHUVPDQ JHQHUDOVXEGLHV/

- 0 DWKHPDWLFV²
- (QJLQHHULQJ SUREDELOLWDQG VWDWLVWLFV²
- &KHPLVWU²
- &RPSXWHUV²
- (WKLFVDQGEXVLQHVVSUDFWLFH²
- (QJLQHHULQJ HFRQRPLFV²
- (QJLQHHULQJ PHFKDQLFV²

- 6\VWHQWKRI PDWHULDOV²
- 0 DWHULDOSURSHUWLHV²
-) OXLGPHFKDQLFV²
- (OHFWULFLWDQGPDJQHWLVP²
- 7KHUPRG\QDPLFV²

) RUWKH SUREOHP IRXUKRXUDWWHQGRQVHVVLRQ\RXPD\ FKRRVHRQHRI VHYHQPRGXOHV FKHPLFDO FLYLO HOHFWULFDO HQYLURQPHQWDO LQGXVWULDO PHFKDQLFDO DQG RWKHUJHQHUDOHQJLQHHULQJ 1 RWHWKDWVXO PHFKDQLFV LV DQ LQWURGXFWRU\ SRSF RI WKH H[DPLQDWLRQ 7KHUHIRUH SUDFWLFH VWXGHQWV DQXPEHU RI HQG RI FKDSWU) (SUREOHPV ZKHUHDSSURSULDWH

7KH IRUPDWRUWKH) (H[DP TXHVWLRQV LV PXOWLSOH FKRLFH XVXDOO ZLWK ILYH VHOHFWLRQV FKRVH FDUHXOO VR WKSWPSWRX ZLWKSDXVLEOH DQZHUV LI \RX XVHG LQFRUHFWXQLW IRUJRWWR GRXEOH RUKDOI VRPHWLQJ DUH PLVLQJ D IDFWRUR CE RUWKH QNH ,Q VRPH FDVHV WKH VHOHFWLRQV DUH XQLQWQWRQDO DP EJXRXV VXFK DVWKHIRORZLQJ H[DPSOHIURP DSUHYLRXV H[DP

7UDQVLWRQIURP ODP LQDUWRWXEXOQWORZ RFFXUVDWD5H\QROGVQXPEHURI
$ % & ' (

7KH³FRUUHFW DQZHU ZDV JUDGHG DV ' 5H &OHDU WKH H[DPLQHUZDVWLQJ EX IRUJRWWR VSHFLI\ 5H IRUIORZ LQ D VPRRWK FLUFXODUSLSH VLQH VHH&KDSV DQG WDQWLWRQ KJKO GHSHQGHQWRQJHRPHWV VXUDFHURXJKQHVV DQG WKHOHQJWK VFDOHXVHG LQWKHGHILQLWRQ RI 5H 7KHPRUDOLV QRWWR JHWSHYLK DERXWKH H[DP EXWLPSO WR JR ZLWKWKHIORZ SXQLQWQG DQ GHFLGHZKFK DQZHUEHWWLWDQXQGHUJUDGXDWWDQQJ VLXDWRQ (YHU HIRUWKDVEHHQPDGHWRNHH VKH) (H[DP TXHVWLRQVLQWKLV WXQDP ELJXRXV

3UREOHPV

0 RVWRI WKHSUREOHPVKHUHQDUHIDLO VWDLJKWRUZDG 0 RUHGLIILFXOWURSHQHGHGDWLJQPHQ

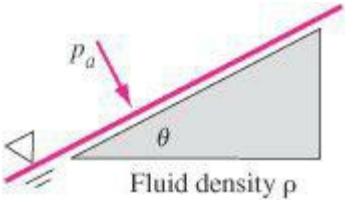
DUHODEHOHGZLWKDQDXWKRUVLQ3URE 3UREOHPVODEHOHGZLWKDQ((6 LFRQ IRUH[DPSO 3URE ZLOOEHQHLWURP WKHXVHRI WKH(QJLQHHULQJ (TXDWRQ6RQYHU ((6 ZKLOHSUREOP ODEHOHGZLWKDFRPSXWHUPD\ UHTXLUHWKHXVHRI DFRPSXWU 7KHVWDQGDUGHQGRI FKDSWU SUREOHPV WR FDWRUL]HGLQWKHSUREOP OLVWEHORZ DUHIROORZHGE\IXQGDPHQWDOV R HQJLQHHULQJ)(H[DP SUREOHPV)(WR)(DQGFRPSUHKHQVLYHSUREOPV& WR&

3UREOHP ' LVWXEXWRQ

Section	Topic	Problems
1.1, 1.4, 1.5	Fluid continuum concept	1.1–1.4
1.6	Dimensions and units	1.5–1.23
1.8	Thermodynamic properties	1.24–1.37
1.9	Viscosity, no-slip condition	1.38–1.61
1.9	Surface tension	1.62–1.71
1.9	Vapor pressure; cavitation	1.72–1.74
1.9	Speed of sound, Mach number	1.75–1.80
1.11	Streamlines	1.81–1.83
1.2	History of fluid mechanics	1.84–1.85*a–n*
1.13	Experimental uncertainty	1.86–1.90

7KHFRQFHSWRI DIOXLG

- 3 $ JDV DW *f*& PD\ EH FRQVLGHUHG UDUHILHG GHYLDWQJ IURP WKH FRQWLQXXP FRQFHSW ZKHQLWFRQGLQV/HVWKDQ PROHFXOHVSHUFXELF PLOOLPHWU,I $YRJDGUR¶VQXPEHU LV (PROHFXOHVSHUPROH ZKDWDEVROXWSUHVXUH LQ3D IRUDLUGHVWKUHSUHVHQW
- 3 7DEOH$ GLVWVWKHGHQLW RI WKH VDQGDUG DWRSKHUHDV DIXQFWRQ RI DOWWXGH 8V WKHVHYDOXHVWRHVWLPDWH FUKGH2 VD\ ZLWKLQDIDFWRURI ² WKHQXPEHURI PROHFXOH RI DLULQWKHHQWHDWRSKHUHRI WKHHDUW
- 3)RUWKHWDQJXODUHOHPHQW) LJ 3 VKRZ WKDWDWORMG IUHHGTXLG VXUDFH L FRQWDFWZLWKDQDWRSKHUHDWSUHVXUHS PXVWXQGHJRVKDUWHVDQGKHQFHEHJL WRIORZ +LQW$FFRXQWRUWKHZHJKWRI WKHIOLGDQGVKRZ WKDWDQR VKDUFRQGLWR ZLOOFDXVHKRU]RQWDOIRUFHVWREHRXWRI EDODQFH



3

- 3 6DQG DQGRWKHUJUDQXODUPDWHULDOV DSSHDUWRIORZ WKDWLV \RX FDQSRXUWKHP IURP I FRQWDLQHU RU D KRSSHU 7KHUHDUH ZKROH WH[WERRNV RQ WKH ³WDQJSRUW RI JUDQXOD PDWHULDOV> @7KHUHRUH LVVDQGDIXLG'([SODLQ

' LP HQVLRQVDQG XQLWV

- 3 7KHPHDQ IUHH SDWK RI D JDV O LV GHILQHG DV WKH DYHUDJH GLVWDQFH WDYHOHG E PROHFXOHVEHWZHHQFRQVHFXWLYH $ SURSRVHGIRUPXODIRUHVWLPDWQJ DQLGHDJDVLV

$$l = 1.26\frac{\mu}{\rho\sqrt{RT}}$$

: KDWDUHWKHGLPHQVLRQVRI WKHFRQVWDQW " 8VHWKHIRUPXODWRHVWLPDWWKHPHDQIUH SDWKRI DLUDW *f*& DQG N3D : RXOG\RX FRQVLGHUDLUUDUHILHGDWKLVFRQGLWRQ'
- 3 7KH6DERW8QYHUVDO9LVFRPHWU QRZ RXGDWG EXWWXOOVRGLQVFLHQWLF FDOOBRJV PHDVXUHV WKH NLQHPDWF YLVFRVLW RI OXEULFDQWV >5H S @$ \SHFLDO]H

FRQVLGHUHG DV D IXQFWLRQ RI SHUFHQW 0 HDVXUHPHQW
IRU PRUH VHFRQGV IURP DVP DOKROHRVKRWXEHLQVWHERWP 7KLVWP
XQLWFDOHG6DVERWKQYHUVLRQVFRQGV RU68 6 LVFRUHDVG ZLWKNLQHPDWLF YLVFRVL
LQFHQWLVWRNHV VWRNH FP V E\ WKHIROORZLQJ FXUYHILWRUPXOD

$$\nu = 0.215t - \frac{145}{t} \qquad \text{for} \quad 40 < t < 100\ \text{SUS}$$

D &RPPHQWRQ WKH GLPHQVLRQDOLW RI WKLV HTXDWRQ E ,V WKH IRUPXOD SK\VLFDO
FRUUHFW F 6LQFH YDULHVVWRQJO ZLWKWPSHUDXUH KRZ GRHVWPSHUDXUHHQWULQRWK
IRUPXOD' G &DQZHHDVLO FRQYHUW IURP FHQWLVWRNHVVRPP V"

- 3 &RQVHUWWKHIROORZLQJ LQDSSURSULDWHTXDQWLWHVLQRW 6, XQLW D DYHORFLWRI
\DUGVSHUKRXU E DYROXPHIORZ UDWRI DFUHIHHWRI ZDWUSHUZHHN DQG F
WKHPDVVIORZ UDWRI JDOORQVSHUGD\ RI 6\$(: RLODW *&
- 3 6XSSRVHZHNQRZ OLWWHDERXWKHVWHQWKRI PDWULDOVEXWDUHVRGWKDWKHEHQGLQ
VWHVV1 LQDEHDP LVSURSRUWLRQDOWR WKHEHDP KDOWKLFNHVV\ DQGDOR GHSHQGVRQ
WKHEHQGLQJ PRPHQW0 DQGWKHEHDP DUHDPRPHQWRI LQHUWD, : HDOR OHDUQWKDWRI
WKHSDUWFXDUFDH0 LQ GE\ LQDQG, LQ WKHSUHLFWGVWHVL
0 3D 8VLQJ WKLV LQRUPDWRQ DQG GLPHQVLRQDO UHDVRQLQJ RQO ILQG WR WKH
VLJQLILFDQWLJXUHV WKHRQO SRWLEOHGLPHQVLRQDO KRPRJHQHRXVIRUPXOD1 \I 0
,
- 3 $QLQYHUWGFRQFDOFRQYLQHU LQLQGLDPHWUDQG LQKJK LVILOOHGZLWKDQLQ
DW *& DQGZHJKHG 7KHQLQGZHJKWVIRXQG WR EH RXQFHV D : KDWLV WK
GHQVLWRI WKHIOXLG LQNJ P " E : KDWOXLGPLJKWWKLVEH" \$VXPHVWDQGDUGJUDYLW J
P V
- 3 7KH6WRNHV2VHQIORUPXOD> @RUGUDJ IRUFH) RQD VSKHUHRI GLDPHWU' LQDIOXL
VWUHDP RI ORZ YHORFLW9 GHQVLW! DQGYLVFRVLW LV

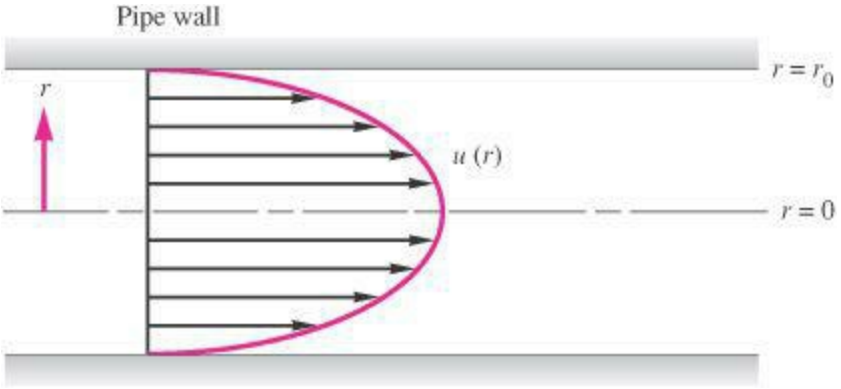
$$F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$$

,VWKLVIRUPXODGLPHQVLRQDOO KRPRJHQHRXV"

- 3 ,Q (QJOLVK (QJLQHHULQJ XQLW WKH VSHFLILF KHDW$S RI DLU DWURRP WPSHUDXUH L
DSSUR[LPDWO %WX OEP *) : KHQZRUNLQJ ZLWKNLQHWF HQHUJ\ UHDWRQ LW
PRUHDSSURSULDWWRH[SUHVV F$S DVDYHORFLW VTXDUHGSHUDEVROXWHGJUHH * LYHWK
QXPHULFDOYDOXH LQWKLVIRUP RI F$S IRUDLULQ D 6, XQLW DQG E %* XQLW
- 3)RUORZ VSHHG ODPLQDU VWUHDG IORZ WKURXJK D FLUFXODU SLSH DV VKRZQ LQ) LJ
3 WKHYHORFLW X YDULHVZLWKUDGLXVDQGVDWLVILHVWKHIRUP

$$u = B\frac{\Delta p}{\mu}(r_0^2 - r^2)$$

ZKHUH LV WKHIOXLG YLVFRVLW DQG ûS LV WKHSUHVVXUHGURS IURP HQWDQFHWR H[LW: KD
DUHWKHGLPHQVLRQVRI WKHFRQVWDQW%'



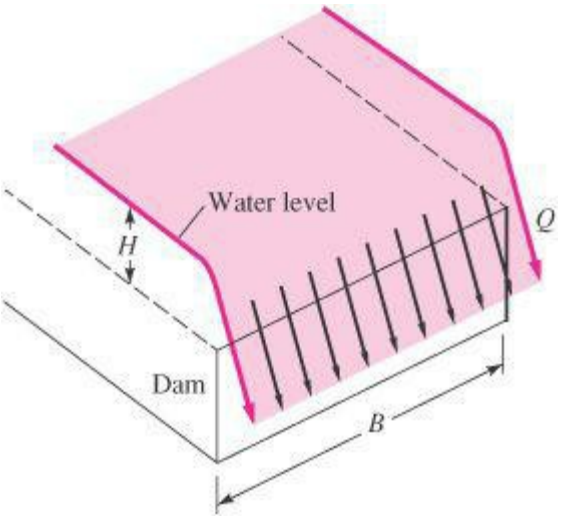
3

- 3 7KH HILFLHQF\ RI D SXPS LV GHILQHG DV WKH GLPHQVLRQOHVV UDWLR RI WKH SRZHU GHOLYHUHG E\ WKH IOXLG WR WKH SRZHU UHTXLUHG WR GULYH WKH SXPS

$$\eta = \frac{Q\Delta p}{\text{input power}}$$

ZKHUH4 LV WKH YROXPHWULF IORZ DQG ûS LV WKH SUHVVXUH ULVH SURGXFHG E\ WKH SXPS 6XSSRVH WKDW D FHUWDLQ SXPS GHYHORSV D SUHVVXUH ULVH RI OEI LQ ZKHQ LW IORZV I W V , I WKH LQSXW SRZHU LV KS ZKDW LV WKH HILFLHQF\"

- 3) LJXUH3 VKRZV WKH IORZ RI ZDWHU RYHU D GDP 7KH YROXPHWULF 4 LV NQRZQ WR GHSHQG RQO\ RQ FHUWDLQ GLPHQVLRQOHVV JURXSV DFFHOHUDWLRQ RI JUDYLW\ DQG XSVWUHDP ZDWHU GHSWK + DERYH WKH GDP FUHVW, WLV IXUWKHU NQRZQ WKDW 4 LV SURSRUWLRQDO WR % : KDW LV WKH IRUP RI WKH RQO\ SRVVLEOH GLPHQVLRQOHVV KRPRJHQHRXV UHODWLRQ IRU WKLV IORZ UDWH"



3

- 3 0 RWW> @UHFRPPHQGV WKH IROORZLQJ IRUPXOD IRU WKH IULFWLRQ KHDG ORVV, LQ IWV IORZ WKURXJK DSLSH RI GHQJWK/ DQG GLDPHWHU' ERWK PXWWEHLQ IW

$$h_f = L\left(\frac{Q}{0.551AC_hD^{0.63}}\right)^{1.852}$$

ZKHUH4 LV WKH YROXPHWULF IORZ UDWH LQ IWV $ LV WKH SLSH FURVV VHFWLRQDO DUHD LQ IW DQG &+ LV D GLPHQVLRQOHVV FRHILFLHQW ZKRVH YDOXH LV DSSUR[LPDWHO\ ' HWHUPLQH WKH GLPHQVLRQV RI WKH FRQVWDQW

- 3 $QRWKHU HTXDWLRQ VXFK DV %HUQRXOOL¶V UHODWLRQ (T RI ([DPSOH DU GLPHQVLRQOHVV FRQVWDQWV EXW ZKDW DERXW GLIIHUHQW HTXDWLRQV" &RQVLGHU IR H[DPSOH WKH ERXQGDU\ OD\HU[PRPHQWXP HTXDWLRQ ILUVW GHULYHG E\ / XGZLJ 3UDQG

LQ

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\rho g_x+\frac{\partial \tau}{\partial y}$$

ZKHUH2 LV WKH HERXQGDU\ OD\HU VKHDU VWUHVV DQG J[LV WKH FRPSRQHQW RI JUDYLW\ LQ WKH[GLUHFWLRQ ,V WKLV HTXDWLRQ GLPHQVLRQDOO\ FRQVLVWHQW&DQ\RXGUDZ DJHQHUDOFRQFOXVLRQ

- 3 7KH+DJHQ: LOOLDPV VKRZGUDXOLF VIRUPXOD IRUYROXPHWULF IORZ 4 WKURXJK DSLSHR GLDPHWHU' DQG OHQJWK/ LVJLYHQE\

$$Q \approx 61.9D^{2.63}\left(\frac{\Delta p}{L}\right)^{0.54}$$

ZKHUHûS LV WKH SUHVVXUH GURS UHTXLUHG WR GULYH WKH IORZ : KDWDUH WKH GLPHQVLRQV R VKHFRQVWDQW " &DQ WKLV IRUPXOD EH XVHGZLWKFRQLGHQFH IRUYDULRXVOTXLGVDQGJDVHV"

- 3)RUVPDOSDUWFOHVDWORZ YHORFLWLHV WKH ILWWLQJ LQ WKH6WRNHV2VHQGUDJ ODZ 3UREOHP LVGRPLQDQWHQFH) §.9 ZKHUH. LVDFRQVWDQW6XSSRVHDSDUWFO RI PDVV P LVFRQVWDQWDFFHOHUDWHV IURP WKHLQLWLDOSRVLWLRQ[ZLW LQLWLDOYHORFLW9 6KRZ D WKDWWWYHORFLWZLOOGHFUHDVHH[SRQHQWLDOO ZLWKWLPH DQG E WKDWWWZLOOWDSDWWUWDYHDQJDGLVWDQFH[P9 .

- 3 ,QKLV WXG\ RI WKHFLUFXODUK\GUDXOF MXPS IRUPHGE\ DIDXFHWWORZ LQJ LQR DVLQ : DWRQ> @SURSRVHGDSDUDPHWUFRPELQLQJ YROXPHIORZ UDW4 GHQVLW! DQ YLVFRVLW RI WKHIOXLG DQG GHSWK RI WKHZDWULQ WKHVLQN +HFDLP VWKWK JURXSLQJ LVGLPHQVLRQOHVV ZLWK4 LQWKHQXPHUDU &DQ\RYHUI\ WKLV"

- 3 %RRNV RQ SRURXV PHGLD DQG DWPL]DWRQ FDLP WKDWWKH YLVFRVLW DQG VXUIDF WHQVLRQ RI DIXLG FDQEHFRPELQHG ZLWK DFKDUDFWULVWF YHORFLW8 WR IRUP D LPSRUDQWGLPHQVLRQOHVV SDUDPHWU D 9HUI\ WKDWWKLV LV VR E (YDOXDWH WKL SDUDPHWUIRUZDWUDW *f& DQG DYHORFLWRI FP V1RW <RXJHW[WDFHG LI \RXNQRZ WKHQDPHRI WKLVSDUDPHWU

- 3 ,Q 3UDQGWVWXGHQW+HQULFK%ODVLXV SURSRVHGWKHIRQOWRZLQJ IRUPXODIRUWK ZDOOWKHUWWV2Z DWDSRVLWLRQ[LQYLVFRXVIORZ DWYHORFLW9 SDWWDIODWXUFH

$$2_Z \quad ! \quad 9 \quad [^{\pm}$$

' HWHUPLQHWKHGLPHQVLRQVRI WKHFRQWDQW

- 3 7KH(NPDQQXPEHU (N DULVHVLQJHRSK\VLFDOO\GLPLFV ,WV DGLPHQVLRQOHV SDUDPHWU FRPELQLQJ VHDZDWU GHQVLW! D FKDUDFWULVWF OHQJWK/ VHDZDW YLVFRVLW DQG WKH&RUROLVIUHTXHQF\ 9 VLQ ZKHUH9 LV WKHURWDWRQUDWHRI WK HDUWKDQG Z LV WKHODWLWXGHDQJOH ' HWHUPLQHWKHFRUUHFWRUP RI (NLI WKHYLVFRVLWL LQWKHQXPHUDU

- 3 ' XULQJ : RUOG : DU ,, 6LU * HRIUH\ 7D\ORU D %ULWLVK IOXLG G\QDPLFLVW XVH GLPHQVLRQDODQDOVLV WRHVWLPDWWKHHQHUJ\ UHOHDVHGE\ DQDWPLF ERPE H[SORVLRQ +HDVXPHGWKDWWKHHQHUJ\ UHOHDVHG(ZDV DIXQFWLRQ RI EODVWZDYHUDGLXV5 DL GHQVLW! DQG WPHW$UDQJH WKHVH YDULDEOHV LQWR D VLQJOH GLPHQVLRQOHVV JURXS ZKLFKZHPD\ WKHEODVWZDYHQXPEHU

7KHUPRG\QDPLFSURSHUWLHV

- 3 $LUDWXPHGWRENHDQLGHDOJDVZLWKN IORZVLVHQWURSLFDOO\WKURXJKDQR]OH $VWHDG\RQ FRQGLWLRQVDUHVHDOHGYHVVHOVDQGGUG VHH7DEOH$ $VWHDG\RQ WK WAPSHUDWXUHLVí *& (WAPDWD WKHSUHVVXUHDQG E WKHGHQVLWRI WKHDLUD VHFWLRQ
- 3 $ VDQNFRQWLQV P RI KHDXP DW N3DDQG *& (WAPDWWKHWRWDOPDVRI WK JDV LQNJ D RQHDWK DQG E RQWKHPRRQ $OR F KRZ PXFK KHDWWDQVHU L 0 - LVUHTXLUHGWRH[SDQGWKLVJDVDWFRQWDQWWAPSHUDWXUHWRDQHZ YROXPHRI P "
- 3 : KHQZHLQWKH8QLWHG6WDWHVVD DFDJ¶VWUHLVILOOHG³WR GE´ ZHPHDQWKDWW LQWHUQDOSUHVXUHLV GE LQ DERYHWKHPDVSKHUH ,I WKHWUHLVDWH OHYHOKDVDYROXPHRI IW DQGLVDW *) HWAPDWWKHWRWDOZHLJKWRI DLU LQGE LQWLGHWKHWUH
- 3)RUWAPDW GE LQ VRPHYDOXHVRI WAPSHUDWXUHDQG VSHFLILF YROXPHDUHD IRGBRZVIURP 5H

T, °F	400	500	600	700	800
v, ft³/lbm	12.624	14.165	15.685	17.195	18.699

,VWAPIRUWKHVHFRQGLWRQV QHDU D SHUHFWJDV RU LV LWZLGBO QRQLGHDO ,UHDVRQDEO SHUHFWILQG D OHDWTXDUH, YDOXHIRUWKH JDV FRQWDQW LQP V . HWAPDWWKHSHUHQWDHHURULQWKLVDSSUR[LPDWRQ DQGFRPSDUHZLWK7DEOH$
- 3 :HWDWRSHULF DLUDW SHUHQWHDWYHKXPLGLWFRQWLQVDXUDWHGZDWUYDSR DQG E\ ' DOWRQ¶VODZ RI SDUWLDOSUHVXUHV

$SDW $GU DLU $ZDWUYDSRU

6XSSRVHWKLVZHWDWRSHULVDW *& DQG DW &DOFXODWWKHGHQVLWRI WKLV SHUHQWKXPLGDLU DQGFRPSDUHLWZLWKWKHGHQVLWRI GU DLUDWWKHVDPHFRQGLWRQV

- 3 $ FRPSUHVVHG DLU VDQN KRGBV IW RI DLU DW GE LQ ³JDJH´ WKDWLV DERYH DWRSHULF SUHVXUH (WAPDWWKHHQHUJ\ LQIWGE UHTXLUHGWR FRPSUHVV WKLV DL IURP WKHDWRSHUH DWXP LQJ DQGLDOLRWKHUPDOSURFHVV
- 3 5HSHDW3URE LI WKHVDQNLVILOOHGZLWKFRPSUHVVHGZDWUULQWRDGRI DLU : K\ L WKHUHVXOWRXDQGVRI WAPHVOHWVWKDQWKHUHVXOWR IW GE LQ3URE "
- 3 2QHFXELF IRRWRI DUJRQJDV DW *& DQG DW LVFRPSUHVVHGLVHQWURSLFDOO WR SUHVXUHRI NPD D : KDWZLOOEHLWVQHZ SUHVXUHDQGWAPSHUDWXUH' E ,I LWL DGBRZHGWRFRRODWKLVQHZ YROXPHEDFNWR *& ZKDWZLOOEHWKHILQDOSUHVXUH'
- 3 $ EOPS LV DSSUR[LPDWG E\ D SURDOWVSKHULG P ORQJ DQG P LQGLDPHWU (WAPDWWKHZHJKWRI *& JDVZLWKLQWKHEOPS IRU D KHDXP DW DW DQG E DLUDW DW : KDWPLJKWWKHGLIIHUHQFHEHWZHHQWKHVHWZR YDOXHVUHSUHVHQWVH &KDS "
- 3 ([SHULPHQWDOGDWD> @RUWKHGHQVLWRI QSHQWDQHGTXLGIRUKLJK SUHVXUHV D *& DUHGLVWAGDVIROBRZV

Pressure, MPa	0.01	10.23	20.70	34.31
Density, kg/m^3	586.3	604.1	617.8	632.8

,QWHUSRODWLQJ WKLV GDWD GRHV QRW LWWKH DXWRUV VXJJHVWHG QTXLG WDWH UHDWRQ (T YHU ZHOO 7KHUHRUH D ILWWKHGDWD DVEHWWRX FDQ WR DVHFRQG RUGHUSRO QRPLDO 8VH\RXFXUYH ILWWRHWKPDW E WKHEXNIPRGXOVRI QSHQDQHDW DW DQG F WKHVSHH RI VRXQGRI QSHQDQHDWDSUHVXUHRI 0 3D

- 3 &RQVLGHUWKDP DWWKHIROORZLQJ WDWHQHDUWKHVDXUDWRQ S 7 0 3D *f*& &DOFXODWH DQG FRPSDUH IRUDQLGHDOJDV 7DEOH$ DQG WKH WKDP WDEOH RUWKH((6 VRIWZDUH D WKHGHQVLW! DQG E WKHGHQVLW! LI WKHWKDP H[SDQG LVHQWURSLFDOO WR DQHZ SUHVXUHRI N3D ' LVFXVV\RXUHVXOW
- 3 ,Q7DEOH$ PRWWFRPPRQJDV DLU QLWURJHQ R[\JHQ K\GURJHQ KDYH DVSHFLLF KHDWDWDWR N§ : K\ GR DJRQDQG KHOXP KDYHVXFK KLJK YDOXHV" : K\ GR1+ KDYHVXFK DORZ YDOXH" : KDWLV WKHORZHVWWIRUDQ JDV WKDWRXNQRZ RI"
- 3 7KH LVHQWURSLF EXONPRGXOV%RI DIOLG LV GHILQHG LQ(T D : KDWDUH LV GLPHQVLRQV" 8VLQJ WKHRUHWFDOS ! UHDWRQVIRU DJDV RU LQG HVWLPDW WKH EXO PRGXOV LQ3D RI E FKORULQH DW *f& DQG DW DQG F ZDWU DW *f& DQ DW
- 3 $ QHDUOLGHDOJDV KDV DPROHFXODUZHJKWRI DQG DVSHFLLF KHDWF\ - NJ . : KDWDUH D LWV VSHFLLF KHDWDWDWR N DQG E LWV VSHHGRI VRXQGDW *f&"

9 LVFRVLW\ QR VOS FRQGLWRQ

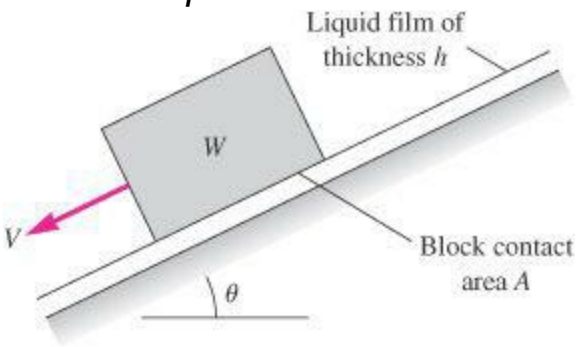
- 3 ,Q) LJ LI WKHIOXLG LVJOFHULQDW *f& DQG WKHZLGWKEHWZHHQSODWHV LV PP ZKDWWKHDUWWHV LQ3D LVUHTXLUHGWRPRYHWKHXSSHUSODWDW P V" : KDWLV WK 5H\QROGV QXPEHULI / LV\DNHQWREHWKHGLVWDQFHEHWZHHQSODWH"
- 3 . QRZ LQJ IRUDLUDW *f& IURP 7DEOH HWXP DWLWV YLVFRVLW\DW *f& E\ D WK SRZHUODZ DQG E WKH6XWKHUODQGODZ $OVR PDNHDQHVWLPDWH IURP F) LJ &RPSDUHZLWKWKHDFFHSWHGYDOXHRI § (NJ P V
- 3 RU OTXLG YLVFRVLW DV D IXQFWLRQ RI WHPSHUDWXUH D VLPSOLFDWLRQ RI WKHORJ TXDGUDWF ODZ RI (T LV$QGUDGH¶VHTXDWLRQ> @ §$H[S %7 ZKHUH $ % DUHFXUYHILWFRQVWDQWV DQG7 LVDEVROXWWHPSHUDWXUH) LWWKLV UHODWLRQWR WKHGDWD IRUZDWULQ7DEOH$ DQGHVWLPDWWKHSHUFHQWDJHUURUR WKHDSSUR[LPDWLRQ
- 3 $QDOXPLQXP FXEHGHUZHJKLQJ 1 FP LQGLDPHWUDQG FP ORQJ LVIDOOLQ FRQHQWLFDO WKURXJK DORQJ YHUWFDOVOHHYHRI GLDPHWU FP 7KHFHDUDFHLV ILOOHG ZLWK 6$(RLO DW *f& (VWXP DWWKHWUPLQDO]HUR DFFHOHUDWRQ IDC YHORFLW\ 1HJOHFWDLUGUDJ DQG DVXPHDQQHUYHORFLW\GLVWULEXWRQLQWKHRLO+LQ <RX DUHJLYHQGLDPHWUV QRWUDGL

- 3 ([SHULPHQWDOYDOXHVIRUWKHYLVFRVLWRI KHOXP DW DW DUHDVIROORZV

T, K	200	400	600	800	1000	1200
μ, kg/(m·s)	1.50 E-5	2.43 E-5	3.20 E-5	3.88 E-5	4.50 E-5	5.08 E-5

) LWWKHVHYDOXHVWRHVWLPDWH D DSRZHUODZ RU E WKH6XWKHUODQGODZ (T

- 3) RUWKHIORZ RI JDVEHWZHHQWZR SDUDOOHOSODWHV RI) LJ UHDGV]HIRUWKHFDVHR VOSIORZ DWERWKZDOOV 8VHWKHVLPSOHVOSFRQGLWLRQ / X_{ZDO} • GXG_{ZDO} ZKHUH• LVWKHPHDQIUHSDWKRI WKHIOXLG 6NHWFK WKHH[SHFWHG YHORFLWSURILOHDQGILQGD H[SUHVVLRQIRUWKHVKHDUVWUHVVDWWKHFKZDOO
- 3 6\$(RLODW *f*& ILOOVWKHFRQFHQWULFDQQXODUSDFHEHWZHHQDQLQQHUF\OLQGHUU FP DQGDQRXWHUF\OLQGHU U_R FP 7KHOHQJWKRI WKHF\OLQGHUVLV FP ,I WK RXWHUF\OLQGHULVIL[HGDQGWKHLQQHUF\OLQGHUURWDWHVDW UHY PLQ XVHWKHQHZ SURILOHDSSURSLPDWRQ WR HVWLPDWH WKH SRZHU LQ ZDWW UHTXLUHG WR PDLQWDLQ WK URWDWRQ 1HJOHFWDQ WHSHUDXUHFKDQJHRI WKHRLO
- 3 \$ EORFNRI ZHJKW VOLGHVGRZQDQLQFOLQHGSODQHZKLOHXEUFDWHGE\ DWKLQILOP R RLO DVLQ) LJ 3 7KHILOP FRQWDFWDUHDV\$ DQGLWWKLFNQHVVVK \$VXPLQJ I QHDU YHORFLW GLVWULEXWRQ LQ WKH ILOP GHULYH DQ H[SUHVVLRQ IRUWKH ³WUPLQDO]HUR DFFHOHUDWRQ YHORFLW9 RI WKHEORFN) LQG WKHWUPLQDOYHORFLWRI WKHEORFN LI WKHEORFNPDVVLV NJ \$ FP *f* DQG WKHILOP LV PP WKLFN6\$(RLODW *f*&



3

- 3 \$ VLPSOHDQG SRSXODUPRGHOIRUWZR QRQQHZWRQLDQIOXLGVLQ) LJ DLVWKHSRZHL ODZ

$$\tau \approx C\left(\frac{du}{dy}\right)^n$$

ZKHUH& DQGQ DUHFRQVWDQWVILWWRWKHIOXLG> @) URP) LJ D GHGXFHWKHYDOXHVR WKH H[SRQHQWQ IRUZKLFK WKHIOXLG LV D QHZWRQLDQ E GLODWDQWDQG F SVHXGRSODVWF &RQVLGHUWKH VSHFLILF PRGHOFRQVWDQW& 1 V^P ZLWWKHIOXLG EHLQJ VKHDUH EHWZHHQWZR SDUDOOHOSODWHVDVLQ) LJ ,I WKHVKHDUVWUHVVLQWKHIOXLGLV 3D ILQGWK YHORFLW9 RI WKHXSSHUSODWHIRUWKHFDVHV G Q H Q DQG I Q

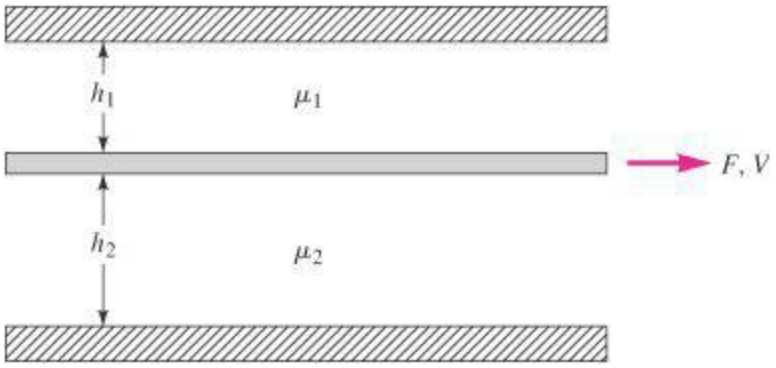
- 3 ' DWDIRUWKHDSSDUHQWYLVFRVLWRI DYHUDJHKXPDQEORRG DWQRUPDOERG WHPSHUDWU RI *f*& YDULHVZLWKVKHDUWHQUDWH DVVKRZQLQWKHIROORZLQJWDEOH

Strain rate, s^{-1}	1	10	100	1000
Apparent viscosity, kg/(m · s)	0.011	0.009	0.006	0.004

D ,VEORRGDQRQQHZWRQLDQIOXLG' E ,I VR ZKDWWSHRI IOXLGLVLW F +RZ GR WKHV YLVFRVLWWVFRPSDUHZLWKSODLQZDWUDW *f*&"

- 3 \$ WKLQSODWHLV VHSDUDWHGIURP WZR IL[HGSODWHV E\ YHUWLFDOGDQ UHSHFWLYHO DVLQ) LJ 3 7KHSODWHVSDFLQJVK DQGK DUHXQHTXDO DVVKRZQ

7KHFRQFHSWRIDUHDLV$ EHZHHQWKHFRQWUSDWDQGHDFKIOLG D $VXPLQJ DOOHD YHORFLWV GLVWULEXWRQ LQHDFK IOLG GHULYHWKHIRUFH) UHTXLUHG WR SXOOWKHSODWH YHORFLW9 E ,VWKHUHDQFHVWDU UHODWRQEHZHHQWKHZRYLVFRVLWLHV DQG "



3

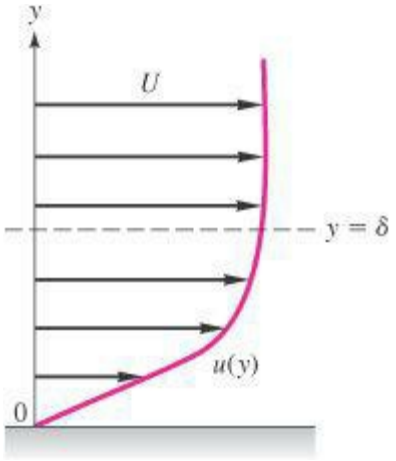
- 3 $QDPDJLQJ QXPEHURI FRPPHUFLDOODQGODERUDWRU GHYLFHVKDYHEHHQGHYHORSHGV PHDVXUH IOLG YLVFRVLW DV GHVFULEHG LQ5HV DQG &RQVLGHUDFRQFHQWU VKDWWLL[HGDLDO DQGURWDWG LQVLGHWKHVOHYH/ HVWKHLQQHUDQGRXWUFLQGHU KDYHUDGLL DQGU UHVSHFWLYHO ZLWKRWDOOHYHOQJWK/ / HVWKHURWDWRQDOOUD EH 9 UDGV DQG WKH DSSOLHG WRUTXH EH0 8VLQJ WKHVH SDUDPHWUV GHULYH I WKHRUHWFDOUHODWRQIRUWKHYLVFRVLW RI WKHIOLGEHWZHHQWKHF\OLQGHUV
- 3 $ VLPSOHYLVFRPHWUPHDVXHV WKH WPHWRUD VROG VSKHUH WR IDOOD GLVWDQFH/ WKURXJKDWWWOLGRI GHQVLW! 7KHIOLGYLVFRVLW LVWKHQJLYHQE\

$$\mu \approx \frac{W_{\text{net}} t}{3\pi DL} \quad \text{if} \quad t \geq \frac{2\rho DL}{\mu}$$

ZKHUH' LVWKHVSKHUHGLDPHWUDQG: $_{QHW}$LVWKHVSKHUHQHWZHLJKWLQWKHIOLG D 3URY WKDWEHWKRI WKHVH IRUPXODV DUH GLPHQVLRQDOO KRPRJHQHRXV E 6XSSRVHWKDWD PP GLDPHWUDOXPLQXP VSKHUH GHQVLW NJ P IDOOVLQDQRLORI GHQVLW NJ P ,I WK WPHWRIDOO FP LV V HWXPDWWKHRLOYLVFRVLWDQGYHUL\ WKDWWKHLQHTXDOLWVLVYDOLG
- 3 $Q DSSUR[LPDWRQ IRU WKH EHRXQGDU OD\HU VKDSH LQ) LJV E DQG3 LV WK IRUPXOD

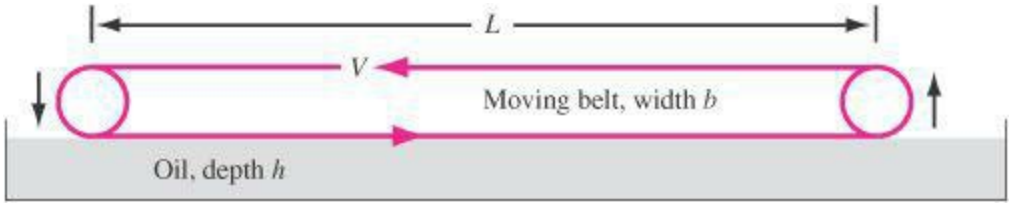
$$u(y) \approx U \sin\left(\frac{\pi y}{2\delta}\right), \qquad 0 \leq y \leq \delta$$

ZKHUH8 LVWKHVWUHDP YHORFLWIDUIURP WKHZDOODQG/ LVWKHERXQGDU OD\HUWKLFNQHVV DVLQ) LJ 3 ,I WKHIOLGLVKHDXP DW *&* DQG DWP DQGL 8 P VDQG/ FP XVHWKHIRUPXODWR D HWXPDWWKHZDOOVKHUWHVW2 LQ3D DQG E ILQGWKHSRVLWRQLQWK ERXQGDU OD\HUZKHUH2LVRQHKDOI RI 2



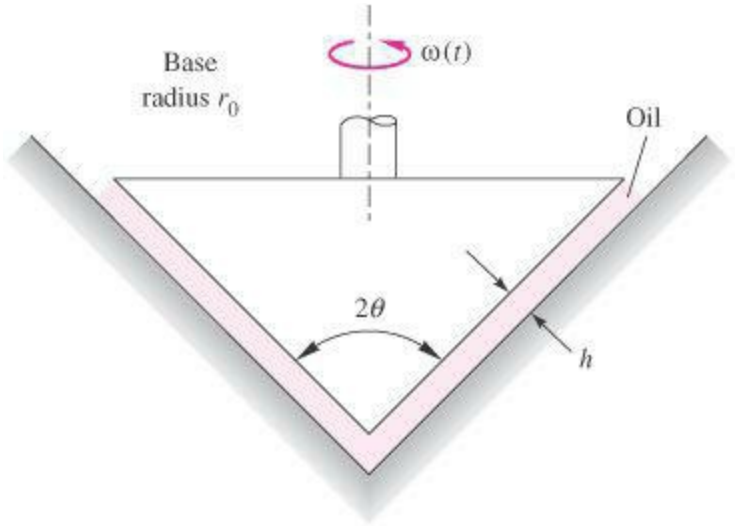
3

- 3 7KHEHOWQ) LJ 3 PRYHVDWDVWDG YHORFLW9 DQGVNLPVWKHVRS RI DVDQNRI RL RI YLVFRVLW DV VKRZQ $ VXPLQJ DQDOHDUYHORFLW SURILOHLQWKHRLO GHYHORS I VLPSOHIRUPXODIRUWKHUHTXLUHGEHOWGULYHSRZHU3 DVDIXQFWLRQRI K / 9 E : KDWEHOWGULYHSRZHU3 LQZDWW LV UHTXLUHG LI WKHEHOWPRYHVDW P VRYH 6\$(: RLOGW *& ZLWK/ P E FP DQGK FP"



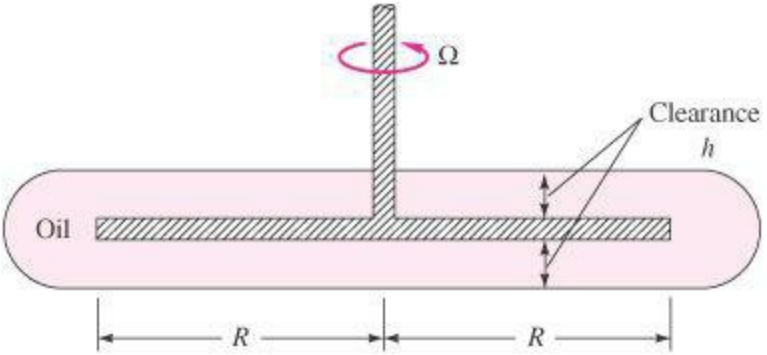
3

- 3 \$ VROG FRQH RI DQJOH EDVHU DQG GHQVLW! $F LV URWDWLQJ ZLWK LQLWLDO DQJXOD YHORFLW& LQVLGH D FRQLFDO VHDWDVVKRZQ) LJ 3 7KHFHQWUDOFHK LVILOOH ZLWKRLORI YLVFRVLW 1HJOHFWLQJ DLUGUDJ GHULYHDQDQDOWLFDOH[SUHVVLRQIRUWK FRQH¶VDQJXODUYHORFLW& WLI WKHUHLVQRDSSOLHGWRUTXH



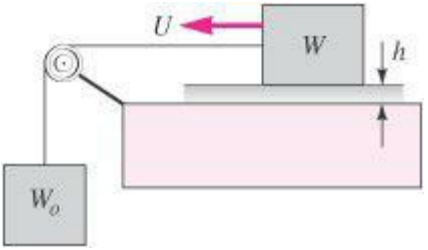
3

- 3 \$ GLVNRI UDGLXV5 URWDWHVDWDQDQJXODUYHORFLW9 LQVLGHDGLVNVKDSHGFRQWDLQ ILOOHGZLWKRLORI YLVFRVLW DV VKRZQ) LJ 3 \$VXPLQJ DQDOHDUYHORFLW SURILOHDQGQHJOHFWLQJ VKHDUVWUHVVRQWKHRXWHUGLVNHGJHV GHULYHDIRUPXODIRUWK YLVFRXVWRUTXHRQWKHGLVN



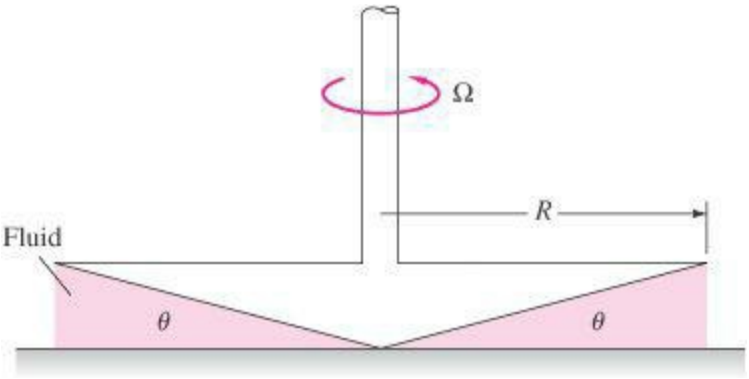
3

- 3 $ EORFN RI ZHLJKW LV EHLQJ SXOOHG RYHU D VPRRWK ZHLJKW R DV VKRZQ L) LJ 3) LQG DQ DQDO\WLFDO IRUPXOD IRU WKH VWHDG\ YHORFLW\ 8 RI WKH EORFN LI L VOLGHV RQ DQ RLOILOP RI WKLFNQHVV K DQG YLVFRVLW\ 7KH EORFN ERWWRP DUHD $ LV L FRQQHFWHG WR WKH RLO 1HJOHFW WKH FRUG ZHLJKW DQG WKH SXOOH\ IULFWLRQ $VVXPH L OLQHDU YHORFLW\ SURILOH LQ WKH RLO ILOP



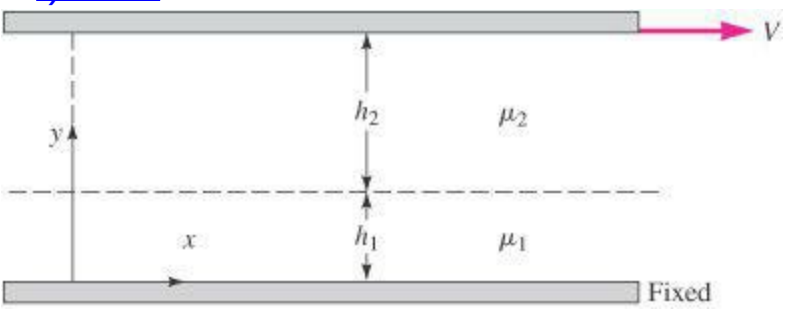
3

- 3 7KH GHYLFH LQ) LJ 3 LV FDOOHG DFRQH SODWH YLVFRPHWHU > @7KH DQJOH RI WK FRQH LV VPDOO VR WKDW WKH JDS VILOOHG ZLWK WKH WHVW IOXLG 7K VPRRWK RQH0 VR URWDWHV WKH FRQH DW UDWH LV PHDVXUHG $VVXPLQJ D OLQHDU YHORFLW SURILOH LQ WKH IOXLG ILOP GHULYH DQ H[SUHVVLRQ IRU IOXLG YLVFRVLW DV D IXQFWLRQ R 0 5



3

- 3 ([WHQG WKH WHVW IOXLG IORZ EHWZHHQ DI[HG ORZHU SODWH DQG D PRYLQJ XSSHU SODWH IURP) LJ WR WKH FDVH RI WZR LPPLVFLEOH OLTXLGV EHWZHHQ WKH SODWHV DV LQ) LJ 3



3

D 6NHWFK WKH H[SHFWHG QR VOLS YHORFLW GLVWULEXWLRQ X \ EHWZHHQ WKH SODWHV E) LQ DQDO\WLF H[SUHVVLRQ IRU WKH YHORFLW 8 DW WKH LQWHUIDFH EHWZHHQ WKH WZR OLTXLGV D\HUV F : KDW LV WKH HUHXOWR E LI WKH YLVFRVLWLHV DQG D\HUWKLFNQHVVHV DUH HTXDO"

- 3 7KH OLPLQJ SLSH IORZ H[DPSOH R 3URE FDQ EH XVHG WR GHVLJQ DFDSLOODU

YLVFRPHWU> @,I4 LVWKHYROXPHIORZ UDWH/ LVWKHSLSHOHQJWK DQG ûS LVWK
SUHVVXUH GURS IURP HQWUDQFH WR H[LWVWKHWKRU RI &KDS \LHOGV DIRUPXODIR
YLVFRVLW\

$$\mu = \frac{\pi r_0^4 \Delta p}{8LQ}$$

3LSHHQG HIHFWVDUHQHJOHFWHG> @6XSSRVHRXUFDSLOODU KDVU PP DQG/
FP 7KHIRORZLQJIORZ UDWHDQGSUHVVXUHGURSGDWDDUHREWDLQHGIRUDFHWDLQIXLG

Q, m^3/h	0.36	0.72	1.08	1.44	1.80
Δp, kPa	159	318	477	1274	1851

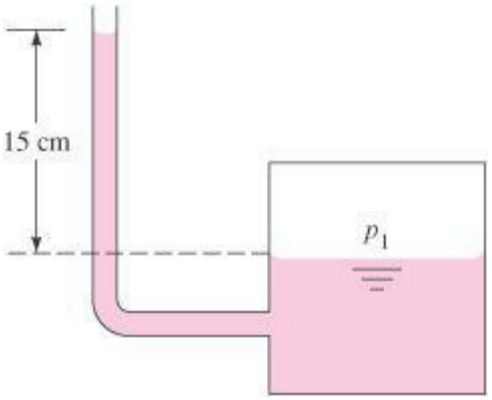
: KDWLV WKH YLVFRVLW RI WKH IOXLG' 1RWH 2QH WKH ILUVWKUHH SRLQWV JLYH WKH SURSH-
YLVFRVLW : KDWVSHFXODUDERXWKHODVWZRSRLQWV ZKLFKZHUHPHDVXUHGDFFXUDWH"

- 3 $ VROGFRQGHURI GLDPHWU' OHQJWK/ DQG GHQVLW!$IDOOVGXHWRJUDYLWLQGHI
WKEHRI GLDPHWU' 7KHFODUDQFH' í ' LVILOOHGZLWKIOXLGRI GHQVLW!
DQGYLVFRVLW 1HJOHFWWKHDLUDERYHDQGEHORZ WKHFRQGHU' HULYHDIRUPXODIR
WKHWUPLQDOIDOOYHORFLW\RI WKHFRQGHU$SSO\RXUIRUPXODWRWKHFDVHRI DVWH-
FRQGHU' FP ' FP / FP ZLWKDILOP RI 6$(RLODW *&
- 3 3LSHOLQHVDUHFOHDQHGE\ SXVKLQJ WKURXJK WKHP DFORVHILWWLQJ FRQGHUFDOOHG DSLJ
7KH QDPH FRPHVIURP WKH VTXHDNLQJ QRLVH LWPDNHV VOLGLQJ DORQJ 5HHUHFH
GHVFULEHVDQHZ QRQOLF SLJ GULYHQE\ FRPSUHVVHGDLUFOHDQLQJ FRVPHWF DQ
EHYHUDJH SLSHV 6XSSRVH WKH SLJ GLDPHWU LV LQDQG LW OHQJWK LQ ,
FOHDQV D LQGLDPHWUSLSHDWD VSHHGRI P V ,I WKHFOHDUDQFH LV ILOOHG ZL
JHFHULQDW *& ZKDWSUHVVXUHGLIHUHQFH LQSDVFDOV LVQHHGHGWRGULYHWKHSLJ'
$VXPHDQQXODUYHORFLW\SURILOHLQWKHROLDQGQHJOHFWDLUGUDJ
- 3 EES $QDLUKRFNH\ SXFNKDVDPDVVRI J DQGLV FP LQGLDPHWU: KHQSODFH
RQWKHDLUWDEOH D *& DLUILOP RI PP WKLFNQHVV IRUPVXQGHUWKHSXFN 7K
SXFN LV VWUFN ZLWK DQ LQLWLDO YHORFLW\ RI P V $VXPLQJ D QQHYDLV
GLVWULEXWRQLQWKHDLUILOP KRZ ORQJ ZLOLWWDNHWKHSXFNWR D VORZ GRZQWR P V
DQG E VWRSFRPSOHWHO" $OVR F KRZ IDUDORQJ WKLV[WHPHO ORQJ WRE\HZLOOWK
SXFNKDYHWUDYHOHGIRUFRQGLWLRQ D"

6XUIDFHWHQVLRQ

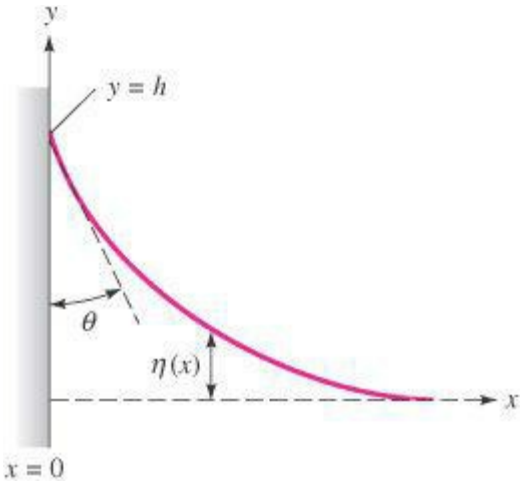
- 3 7KH K\GURJHQ EXEEOHV WKDWSURGXFHG WKH YHORFLW\ SURILOHV LQ) LJ DUH TXLW
VPDOO' § PP ,I WKH K\GURJHQZDVULQVUFHG LVFRPSDUDEOH WR DLUZDV
DQG WKHZDVUWHPSHUDWXUHLV *& HVWLPDWWKH[FHVVSUHVXUHZLWKLQWKHEXEEOH
- 3 ' HULYH(T E\ PDNLQJ DIRUFHEDODQFHRQWKHIOXLGLQWKHQF) LJ F
- 3 ' HWHUPLQHWKHPD[LPXP GLDPHWUOPOOLPHWU RI DVRQG DXPLQXP EDOO GHQVLW!
NJ P ZKLFKZLOOIORDWRQDFOHDQZDWHUXUIDFHDW *&
- 3 7KHVXVWHP LQ) LJ 3 LVXVHGWRFDOFXODWHWKHSUHVXUHS LQWHWDQNE\ PHDVXULQ

VKH FP KHLJKWRI OLTXLG LQ VKH PP GLDPHWHU WXEH 7KH IOXLG LV DW f&
&DOFXODWH VKH WXKHIOXLG KHLJKW LQ VKH WXEH DQG VKH SHUFHQWDJH HUURU GXH WR FDSLOODULW
LI VKH IOXLGLV D ZDWHU E PHUFXU



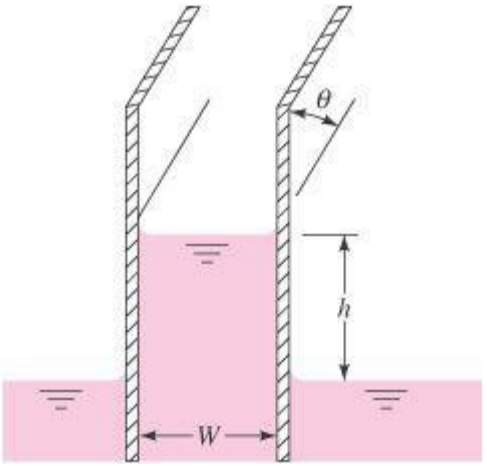
3

- 3 $ VKLQZ LUHULQJ FP LQGLDPHWHU LVOLGIURP DZDWHUXUDFHDW f& 1HJOHFWLQ
VKH ZLUHZHLJKWZKDWLV VKH IRUFH UHTXLUHG WR OLIW VKH ULQJ" ,V VKLV DJRRG ZD\ W
PHDVXUH VXUDFHWHQVLRQ" 6KRXOG VKH ZLUHBHPDGHRI DQ SDUWFODUPDWHULDO"
- 3 $ YHUWFDOFRQHQWF DQQXOV ZLWK RXWUDGLXV R DQG LQQHU UDGLXV L ORZHUH
LQWRDIOXLGRI VXUDFHWHQVLRQ< DQGFRQWDFWDQJOH f ' HULYHDQH[SUHVVLRQIR
VKHFDSLOODU\ULVHKLQVKHDQQXODUJDSLI VKHJDSLVYHU\ QDUURZ
- 3 0DNHDQDQDOVLVRI VKHVKDSH [RI VKHZDWHU±DLULQWHUIDFHQHDUDSODQHZDOOD
LQ) LJ 3 DVXPLQJ WKDWVKHVORSHLVVPDOO5í § G G[$OVRDVXPHWKDWVK
SUHVXUHGLIIHUHQFHDFURVVKHLQWHUIDFHLVEDODQFHGE\ VKHVSHFLILF ZHLJKWDQG VK
LQWHUIDFHKHLJKWûS§!J 7KHERXQGDU\ FRQGLWLRQVDUHDZHWWLQJ FRQWDFWDQJOH D
[DQGDKRUL]RQWDOVXUIDFH DV[: ⊁ : KDWLV VKHPD[LPXP KHLJKWK DWVKH
ZDOO



3

- 3 $ VROGFA\OLQGULFDOQHGGHR GLDPHWUG GHQWK/ DQG GHQLW! Q PD\ IORDWLQQOTXL
R VXUDFHWHQVLRQ< 1HJOHFWWEXR\DQF\ DQGDXXPHDFRQWDFWDQJOHR f ' HULYH
IRUPXODIRUWKHPD[LPXP GLDPHWUG PDI DEOHWR IORDWLQWKHOTXLG &DOFXODWG PDI
IRUDVWHOQHGGH 6* LQZDWUDW f&
- 3 ' HULYHDQH[SUHVVLRQIRUWKHFDSLOODU\ KHLJKWWKDQJHK IRUDIOXLGR VXUDFHWHQVLR
< DQGFRQWDFWDQJOH EHWZHHQWZR YHUWFDOSODWHGDSODUWDVGLVWDQFH: DSDUWDV/L
) LJ 3 : KDWZLOOK EHIRUZDUDW f& LI : PP"



3

- 3 $ VRDS EXEEOH RI GLDPHWU' FRDOHVFH VZLWK DQRWKHU EXEEOH RI GLDPHWU' W IRUP D VLQJOH EXEEOH' ZLWK WKH VDPH DPRXQW RI $ VWXPLQJ DQ LVRWKHUPDO SURFHVV GHULYHDQH[SUHVVLRQ IRU ILQGLQJ' DV D IXQFWLRQ RI ' ' $W DQG<

9 DSRUSUHVVXUH

- 3 (DUO PRXQWDLQHHUV EORZHG ZDWHU WR HVWLPDWH WKHLU DOWLWXGH ,I WKH UHDFK WKH VRS DQ ILQG WKDW ZDWHU EORLOV DW *f*& DSSUR[LPDWH KRZ KLJK LV WKH PRXQWDLQ'
- 3 $ VPDOO VXEPHUVLEOH PRYHV DW\HORFLW9 LQ IUHVK ZDWHU DW *f*& DQG P GHSWK ZKHUH DPELHQW SUHVVXUHV N3D ,WV FULWLFDO FDYLWDWLRQ QXPEHU LV NQRZ Q WR EH&[$W ZKDW\HORFLW ZLOO FDYLWDWLRQ EXEEOHV EHJLQ WR IRUP RQ WKH EORG\" : LO WKH EORG\ FDYLWDWLL 9 P VDQG WKH ZDWHU LV FROG *f*&"
- 3 2LO ZLWK D YDSRU SUHVVXUH RI N3D LV GHOLYHUHG WKURXJK D SLSHOLQH E\ HTXDO VSDFHG SXPSV HDFK RI ZKLFK LQFUHDVHV WKH RLO SUHVVXUH E\ 0 3D) ULFWLRQ ORVVHV LQ WKH SLSH DUH 3D SHU PHWHU RI SLSH : KDW LV WKH PD[LPXP SRVVLEOH SXP VSDFLQJ WR DYRLG FDYLWDWLRQ RI WKH RLO'

6SHHG RI VRXQG 0DFK QXPEHU

- 3 $QDLUSODQH IOLHV DW PLK $W ZKDW DOWLWXGH LQ WKH VWDQGDUG DWPRVSKHUH ZLOO WKH DLUSODQH¶V 0DFK QXPEHU EH "
- 3 (VWLPDWH WKH VSHHG RI VRXQG RI VWHDP DW *f*& DQG N3D D E\ DQ LGHDO JD DSSUR[LPDWLRQ 7DEOH $ DQG E XVLQJ ((6 RU WKH VWHDP WDEOHV DQG PDNH VP DOOLVHQWURSLF FKDQJHV LQ SUHVVXUH DQG GHQVLW DQG DSSUR[LPDWH (T
- 3 EES 7KH GHQVLW RI *f*& JDV RQ H YDULHV ZLWK SUHVVXUH DSSUR[LPDWH DV IROORZV

p, atm	1	500	1000	1500
ρ, lbm/ft^3	42.45	44.85	46.60	47.98

8VH WKH VHGDWD WR HVWLPDWH D WKH VSHHG RI VRXQG P V DQG E WKH EXON PRGXOXV 0 3D RI JDV RQ H DW DW

- 3 6LU ,VDDF 1HZWRQ PHDVXUHG WKH VSHHG RI VRXQG E\ WLPLQJ WKH GLIIHUHQFH EHWZHHQ VHHLQJ D FDQQRQ¶V PXIIOH DQG KHDULQJ LWV ERRP ,I WKH FDQQRQ LV RQ D PRXQWDL

PLDZ D\ HWWDDWWKHDLWPSHUDXUHLQGHJUHHV&HOLXVL WKHWPHGLIHUHQFHL
D VDQG E V

- 3) URP 7DEOH$ WKHGHQLWRI JOFHULQDWWDQGDUGFRQGLWRQVLVDERXW NJ P
$WDYHU KLJKSUHVXUHRI GE LQ LWGHQLWLFHDVHVRDSSUR[LPDWHO
NJ P 8VHWKLVGDWDWRHWWPDWWKHVSHGRI VRXQGRI JOFHULQ LQIW/
- 3 ,Q3UREOP 3 IRUWKHJLYHQGDWDWKHDLUYHORFLWDWWHQLV IW/ : KD
LVWKH0DFKQXPEHUDWKDWWHQLRQ'

6WHDPQQHV

- 3 5HSHDW([DPSOH E\ GHWQJWKHYHORFLWFRPSRQHQWLQFUHDVHQQHUJZLWWPH

$$V = Kxti — Kytj \qquad N$$

)LQGDQGVNHWFKIRUDIHZ UHSUHVHQWDWLYHWPHV WKHLQXDQQHRXVWHPQHV +RZ GI
WKH GLIHUIURP WKHWDG\ IORZ QQHVLQ([DPSOH "

- 3 $ YHORFLWILHGLVJLYHQE\ X 9FRV Y 9VLQ DQGZ ZKHUH9 DQG DU
FRQWDQW' HULYHDIRUPXODIRUWKHWWHPQHVRI WKLVIORZ
- 3 $ VZRGLPHQVLRQDOXQWDG\ YHORFLWILHGLVJLYHQE\ X [W Y \ ) LQGWK
HTXDWRQ RI WKHWPHYDULQJ WWHPQHVWKDWDOSDVWKURXJK WKHSRLQW[\ D
VRPHWPHW6NHWFKDIHZ RI WKHVH

+ LVWRU\ RI IOXLG P HFKDQLFV

- 3 ,Q WKH HDUO V WKH %UWWK FKHPLW6LU &\UL+ LQWKHZRRG TXLSSHG WKDWOXL
G\QDPLFV WXG\ ZDV GLYLGHG LQR ³ZRUN-U ZKR REVHUYHG WKLQJV WKH\ FRXOG QR
H[SODLQ DQG ZRUN-U ZKR H[SODLQHG WKLQJV WKH\ FRXOG QRWREVHUYH´ 7R ZKD
KLWRULFVLWXDWRQZDVKHUHHUULQ"
- 3 ' RVRPHUHDGLQJ DQGUHSRUWWKWKHFDVRQWHQDQDFKHYHPHQW HVSHFLDOO YLV
j YLVIOXLGP HFKDQLFV RI
D (YDQJHOLXV7RUULFHOOL ±
E +HQULGH3LWRW ±
F $QDRLQH&Kp]\ ±
G * RWWKLOI+HQUFK/ XGZLJ+DJHQ ±
H -XOLXV: HLVEDFK ±
I * HRUJH* DEULHO6WRNHV ±
J 0 RUL]V: HEHU ±
K 7KHRGRUYRQ. iUPiQ ±
L 3DXO5LFKDUG+HQUFK%ODVLXV ±
M/ XGZLJ3UDQGWO ±
N 2VERUQH5H\QROGV ±
O -RKQ: LOOLP 6WUXWW/ RUG5D\OHLJK ±
P ' DQLHO%HUQRXOOL ±

Q / HQKDQG(XOHU ±

([SHULP HQWDO XQFHUWDLQWLHV

- 3 $ ULJKWFLUFXODU\ OQGHUYROXPH LV WKH HFDOFXODWHG IURP WKH PHDVXUHG EDUHDLX 5 DQG KHJKW+ ,I WKH XQFHUWDLQWLQ5 LV SHUFHQWDQG WKH XQFHUWDLQWLQ+ LV SHUFHQWHVWLPDWHWKHRYHUDOOXQFHUWDLQWLQWKHFDOFXODWHGYROXPH
- 3 $ GLPHQVLRQOHVSDUDPHWHU LPSRUWDQWLQQDWXUDOFRQYHFWLRQKHDWWUDQVIHU LV WKH* UDVKRI QXPEHU

$$\mathrm{Gr} = \frac{g\,\beta\,\rho^2 L^3\,\Delta T}{\mu^2}$$

Z KHUHJ LV WKH DFFHOHUDWLRQ RI JUDYLW LV WKH WKHUPDO H[SDQVLRQ FRHIILFLHQW WK GHQVLW / DFKDUDFWHULVWLF OHQJWK û7 D WHPSHUDWXUH GLIIHUHQFH DQG WKH YLVFRVLW ,I WK XQFHUWDLQWLQ HDFK RI WKHVH YDULDEOHV“ SHUFHQWGHWHUPLQHWKHRYHUDOOXQFHUWDLQWLQ WK * UDVKRI QXPEHU

- 3 7KHGHYLFHLQ) LJ 3 LVFDOLEUDWHGXVLQJ GLVNYLVFRPHWHU> @6XSSRVHWKDW5 FP DQGK PP D ,I WKHVRUTXHUHTXLUHGWRURWDWWKHGLVNDW UPLQ 1 P Z KDWLV WKHYLVFRVLW\RI WKHIOXLG' E ,I WKHXQFHUWDLQWLQHDFK SDUDPHWHU 0 5 K 9 LV“ SHUFHQWZ KDWLV WKHRYHUDOOXQFHUWDLQWLQWKHYLVFRVLW"
- 3)RUWKHFRQHSODWHYLVFRPHWHUIRU) LJ 3 VXSSRVH5 FP DQG *f* D ,I WK VRUTXHUHTXLUHGWRURWDWWKHFRQHLV 1 P Z KDWLV WKHYLVFRVLW\RI WKHIOXLG E ,I WKH XQFHUWDLQWLQHDFK SDUDPHWHU 0 5 9 LV“ SHUFHQWZ KDWLV WK RYHUDOOXQFHUWDLQWLQWKHYLVFRVLW"
- 3 7KHGLPHQVLRQOHVVGUDJ FRHIILFLHQW& RI DVSKHUH VXEMHFWHGLQ&KDSV DQG LV

$$C_D = \frac{F}{(1/2)\rho V^2(\pi/4)D^2}$$

Z KHUH) LV WKH GUDJ IRUFH ! WKH IOXLG GHQVLW 9 WKH IOXLG YHORFLW DQG' WKH VSKHU GLDPHWHU,I WKH XQFHUWDLQWLQVRI WKHVHYDULDEOHVDUH) “ SHUFHQW! “ SHUFHQW 9 “ SHUFHQW DQG' “ SHUFHQW Z KDWLV WKH RYHUDOO XQFHUWDLQW LQ WKH PHDVXUHG GUD FRHIILFLHQW

) XQGDPHQWDOVRI (QJLQHHULQJ ([DP 3UREOHPV

-) (7KHDEVROXWHYLVFRVLW RI DIOXLGLVSUPDULO DIXQFWLRQRI
  ' HQVLW
  7HPSHUDWXUH
  3UHVVXUH
  9HORFLW
  6XUIDFHWHQVLRQ
-) (&DUERQGLR[LGH DW *f& DQG DW LVFRPSUHVVHGLQWRSLFDO WR DW $VXP

&2 LVDQLGHDOJDV 7KHILQDOWHPSHUDWXUHZRXOGEH

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-)(+HDWP KDVDPRGHFXDUZHJKWRI : KDWLVWKHZHJKWRI P RI KHDWP DW DWP DQG *f*&"

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-)($QRLOKDVDNLQHPDWF YLVFRVLWRI (P VDQG DVSHFLILF JUDYLWRI : KDWLVLWVG\QDPLF DEVROXWH YLVFRVLWLQNJ P V"

-)(&RQVLGHUDSRSEXEEOHRI GLDPHWU PP ,I WKHVXUIDFHWHQVLRQFRHILFLHQWV 1 P DQGH[WHUQDOSUHVVXUHLV 3DJDJH ZKDWVWKHEXEEOHV¶LQWHUQDOGDJHSUHVVXUH

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-)(7KHRQO SRVVLEOHGLPHQVLRQOHVJURXS WKDWFRPELQHV YHORFLW9 ERG\ VL]H/ IOXL GHQVLW! DQGVXUIDFHWHQVLRQFRHILFLHQW1 LV

  /!19

  !9/ 1

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  1/9 !

  !/9 1

-)(7Z RSDUDOOHOSODWHV RQHPRYLQJ DW P VDQGWKHRWKHUIL[HG DUHVHSDUDWHGE\ D PP WKLFNOD\HU RI RLORI VSHFLILF JUDYLW DQG NLQHPDWF YLVFRVLW (

  P V: KDWLVWKHDYHUDJHVKHDUVWUHVVLQWKHRLO"

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-)(&DUERQGLR[LGHKDV DVSHFLILF KHDWDWR RI DQG DJDVFRQVDQWRI - NJ *f*& ,I LWWAMPSHUDWXUHULVHVIURP VR *f*& ZKDWLVLWVLWLQWLQDOHQHUJ\ ULVH'

N+ NJ
N+ NJ
N+ NJ
N+ NJ
N+ NJ

-)($ FHUWLILQZDWUIORZ DW f& KDVDFULWFDOFDYLWLWRQQXPEHUZKHUHEXEEOHVIRUP
  &D§ ZKHUH&D $ S í S_{YDS} !9 ,I S_D DW DQGWKHYDSRUSUHVVXUHLV
  SRXQGVSHUVTXDUHLQFKDEVROXWH SVLD IRUZKDWZDWUYHORFLWZLOOEXEEOHVIRUP"
  PLK
  PLK
  PLK
  PLK
  PLK
-)(([DPSOH JDYHDQDQDOVLVWKDWSUHGLFWHGWKDWWKHYLVFRXVPRPHQWRQDURWDWQ
  GLVN0 CE95 K ,I WKHXQHUWDLQWRI HDFKRI WKHIRXUYDULDEOHV 9 5 K L
  SHUFHQWZKDWLVWKHHVWLPDWHGYHUWDOXQFHUWDLQWRI WKHPRPHQW0"
  SHUFHQW
  SHUFHQW
  SHUFHQW
  SHUFHQW
  SHUFHQW

&RPSUHKHQVLYH3UREOHPV

- & 6RPHWLPHVZHFDQGHYHORSHTXDWLRQVDQGVROYHSUDFWLFDOSUREOHPVE\NQRZLQ
  QRWKLQJPRUHWKDQWKHGLPHQVLRQVRI WKHNH\SDUDPHWHUVLQWKHSUREOHP)RUH[DPSO
  FRQVLGHUWKHKHDWRVVWKURXJKDZLQGRZ LQDEXLOGLQJ : LQGRZ HILFLHQF\ LVUDWHGL
  WKUPVRI ³5 YDOXH´ ZKLFK KDVXQLWVRI IW K f) %WX $ FHUWLQPDQXI
  DGYHUWLVHV D GRXEOHSDQH ZLQGRZ ZLWK DQ5 YDOXH RI 7KH VDPH FRPSDQ
  SURGXFHVD WULSOHSDQH ZLQGRZ ZLWK DQ5 YDOXH RI ,QHLWKHUFDVHWKHZLQGRZ
  GLPHQVLRQVDUH IWE\ IW2 QDJLYHQZLQWUGD\ WKHWHPSHUDWXUHGLIIHUHQFHEHWZHH
  WKHLQVLGHDQGRXWLGHRI WKHEXLOGLQJ LV f)
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\WSLFDOZ LQAUKDV\WKHHTXLYDOHQWRI DERXW KHDLQJ G\VDWDWPSHUDWXUHGLIIHUHQFH
RI f) (DFK WULSOHSDQH ZLQGRZ FRVW PRUH WKDQ WKH GRXEOHSDQH ZLQGRZ
,QRUGHU LQAUHVWDQGLQJDQRQKRZ PDQ\HDUVZLOOLWWDNHWKHKRPHRZQHUWRPDNHX
WKHDGGLWLRQDOFRVWRI WKHWULSOHSDQHZLQGRZ VIURP KHDLQJ ELOOVDYLQJV"

- & : KHQDSHUVRQLFHVNDAV WKHVXUFHRI WKHLFHDFWDOO PHOWEHQDWKWKHEODGHV VI
  WKDWKHRUVKHVNDAVRQDWKLQVKHHWRI ZDAUEHWHQWKEODGHDQGWKHLFH

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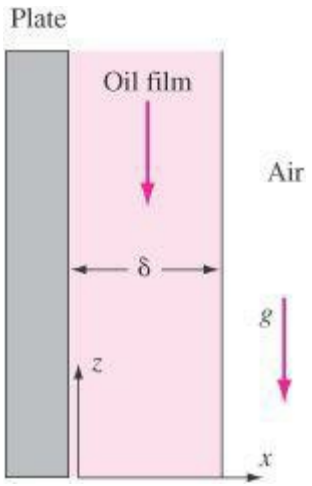
Plate
Oil film
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X\ LVPHDVXUHG ZLWKWKHIROORZLQJUHVXOWV

y, m	0.0	0.003	0.006	0.009	0.012	0.015
u, m/s	0.0	1.99	3.94	5.75	7.29	8.46

8VLQJ\RXUEHVWOQHDUSRODUQJVNLOOV HVWLPDWHWKHVKHDUVWUHVVLQWKHRLO DWWKHZDO
DQG E DW PP

- & $ PHFKDQLFDO GHYLFH WKDWXVHV WKH URWDWLQJ FDQGHUIOLJ & LV WKH6WRUPH
YLVFRPHWHU> @,QWKDGRI EHLQJ GULYHQDWFRQVWDQW9 DFRUGLVZUDSSHGDURXQG
WKH VKDIWDQG DWWDFKHG WR DIDQQQJ ZHJKW 7KH WLPHWRWXUQWKH VKDIWDJLYH
QXPEHURI UHYROXWLRQV XVXDOO ILYH LVPHDVXUHGDQGFRUUHODWHGZLWKYLVFRVLW 7KI
IRUPXODLV

$$t \approx \frac{A\mu}{W - B}$$

ZKHUH$ DQG%DUHFRQVWDQWVWKDWDUHGHWHUPLQHGE\ FDOLEUDWQJ WKHGHYLFHZLWKDNQRZ
IOXLG + HUHDUHFDOLEUDWRQGDWDIRUD6WRUPHUYLVFRPHWHUWXEHQJDWFHURO XVLQJ DZHJK
RI 1

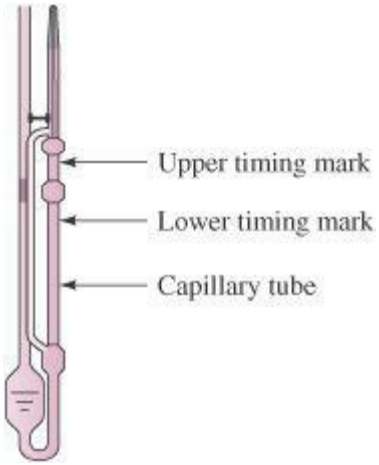
μ, kg/m-s	0.23	0.34	0.57	0.84	1.15
t, sec	15	23	38	56	77

-
-
-
-

PHQLVFXV VRX SDV IURP XSSHU VRX OHZ HUWPLQJ PDUN LV UHFRUGHG 7KH NLQHPDW
YLVFRVLWLV FRPSXWHG E\ WKH VLPSOH IRUPXOD

Y &W

Z KHUH & LV D FDOLEUDWLRQ FRQVWDQW) RU LQ WKH UDQJH RI ± PP V WK
UHFRPPHQGHG FRQVWDQW& PP V ZLWK DQ DFFXUDF\ OHVV WKDQ SHUFHQW



& 7KH &DQRQ8EEHOKGH YLVFRPHWHU >&RXUWHV\ RI &DQQRQ ,QVWUXPHQ
&RPSDQ\ @

D : KDWGDWD IURP 7DEOH$ DUH LQWKLV YLVFRVLWUDQJH'
E ,V WKH FDOLEUDWLRQ IRUPXODH GLPHQVLRQDOO FRQVLVWHQW'
F : KDW XVHP SURSHUWLHV PLJKW WKHRQVDQ& GHSHQGXSRQ'
G : KDV SUREOHP LQWKLV FKDSWUKLQWDWDIRUPXODIRUHVWPDWQJ WKHYLVFRVLW'

- & 0RWW5H S @GLVFXVVHV DVLPSOHIDQQJ EDOOYLVFRPHWHU ZKLFK ZHFD
DQDO]HDWULQ&KDSWU $ VPDOOEDOOR GLPHWU' DQG GHQVLW!E IDOOVWRXJKI
WKEH RI WWWGTXG ! 7KHIDOOYHORFLW9 LV FDOFXODWG E\ WKH WPH WR IDOOI
PHDVXUHG GLVDQFH 7KHIRUPXDIRUFDOFXODWQJ WKHYLVFRVLWRI WKHIOXLGLV

$$\mu = \frac{(\rho_b - \rho)gD^2}{18\,V}$$

7KLV UHVXOW VOPLWGE\ WKHUHTXLUHPHQWWKDWWKH5H\QROGVQXPEHU!9' EHOWVWKD
6XSSRVH DWWHOEDOO 6* RI GLPHWU PP IDOOV LQ6$(: RLO 6*
DW *& 7KHPHDVXUHGIDOOYHORFLWLV FP V D : KDWLVWKHYLVFRVLWRI WKHRLOL
NJ P V' E ,VWKH5H\QROGVQXPEHUPDOOHQRXJKIRUDYDOGHWWPDW'

- & $VRQG DXPLQXP GLVN 6* LV LQLQGLPHWUDQG LQWKLFN ,WWDGH
WWDGLO GRZQD *LQFDQH WKDWLV FRDWG ZLWK DFDWRURLO 6* ILOP RQ
KXQGUHGWK RI DQLQFK WKLFN 7KHWWDG VDGHYHORFLWLV FP V 8VLQJ) LJXUH$
DQG DOQHDUROOYHORFLW SURILOHDWXPSWRQ HWWPDWWKHWWP SHUDWXUH RI WKHFDWR
RLO

5HIHUHQFHV

- & 7DQQHKLOO' $ $QGHUVRQ DQG5 + 3HWHUVHQ&RPSXWDWRQDO) OXLG0 HFKDQLFVDQ
+HDW7UDQVIHU GHG 7D\ORUDQG)UDQFLV %URZRO3$
- 7X * + <HRK DQG& /LX&RPSXWDWRQDO) OXLG' \QDPLFV $ 3UDFWLFDO$SSURDFK
%XWWHUZRUWK+HLQHPDQQ1HZ <RUN
6 9 3DWDQNDU1XPHULFDO+HDW7UDQVIHUDQG) OXLG) ORZ +HPLVSKHUH1HZ <RUN
+ 9HUWHJ DQG: 0 DGDODVHNUD$Q,QWURGXFWRQ&RPSXWDWRQDO) OXLG' \QDPLFV 7K
) LQLWH9ROXPH0 HWKRG 3UHQWLFH+DOO8SSHU6DGGOH5LYHU1-
& - &KHQ DQG6 < -DZ)XQGDPHQWDOV RI 7XUEXOHQFH0 RGHOLQJ 7D\ORU DQG)UDQFLV
%URZRO3$
6 %3RSH7XUEXOHQW)ORZV &DPEULGJH8QLYHUVLW\3UHVV1HZ <RUN
7 %DUDXG' LUHFW1XPHULFDO6LPXODWRQIRU7XUEXOHQW5HDFWLQJ)ORZV(GLWRQV7HFKQLS
3DULV
% * HXUW(OHPHQWVRI ' LUHFWDQG/ DUJH(GG\ 6LPXODWRQ5 7 (GZDUGV,QF)ORULGD(
3$
6 7DYRXODULV³0 HDVXUHPHQWLQ) OXLG0 HFKDQLFV´ &DPEULGJH8QLYHUVLW3UHVV1HZ
<RUN
5 & %DNHU,QWURGXFWRU\ * XLGHWR) ORZ0 HDVXUHPHQW: LOH\ 1HZ <RUN
5 : 0 LOOHU) ORZ0 HDVXUHPHQW(QJLQHHULQJ +DQGERRN G HG 0 F* UDZ +LOO1HZ
<RUN
+ 5RXVHDQG6 ,QFH+LVWRU\ RI +\GUDXOLFV,RZD,QVWLWXWHRI +\GUDXOLF5HVHDUFK8QLY R
,RZD ,RZD&LW\,$ UHSULQWHGE\ ' RYHU1HZ <RUN
+ 5RXVH+\GUDXOLFV LQ WKH8QLWHG6WDWHV ± ,RZD ,QVWLWXWHRI +\GUDXOLF5HVHDUFK8QLY RI ,RZD ,RZD&LW\,$
0 * DGHO+DN³) OXLG0 HFKDQLFVIURP WKH%HJLQQLQJ WR WKH7KLUG0 LOOHQQLXP´,QW- R
(QJLQHHULQJ (GXFDWRQ YRO QR SS ±
&DPEULGJH8QLYHUVLW3UHVV³/ XGZLJ3UDQGWO) DWKHURI 0 RGHUQ) OXLG0 HFKDQLFV´ 85
ZZZ IOXLGPHFKQHWPVFSUDQGWOKWP!
5 , 7DQQHU(QJLQHHULQJ5KHRORJ\ GHG 2[IRUG8QLYHUVLW3UHVV1HZ <RUN
& (%UHQQHQ) XQGDPHQWDOVRI 0 XOWLSKDVH) ORZ &DPEULGJH8QLYHUVLW3UHVV1HZ <RU
6HHDOVR85/ KWWS FDOWHFKERRNDEUDU FDOWHFKHGX PXOWLSKDVH!
* $ $ %LG0 RGHFXODU* DV' \QDPLFVDQGWKH' LUHFW6LPXODWRQRI * DV) ORZV2[IRU
8QLYHUVLW3UHVV1HZ <RUN
) &DUGHUHOO DQG 0 - 6KLHOGV6FLHQWLF 8QW&RQYHUVLRQ $ 3UDFWLFDO* XLGH\
0 HWULFDWLRQ GHG 6SULQJHU9HUODJ 1HZ <RUN
- 3 +RPDQ+HDW7UDQVIHU WKHG 0 F* UDZ +LOO1HZ <RUN
5 & 5HG - 0 3UDXVQLW] DQG7 . 6KHUZRRG7KH3URSHUWLHVRI * DVHVDQG/ LTXLGV WK
HG 0 F* UDZ +LOO1HZ <RUN
- +LVHQUDWHDO ³7DEOHVRI 7KHUPRG\QDPLF DQG7UDQVSRUW3URSHUWLHV´ 8 6 1DW%X
6WDQGDUGV&LUFXODU UHSULQWHGE\ 3HJUDPRQ1HZ <RUN
: 7 3DUU HWDO $60 (,QWHUQDWLRQDO6WDQGDUG7DEOHVIRU,QGXVWULDO8VH$60 (3UHV
1HZ <RUN VRIWZDUHDOVRDYDLODEOH
0 DN3DG &RUS ,QWHUQHW 6WDQGDUGV 7DEOHV &DOFXODWRU 85/
KWWS ZZZ PDNSDGFRP SXEOLF KWPO VPDLQCHW6WQDRKWPO
2 $ +RXJHQDDG. 0 : DWRQ&KHPLFDO3URFHVV3ULQFLSOHV&KDUW: LOH\ 1HZ <RU

) 0 : KLWH 9LVFRXV) OXLG) ORZ GHG 0 F* UDZ +LOO1HZ <RUN
& / <DZV ; / LQDQG/ %X³&DPEULGJH9LVFRVLWHVIRU &RPSRXQGV \$Q(TXDWRQ&D
%H8VHGVR&DPEULGJH/ LTXLG9LVFRVLWDV D) XQFWLRQRI 7HPSHUDWXUH´&KHPLFDO(QJLQHHULQJ
YRO QR \$SULO SS ±
6\$( ) XHOV DQG / XEULFDQWV 6WDQGDUGV 0DQXDO 6RFLHWV RI \$XWRPRWLYH (QJLQHHUV
: DUUHQGDOH 3\$
& / <DZV +DQGERRNRI 9LVFRVLW\ YROV (OHYLHU6FLHQFHDQG7HFKQRORJ\ 1HZ <RUN

\$: \$GDPVRQDQG\$ 3 * DWW3K\VLFDO&KHPLVWU RI 6XUIDFHV : LOH\ 1HZ <RUN
& (%UHQQHQ&DYLWDWLRQ DQG %XEEOH' \QDPLFV 2[IRUG 8QLYHUVLW 3UHVV 1HZ <RU

1DWLRQDO &RPPLWWHH IRU) OXLG 0HFKDQLFV) LOPV,QWURGXFWLRQ ([SHULPHQWV LQ) OX
0HFKDQLFV 0 ,7 3UHVV &DPEULGJH 0\$
, * &XUUH) XQGDPHQWDO0HFKDQLFVRI) OXLGV GHG 0DUFHO HNNHU1HZ <RUN
: -<DQJ HG +DQGERRNRI) ORZ 9LVXDOL]DWLRQ GHG 7D\ORUDQG) UDQFLV1HZ <RUN

) 7 1LHKZ VWDGWHG) ORZ 9LVXDOL]DWLRQDQG,PDJH\$QDOVLV 6SULQJHU1HZ <RUN
7 7 /LP DQG\$ - 6PLWK HGV) ORZ 9LVXDOL]DWLRQ 7HFKQLTXHVDQG([DPSOHV ,PSHULD
&ROOHJH3UHVV /RQGRQ
\$ 6FKUGHUDQG& (: LOHUWHGV 3DWWHUQ,PDJH9HORFLPHWU 1HZ ' HYHORSPHQWVDQ
5HFHQW\$SSOLFDWLRQV 6SULQJHU1HZ <RUN
: HQ-DL<DQJ &RPSXWHU\$VVLVWHG) ORZ 9LVXDOL]DWLRQ %HJHO+RXVH 1HZ <RUN
0 YDQ' \NH \$Q\$OEXP RI) OXLG0 RWLRQ 3DUDERORF 3UHVV 6WDQRUG &\$
< 1DNDPD DQG< 7DQLGD HGV 9LVXDOL]HG) ORZ YRO (OHYLHU 1HZ <RUN
YROV DQG &5& 3UHVV %RFD5DWRQ) /
0 6DPLP\ . 6 %UHXHU / * / HDO DQG 3 + 6WHQ\$ * DOHU RI) OXLG 0RWLR
&DPEULGJH8QLYHUVLW3UHVV 1HZ <RUN
6 < 6RQHVDO ³&RORQ\) ORZ) LHOG0 HDVXUHPHQWLQD7Z R3DV&KDQQHO8VLQJ 3DUWL
,PDJH9HORFLPHWU´ +HDW7UDQVIHU* DOHU -RXUQDORI +HDW7UDQVIHU YRO \$XJXV

% &DUUDQG9 (<RXQJ ³9LGHRVDSHVDQG0RYLHVRQ) OXLG' \QDPLFVDQG) OXLG0 DFKLQHV
LQ+DQGERRN RI) OXLG' \QDPLFV DQG) OXLG0 DFKLQHU YRO ,, - \$ 6FKHWVDQG\$ ( ) X
HGV : LOH\ 1HZ <RUN SS ±
6DQRUG . HLQ DQG : LOOLDP %HFNPDQ(QJLQHHULQJ (TXDWRQ 6RIWZ ((6) &KD
6RIWZDUH 0LGGOHWRQ : ,
+ : &RHP DQ DQG : * 6WHHOH([SHULPHQWDWRQ DQG 8QFHUWDLQWV \$QDOVLV IR
(QJLQHHUV GHG : LOH\ 1HZ <RUN
5 &RRNH 8QFHUWDLQWV\$QDOVLV : LOH\ 1HZ <RUN
\$ 7KRP ³7KH) ORZ 3DVW&LUFXODU&\OLQGHUVDW RZ 6SHHGV´3URF 5R\DO6RFLHWV\$
/RQGRQ SS ±
6 - . OQHDQG) \$ 0F&OQWRFN ³' HVFULELQJ 8QFHUWDLQWLHV6LQJOH6DPSOH([SHULPHQW
0HFKDQLFDO(QJLQHHULQJ -DQXDU SS ±
5 / 0 RWW\$SSOHG) OXLG 0HFKDQLFV 3HDUVRQ 3UHQWLFH+DOO 8SSHU6DGGOH 5LYHU 1

³3XWWQJ 3RUN\ VR: RUN´ 7HFKQRORJ\ ) RFXV 0HFKDQLFDO(QJLQHHULQJ \$XJXVW S

5 0 2 OVRQ DQG 6 - : ULJKW/ VW+QWDOV RI (QJLQHHULQJ) OXLG 0 HFKDQLFV VK HG
+DUSHU&RQOQV 1 HZ <RUN
: / LRX DQG <) DQJ 0 LFURFOXLG 0 HFKDQLFV 3ULQFLSOHV DQG 0 RGHOOQJ 0 F* UDZ +LO
1 HZ <RUN
(- : DWRQ ³7KH 6SUHDG RI D/ LTXLG -HWRYHUD+RU]RQWDO3ODQH´ -) OXLG 0 HFKDQLFV
YRO SS ±
- ' HQQQG $ 5HVLQJHU6DQGV 3RZGHUV DQG* UDLQV $Q,QWURGXFWRQ WR WKH 3K\VLFVR
* UDQXODU0 DWHULDOV 6SULQJHU9HUODJ 1 HZ <RUN
(. LUDQ DQG < / 6HQ ³+LJK 3UHVVXUH ' HQVLW DQG 9LVFRVLW RI QDQDQHV´ ,QW -
7KHUPRSK\VLFV YRO QR SS ±

– 1 XPEHUGUHHUQFHVDSSHDUDWKHQGRI HDFKFKDSWU

– 2 QHDWRVSKHUHTXDO GEI IW 3D

– 7KHPHDQGLVWDQFHWDYHOHGE\ PROHFXOVEHWZHHQFROOLVLQV VHH3URE 3

– ,I HDFWRPDJQHWF HIHFWVDUHLPSRUWDQWDILUVWSULPDU GLPHQVLRQPXWEHLQFOXGHG HDFWL
FXUHQW,` ZKRVH6, XQLWV\WHPSHU $

– 7KDWVDQDZIXOORVWR DVXPSWLRQV ZKLFKQHGIXUWKHUVWXG\ LQ&KDS

– 2 QHH[DPSOHZKHUHIOLGSDUWFHSDWKVDUHLPSRUWDQWVLQZDUTXDQWDQDOVLVR WKHIDWF
FRQWPLQDQWGLVFKDUJHV

– / DPLQDUDQGWXUEXOHQWORZ VDUHVXGLHGLQ&KDSV DQG

– 7KHFRQFHSWR ³HDWWTXDUV´ HURULVYHU LPSRUWDQWDQGVKRXOGEHHDUQHGE\ HYHUVRQH

&KDSWHU 3UHVVXUH' LVWULEXWLRQ LQ D) OXLG

: HOFRPH WR RXU WEO LQ DK\GURVWDWLF FRQGLWLRQ QDPHD RXU DWP RVSKHUH DVSUHVVXUH V RI N3D RU OEV 7KH RFHDQ KRZHYHU SURGXFHV FUXVKLQJ K\GURVWDWLF SUHVVXUHV LQFUHDVLQJ GRZQZDUG E\ DWP HYHU PHWHU 7KH ' HHS 6XEP HUJHQFH 9HKLFOH \$ / 9 , 1 LV WKH GHVFHQGDQW RI ' : LOOLDP %HEHEH¶V FDEOH VXSSRUWHG *Bathysphere* ZKLFK UHDFKHG WKH GHSWK RI PHWHUV LQ WK V \$ / 9 , 1 LV D VHD SURPHOHG YHKLFOH QR FDEOH ZLWK JRRG YLVLELOLW\ YLGHR FDP HUDV LQVWUXP HQWV SUREHV DQG H[WUDFWLRQ DUPV , WFDQ FDUU WKUHH SDVVHQJHUV DV GHHS DV PHWHUV ZKHUH WKH SUHVVXUH LV DWP RU DERXW WRQV SHU VTXDUH LQFK *(Photo courtesy of Mark Spear of the Woods Hole Oceanographic Institution.)*

0 RWLYDWLRQ

0 DQ IOXLG SUREOHP V GR QRW LQYROYH PRWLRQ 7KH FRQFHQWUDWLRQ SUHVVXUH GLVWULEXWLRQ LQ D VWDW IOXLG DQG LWV HIHFWV RQ VROG VXUIDFHV DQG RQ IORDWLQJ DQG VXEP HUJHG ERGLHV

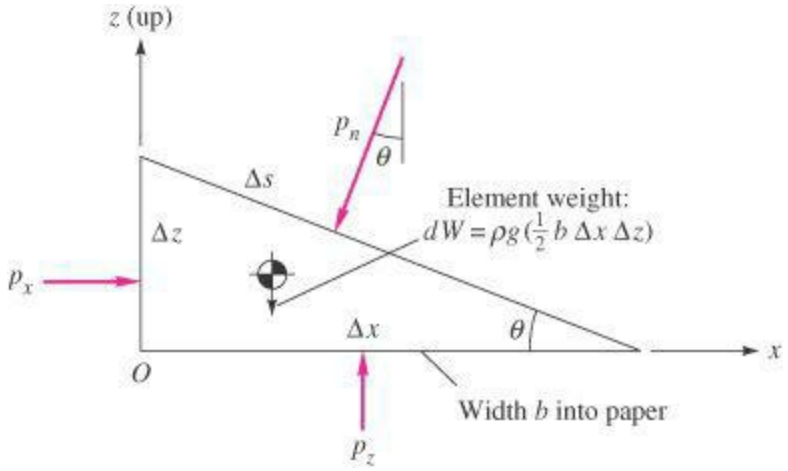
: KHQ WKH IOXLG YHORFLWLV]HUR G HQRWH G DV WKH K\GURVWDWLF FRQGLWLRQ WKH SUHVVXUH YDULDWLRQ L GXH RQO WR WKH ZHLJKW RI WKH IOXLG \$ VXP LQJ DNQRZ QIOXLG LQD JLYHQ UDYLW\ILHOG WKH SUHVVXUH PD HDVLO EHFDOFXODWG E\ LQWUDWLRQ ,P SRUWDQW SSOLFDWLRQV LQWKLV FKDSWUH SUHVVXUHGLVWULEXWLR LQ WKH DWPRVSKHUH DQG WKH RFHDQ WKH GHVLJQ RI PDQ PHFKDQLFDO DQG HOHFWURQF SUHVVXU LQVWUXP HQWV IRUFHV RQ VXEP HUJHG IODWDQG FXUYHG VXUIDFHV EXR\ DQF\ RQ D VXEP HUJHG ERG DQG WKH EHKDYLRUR IORDWLQJ ERGLHV 7KH DVWWZR RUHVXOWLQ\$UFKLP HGHV¶SULQFLSOH ,I WKH IOXLG LV PRYLQJ LQUJLG ERG\ PRWRQ VXFK DV DWDQN RI OLTXLG WKDWKDV EHHQ VSLQQLQJ IRUI

GRQ WKH SUHVVXUH DOR FDQ EH HDVLO FDOFXODWG EHFDXVH WKH IOXLG LV IUHH RI VKHDU VWUHVV : H DSSO WKLV LGHDK HUHVRVLPSOHUJLG ERG\ DFFHOHUDWRQV/LQ6HF 3UHVXUHPHDVXUHPHQWVXVHG DUH GLVFXVVHG LQ6HF $VDPDWWHURI IDFWSUHVVXUHDOR FDQEHDQDO]HGLQDEHWDU QRQUJLG ERG\ PRWRQV9 [\ ] W EXWZHGHUWKDWXEMHFWVWR&KDS

3UHVVXUHDQG3UHVVXUH* UDGLHQWV

,Q) LJ ZHVDZ WKDWDIOXLGDWUHVWFDQQRWXSSRUWVKHDUVWUHVVDQGWKXV0 RKU¶VFLUFOHUHGXFH VRDSRLQW,QRWKHUZRUGV WKHQRUPDOVWUHVVRQDQ SODQHWKURXJKDIOXLGHOHPHQWDWUHVWVDSRLQ SURSHUWVFDOHGWKHIOXLGSUHVXUHS WDNHQSRVLWLYHIRUFRPSUHVVLRQE\ FRPPRQFRQYHQWLRQ 7KLVL VXFKDQLPSRUWDQWFRQFHSWWKDWZHVKDOOUHYLHZ LWZLWKDQRWKHUDSSURDFK

) LJXUH VKRZVDVPDOZHGJHRI IOXLGDWUHVWWR VL]HûV E\ û] E\ ûV DQG GHSWKE LQR WKH SDSHU 7KHUHLV QR VKHDUE\ GHILQLWLRQ EXWZHSRVXODWWKDWWKH SUHVVXUHS[S] DQGS PD\ EH GLIIHUHQWRQ HDFK IDFH 7KHZHJKWRI WKHHOHPHQWDOR PD\ EHLPSRUWDQW7KHHOHPHQWLV DVXPH VPDOOVR WKHSUHVVXUHLVFRQVWDQWRQHDFKIDFH 6XPPDWLRQRI IRUFHVPXVWHTXDO]HUR QRDFFHOHUDWRQ LQERWKWKH[DQG] GLUHFWLRQV



) LJ (TXLOLEULXP RI DVPDOOZHGJHRI IOXLGDWUHVW

$$\begin{aligned} \textstyle\sum F_x = 0 &= p_x b\,\Delta z - p_n b\,\Delta s \sin\theta \\ \textstyle\sum F_z = 0 &= p_z b\,\Delta x - p_n b\,\Delta s \cos\theta - \tfrac{1}{2}\rho g b\,\Delta x\,\Delta z \end{aligned} \qquad (2.1)$$

%XWWKHJHRPHWU RI WKHZHGJHLVVXFKWKDW

$$\Delta s \sin\theta = \Delta z \qquad \Delta s \cos\theta = \Delta x \qquad (2.2)$$

6XEVWLWXWLRQLQWR(T DQGUHDUUDQJHPHQWLYH

$$p_x = p_n \qquad p_z = p_n + \tfrac{1}{2}\rho g\,\Delta z \qquad (2.3)$$

7KHVHUHDWRQVOLQXWDWWRLPSRUWDQWSULQFLSOHVRI WKHK\GURVWDWF RUWKHDUIUHH FRQGLWLRQ 7KHUHLVQR SUHVVXUHFKDQJHLQWKHKRUL]RQWDOGLUHFWLRQ DQG WKHUHLV DYHUWFDOFKDQJHLQSUHVVXU SURSRUWLRQDOWR WKH GHQVLW JUDYLW DQG GHSWFKDQJH : HVKDOOH[SORLWWKHVHUHVXOWV WR WKHIXOBW

VWDWLQJ LQ6HF

,QWKHOLPLWDVWKHIOXLGZHGJHVKULQNVWRD³SRLQW û]: DQG(TV EHFRPH

$$p_x = p_z = p_n = p \tag{2.4}$$

6LQFH LV DUELWUDU\ ZH FRQFOXGH WKDWWKH SUHVVXUH S LQ D VWDWF IOXLG LV D SRLQWSURSHUW\ LQGHSHQGHQWRI RULHQWDWLRQ

3UHVVXUH)RUFHRQ D) OXLG(OHPHQWV

3UHVVXUH RUDQ RVKHUVWUHVV IRUWKDWP DWWU FDXVHVDQHWRUFHRQDIOXLGHOHPHQWZKHQLWYDUH VSDWDOO 7RVHHWKLV FRQVLGHUWKHSUHVVXUHDFWLQJ RQWKHWZR[IDFHV LQ) LJ / HWWKHSUHVVXU YDU DUELWUDULO

S S[\ ] W

7KHQHWRUFHLQWKH[GLUHFWLRQRQWKHHOHPHQWQ) LJ LVJLYHQE\

$$dF_x = p\,dy\,dz - \left(p + \frac{\partial p}{\partial x}dx\right)dy\,dz = -\frac{\partial p}{\partial x}dx\,dy\,dz$$

,QOLNHPDQQHUWKHQHWRUFHG)\ LQYROYHVí %S% DQGWKHQHWRUFHG)] FRQFHLQVí %S%p 7KHVWDO QHWRUFHYHFWRURQWKHHOHPHQWGXHWRSUHVVXUHLV

$$d\mathbf{F}_{\text{press}} = \left(-\mathbf{i}\frac{\partial p}{\partial x} - \mathbf{j}\frac{\partial p}{\partial y} - \mathbf{k}\frac{\partial p}{\partial z}\right)dx\,dy\,dz \tag{2.5}$$

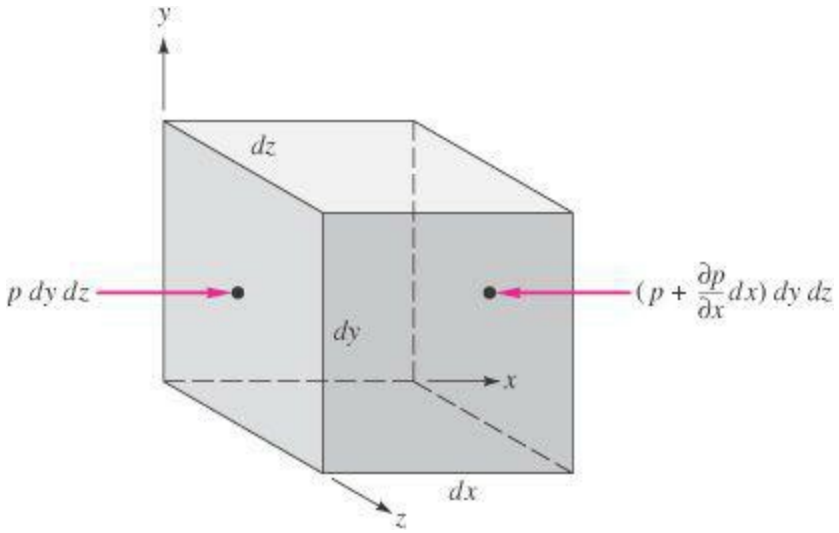
: HUHFRJQL]HWKHWHUP LQSDUHQWKHVHVDVWKHQHJDWLYHYHFWRUJUDGLHQWRI S ' HQRWLQJ I DVWKHQH IRUFHSHUXQLWYROXPH ZHUHZ ULWW(T DV

$$\mathbf{f}_{\text{press}} = -\nabla p \tag{2.6}$$

ZKHUH

$$\nabla = \text{gradient operator} = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

7KXVLWLVQRWWKHSUHVVXUHEXWWKHSUHVVXUHJUDGLHQWWFDXVLQJ DQHWRUFHWKDWP XVWEHEDODQFHGE\ JUDYLWRUDFFHOHUDWLRQRUVRPHRWKHUIHFWLQWKHIOXLG



) LJ 1HW IRUFH RQ DQ HOHPHQWDU\ SUHVVXUH YDULDWLRQ

(TXLOLEULXP RI D) OXLG(OHP HQWV

7KH SUHVVXUH JUDGLHQW DVXUDFH IRUFH WKDW DFWV RQ WKH VLGHV RI WKH HOHP HQW7KHUHP DY DOVR EH
ERG\ IRUFH GXH WR HOHFWURPDJQHWF RU JUDYLWDWLRQDO SRWHQWLDO DFWLQJ RQ WKH HQWLUH PDVV RI WK
HOHP HQW+ HUHZ HFRQVLGHURQO WKH JUDYLW IRUFH RUZ HLJKW RI WKHHOHP HQW

$$d\mathbf{F}_{\text{grav}} = \rho \mathbf{g}\, dx\, dy\, dz \qquad (2.7)$$

or

$$\mathbf{f}_{\text{grav}} = \rho \mathbf{g}$$

,Q DGGLWLRQ WR JUDYLW DIXLG LQ PRWLRQ Z LOOKDYHVXUDFH IRUFHV GXH WR YLVFRXV VWUHVVHV %
1HZ WRQ¶V ODZ (T WKHVXP RI WKHVH SHUXQLWYROXP HIRUFHV HTXDOV WKH PDVV SHUXQLWYROXP I
GHQVLW WLPHV WKHDFFHOHUDWLRQ RI WKHIOXLGHOHP HQW

$$\sum \mathbf{f} = \mathbf{f}_{\text{press}} + \mathbf{f}_{\text{grav}} + \mathbf{f}_{\text{visc}} = -\nabla p + \rho \mathbf{g} + \mathbf{f}_{\text{visc}} = \rho \mathbf{a} \qquad (2.8)$$

7KLVJHQHUDOHTXDWRQZ LOOEHVXGLHGLQGHWDLOLQ&KDS 1 RWHWKDW(T LVDY HFWRUHTXDWRQ
DQG WKHDFFHOHUDWRQ PD\ QRWEH LQ WKHVDP H YHFWRUGLUHFWLRQ DV WKH YHORFLW) RUFRXUSRVHQWRSLF
K\GURVWDWLFV WKHYLVFRXVWHUPVDQG WKHDFFHOHUDWRQDUH]HUR

* DJH3UHVVXUHDQG9DFXXP 3UHVVXUH 5HODWLYH7HUP V

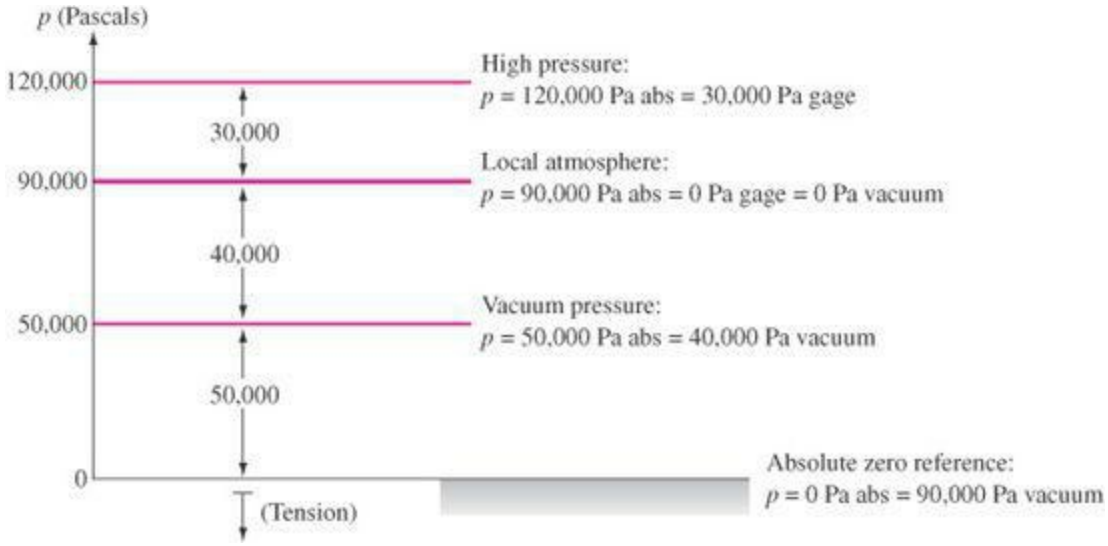
%HRUHHPEDUNLQJ RQH[DPSOHV Z HVKRXOG QRWWKDWHQJLQHHUV DUH DSWRVSHFL\ SUHVVXUHV DV
WKHDEVROXWHRUWRWDOP DJQLWXGHRU WKHYDOXHUHODWLYH WR WKHORFDOP EHQWDWPRVSKHUH 7KHVHFRQ
FDVHRFFXUVEHFDXVHP DQ SUHVVXUHLQWKHP HQW DUH RI GLIHUHQWDO\SH DQG UHFRUG QRWDQ DEVROX
P DJQLWXGH EXWWKHGLIIHUHQFHEHWZHHQWKHIOXLG SUHVVXUHDQG WKHDWPRVSKHUH 7KHP HDVXUHG SUHVVU
P D\ EHHLWKHUKJKHURUORZHUWKDQWKHORFDODWPRVSKHUH DQGHDFKFDVHLVJLYHQDQ DP H

S! S$_D$* DJHSUHVVXUH S JDJH S S$_D$

S S$_D$9DFXXP SUHVVXUH S YDFXXP S$_D$ S

7KLV LV D FRQYHQLHQW VKRUWKDQG QRWH ODWHU RQ VXEVFULSW DWPRVSKHULF SUHVVXUH V
GHWHUPLQHV WKH DEVROXWH OLQH SUHVVXUH

$ WSLFDO VLWXDWLRQ LV VKRZQ LQ) LJ 7KH GRFDODWRSKHUH DWPRVSKHULF 3D Z KLFK PLJK
UHGFH WR DERXW FRQGLWLRQ LQ D VHD OHYHO GRFDWLRQ RU QRUPDO FRQGLWLRQV DW DQ DOWLWXGH RI P 7KXV
RQ WKLV GD\ S D 3D DEVROXWH 3D JDJH 3D YDFXXP 6XSSRVH JDJH LQ D GDERUDWRU
UHDGV S 3D DEVROXWH 7KLV YDOXH P D EH UHSRUWHG DV DJDJH SUHVVXUH S í
3DJDJH 2QH PXVW DOR UHFRGQL WKH DWPRVSKHULF SUHVVXUH LQ WKH GDERUDWRU VLQFH S
FKDQJHV JUDGXDOO 6XSSRVH JDJH UHDGV S 3D DEVROXWH / RFDOO WKLV LV DYDFXXP SUHVVXU
DQG PLJKW EH UHSRUWHG DV S í 3D YDFXXP 2FFDVLRQDOO LQ WKH SURFHVV
VHFWLRQ Z H Z LOO VSHFLI\ JDJH RU YDFXXP SUHVVXUH WR N HHS \RX DOHUW WR WKLV FRQ P RQ HQJLQHHULQ
SUDFWLFH ,I D SUHVVXUH LV OLVWHG Z LWKRXW WKH P RGLILHU JDJH RU YDFXXP Z H DVXP H LW LV DEVROXW
SUHVVXUH



) LJ ,OOXVWUDWLRQ RI DEVROXWH JDJH DQG YDFXXP SUHVVXUH UHDGLQJV

+ \ GURVWDWLF 3 UHVVXUH' LVWULEXWLRQV

,I WKH IOXLG LV DW UHVW RU DW FRQVWDQW YHORFLW D DQG I $ (TXDWLRQ IRU WKH SUHVVXU
GLVWULEXWLRQ UHGXFHV WR

$$\nabla p = \rho \mathbf{g} \qquad (2.9)$$

7KLV LV DK\GURVWDWLF GLVWULEXWLRQ DQG LV FRUUHFW RU DOO IOXLGV DW UHVW UHJDUGOHVV RI WKHLU YLVFRV
EHFDXVH WKH YLVFRXV WUP YDQLVKHV LGHQWLFDOO
5HFDOO IURP YHFWRU DQDOVLV WKDW WKH YHFWRU S H[SUHVVHV WKH PDJQLWXGH DQG GLUHFWLRQ RI WK
PD[LPXP VSDWLDO UDWH RI LQFUHDVH RI WKH VFDODU SURSHUW S $V D UHVXOW S LV SHUSHQGLFXOD
HYHUZ KHUH WR VXUIDFHV RI FRQVWDQW S 7KXV(T VWDWHV WKDW D IOXLG LQK GURVWDWLF HTXLOLEULXP Z LO
DOZD LQW FRQVWDQW SUHVVXUH VXUIDFHV HYHUZ KHUH QRUP DO WR WKH GRFDOJUDYLW YHFWRU 7KH PD[LPX
SUHVVXUH LQFUHDVH Z LOO EH LQ WKH GLUHFWLRQ RI JUDYLW² WKDW V ³GRZ Q´ ,I WKH IOXLG LV DOTXLG LW IUH
VXUIDFH EHLQJ DW DWPRVSKHULF SUHVVXUH Z LOO EH QRUP DO WR GRFDOJUDYLW RU³KRUL]RQWDO´ <RX SUREDEO

NQHZ DOOWKLVEHRUH EXW(T LVWKHSURRI RI LW
,QRXUFXWWPDU FRRUGLQDWH VWWP] LV³XS´ 7KXV WKH GRFDOJUDYLW YHFWRUIRUVPDOOVFD
SUREOPVLV

$$\mathbf{g} = -g\mathbf{k} \tag{2.10}$$

ZKHUHJ LVWKHPDJQLWXGH RI GRFDOJUDYLW IRUH[DPSOH P V) RUVWKHVH FRRUGLQDWV(T
KDVWKHFRPSRQHQWV

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\rho g = -\gamma \tag{2.11}$$

WKHILUVWWZR RI ZKLFK WHOOXVWKDWS LVLQGHSHQGHQWRI [DQG\ +HQFH%S %\ FDQEHUHSODFHGE\ WK
WRWDOGHULYDWLYHGS G] DQGWKHK\GURVWDWLF FRQGLWLRQUHGXFHVWR

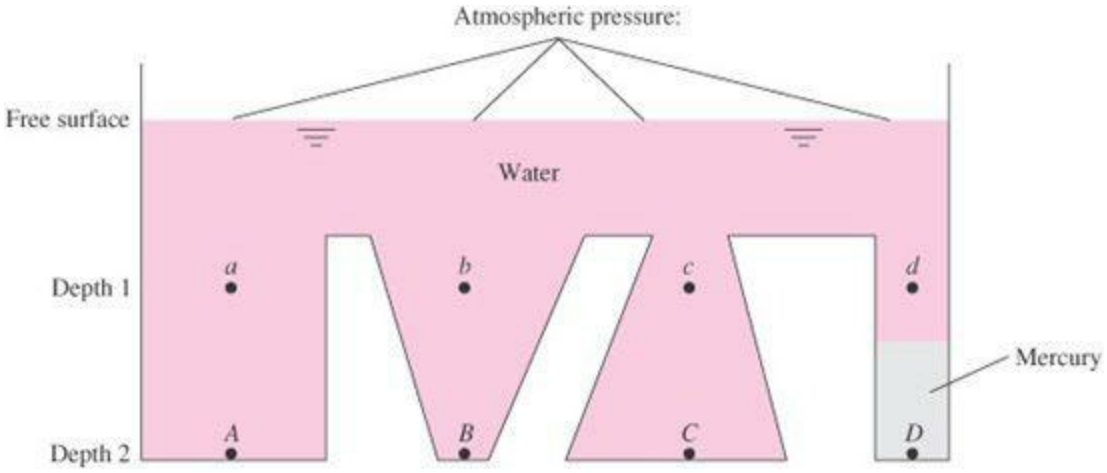
$$\frac{dp}{dz} = -\gamma$$

or

$$p_2 - p_1 = -\int_1^2 \gamma\, dz \tag{2.12}$$

(TXDWLRQ LVWKHVRXJKWUHODWLRQVKLS7KHLQWHJUDWLRQUHTXLUHVDQDVVXPSWLR
DERXWWKHGHQVLW\DQGJUDYLW\GLVWULEXWLRQ * DVVXPLQJWKDWWKHIROORZLQJ FRQGLWLRQVDUH
: HVWDUWZLWKWKHIROORZLQJ FRQFOXVLRQVDERXWDK\GURVWDWLF FRQGLWLRQ
$QLQWXLWLYHLQWHUSUHWDWLRQRI WKLV LV WKDW) LJ 7KHIUHHVXUIDFHRI WKHFRQWLQHULV DWWRSSKHULF DQ
IRUPV D KRUL]RQWDOSODQH 3RLQWVD E F DQGG DUH DWHTXDOGHSWK LQ D KRUL]RQWDOSODQH DQG DU
LQWHUFRQQHFWHGE\ WKHVDPHIOXLG ZDWU WKHUHRQO\SRLQWVKDYHWKHVDPHSUHVVXUH 7KHVDPHLVWUX
RI SRLQWV$ % DQG& RQWKHERWWRP ZKLFK DOOKDYHWKHVDPHKLJKHUSUHVVXUHWKDQDQ D E F DQGG
+RZHYHU SRLQW' DOWKRXJK DWWKHVDPHGHSWKDV$ % DQG& KDVDGLIIHUHQWSUHVVXUHEHFDXVHLWLVDW
EHQHDWKDGLIIHUHQWOLG PHUFXU

3UHVVXUHLQDFRQWLQXRXVO GLVWULEXWHGXQLRUP VWDWF IOLG YDUHVRQ ZLWKYHUWFDOGLVWDQF
DQGLVLQGHSHQGHQWRI WKHVKDSHRI WKHFRQWLQHU 7KHSUHVVXUHLVWKHVDPHDWDOOSRLQWVRQDJLYH
KRUL]RQWDOSODQHLQWKHIOLG 7KHSUHVVXUHLQFUHDVHVZLWKGHSWKLQWKHIOLG



) LJ + \GURVWDWLF SUHVVXUH GLVWULEXWLRQ 3RLQWV *a*, *b*, *c* DQG *d* DUH DW HTXDO GHSWKV LQ ZDWHU DQG WKH UHRUHKDYH LGHQWLFDO SUHVVXUHV 3RLQWV *A* DQG *C* DUH DOVR DW HTXDO GHSWKV LQ ZDWUDQ KDYH LGHQWLFDO SUHVVXUHV VXUHV KLJKHU WKDQ *a*, *b*, *c* DQG *d* 3RLQW *D* KDV D GLIIHUHQW SUHVVXUH IURP *A* DQG *C* EHFDXVH LW LV QRW FRQQHFWHG WR WKHP E\ DZDWUSDWK

(IIHFWRI 9DULDEOH* UDYLW\

) RU D VSKHULFDO SODQHWRI XQLIRUP GHQVLW\ WKH DFFHOHUDWRQ RI JUDYLW\ YDULHV LQYHUVHO DV WKH VTXDUHRI WKHUDGLXVIURP LWVFHQWU

$$g = g_0\left(\frac{r_0}{r}\right)^2 \qquad (2.13)$$

ZKHUH LV WKH SODQHWUDGLXV DQG J LV WKH VXUIDFH YDOXHRI J) RUHDUWK U § VDWXUQ § NP ,QWSLFDOHQJLQHULQJ SUREOPV WKH GHYLDWRQIURP U H[WQGVIURP WKHGHSHWRFDQ DERX NP WR WKH DWPRVSKHULF KHLJKWRI VXSHUVRQLF VWDQSRUWSHUDWRQ DERXW NP 7KLV JLYHV I PD[LPXP YDUDWRQLQJ RI RU SHUFHQW: HWKHUHRHQHJOHFWWKH YDUDWRQRI J L PRWSUREOPV

+ \GURVWDWLF 3UHVVXUHLQ/ LTXLGV

/ LTXLGVDUH VRQHDUO LQFRP SUHVVLEOHWKDWZ HFDQQHJOHFWWKHUHGHQVLW YDUDWRQLQK\GURVWDWV , ([DPSOH ZHVDZ WKDWZDWUGHQVLW VLQFUHDVHVRQ SHUFHQWDWWKHGHHSHWSDUWRI WKHRFHDQ ,V HIIHFWRQK\GURVWDWVZRXOGEHDERXWKDO RI SHUFHQW7KXVZHDVXPHFRQVWDQWGHQVLW VLQTXLGK\GURVWDWFDOFXODWRQV IRUZKLFK(T LQWUDUVVR

Liquids:	$p_2 - p_1 = -\gamma(z_2 - z_1)$
or	$z_1 - z_2 = \frac{p_2}{\gamma} - \frac{p_1}{\gamma}$

(2.14)

: HXVHWKHILWWRUP LQPRVWSUREOPV 7KHTXDQWW LVFDOOHG WKHVSHFLILF ZHJKWRI WKHIOXLG ZLWKGLPHQVLRQV RI ZHJKWSHUXQLWYROXPH VRPHYDOXHVDUHXDGLQ7DEOH 7KHTXDQWWS LVDOHQJWKFDOOHG WKHSUHVVXUHKHDGRI WKHIOXLG

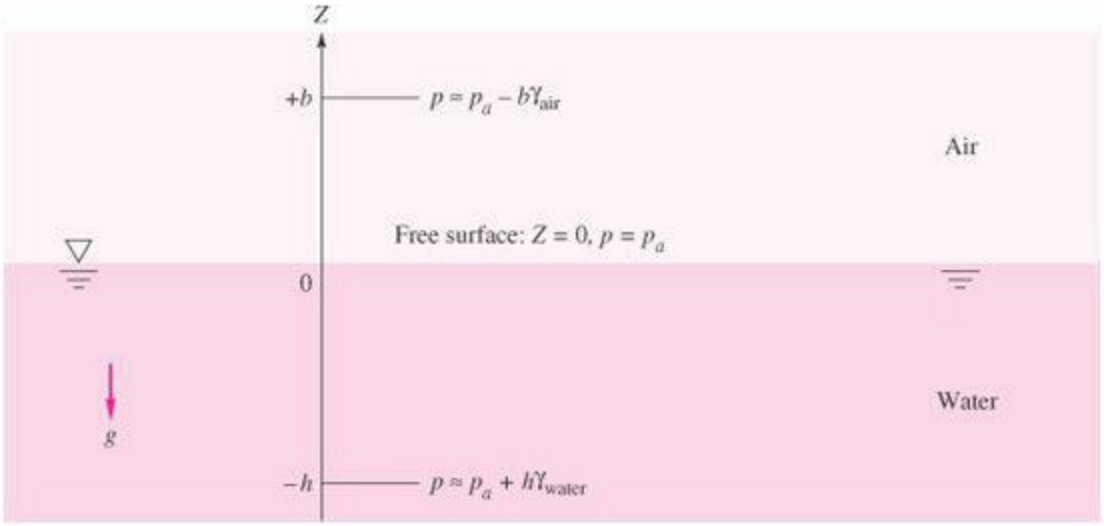
7DEOH 6SHFLILF: HJKWRI 6RPH&RPPRQ) OXLGV

Fluid	Specific weight γ at 68°F = 20°C	
	lbf/ft^3	N/m^3
Air (at 1 atm)	0.0752	11.8
Ethyl alcohol	49.2	7,733
SAE 30 oil	55.5	8,720
Water	62.4	9,790
Seawater	64.0	10,050
Glycerin	78.7	12,360
Carbon tetrachloride	99.1	15,570
Mercury	846	133,100

) RUOLTXLGV DQG RFHDQV WKH FRRUGLQDWH V\VWHP LV XVXDOO\ FKRVHQ DV LQ) LJ ZLWK] DWWKH IUHH VXUIDFH ZKHUH S HTXDOV WKH VXUIDFH DWPRVSKHULF SUHVVXUH S$: KHQ ZH LQWURGXFH WKH UHHQWU YDOXH S] S$ (T EHFRPHV IRUS DWDQ QHJDWLYH GHSWK]

$$p = p_a - \gamma z \qquad (2.15)$$

ZKHUH LV WKH DYHUDJH VSHFLILF ZHLJKW RI WKH OLNHRUHDQ $ VZHVKDOOVHH (T KRGGVL WKHDWPRVSKHUHOR ZLWKDQDFFXUDF\ RI SHUFHQWRUKLJKW] XS WR P



) LJ +\GURVWDWLF SUHVVXUH GLVWULEXWLRQLQRFHDQVDQGDWPRVSKHUHV

(; $0 3/ (

1HZIRXQG/ DNH DIUHVKZDWUDNHQHDU%ULWWRO1HZ +DPSVKLUH KDVDPD[LPXP GHSWKRI P DQGWKHPHDQDWRVSKHULF SUHVVXUHLV N3D (VWLPDWHWKHDEVROXWSUHVVXUHLQN3DDWWKLVPD[LPXP GHSWK

6ROXWLRQ

- 6\VWHP VNHWFK ,PDJLQH WKDW) LJ LV 1HZIRXQG/ DNH ZLWKK P DQG] DWWKH

VXUIDFH

- 3URSHUWYDXHV)URP 7DEOH ZDWHU 1 P : HDUHJLYHQWKDWS$RV N3D
- 6RXWRQVWSV $SSO(T VRWKHGHSHWSRLQW8VH6, XQLW SDVFDOV QRWKLQJRSDVFDOV

$$p_{max} = p_a - \gamma z = 91{,}000\ \text{Pa} - (9790\ \frac{\text{N}}{\text{m}^3})(-60\ \text{m}) = 678{,}400\ \text{Pa} \approx 678\ \text{kPa} \quad Ans.$$

- &RPPHQWV . LQRSDVFDOVDUHDZNZDUG 8VHSDVFDOVLQWKHIRUPXOD WKHQFRQYHUWWKHDQZHU

7KH0 HUFXU\ %DURP HWHU

7KHVLPSOHWWSUDFWLFDOGSSOLFDWLRQRI WKHK\GURVWDWLF IRUPXOD LV WKHEDURP HWHU)LJ ZKLFKPHDVXUHVDWPRVSKHULF SUHVVXUH $ WXEHLVILOOHGZLWKPHUFXU DQGLQYHUWHGZKLOHVXEPHUJHGL DUHVHUYRLU 7KLVFDXVHVDQHDUYDFXXP LQWKHFORVHGXSSHUHQG EHFDXVHPHUFXU KDVDQH[WHPHO VPDOOYDSRUSUHVVXUHDWURRP WHPSHUDWXUH 3DDW *& 6LQFHDWPRVSKHULF SUHVVXUHIRUFHV PHUFXU FROXPQWRULVHDGLVWDQFHKLQWKHWXEH WKHXSSHUPHUFXU VXUIDFHLVDWJHURSUHVVXUH)URP)LJ (T DSSOHVZLWKS DWJ KDQGS S$ DWJ

$$S_D \quad _0 \quad K$$

RU

$$h = \frac{p_a}{\gamma_M} \qquad (2.16)$$

$WWHDOHYHOVWDQGDUG ZLWKS$ 3D DQG 0 1 P IURP 7DEOH WK EDURP HWHUF KHJKWLVK P RU PP ,QWKH8QLWHG6WDWHVWKHZHDWKHUHUHIHUUHGWRWKLVDVDQDWPRVSKHUF SUHVVXUH RI LQ-J LQFKHVRI PHUFXU 0HUFXU LVXVHEHFDXVHILWVWKHKDYLHVWFRPPRQOLTXLG $ ZDWUEDURP HWUZRXOGEH IWKLJK

+\GURXWDWF3UHVVXUHLQ* DVHV

* DVHV DUH FRPSUHVVLEOH ZLWK GHQVLW QHDU SURSRUWRQDO WR SUHVVXUH 7KXV GHQVLW PXWHG DV D YDULDEOH LQ(T LI WKH LQWHUDFWLRQ FDUULHV RYHU GLJH SUHVVXUH FKDQJHV ,WV VXIILFLHQW DFFXUDWH WR LQWURGXFH WKH SHUHFWDVOZ S !57 LQ(T

6HSDUDWH WKH YDULDEOHV DQG LQWHJUDWH EHWZHHQ SRLQWV DQG

7KH LQWHJUDO RYHU] UHTXLUHV DQ DVVXPSWLRQ DERXW WKH WHPSHUDWXUH YDULDWLRQ7] 2QH FRPPR DSSUR[LPDWLRQ LV WKH LVRWKHUP DODWF RVSKHUH ZKHUH7 7

7KH TXDQWLW LQ EUDFNHWV LV GLPHQVLRQOHVV 7KLQN WKDW RYHU LWV XVEH GLPHQVLRQOHVV UJKW (TXDWLRQ LV D IDLU DSSUR[LPDWLRQ IRU HDUK EXW DFWXDOO WKH HDUKV PHDQ DWF RVSKHUH WHPSHUDWXUH GURSV RI I QHDU0 QQHDU0 ZLWK] XS WR DQ DOWLWXGH RI DERXW IW P

+HUH7 LVVHD OHYHO DWP SHUDWXUH DEVROXW DQG%LVWKH ODSVH UDWH RIWKH ZKLFK YDU VRP HZ KD IURP GD\ WR GD\

7KH6WDQGDUG $WP RVSKHUH

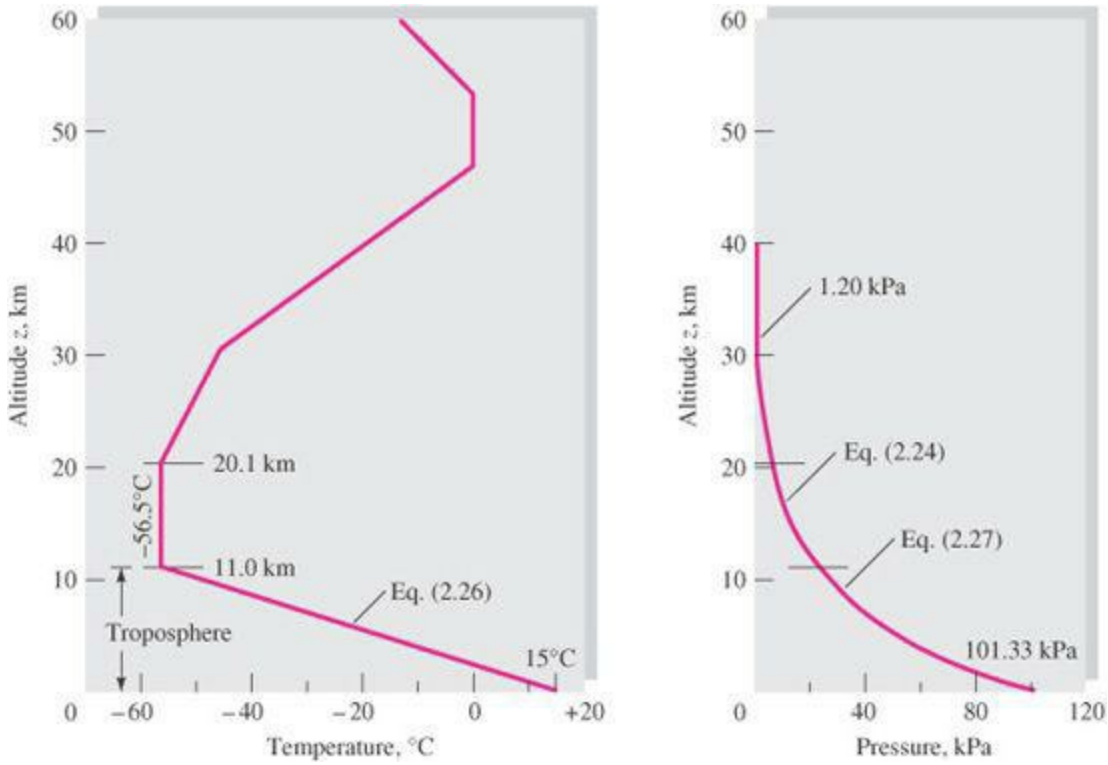
%\ LQWHJUDWLQJ RQ DJUHHPHQW> @WKH IROORZLQJ WDQGDUG YDOXHV DUH DWXPHG WR DSSO IURP VI IW

7 *f*5 . *f*&
% *f*5 IW . P

7KLV OHZ HUSRUWRQ RI WKH DWF RVSKHUH LV FDOOHG WKH WURSRSKHUH ,QWURGXFLQ(T LQW DQG LQWHJUDWLQJ ZHREWLQ WKH P RUHDFFXUDWH UHDWLQ

LQWKHWURSRVSKHUH ZLWK] DVWHDGHYHO 7KHH[SRQHQW 5% LVGLPHQVLRQOHVV DJDLQLWPXV EH DQGKDVWKHVDQGDUGYDOXHRI IRUDLU ZLWK5 P V .

7KH8 6 VWDQGDUGDWPRVSKHUH> @LVVNHWKHGLQ) LJ 7KHSUHVVXUHLVVHHQWREHQHDUO] HUDW NP)RUDEXODWGSURSHUWHVVHH7DEOH\$



) LJ 7HPSHUDWXUHDQGSUHVVXUHGLVWULEXWLRQLQWKH8 6 VWDQGDUGDWPRVSKHUH*From Ref. 1.)*

(;\$03/(

,I VHDOHYHOSUHVVXUHLV 3D FRPSXWWKHVWDQGDUGSUHVVXUHDWDQDOWLWXGHRI P XVLQ D WKHH[DFWRUPXODQG E DQLVRWKHUPDODWPRVSKHUDQDVWDQGDUGVHDOHYHOWHPSHUDWXUHRI *f*& , WKHLVRWKHUPDOSSUR[LPDWLRQDQGHTXDW

6RROXWLRQ

3DUWD

8VHEVROXWWPSHUDWXUHLQWKH[DFWRUPXOD (T

$$p = p_a\left[1 - \frac{(0.00650\ \text{K/m})(5000\ \text{m})}{288.16\ \text{K}}\right]^{5.26} = (101{,}350\ \text{Pa})(0.8872)^{5.26}$$

$$= 101{,}350(0.5328) = 54{,}000\ \text{Pa} \qquad \textit{Ans. (a)}$$

7KLVLVWKHWDQGDUGSUHVVXUHUHVXOWLYHQDW P LQ7DEOH$

3DUWE

,IWKHDWPRVSKHUHZHUHLVRWKHUPDODW . (T ZRXOGDSSO

$$p \approx p_a \exp\left(-\frac{gz}{RT}\right) = (101{,}350\ \text{Pa}) \exp\left\{-\frac{(9.807\ \text{m/s}^2)(5000\ \text{m})}{[287\ \text{m}^2/(\text{s}^2 \cdot \text{K})](288.16\ \text{K})}\right\}$$
$$= (101{,}350\ \text{Pa}) \exp(-0.5929) \approx 56{,}000\ \text{Pa} \qquad \textit{Ans. (b)}$$

7KLV LV SHUFHQWKLJKHU WKDQ WKH H[DFWUHVXOW 7KH LVRWKHUPDO IRUPXOD LV LQDFFXUDWH LQ WK
WURSRVSKHUH

,VWKH/ LQHDU) RUPXOD$GHTXDWHIRU* DVHV"

7KHOQHDUDSSUR[LPDWRQIURP (T /Sí!J/] LVVDLGWDFWRU IRUOQTXLGV ZKLFK DUHQDU
LQFRPSUHVVLEOH) RUJDVHV LWVLQDFFXUDWXQDWV/] LVUDWKHUPDOO3UREOP 3 DVV\RXVRKRZ
E\ ELQRPLDOH[SDQVLRQRI (T WKDWWKHHUURULQXVLQJ FRQVWDQWDVGHQVLWVWHXPDW/SIURP
(T LVVPDOOL

$$\delta z \ll \frac{2T_0}{(n-1)B} \qquad (2.21)$$

ZKHUH7R LVWKHORFDOEVROXWHWHPSHUDXUH %LVWKHODSVHUDWIURP (T DQGQ J 5% LV
WKHH[SRQHQWQ(T 7KHHUURULVOHVVWKDQ SHUFHQWL/] P

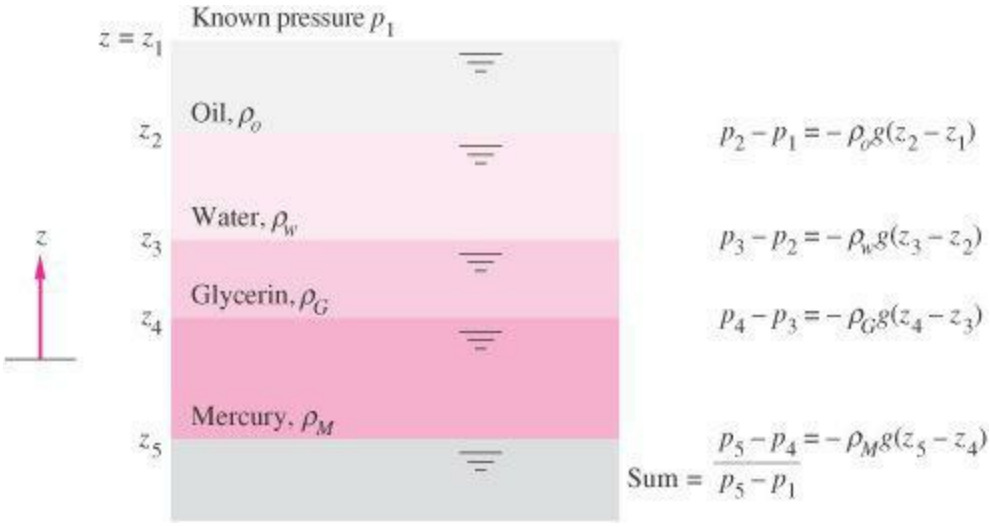
$SSOLFDWLRQVWR0 DQRP HWU

)URP WKHK\GURVWDWF IRUPXOD DFKDQJHLQHOHYDWLRQ] í] RI DOTXLGLVHTXLYDOHQWWR I
FKDQJHLQSUHVVXUH S í S 7KXVDVWDWF FROXPQRI RQHRUPRUHOTXLGVRUJDVHVFDQEHXVHGV
PHDVXUHSUHVVXUHGLIIHUHQFHVEHWZHHQWZRSRLQWV 6XFKDGHYLFHLVFDOOHGDPDQRPHWU ,I PXOWSO
IOXLGVDUHXVHG ZHPXVWFKDQJHWKHGHQVLWVLQWKHIRUPXODVZHPRYHIURP RQHIOXLGWRDQRWKH
)LJXUH LOOXVWUDWVWKHXVHRI WKHIRUPXODZLWKDFROXPQRI PXOWSOHIOXLGV 7KHSUHVVXUHFKDQJH
WKURXJKHDFKIOXLGLVFDOFXODWGVHSDUDWO ,IZHZLVKWRNQRZ WKHWRWDOFKDQJHS í S ZHDGGWK
VXFFHVVLYHFKDQJHVS í S S í S S í S DQGS í S 7KHLQWHUGDWYDOXHVRI SFDQFHODQGZI
KDYHIRUWKHH[DPSOHR) LJ

$$p_5 - p_1 = -\gamma_0(z_2 - z_1) - \gamma_w(z_3 - z_2) - \gamma_G(z_4 - z_3) - \gamma_M(z_5 - z_4) \quad (2.22)$$

1RDGGLWLRQDOLPSOLFDWLRQLVSRVVLEOHRQWKHULJKWKDQGVLGHEHFDXVHRI WKHGLIIHUHQWGHQVLWV

1 RWH WKDW HKDYH SODFHG WKH IOXLGV LQ RUGHU IURP WKHLU NQRZQ VSHFLILF ZHLJKWV WRS WR ERWWRP 7KL
LV WKH RQO\ VLWXDWLRQ , ZLOO XVH , ZLOO DOZD\V WDNH WKH PDQRPHWHU UHDGLQJ RXW IURP WKH IOXLGV ZLO
RYHUO\LQJ DQG VHHN WKH VLWXDWLRQV , ZLOO HPSOR\ WR GHWHUPLQH WKH



) LJ (YDOXDWLQJ SUHVVXUH FKDQJHV WKURXJK D FROXPQ RI PXOWLSOH IOXLGV

3UHVVXUH, QFUHDVHV' RZQZDUG

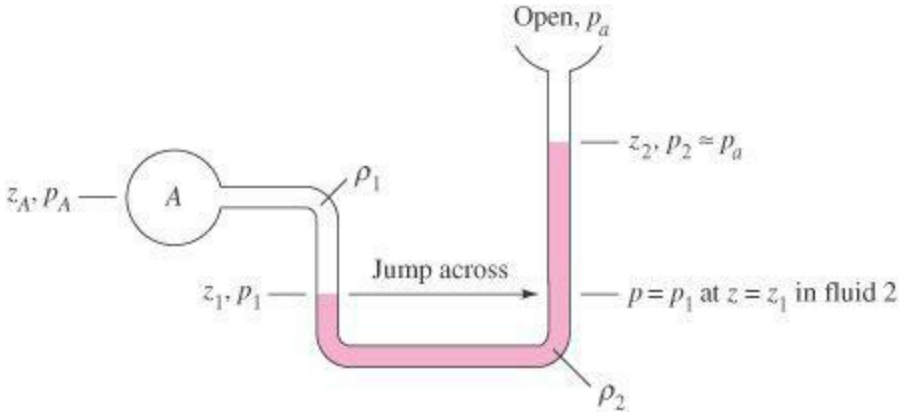
7KH EDVLF K\GURVWDWLF UHODWLRQ (T LV PDWKHPDWLFDOO\ FRUUHFW EXW YH[LQJ WR HQJLQHHUV EHFDXVH LW FRPELQHV WZR QHJDWLYH VLJQV WR NHHS WKH SUHVVXUH LQFUHDVHG GRZQZDUG : KHQ FDOFXODWLQJ K\GURVWDWLF SUHVVXUH FKDQJHV HQJLQHHUV ZRUN LQWXLWLYHO\ E\ VLPSO\ KDYLQJ WKH SUHVVXUH LQFUHDVH GRZQZDUG DQG GHFUHDVH XSZDUG ,I SRLQW 1 DQG SRLQW 2 DUH DW WKH VDPH HOHYDWLRQ LQ D XQLIRUP OTXLG WKHQ S ! JK ,QWKH PHDQLQJ RI (T UHPDLQV DFFXUDWH DQG VDIHLI XVHG SURPHUO\ ) RUH[DPSOH (T LV FRUUHFW DV VKRZQ RU LW FRXOG EH UHZULWWQ LQ WKH IRORZLQJ ³PXOWLSOH GRZQZDUG LQFUHDVH PRGH

$$p_5 = p_1 + \gamma_0 |z_1 - z_2| + \gamma_w |z_2 - z_3| + \gamma_G |z_3 - z_4| + \gamma_M |z_4 - z_5|$$

7KDWLV NHHS DGGLQJ RQ SUHVVXUH LQFUHPHQWV DV \RX PRYH GRZQ WKURXJK WKH OD\HUHG IOXLG $
GLIIHUHQWLDSSOLFDWLRQVLP DQRPHWHU ZKLFK LQYROYHVERWK³XS´ DQG³GRZQ´ FDOFXODWLRQV

$SSOLFDWLRQ $ 6LPSOH0 DQRPHWHU

) LJXUH VKRZV DVLPSOH8 VHEHRSHQPDQRPHWHUWKDWPHDVXUHVWKHJDJHSUHVVXUHS$ UHODWLYHWR WKHDWPRVSKHUH S' 7KHFKDPEHUIOXLG! LVVHDUDWGIURP WKHDWPRVSKHUHE\ DVHFRQG KHDYLHUIOXLG! SHUKDSVEHFDXVHIOXLG$ LVFRUURVLYH RUPRUHQN0 EHFDXVHDKHDYLHUIOXLG! ZLOONHS] VPDO DQGWKHRSHQWXEHFDQEHVKRUWHU



) LJ 6LP SOH IRSHQ PDQRP HWHU IRU PHDVXULQJ p_A UHDGLQJ YHUVXV DQ RSHQ PDQRPHWHU

: HILUVW DSSO\ WKH K\GURVWDWLF IRUPXOD IURP \$GRZQ WR] 1RWH WKDW WKH GHQVLW\ FDQ WKHQ JR GRZQ WR WKH ERWWRP RI WKH 8 WXEH DQG EDFN XS RQ WKH ULJKW WR] DQG WKH SUHVVXUH ZLOO EH WKH VDPH S S 7KXV ZH FDQ³MXPS DFURVV´ DQG WKHQ XS WR OHYHO]

$$p_A + \gamma_1 |z_A - z_1| - \gamma_2 |z_1 - z_2| = p_2 \approx p_{\text{atm}} \qquad (2.23)$$

\$QRWKHU SK\VLFDO UHDVRQ WKDW ZH FDQ³MXPS DFURVV´ DW WHVWRQ LV WKDW DFRQWLQXRXV OHQJWK RI WK VDPH IOXLG FRQQHFWV WKH VHWZR HTXDO HOHYDWLRQV 7KH K\GURVWDWLF UHDWLRQ UHTXLUHV WKLV HTXDO DV DIRUP RI 3DVFDO¶V ODZ

7KLV LGHD RI MXPSLQJ DFURVV VR HTXDO SUHVXUHV IDFLOLWDWHV PXOWSOH IOXLG SUREOHPV ,W ZLOO E LQDFFXUDWH KRZ HYHUL WKH UHDUHEXEEOHV LQ WKH IOXLG

\$Q VZR SRLQWV DW WKH VDPH HOHYDWLRQ LQ DFRQWLQXRXV PDVV RI WKH VDPH VWDWLF IOXLG ZLOO EH WKH VDPH SUHVXUH

(; \$0 3/ (

7KH FDVLF XVH RI DP DQRP HWHU LV ZKHQ WZR 8 WXEH GHJ VDUH RI HQJWK DV LQ) LJ (DQ WKH PHDVXUHPHQW YROYHV DSUHVXUH GLIIHUHQFH DFURVV WZR KRU] RQWDO SRLQW 7KH WWSLFDO DSSOLFDWLR LV WR PHDVXUH SUHVXUH FKDQJH DFURVV DIORZ GHYLFH DV VKRZ Q ' HULYH D IRUPXOD IRU WKH SUHVXUH GLIIHUHQFH S D í S E LQWHUPV RI WKH VSHFLPHWLV LQ) LJ (

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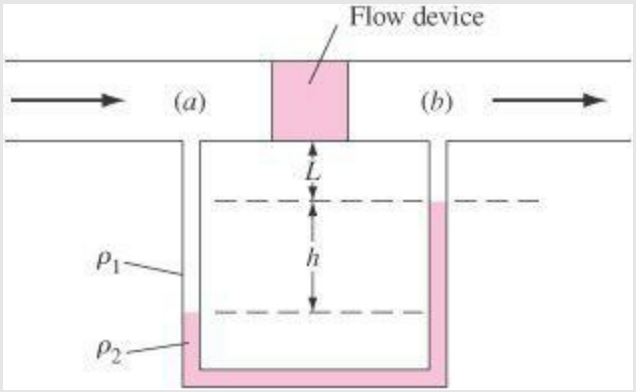


6RO XWLRQ

8VLQJ (T VWDWLFDWD HYDOXDWH SUHVVXUHFKDQJHVDURXQGWKH8 VKEH DQGHQGXS DWE

$$p_a + \rho_1 gL + \rho_1 gh - \rho_2 gh - \rho_1 gL = p_b$$

$$p_a - p_b = (\rho_2 - \rho_1)gh \qquad Ans.$$

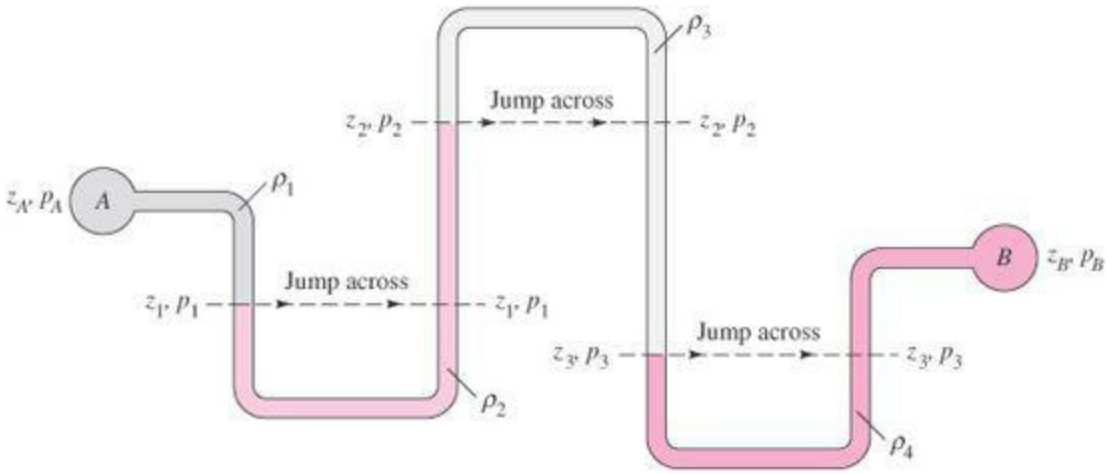
RU

7KHPHDVXUHPHQWRQ LQFXGHVK WKHPDQRPHWUHDGLQJ 7HUPVLQYROYLQJ / GURS RXW1 RWWK DSSHDUDQFHRI WKHGLIIHUHQFHLQGHQVLWHVEHWZHQPDQRPHWUIOXLGDQGZRUNLQJ IOXLG ,WLVDFRPPR VXGHQWHUURUWRIDLOWRVXEWDFWRXWKHZRUNLQJ IOXLG GHQVLW! ² DVHUURXVHUURULI EWKIOXLGVDU QTXLGVDQGGHWVGLVDXWRXVQPHUFDO LI IOXLG LVDJDV $FDGHPLFDO RI FRXUVH VXFKDQHUURULV DQZDVFRQVLGHUHGVHUURXVE\ IOXLGPHFKDQFVLQWXKFWRUV

$QWRXJK([DPSOH EHFDXVH RI LWV SRSXODULW LQ HQJLQHULQJ H[SHULPHQW LV VRPHWPH FRQVLGHUHGWR EHWKH³PDQRPHWUIRUPXOD´ LWLVEHWQRWWRPHPRU]HLWEXWDKHUWRDGDSW(T WR HDFKQHZ PXOWSOH IOXLGK\GURVWDWFVSUREOP)RU[DPSOH) LJ LOOXWDWV DPXOWSOH IOXLG PDQRPHWU SUREOP IRU ILQGLQJ WKH GLIIHUHQFH LQ SHWXUE EHWZHQ WZR FKDPEHUV$ DQG% : H UHSHDWGO DSSO(T MXPSLQJ DFURVWDWHTXDOSUHVXUHVZKHQZHFRPHWR DFRQWQXRXVPDV RI WKHVDPHIOXLG 7KXV LQ) LJ ZHFRPSXWHIRUSUHVXUHGLIIHUHQFHVZKLHPDNLQJWKUHHMXPSV

$$\begin{aligned} p_A - p_B &= (p_A - p_1) + (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_B) \\ &= -\gamma_1(z_A - z_1) - \gamma_2(z_1 - z_2) - \gamma_3(z_2 - z_3) - \gamma_4(z_3 - z_B) \end{aligned} \tag{2.24}$$

7KHLQWUPHGLDWSUHVXUHVS FDQFHO,WBRNVFRPSOFDWG EXWUHDOO LWLVPHUHO VHTXHQWDO 2QHVWDWW DV$ JRHVGRZQWR MXPSVDFURVV JRHVXS WR MXPSVDFURVV JRHVGRZQWR MXPS DFURVV DQGILQDOO JRHVXS WR%

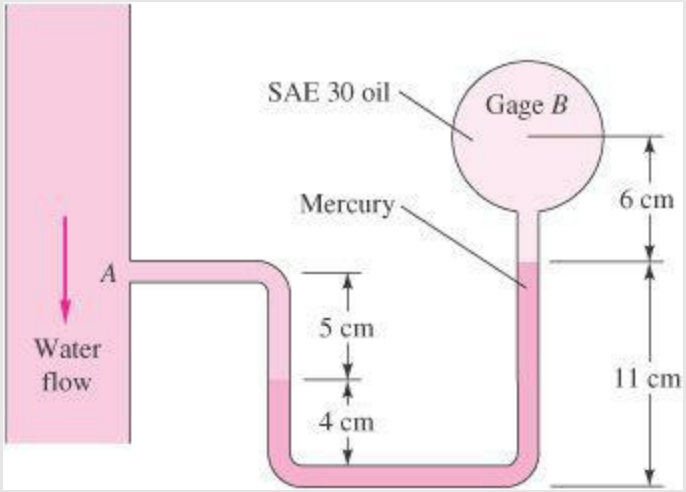


) LJ $ FRPSOFDWG PXOWSOH IOXLG PDQRPHWU WR UHDWp$_A$ WRp$_B$ 7KLV VWXWMP LV QR HVSHFLDOO SUDFWFDOEXWP DNHVDJRRGKRPHZRUNRUH[DPLQDWRQSUREOP

(; $0 3/ (

3UHVVXUH JDJH %LV VRPHDVXUH WKH SUHVVXUH DW SRLQW $ LQ D ZDWHU IORZ ,I WKH SUHVVXUH DW %LV NQD HWLP DW WKH SUHVVXUH DW $ LQNQD $ VXP HDQG IOXLG VDUH DW f& 6HH) LJ (

(



6RXWLRQ

- 6\VWHP VNHWFK 7KH\VWHP LV VKRZQ LQ) LJ (
- $VVXPSWLRQV +\GURVWDWLF IOXLGV QRPL[LQJ YHUWLFDO³XS´ LQ) LJ (
- $SSURDFK 6HTXHQWLDO XVH R (T VRJRIURP $ VR %
- 3URSHUWLHV) URP 7DEOH RU7DEOH $

$$\gamma_{water} = 9790\ \text{N/m}^3; \qquad \gamma_{mercury} = 133{,}100\ \text{N/m}^3; \qquad \gamma_{oil} = 8720\ \text{N/m}^3$$

- 6RXWLRQ VWHSV 3URFHHG IURP $ VR % ³GRZQ´ VKHQ³XS´ NXPSLQJ DFURVV DVWKH OHIWPHUFXU PHQLVFXV

$$p_A + \gamma_w |\Delta z|_w - \gamma_m |\Delta z_m| - \gamma_o |\Delta z|_o = p_B$$

RUS$ 1 P P 1 P P 1 P P

RUS$ 6RYHIRUS$ 1 P § NQD$QV

- &RPPHQW 1RWLFH DEEUHYLDWLRQV 1 P VRSDFDO RU3D 7KHLQWHUQDWLRQDO ILYH-ILJXUH UHVXOWS$ 3D LV XQHDOWHF VLQFH WKH GDWD DUH NQRZQ WR RQO DERXWKUH VLJQLILFDQWLJXUHV

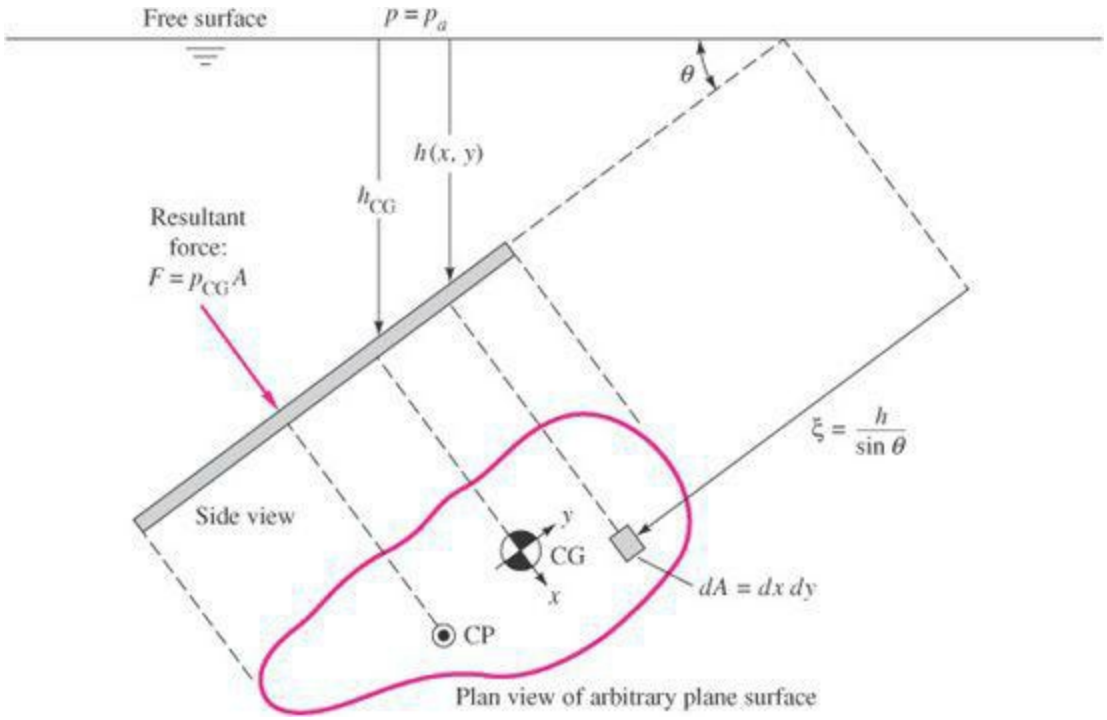
,QPDNLQJ WKHVHPDQRPHWUFDOFXODWRQVZHKDYHQHJOHFWHG WKHFDSLOODU KHLJKWFKDQJHVGXHV VXUDFH WHQVLRQ ZKLFK ZHUH GLVFXVVHG LQ([DPSOH 7KHVH HIHFWV FDQFHOLL WKHUH LV D IXL LQWHUIDFH RUPHQVFXV EHWZHQVLPLODUIOXLGVRQERWKVLGHVRI WKH8 VXEH 2WKHUZLVH DVLQWHUJK KDQG8 VXEHRI)LJ DFDSLOODU FRUUHFWRQFDQEHPDGHRUWKHHIHFWFDQEHPDGHQHJOLJLEOHE XLQJ ODUJHERUH FP VXEHV

+\GURVWDWLF)RUFHVRQ3ODQH6XUIDFHV

7KHGHVLJQRI FRQWDLQPHQWWDQNVUHTXLUHV FRPSXWDWLRQRI WKHK\GURVWDWLF IRUFHVRQ YDULRX VRQG VXUDFHVDGMDFHQWWR WKHIOXLG 7KHVHIRUFHVUHDWWR WKHZHLJKWRI IOXLGEHDULQJ RQWKHVXUDF) RUH[DPSOH DFRQWDLQHUZLWKDIODWKRUL]RQWDOERWWRP RI DUHD$E DQGZDWUGHSWK+ ZLOOH[SHULHQF DGRZQZDUGERWWRP IRUFH)E +$E ,I WKH VXUDFHLV QRWKRUL]RQWDO DGGLWLRQDOFRPSXWDWLRQVDU QHHGHGWRILQGWKHKRUL]RQWDOFRP SRQHQWRI WKHK\GURVWDWLF IRUFH

,I ZHQHJOHFWHGWHQVLWFKDQJHVLQWKHIOXLG(T DSSOHVDQGWKHSUHVXUHRQDQ VXEPHUJHG VXUDFH YDULHV OLQHDUO ZLWK GHSWK) RU D SODQH VXUDFH WKH OLQHDU VWUHVV GLVWULEXWLRQ LV H[DFW DQDORJRXV WR FRPELQHG EHQGLQJ DQG FRPSUHVVLRQ RI D EHDP LQ VWUHQJWK RI PDWHULDOV WKRU 7K K\GURVWDWLF SUREOHP WKXVUHGXFHVWR VLPSOHIRUPXODVLQYROYLQJ WKHFHQWURLG DQGPRPHQWRI LQHUWLDR WKHSODWHFURVVVHFWLRQDODUHD

)LJXUH VKRZVDSODQHSDQHORI DUEWUDU\ VKDSHFRPSOHWHO VXEPHUJHGLQDOLTXLG 7KHSDQ SODQHPDNHVDQ DUEWUDU\ DQJOH ZLWK WKH KRUL]RQWDOIUHHVXUDFH VR WKDWWKH GHSWKYDULHVRYHUWK SDQHOVXUDFH ,IKLVWKHGHSWKWRDQ HOHPHQWDUHDG$ RI WKHSODWH IURP (T WKHSUHVXUHWKHU LVS S$ K



) LJ + \ GURVWDWLF IRUFH DQG FHQWURLG SUHVVXUH RQ DQ DUELWUDU SODQH VXUIDFH RI DUHD1 LQFOLQHG DWDQDQJOH EHORZ WKH IUHH VXUIDFH

7R GHULYH IRUPXODV LQYROYLQJ WKH SDUWVKDSH HVDEOVKDQ[\ FRRUGLQDWH\ VWHP LQWKH SODQH R VWHSODWHZ LWKWKHRULJLQDWWKHFHQWURLG SOXVDGXPP\ FRRUGLQDW GRZQIURP WKHVXUIDFHLQWKHSODQ RI WKHSODW 7KHQWKHVWDWLFKGURVWDWLF IRUFHRQRQHVLGHRI WKHSODWLVJLYHQE\

$$F = \int p\,dA = \int (p_a + \gamma h)\,dA = p_a A + \gamma \int h\,dA \qquad (2.25)$$

7KHUHP DLQLQJ LQWHJUDOLVHYDOXDWHGE\ QRWLFLQJ IURP) LJ WKDWK VLQ DQG E\ GHILQLWLRQ VWHFHQWURLGDOVDQWGLVWDQFHIURP WKHVXUIDFHWR WKHSODWLV

$$\xi_{CG} = \frac{1}{A}\int \xi\,dA$$

7KHUHRUH VLQFH LVFRQVWDQWRQ WKHSODW (T EHFRPHV

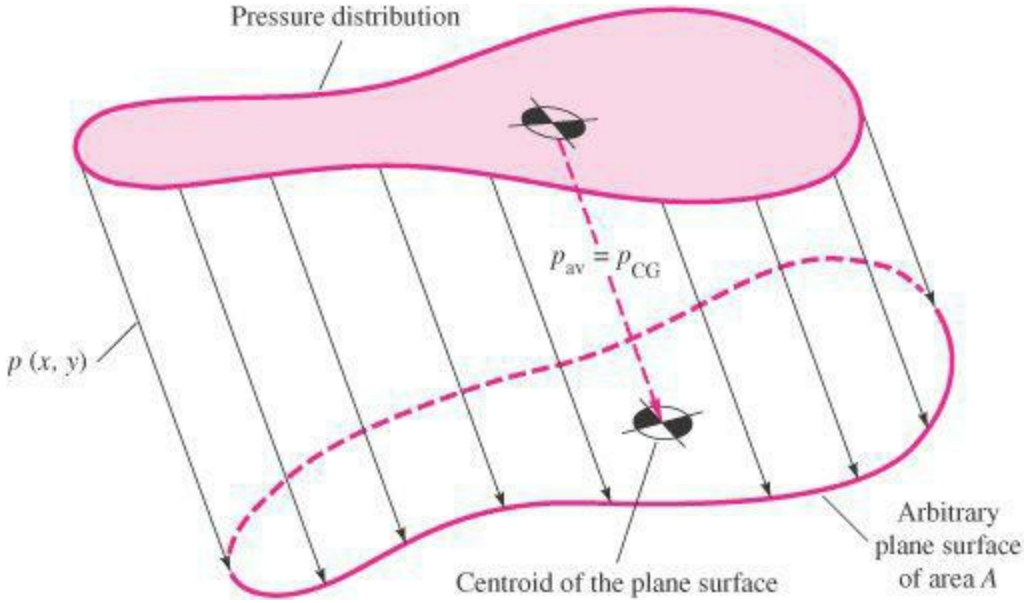
$$F = p_a A + \gamma \sin\theta \int \xi\,dA = p_a A + \gamma \sin\theta\, \xi_{CG} A$$

) LQDOO XQLDYHOWKLVE\ QRWLFLQJ WKDW$_{&*}$ VLQ K$_{&*}$ WKHGHSWKWKGRZQIURP WKHVXUIDFHWR WKHSODWFHQWURLG 7KXV

$$\boxed{F = p_a A + \gamma h_{CG} A = (p_a + \gamma h_{CG})A = p_{CG} A} \qquad (2.26)$$

7KHIRUFHRQRQHVLGHRI DQ SODQHVXEP HUJHGVXUIDFHLQDXQLRUP IOXLGHTXDOVWKHSUHVVXUHDWWK SODWFHQWURLGWLP HVWKHSODWDUHD LQGHSHQGHQWRI WKHVKDSHRI WKHSODWRUWKHDQJOH DWZKLFKLWLV VODQWHG

(TXDWLRQ FDQ EH YLVXDOL]HG SK\VLFDOO LQ) LJ DV WKH UHVXOWDQWRI D GLVWULEXWHG GLVWULEXWLRQRYHUWKHSODWDUHD 7KLVVLP XODWHVFRP SRVLWLRQHG FRP SRQHQW RI DEHDP RI WK VDP H FURVV VHFWLRQ ,WSRLQWVWKDWWKH³EHQGLQJ´ SRUWLRQ RI WKHVXUHV FDXVHVQR IRUFHLI LW³QHXWD[LV´ SDVVHVWKURXJK WKHSODWFHQWURLGRI DUHD 7KXVWKHUHP DLQLQJ³FRP SRQHQW SDUWP XVWHTXDOWK FHQWURLGVUHVWVP HVWKHSODWDUHD 7KLVLVWKHUHVXOWRI (T



) LJ 7KH K\GURVWDWLF SUHVVXUH IRUFH RQ D SODQH VXUIDFH LV HTXDO UHJDUGOHVV RI LWV VKDSH WR WKH UHVXOWDQW VKHDULQJ IRUFH GLVWULEXWLRQ RQ WKH VXUIDFH $ p &*A

+RZHYHU WR EDODQFH WKH EHQGLQJ PRPHQWSRUWRQ RI WKH VWUHVV WKH UHVXOWDQW RFFXUV EHORZ WKURXJK WKH FHQWURLG EXW EHORZ LWWRZDUG WKH KLJK SUHVVXUH VLGH ,WV OLQH RI DFWLRQ SDVVHV WKURXJK WKH FHQWUH RI SUHVVXUH &3 RI WKH SODWH DV VNHWFKHG LQ) LJ 7R ILQG WKH FRRUGLQDWHV [&3 \&3 ZH VXP PRPHQWV RI WKH HOHPHQWDO IRUFHS G$ DERXW WKH FHQWURLG DQG HTXDWH WR WKH PRPHQW RI WKH UHVXOWDQ) 7R FRPSXWH \&3 ZH HTXDWH

$$Fy_{\mathrm{CP}} = \int yp\,dA = \int y(p_a + \gamma\xi\sin\theta)\,dA = \gamma\sin\theta\int y\xi\,dA$$

7KH WHUP HS D\ G$ YDQLVKHV EHFDXVH WKH RULJLQ RI FRRUGLQDWHV ,QWURGXFLQJ &* í \ ZH REWDLQ

$$Fy_{\mathrm{CP}} = \gamma\sin\theta\left(\xi_{\mathrm{CG}}\int y\,dA - \int y^2\,dA\right) = -\gamma\sin\theta\,I_{xx}$$

ZKHUH DJDLQ \ G$ DQG ,[[LV WKH DUHD PRPHQW RI LQHUWLD RI WKH SODWH DUHD DERXW WKH FHQWURLGDO [D[LV FRPSXWHG LQ WKH SODQH RI WKH SODWH 6XEVWLWXWLQJ IRU) JLYHV WKH UHVXOW

$$y_{\mathrm{CP}} = -\gamma\sin\theta\,\frac{I_{xx}}{p_{\mathrm{CG}}A} \qquad (2.27)$$

7KH QHJDWLYH VLJQ LQ(T VKRZV WKDW &3 LV EHORZ WKH FHQWURLG DVD GHSHUGHQGHQFH) GHSHQGV RQ DQJOH ,I ZH PRYH WKH SODWH GHSHU \&3 DSSURDFKHV WKH FHQWURLG EHFDXVH HYHU WHUP LQ(T UHPDLQV FRQVWDQW[FHSW S&* ZKLFK LQFUHDVHV

7KH GHWHUPLQDWLRQ RI [&3 LV H[DFWO VLPLODU

$$Fx_{CP} = \int xp\, dA = \int x[p_a + \gamma(\xi_{CG} - y)\sin\theta]\, dA$$

$$= -\gamma \sin\theta \int xy\, dA = -\gamma \sin\theta\, I_{xy}$$

Z KHUH,[\ LV WKH SURGXFWRI LQHUWLD RI WKH SODWH DJDLQ FRPSXWHG LQ WKH SODQH RI WKH SODWH
6XEVWLWXWLQJ IRU) JLYHV

$$x_{CP} = -\gamma \sin\theta \frac{I_{xy}}{p_{CG}A} \qquad (2.28)$$

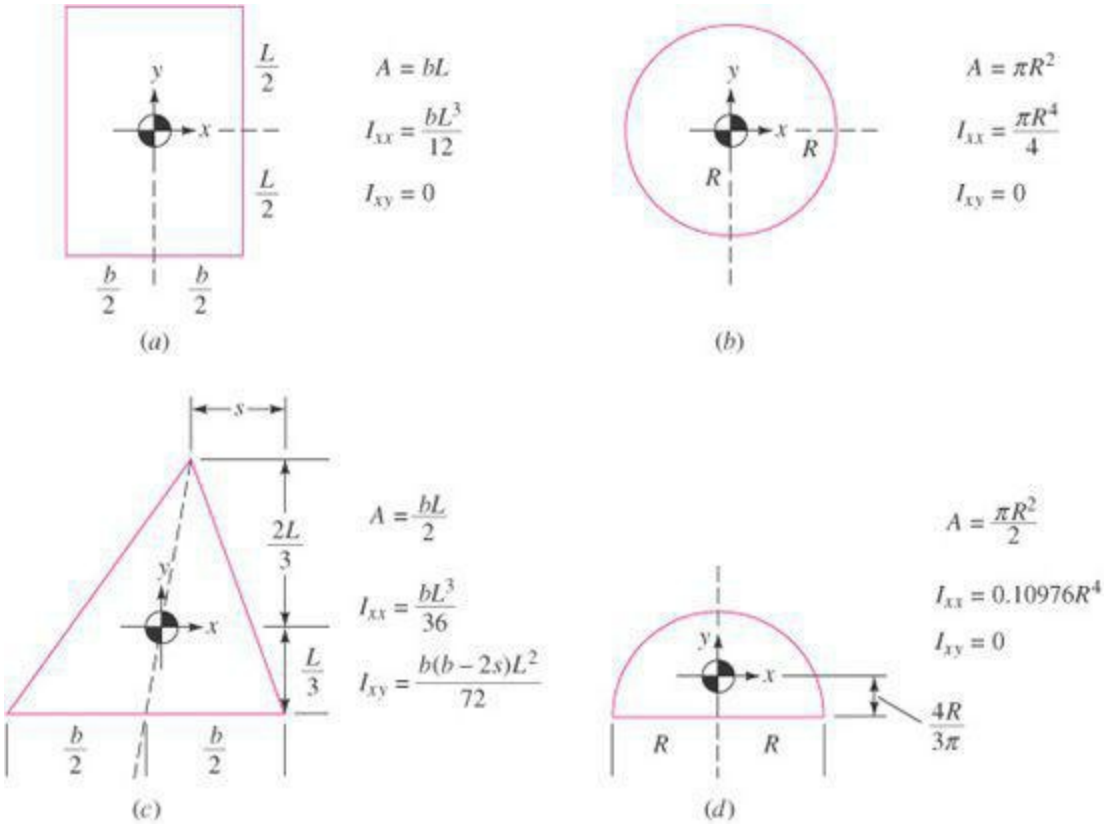
) RUSRVLWLYH,[\ [&3 LV QHJDWLYH EHFDXVH WKH GRPLQDQW SUHVVXUH IRUFH DFWV LQ WKH WKLUG RUGRZH
OHWTXDGUDQWRI WKH SDQHO ,I,[\ XVXDOO LPSOLQJ \PPHWU [&3 DQG WKH FHQWURI SUHVVXU
OLHVGLUHFWO EHORZ WKHFHQWURLGRQWKH\ D[LV

* DJH3 UHVXUH) RUP XODV

,QPRVWFDVHV WKH DPELHQWSUHVXUHS D LV QHJOHFWHG EHFDXVHLWDFWVRQERWKVLGHVRI WKHSODWH IR
H[DPSOH WKHRWKHUVLGHRI WKHSODWHLV LQLWLDOVKLS RUQWKHGU VLGHRI DJDWRUGDP ,QWKLVFDV
S&* K&* DQG WKHFHQWURI SUHVVXUEHFRPHVLQGHSHQGHQWRI VSHFLILF ZHJKW

$$F = \gamma h_{CG} A \qquad y_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} \qquad x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} \qquad (2.29)$$

) LJXUH JLYHV WKH DUHD DQG PRPHQWRI LQHUWLD RI VHYHUDO FRPPRQFURVVVHFWLRQV IRUXV
ZLWK WKHVH IRUPXODV 1RWH WKDW LV WKH DQJOH EHWZHHQ WKH SODWH DQG WKH KRUL]RQ



) LJ &HQWURLGDO PRPHQWV RI LQHUWLD IRU YDULRXV FURVV VHFWLRQV *a* UHFWDQJOH *b* FLUFOH *c* WULDQJOH DQG *d* VHPLFLUFOH

(; $0 3/ (

7KH JDWH LV KLQJHG DW SRLQW % DQG UHVWV DJDLQVW D VPRRWK ZDOO DW SRLQW $ &RPSXWH D WKH IRUFH RQ WKH JDWH GXH WR VHDZDWHU SUHVVXUH E WKH KRUL]RQWDO IRUFH3 H[HUWHG E\ WK ZDOO DW SRLQW$ DQG F WKH UHDFWLRQ DW WKH KLQJH%

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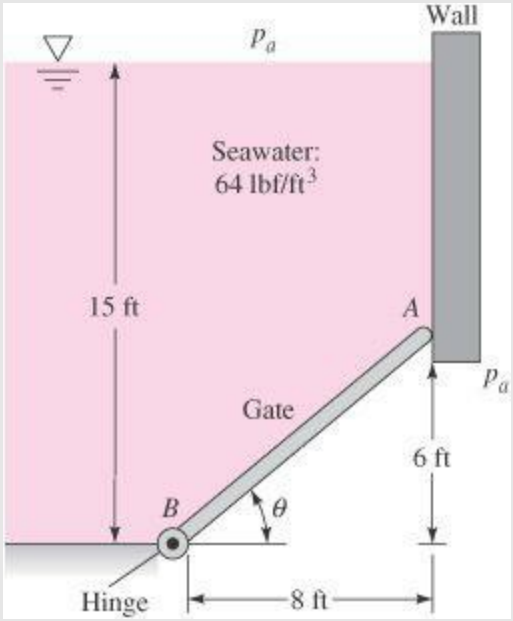


6RO XWLRQ

%\ JHRPHWU\ WKH JDWH LV IWDQG IURP $ WR % DQG LWV FHQWURLG LV KDOIZD\ EHWZHHQ RUDWHOHYDWR IWDERYH SRLQW% 7KHGHSWKK&* LVWKXV í IW7KHJDWHDUHDLV IW1HJOHFWS D DFWLQJ RQERWKVLGHVRI WKHJDWH) URP (T WKHK\GURVWDWLF IRUFHRQWKHJDWHLV

$$F = p_{CG}A = \gamma h_{CG}A = (64\ \text{lbf/ft}^3)(12\ \text{ft})(50\ \text{ft}^2) = 38{,}400\ \text{lbf} \qquad Ans.\ (a)$$

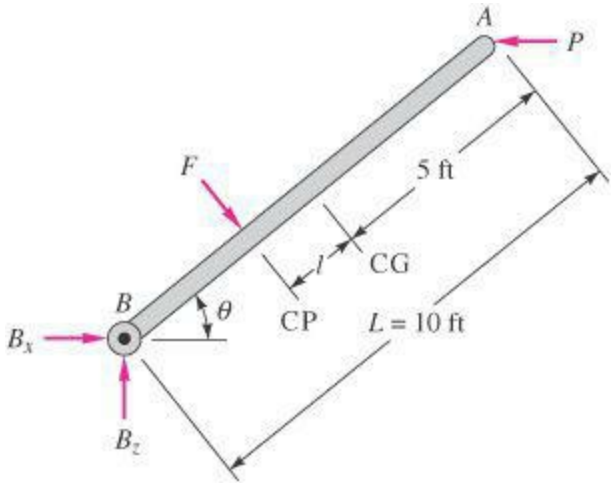
) LUVWZ HPXVWILQG WKHFHQWURI SUHVVXUH R) \$ IUHERG\ GLDJUDP RI WKHJDWHLV VKRZ QLQ) LJ (E 7KHJDWHLVDUHFWDQJOH KHQFH

$$I_{xy} = 0 \quad \text{and} \quad I_{xx} = \frac{bL^3}{12} = \frac{(5\ \text{ft})(10\ \text{ft})^3}{12} = 417\ \text{ft}^4$$

7KHGLVWDQFHIURP WKH&* WR WKH&3 LVJLYHQE\ (T VLQFHS D LVQHJDWLYH

$$l = -y_{CP} = +\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(417\ \text{ft}^4)(\frac{6}{10})}{(12\ \text{ft})(50\ \text{ft}^2)} = 0.417\ \text{ft}$$

(E



7KH GLVWDQFH IURP SRLQW% WR IRUFH) LV WKXV í Oí IW6XPPLQJ WKH PRPHQW FRXQWHUFORFNZLVHDERXW%JLYHV

$$PL\sin\theta - F(5 - l) = P(6\ \text{ft}) - (38{,}400\ \text{lbf})(4.583\ \text{ft}) = 0$$

or

$$P = 29{,}300\ \text{lbf} \qquad Ans.\ (b)$$

3DUWF

: LWK) DQG3 NQRZ Q VWHUHDFWLRQV%[DQG%\ DUH IRXQG E\ VXPPLQJ IRUFHV RQ WKH JDWH

$$\sum F_x = 0 = B_x + F \sin\theta - P = B_x + 38{,}400 \text{ lbf } (0.6) - 29{,}300 \text{ lbf}$$

or

$$B_x = 6300 \text{ lbf}$$

$$\sum F_z = 0 = B_z - F \cos\theta = B_z - 38{,}400 \text{ lbf } (0.8)$$

or

$$B_z = 30{,}700 \text{ lbf} \qquad Ans.\ (c)$$

7KLV H[DPSOH VKRZV GKDYHUHYLHZHG\RXUNQRZOHGJHRI VWDWLFV

7KH VROXWLRQ RI ([DPSOH ZDV DFKLHYHG ZLWK WKH PRPHQWV RI LQHUWLD IRUPXODV (TV 7KH\ VLPSOLI\ WKH FDOFXODWLRQV EXW RQH RVHV DSK\VLFDOLHOLQJ IRUWKHIRUFHV / HVXVUHSHDW3DUW D DQG E RI ([DPSOH XVLQJ DPRUHYLVXDODSSURDFK

(;$03/(

5HSHDW([DPSOH VR VNHWFK WKH SUHVXUHGLVWULEXWLRQ RQ SODW\$% DQG EUHDN WKLV GLVWULEXWLRQ LQR UHFWDQJXODU DQG WULDQJXODU SDUW VR VROYH IRU D WKH IRUFH RQ WKH SODW DQG E WKH FHQWURSUHVXUH

6RXWLRQ

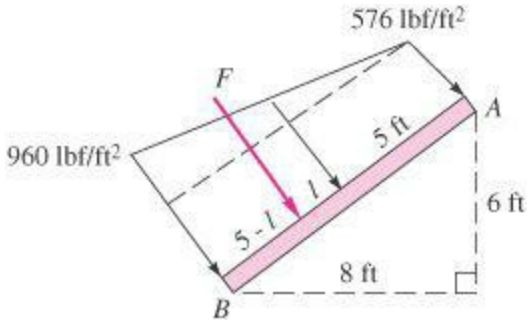
3DUWD

3RLQW$ LV IWGHHS KHQFH S$ K$ GE IW IW GE IW 6LPLODUO 3RLQW% LV I GHHS KHQFH S% K% GE IW IW GE IW 7KLV GHILQHV WKH OQHDU SUHVXUHGLVWULEXWLRQ

)LJ (7KHUHDQJOHV E\ IWE\ IWLQR WKHSDSHU 7KHWULDQJOHV í GE IW î IWE\ IW7KHFHQWURLGRI WKHUHDQJOHV IWGRZQWKHSDSHUIURP $ 7KHFHQWURLGRI WKHWULDQJOLV IWGRZQIURP $ 7KHVROXWLRQIRUFHVWKHUHDQJOHIRUFHSOXVWKHWULDQJOHIRUFH

$$F = \left(576\frac{\text{lbf}}{\text{ft}^2}\right)(10 \text{ ft})(5 \text{ ft}) + \left(\frac{384}{2}\frac{\text{lbf}}{\text{ft}^2}\right)(10 \text{ ft})(5 \text{ ft})$$

$$= 28{,}800 \text{ lbf} + 9600 \text{ lbf} = 38{,}400 \text{ lbf} \qquad Ans.\ (a)$$



3DUWE

7KHPRPHQWVRI WKHVHIRUFHVDERXWSRLQW$ DUH

$$\Sigma M_A = (28{,}800\ \text{lbf})(5\ \text{ft}) + (9600\ \text{lbf})(6.67\ \text{ft}) = 144{,}000 + 64{,}000 = 208{,}000\ \text{ft}\cdot\text{lbf}$$

Then $$5\ \text{ft} + l = \frac{M_A}{F} = \frac{208{,}000\ \text{ft}\cdot\text{lbf}}{38{,}400\ \text{lbf}} = 5.417\ \text{ft} \quad \text{hence } l = 0.417\ \text{ft} \qquad \textit{Ans. (b)}$$

&RPPHQW: HREWDLQ WKH VDPH IRUFH DQG FHQWHU RI SUHVVXUH DV LQ([DPSOH EXW ZLWK PRU XQGHUWDQGLQJ +RZHYHU WKLV DSSURDFK LV DZNZDUG DQG GDERURXV LI WKH SODWH LV QRW DUHFWDQJH, ZRXOG EH GLIILFXOW WR VROYH([DPSOH ZLWK WKH SUHVVXUH GLVWULEXWLRQ DORQH EHFDXVH WKH SODWH LV WLOWHG 7KXV PRPHQWV RI LQHUWLD FDQEH XVHXOLP SOLFDWRQ

(;$03/(

$ WDQNRI RLOKDV DULJKWWULDQJXODUSDQHODQHDUWKHERWWRP DVLQ)LJ (2PLWWQJ S$ ILQGWKH D K\GURVWDWF IRUFHDQG E &3 RQWKHSDQHO

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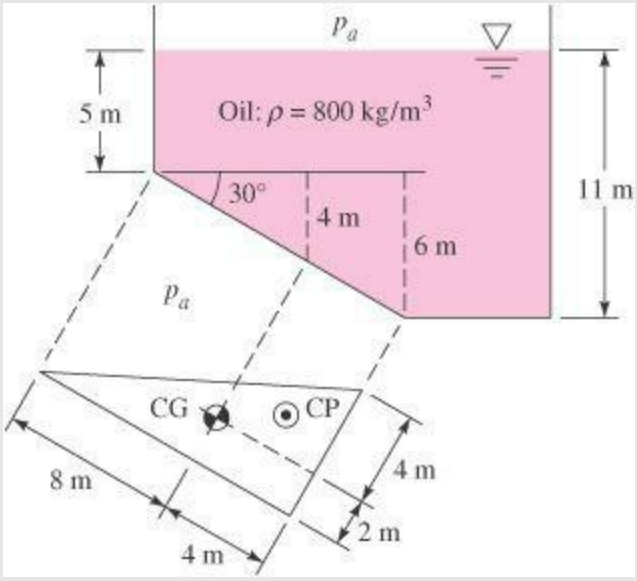


6RXWLRQ

3DUWD

7KHWUDQJHKDVSURSHUWLHVJLYHQLQ) LJ F 7KHFHQWURLGLVRQHWKLUGXS P DQGRQHWKLU RYHU P IURP WKHOHIWHUGHWHUPLQHU DVVKRZQ 7KHDUHDLV

$$\tfrac{1}{2}(6\text{ m})(12\text{ m}) = 36\text{ m}^2$$

7KHPRPHQWVRI LQHUWLDDUH

$$I_{xx} = \frac{bL^3}{36} = \frac{(6\text{ m})(12\text{ m})^3}{36} = 288\text{ m}^4$$

and

$$I_{xy} = \frac{b(b-2s)L^2}{72} = \frac{(6\text{ m})[6\text{ m} - 2(6\text{ m})](12\text{ m})^2}{72} = -72\text{ m}^4$$

7KHGHSWKWRWKHFHQWURLGLVK&* P WKXVWKHK\GURVWDWLFIRUFHIURP (T LV

$$F = \rho g h_{CG} A = (800\text{ kg/m}^3)(9.807\text{ m/s}^2)(9\text{ m})(36\text{ m}^2)$$
$$= 2.54 \times 10^6\,(\text{kg}\cdot\text{m})/\text{s}^2 = 2.54 \times 10^6\text{ N} = 2.54\text{ MN} \qquad \textit{Ans. (a)}$$

3DUWE

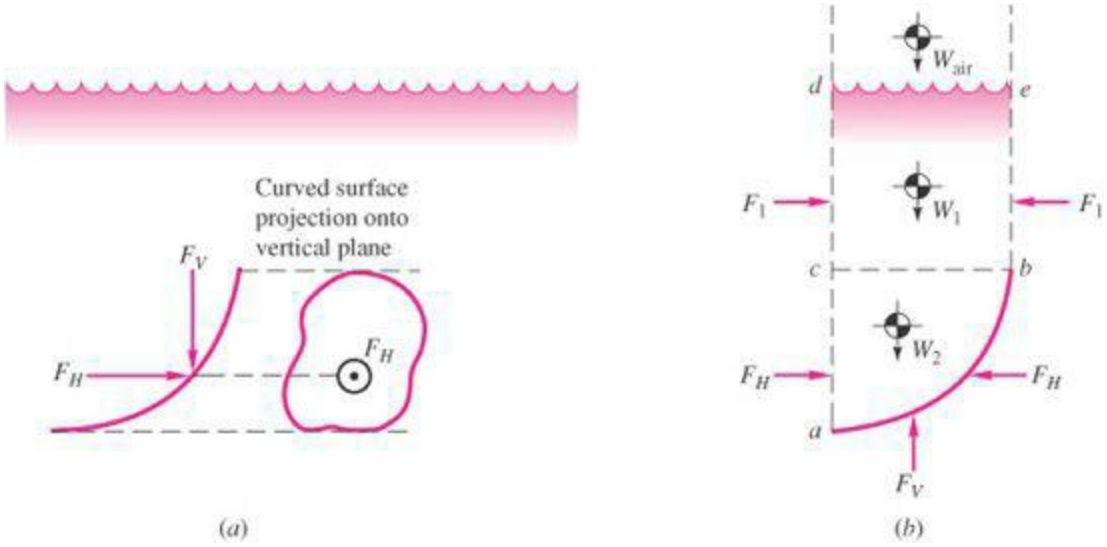
7KH&3 SRVLWLRQLVJLYHQE\ (TV

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(288\text{ m}^4)(\sin 30^\circ)}{(9\text{ m})(36\text{ m}^2)} = -0.444\text{ m}$$
$$x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} = -\frac{(-72\text{ m}^4)(\sin 30^\circ)}{(9\text{ m})(36\text{ m}^2)} = +0.111\text{ m} \qquad \textit{Ans. (b)}$$

7KHUHVXOWDQWRUFH) 0 1 DFWVWKURXJKWKLVSRLQWZKLFKLVGRZQDQGWRWKHULJKWRI WK FHQWURLG DVVKRZQLQ) LJ (

+\GURVWDWLF)RUFHVRQ&XUYHG6XUIDFHV

7KHUHVXOWDQWSUHVVXUHIRUFHRQDFXUYHGVXUIDFHLVPRVWHDVLO\FRPSXWHGE\VHSDUDWLQJLWLQWR KRUL]RQWDODQGYHUWLFDOFRPSRQHQWV&RQVLGHUWKHDUELWUDU\FXUYHGVXUIDFHVKRZQLQ)LJ D 7KH LQFUHPHQWDOSUHVVXUHIRUFHV EHLQJ QRUPDOWR WKH ORFDO DUHD HOHPHQWYDU LQ GLUHFWLRQ DQGQJ WKH VXUIDFHDQGWKXVFDQQRWEHDGGHGQXPHULFDOO\ :HFRXOGVXP WKHVHSDUDWHFRPSRQHQWVRI WKHVH HOHPHQWDOSUHVVXUHIRUFHV EXWLWWXUQVRXWWKDWZHQHHGQRWGRSHUIRUP DOOERURXVWKHZD\ LQMUDWLRQ



)LJ &RPSXWDWLRQ RI K\GURVWDWLF IRUFH RQ D FXUYHG VXUIDFH *a* VXEMHFWHG FXUYHG VXUIDFH *b* IUHHERG\ GLDJUDP RI ILXLGDERYH WKH FXUYHG VXUIDFH

)LJXUH E VKRZV D IUHHERG\ GLDJUDP RI WKH FROXPQ RI ILXLG FRQWDLQHG LQ WKH YHUWLFDO SURMHFWLRQDERYH WKH FXUYHG VXUIDFH 7KH GHVLUHG IRUFHV)+ DQG)9 DUH[HUWHG E\ WKH VXUIDFHRQWKH ILXLG FROXPQ 2WKHU IRUFHV DUH VKRZQGXH WR ILXLG Z HLJKWDQG KRUL]RQWDOSUHVVXUHRQWKH YHUWLFDOVLGH RI WKLV FROXPQ 7KH FROXPQ RI ILXLG PXVWEH LQ VWDWLF HTXLOLEULXP 2QWKH XSSHUSDUWRI WKH FROXPQ EFGH WKH KRUL]RQWDOFRPSRQHQW) H[DFWO EDODQFH DQG DUH QRWUHOHYDQWWR WKH GLVFXVVLRQ 2QWKH ORZ HU LUHHJXODUSRUWLRQ RI ILXLG DEF DGHVLQJ WKH VXUIDFH VXP PDWLRQ RI KRUL]RQWDOIRUFHV VKRZ V WKDW WKH GHVLUHG IRUFH)+ GXHWR WKH FXUYHG VXUIDFHLV H[DFWO HTXDOWR WKH IRUFH)+ RQWKH YHUWLFDOFHVWLG RI WKH ILXLG FROXPQ 7KLV OHWVLGH IRUFH FDQ EH FRPSXWHG E\ WKH SODQH VXUIDFH IRUPXOD(T EDVHGRQD YHUWLFDOSURMHFWLRQ RI WKH DUHD RI WKH FXUYHG VXUIDFH 7KLV LV DJHQHUDOUXOHDQG VLPSOLILH WKH DQDOVLV

,I WKHUH DUH WZR RKRUL]RQWDOFRPSRQHQWV ERWK FDQ EH FRPSXWHG E\ WKLV VFKHPH 6XPPDWLRQ RI YHUWLFDOIRUFHV RQ WKH ILXLG IUHHERG\ WKHQ VKRZ V WKDW

$$F_V = W_1 + W_2 + W_{air} \tag{2.30}$$

: HFDQ VXPPDUL]H WKLV LQ Z RUGV DV RXU VHFRQG JHQHUDOUXOH

7KXV WKH FDOFXODWLRQ RI)9 LQYROYHV GHWHUPLQLQJ WKH PDVV RI DFROXPQ RI ILXLG SHUKDSV DQDO WHLQJ UDWLRQ LI WKH ORZ HUSRUWLRQ DEF LQ)LJ E KDV SDUWFXUDO YH[LQJ VKDSH

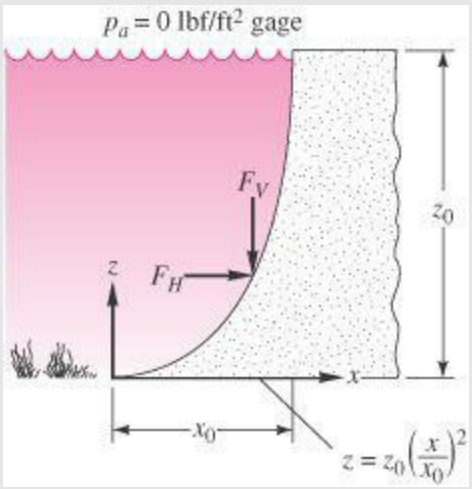
7KH KRUL]RQWDOFRPSRQHQW RI IRUFH RQ D FXUYHG VXUIDFH HTXDOV WKH IRUFH RQ WKH SODQH DUH IRUPHG E\ WKH SURMHFWLRQ RI WKH FXUYHG VXUIDFH RQWR D YHUWLFDOSODQH RUP DOWR WKH FRPSRQHQW

7KH YHUWLFDO FRPSRQHQW RI SUHVVXUH IRUFH RQ D FXUYHG VXUIDFH HTXDOV LQ PDJQLWXGH DQG GLUHFWLRQ WR WKH ZHLJKW RI WKH HQWLUH FROXPQ RI ILXLG ERYH WKH FXUYHG VXUIDFH

(; $ 0 3/ (

$ GDP KDV D SDUDEROLF VKDSH]] [[DV VKRZQ LQ) LJ (D ZLWK[IWDQG] IW 7KH ILXLG LV ZDWHU GE IW DQG DVVXPH WKH IRUFH SUHVVXUH PD\ EH RPLWWHG &RPSXWH WKH IRUFHV) + DQG) 9 RQ WKH GDP DQG WKH LQWHQVLW\ RI DFWLRQ 7KH ZLGWK RI WKH GDP LV IW

(D



6RXWLRQ

- 6\VWHP VNHWFK) LJXUH(E VKRZ VWKH YDULRXV GLPHQVLRQV 7KH GDP ZLGWK LV IW
- $SSURDFK &DOFXODWH) + DQG LWV OLQH RI DFWLRQ IURP (TV DQG &DOFXODWH) 9 DQG LWV OLQH RI DFWLRQ E\ ILQGLQJ WKH ZHLJKW RI ILXLG DERYH WKH SDUDEROD DQG WKH FHQWURLG RI WKLV ZHLJKW
- 6ROXWLRQ VWHSV IRU WKH KRUL]RQWDO FRPSRQHQW 7KH YHUWLFDO SURMHFWLRQ RI WKH SDUDEROLF DQG WKH] D[LV LQ) LJ (E DQG LV DUHD DQJOH IWKLJK DQG IWZLGH ,WV FHQWURLG LV KDOIZ D GRZQ RUK&* IW,WV DUHD LV $SURM IW IW IW 7KHQ IURP (T

$$F_H = \gamma h_{CG} A_{proj} = \left(62.4 \frac{\text{lbf}}{\text{ft}^3}\right)(12 \text{ ft})(1200 \text{ ft}^2) = 898{,}560 \text{ lbf} \approx 899 \times 10^3 \text{ lbf}$$

7KH OLQH RI DFWLRQ RI) + LV EHORZ WKH FHQWURLG RI $SURM DV JLYHQ E\ (T

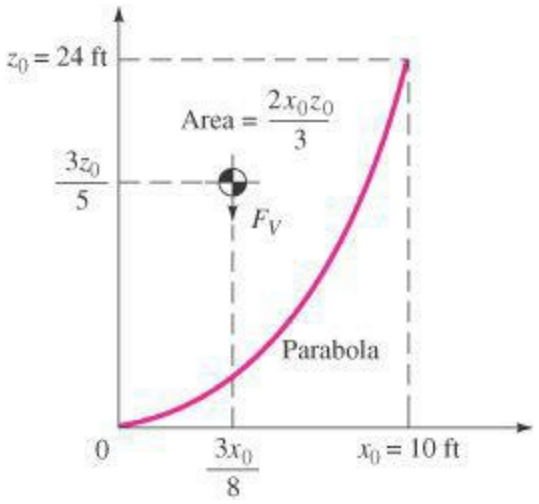
$$y_{CP,\,proj} = -\frac{I_{xx}\sin\theta}{h_{CG}A_{proj}} = -\frac{(1/12)(50\text{ ft})(24\text{ ft})^3\sin 90^\circ}{(12\text{ ft})(1200\text{ ft}^2)} = -4\text{ ft}$$

7KXV) + LV IWRUWZR WKLUGV RI WKH ZD\ GRZQ IURP WKH VXUIDFH IWXS IURP WK EHWWRP

- &RPPHQWV 1RWH WKDW RX FDOFXODWH) + DQG LWV GLVWDQFH RI DFWLRQ IURP WKH YHUWLFDO SURMHFWLRQ R WKH SDUDEROD QRW URP WKH SDUDEROD LWHO 6LQFH WKLV SURMHFWLRQ LV YHUWLFDO LWV DQJOH *f*
- 6RXWLRQ VWHSV IRU WKH YHUWLFDO FRPSRQHQW 7KH YHUWLFDO FRUFH) 9 HTXDOV WKH ZHLJKW RI ZDW DERYH WKH SDUDEROD $OVR D SDUDEROD F VHFWLRQ LV QRWLQ) LJ VR ZH HDG WR RRN LWXS L DQRWKHU ERRN 7KH DUHD DQG FHQWURLG DUH VKRZ Q LQ) LJ (E 7KH ZHLJKW WKLV SDUDEROD DP RXQWV ZDWHU

$$F_V = \gamma A_{section} b = \left(62.4\frac{\text{lbf}}{\text{ft}^3}\right)\left[\frac{2}{3}(24\text{ ft})(10\text{ ft})\right](50\text{ ft}) = 499{,}200\text{ lbf} \approx 499 \times 10^3\text{ lbf}$$

(E



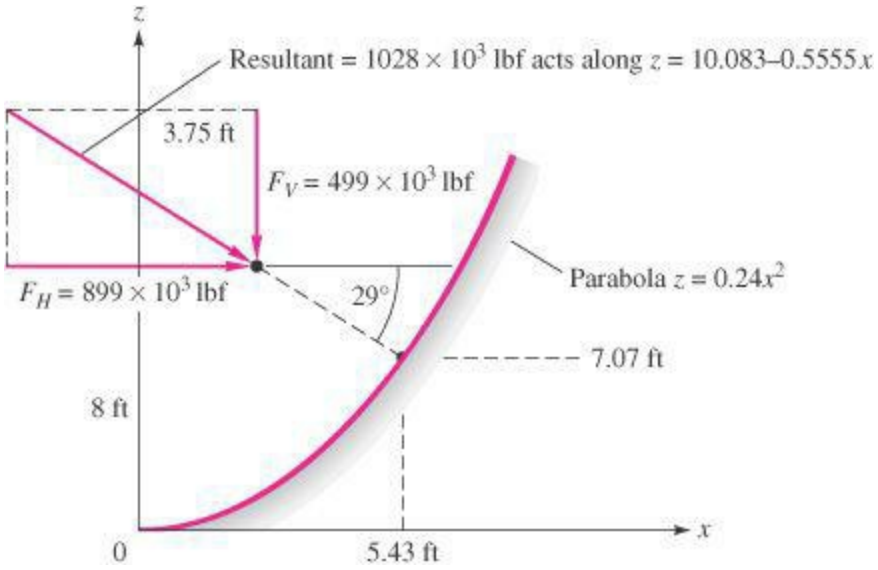
7KLV IRUFH DFWV GRZQZDUG WKURXJK WKH FHQWURLG RI WKH SDUDEROD F VHFWLRQ RU DWD GLVWDQFH [IWRYHUIURP WKH RUJLQ DV VKRZ Q LQ) LJV (E F 7KH UHVXOWDQW KDURXDWF IRUF RQ WKH GDP LV

$$F = (F_H^2 + F_V^2)^{1/2} = [(899\text{E}3\text{ lbf})^2 + (499\text{E}3\text{ lbf})^2]^{1/2} = 1028 \times 10^3\text{ lbf at } 29^\circ \text{ Ans.}$$

7KLV UHVXOWDQW V VKRZ Q LQ) LJ (F DQG SDVVHV WKURXJK DSRLQW IWXS DQG IWRYHUIURF WKH RUJLQ ,WXUNHV WKH GDP DWD SRLQW IWRYHUDQG IWXS DV VKRZ Q

- &RPPHQWV 1RWH WKDW WHQWHU GLIHUHQW IRUP XODV DUH XVHG WR FDOFXODWH) + DQG) 9 7K FRQFHSWRI FHQWURLG RI SUHVVXUH &3 LV LQ WKH ZULWLQJ RSLQLRQ VWHWHKG WR IDZ KHQ DSSOHG V FXUYHG VXUIDFHV

(F



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) LQG DQ DOJHEUDLF IRUP XOD IRU WKH QHWYHUWDOIRUFH) RQ WKH VXEPHUJHG VHPLFLUFXODU SURMFWLQ VWUKFWXUH&' (LQ) LJ (7KH VWUKFWXUH KDV XQIRUP ZLGWKE LQR WKH SDSHU 7KH QDTXLGKDV VSHFLILF ZHJKW

6RXWLRQ

7KH QHWRUFH LV WKH GLIIHUHQFH EHWZHHQ WKH XSZDUG IRUFH) / RQ WKH ORZHU VXUDFH' (DQG WK GRZQZDUG IRUFH) 8 RQ WKH XSSHU VXUDFH&' DV VKRZQ LQ) LJ (7KH IRUFH) 8 HTXDOV WKH VWK YROXPH$% & DERYH VXUDFH&' 7KH IRUFH) / HTXDOV WKH VWK HYROXPH$% (& DERYH VXUDFH' (7KH QDWXUH OI FHDUO ODUJHU 7KH GLIIHUHQFH LV WKH VWK HYROXPH RI WKH VWUKFWXUH LWHO 7KXV WKH QHXXP DUGIOLG IRUFH RQ WKH VHPLFA\OQGHUV

$$F = \gamma_{\text{fluid}}\,(\text{volume } CDE) = \gamma_{\text{fluid}}\,\frac{\pi}{2}R^2 b \qquad Ans.$$

7KLV LV WKH SULQFLSOH XSRQ ZKLFK WKH OZ VRI EXR\DQF\ 6HF DUH IRXQGHG 1 RWH WKDW WKH UHVXO LV LQGHSHQGHQW RI WKH GHSWK RI WKH VWUKFWXUH DQG GHSHQGV XSRQ WKH VSHFLILF ZHJKW RI WKH IOLG QRW WK PDWHULDO ZLWKLQ WKH VWUKFWXUH

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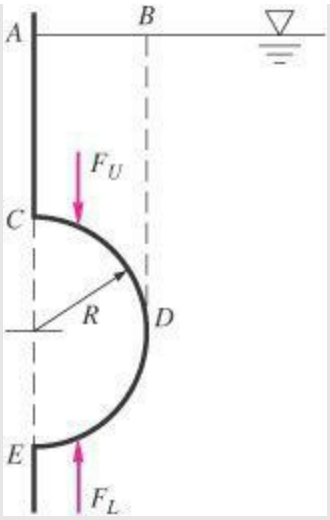
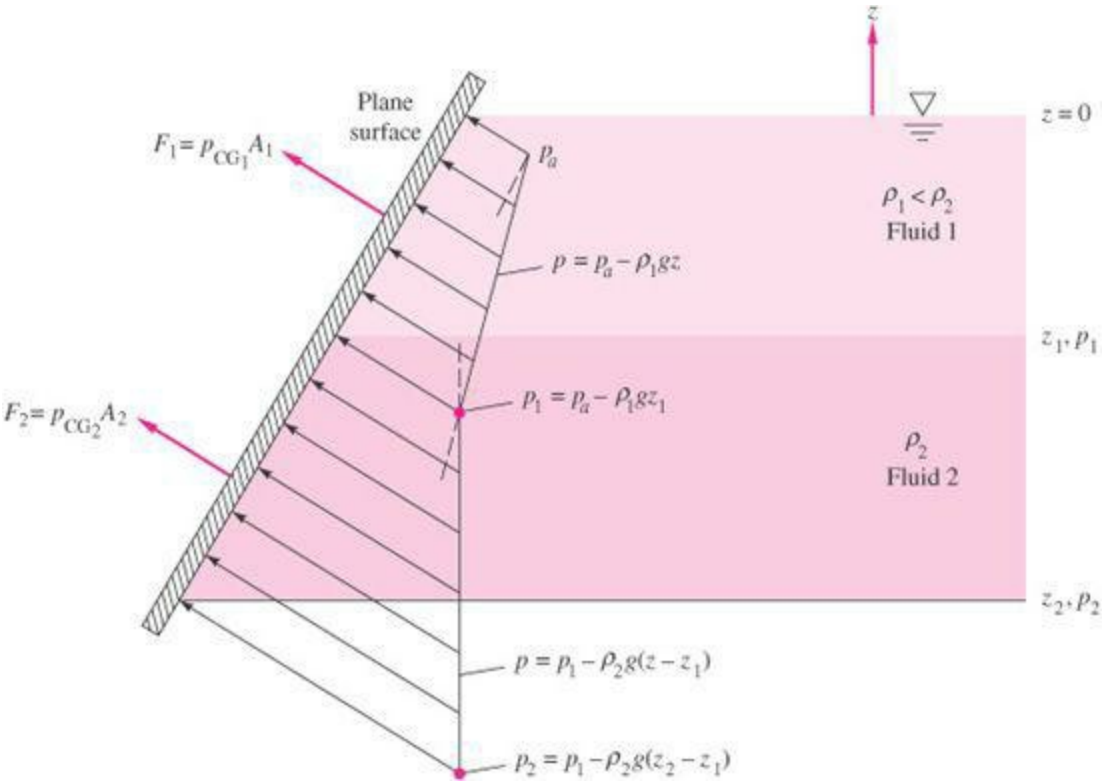


+\GURVWDWLF)RUFHVLQ/DLHUHG)OXLGV

7KHIRUPXODVIRUSODQHDQGFXUYHGVXUIDFHVLQ6HFV DQG DUHYDOLGRQO IRUDIOXLGR
XQLIRUP GHQVLW ,I WKHIOXLGLVOD HUHGZLWKGLIIHUHQWGHQVLWLHV DVLQ) LJ DVLQJHIRUPXODFDQF
VROYH WKH SUREOP EHFDXVH WKH VORSH RI WKH OLQHDU SUHVXUH GLWULEXWLRQ FKDQJHV EHWZHHQ OD HU
+RZHYHU WKHIRUPXODVDSSO VHSDUDWO WRHDFK OD HU DQG WKXV WKHDSSURSULDWH GLVWRFRPSXW
DQG VXP WKHVHSDUDWOD HUIRUFHVDQGPRPHQWV



) LJ +\GURVWDWLF IRUFHVRQ D VXUIDFH LPPHUVHG LQ D OD HU HG IOXLG PXVWEH VXPPHG L
VHSDUDWO SLHFHV

&RQVLGHUWKH VDQG SDQH VXUIDFHLPPHUVHG LQDWZR OD HUIOXLG LQ) LJ 7KHVORSH RI WK
SUHVXUHGLWULEXWLRQEHFRPHVVWHSHUDZHPRYHGRZQLQWKHGHQVHUFRQG OD HU7KHVAOD IRF

RQ WKH SODWH GRHV QRW HTXDO WKH SUHVVXUH DW WKH FHQWURLG WLPHV WKH SODWH DUHD EXW WKH SODWH SRUWRQ LQ HDFK OD\HU GRHV \ WKH IRUPXOD VR WKDW ZH FDQ VXP IRU FHV WR ILQG WKH WRWDO

$$F = \sum F_i = \sum p_{CG_i} A_i \qquad (2.31)$$

6LPLODUO WKH FHQWURLG RI WKH SODWH SRUWRQ LQ HDFK OD\HU FDQ EH XVHG WR ORFDWH WKH FHQWUR SUHVVXUHRQWKDVSRUWRQ

$$y_{CP_i} = -\frac{\rho_i g \sin\theta_i I_{xx_i}}{p_{CG_i} A_i} \qquad x_{CP_i} = -\frac{\rho_i g \sin\theta_i I_{xy_i}}{p_{CG_i} A_i} \qquad (2.32)$$

7KHVH IRUPXODV ORFDWH WKH FHQWUR SUHVVXUH RI WKDVSDUWFXODU) L ZLWK UHVSHFWWR WKH FHQWURLG R WKDVSDUWFXODUSRUWRQ RI SODWH LQ WKH OD\HU QRWZ LWK UHVSHFWWR WKH FHQWURLG RI WKH HQWLUH SODWH 7KI FHQWUR SUHVVXUH RI WKH VRLO IRUFH)) L FDQ WKHQ EH IRXQG E\ VXPPLQJ PRPHQWV DERXW WKH FRQYHQLHQW SRLQW XVLQJ WKH WHFKDVWKH WXUDFH 7KHIROORZLQJ H[DPSOH ZLOOLOXVWUDWH WKLV

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$ VDQN IWGHHS DQG IWZLGH LV OD\HUHG ZLWK IWRI RLO IWRI ZDWHU DQG IWRI PHUFXU &RPSXWH D WKH VRLO K\GURVWDWLF IRUFH DQG E WKH UHVXOWDQWFHQWUR SUHVVXUH RI WKH ILXLG RQ WKH ULJKWKDQG VLGH RI WKH VDQN

6RXWLRQ

3DUW D

' LYLGH WKH HQG SDQHO LQWR WKUHH SDUWV DV VKRZQ) LJ (DQG ILQG WKH K\GURVWDWLF SUHVVXU DWWKH FHQWURLG RI HDFK SDUW XVLQJ WKH UHODWLRQ LQ WKH VDPH) LJ (

S&* GEI IW IW GEI IW

S&* GEI IW

S&* GEI IW

7KHVH SUHVVXUHV DUH WKHQ PXOWLSOLHG E\ WKH UHVSHFWLYH SDQHO DUHDV WR ILQG WKH IRUFH RQ HDF SRUWLRQ

$$F_1 = p_{CG_1} A_1 = (220 \text{ lbf/ft}^2)(8 \text{ ft})(7 \text{ ft}) = 12{,}300 \text{ lbf}$$
$$F_2 = p_{CG_2} A_2 = 627(6)(7) = 26{,}300 \text{ lbf}$$
$$F_3 = p_{CG_3} A_3 = 2506(4)(7) = 70{,}200 \text{ lbf}$$
$$F = \sum F_i = 108{,}800 \text{ lbf} \qquad \textit{Ans. (a)}$$

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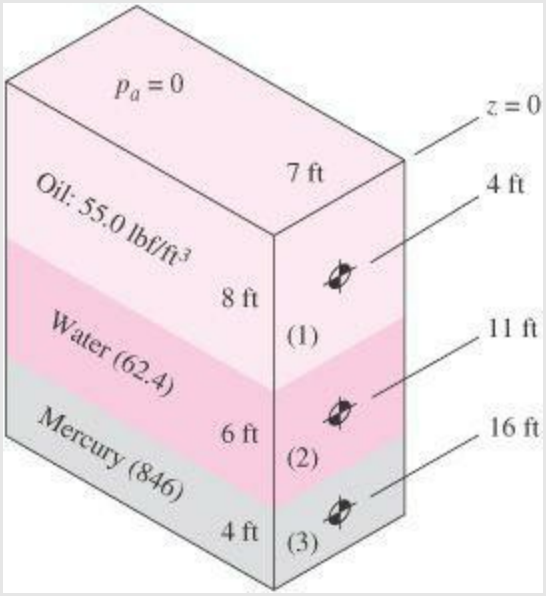


3DUWE

(TXDWLRQV FDQEHXVHGWRGHFDWWKH&3 RI HDFKIRUFH) L QRWLQJ WKDW f DQGVLQ IRUDOOSDUW 7KHPRPHQWRI LQHUWLD DUH $I_{xx_1} = (7\text{ ft})(8\text{ ft})^3/12 = 298.7\text{ ft}^4$, $I_{xx_2} =$ IW DQG $I_{xx_3} = 7(4)^3/12 = 37.3\text{ ft}^4$. 7KHFHQWURLGVRI SUHVVXUHDUHWKXVDW

$$y_{CP_1} = -\frac{\rho_1 g I_{xx_1}}{F_1} = -\frac{(55.0\text{ lbf/ft}^3)(298.7\text{ ft}^4)}{12{,}300\text{ lbf}} = -1.33\text{ ft}$$

$$y_{CP_2} = -\frac{62.4(126.0)}{26{,}300} = -0.30\text{ ft} \qquad y_{CP_3} = -\frac{846(37.3)}{70{,}200} = -0.45\text{ ft}$$

7KLVGRFDWH $z_{CP_1} = -4 - 1.33 = -5.33\text{ ft}$, $z_{CP_2} = -11 - 0.30 = -11.30\text{ ft}$, DQG $z_{CP_3} =$ í í í IW6XPPLQJ PRPHQWVDERXWKHVXUIDFHWKHQJLYHV

$$\Sigma F_i z_{CP_i} = F z_{CP}$$

or $$12{,}300(-5.33) + 26{,}300(-11.30) + 70{,}200(-16.45) = 108{,}800 z_{CP}$$

or $$z_{CP} = -\frac{1{,}518{,}000}{108{,}800} = -13.95\text{ ft} \qquad \textit{Ans. (b)}$$

7KHFHQWURI SUHVVXUHRI WKHWRWDOIRUFHDFWVXQGHUQHDWKWKHVDQNOHV IWEHORZ WKHVXUIDFH

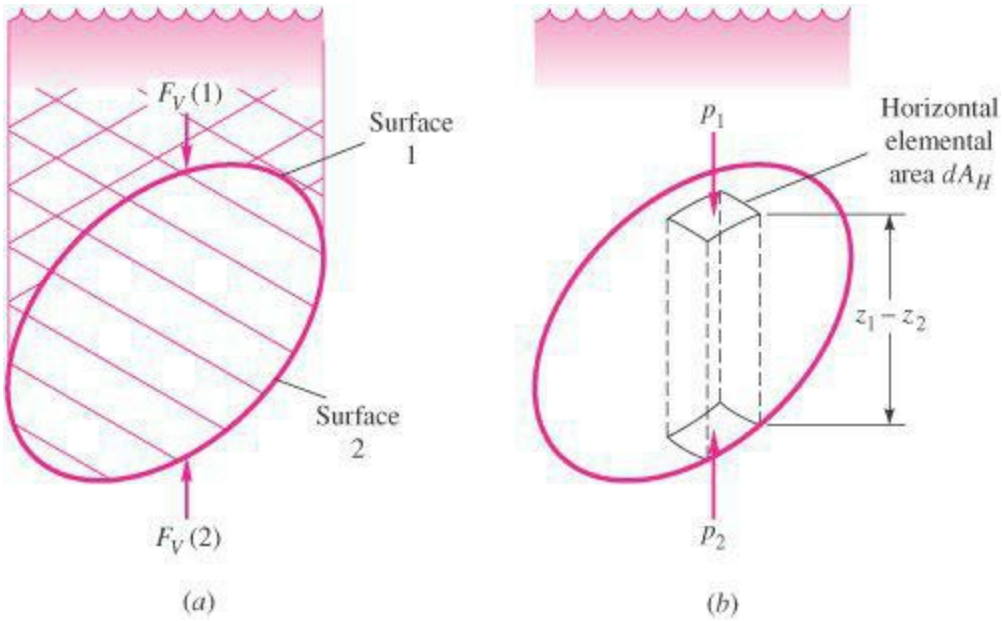
%XR\DQF\ DQG 6WDELOLW\

7KH VDPH SULQFLSOH XVHG WR FRPSXWH K\GURVWDWLF IRUFHV RQ VXUIDFHV FDQ EH DSSOLHG WR WKH QH-
SUHVVXUH IRUFH RQ D FRPSOHWHO VXEPHUJHG RUIDFH LQ DQ\ IOXLG 7KHUHVXOWDQW IRUFH WKH VZR OD\ VRI EXR\DQF
GLVFRYHUHG E\ $UFKLPHGHV LQ WKH WKLUG FHQWXU %&

$ ERG\ LPPHUVHG LQ D IOXLG H[SHULHQFHV D YHUWLFDO EXR\DQW IRUFH HTXDO WR WKH ZHLJKW RI WK
IOXLG LW GLVSODFHV

$ IORDWLQJ ERG\ GLVSODFHV LWV RZ QZHLJKW LQ WKH IOXLG LQ ZKLFK LW IORDWV

$UFKLPHGHV ± %& ZDV ERUQ DQG GZHOW LQ WKH * UHHN FLW VWDWH RI 6\UDFXVH RQ ZKDW
QRZ WKH LVODQG RI 6LFLO + HZ DV DEULOOLDQW PDWKHPDWLFLDQ DQG HQJLQHHU WR PLOHVWQH DKHDG RI KL
WLPH + HFDOFXODWHG DQ DFFXUDWH YDOXH IRU SLQ DSSUR[LPDWHG DUHDV DQG YROXPHV RI YDULRXV ERGLH
E\ VXPPLQJ HOHPHQWDO VKDSHV ,Q RWKHU ZRUGV KH LQYHQWHG WKH LQWHJUDFDOFXOXV + HGHYHORSHG OHY
SXOOH\V FDOSXOW DQG DVFUHZ SXPS $UFKLPHGHV ZDV WKH ILUVW WR ZULWH OLJHQPEHUV DSSOH\ WR
DYRLGLQJ 5RPDQ QXPHUDOV $QG KH GHGXFHG WKH SULQFLSOHV RI EXR\DQF\ ZKLFK ZH VWXG KHU-
ZKHQKHUHDG]HGKRZ OJKWKWHZDVZKHQVLWWQJ LQDEDWKXE



) LJ 7ZR GLIIHUHQW DSSURDFKHV WR WKH EXR\DQW IRUFH RQ DQ LPPHUVHG ERG\ ***a*** IRUFHV RQ XSSHU DQG ORZHU FXUYHG VXUIDFHV ***b*** VXPPDWLRQ RI HOHPHQWDO YHUWLFDO SUHVVXUH IRUFHV

$UFKLPHGHV¶ VZR OD\ DSSURDFK GHULYHG E\ UHIHUULQJ WR) LJ ,Q) LJ D WKH ERG\ GH
EHZHHQ DQ XSSHU FXUYHG VXUIDFH DQG D ORZHU FXUYHG VXUIDFH) URP (T IRUY HUWLFDO IRUFH
WKH ERG\ H[SHULHQFHV DQ HXSZ DUG IRUFH

$$
\begin{aligned}
F_B &= F_V(2) - F_V(1) \\
&= (\text{fluid weight above } 2) - (\text{fluid weight above } 1) \\
&= \text{weight of fluid equivalent to body volume} \qquad (2.33)
\end{aligned}
$$

$ QHWGRZQ IRUFH E ZHFDQ VXP WKH YHUWLFDO IRUFHV RQ HOHPHQWDO YHUWLFDO FROXPQV WKURXJ
WKH LPPHUVHG ERG\

$$F_B = \int_{\text{body}} (p_2 - p_1)\, dA_H = -\gamma \int (z_2 - z_1)\, dA_H = (\gamma)(\text{body volume}) \qquad (2.34)$$

7KHVH DUH LGHQWLFDO UHVXOWV DQG HTXLYDOHQW WR \$UFKLPHGHV¶ ODZ

(TXDWLRQ DVVXPHV WKDW WKH IOXLG KDV XQLIRUP VSHFLILF ZHLJKW 7KH OLQH RI DFWLRQ RI WK EXR\DQW IRUFH SDVVHV WKURXJK WKH FHQWURLG RI YROXPH RI WKH GLVSODFHG ERG\ WKDW LV WKH FHQWURLG RI PDV FRQVWLWXWHG DV LI LW KDG XQLIRUP GHQVLW 7KLV SRLQW WKURXJK ZKLFK) % DFW LV FDOOHG WKH FHQWUR EXR\DQF\ FRQVWDQW OHEHOHG% RU &% RQ D GUDZ LQJ 2I FRXUVH WKH SRLQW% PD\ RU PD\ QR FRUUHVSRQG WR WKH DFWXDO FHQWURLG RI PDVV RI WKH ERG\ ¶VRZ QP DWULDO ZKLFK PD\ KDYH YDULDEOH GHQVLW

(TXDWLRQ FDQ EH JHQHUDOL]HG WR DOO\ HUHG IOXLG /) E\ VXPPLQJ WKH Z HLJKWV RI HDFK OD\ H RI GHQVLW! L GLVSODFHG E\ WKH LPPHUVHG ERG\

$$(F_B)_{LF} = \sum \rho_i g(\text{displaced volume})_i \qquad (2.35)$$

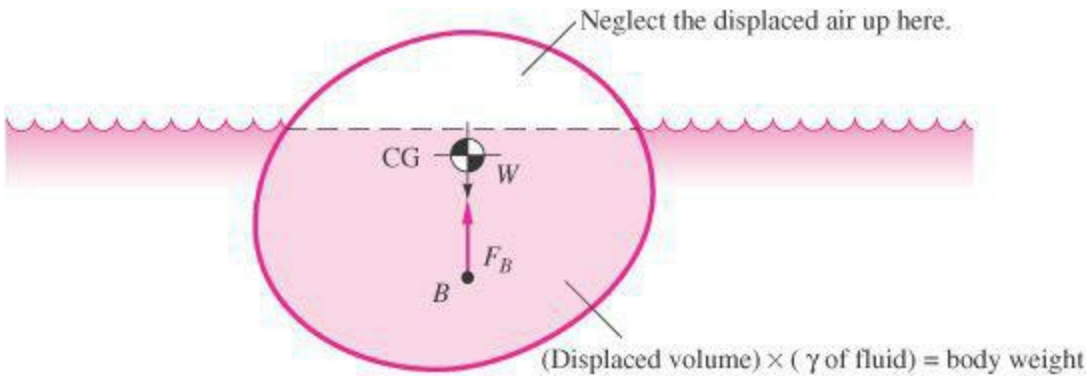
(DFK GLVSODFHG OD\ HU Z RXOG KDYH LWV RZ QF HQWUR RI YROXPH DQG RQH Z RXOG KDYH WR VXP PRPHQ RI WKH LQFUHPHQWDO EXR\DQW IRUFHV WR ILQG WKH FHQWURLG RI EXR\DQF\ RI WKH LPPHUVHG ERG\

6LQFH GTXLGV DUH UHDGLO\ KHDY\ Z HDUH FRQFLRXV RI WKH UEXR\DQW IRUFHV EXWJDVHV DOR H[EXR\DQF\ RQDQ ERG\ LPPHUVHG LQ WKH P) RUH[DPSOH KXPDQEHLQJV KDYH DQDYHUDJH VSHFLILF Z HLJK RI DERXW GEI IW : HPD\ UHFRUG WKH Z HLJKW RI D SHUVRQ DV GEI DQG WKXV HVWLPDWH WKH SHUVRQ¶V VWDWLF YROXPH DV IW + RZ HYHU LQ VR GRLQJ Z H DUH QHJOHFWLQJ WKH EXR\DQW IRUFH RI WKH DL VXUURXQGLQJ WKH SHUVRQ \$WWDQGDUG FRQGLWLRQV WKH VSHFLILF Z HLJKW RI DLU LV GEI IW KHQFH WK EXR\DQW IRUFH LV DSSUR[LPDWHO GEI ,I PHDVXUHG LQ DYDFXXP WKH SHUVRQ Z RXOG Z HLJK DERXW GEI PRUH) RU EDOORRQV DQG EOLPSV WKH EXR\DQW IRUFH LV VXEVWDQWLDO RI DLU LQWHG RI EHLQJ QHJOLJLEOH LV WK FRQWULEXWLQJ IDFWRU LQ WKH GHVLJQ \$OR PDQ IORZ SKHQRPHQD VXFK DV QDWXUDO FRQYHFWLRQ RI KHDWHG YHUWLFDO P[LQJ LQ WKH RFHDQ DUH VWURQJO GHSHQGHQW RQ VHPLQJO VP DOO EXR\DQW IRUFHV

) ORDWLQJ ERGLHV DUH D VSHFLDO FDVH RQ D SRUWLRQ RI WKH ERG\ LV VXEP HUJHG Z LWK WKH UHP DLQG SRNLQJ XS RXW RI WKH IUHH VXUIDFH 7KLV LV LOOXVWUDWHG LQ) LJ Z KHUH WKH VKDGHG SRUWLRQ LV WK GLVSODFHG YROXPH (TXDWLRQ LVP RGLILHG WR DSSO WR WKLV VP DOOEXR\DQW YROXPH

$$F_B = (\gamma)(\text{displaced volume}) = \text{floating-body weight} \qquad (2.36)$$

1 RWRQO GRHV WKH EXR\DQW IRUFH HTXDO WKH ERG\ Z HLJKW EXWDOR WKH DUHFRQGLWLRQHG WKUHFD EH QR QHWPRPHQW IRU VWDWF HTXLOEULXP (TXDWLRQ LV WKH PDWKHPDWFDO HTXLYDOHQWR \$UFKLPHGHV¶ SULQFLSOH



) LJ 6WDWF HTXLOEULXP RI DI ORDWLQJ ERG\

(; $0 3/ (

$ EORFN RI FRQFUHWH ZHLJKV OEI LQDLUDQG³ ZHLJKV´ RQ OEI ZKHQ LPPHUVHG LQ IUHVK ZDWHU LW : KDW LV WKH DYHUDJH VSHFLILF ZHLJKW RI WKH EORFN"

6RXWLRQ

$ IUHH ERG\ GLDJUDP RI WKH VXEPHUJHG EORFN VHH) LJ (VKRZV D EDODQFH EHWZHHQ WKH SSSDUHQW ZHLJKW WKH EORFN DQG WKH DFWXDO ZHLJKW

$$\sum F_z = 0 = 60 + F_B - 100$$

or

$$F_B = 40 \text{ lbf} = (62.4 \text{ lbf/ft}^3)(\text{block volume, ft}^3)$$

6RO YLQJ JLYHV WKH YROXP H RI WKH EORFN DV IW 7KHUHRUH WKH VSHFLILF ZHLJKW RI WK EORFN LV

$$\gamma_{\text{block}} = \frac{100 \text{ lbf}}{0.641 \text{ ft}^3} = 156 \text{ lbf/ft}^3 \qquad \textit{Ans.}$$

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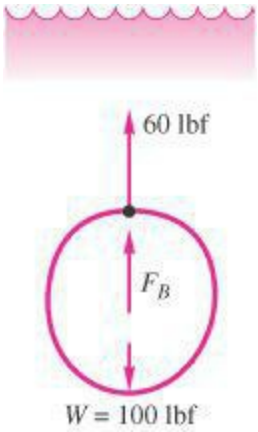


2 FFDVLRQDOO D ERG\ ZLOO KDYH H[DFWO WKH ULJKW ZHLJKW DQG YROXP H IRU LW UDLR VR HTXDO WK VSHFLILF ZHLJKW RI WKH IOXLG ,I VR WKH ERG\ ZLOOEH QHXWUDOO EXR\DQW DQG ZLOOUHP DLQ DW UHVW DW DQ SRLQW ZKHUHLWV LPPHUVHG LQWKH IOXLG 6P DOOQHXWUDOO EXR\DQWSDUWFOHV DUH VRP HWP HV XVHG LQ IOXLG YLVXDOL]DWLRQ DQG DQHXWUDOO EXR\DQWERG\ FDOOHG D6ZDOORZ IORDW> @LV XVHG WR WDFNRFHDQRJUDSKL FXUUHQW $ VXEP DULQH FDQ DFKLHYH SRVLWLYH QHXWUDO RUQHJDWLYH EXR\DQF\ E\ SXP SLQJ Z DWHU LQRURX RI LWVEDOODVWWDQNV

6WDELOLW\

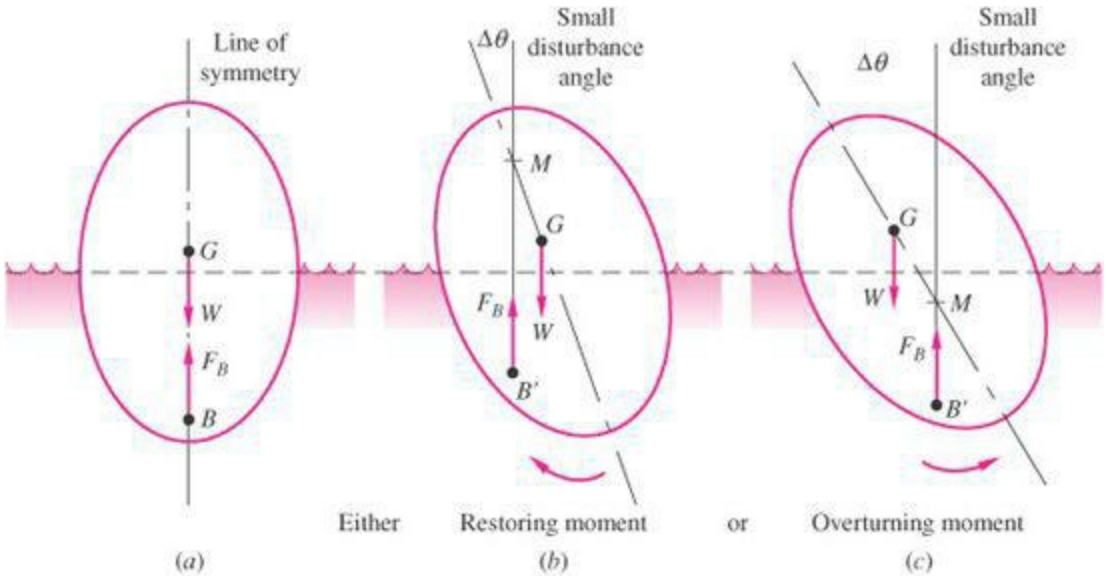
$ IORDWLQJ ERG\ DVLQ) LJ PD\ QRWDSSURYHR VWHSRVLWLRQLQZKLFKLWLV IORDWLQJ ,I VR LWZ LO RYHUWXUQ DWWKH ILUVWRSSRUWXQLW\ DQG LV VDLG WR EH XQVWDEOH QNH D SHQFLOEDODQFHG RQ LV SRLQW7KHGLVWDQFHZLOOFDXVHLWWR VHHNDQRWKHUHTXLOLEULXP SRVLWLRQWKDWV VWDEOH (QJLQHU PXVWGHVLJQ WR DYRLG IORDWLQJ LQXVHLDW 7KHRQ ZD\ WR WHVW IRUVXUHZKHWKHUD IORDWLQJ SRVLWLRQ LV VWDEOHLV WR³GLVWXUE´ WKHERG\ DVOLJKWOPRXQWP DWKHPDWFDO DQG VHZKHULWGHYHORSVDUHWXUQ PRPHQWWKDWZLOOUHWXUQLWWR LWVRULJLQDOSRVLWLRQ ,I VR LWV VWDEOH LI QRWXQXVDEOH 6XFK FDOFXODWLRQ IRUDUELWUDU IORDWLQJ ERGLHVKDYHEHHQKRQHGWR DILQHDUWE\ QDYDODUFKLWHFWV> @EXWZHFDQDWOHDV RXQGHWKHEDVLF SULQFLSOHR WKHVWDWF VWDELOLW\ FDOFXODWRQ) LJXUH LOOXVWUDWHV WKHFRPSXWDWR IRUWKHXVXDOFDVHR DV\PPHWULF IORDWLQJ ERG\ 7KHVWSVDUHDVIROOOZV

7KHEDVLF IORDWLQJ SRVLWLRQLV FDOFXODWHG IURP (T 7KHERG\¶V FHQWURLG PDW* DQ FHQWURLG EXR\DQF\ %DUHFRPSXWHG

7KHERG\ LV WLOWHG DVPDOODQJOHû DQG DQHZ ZDWHUOLQH LV HVWDEOLVKHG IRUWKHERG\ WR IORDWLQ DWWKLV DQJOH 7KHQHZ SRVLWLRQ%• RI WKHFHQWURLG EXR\DQF\ LV FDOFXODWHG $ YHUWFDOOLQH GUDZ IURP %• LQWHUVHFW WKH OLQH RI V\PPHWU DVD SRLQW0 FDOHG WKHPHWDFHQWU ZKLFK LV LQGHSHQGHQWRû IRUVPDOODQJOHV

,I SRLQW0 LVDERYH* WKDWV LI WKHPHWDFHQWULF KHLJKW $\overline{MG}$ LVSRVLWLYH DUHWRULQJ PRPHQ LVSUHVHQWDQG WKHRULJLQDOSRVLWLRQLVVWDEOH ,I 0 LVEHORZ * QHJDWLYH $\overline{MG}$ WKHERG\ LVXQVWDEO DQGZLOORYHUWXUQLI GLVWXUEHG 6WDELOLWVLQFUHDVHVZLWKLQFUHDVLQJ $\overline{MG}$

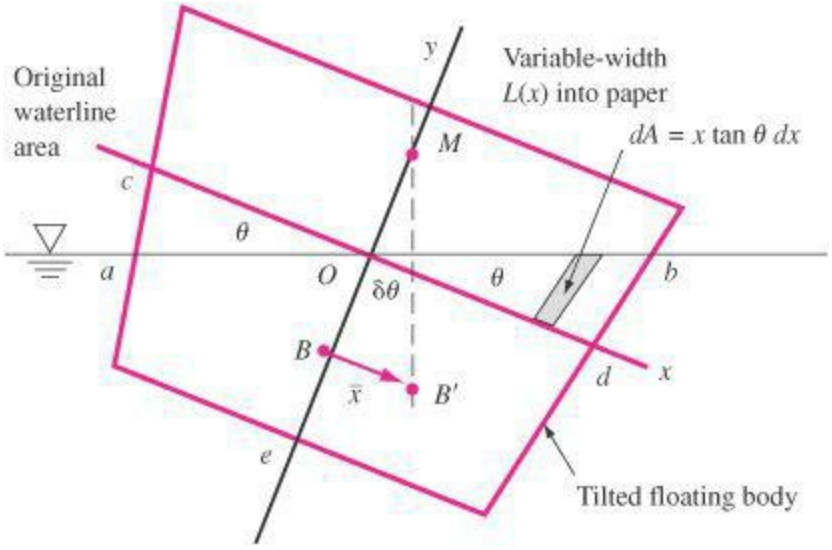
7KXV WKHPHWDFHQWULF KHLJKWWV D SURSHUW RI WKHFURVV VHFWLRQ IRUWKHJLYHQ ZHLJKWDQG LWV YDOX JLYHVDQLQGLFDWRQ RI WKHVWDELOLW RI WKHERG\ ) RUDERG\ RI YDULQJ FURVV VHFWLRQ DQG GUDWWXFK D DVKLS WKHFRPSXWDWRQ RI WKHPHWDFHQWUFDQEHYHU LQYROYHG



) LJ &DOFXODWRQ RI WKHPHWDFHQWULM RI WKHIORDWLQJ ERG\ VKRZQ LQ $ 7LOWWKHERG\ D VP DOODQJOHû (LWKHU $b $• PRYHVIDURXW SRLQWM DERYH* GHQRWHV VWDEOLW RU $c $• PRYHV WRJHWKHU SRLQWM EHORZ* GHQRWHV LQVWDEOLW

6WDELOLW\ 5HODWHG WR: DWHUOLQH$UHD

1DYDODUFKLWHFWV> @KDYH GHYHORSHG WKH JHQHUDO VWDELOLW\ FRQGLWLRQ IRU DQ LPSO FRPSXWDWLRQ LQYROYLQJ WKH DUHD PRPHQWV RI LQHUWLD RI WKH ZDWHUOLQH DUHD DERXW WKH [LV RI WKH V7K GHULYDWLRQ DVVXPHV WKDW WKH ERG\ KDV D PRRWK VKDSH YDULDWLRQ QR GLVFRQWLQXLWLHV QHDU WKH ZDWHUOLQ DQG LV GHULYHG IURP) LJ



) LJ $ IORDWLQJ ERG\ WLOWHG WKURXJK D VPDOO DQJOH θ 7KH PRYHPHQW RI WKH FHQWURLG EXR\DQF\ B LV UHDGLO\ GHWHUPLQHG IURP WKH ZDWHUOLQH DUHD PRPHQWV RI LQHUWLD

7KH\ [LV RI WKH ERG\ LV DVVXPHG WR EH D QHJDWLYH YDOXH 7LOWLQJ WKH ERG\ DVPDOO DQJOH WKH VXEPHUJHG VKDSH FKDQJHV WKH FHQWUH RI EXR\DQF\ IURP % WR WKH FHQWURLG RI WKH IRUPHUO\ GU2 EG DQG GXQFRYHUV DQHTXDOZ HGJH2 EG DQG GXQFRYHUHG DV WKH FHQWURLG RI EXR\DQF\ LV FDOFXODWHG IURP WKH VXEPHUJHG SRUWLRQ2 EG RI WKH ERG\

$$\bar{x}\, v_{aObde} = \int_{cOdea} x\, dv + \int_{Obd} x\, dv - \int_{cOa} x\, dv = 0 + \int_{Obd} x\,(L\, dA) - \int_{cOa} x\,(L\, dA)$$

$$= 0 + \int_{Obd} x\, L\,(x \tan\theta\, dx) - \int_{cOa} xL\,(-x \tan\theta\, dx) = \tan\theta \int_{\text{waterline}} x^2\, dA_{\text{waterline}} = I_O \tan\theta$$

ZKHUH,2 LV WKH DUHD PRPHQW RI LQHUWLD RI WKH ZDWHUOLQH DERXW WKH EXR\DQF\ 7KH IILWWLQJ RUGU DUHV WKH VXEPHUJHG SRUWLRQ RI WKH RULJLQDO SXEP HUJHG SRUWLRQ2 GD 7KH UHPDLQLQJ WR³ZHGJH LQWR FRQFELQH LQWR,2 ZKHQ ZH QRWH WKDW GL HTXDO DQ HOPHQWR ZDWHUOLQH DUHD 7KXV ZH GHWHUPLQH WKH GHVLUHG GLVWDQFH IURP 0 WR%

$$\frac{\bar{x}}{\tan\theta} = \overline{MB} = \frac{I_O}{v_{\text{submerged}}} = \overline{MG} + \overline{GB} \qquad \text{or} \qquad \overline{MG} = \frac{I_O}{v_{\text{sub}}} - \overline{GB} \tag{2.37}$$

7KH HQJLQHHUZRXOG GHWHUPLQH WKH GLVWDQFH IURP * WR%IURP WKH EDVLF VKDSH DQG GHVLJQRI WK IORDWLQJ ERG\ DQG WKHQ PHDVXUH WKH FDOFXODWRQ RI ,2 DQG WKH VXEPHUJHG YROXPH%%XE ,I WKH PHUDFHQWU KHJKW0 * LVSRVLWLYH WKH ERG\ LV VWDEOH2IRUVPDOOGLVWXUEDQFHV 1RWHWKDWWKH $\overline{GB}$ LVQHJDWLYH WKDWWV %

LVDERYH* WKHERG\ LVDOZD\V VWDEOH

(; $0 3/ (

$ EDUJHKDVDXQLRUP UHFWDQJXODUFURVV VHFWLRQRI ZLGWK / DQG YHUWLFDOGLVWDQFH KHLJKW+ DVLO
)LJ (' HWHUPLQH D WKHPHWDFHQWULF KHLJKWRUDVPDOOLVWDQJHDQG E WKHUDQJHRI UDLR/ +
IRUZKLFKWKHEDUHLVVWDEOHDWDOO VWDEOHLI * LVH[DFWO DWWKHZDWHUOLQHDVVKRZQ

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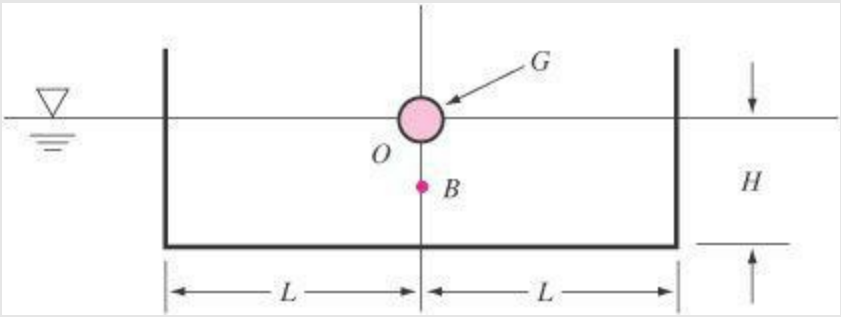


6RXWLRQ

,I WKHEDUHKDVGHQJWKE LQRWKHSDSHUWKHZDWHUOLQHDUHD UHDLVYHWRWDO[LV2 KDVDEDVHE DQ
DKHLJKW/ WKHUHRUH ,2 E / 0 HDQZKLOH %%XE / E+ (TXDLRQ SUHGLFWV

$$\overline{MG} = \frac{I_o}{v_{sub}} - \overline{GB} = \frac{8bL^3/12}{2LbH} - \frac{H}{2} = \frac{L^2}{3H} - \frac{H}{2} \qquad Ans.\ (a)$$

7KHEDUHFDQWKXVEHVWDEOHRQO LI

$$L^2 > 3H^2/2 \quad \text{or} \quad 2L > 2.45H \qquad Ans.\ (b)$$

7KHZ LGHUWKHEDUHUHDLVYHWRLWGUDWWKHPRUHVWDEOHLWLV / RZHULQJ * ZRXOGKHOSDOR

(YHQ DQ H[SHUWZ LOOKDYH GLIILFXOW GHWHUPLQLQJ WKH IORDWLQJ VWDELOLW RI D EXR\DQWERG\ R
LUUHJXODUVKDSH 6XFK ERGLHV PD\ KDYHZR RUPRUHVWDEOHSRVLWLRQV)RU[DPSOH DVKLS PD\ IORD
WKHZD\ ZHOONHLWVR WKDWZHFDQVLRQ WKHGHFN RULWPD\ IORDWXSVLGHGRZQ FDSVL]HG $ (
LQWUHVWLQJ PDWKHPDWLFDOSSURDFKWRIORDWLQJ VWDELOLW LVJLYHQLQ5H 7KHDXWKRUR WKLVUHHUHQF
SRLQWVRXWKDWHYHQVLPSOHVKDSHV VXFK DV DFXEHR XQLRUP GHQVLW PD\ KDYHDJUHDWP DQ VWDEG
IORDWLQJ RUHQDWLRQV QRWQHFHVVDULO \PPHWULF +RPRJHQHRXV FLUFXODU\OLQGHUV FDQ IORDWZ LWK WK
D[LVRI \PPHWU VWDGIURP WKHYHUWLFDO

) ORDWLQJ LQVWDELOLW RFFXUVLQQDWXUH) LVK JHQHUDOO VZLP ZLWKWKHLUSDQHVRI \PPHWU YHUWLFD
$IWUGHDWK WKLV SRVLWLRQ LV XQVWDEOH DQG WKH IORDWZ LWK WKHLUIODWVLGHV XS * LDQWLFHEHJV PD

RYHUWXUQ DWWU EHFRPLQJ XQWDEOH ZKHQ WKHLU VKDSHV FKDQJH GXH WR XQGHUZDWHU PHOWLQJ ,FHEHUJ
RYHUWXUQLQJ LVDGUDPDWF UDUHO VHHQHYHQW

) LJ $ 1 RUWK $WODQWF LFHEHUJ IRUPHG E\ FDOYLQJ IURP D* UHHQODQG JODFLHU 7KHVH DQ
WKHLU HYHQ ODUJHU $QWDUFWF VLWWLV DUH WKH ODUJHVW RDWLQJ ERGLHV LQ WKH ZRUOG 1RWH WKH
HYLGHQFH RI XUWKHUFDOYLQJ IUDFWXUHV RQ WKH IURQWXUDFH*(© Corbis.)

) LJXUH VKRZV DWSLFDO1 RUWK$WODQWF LFHEHUJ IRUPHG E\ FDOYLQJ IURP D* UHHQODQG JODFL
WKDWSURWXGHG LQR WKH RFHDQ 7KH H[SRVHG VXUDFH LV URXJK LQGLFDWLQJ WKDWWKDV XQGHUJRQH IXUWK
FDOYLQJ ,FHEHUJV DUH IUH]HQ IUHVK EXEEO JODFLDOZDWHU DQG DYHUDJH GHQVLW NJ P 7KXV ZKHQ D
LFHEHUJ LV IRDWLQJ LQ VHDZDWHU ZKRVH DYHUDJH GHQVLW LV NJ P DSSUR[LPDWHO R
VHYHQ HLJKWV RI LWV YROXPH GHSUHVEHORZ WKHZDWHU

3UHVVXUH' LVWULEXWLRQ LQ5LJLG %RG\ 0RWLRQ

,Q ULJLG ERG\ PRWLRQ DOO SDUWFOHV DUH LQ FRPELQHG WDQVODWLRQ DQG URWDWLRQ DQG WKHUH LV Q
UHODWLYH PRWLRQEHWZHHQSDUWFOHV : LWKQR UHODWLYH PRWLRQ WKHUHDUHQR VWUDLQVRUVWUHVVHV WKH
WKH YLVFRXV WUP LQ(T YDQLVKHV OHDYLQJ D EDODQFH EHWZHHQ SUHVVXUH JUDYLW DQG SDUWFH
DFFHOHUDWLRQ

$$\nabla p = \rho(\mathbf{g} - \mathbf{a}) \qquad (2.38)$$

7KH SUHVXUH JUDGLHQWDFWLQWKHGLUHFWLRQ í D DQGOQHVRI FRQWDQWSUHVXUH LQFXGLQJ WKHIUH
VXUDFH LI DQ DUH SHUSHQGLFXODU WR WKLV GLUHFWLRQ 7KH JHQHUDFDVH RI FRPELQHG WDQVODWLRQ DQ
URWDWLRQ RI DULJLG ERG\ LVGLVFXVVHGLQ&KDS) LJ
) OXLGV FDQ UDUHO PRYH LQ ULJLG ERG\ PRWLRQ XQGHUV UHVWULFWHG E\ FRQLQLQJ ZDOOV IRU D ORQ
WP H) RUH[DPSOH VXSSRVH DWDQN RI ZDWLV LQDFFXUWKDWWDWWDFRQWDQWWFHOHUDWLRQ 7KHZDWL
WKHWDQN ZRXOG EHJLQ WR VORVK DERXWDQG WKDWVORVKLQJ ZRXOG GDP S RXWYHU VORZ O XQWLOILQDOO WK
SDUWFHV RI ZDWU ZRXOG EH LQDSSUR[LPDWH ULJLG ERG\ DFFHOHUDWLRQ 7KLV ZRXOG WDNH VR ORQJ WK
WKH FDU ZRXOG KDYH UHDFKHG K\SHUVRQF VSHHGV 1HYHUWKHOHVV ZH FDQ DWOHDWWDFK WKH SUHVXU
GLVWULEXWLRQ LQDWDQN RI ULJLG0 DFFHOHUDWLQJ ZDWU

8QLIRUP / LQHDU$FFHOHUDWLRQ

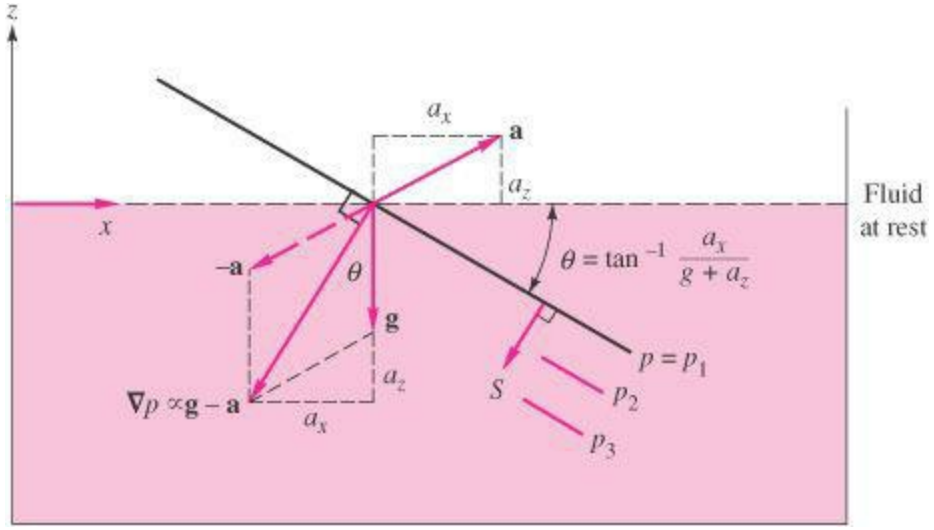
,Q WKH FDVH RI XQLIRUP ULJLG ERG\ DFFHOHUDWLRQ (T DSSOLHV ED E KDYLQJ WKH VDPH PDJQLWXGH DQG GLUHFWLRQ IRU DOO SDUWLFOHV : LWK UHIHUHQFH WR) LJ WKH SDUDOOHORJUDP VXP RI EJ E DQG í ED E JLYHV WKH GLUHFWLRQ RI WKH SUHVVXUH JUDGLHQW RU JUHDWHVW UDWH RI S 7KHVH UHVR FRQWDQW SUHVVXUH PXVW EH SHUSHQGLFXODU WR LWV DQG DUH WKXV OLQH GDWD GRZQ DUG DQG H VXFK WKDW

$$\theta = \tan^{-1}\frac{a_x}{g + a_z} \qquad (2.39)$$

2QH RI WKHVH OLQHV GHQRWHV LVRWKHUPV ZKLFK LV RXQG E\ WKH UHTXLUHPHQW WKDW WKH IOXLG UHDL LWV YROXPH XQGHU VWSLOOV RXW 7KHUHIRUH LQFUHDVHV SUHVVXUH LQ WKH GLUHFWLRQ J í DLV JUHDWHVW WKDQ LQ RUGLQDU\ KGURVWDWLF VDQG LV JLYHQ E\

$$\frac{dp}{ds} = \rho G \qquad \text{where } G = [a_x^2 + (g + a_z)^2]^{1/2} \qquad (2.40)$$

7KHVH UHVXOWV DUH LQGHSHQGHQW RI WKH VL]H RU VKDSH RI WKH FRQWDLQHU DV ORQJ DV WKH IOXLG L FRQWLQXRXV) FRQQHFWHG WKURXJKRXW WKH FRQWDLQHU



) LJ 7LOWLQJ RI FRQWDQW SUHVVXUH VXUIDFHV LQ DQ DFFHOHUDWLQJ ULJLG ERG\ DFFHOHUDWLRQ

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$ GUDJ UDFHU UHVWV KHUFRIHHP XJ RQ DKRUL]RQWDO WD ZKLHV KHDFFHOHUDWHV DW P V 7KHP XJ L FP GHSS DQG FP LQ GLDP HWHU DQG FRQWDLQV FRIIHH FP GHSS DWUHVW D $VXPLQJ ULJLG ERG DFFHOHUDWLRQ RI WKH FRIIHH GHWHUPLQH ZKHWKHU LW ZLOO VSLOO RXW RI WKH PXJ E &DOFXODWH WKH JDJ SUHVVXUH LQ WKH FRQWDLQHU SRLQW $ LI WKH GHQWLVW RI FRIIHHLV NJ P

6RXWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZ VWKHFRIIHHVLGHGXULQJ WKHDFFHOHUDWLRQ

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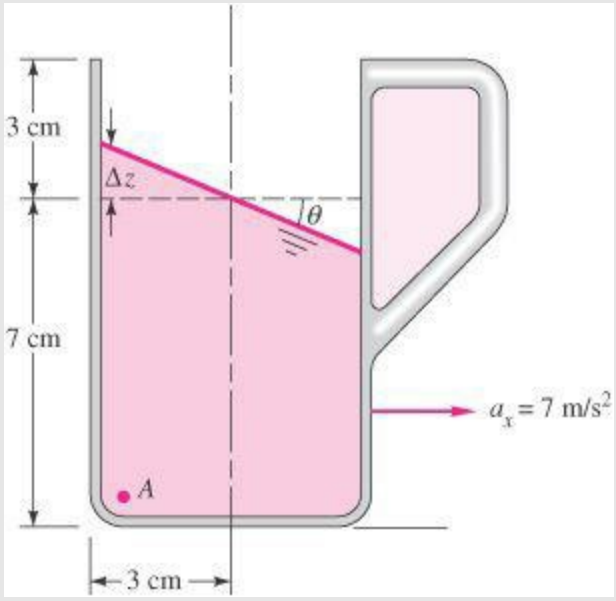


- $VVXPSWLRQV 5LJLG ERG\ KRUL]RQWDODFFHOHUDWLRQ D[P V 6\PPHWULF FRIIHHFXS
- 3URSHUWLHV ' HQVLWLRI FRIIHHJLYHQDV NJ P
- $SSURDFK D ' HWHUPLQHWKHDQJOHRI WLOWIURP WKHNQRZQDFFHOHUDWLRQ WKHQILQG WKHKHLJK
ULVH
- 6ROXWLRQVWHSV) URP (T WKHDQJOHRI WLOWLVJLYHQE\

$$\theta = \tan^{-1}\frac{a_x}{g} = \tan^{-1}\frac{7.0\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 35.5^\circ$$

,I WKHPXJ LV V\PPHWULF WKHWLOWHG VXUIDFHZLOOSDVV WKURXJK WKHFHQWUSRLQWRI WKHUHV
SRVLWRQ DV VKRZQLQ) LJ (7KHQ WKHUHLVGHRI WLOWHG FRIIHHIUHH VXUIDFHZLOOULVHD
DPRXQW] JLYHQE\

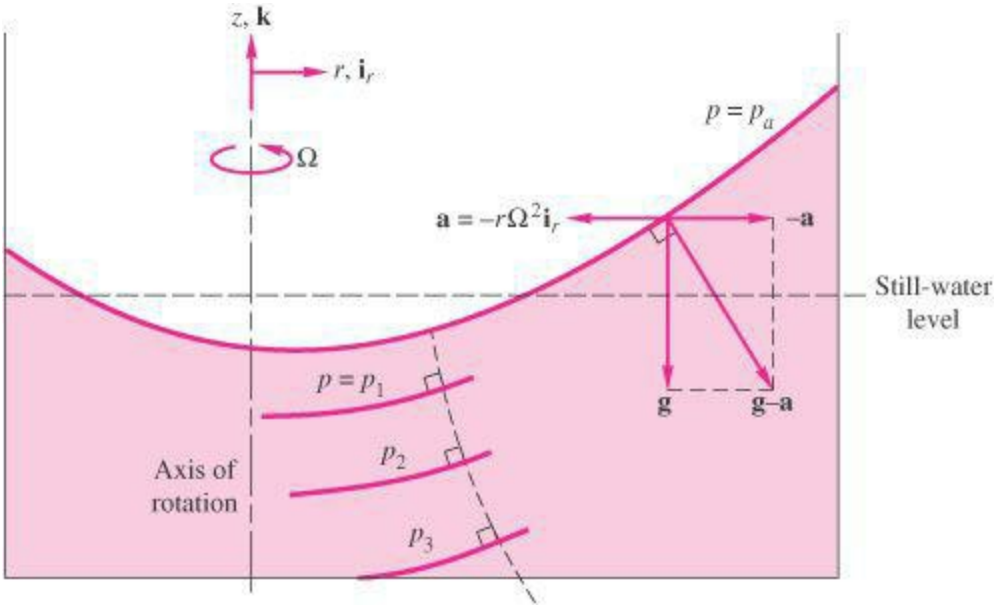
$$\Delta z = (3\ \text{cm})(\tan 35.5^\circ) = 2.14\ \text{cm} < 3\ \text{cm} \quad \text{therefore no spilling} \quad \textit{Ans. (a)}$$

- &RPPHQWD 7KLVVROXWLRQQHJOHFWVWVORVKLQJ ZKLFKPLJKWRFFXUL WKHVWDUWXSLVXQHYHQ
- $SSURDFK E 7KHSUHVXUHDW$ FDQEHFRPSXWHGIURP (T XVLQJ WKHSHUSHQGLFXODU
GLVWDQFHûVIURP WKHVXUIDFHWR$: KHQDWWHWVWS$!JK$UHW NJ P P V P
3D : KHQDFFHOHUDWLQJ

$$p_A = \rho G\,\Delta s = \left(1010\frac{\text{kg}}{\text{m}^3}\right)\left[\sqrt{(9.81)^2 + (7.0)^2}\right][(0.07 + 0.0214)\cos 35.5^\circ] \approx 906\ \text{Pa} \quad \textit{Ans. (b)}$$

- &RPPHQWE 7KHDFFHOHUDWLRQKDVLQFUHDVHGWKHSUHVXUHDW$ E\ SHUFHQW7KLQNDERX

5LJLG %RG\ 5RWDWLRQ



) LJ ' HYHORSPHQWRI SDUDERORLG FRQWRXUSUHVVXUH VXUIDFHV LQ D IOXLG LQ ULJLG ERG
URWDWRQ 7KH GDVKHG OLQHV DOVR GHILQH WKH GLUHFWLRQ RI P D[LPXP SUHVVXUH LQFUHDVH LV DQ H[SRQHQWLDO
FXUYH

$ VDVHFRQGVSHFLDOFDVH FRQVLGHUURWDWRQRI WKHIOXLGDERXWWKH] D[LVZLWKRXWDQ WDQVODWRQ D
VHWWKHGLQ) LJ : HDVXPHWKDWWKHFRQWDLQHUKDVEHHQURWDWLQJORQJHQRXJKDWFRQVWDQW IRUWKH
IOXLGWRKDYHDWWDLQHGULJLG ERG\ URWDWRQ 7KHIOXLGDFFHOHUDWRQZLOOWKHQEHDFRQVWDQWSHUSHQGLFXODU,QWK
FRRUGLQDWHVRI) LJ WKHDQJXODUYHORFLWDQGSRVLWRQYHFWRUVDUHJLYHQE\

$$\boldsymbol{\Omega} = \mathbf{k}\Omega \quad \mathbf{r_0} = \mathbf{i}_r r \tag{2.41}$$

7KHQWKHDFFHOHUDWRQLVJLYHQE\

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r_0}) = -r\Omega^2 \mathbf{i}_r \tag{2.42}$$

DVP DUNHGLQWKHILJXUH DQG(T IRUWKHIRUFHEDODQFHEHFRPHV

$$\nabla p = \mathbf{i}_r \frac{\partial p}{\partial r} + \mathbf{k} \frac{\partial p}{\partial z} = \rho(\mathbf{g} - \mathbf{a}) = \rho(-g\mathbf{k} + r\Omega^2 \mathbf{i}_r)$$

(TXDWLQJ FRPSRQHQWV ZHILQGWKHSUHVVXUHILHOGE\ VROYLQJ WZR ILUVWRUGHUSDUWLDOGLIIHUHQWL

HTXDWRQV

$$\frac{\partial p}{\partial r} = \rho r\Omega^2 \qquad \frac{\partial p}{\partial z} = -\gamma \tag{2.43}$$

7KH ULJKWKDQG VLGHV RI DUH NQRZ Q IXQFWLRQV RI U DQG] 2 QH FDQ SURFHHG DV IRQORZ V ,QWUDWH WKH ILUVW HTXDWLRQ³ SDUWLDOO ´ KROGLQJ] FRQVWDQWZ LWKUHVSHFWWR U 7KHUHVXOWLV

$$p = \tfrac{1}{2}\rho r^2\Omega^2 + f(z) \tag{2.44}$$

Z KHUH WKH³ FRQVWDQW´RI LQWHJUDWLRQLV DFWXDOO DIXQFWLRQ I] 1RZ GLIIHUHQWLDWH WKLVZ LWKUHVSHFWV] DQG FRP SDUHZ LWK WKH VHFRQG HTXDWLRQ

$$\frac{\partial p}{\partial z} = 0 + f'(z) = -\gamma$$

RU

I]] &

Z KHUH& LVDFRQVWDQW7KXV(T QRZ EHFRPHV

$$p = \text{const} - \gamma z + \tfrac{1}{2}\rho r^2\Omega^2 \tag{2.45}$$

7KLVLV WKH SUHVXUHGLVWULEXWLRQLQWKHIOXLG 7KHYDOXHRI & LVIRXQGE\ VSHFLI\LQJ WKH SUHVVXUHDW RQHSRLQW,I S S DWU] WKHQ& S 7KHILQDOGHVLUHGGLVWULEXWLRQLV

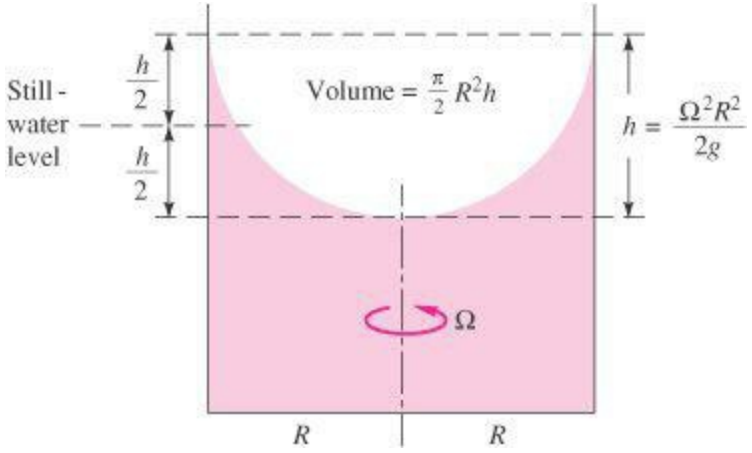
$$\boxed{p = p_0 - \gamma z + \tfrac{1}{2}\rho r^2\Omega^2} \tag{2.46}$$

7KHSUHVVXUHLVQDUL] DQGSDUDEROF LQU ,I Z HZ LVK WR SORWDFRQWDQWSUHVVXUHVXUIDFH VD\ S S (T EHFRPHV

$$z = \frac{p_0 - p_1}{\gamma} + \frac{r^2\Omega^2}{2g} = a + br^2 \tag{2.47}$$

7KXV WKH VXUIDFHV DUH SDUDEROLGV RI UHYROXWLRQ FRQFDYH XSZ DUG Z LWK WKH LUP LQLPXP SRLQW WR WKHD[LVRI URWDWLRQ 6RPHH[DPSOHVDUHVNHWFKHGLQ) LJ

$\ VLQ WKH SUHYLRXVH[DPSOHRI OLQHDUDFFHOHUDWLRQ WKH SRVLWLRQ RI WKH IUHHVXUIDFH LV IRXQG E\ FRQVHUYLQJ WKH YROXPH RI IXLG) RUDQRQLQHUWLDOFRQWDLQHUZ LWK WKHD[LVRI URWDWLRQRI I FHQWUDVLV) LJ DGRVWRI ODEORUYVPHQWUDWLRQVUHTXLUHG DQGDVLQJOHSUREOP ZLOOWDNH\RXDOZ HHNHQG+RZHYHU WKHFDOFXODWLRQLVHDV\ IRUDF\OLQGHUURWDWLQJ DERXWWKFHQWUDOD[LV DVLQ) LJ 6LQFH WKHYROXPH D SDUDEROLGLVRQHKDOI WKHEDVHDUHDWLPHVLWVKHLJKWWKHVWUDLJKWHGJHRI WKHIUHHVXUIDFH 7KHFHQWURI WKHIUHHGURSVDQDP RXQ K ÷ 5 J DQG\WKHHGJHVULHDQHTXDOLP RXQW



) LJ ' HWHUPLQLQJ WKH IUHH VXUIDFH SRVLWLRQ IRU URWDWLRQ RI D FRQWDLQHU RI OLTXLG DERXW LWV FHQWUDOO[LV

(; $ 0 3/ (

7KH FRIIHH FXS LQ ([DPSOH LV UHPRYHG IURP WKH GULJ UDFN SODFHG RQ D WXUQWDEOH DQG URWDWH DERXW LWV FHQWUDOO[LV XQWLO D UHJLG ERG\ PRWLRQ)LQG D WKH DQJXODU YHORFLW\ WKDW ZLOO FDXVH WKH FRIIHH WR MXVW UHDFK WKH OS RI WKH FXS DQG E WKH JDJH SUHVVXUH DW SRLQW $ IRU WKLV FRQGLWLRQ

6RXWLRQ

3DUW D

7KH FXS FRQWDLQV FP RI FRIIHH 7KH UHPDLQLQJ GLVWDQFH RI FP XS WR WKH OS PXVW HTXDO WK GLVWDQFH K LQ) LJ 7KXV

$$\frac{h}{2} = 0.03\ \text{m} = \frac{\Omega^2 R^2}{4g} = \frac{\Omega^2 (0.03\ \text{m})^2}{4(9.81\ \text{m/s}^2)}$$

6RYLQJ ZKHEDLQ

$$\Omega^2 = 1308 \quad \text{or} \quad \Omega = 36.2\ \text{rad/s} = 345\ \text{r/min} \qquad \textit{Ans. (a)}$$

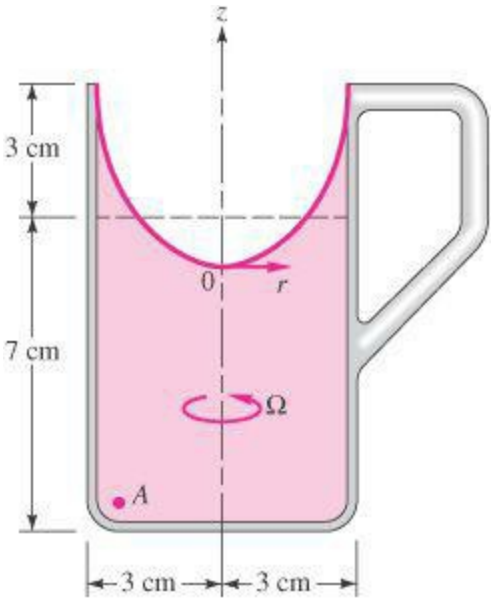
3DUW E

7R FRPSXWH WKH SUHVVXUH LWV FRQYHQLHQW WR SXW WKH RULJLQ RI FRRUGLQDWHV UDQG] DW WKH ERWWRP R WKH IUHH VXUIDFH GHSUHVVLRQ DV VKRZ QLQ) LJ (7KH JDJH SUHVVXUH KHUH LV S DQG SRLQW $ L DWU] FP í FP (TXDWLRQ FDQ WKHQ EH HYDOXDWHG

$$p_A = 0 - (1010\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-0.04\ \text{m}) + \tfrac{1}{2}(1010\ \text{kg/m}^3)(0.03\ \text{m})^2(1308\ \text{rad}^2/\text{s}^2)$$
$$= 396\ \text{N/m}^2 + 594\ \text{N/m}^2 = 990\ \text{Pa} \qquad \textit{Ans. (b)}$$

7KLVLVDERXW SHUFHQWJUHDWHUWKDQWKHYDOXHZDVUSUHVXUHS$ 3D

(

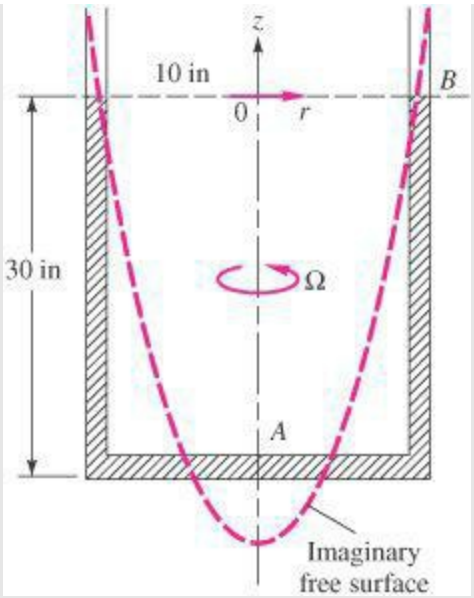


+HUH DVLQWKHOQHDUFHOHUDWRQFDVH LWWKRXGE HHPSKDVL]HG WKDWWKH SDUDERORLG SUHVVXU GLVWULEXWLRQ VHWXSLQDQ IOXLGXQGHUURWDWLRQ UHJDUGOHVVRI WKHVKDSHRUVL]HRI WK FRQWDLQHU7KHFRQWDLQHUPD\ HYHQEHFORVHGDQGILOOHGZLWKIOXLG ,WVRQO QHFHVVDU WKDWWKHIOXLGE FRQWLQXRXVO LQWUFRQHFWHG WKURXJKRXWWKHFRQWDLQHU7KHIROORZLQJ H[DPSOHZLOOLOOXVWUDWHSFXOD FDVH LQ ZKLFK RQH FDQ YLVXDOL]H DQ LPDJLQDU IUHH VXUFH H[WHQGLQJ RXWLGH WKH ZDOOV RI WK FRQWDLQHU

(; $0 3/ (

$ 8 WEHZLWKDUDGLXVRI LQDQGFRQWDLQLQJ PHUFXU WR DKHJKWRI LQLVURWDWHG DERXWV FHQWUDW UPLQXQGODULGERG PRGHOVDFKLHYHG 7KHGLPHQVLRI WKHWELQ LVQHJOLEO $WRSKHULF SUHVVXUHV GEI IW) LQGWKHSUHVVXHDWSRLQW$ LQWKHURWDWLQJ FRQGLWLRQ 6HH) LJ (

(



6RXWLRQ

&RQVLGHUWKHDQJXODUYHORFLWLHVUDGLDQ/SHUVHFRQG

$$\Omega = (180 \text{ r/min})\frac{2\pi \text{ rad/r}}{60 \text{ s/min}} = 18.85 \text{ rad/s}$$

) URP 7DEOH ZHILQGIRUPHUFXU WKDW GEI IWDQGKHQFH! VOXJVIW
$ WWKLV KJK URDWRQUDW WKHIUHVXUFHZLOOVDQWSZDGDWDILHUHDQJOH>DERXW ƒ FKHFNWKL
IURP (T @EXWWKHWELQJ LVVRWLQWKDWWKHIUHVXUFHZLOOUHPDLQDWDSSUR[LPDWH WKHVDPH
LQKHJKWSRLQW% 3DFLQJ RXURULJLQRI FRRUGLQDWHVDWWKLVKHJKWZHFDQFDOFXODWHWKHFRQWDQW
LQ(T IURP WKHFRQGLWLRQS% GEI IWDWU] LQ

$$p_B = 2116 \text{ lbf/ft}^2 = C - 0 + \tfrac{1}{2}(26.3 \text{ slugs/ft}^3)(\tfrac{10}{12} \text{ ft})^2(18.85 \text{ rad/s})^2$$

RU

& GEI IW

: HWKHQREWDLQS$ E\ HYDOXDWQJ (T DWU] í LQ

$$p_A = -1129 - (846 \text{ lbf/ft}^3)(-\tfrac{30}{12} \text{ ft}) = -1129 + 2115 = 986 \text{ lbf/ft}^2 \qquad \textit{Ans.}$$

7KLV LV GHW WKDQ DWPRVSKHULF SUHVXUH DQG ZH FDQ VHH ZK\ LI ZH IRQRZ WKH IUHVXUFH
SDUDERRLG GRZQ IURP SRLQW% DORQJ WKHGDVKHG OLQH,WZ LOOFURVV WKHKRUL]RQWDOSRUWR
RI WKH8 WXEH ZKHUHS ZLOOEH DWPRVSKHULF DQG IDOOEHORZ SRLQW$) URP) LJ WKHDFWXDOGUR
IURP SRLQW%ZLOOEH

$$h = \frac{\Omega^2 R^2}{2g} = \frac{(18.85)^2(\frac{10}{12})^2}{2(32.2)} = 3.83 \text{ ft} = 46 \text{ in}$$

7KXVS$ LVDERXW LQ+ JEHORZ DWRVSKHULF SUHVVXUH RUDERX[16/12/ GEI IWEHORZ S'

GEI IW ZKLFKFKHFNVZLWKWKHDQZHUDERYH : KHQWKHVXEHLVDWUHVW

$$p_A = 2116 - 846(-\tfrac{30}{12}) = 4231 \text{ lbf/ft}^2$$

+HQFHURWDWRQKDVUHGXFHGWKHSUHVVXUHDWSRLQW$ E\ SHUFHQW) XUWKHUURWDWRQFDQUHGXFHS$ VRQHDU]HURSUHVXUH DQGFDYLWDWRQFDQRFFXU

$QLQWHUHVWLQJ E\ SURGXFWRI WKLV DQDOVLV IRUULJLG ERG\ URWDWRQ LV WKDWWKH OQHV HYHUZKHU SDUDOHOWR WKH SUHVXUH JUDGLHQWRUP DIDP LO RI FXUYHG VXUIHFHV DV VNHWFKHG LQ) LJ 7KH DU HYHUZKHUHORWKRJRQDOWR WKHFRQWDQWSUHVXUHVXUIHFHV DQGKHQFHWKHLUVRSHLVWKHQHJDWLYHLQYHUV RI WKHVORSHFRP SXWHGIURP (T

$$\left.\frac{dz}{dr}\right|_{GL} = -\frac{1}{(dz/dr)_{p=\text{const}}} = -\frac{1}{r\Omega^2/g}$$

ZKHUH* / VWDQGVIRUJUDGLHQWZDQH

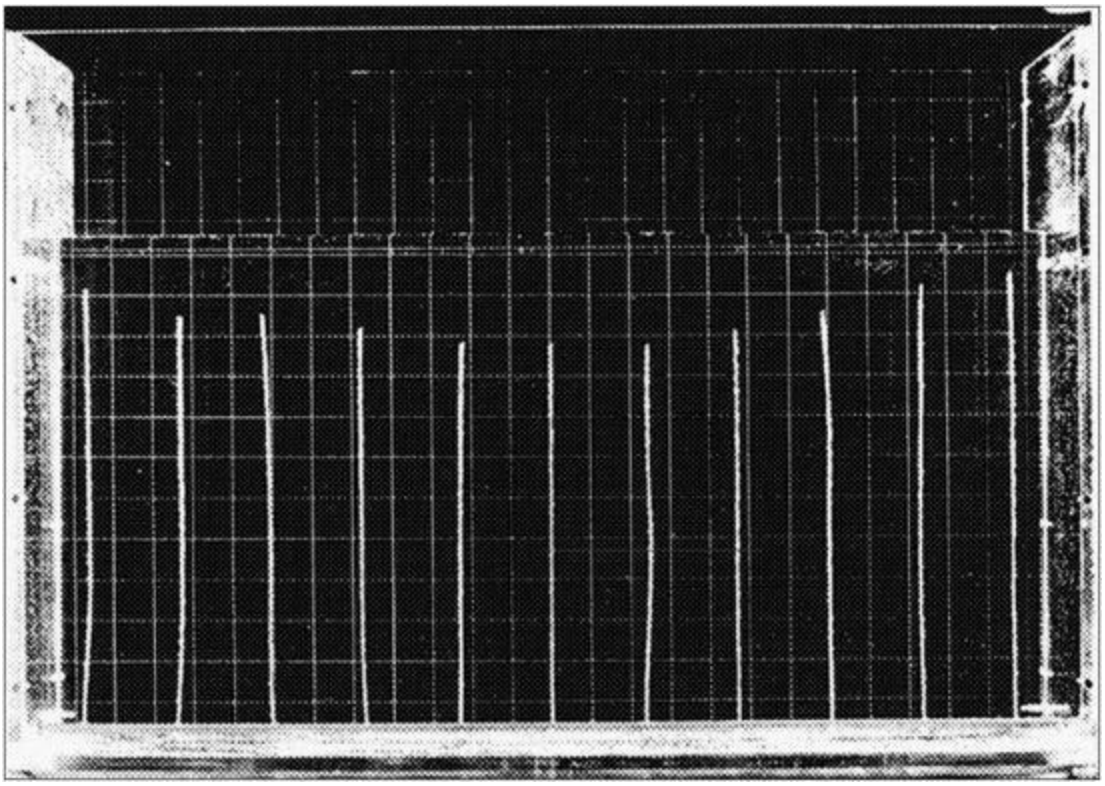
$$\frac{dz}{dr} = -\frac{g}{r\Omega^2} \qquad (2.48)$$

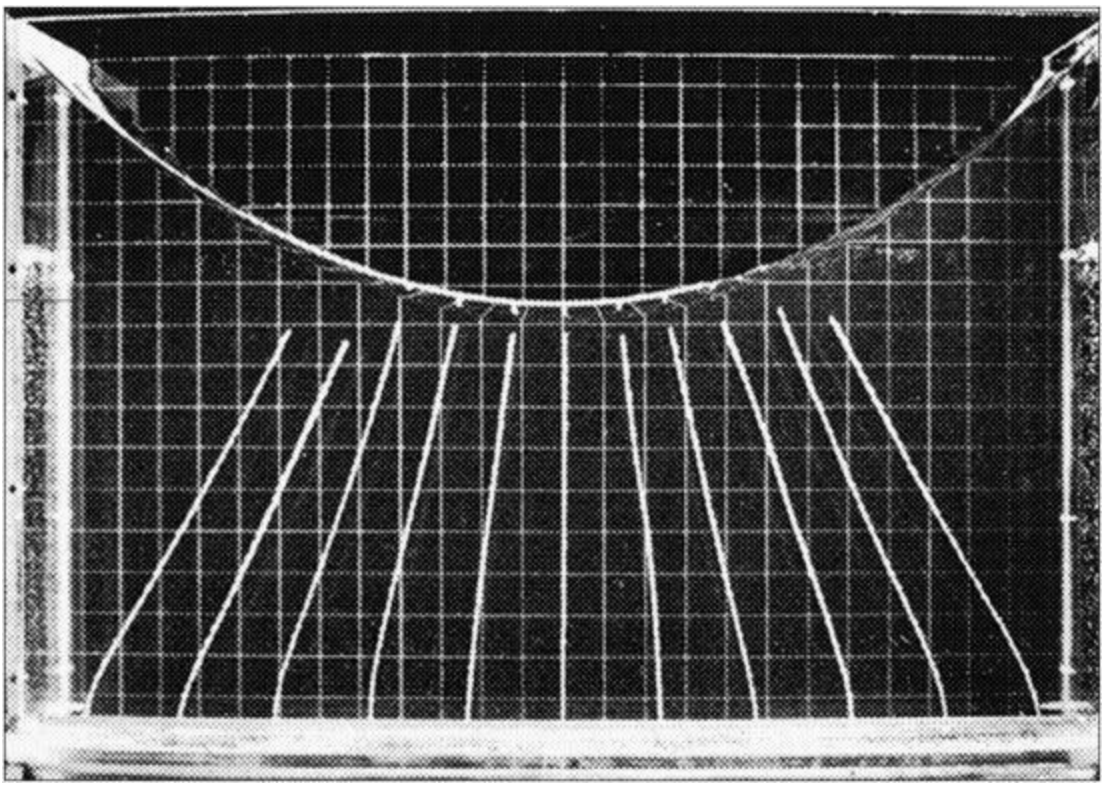
RU

6HSDUDWLQJ WKHYDULDEOHVDQGLQWHJUDWLQJ ZHILQGWKHHTXDWLRQRI WKHSUHVVXUHJUDGLHQWVXUIDFHV

$$r = C_1 \exp\left(-\frac{\Omega^2 z}{g}\right) \qquad (2.49)$$

1RWHWKDWWKLVUHVXOWDQG(T DUHLQGHSHQGHQWRI WKHGHQVLWLRI WKHIOXLG ,QWKHDEVHQFHR IUFWLRQDQG &RULROLV HIHFW(T GHILQHV WKHDQJHV ZKLFK WKHDSSDUHQWZHLJUDYLWDWLRQD ILHOGZRXOGDFWRQDSDUWFOH ' HSHQGLQJ RQLWVGHQVLW DVP DOOSDUWFOHRUEXEEOHZRXOGWHQG WRULV RUIDOOLQ WKHIOXLG DORQJ WKHVH[SRQHQWLDOOLQHV DV GHP RQVWUDWHG H[SHULP HQWDOO LQ5H $OVF EXR\DQWWXEP HUZRXOGDOJQWKPVHOYHVZLWKWKHVH[SRQHQWLDOOLQHV WKXVDYRLGLQJ WXWHVRWKH WKDQSXHWHQVLRQ)LJXUH VKRZ VWKHFRQLJXUDWRQ RI VXFK WXWHP HUVEHRUHDQGGXULQJ URWDWRQ





) LJ ([SHULP HQWDO GHP RQVWUDWLRQ ZLWK EXR\DQWWWHDP HUVRI WKH IOXLG IRUFH ILHOG GLQULJ ERG URWDWLRQ *top* IOXLG DWUHVW WWHDP HUV KDQJ YHUWLFDOO XSZDUG *bottom* ULJLG ERG URWDWLRQ WWHDP HUV DUH DQJQHG ZLWK WKH GLUHFWLRQ RI P D[LP XP SUHVXUH JUDGLHQW*(© The American Association of Physics Teachers. Reprinted with permission from "The Apparent Field of Gravity in a Rotating Fluid System" by R. Ian Fletcher. American Journal of Physics vol. 40, pp. 959–965, July 1972.)*

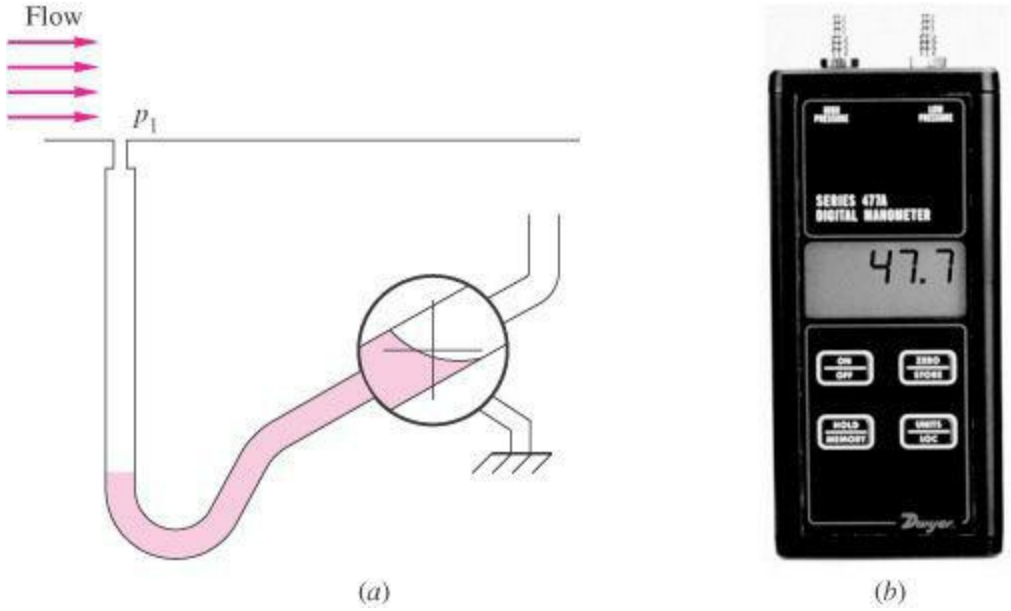
3UHVVXUH0 HDVXUHPHQW

3UHVVXUH LV D GHULYHG SURSHUW ,WLV WKH IRUFH SHU XQLW DUHD DV UHODWHG WR IOLG PRWFXOH ERPEDUGPHQWR DVXUIDFH 7KXV PRWSUHVVXUHLQWXPHQWRQ LQHUWKHSUHVVXUHE\ FDOEUDWRQZL D SULPDU GHYLFHV XFK DV D GHDGZHLJKWSLVWRQ WWWU 7KHUH DUH PDQ VXFK LQWXPHQW IRUERWK WDWF IOLG DQG DPRVLQJ WUHDP 7KHLQWXPHQDWRQWLQ5HV WR DQG ± GV RYHU GHVLJQV IRUSUHVVXUHPHDVXUHPHQWLQWXPHQW 7KHVHLQWXPHQWV PD\ EHJURXSHGLQRIRX FDWHJRULHV

* UDYLW EDVHG EDURPHWU P DQRPHWU GHDGZHLJKWSLVWRQ
(ODVWF GHRUPDWRQ ERXUGRQ WXEH PHDO DQG TXDUW GLDSKUDJP EHOORZ V VWDLQJDJH RSWFDOEHDP GLVSODFHPHQW
* DV EHKDYLRU JDV FRPSUHVVLRQ 0 F/ HRG JDJH WKHUPDO FRQGXFWDQFH 3LUDQL JDJH PROHFXDULP SDFW. QXGVHQJDJH LRQ]DWRQ WKHUPDOFRQGXFWLYLW DLUSLWRQ
(OHFWUF RXWSXW UHVLVWDQFH %UGJP DQ ZLWH JDJH GLIXVHG VWDLQ JDJH FDSDFLWDWYH SLH]RHOHFWF SRWQWLRPHWF PDJQHWF LQGXFWDQFH PDJQHWF UHOXFWDQFH QDU YDULEOH GLIHUHQWDOWRUP / 9' 7 UHVRQDQWUHTXHQF\ / XPLQHVFHQWFRDWQJV IRUVXUIDFHSUHVVXUHV> @

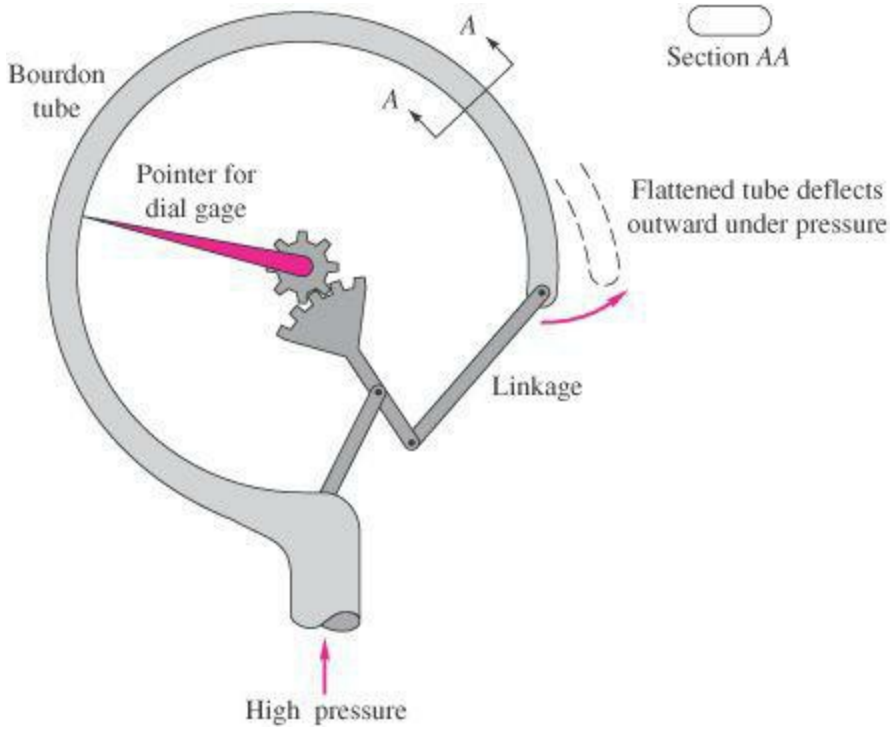
7KH JDVEHKDYLRU JDJHV DUH PRVW VSHFLDOSXUSRVH LQWXPHQW XVHG IRU FHUDLQ VFLHQWL H[SHULPHQW 7KHGHDGZHLJKWWWXULV WKHLQWXPHQ/XVHG PRVWWQIRUFDOEUDWRQ/ IRUH[DPSOH LV XVHG E\ WKH 8 6 1DWRQDO,QWXWXW IRU 6WDQGDUGV DQG 7HFKQRORJ\ 1,67 7KH EURPHWU GHVFUEHGLQ) LJ

7KHPDQRPHWU DQDO]HGLQ6HF LV D VLPSOHDQG LQH[SHQVLYH K\GURVWDWF SULQFLSOHGHYLFH ZLWK QR PRYLQJ SDUWH[FHSWWKH OTXLG FROXPQLWHO 0 DQRPHWUPHDVXUHPHQWPXWQRWGLVXLE WK IORZ 7KHEHWWZD\ WR GR WKLVLV WR DNHWKHPHDVXUHPHQWWKURXJK DVWDWF KROHLQWKHZDOORI WKHIORZ DVLOOXWUDWGLQ) LJ D 7KHKROHVKRXOGEHQRUPDOWR WKHZDOODQGEXUUVKRXOGEHDYRLGHG ,I WK KROHLV VPDOOHQRXJK \WSLFDO PP GLDPHWU WKHUHZLOOEHQR IORZ LQWR WKHPHDVXULQJ WXEHRQ WKHSUHVVXUHKDV DGYXWG WR DVWDG\ YDOXH 7KXV WKHIORZ LV DOPRVWXQGLVWXUEHG $QRVFLOODWQJ IORZ SUHVVXUH KRZHYHU FDQ FDXVH D ODJH HUURUGXHWR SRVLEOHGQDPLF UHVSRQVH RI WKH WXELQJ 2WKH GHYLFHVRI VPDOOHUGLPHQVLRQV DUH XVHGIRUG\QDPLF SUHVVXUHPHDVXUHPHQW 7KHPDQRPHWULQ) LJ DPHDVXUHVWKHJDJHSUHVVXUHS 7KHLQWXPHQWQ) LJ EL VDGLJLWDOGLIHUHQWDOPDQRPHWU ZKLFK FDQ PHDVXUHWKHGLIHUHQFHEHWZHHQWZR GLIHUHQWSRLQWLQ WKHIORZ ZLWKWXEHG DFFXUDF\ R SHUFHQWR IXOOVFDOH 7KHZRUGRI LQXWXPHQDWRQSRLQWTXLFNO WRZDUGGLJLWDOUHDGLQJV



) LJ 7ZR WSHV RI DFFXUDWH PDQRPHWHUV *a* WDSHG WXEH ZLWK HVHSLHFH *b* DFDSDFLWLYH WSHGLJLWDO PDQRPHWHU RI UDQGDFFXUDF\ " SHUFHQW*(Courtesy of Dwyer Instruments, Inc.)*

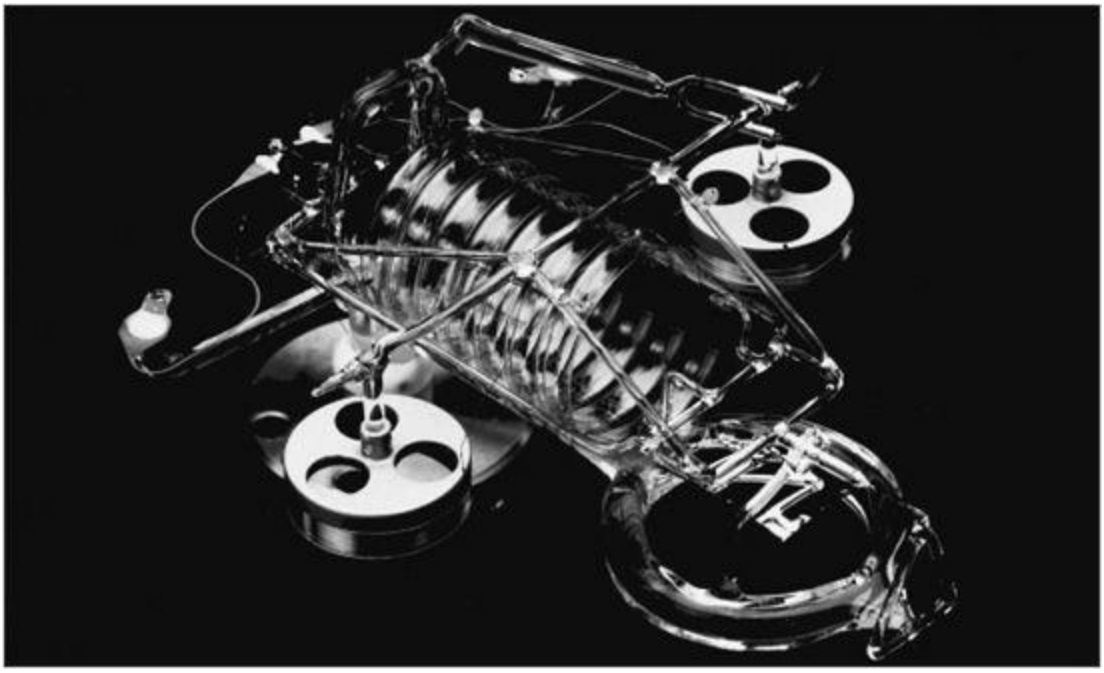
,QFDWURU HOHPHQW DSSOLFDWLRQV DSSOXDULQ[SHQVLYH DQGUHOLDEOHGHYLFHVWK EHUXGRQWXEH VNHWKHGLQ) LJ : KHQSUHVXUL]HGLQWRQGDD DFXUYHGWXEHZLWKIODWWQHGFURVVHFWLRQZLOOGHIOHFWRXWZDUG 7KHGHIOHFWLRQFDQEHPHDVXUHG E\ DQQDJHDWWDFKHGWR DFDOEUDWHGGLDOSRLQWHU VKRZQ 2 UWKHGHIOHFWLRQFDQEHXVHGWRGULYHHOHFWULFRXSXWHQWRUV VXFK DV I YDULDEOHWDQVRUP HU6LPLODU DPHPEUDQHORGLDSKUDJP ZLOOGHIOHFWXQGHUSUHVXUHDQGFDQHLWKHEHVHQVHGGLUHFWO RUXVHGWRGULYHDQRWKHUVHQVRU



) LJ 6FKHP DWLF RI DERXUGRQ WXEH GHYLFH IRUP HFKDQLFDOP HDVXUHP HQWRI KJK SUHVVXUH

$ QLQWHUHVWLQJ YDULDWRQ RI) LJ LV WKHIXVHG TXDUW IRUFHEDOODQHG ERXUGRQ WXEH VKRZ

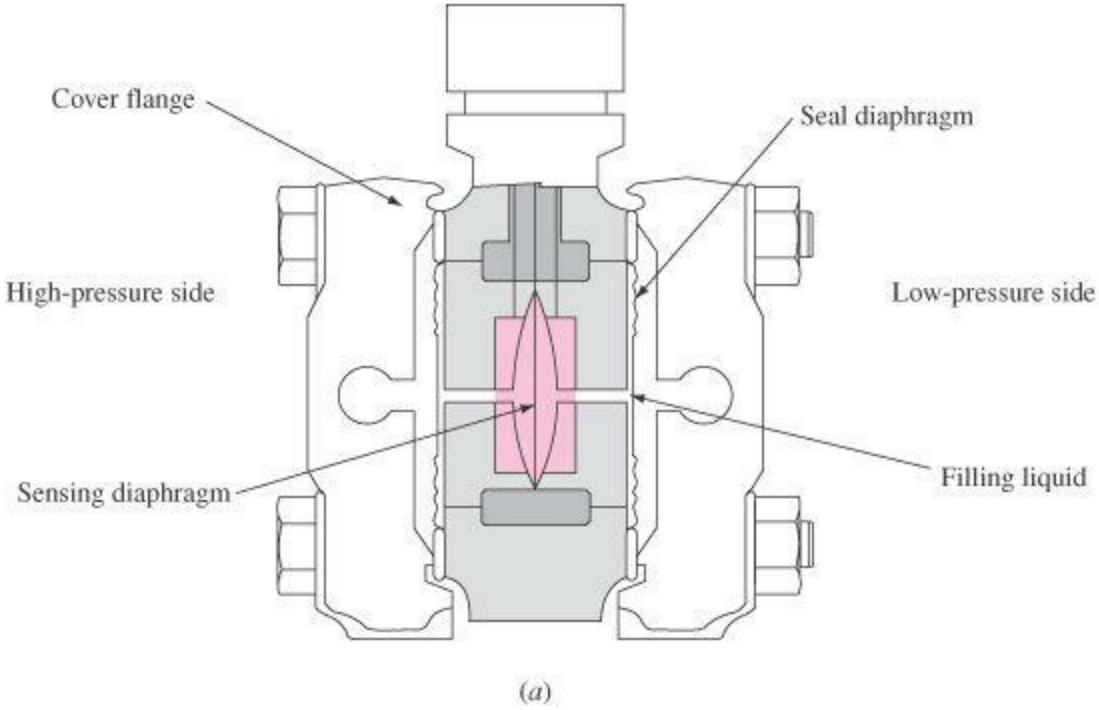
LQ) LJ ZKRVHVSLUDOVXEHGHOHFWURQLVVHQVHGRSWFDOO DQGUHXQLHGWRD]HURUHHUHQFHVDWE
D PDJQHWF HOPHQWZKRVH RXWSXWLV SURSRUWRQDOWR WKH IOXLG SUHVXUH 7KH IXVHG TXDUW] IRUF-
EDODQFHG ERXUGRQ WEHLVUHSRUWHGWRHRQHRI WKHPRVWDFFXUDWHSUHVXUHVHQVRUVHYHUGHYLVHG ZL
XQFHUWDLQWRQWKHRUGHURI “ SHUFHQW



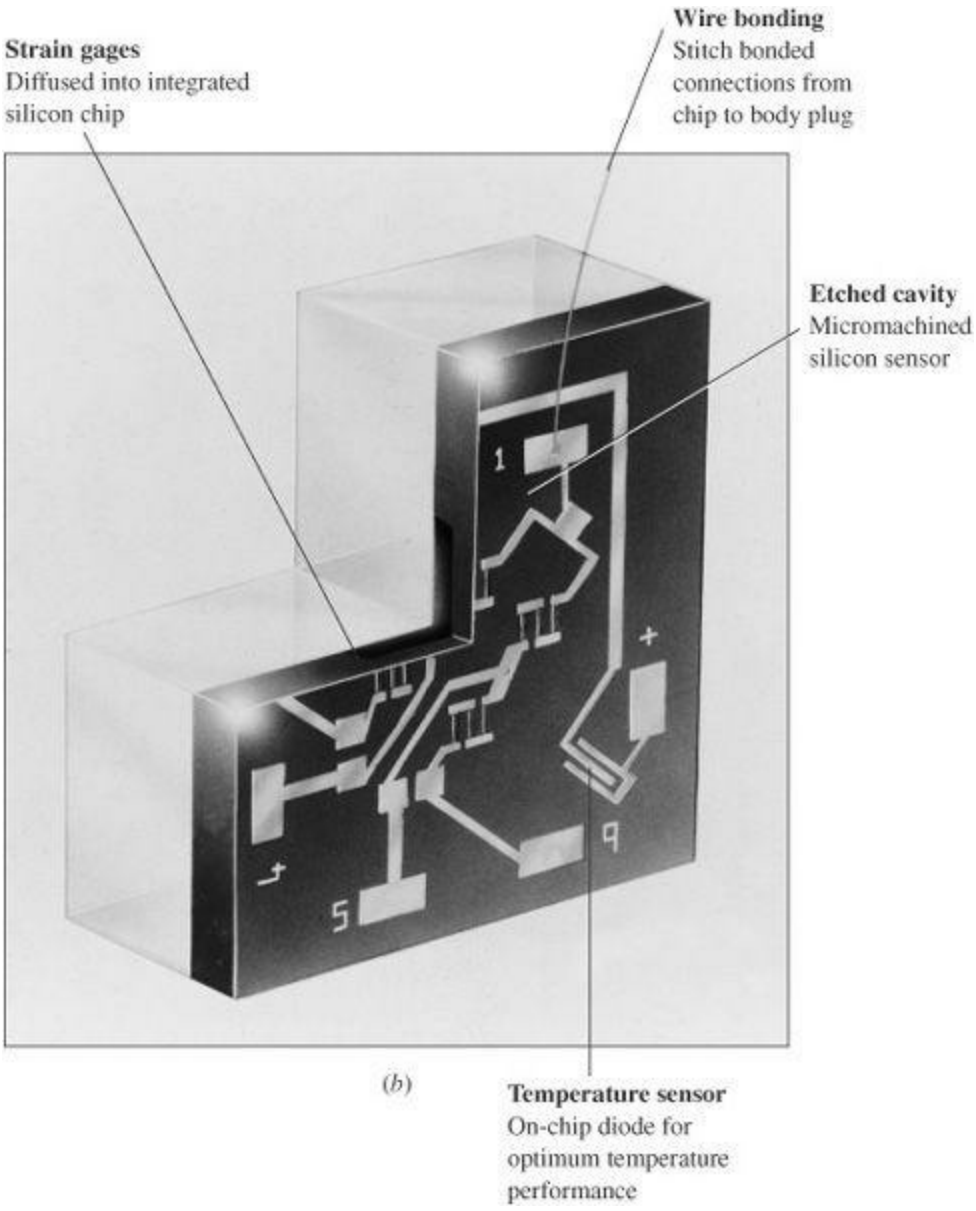
) LJ 7KH IXVHG TXDUW] IRUFH EDODQFHG ERXUGRQ WXEH LV WKH PRVWDFFXUDWH SUHVXU
VHQVRU XVHG LQ FRPPHUFLDO DSSOLFDWLRQV WRGD\ ***(Courtesy of Ruska Instrument Corporation, Houston, TX.)***

7KH TXDUW] JDJHV ERWK WKH ERXUGRQ WXEH DQG WKH UHVRQDQWWXEH H[SHULYH EXWH[WHPHC
DFFXUDWH VWDEOH DQGUHDEOH> @7KH\ DUH RIWQXVHG IRUGHHS RFHDQSUHVXUHPHDVXUHPHQW ZKLF
GHWHFWVRQJ ZDYHV DQG WXQDPLDFWLYLWRYHU[WQVLYHWPHSHUGV

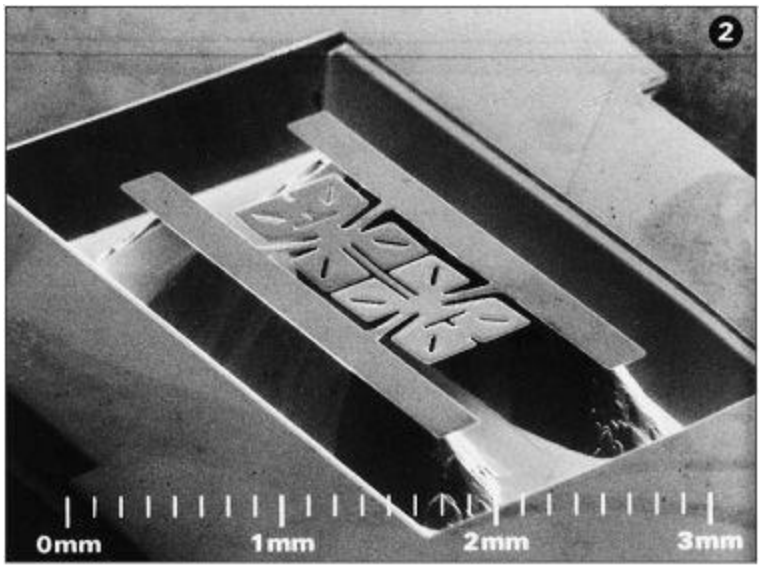
7KH ODXVFDWURU HOFWUF RXSXWWHQVRUV LV H[WPHO LPSRUDQWLQHQJLQHULQJ EHFDXVH WK
GDWFDQ EHVRUHG RQFRPSXWHUV DQG IUHO PDQSXODWG SORWWG DQG DQDO]HG 7KUHH H[DPSOHV DU
VKRZ Q LQ) LJ WKH ILUWEHLQJ WKFDSDFLWYH VHQVRU LQ) LJ D 7KH GLIHUHQWLDO SUHVXU
GHOHFWV WKH VLQFRQGLDSKUDJP DQG FKDQJHV WKH FDSDFLWDQFH RI WKH GTXLG LQ WKH FDYLW 1 RWWKDWWK
FDYLWKDV VSKHULFDOHQG FDSV WR SURPHQWRYHUSUHVXUHGP DJH ,QWKH VFRQG WSH) LJ E VWDLO
JDJHV DQG RWKHU VHQVRUV DUH FKHPLFDOO GLIXVHG RU HWKHG RQWR D FKLS ZKLFK LV WXHWHG E\ WK
DSSOHG SUHVXUH) LQDOO LQ) LJ F DP LFURP DFKLQHG VLOFRQ VHQVRULV DUUDQJHG WR GHRUP XQGH
SUHVXUHVXFK WKDWWWQDWXDOYLEUDWRQ IUHTXHQF\ LV SURSRUWRQDO WR WKH SUHVXUH $QRVFLOODWRU[FLW
WKH HOPHQWV UHVRQDQWUHTXHQF\ DQG FRQYHUWVLWQRDSSURSULDWSUHVXUHXQLW



) LJ 3UHVVXUH VHQVRUV ZLWK HODFWULF RXWSXW *a* D VLQJOH FRQ GLDSKUDJP ZKRVH GHIOHFWLRQ FKDQJHV VWKH FDSDFLWDQFH *(Courtesy of Johnson-Yokogawa Inc.);*



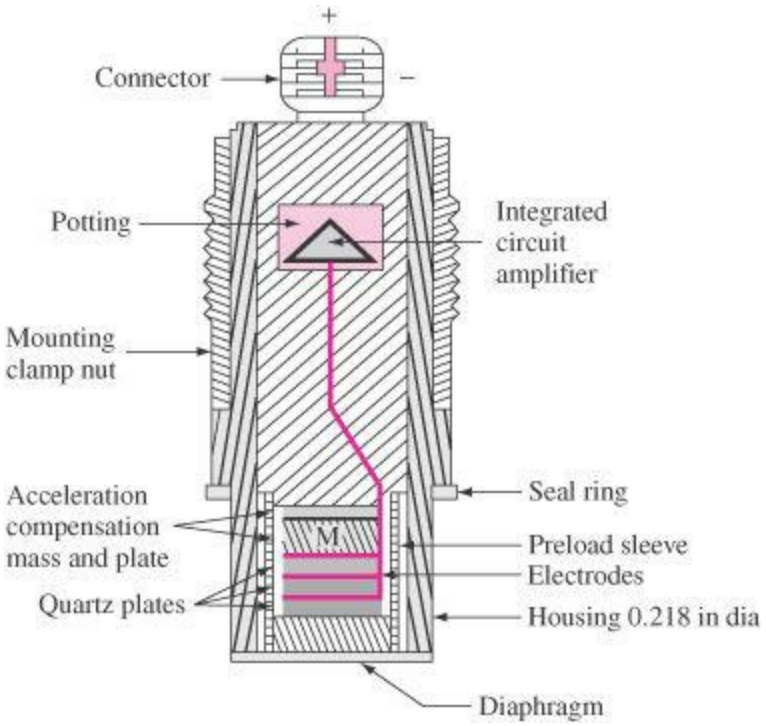
(*b*)



(*c*)

b D VLOLFRQ WUDQVLVWRUV VWUHVVHG E\ DSSOLHG SUHVVXUH ***c*** D PLFURPDFKLQHG VLOLFRQ HOHPHQWWKDWUHVRQDWHVDWDIUHTXHQF\ SURSRUWLRQDOWR DSSOLHG SUHVVXUH***[(b) and (c) are courtesy of Druck, Inc., Fairfield, CT.]***

$QRWKHUNLQG RI GLDPLF HOHFWULF RXSXWVHQVRULV WKHSLH]RHOHFWULF WUDQVGXFHU VKRZQ LQ) LJ 7KH VHQVLQJ HOHPHQWV DUH WKLQ GLVNV RI TXDUW] ZKLFK JHQHUDWH DQ HOHFWULF FKDUJH ZKH VXEMHFWHG WR VWUHVV 7KH GHVLJQ) LJ LV IOXVK PRXQWHG RQ D VROG VXUIDFH DQG FDQ VHQVH XQLDO SUHVXUH YDULQJ SUHVXUHV VXFK DV EODVWZDYHV 2WKHU GHVLJQV DUH RI WKH FDYLW\WSH 7KLV WSH RI VHQVR SUP DULO GHWHFWV WUDQVLHQWSUHVXUHV QRWXWDG VWUHVV EXW KLJKO LQXXOHWG FDQ DOR EH XVHG IR VKRUWWHUP VWDQLHQWSUHVXUHV 1RWH DOR WKDWLWP HDVXUHVJDJH SUHVXUH WKDWLV LWGHWHFWRQ D FKDQJI IURP DP ELHQWSRQGLWRQV



) LJ $ SLH]RHOHFWULF WUDQVGXFHUP HDVXUHVUDSLGO FKDQJLQJ SUHVXUHV*(Courtesy of PCB Piezotronics, Inc. Depew, New York.)*

6XPP DUI

7KLV FKDSWU KDV EHHQ GHYRWHG WR WKH FRPSXWDWQ RI SUHVXUH GLVWULEXWQV DQG WK UHVXOWQJ IRUFHV DQG PRPHQWV LQ DVWDWF ILXLG RU DILXLG ZLWK DNQRZ QYHORFLW ILHG $ODK GURVWDW 6HFV WR DQG UJLGERG 6HF SUREOPV DUH VROYHG LQWKLV PDQQHU DQG DUH FODVLF FDVH WKDWHYHU VXJHQWKRXG XQGHUWDQG ,Q DUELWUDU YLVFRXV IORZ V EHWK SUHVXUH DQG YHORFLW DU XQNQRZ QV DQG DUH VROYHG WRJHWKHU DV VWP RI HTXDWQV LQ WKH FKDSWU WKDWVROORZ

3UREOPV

0 RVWQ WKH SUREOPV KHUHQ DUH IDLO VWDLJKWUZ DG 0 RUHGLILFXOWURSHQHGHDVLJQP HQ DUH LQGLFDWG ZLWK DQ DXWULN DVLQ3URE 3UREOPV OBEHOHG ZLWK DQ((6 LFRQ IRU[DPSOH 3URE ZLOOEHQHLWURP WKH XVH RI WKH (QJLQHULQJ (TXDWQ6RQYHU ((6 ZKLOH SUREOPV OBEHOHG ZLWK D FRP SXWU LFRQ PD UHTXLUH WKH XVH RI D FRP SXWU 7KH VWDQGDUG HQG RI FKDSWU SUREOPV WR FDWUR]HG LQ WKH SUREOP GLVWULEXWQ DUH IROORZHG E\ ZRUGSUREOPV : WR: IXQGDP HQWDO RI HQJLQHULQJ H[DP SUREOPV)(WR)(FRP SUHKHQLYH

3UREOHP ' LVWULEXWLRQ

Section	Topic	Problems
2.1, 2.2	Stresses; pressure gradient; gage pressure	2.1–2.6
2.3	Hydrostatic pressure; barometers	2.7–2.23
2.3	The atmosphere	2.24–2.29
2.4	Manometers; multiple fluids	2.30–2.47
2.5	Forces on plane surfaces	2.48–2.80
2.6	Forces on curved surfaces	2.81–2.100
2.7	Forces in layered fluids	2.101–2.102
2.8	Buoyancy; Archimedes' principles	2.103–2.126
2.8	Stability of floating bodies	2.127–2.136
2.9	Uniform acceleration	2.137–2.151
2.9	Rigid-body rotation	2.152–2.159
2.10	Pressure measurements	None

6WUHVVHV SUHVVXUH JUDGLHQW JDJH SUHVVXUH

- 3) RUWKHVWR GLPHQVLRQDOVWDWHVILHOGVKRZQLQ) LJ 3 LWLVIRXQGWKDWRUWKHVWR GLPHQVLRQDOVWDWHVILHOGVKRZQ

  1[[GEH IW1\\ GEH IW1[\ GEH IW

) LQGWKHVKHDUDQGQRUPDOVWUHVV LQGEH IW DFWLQJ RQSODQH$$ FXWWLQJ WKURXJK WK HOHPHQWDO *DQGHDVKRZQ

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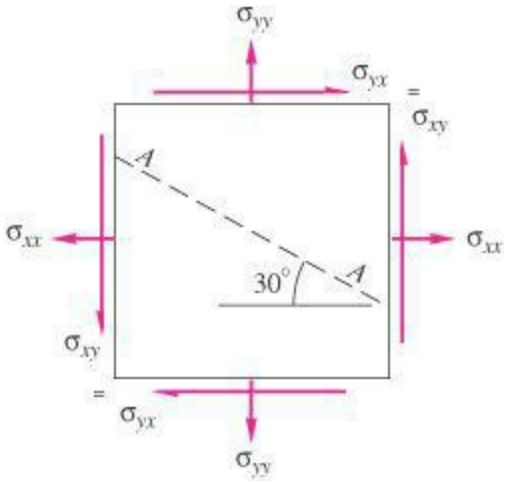

- 3) RUWKHVWR GLPHQVLRQDOVWDWHVILHOGVKRZQLQ) LJ 3 VXSSRVHWKDW

  1[[GEH IW1\\ GEH IW1Q $$ GEH IW

  &RPSXWH D WKHVKHDUVWUHVV1[\ DQG E WKHVKHDUVWUHVVRQSODQH$$

- 3 $ YHUWLFDO FOHDQ JODVV SLS] RPHWHU WXEH KDV DQ LQVLGH GLDPHWHU RI PP : KH

SUHVVXUH LV DSSOLHG ZDWHU DW *f*& ULVHV LQWR WKH WEH WR D KHLJKW RI FP $IW
FRUUHFWLQJ IRU VXUIDFH WHQVLRQ HIIHFWV DW WKH DSSOLHG SUHVVXUH LQ3D

- 3 3UHVVXUH JDJHV VXFK DV WKH %RXUGRQ JDJH LQ)LJ 3 DUH FDOLEUDWHG ZLWK I
GHDGZHLJKWSLVWRQ ,I WKH%RXUGRQJDJHLVGHVLJQHGWRURWDWHWKHSRLQWHU GHJUHHVIR
HYHU SVLJ RI LQFUHDVHSUHVVXUH KRZ PDQ GHJUHHV GRHV WKH SRLQWHU URWDWH LI WK
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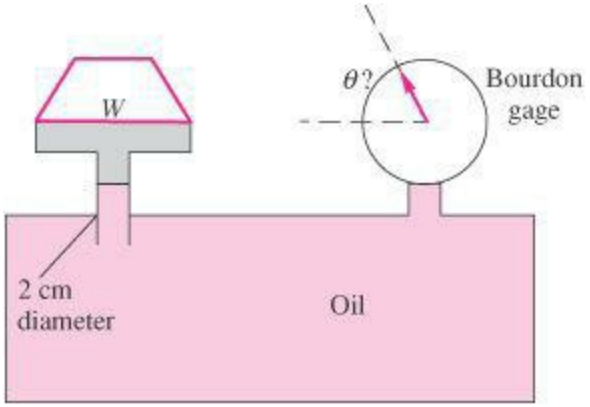


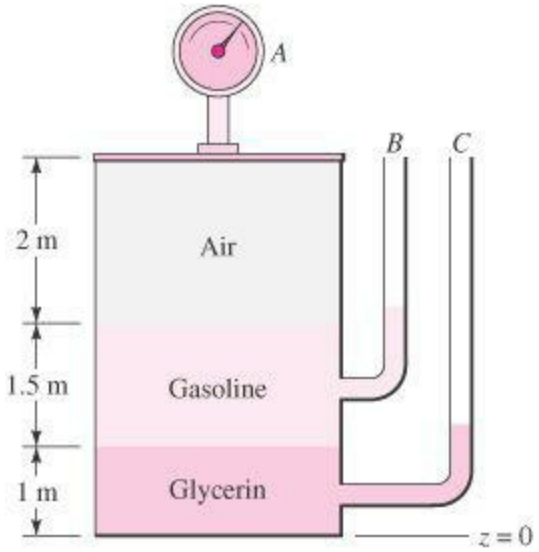
- 3 ' HQVH&RORUDGR KDVDQDYHUDJHDOWLWXGHRI IW2QDWLRQDOGLG\ 7DEOH$
SUHVVXUHJDJH$ LQDODERUDWRU\ H[SHULPHQWV NBDQGJDJH%UHDGV NBD
([SUHVV WKHVH UHDGLQJV LQJDJH SUHVVXUH RU YDFXXP SUHVVXUH 3D ZKLFKHYHU L
DSSURSULDWH
- 3 $Q SUHVVXUHUHDGLQJFDQEH[SUHVVHGDVDOHQJWKRUKHDG K S!J : KDWLVWKDQGDU
VHDOHYHOSUHVVXUH H[SUHVVHGLQ D IWRI JOFHULQ E LQ+J F P RI ZDWHU DQG G
PP RI HKDQRO"$VXPHDOOIOXLGVDUHDW *f&

+\GURVWDWLF SUHVVXUH EDURPHWHUV

- 3 /D 3D] %ROLYLD LV DWDQ DOWLWXGH RI DSSUR[LPDWH IW$VXPH D VWDQGDU
DWPRVSKHUH +RZ KLJK ZRXOG WKH OLTXLG ULVH LQ DP HKDQROEDURPHWHU DVXPHG D
f&"
+LQW' RQJWVRUJHWWKHYDSRUSUHVVXUH
- 3 $ GLDPRQG PLQHLV WZR PLOHVEHORZ VHD OHYHO D (VWLPDWH WKH DLUSUHVVXUHDWWKL
GHSWK E ,I DEDURPHWHU DFFXUDWH WR PP RI PHUFXU LV FDUULHG LQWR WKLV PLQH KRZ
DFFXUDWH FDQ LWV\VWHP DWWKHGHSWK RI WKHPLQH' / LVWVRXUDVXPSWRQVFDUHXOO
- 3 $ VWRUDJHWDQN IWLQGLDPHWHUDQG IWKLJK LV ILOOHGZLWK6$(: RLODW *f&
D : KDWLV WKH JDJHSUHVVXUH LQGE LQ DWWKHERWWRP RI WKHWDQN" E +RZ GRHV\RX
UHXOWLQ D FKDQJHLI WKHWDQNGLDPHWHULV UHGXFHGWR IW F 5HSHDWD LI OHDNDJI
KDVFDXVHGDOO\HURI IWRI ZDWHUWRUHWDWWKHERWWRP RI WKH IXOO WDQN
- 3 $ FORVHGWDQNFRQWDLQV P RI6$(RLO P RI ZDWU FP RI PHUFXU DQGD
DLUVSDFHRQWRS DOODW *f& 7KHDEVROXWSUHVVXUHDWWKHERWWRP RI WKHWDQNLV
NBD: KDWLVWKHSUHVVXUHLQWKHDLUVSDFH"
- 3 ,Q)LJ 3 SUHVVXUH JDJH$ UHDGV NBD JDJH 7KH IOXLGV DUH DW *f&
' HWHUPLQH WKH HOHYDWRQV] LQ PHWHUV RI WKH OTXLG OHYHOV LQ WKH RSHQ SLSH] RP HW

VXEHV%DQG&

3



- 3 ,Q) LJ 3 VKHVDQNFRQDLQVZDWUDQGLPPLVFLEOHRLODW f& : KDWLVKLQFP L VKHGHQVLWRI VKHRLOLV NJ P "

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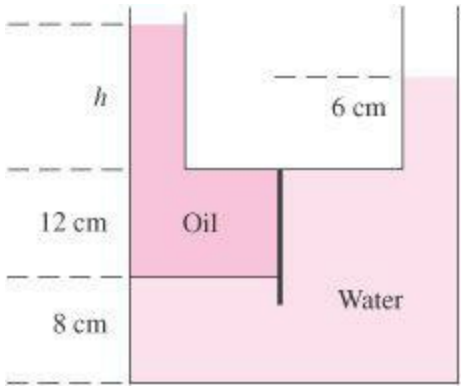

- 3 ,Q) LJ 3 VKH f& ZDWUDQGJDVROLQHVXUFHVDUHRSHQWRWKHDWPRVSKHUHDQGD VKHVDPHHOHYDWLRQ : KDWLVWKHKHLJKWRI VKHVXUIDFHRIWKHJDVROLQHUHODWLYHWR

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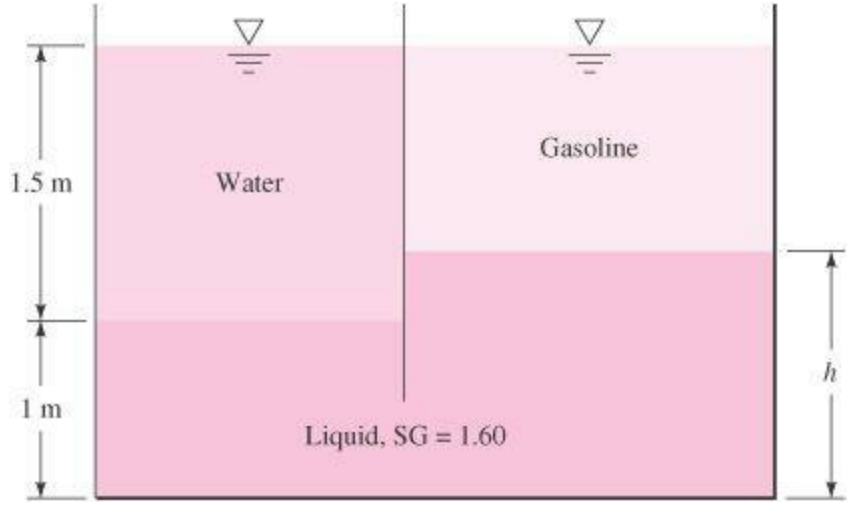

- 3) RUWKHWKUHHTXLGVVWDWLF VKRZQ FRPSXWH K DQGK 1HJOHFWWKHDLUGHQVLW

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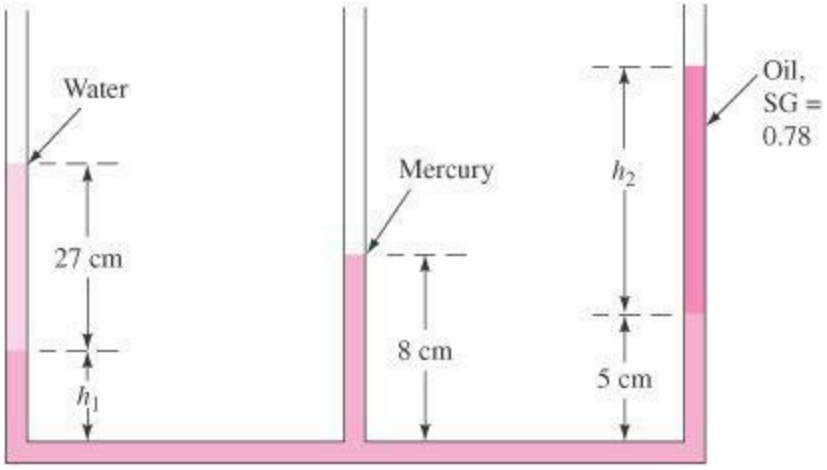
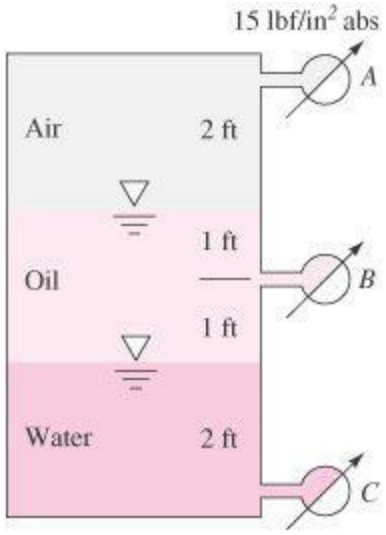


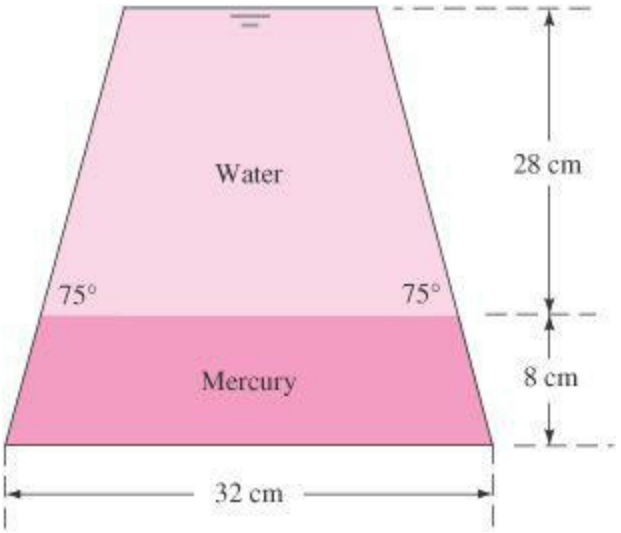
- 3 7KHDLUHRLGZDWUVVWDWLQ) LJ 3 LVDW f& . QRZLQJ WKDWJDJH$ UHDGV GEI LQ DEVROXWH DQG JDJH% UHDGV GEI LQ GHVV WKDQ JDJH& FRPSXWH D WK VSHFLILF ZHJKWRI WKHROLQGEI IWDQG E WKH DFWXDOUHDGLQJ RI JDJH& LQGEI LQ DEVROXWH

3



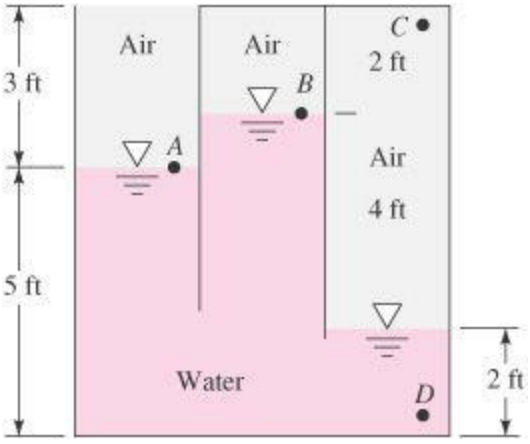
- 3 ,I WKH DEVROXWH SUHVVXUH DW WKH ORZHU HQG RI WKH FRQGXLW LQ) LJ 3 LV N3D ZKDW LQ IW LV D WKH SUHVVXUH DW WKH VXUIDFH DQG E WKH SUHVVXUH DW WKH ERWWRP RI WKH FRQGXLW

3



- 3 7KH VWXPP LQ) LJ 3 LV DW f& ,I WKH SUHVVXUH DW SRLQW$ LV GE IW GHWHUPLQH WKH SUHVVXUH DW SRLQWV% & DQG' LQGE IW

3



- 3 7KH VWXPP LQ) LJ 3 LV DW f& ,I DFFRVSKHULF SUHVVXUH LV N3D DQG WK SUHVVXUH DW WKH ERWWRP RI WKH WDQNLV N3D ZKDW LV WKH VSHFLILF JUDYLW RI IOXLG; "

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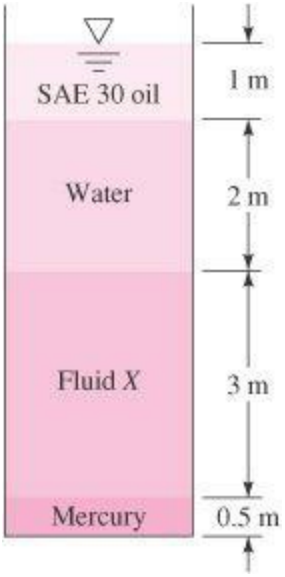

- 3 7KH8 WXEHLQ) LJ 3 KDVD FP ,' DQG FRQWDLQV PHUFXU DV VKRZQ ,I FP R ZDWHU LV SRXUHG LQWR WKH ULJKW KDQG OHJ ZKDW LV WKH IUHH VXUIDFH HLJKWQHDFK OH EHDWWUHH WKH VRVKLQJ KDV GLHGRZQ'

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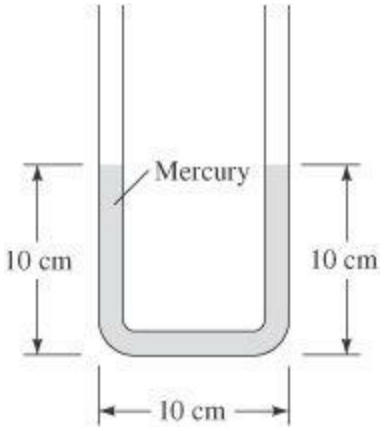

- 3 7KH K\GUDXOLF MDFNLQ) LJ 3 LV ILOOHG ZLWK RLO DW GE IW 1 HJOHFWQJ WKH ZHJK RI WKH VZR SLWRQV ZKDWRUFH) RQ WKH KDQGOH LV UHTXLUHG WR VXSSRUW WKH GE ZHJKWRUWKLVGHVLJQ'
- 3 $W f& JDJH$ UHDGV N3D DEVROXWH : KDWLV WKH KHLJKW RI WKH ZDWHU LQ FP' : KDWWKRXOGJDJH%UHDGLQN3D DEVROXWH 6HH) LJ 3

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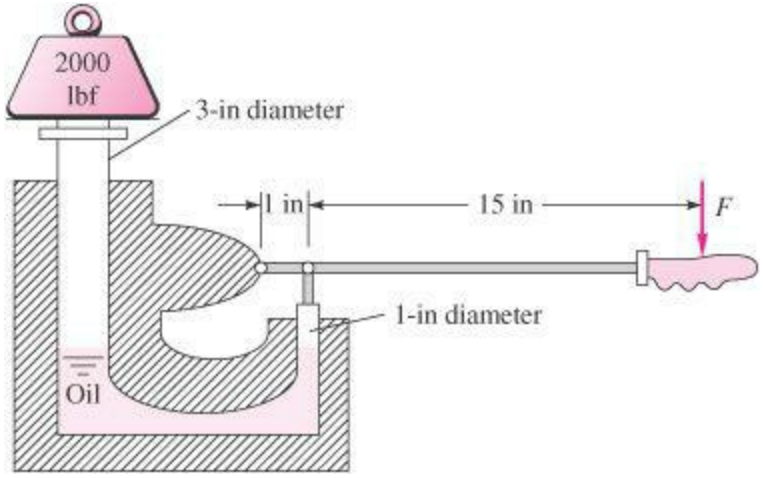

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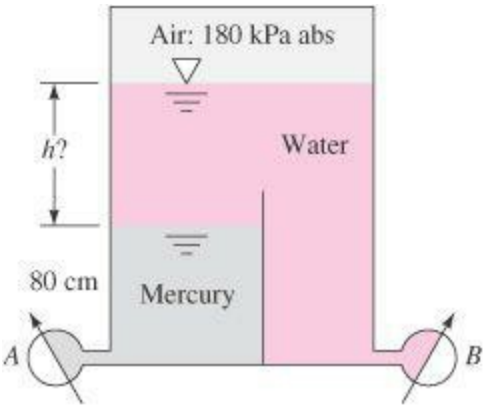

- 3 7KH IXHO JDJH IRU D JDVROLQH WDQN LQ D FDU UHDGV SURSRUWLRQDO WR WKH EROWP JDJ SUHVVXUH DV LQ) LJ 3 ,I WKH WDQN LV FP GHHS DQG DFFLGHQWDO FRQWLQV FF RI Z DWUSOXV JDVROLQH KRZ PDQ FHQWLPHWHUV RI DLUHP DLQ DWWKH WRS Z KHQ WKH JDJ HUURQHRXV) UHDGV³IXOO"

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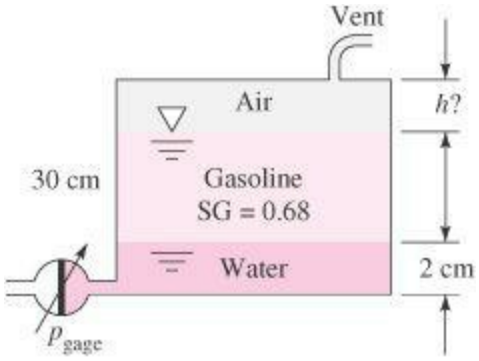

- 3 ,Q) LJ 3 EROWIOXLGVDUHDW *f*& ,I VXUIDFHWHQVLRQHIHFWVDUHQHJOLJLEOH Z KDW VWKHGHQLWVRI WKHRLOLQNJ P "

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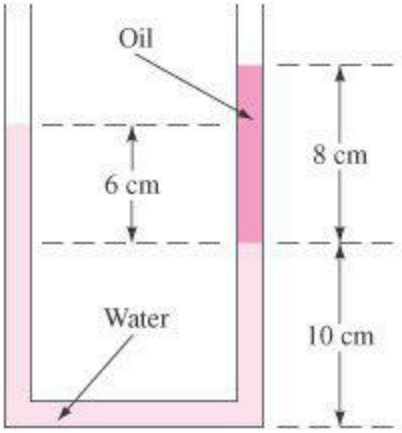

7KHDWP RVSKHUH

- 3 ,Q3UREOHPDFXGHLQWHUDFWLRQRI WKHGHQVLWGLVWULEXWLRQ] LQ7DEOH$ DQGHXWPDWGWKHPDVVRI WKHHDUWK¶VDWPRVSKHUHWREHP § (NJ &DOWKLVUHVXOEHXVHGWRHXWPDWVHDOHYHOSUHVXUHRQWKHHDUWK' &RQVLGHU FDQWKHDFWXDOVHDOHYHOSUHVXUH RI NJ D EH XVHG WR PDNH D PRUH DFFXUDWHXWPDWRI WKDWRVSKHULF PDW"
- 3 $VPHDVXUHG E\ 1$6$¶V 9LNLQJ ODQGHUV WKH DWPRVSKHUH RI 0DUV ZKHUHJ P V LVDOPRVWHQWLUHO FDUERQGLR[LGH DQG WKH VXUIDFHSUHVXUHDYHUDJHV 3D 7KHWMPSHUDXUHLVFRQGDQGGURSVRI H[SRQHQWLDOO 7 $7_R Hí&] ZKHUH& (Pí DQG7_R .)RUH[DPSOH DW P DOWLWXGH 7"""" . D)LQG D DQDOWLF IRUPXOD IRUWKH YDULDWRQ RI SUHVXUH ZLWK DOWLWXGH E)LQG WKH DOWLWXG ZKHUHSUHVXUHQ0DVKDVGURSSHGWR SDFDO
- 3)RUJDHVWKDWXQGHUJRDOJHFKDQJHVLQKHLJKWWKHQQHDUDSSUR[LPDWRQ (T LVLQDFFXUDWH ([SDQG WKHWRSRSVKHUHSRZHUOD[(T LQWRD SRZHUVHULHV DQG VKRZ WKDWWKHQQHDUDSSUR[LPDWRQS§ S_D í !_D J] LVDGHTXDWZKHQ

$$\delta z \ll \frac{2T_0}{(n-1)B} \quad \text{where } n = \frac{g}{RB}$$

- 3 &RQGXFWDQH[SHULPHQWR LOOXVWDWH DWPRVSKHUF SUHVXUH 1RWH ' RWKLVRYHUDVLQ RU\RX PD\ JHWZHW) LQG D GULQNLQJ JODVV ZLWK D YHU VPRRWK XQIRUP UIP DWWK VRS)LOOWKHJODVVQHDUO IXOOZLWKZDWU 3ODFHD VPRRWK QJKWODWSODWHRQVRS R WKHJODVVVXFK WKDWWKHQQHUIP RI WKHJODVVLVFRYHUHG $ JRXW SRVWDUGZRUN EHWW$ VPDOOLQH[FDUGRUQH IODS RI DJUHHLQJ FDUGZLOODOR ZRUN 6HH) LJ 3 D

D +RGWKHFDUGDJDLQVWWKHUIP RI WKHJODVV DQGWXQWKHJODVVXSVLGHGRZQ 6HOHDVH SUHVXUH RQ WKH FDUG ' RHV WKH ZDWU IDOO RXWRI WKH JODVV" 5HFRUG \RX H[SHULPHQWDOREVHUYDWRQV E)LQG DQH[SUHVLRQIRUWKHSUHVXUHDWSRLQW DQG LQ) LJ 3 E 1RWWKDWWKHJODVVLVQRZ LQYHUWHG VR WKHRULJLQDOWRS UIP RI WKHJODVVLV DWWKH ERWWRP RI WKH SLFWXUH DQG WKHRULJLQDOERWWRP RI WKHJODVVLV DWWKHWRS RI WKH SLFWXUH 7K ZHLJKWRI WKHFDUG FDQEHQHJOHFWHG F (VWLPDWWKH WKRUHWLFDOPD[LPXP JODVVKHLJKWL

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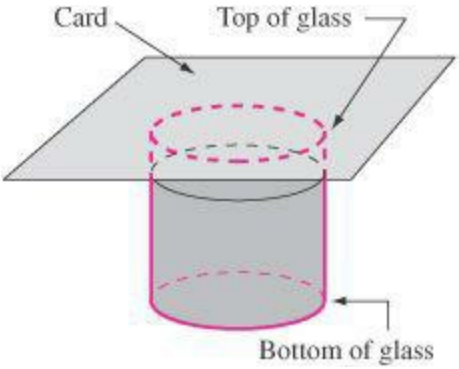
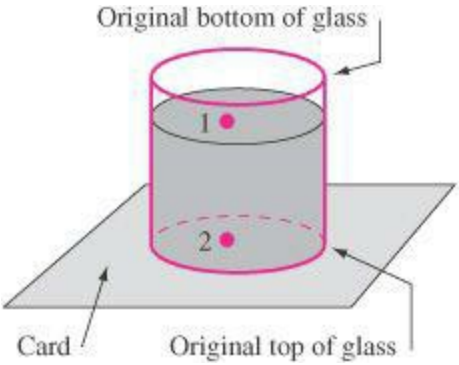
3 D



3 E



- 3 $ FRUHOD WRQ RI QXP HULFDO FDOFXODWRQV LQGLFDWH WKDW DOO RWKHU WKLQJV EHLQJ HTXDO WKH GLVWDQFH WDYHOHG E\ DZ HOKLWH EDOO EHORZ WKH FXEH URRW RI WKH DLU GHQVLW ,I DKRPH UXQ EDOKLWQ1HZ <RUN &LW WDY HO IWHWXP DW WKH GLVWDQFH LWZ RXOG WDYHOLQ D ' HQYHU &RORUDGR DQG E / D3D] %RQYLD
- 3 $QDLSODQH IOLHV DW D0 DFK QP EHUR DW WDO WDQGDUG DOWLWXGH IW D : KDWLV WKH SODQH¶V YHORFLW LQ PLK" E : KDWLV WKH WDQGDUG GHQLW DW WKDW DOWLWXGH"

0 DQRP HWHUV P XOWLSOH FKLGV

- 3) RUWKH WDGLWRQDO HTXDOOHY HOP DQRP HWHUP HDVXUHP HQWQ) LJ (Z DWHUDW *f&* IORZV WKURXJK WKH SXJ GHYLFH IURP D WR E 7KHP DQRP HWHUIOXLGV PHUFXU ,I/ FP DQG K FP D ZKDWV WKH SUHVVXUHGURS WKURXJK WKH GHYLFH" E ,I WK Z DWHUIORZV WKURXJK WKH SLSH DW D YHORFLW9 IWV ZKDWV WKH GLPHQVLRQOHVV RRW FRHILFLHQWRI WKH GHYLFH GHILQHG E\ . %S !9 " : H ZLOO VXG\ ORV FRHILFLHQWQ&KDS
- 3 ,Q) LJ 3 DOOIXLGV DUH DW *f&* ' HWHUPLQH WKH SUHVVXUHGLIIHUHQFH 3D EHWZH SRLQWV\$ DQG%

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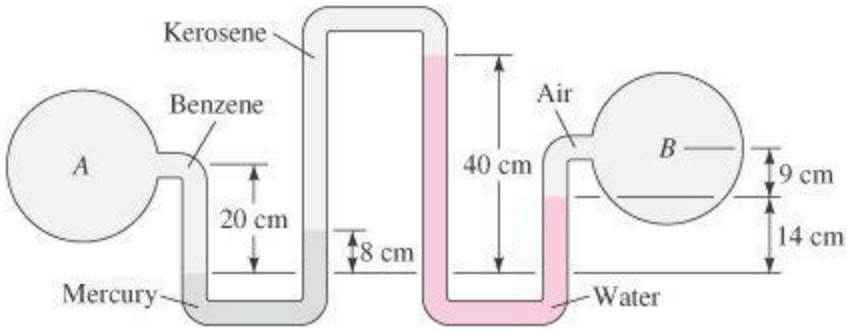

- 3) RUWKHLQYHUWHGPDQRPHWHU) LJ 3 DOOLQGVDUHDW *f*& ,I S% í S$ NP ZKDWPXVWWKHKHLJKW- EHLQFP"

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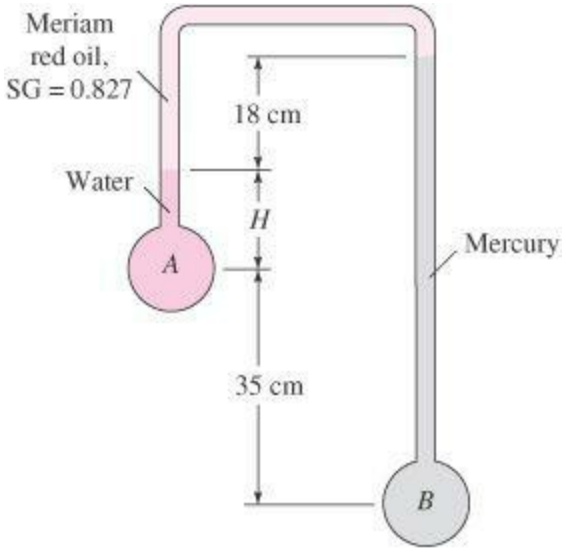

- 3 ,Q) LJ 3 WKHSUHVVXUHDWSRLQW$ LV GE LQ $OOLQGVDUHDW *f& : KDWLVWK DLUSUHVVXUHLQWKHFORVHGFKDPEHU% LQ3D'

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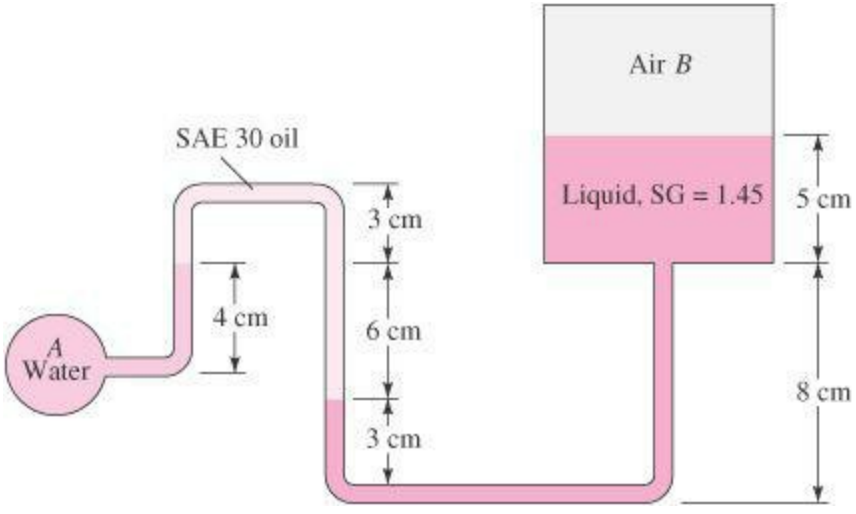

- 3 6RPHLPPHUVPDQRPHWUGLPHQVLRQVKDYHDVJQLFDQWHIHFW,Q) LJ 3 FRQVLGHU D DQG E DUHFWDQJXODUDQGFRQGLWLRQVDUHVXFKWKDWS D S E ' HULYHDIRUPXO

IRUWKHSUHVVXUHGLIIHUHQFHS$ íS% ZKHQWKHRLOLZDWHULQWHUIDFHRQWKHULJKWOLVHV
GLVDSSHDUV K IRU D G ' DQG E G ' : KDWLVWKHSUHVVXUHFKDQJHL
WKHYDOXHRI ûS'

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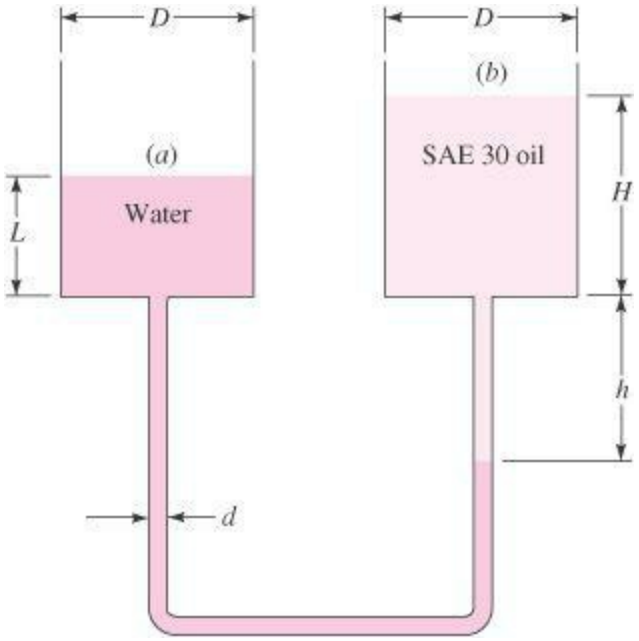

- 3 : DWHU IORZV XSZDUG LQ D SLSH VDQGWG DW f DV LQ) LJ 3 7KH PHUFXU
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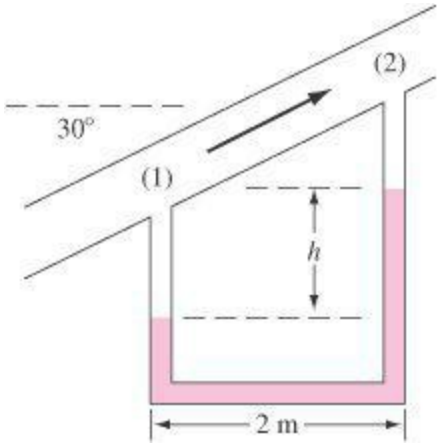

- 3 ,Q) LJ 3 ERWKWKHVDQNDQGWKHVXEHDUHRSHQWRWKHDWPRVSKHUH ,I / P
ZKDWLVWKHDQJOHRI WLOW RI WKHVXEH'
- 3 7KH LQFOLQHG PDQRPHWHU LQ) LJ 3 FRQWDLQV 0 HUFXU UHG PDQRPHWURLO 6*
$ \WXPH WKDWWKH UHVHUYRLU LV YHU ODUJH ,I WKH LQFOLQHG DUP LV ILWWHG ZLW
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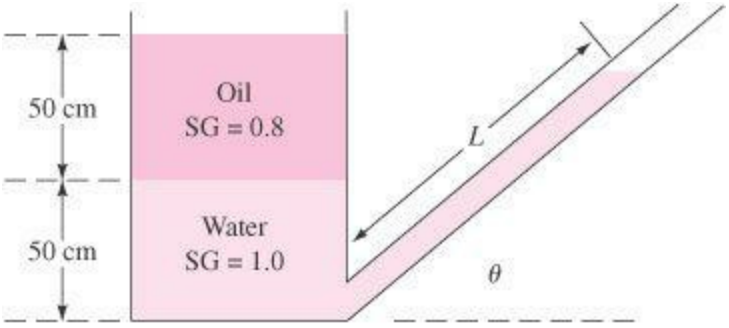

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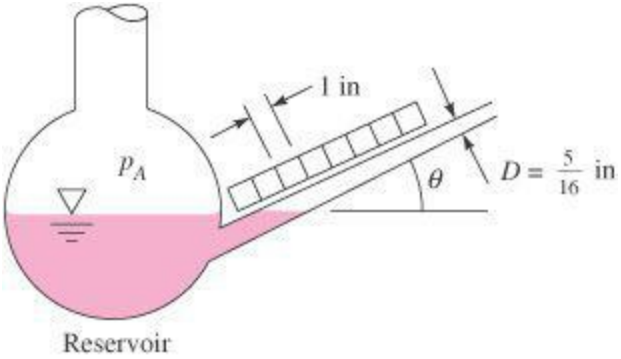

- 3 ,I WKH SUHVVXUH LQ FRQWDLQHU $ LQ) LJ 3 LV N3D FRP SXWH WKH SUHVVXUH L FRQWDLQHU %

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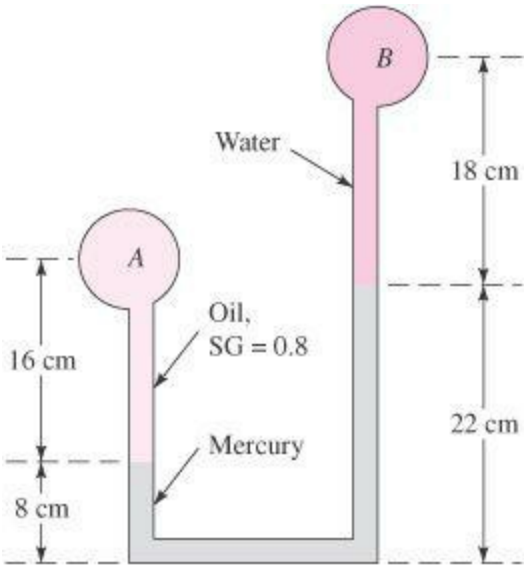

- 3 ,Q) LJ 3 WKH ULJKWOHJ RI WKH PDQRPHWULV RSHQ WR WKH DWPRVSKHUH) LQG WKH JDJ SUHVVXUH LQ3D LQWKH DLUJDS LQWKH WDQN
- 3 ,Q) LJ 3 WKH SUHVVXUHV DW$ DQG%DUH WKH VDPH N3D ,I Z DWULV LQWRGXFHG D $ WR LQFUHDVHS$ WR N3D ILQG DQG VNHWK WKH QHZ SRVLWRQV RI WKH PHUFXU PHQLVFL 7KHFRQ¿JXUDWLRQ WKEHLV DXQLRUP FP GLDPHWU $ VXPHQR FKDQJHLQWK GTXLGGHQVLWLHV

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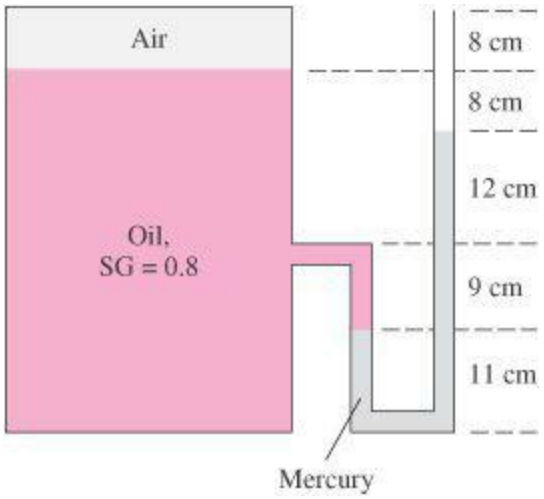

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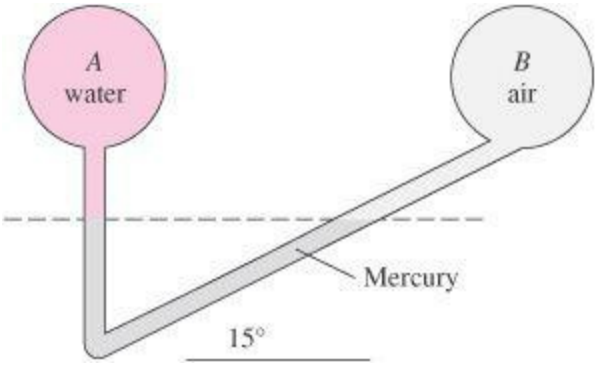

- 3 7KH VWXWP LQ) LJ 3 LV DW *f*& &RPSXWH WKH SUHVXUH DWSRLQW LQ GE IV DEVROXWH

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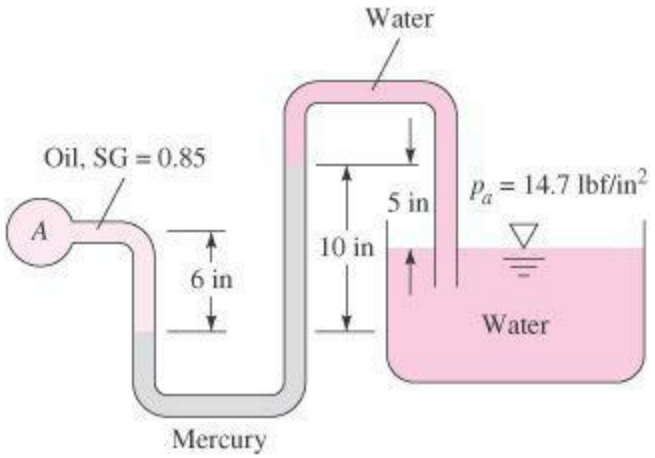

- 3 9HUI\ VPDOOSUHVXUHGLIIHUHQFHVS$ í S% FDQEHPHDVXUHGDFFXUDWHO\ E\ WKHVZR IOXL GLIIHUHQWDOPDQRPHWHULQ) LJ 3 ' HQVLW! LVRQO\ VOJKWO\ GLJHUWKDQWKDWR WKHXSSHUIOXLG! ' HULYHDQH[SUHVVLRQIRUWKHSURSRUWLRQDOLWZHHQK DQGS$ í S% LI WKHUHVHUYRLUVDUHYHU\ODUJH

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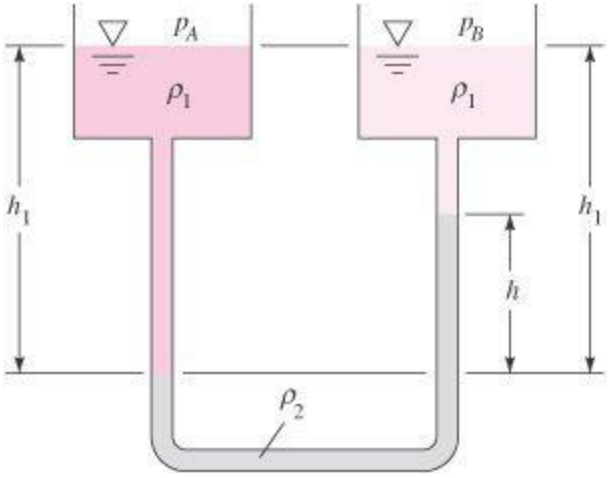

- 3 7KHWDGLWRQDOPHWRGRI PHDVXULQJ EORRGSUHVVXUHVDVK\JPRPDQRPHWU ILOV UHFRUGLQJWKHKLJKHVWVVVWROF DQGWKHORZHVWGLDVWROF SUHVVXUHIURP ZKLFK ICRZLQJ³. RURWRI´ VRXQGVFDQEHKHDUG 3DWHQWZLWKGDQJHURXVK\SHUWHQVLRQFD H[KLELWV\VWROF SUHVVXUHVDVKJKDV GEHLQ 1RUPDOOHYHOV KRZHYHUDUH DQ GEHLQ UHVSHFWLYHO IRUV\VWROF DQGGLDVWROF SUHVVXUHV 7KHPDQRPHWUXVH PHUFXU DQGDLUDVIOXLGV D +RZ KJK LQFP VKRXOGWKHPDQRPHWUXEHEH' E ([SUHVVQRUPDO\VWROF DQGGLDVWROF EORRGSUHVVXHLQPLOOLPHWVRI PHUFXU
- 3 : DWUICRZ VGRZ QZ DUGLDSLSHDW ƒ DVKRZQLQ)LJ 3 7KHSUHVXUHGUR S í S LVSDUW GXHWRJUDYLWDQGSDUW GXHWRIULFWRQ 7KHPHUFXU PDQRPHWU UHDGVD LQKHJKWGLIHUHQFH : KDWLV WKHVRXSSUHVXUHGURSS í S LQGEI LQ' : KDWLV WKHSUHVXUHGURS GXHWRIULFWRQRQO EHWZHHQ DQG LQGEI LQ" ' RHVWK PDQRPHWUUHDGLQJFRUUHVSRQGRQO WRIULFWRQGURS": K\"

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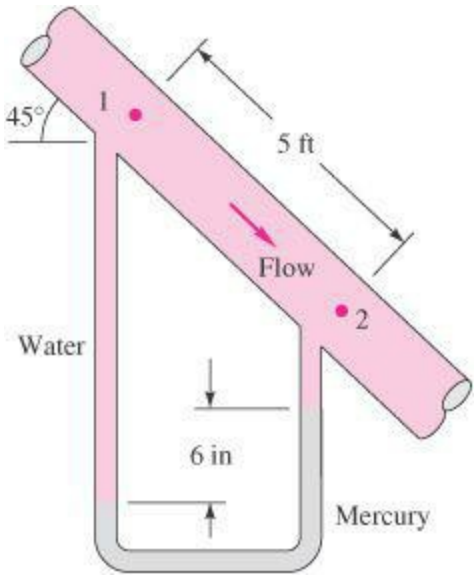

- 3 ,Q)LJ 3 GHWHUPLQHWKHJDJHSUHVVXUHDWSRLQW$ LQ3D ,VLWKJKURUOWHUWKD DWPRVSKHUF'

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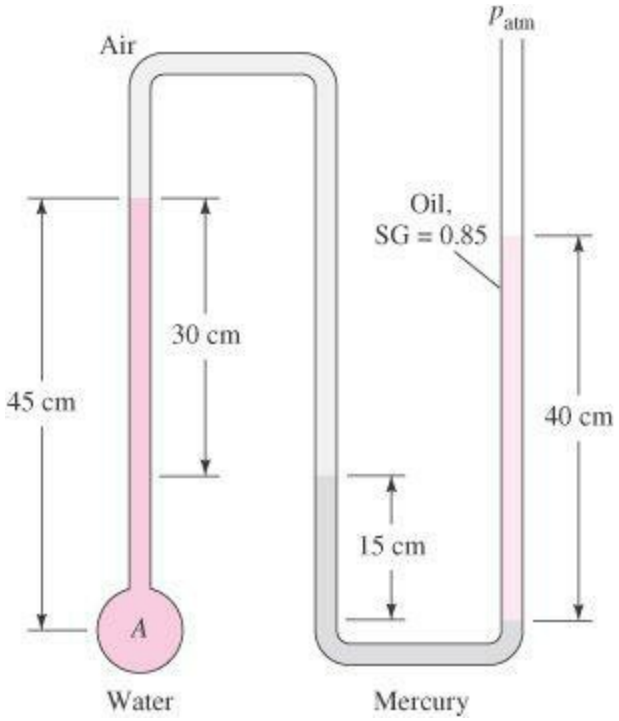

- 3 ,Q) LJ 3 ERWKHQGVRI WKHPDQRPHWHUDUHRSHQWRWKHDWPRVSKHUH (VWLPDWH WK VSHFLILF JUDYLWRI IOXLG;

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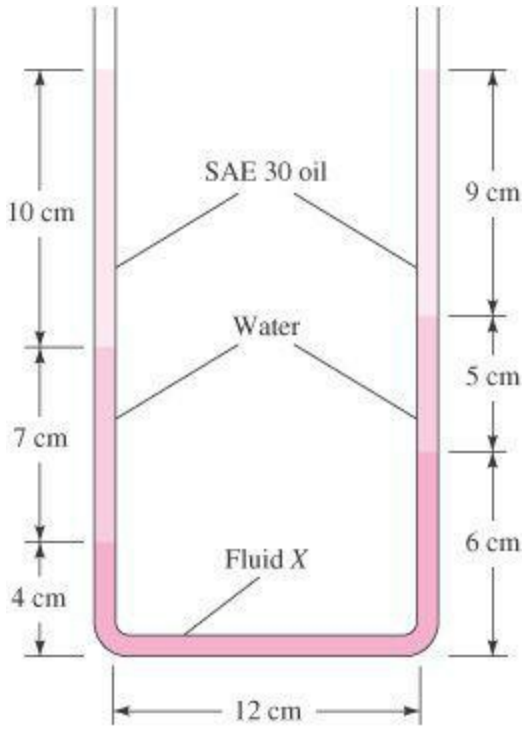

- 3 EES 7KHFRQGUFDOWQNLQ) LJ 3 LVEHLQJ ILOOHGZLWKZDWUDW *f*& E\ DSXP GHYHORSLQJ DQH[LVWSUHVXUHRI N3D $WWKHLQWDQWKRZQWKHDLUSUHVXUHLV N3DDQG+ FP 7KHSXPSVWRSSVZKHQLWFDQQRORQJHUUDLVHWKHZDWUSUHVXU) RULVRWKHUPDOLUFRPSUHVVLRQ HWWP DW+ DWKDWWP H

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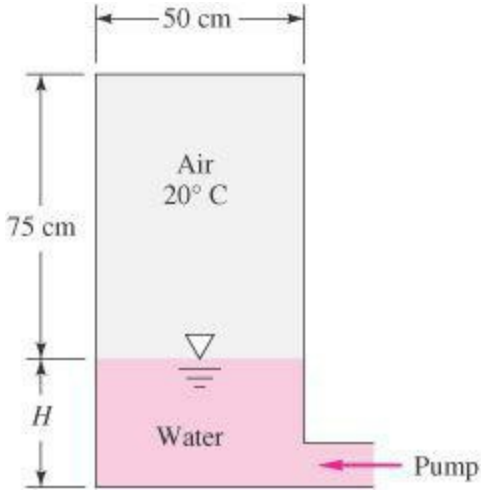

- 3 7KHV\VWHP LQ) LJ 3 LVRSHQWR DWP RQWKHULJKWVLGH D ,I / FP ZKDWV WKHDLUSUHVVXUHLQFRQGLQHU$" E &RQYHUWHG LI S$ NBD ZKDWVWKHOHQJWK /"

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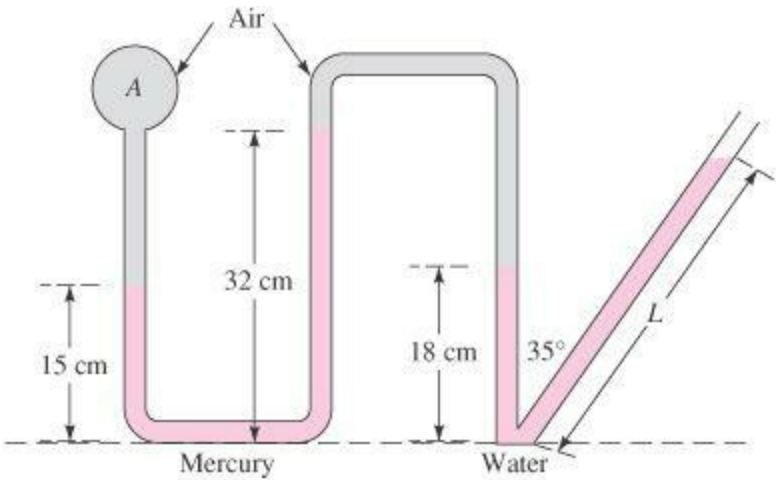

) RUFHVRQ SODQHVXUIDFHV

- 3 &RQGXFWWKHIROORZLQJ H[SHULPHQWRLOOXVWUDWHSUHVVXUH) LQGDWKLQZRRGHQUKE DSSUR[LPDWH IWLQHQJWK RUDWKLQZRRGHQSDQWXUHU 3ODFHLWRQWKHHGJHR DGHVNRUUDEOHZLWKDOLWWOHWWKDQKDOI RI LWKDQJLQJ RYHUWKHHGJHOHQJWKZLVH * H WZR IXOOVL]HVKHHWV RI QHZVSDSHU RSHQWKP XS DQG SODFHWKP RQWS RI WKHUXOHU FRYHULQJ RQO WKHSRUWRQ RI WKHUXOHUHWQJ RQWKHGHVNDVLGXWDMGLQ) LJ 3 D (WLP DWWKHWRSRI RFHRQWS RI WKHQHZSDSHUGXHWR DLUSUHVVXUHLQWKHURRP E &DUHXO 7R DYRLG SRWQWLDOLQXU PDNHVXUH QRERG\ LV WDQGLQJ GLUHFW LQ IURQW WKHGHVN 3HUFRP DNDUDWFKRS RQWKHSRUWRQ RI WKHUXHUWWFNLQJ RXWRYH WKHGJHR WKHGHVN 5HFRUG\RXUHVXOW F ([SODLQ\RXUHVXOW

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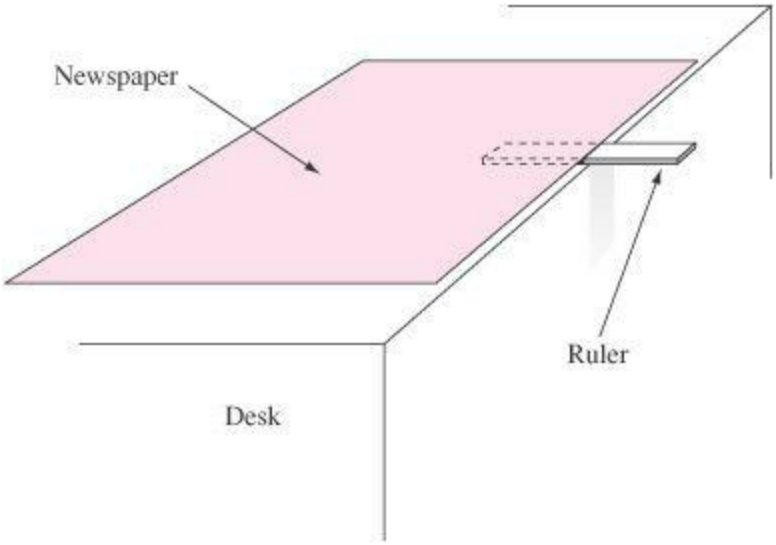

- 3 $ VP DOOVXEP DULQH Z LWK DKDWK GRRU LQLQGLDP HWU LV VXEP HUJHG LQVHDZ DWL
  D ,I WKH Z DWUK\ GUR VDWF IRUFH RQ WKH KDWK LV GE KRZ GHHS LV WKH VXE
  E ,I WKH VXE LV IWGHHS Z KDWLV WKH K\ GUR VDWF IRUFH RQ WKH KDWK'
- 3 * DW$%LQ) LJ 3 LV P ORQJ DQG P LQ WKH SDSHU 1 HJOHFWLQJ DWPRVSKHUL
  SUHVVXUH FRP SXWH WKH IRUFH) RQ WKH JDWH DQG LWV FHQWUH RI SUHVVXUH SRVLWLRQ;

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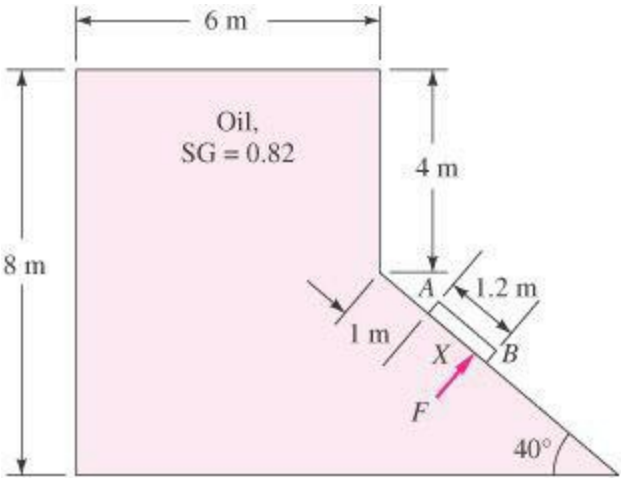

- 3 ([DPSOH FDOFXODWH WKH IRUFH RQ SODW$% DQG LWV OLQH RI DFWLRQ XVLQJ WK
  PRPHQWV LQHUWLD DSSURDFK 6RPH WHFKHVVV LWLV PRUH LQWXLWLVH WR FDOFXODW
  WKHVHE\ GLUHFWLQWUDWRQ WKHSUHVVXUHIRUFHV 8VLQJ) LJV 3 DQG(D D
  ILQG DQH[SUHVVLRQ IRUWKHSUHVVXUH YDULDWRQS M DORQJ WKHSODW E LQWUDWWKL
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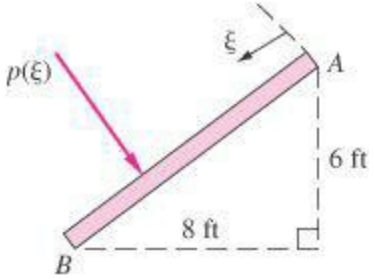

- 3 3DQHO$%& LQWKH VOXLFHG VLGH RI DZDWHUWLJKW WDQN LV DQLVRVFHOHV WULDQJOH ZLWK WKH YHUWH DV$ DQG WKH EDVH%& P DV LQ) LJ 3) LQG WKH ZDWHU IRUFH RQ WKH SDQHO DQ LWV OLQH RI DFWLRQ

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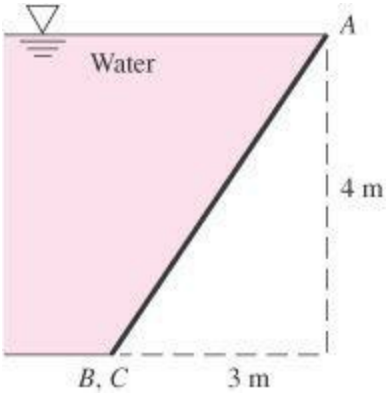

- 3 ,Q) LJ 3 WKH K\GURVWDWLF IRUFH) LV WKH VDPH RQ WKH ERWWRP RI DOO WKUH FRQWDLQHUV HYHQ WKRXJK WKH ZHLJKWV RI OTXLG DERYH DUH TXLWH GLIIHUHQW7KH WKUH ERWWRP VKDSHV DQG WKH IOXLG VDUH WKH VDPH 7KLV LV FDOOHG WKH K\GURVWDWLF SDUDGR[([SODLQ ZK\ LWLV WUXH DQG VNHWFK DIUHHERG\ RI HDFK RI WKH OTXLG FRQWDLQHUV

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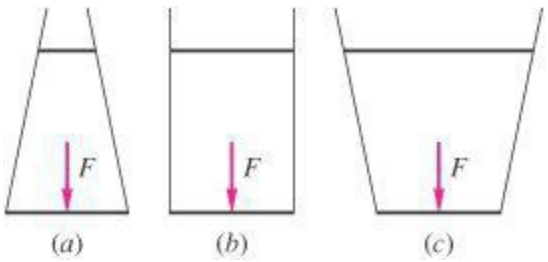

- 3 * DWH$%LQ) LJ 3 LV IWZ LGH LQR WKH SDSHU KLQJHG DW$ DQG UHVWLQHG E\ I VWRS DW% 7KH ZDWHU LV DW *& &RPSXWH D WKH IRUFH RQ VWRS % DQG E WK UHDFWLRQV DW$ L WKH ZDWHU GHSWK IW
- 3 ,Q) LJ 3 JDWH$%LV IWZ LGH LQR WKH SDSHU DQG VWRS %ZLOO EUHDN L WKH ZDW IRUFH RQLWHTXDOV OEI) RUZ KDWZ DWUGHSWK KLV WKLV FRQGLWLRQ UHFKHG'
- 3 7KH WDQN) LJ 3 LV P ZLGH LQR WKH SDSHU 1HJOHFWLQJ DWPRVSKHULF SUHVVXU ILQG WKH UHVXOWDQW K\GURVWDWLF IRUFH RQ SDQHO%& D IURP DVLQJOH IRUP XOD DQG E E\ FRPSXWLQJ K\GUDXOLF RQDODQG YHUWLFDO IRUFHV VHSDUDWHO LQWKH VSLULWRI 6HFWLRQ

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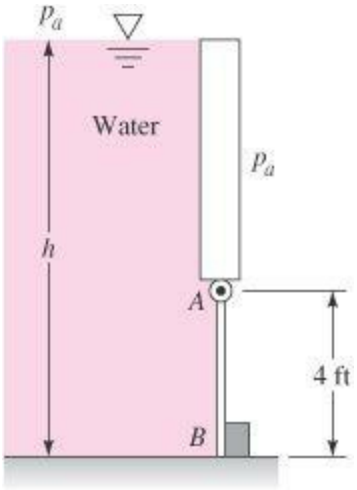

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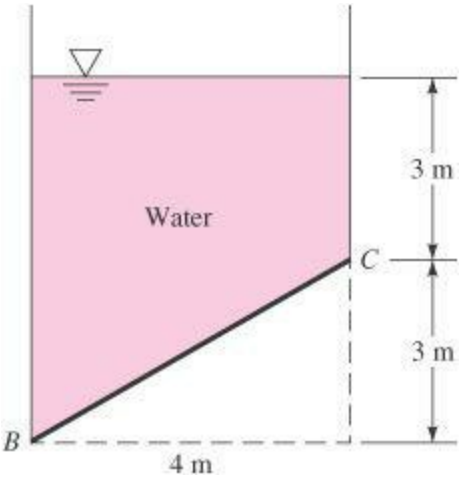

- 3 ,Q) LJ 3 VKH FRYHU JDWH $% FRQVLVWV RI D FLUFXODU RSHQLQJ FP LQ GLDPHWHU 7KH JDWH LV KLQJHG FRQVHG E\ D NJ PDVV DV VKRZQ $VVXPH WKDW WKH JDWH LV VPRRWK DQG GUDZ LWV DW *& $VZKDWZDWUDYHOVKZLOOWKHJDWHEHGLVRGJHG' 1HJOHFWWKHZHLJKWRI WKHJDWH

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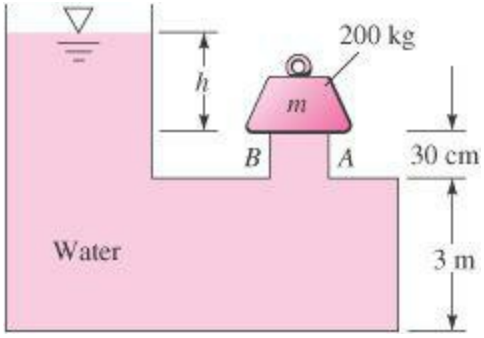

- 3 * DW$%KDVOHQJWK/ DQG ZLGWK E LQRR WKH SDSHU LVKLQJHG DW% DQG KDVQHJOLJLEO ZHJKW7KH OLTXLG OHYHOK UHPDLQV DWWKH VRS RI WKH JDWH IRUDQ DQJOH) LQG D DQDOWLF H[SUHVVLRQ IRUWKHIRUFH3 SHUSHQGLFXODUWR$% UHTXLUHGWRNHHSWKHJDWHL HTXLOLEULXP LQ) LJ 3

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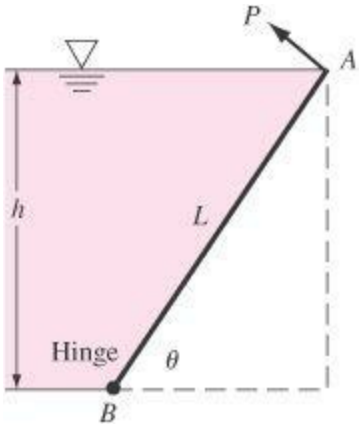

- 3 ' HWHUPLQHWKHZDWHUKHLJKWDWZKLFKWKHJDWHZLOOWXUQDERXWWKHKLQJH SDQHO%&' LQ) LJ 3 1HJOHFWWKHZHLJKWRIWKHSODWH

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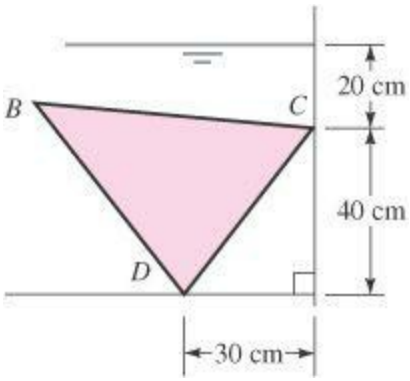

- 3 * DWH$%LQ) LJ 3 LVDKRPRJHQHRXVPDWHULDO NJ P ZLGHLQWKHSDSHU KLQJHGDW$ DQGUHVWLQJ RQDVPRRWKERWWRP DW% $OOGLPHQVLRQVDUHLQ f&) RUZKHQ ZDWHUGHSWKZLOOWKHJDWHRSHQ

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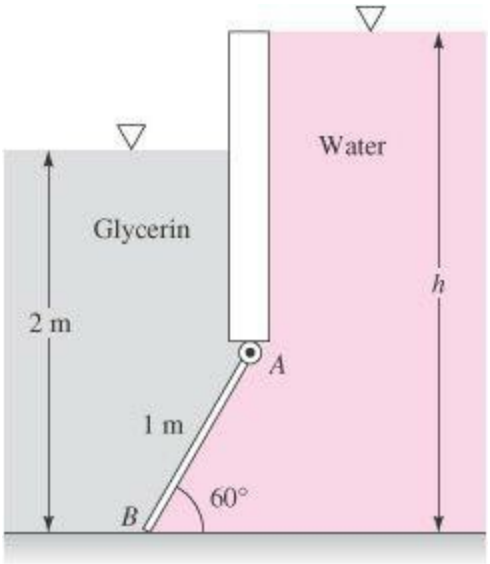

- 3 EES * DW$%LQ) LJ 3 LV IWORQJ DQG IWZLGHLQWKHSDSHUDQGLVKLQJHGDW% ZLWK D VPRS DW$ 7KHZDWHULV DW f& 7KHJDWHLV LQWKLFN VWHHO 6*

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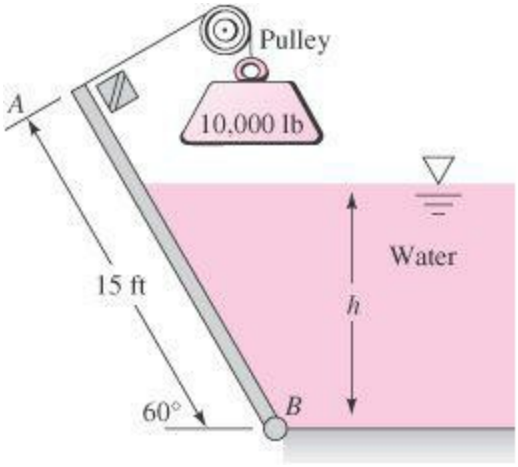

- 3 7KHWDQNLQ)LJ 3 KDVD FP GLDPHWUSOXJ DWWKHERWWRP RQWKHULJKW\$ODIOXLG DUHDW f& 7KHSOXJ ZLOOSRS RXWL WKHK\GURVWDWF IRUFHRQLWV 1)RUWK FRQGLWLRQ ZKDWZLOOEHWKHUHDGLQJ KRQWKHPHUFXU PDQRPHWURQWKHOHIWOLG'

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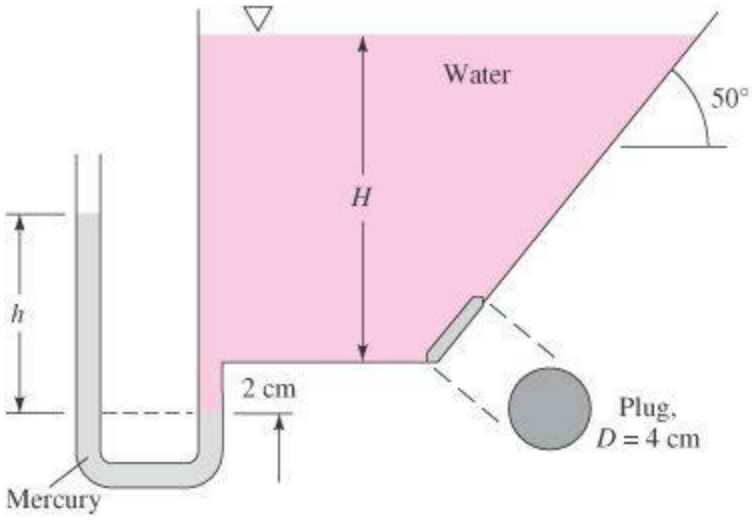

- 3 * DWP\$%& LQ)LJ 3 KDVDIL[HGKLQJHOQHDW%DQGLV P ZLGHLQR WKHSDSH 7KHJDWHZLOORSHQDW\$ WRUHOHDVHZDWHULWKHZDWHUGHSWKLVKJKHQRXJK &RPSXW WKHGHSWKIRUZKLFKWKHJDWHZLOOEHJLQWRRSHQ

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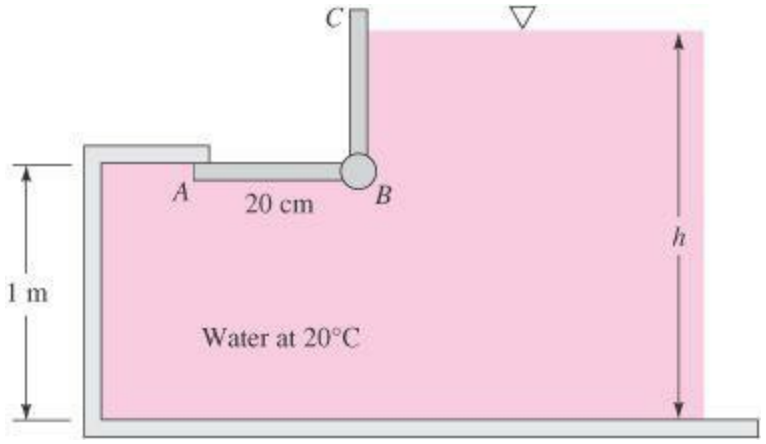

- 3 * DWP$%LQ) LJ 3 LVVHPLFLUFXODUKLQJHGDW% DQGKHOGE\ DKRUL]RQWDOIRUFH3 DW$: KDWRUFH3 LVUHTXLUHGIRUHTXLOLEULXP"

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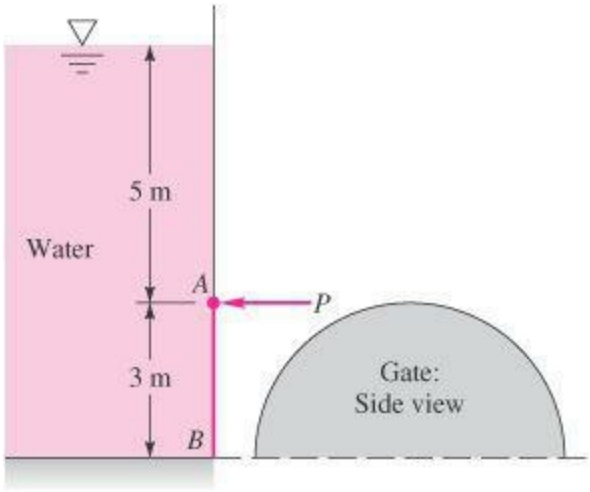

- 3 ' DP $%& LQ) LJ 3 LV P ZLGHLQWKHSDSHUDQGPDGHRI FRQFUHWH 6*) LQGWKHK\GURVWDWLFIRUFHRQVXUIDFH$%DQGLWVPRPHQWDERXW& $VVXPLQJ QI VHHSDJHRI ZDWHUXQGHUWKHGDP FRXQGWKLVIRUFHWSWKHGDP RYHU' +RZ GRHV\RX DJXPHQWFKDQJHLI WKHUHLVVHHSDJHXQGHUWKHGDP"

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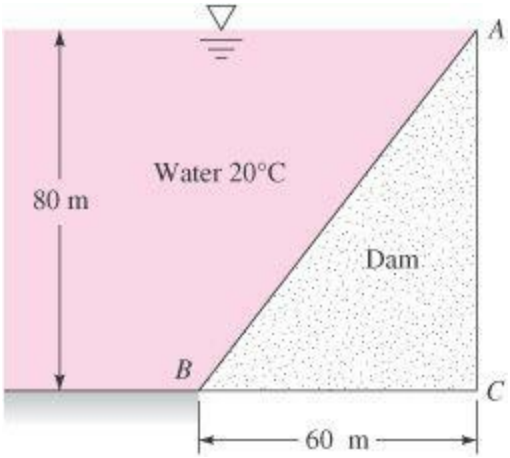

- 3 * HQHUDWH3URE 3 DVIRORZV ' HQRWHGHQWK$% DV+ GHQWK%& DV/ DQ

DQJOH$%& DV / HWWKH GDP PDWHULDOKDYH VSHFLILF JUDYLW 6* 7KH ZLGWK RI WK
GDP LVE $ VXPH QR VHHSDJH RI ZDWHU XQGHUWKH GDP) LQG DQ DQDOWF UHDWR
EHWZHHQ 6* DQG WKH FULWFDODQJOH F IRU ZKLFK WKH GDP ZLOOWXWWS RYHUWR WK
ULJKW8VH\RXUHDWRQWRFRPSXWH F IRUWKHVSHFLDOFDVH6* FRQFUHWH

- 3 ,VRVFHOHV WULDQJOH JDWH$% LQ) LJ 3 LV KLQJHG DW$ DQG ZHLJKV 1 : KD
KRUL]RQWDO IRUFH3 LVUHTXLUHG DWSRLQW%IRUHTXLOLEUXP"

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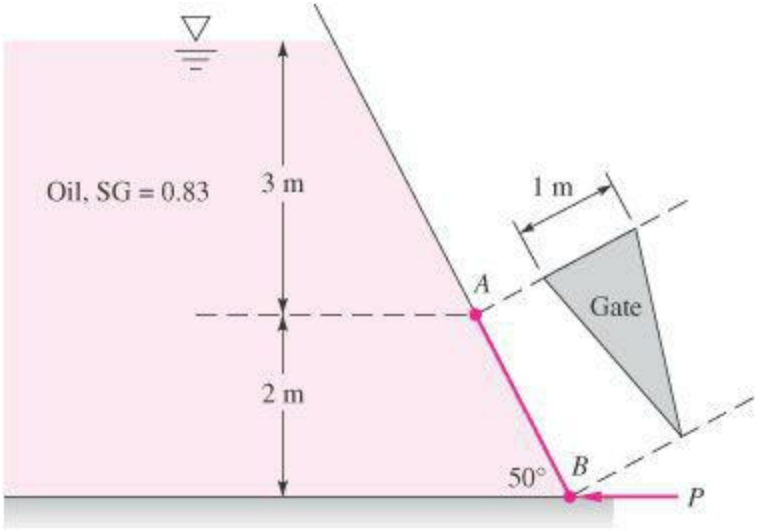

- 3 &RQVLGHUWKHVDQAGSODWH$%RI OHQJWK/ LQ) LJ 3 D ,VWKHK\GURVWDWF IRUFH)
RQWKHSODWHTXDOWR WKHZHLJKWRI WKHPLVWLQJ ZDWUDERYHWKHSODWH' ,I QRWFRUUHF
WKLV K\SRWKHVLV 1HJOHFWWKH DWPRVSKHUH E &DQ D³PLWLQJ ZDWÚ WKHRU E
JHQHUDQ]HGWR FXUYHG VXUFHVRI WKLVWSH'

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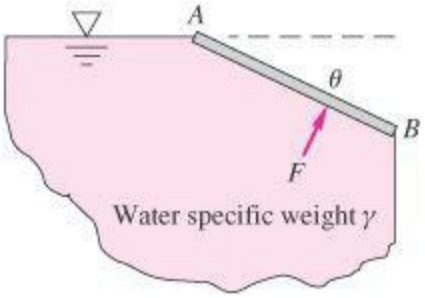

- 3 7KH VZLQJ FKHFN YDOYH LQ) LJ 3 FRYHUV D FP GLDPHWUSHQLQJ LQ WK
VDQAGZDOO7KHKLQJHLV FP IURP WKHFHQWUDQH DVVKRZQ 7KHYDOYHZLOORSH
ZKHQWKHKLQJHPRPHQWV 1 P) LQGWKHYDOXHRI K IRUWKHZDWUWR FDXVHWKL
FRQGLWRQ

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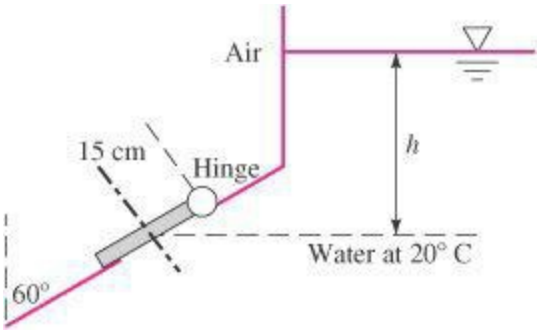

- 3 ,Q) LJ 3 JDWH$%LV P ZLGH LQRWKHSDSHUDQG LV FRQQHFWHG E\ DURG DQ SXOOH\ WR D FRQFUHWH VSKHUH 6* : KDWGLDPHWHU RI WKH VSKHUH LV MXVWLILHG WR NHHS WKH JDWH FORVHG'

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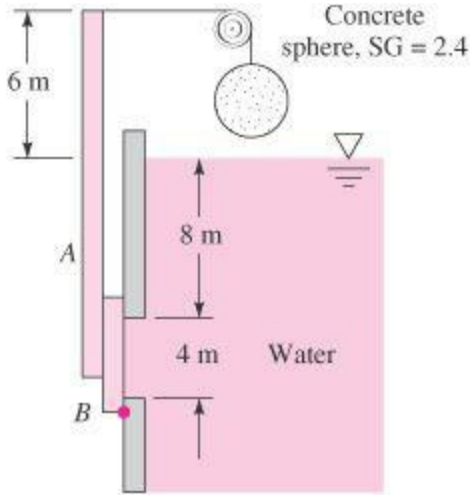

- 3 * DWH%LQ) LJ 3 LV FP KLJK FP ZLGH LQRWKHSDSHUDQG KLQJHG DWWKHVRS : KDWZDWHUGHSWKKZLOOILUVWFDXVHWKHJDWHWRRSHQ'

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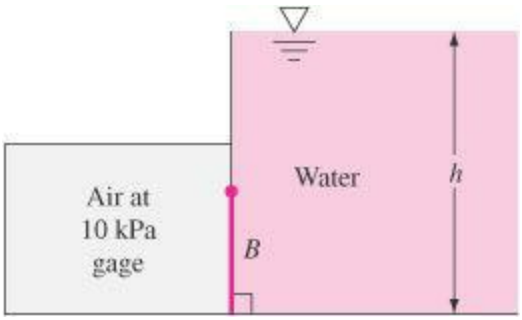

- 3 * DWH$%LV IWZ LGH LQRWKHSDSHUDQG RSHQV WR OHWUHVKZDWHU RQWKHRFHDQ VLGH LV GURSSLQJ 7KHKLQJHDV$ LV IWDERYHWKHIUHVKZDWHUOHYHO $WZKDWRFHDQ OHYHOKZLOOWKHJDWHILUVWRSHQ' 1HJOHFWWKHJDWHZHLJKW

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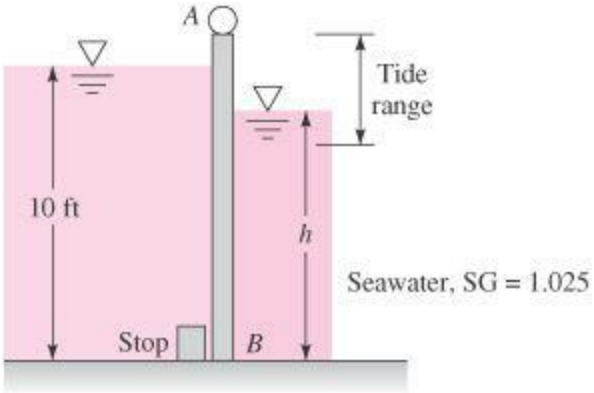

- 3) LQG WKH KHLJKW LQ) LJ 3 IRU Z KLFK WKH K\GURVWDWLF IRUFH RQ WKH UHFWDQJXODU SDQHOLV WKH VDPH DV WKH IRUFH RQ WKH VHPLFLUFXODU SDQHOEHORZ
- 3 7KH FDS DWSRLQW% RQ WKH FP GLDPHWHU WXEHLQ) LJ 3 Z LOOEH GLVORGJHG Z KH WKH K\GURVWDWLF IRUFH RQ LWV EDVH UHDFKHV GBI) RUZ KDWZ DWUGHSWK K GRHV WKLV RFFXU

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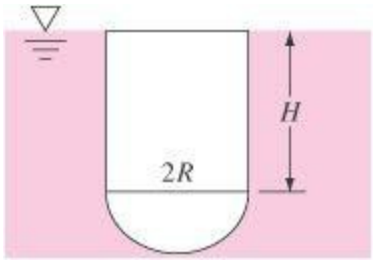

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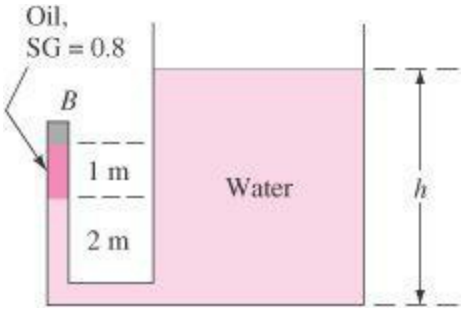

- 3 3DQHO%& LQ) LJ 3 LVFLUFXODU &RPSXWH D WKH K\GURVWDWLF IRUFH RI WKH Z DWUR WKHSDQHO E LWVFHQWURLG SUHVVXUH DQG F WKHP RPHQWRI WKLVIRUFHDERXWSRLQW%

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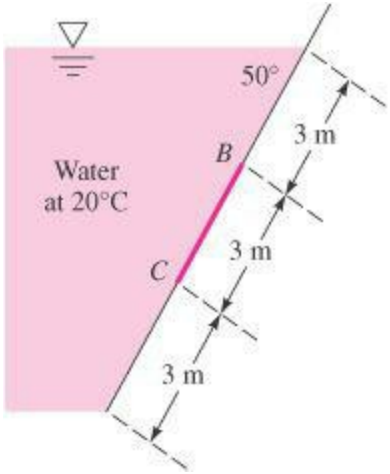

- 3 7KHFLUFXODUJDWH$%& LQ) LJ 3 KDVD PUDGLXVDQGLVKLQJHGDW% &RPSXWHWK IRUFH3 NXWWXIILFLHQWWR NHHS WKH JDWH IURP RSHQLQJ ZKHQK P 1HJOHF DWRVSKHULF SUHVVXUH

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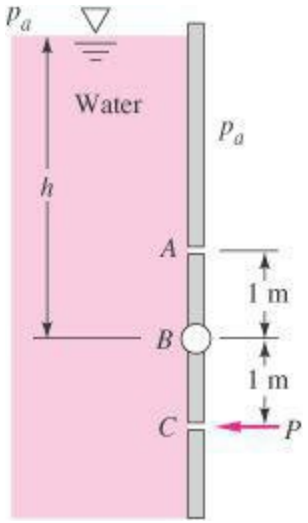

- 3 3DQHOV$%DQG&' LQ) LJ 3 DUHHDFK FP ZLGHLQRWKHSDSHU D &DQ\R GHGXFH E\ LQVSHFWLRQ ZKLFK SDQHOKDV WKH ODUJHUZDWUFRFH' E (YHQLI \RX GHGXFWLRQLVEULOOLDQWFDOFXODWWKHSDQHOIRUFHVDQZD\

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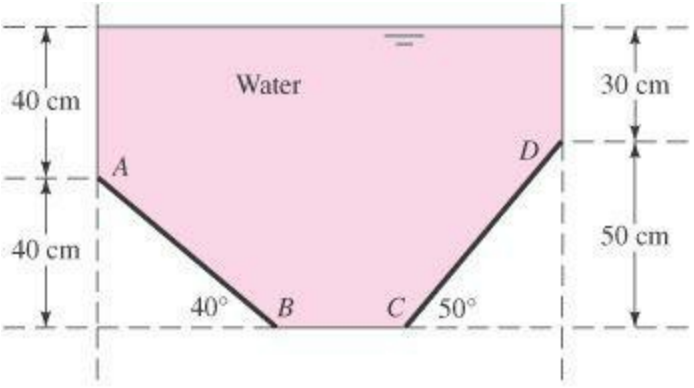

- 3 * DW$%& LQ) LJ 3 LV P VTXDUHDQGLVKLQJHGDW% ,WZLOORSHQDXWRPDWFDO

Z KHQWKHZ DWUDQVPLWWHGZ DYHOHQJWKRIWKHZDYHLVWKHGLVWDQFHEHWZHHQWZRFRQVHFXWLYHSRLQWVRQWKHZDYHWKDWDUHLQWKHVDPHSKDVH ,VWKLVUHVXOWLQGHSHQGHQWRIWKHDPSOLWXGH"

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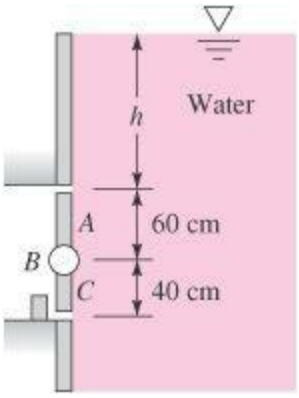

- 3 $ FRQFUHWHGDP 6* LVPDGHLQWKHVKDSHRI DQLVRVFHOHVWUDQJXODU) LJ 3 $QDO]HWKLVJHRPHWU\WRILQGWKHUDQJHRI DQJOHV IRUZKLFKWKHK\GURVWDWLF IRUFHZLOOWHQGWRWLS WKHGDP RYHUDVRLQW%7KHZLGWKLQWKHSDSHULVE

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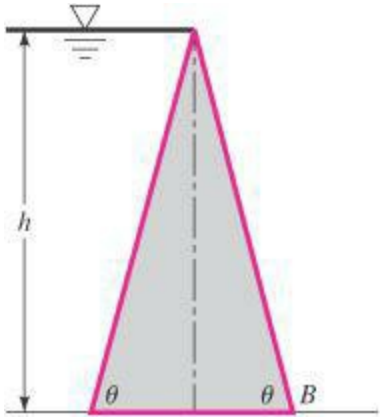

) RUFHVRQ FXUYHG VXUIDFHV

- 3) RUWKHVHPLFLUFXODUF\OLQGHU&' (LQ([DPSOH ILQGWKHYHUWLFDOK\GURVWDWF IRUFH E\ LQWHJUDWLQJ WKHYHUWLFDOFRPSRQHQWRI SUHVVXUHDURXQGWKHVXUIDFHIURP WR CE
- 3 7KHGDP LQ) LJ 3 LVDTXDUWHUFLUFOH P ZLGHLQWKHSDSHU' HWHUPLQHWKH KRUL]RQWDODQG YHUWLFDOFRPSRQHQWVRI WKHK\GURVWDWF IRUFHDJDLQVWWKHGDP DQG WKH SRLQW&3 ZKHUHWKHUHVXOWDQWWUN-V\WKHGDP

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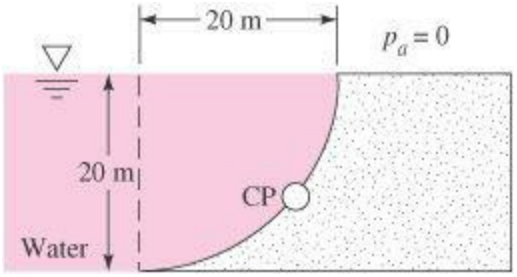

- 3 * DWH$%LQ) LJ 3 LV D TXDUWHU FLUFOH IWZ LGH HLQR VKH SDSHUDQG KLQJHG DW(
) LQG VKH IRUFH) IXVWXIILFLHQWR NHHS VKH JDWH IURP RSHQLQJ 7KH JDWH LV XQLIRU
DQG Z HLJKV CE

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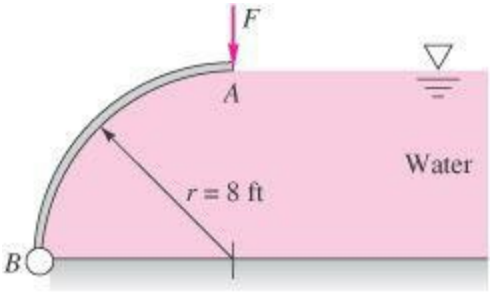

- 3 3DQHO$%LQ) LJ 3 LV D SDUDERODZ LWK LWV PD[LPXP DWSRLQW$,WV FP Z LG
LQR VKH SDSHU 1HJOHFWDWRVSKHUF SUHVXUH) LQG D VKH YHUWLFDO DQG E VK
KRUL]RQWDOZ DWUIRUFHVRQVKHSDQHO

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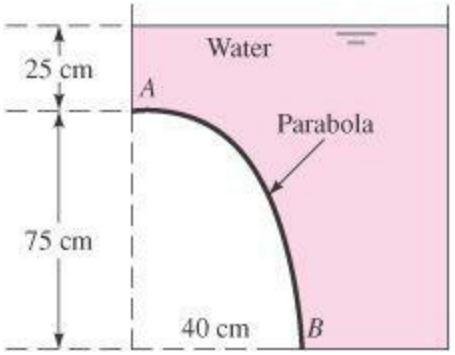

- 3 &RPSXWH VKH KRUL]RQWDO DQG YHUWLFDO FRPSRQHQWV RI VKH K\GURVWDWF IRUFH RQ VK
TXDUWHUFLUFOHSDQHODVKHERWWRP RI VKHZ DWUWDQNLQ) LJ 3

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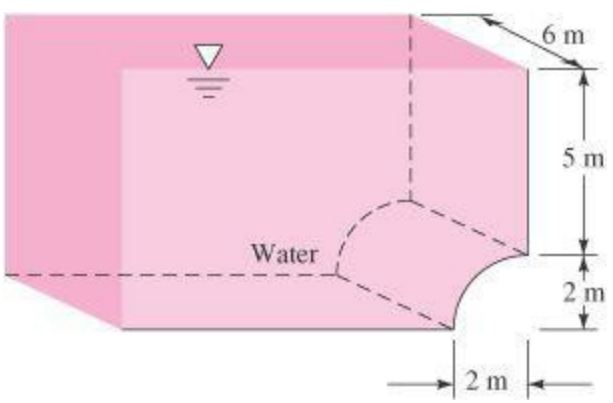

- 3 7KH TXDUWHU FLUFOH JDWH%& LQ) LJ 3 LQKLQJHG DW&) LQG WKH KRUL]RQWDO IRUFH3 UHTXLUHG WR KROG WKH JDWH VWDWLRQDU\ 1HJOHFW WKH ZHLJKW RI WKH JDWH

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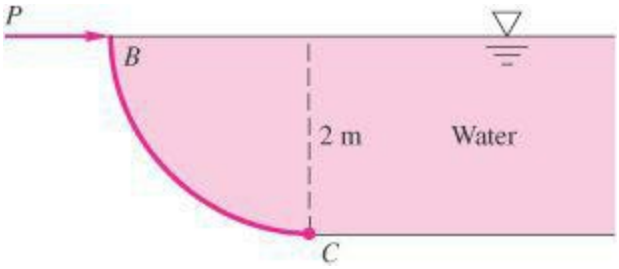

- 3 7KH EORFN RI FKDPSDJQH 6* LQ) LJ 3 LV XQGHU SUHVVXUH DV VKRZ QE\ WKH PHUFXU PDQRPHWHU UHDGLQJ &RPSXWH WKH QHWIRUFH RQ WKH LQUDGLX KHPLVSKHULFDO HQGFDS DW WKH EORWWRP RI WKH EORWWH

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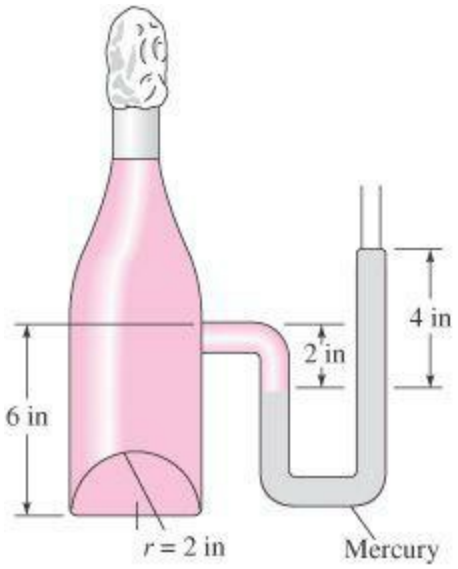

- 3 * DW$%& LV D FLUFXODU DUF VRP HLWP HV FDQHG D7DLQWJDW ZKLFK FDQ EH UDLVH DQG ORZHUHG E\ SLYRWLQJ DERXW SRLQW2 6HH) LJ 3) RU WKH SRVLWLRQ VKRZ G GHWHUPLQH D WKH K\GURVWDWF IRUFH RI WKH ZDWHU RQ WKH JDWH DQG E LWV OLQH RI DFWLR ' RHV WKH IRUFH SDVV WKURXJK SRLQW2"

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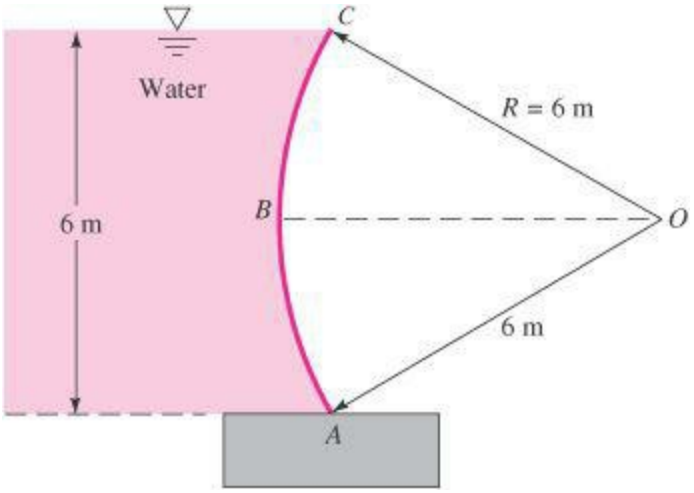

- 3 7KH WDQN LQ) LJ 3 FRQWDLQV EHQ]HQH DQG LV SUHVVXUL]HG WR N3D JDJH LQWR DLUJDS ' HWHUPLQH WKH YHUWLFDO KGURVWDWLF IRUFH RQ FLUFXODU DUF VHFWLRQ$%DQG LV OQHR DFWLRQ

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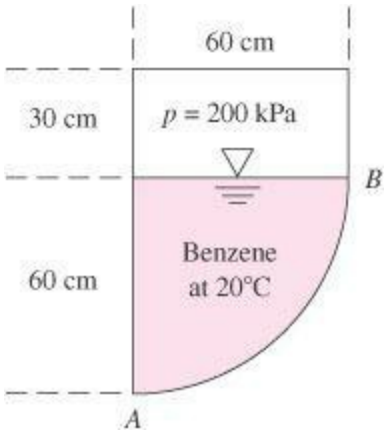

- 3 7KH WDQN LQ) LJ 3 LV FP ORQJ LQR WKH SDSHU ' HWHUPLQH WKH KRUL]RQWDO YHUWLFDO KGURVWDWLF IRUFHV RQ WKH TXDUWHUFLUFOH SDQHO$% 7KH IOXLG LV Z DWHU D *& 1HJOHFWDQVRVSKHULF SUHVVXUH

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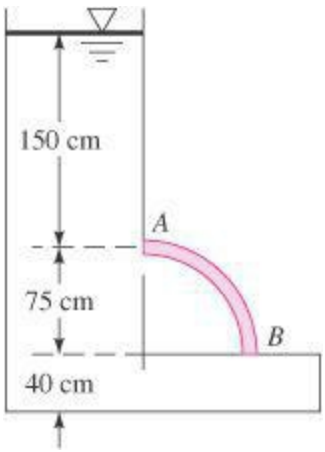

- 3 7KH KHPLVSKHULFDO GRPH LQ) LJ 3 Z HJKV N1 DQG LV ILOOHG Z LWK Z DWHU DQ

DWWDFKHG WR WKH IORRU E\ VL[HTXDO VSDFHG EROWV : KDWLV WKH IRUFH LQ HDFK ER
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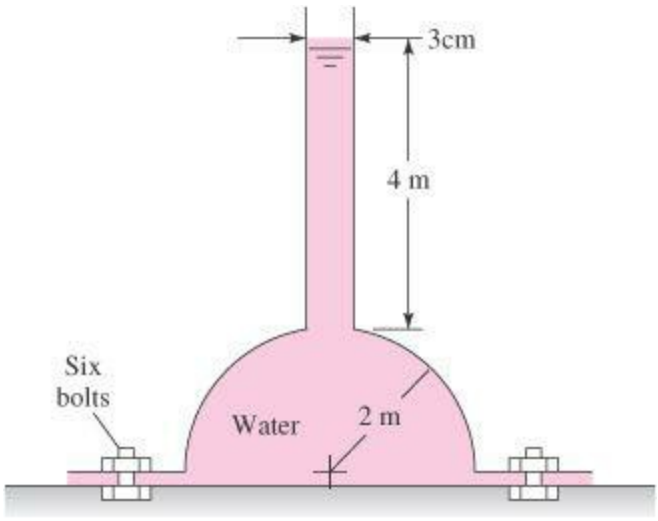

- 3 $ P GLDPHWUZ DWUVDQ FRQLWVWR RI VZR KDOI F\OLQGHUV HDFK ZHJKLQJ NL P
ERONG WRJHWKHUDV VKRZ QLQ) LJ 3 ,I WKH VXSSRUWR WKHHQG FDSVLVQHJOHFWHG
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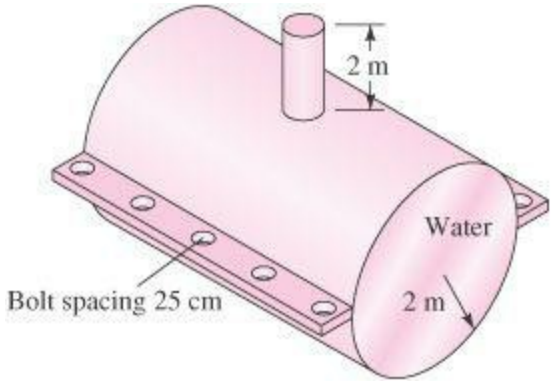

- 3 ,Q) LJ 3 DRQH TXDGUDQWSKHULFDOVKHOOR UDGLXV5 LV VXEP HJHGLQOLTXLG R
VSHFLLF ZHJKW DQG GHSWKK ! 5) LQG DQ DQDOWF H[SUHVVLRQ IRU WKH UHVXOWDQ
K\GURVWDWF IRUFH DQGLWVOLQHRI DFWLRQ RQWKHVKHOVXUFH

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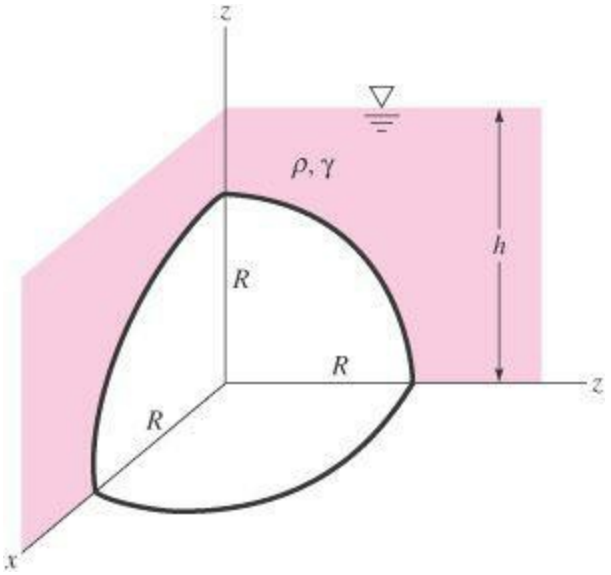

- 3) LQG DQ DQDO WF IRUPXOD IRU WKH YHUWFDO DQG KRUL]RQWDO IRUFHV RQ HDFK RI WK VHPLFLUFXODUSDQHOV$%LQ) LJ 3 7KHZLGWKLQRWKHSDSHULVE : KLFKIRUFHLV ODUJHU : K\"

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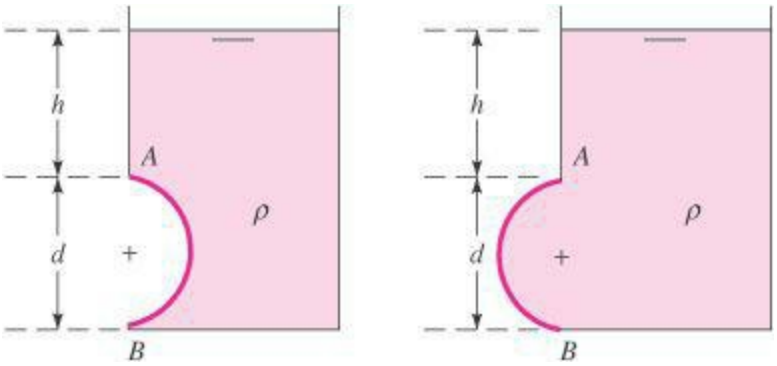

- 3 7KH XQLIRUP ERG\ $ LQ) LJ 3 KDV ZLGWKE LQRX WKH SDSHU DQG LV LQ WDOH HTXLOLEULXP ZKHQSLYRWHGDERXWKLQH2 : KDWLVWKHVSHFLILF JUDYLWWR WKLVERG\ L D K DQG E K 5"

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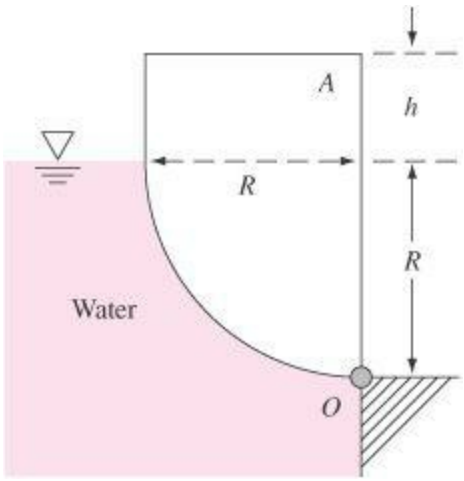

- 3 &XUYHG SDQHO %& LQ) LJ 3 LV D ƒ DUF SHUSHQGLFXODU WR WKH ERWWRP DW & ,I WK SDQHO LV P ZLGH LQ WKH SDSHU HWHUP DW WKH UHVXOWDQW KGURVWDWLF IRUFH RI WK ZDWHU RQ WKH SDQHO

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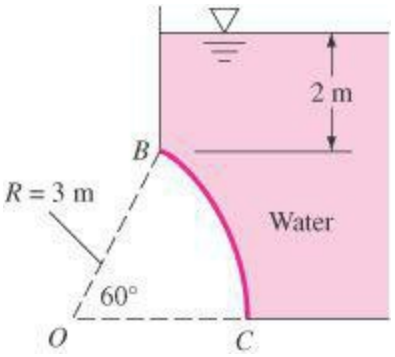

- 3 7KH FRQFDYH VXUIDFH RXWHU JXQLWH[VKUHDGG ILQLVKHG WKH GHHS FRUQHU RI D P ZLG VZLPPLQJ SRRO ZLWK D TXDUWHU FLUFOH SLHFH RI 39 & SLSH OEHOHG $%LQ) LJ 3 &RPSXWH WKH KRUL]RQWDO DQG YHUWLFDO ZDWHU IRUFHV RQ WKH FXUYHG SDQHO $%

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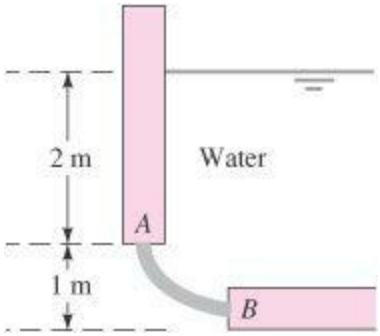
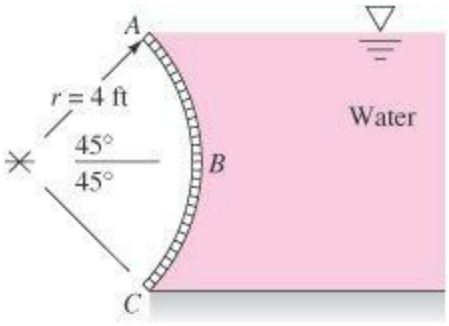


- 3 * DWH $%& LQ) LJ 3 LV D TXDUWHU FLUFOH IWZ LGH LQ WKH SDSHU &RPSXWH WKH KRUL]RQWDO DQG YHUWLFDO KGURVWDWLF IRUFHV RQ WKH JDWH DQG WKH OLQH RI DFWLRQ RI WK UHVXOWDQW RUFH

3



- 3 7KH PHJD PDJQHW FRQWDLQHU LQ) LJ 3 KDV D KHPLVSKHULFDO ERWWRP DQG LV SURWXUL]HG ZLWK DLUWR NBD JDJH DWWKH WRS ' HWHUPLQH D WKH KRUL]RQWDO DQG E WKH YHUWLFDO KGURVWDWLF IRUFHV RQ WKH KHPLVSKHUH LQGE

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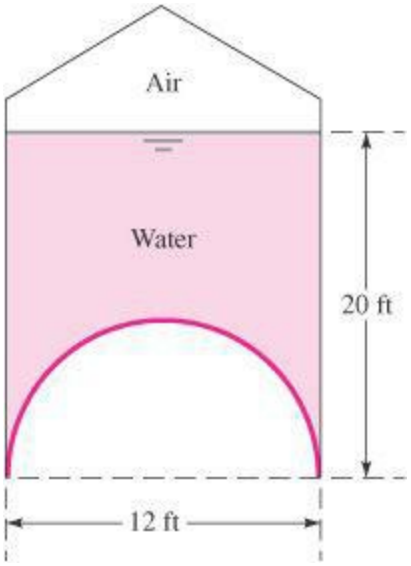
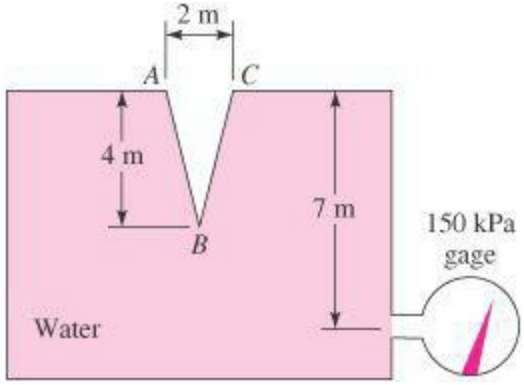


- 3 3UHVVXUL]HGZDWHULVWKHVDPHLQ) LJ 3 &RPSXWHWKHQHWKGURVWDWLFIRUFHRQ WKHFRQFDYHVXUIDFH$%&

3



) RUFHVRQ&XUYHG6XUIDFHV

- 3 7KH FORVHG FXUYHG ER[LQ) LJ 3 KDV VTXDUH KRUL]RQWDO FURVV VHFWLRQ HYHUZKHUH $ FOLQGULFDO VXUIDWH f& (WXPDWWKHJDJH SUHVXUHRI WKHDLUL D WK KGURVWDWLFIRUFHRQSDQHO$%LV N1 RU E WKHKGURVWDWLFIRUFHRQWKHERWWRP SDQHO%&LV N1

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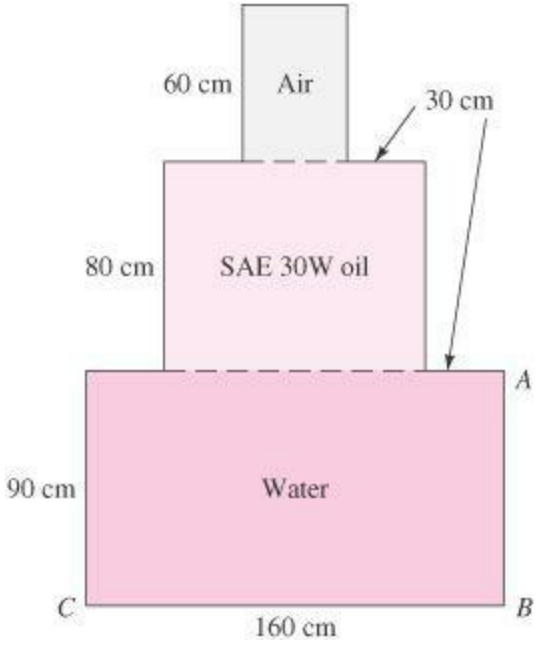

- 3 $ FXEHFDOWDQNLV î î P DQGLVOD\HUHGZLWK PHWURI IOXLGRI VSHFLILF JUDYLW PHWURI IOXLGZLWK6* DQG PHWURI IOXLGZLWK6* 1HJOHF DWPRVSKHULF SUHVVXUH) LQG D WKH K\GURVWDWLF IRUFHRQWKHERWWRP DQG E WKHIRUF RQDVLGHSDQHO

%XR\DQF\ $UFKLPHGHV¶SULQFLSOH

- 3 $ VROLGEORFN RI VSHFLILF JUDYLW IORDWVXSFKWKDW SHUFHQWRI LWVYROXPHLVL ZDWUDQG SHUFHQWRI LWVYROXPHLVLQIOXLG; ZKLFKLVOD\HUGDERYHWKHZDWL : KDWLVWKHVSHFLILF JUDYLWRI IOXLG; "
- 3 7KHFDQLQ) LJ 3 IORDWVLQWKHSRVLWRQVKRZQ: KDWLVLWZHLJKWQ1 "

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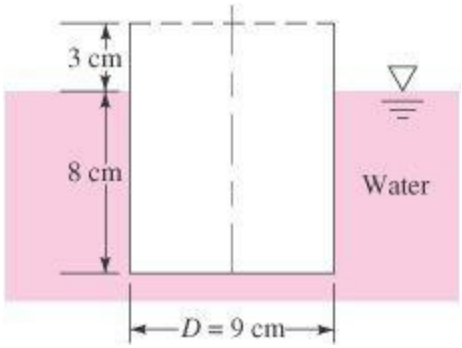

- 3 ,WLVVDLGWKDW$UFKLPHGHVGLVFRYHUHGWKHEXR\DQF\ ODZ VZKHQDVNHGE\ . LQJ + LH RI 6\UDFXVH WR GHWHUPLQH ZKHWKHU FURZQ ZDV SXUH JROG 6* $UFKLPHGHVPHDVXUHGWKHZHLJKWRI WKHFURZQLQDLUWREH 1 DQGLWZHLJKWV ZDWHUWREH 1 : DVLWSXUHJROG"
- 3 $ VSKHULFDONHDXP EDOORQ V P LQGLDP HWUDQG KDV DWRWDOP DVV RI NJ : KHQUHOHDVHGLQWR WKH8 6 VDQGDUGDWPRVSKHUH DWZKDWDOWLWXGHZLOLWVWHWOH"
- 3 5HSHDW3URE DVVXPLQJ WKDWWKH OEI ZHLJKWLV DOXPLQXP 6* DQGLVKDQJLQJ VXEPHUGHGLQWKHZDWU

- 3 7KH LQE\ LQE\ IWWSDUEXR\ IURP) LJ 3 KDV GEP RI VWHODWDFKH DQGKDVJRQHDJURXQGRQDURFN DVLQ) LJ 3 &RPSXWHWKHDQJOH DWZKLFKWK EXR\ ZLOOOHDQ DWXPLQJWKDWWKHURFNH[HUWVQR PRPHQWRQWKHVSDU

3

- 3 7KHGHSVXEPHUJHGYHKLFOH$/9,1 LQWKHFKDSWURSHQHUSKRWKDVDKROORZ VWDQXP 6* VSKHULFDOSDUWHQGHUFRPSDUWHQWZLWKDQLQVLGHGLPHWUF LQDQGDZDOOWKLFNQHVV RI LQ D : RXOG WKHHPSW VSKHUHIORDWL VHDZDU' E : RXOGLWORDWLLWFRQWLQHG GEP RI SHRSOHDQGHTXLSPHQ LQVLGH' F : KDWZDOOWKLFNQHVZRXOGFDXVHWKHHPSW VSKHUHWREHQHXWDO EXR\DQW

- 3 7KHEDOORRQLQ) LJ 3 LVILOOHGZLWKKHOXP DQGSUHVXUL]HGWR N3DDQ *f& 7KHEDOORRQPDWULDOKDVDPDVV RI J P (VWLPDWH D WKHWHQVLRQLQWK PRRULQJOLQHDQG E WKHKHLJKWWQWKHVXUQGDU DWRSSKHUHWRZKLFKWKEDOORRQZLO UHLWHPRRULQJOLQHLVFXW

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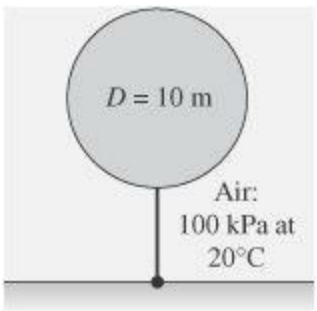

- 3 $ Q LQWHUHVWLQJ WHDXUH VDOYDJH JURXS KDV GLVFRYHUHG D VWHOEER[FRQWDLQLQJ JRO GRXEORRQV DQG RWKHU YDOXDEOHV UHWULQJ LQ IWRI VHDZDUU 7KH HVWLP DWH WKH ZHLJKWRI WKHER[DQG WUHDXUH LQDLU DW GEI 7KHLUSODQVWRDWWDFKWKHER[V DWXUGEDOORRQVLQDWDGZLWKDLUWR DW SUHVXUH 7KHHPSWEDOORRQZHJKV GEI 7KHER[LV IWZLGH IWORQJ DQG LQKLJK : KDWLVWKHSURSHUGHPHUF WKHEDOORRQVRHQXUHDQXSZDUGDFFRUFHRQWKHER[WKDWLV SHUFHQWPRUHWKD UHTXLUHG'
- 3 : KHQD GEI ZHJKWLVSODFHGRQWKHHQGRI WKHXQIRUP IORDWLQJ ZRRGHQEHDP L) LJ 3 WKHEHDP WLOWVDWDQDQJOH ZLWKLWXSSHUULJKWFRUQHUDWWKHVXUIDFH D VKRZQ' HWHUPLQH D WKHDQJOH DQG E WKHVSHFLILF JUDYLW RI WKHZRRG +LQ %RWKWKHYHUWLFDOFRUFHVDQGWKHPRPHQWDERXWWKHEHDP FHQWURLGPXWEHEDODQFHG

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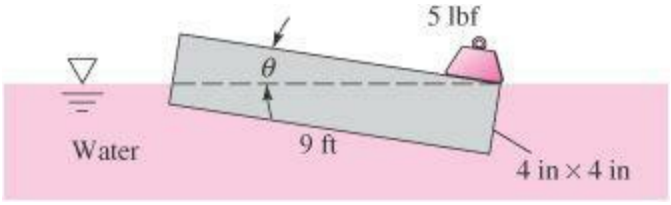
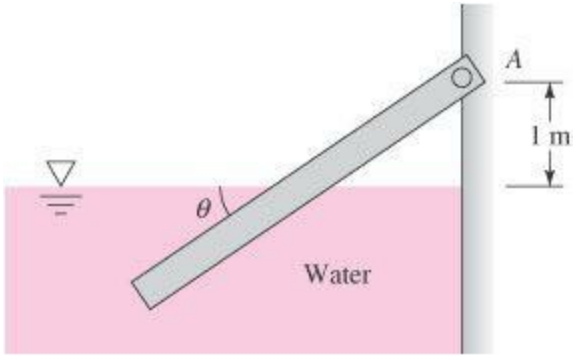


- 3 $ XQLRUP ZRRGHQEHDP 6* LV FP E\ FP E\ P DQGLVKLQJHGDW$ DVLQ) LJ 3 $WZKDWDQJOH ZLOOWKHEHDP IORDWLQWKH *& ZDWU

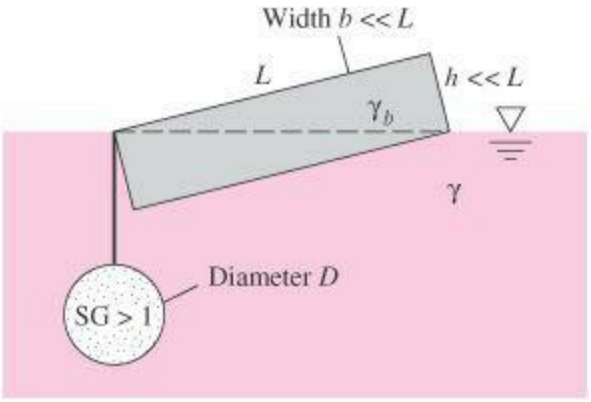
3



- 3 7KHXQIRUP EHDP LQ) LJ 3 RI VL]H/ E\ K E\ E DQG ZLWKVSHFLILF ZHJKW$ IORDWVH[DFWO RQLWVGLDJRQDOZKHQDKHDY\ XQIRUP VSKHUHLVWHGWRWKHOHIWRQH DVVKRZQ 6KRZ WKDWWKLVFDQKDSSHQRQO D ZKHQ E DQG E ZKHQWKHVSKHU KDVVL]H

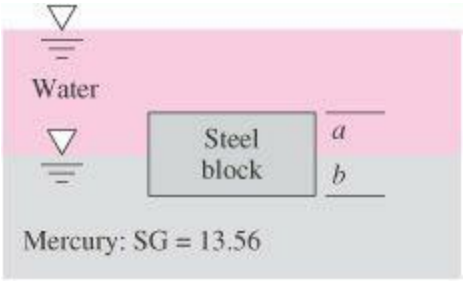
$$D = \left[\frac{Lhb}{\pi(\mathrm{SG}-1)}\right]^{1/3}$$

3



- 3 $ XQLRUP EORFNRI VWHHO 6* ZLOO³IORDWDWDPHUFXU±ZDWULQWUDFHDVLG)LJ 3 : KDWLVWKHUDWRRI WKHGLVWDQFHVDQGEIRUWKLVFRQGLWLRQ'

3



- 3 $ EDUJHKDVWKHWDSH]RLGDOVKDSHVKRZQLQ)LJ 3 DQGLV P ORQJ LQWR WKH SDSHU,I WKHVRXDOZHJKWRI EDUJHDQGFDUJRLV WRQV ZKDWLVWKHGUDIW RI WKH EDUJHZKHQIORDWLQJ LQVHDZDWHU'

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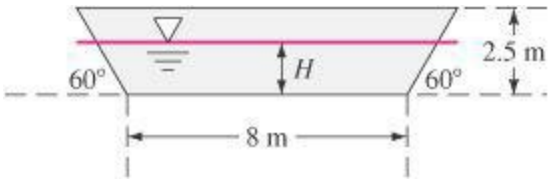

- 3 $ EDOORRQZHJKLQJ GEI LV IWLQGLDPHWHU,WLVILOOHGZLWKK\GURJHQDW GEI LQ DEVROXWHDQG *f) DQGLVUHOHDVHG$WZKDWDOWXGHLQWKH8 6 VWDQGDUGDWPRVSKHU ZLOOWKLVEDOORRQEHQHXWUDO EXR\DQW
- 3 $ VROG VSKHUH RI GLDPHWHU FP KDV D VSHFLILF JUDYLW RI D : LOOWKLV VSKHUH IORDWLQ *f& 6$(: RU' ,I VR E KRZ PDQ FXELF FHQWLPHWHUV DU H[SRVHG DQG F KRZ KLJK ZLOOD VSKHUHFDOSSURWXGHDERYHWKHVXUIDFH' 1RWH ,\RXUNQRZOHGJHRI RIIEHDWSKHUHIORUPXODVLVODFNLQJ \RX FDQ³$VN' U0 DWK' ' UH[HO8QLYHUVLW KWWS PDWKRUXP RUJ GUPDWK ((6 LVUHFRPPHQGHG IRUVWR VROXWRQ
- 3 $ EORFNRI ZRRG 6* IORDWVLQIXLG; LQ)LJ 3 VXFKWKDW SHUFHQWR LWVYROXPHLVVXEPHUJHGLQIXLG; (VWLPDWWKHYDFXXP SUHVVXUHRI WKHDLULQWK WDQN

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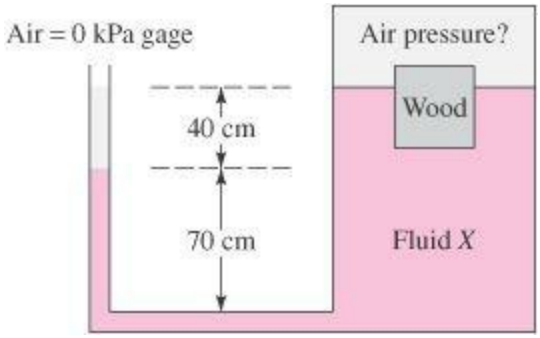

6WDELOLDWV RI IORDWLQJ ERGLHV

- 3 &RQVLGHU D F\OLQGULFDO VSHFLILF JUDYLW6 IORDWLQJ YHUWLFDOO LQZDWU 6 DVLQ) LJ 3 ' HULYH DIRUPXODIRUWKHWDEOHYDOXHVRI ' / DVDIXQFWLRQRI6 DQGDSSO\ LWWRWKHFDVH' /

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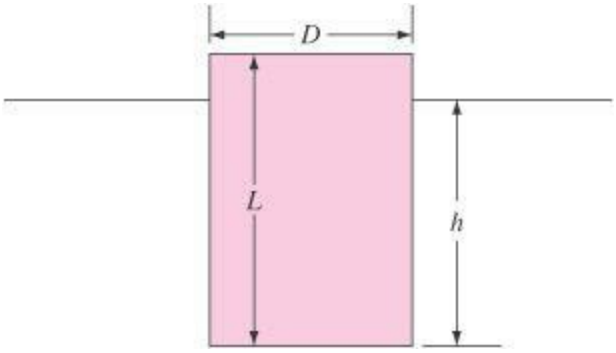
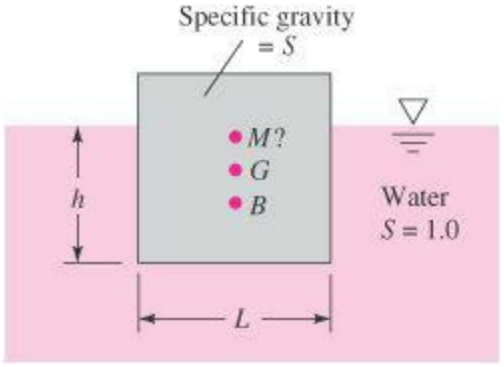


- 3 $QLFHEHUJ FDQEH LGHDOL]HG DV D FXEH RI VLGH OHQJWK/ DV LQ) LJ 3 , VHDZDWU LV GHQRWHG E\ 6 WKHQJODFLHUICH ZKLFK IRUPV LFHEHUJV KDV6 ' HWHUPLQH LI WKLV ³FXEH´ LFHEHUJ LV VWDEOH IRU WKH SRVLWLRQ VKRZQ LQ) LJ 3

3



- 3 7KHLFHEHUJ LGHDOL]DWRQLQ3UREOHP 3 PD\ EHFRPH XQVWDEOH LI LWV VLGH-V PHDVXUH LWVKHLJKWH[FHHGV LWZLGWK ,Q) LJ 3 VXSSRVHWKDWWKHKHLJKW/ DQGWKHGHQVL

LQRWKHSDSHUV/ EXWKHZLGWKLQWKHSODQHRI WKHSDSHUV+ / \$VXPLQJ6
IRUWKHLFHEHUJ ILQGWKHUDWR+ / IRUZKLFKLWEHFRPHVQHXWUDO WKDWEH DERX
VRRYHUWXLQ

- 3 &RQVLGHUDZRRGHQF\OLQGHU6* P LQGLDPHWUDQG P ORQJ : RXOGWKL
F\OLQGHUEHVWDEOHLI SODFHGWRIORDWZLWKLWD[LVYHUWLFDOLQRLO6* "
- 3 \$ EDUJHLV IWZLGHDQG IWORQJ DQGIORDWZLWKDGUDIWRI IW,WLVSLQHGVRKJI
ZLWKJUDYHOWKDWVXFHQWURI JUDYLWLV IWDERYHWKHZDWHUOLQH ,VLWVWDEOH"
- 3 \$ VROGUHJKWFLUFXODUFRQHKDV6* DQGIORDWYHUWFDOO DVLQ) LJ 3 ,'
WKLVDVWDEOHSRVLWLRQIRUWKHFRQH"

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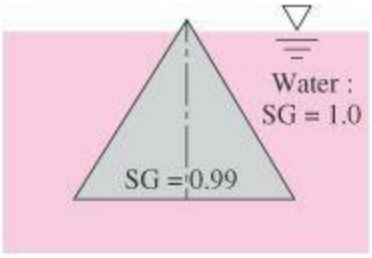

- 3 &RQVLGHUDXQLRUP UHJKWFLUFXODUFRQHRI VSHFLILF JUDYLW6 IORDWQJ ZLWKLV
YHUWI[GRZQ LQ ZDWU 6 7KHEDVH UDGLXV LV5 DQG WKH FRQH KHJKWLV+
&DOFXODWHDQG SORWWKHVWDELOLW0 * RI WKLVFRQH LQGLPHQVLRQOHVVIRUP YHUXV+ 5
IRUDUDQJHRI 6
- 3 : KHQIORDWQJ LQZDWU 6* DQHTXLOLEUDWLQJXDUERG 6* PLJ
WDNHRQH RI WKH VZR SRVLWRQV VKRZQ LQ) LJ 3 : KLFK LV WKH PRUH VWDEO
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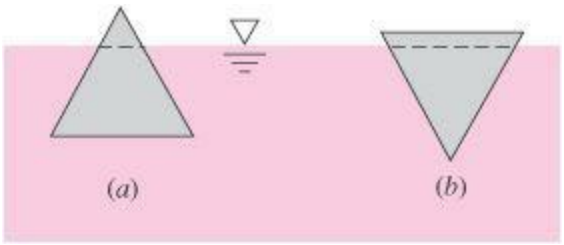

- 3 &RQVLGHUDKRPRJHQHRXVUJKWFLUFXODUF\OLQGHURI OHQJWK/ UDGLXV5 DQG VSHFLL
JUDYLW6* IORDWQJ LQZDWU 6* 6KRZ WKDWWKHERG\ ZLOOEHVWDEOHZLWKLV
D[LVYHUWFDOLI

$$\frac{R}{L} > [2SG(1 - SG)]^{1/2}$$

- 3 &RQVLGHUDKRPRJHQHRXVUJKWFLUFXODUF\OLQGHURI OHQJWK/ UDGLXV5 DQG VSHFLL
JUDYLW6* IORDWQJ LQZDWU 6* 6KRZ WKDWWKHERG\ ZLOOEHVWDEOHZLV
LWD[LVKRU]RQWDOLI / 5 !

8 QLIRUP DFFHOHUDWLRQ

- 3 $ VDQN RI ZDWU P GHHS UHFHLYHV D FRQVWDQW ZDUG DFFHOHUDWLRQ D ' HWHUPLQH WKH JDJH SUHVVXUH DW WKH VDQN ERWWRP LI D P VDQG E WKH YDOXH RI D WKDW FDXVHV WKH JDJH SUHVVXUH DW WKH VDQN ERWWRP WR EH DW
- 3 $ IOR[JDXJH RI LQGLDPHWHU SDUW IXOO RI ZDWHU LV DWWDFKHG WR WKH HGJH RI D IWGLDPHWHU PHUU JR URXQG ZKLFK LV URWDWHG DW UPLQ +RZ IXOO FDQ WKH JDJH EH EHIRUH ZDWHU VSLOOV" +LQW$ VXPH WKDW WKH JDJH LV PXFK VPDOOHU WKDQ WKH UDGLXV RI WKH PHUU JR URXQG
- 3 7KH VDQN RI) LJ 3 DFFHOHUDWHV WR WKH ULJKW ZLWK WKH IXLGLQUULJ ERG PRWLRQ D &RPSXWH D LQP V E : K\ GRHV TMWKH VRXWLRQ WR SDUW D GHSHQG RI WKH GHQVLW RI WKH IXLG' F ' HWHUPLQH WKH JDJH SUHVVXUH DW SRLQW$ LI WKH IXLG LV JO\FHULQDW *

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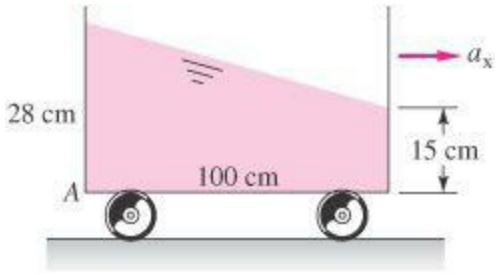
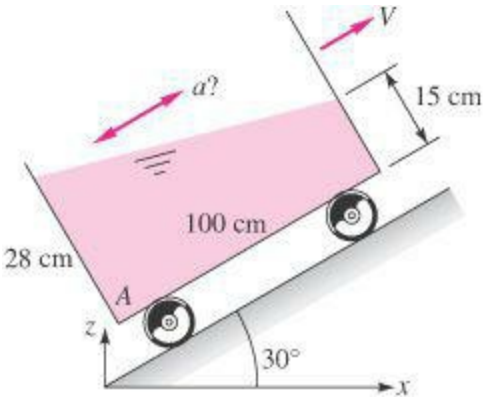


- 3 6XSSRVH DQ HQSWFDOHQG IXHOODQN WKDWV P GRJ DQG KDV D P KRUL]RQWDOP DW D[LV DQG P YHUWFDOP DWRUD[LV LV ILOOHG FRPSOHWHO ZLWK IXHORLO! NJ P / HWWKH VDQN EH SXOOHG DORJ D KRUL]RQWDO URDG) RU ULJLG ERG PRWLRQ ILQG WK DFFHOHUDWLRQ DQG LWV GLUHFWLRQ IRU ZKLFK D D FRQWDQWSUHVXUH VXUIDFH H[WHQGV IURP WKH WRS RI WKH IURQWHQG ZDOO WR WKH EDFN RI WKH EDFNHQG DQG E WKH WRS RI WKH EDFNHQG LV DW SUHVXUH DW WKH WRS RI WKH IURQWHQG
- 3 7KH VDPH VDQN IURP 3URE 3 LV QRZ PRYLQJ ZLWK FRQVWDQW DFFHOHUDWLRQ XS I f LQFOLQHG SODQH DV LQ) LJ 3 $VVXPLQJ ULJLG ERG PRWLRQ FRPSXWH D WK YDOXH RI WKH DFFHOHUDWLRQ D E ZKHWKHU WKH DFFHOHUDWLRQ LV XS RU GRZQ DQG F WK JDJH SUHVVXUH DW SRLQW$ LI WKH IXLG LV PHUFXU DW f&

3



- 3 7KH WDQN RI ZDWHU LQ) LJ 3 LV FP ZLGH LQRR WKH SDSHU ,I WKH WDQN LV DFFHOHUDWHG WR WKH ULJKW LQ ULJLG ERG\ PRWLRQ DW P V FRPSXWH D WKH ZDWHU GHSWK RQ VLGH $% DQG E WKH ZDWHU SUHVVXUH IRU FHQSDQHO $% $VVXPH QR VSLOODJH

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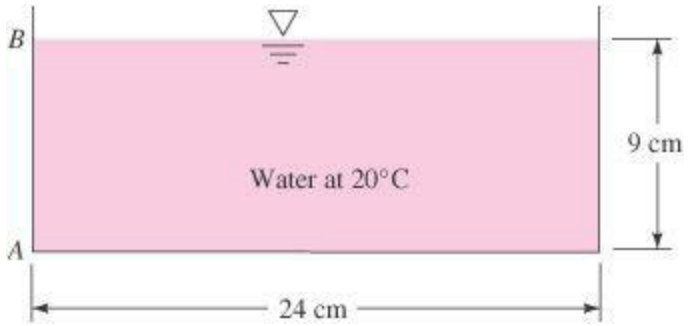

- 3 7KH WDQN RI ZDWHU LQ) LJ 3 LV IXOO DQG RSHQ WR WKH DWPRVSKHUH DW SRLQW $) R ZKDW DFFHOHUDWLRQ [LQIWV ZLOO WKH SUHVVXUH DW SRLQW % EH D DW PRVSKHULF DQG E]HUR DEVROXWH

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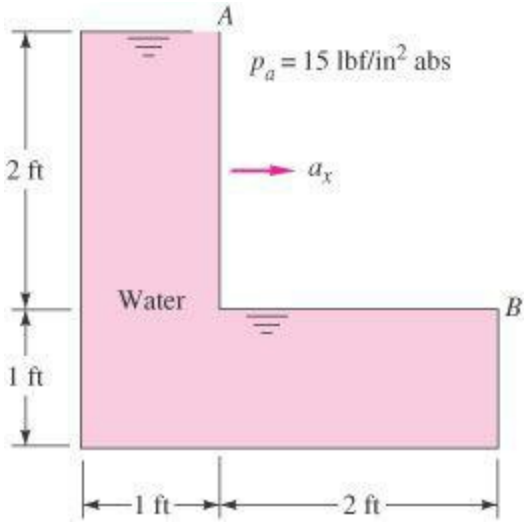

- 3 &RQVLGHU DKRUBRZ FXEH RI VLGH OHQJWK FP ILOOHG FRPSOHWHO ZLWK ZDWHU DW *&
7KH WRS VXUIDFH RI WKH FXEH LV KRUL]RQWDO 2QH WRS FRUQHU SRLQW $ LV RSHQ WR WKH URXJK I VPDOO KROH WR DWPRVSKHULF SUHVVXUH DW ' LDJRQDOO RSSRVLWH WR SRLQW $ LV WRS FRUQHU % ' HWHUPLQH DQG GLVFXVV WKH YDULRXV ULJLG ERG\ DFFHOHUDWLRQV IRU ZKLFK WKH ZDWHU SRLQW % EHJLQV WR FDYLWDWH IRU D KRUL]RQWDO PRWLRQ DQG E YHUWLFDO PRWLRQ
- 3 $ ILVK WDQN LQGHSE\ E\ LQLV WR EH FDUULHG LQ D FDUWKDWPD H[SHULHQF DFFHOHUDWLRQ V DV KLJK DV P V : KDW LV WKH PD[LPXP ZDWHU GHSWK WKDW ZLOO DYRLG VSLOODJ LQ ULJLG ERG\ PRWLRQ' : KDW LV WKH SURSHUDQJOH QHQW RI WKH WDQN ZLWK UHVSHF WR WKH FDUPRWLRQ'
- 3 7KH WDQN LQ) LJ 3 LV ILOOHG ZLWK ZDWHU DQG KDV DYHQWKRGH DW SRLQW $ 7KH WDQ LV P ZLGH LQ WKH SDSHU ,QVLGH WKH WDQN D FP EDOORRQ ILOOHG ZLWK KHOXP D N3D LV WWKH UHGFHQDOO E\ DVWULQJ ,I WKH WDQN DFFHOHUDWHV WR WKH ULJKW DW P V

LQULJLG ERG\ PRWRQ DWZ KDWDQJOHZ LOOWKHEDOORRQOHDQ : LOOLWOHDQWRWKHULJKWR
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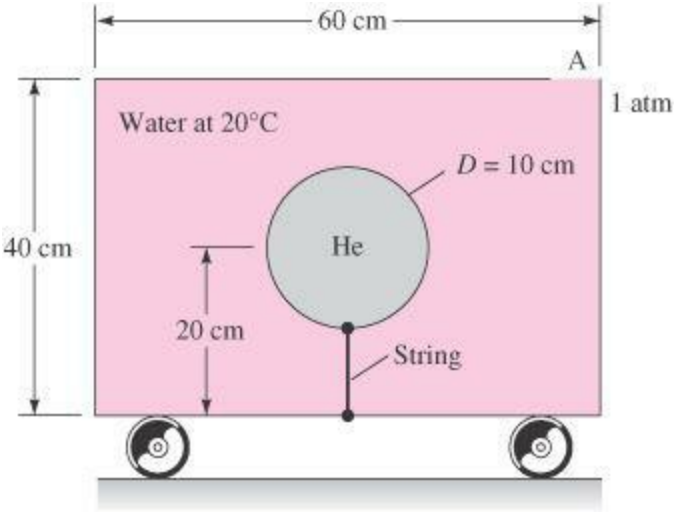
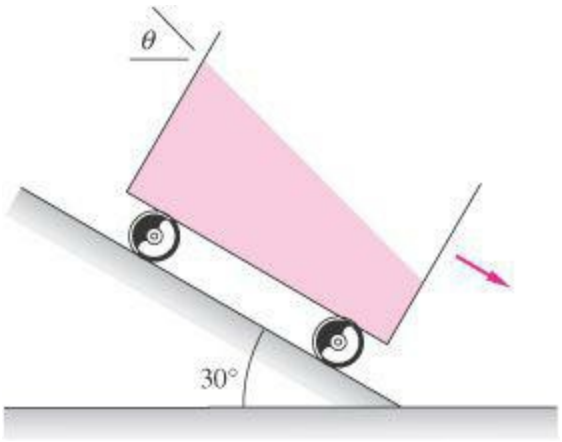


- 3 7KHWDQNRI ZDWULQ)LJ 3 DFFHOHUDWHVXQLIRUPO\ E\ IUHO URODQJGRZQD
LQFOLQH ,I WKH ZKHHOV DUH IUFWLRQOHVV ZKDWLV WKH DQJOH " &DQ\RX H[SODLQWK
LQWHUWLQJ UHVXOWV

3



- 3 $ FKLOGLVKRORLQJ DWWLQJ RQWR ZKLFK LV DWWDFKHGDKHOXP ILOOHGEDOORRQ D 7K
FKLOG LV WDQGLQJ WOOODQG VXGGHQO DFFHOHUDWV IRUZDUG ,QDIUDPHRI UHHUHQF
PRYLQJ ZLWK WKH FKLOG ZKLFK ZD\ ZLOOWKH EDOORRQ WOOWIRUZDUG RUEDFNZDUG
([SODLQ E 7KHFKLOG LVQRZ VLWWLQJ LQDFDUWKDWLV VRSSHGDWDUHGQJKW7K
KHOXP ILOOHGEDOORRQLVQRWQFRQDFWZLWKDQ SDUWRI WKHFDU VHDW FHLQJ HW
EXWLVKHOGLQSODFHE\ WKHWXLQJ ZKFKLVLQWKQKHGE\ WKHFKLOG $OWKHZLQGRZ
LQWKHFDUDUHFORVHG : KHQWKHWDILFQJKWXLQVJUHQ WKHFDUDFFHOHUDWVIRUZDUG
,QDIUDPHRI UHHUHQFHPRYLQJ ZLWKWKHFDUDQGFKLOG ZKFK ZD\ ZLOOWKHEDOORR
WOOWIRUZDUGRUEDFNZDUG' ([SODLQ F 3UFKDVHRUERUZ DKHOXP ILOOHGEDOORRQ
&RQGXFWDVFLHQWLF H[SHULPHQWRVHHLI \RXSUHGLFWRQVLQSDW D DQG E DERYH
DUHFRUUHFW,I QRW H[SODLQ

- 3 7KH IWUDGLXV ZDWULZKHHOLQ)LJ 3 LVEHLQJ XVHGWRQWZDWUZLWKLW IV

GLDPHWHU KDOI FLQQGHUEODGHV ,I WKH ZKHHOUHYROXWHV DW UPLQ DQG ULJLG ERG
PRWLRQLVDVVXPHG ZKDWLVWKHZDWHUVXUIDFHDQJOH DVSRVLWLRQ$"

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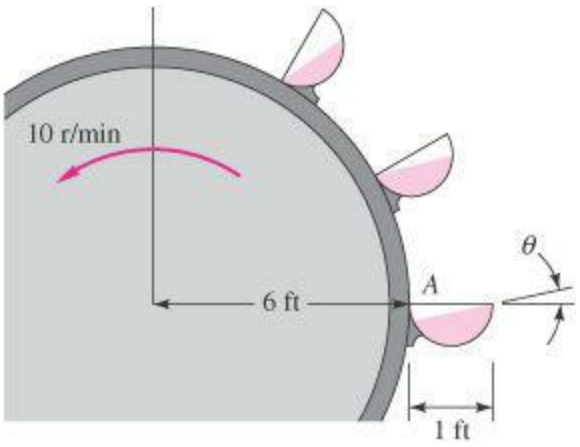
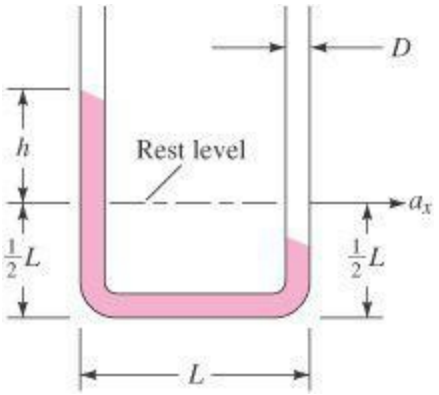


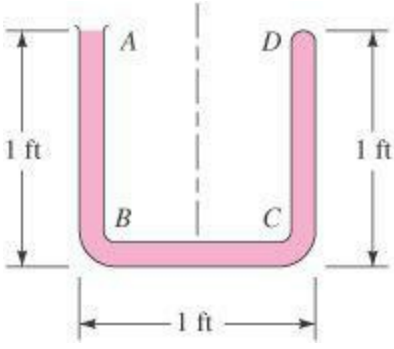
- 3 $ FKHDS DFFHOHURPHWHU SUREDEO ZRUNVWKHSUHF FDQEHPDGHIURP D8 VWEHDVL
) LJ 3 ,I / FP DQG' PP ZKDWZLOOK EH LI D P V" &DQWK
VFDOHPDUNLQJVRQWKHWEHEHOQHDUPXOWSOHVR D"

3



- 3 7KH8 WEHLQ) LJ 3 LVRSHQDW$ DQGFORVHGDW ,I DFFHOHUDWHGWKHULJKWL
XQLRUP D ZKDWDFFHOHUDWLRQZLOOFDXVHWKHSUHVVXUHDWSRLQW& WREHDWPRVSKHULF
7KHIOXLGLVZDWU 6*

3



5 LJLG ERG\ P RWLRQ

- 3 $ FP GLDP HWHU RSHQ F\OLQGHU FP KLJK LV IXOO RI ZDWHU &RP SXWH WKH ULJLG ERG\ URWDWLRQ DO LQHUWLD DERXW LWV FHQWUDO D[LV LQ UPLQ D IRUZKLFK RQH WKLUG RI WKH ZDWHU ZLOO VSLOO RXW DQG E IRUZKLFK WKH HERWWRP ZLOO EH EDUHO H[SRVHG
- 3 $ VROLG F\OLQGULFDO FRQWDLQHU LQ LQGLDP HWHU LV XVHG WR PDNH D PRGHO IRU IRUP LQ LQVWDOODWLRQV 7KH HERZ OV DUH WR EH LQGHS 7KH F\OLQGHULV KDOI ILOOHG ZLWK PRQWQSODWLF NJ P V URWDWHG WKURXJK DERXW WKH FHQWUDO D[LV WKH QFRRO HV ZKLOH URWDWLQJ : KDW V WKH DSSURSULDWH URWDWLRQDO LQUPLQ'
- 3 $ YHUWLFDO FP GLDP HWHU YDVH FRQWDLQV FP RI ZDWHU : KHQ VSXQ XS DW D FHUWDLQ URWDWLRQDO VSHHG WKH ZDWHU GUDZV DZD\ IURP WKH ERWWRP RI WKH YDVH : KDW V WKH URWDWLRQDO LQUPLQ IRU WKLV FRQGLWLRQ'
- 3) RUZKDW XQIRUP URWDWLRQDO LQUPLQ DERXW D[LV& ZLOO WKH 8 WXEH LQ) LJ 3 WDNH WKH FRQLJXUDWLRQ VKRZ Q' 7KH IOXLG VPHUFXU DW *f*&

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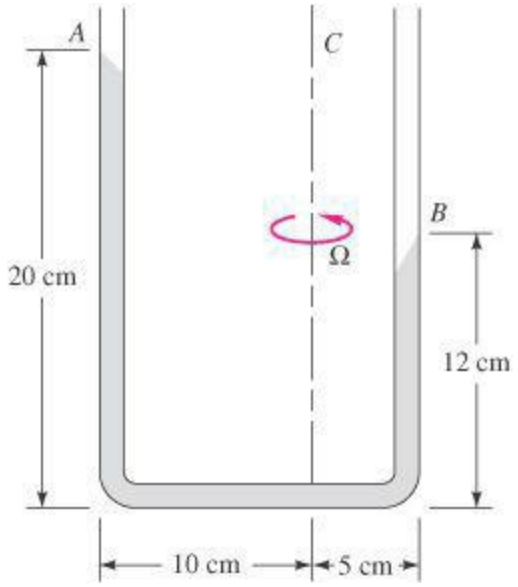

- 3 6XSSRVH WKDW WKH8 WXEH RI) LJ 3 LV URWDWHG DERXW D[LV' & ,I WKH IOXLG LVZDW DW *f*) DQG DWPRVSKHULF SUHVVXUH LV GEI IW DEVROXWH DWZKDW URWDWLRQDO ZLOO WKH IOXLG ZLWKLQ WKH WXEH EH JLQ WR YDSRU[H' $VZ KDW SRLQW ZLOO WKLV RFFXU'
- 3 7KH *f* 9 WXEH LQ) LJ 3 FRQWDLQV ZDWHU DQG LV RSHQ DW$ DQG FORVHG DW& : KDW XQIRUP URWDWLRQDO VSHHG LQ UPLQ DERXW D[LV$% ZLOO FDXVH WKH SUHVVXUH WR EH HTXDO DW SRLQW% DQG&") RU WKLV FRQGLWLRQ DWZKDW SRLQWLQ HJ %& ZLOO WKH SUHVVXUHEHDPLQLPXP"

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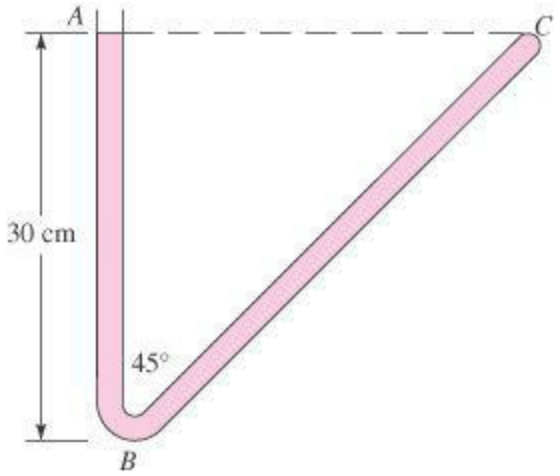

- 3 ,WLV GHVLUHG WR PDNH D P GLDPHWHU SDUDEROLF WURSHPLUURU E\ URWDWLQ PRQWQJODVV LQ ULJLG ERG\ PRWRQ XQWLO WKH GHVLUHG VKDSH LV DFKLHYHG DQG WKH FRRQQJ WKH JODVV WR D VROG 7KHIRFXVRI WKHPLUURULV WR EH P IURP WKHPLUURU PHDVXUHG DORQJ WKH FHQWUDOLQH : KDWLV WKH SURPHUPLURURUDWRQDW LQUPLG IRUWKLVWDN'
- 3 7KHWKUHHGJHGPDORPHWULQ) LJ 3 LVILOOHGZLWKZDWHUWR D GHSWKRI FP $ COWEHV DUH ORQJ DQG KDYH HTXDOVPDOGLPHWHUV ,I WKH VWWP VSLQV DWDQJXOD YHORFLW9 DERXWWKHFHQWDOWEH D GHULYHDIRUPXODWRILQGWKHFKDQJHRI KHJKWW WKHWEHV E ILQGWKHKHJKWQFP LQHDFKWEHLI 9 UPLQ+LQW7KHFHQWL WEHPXWXSSO ZDWUWRERWWKHRXWUHJV

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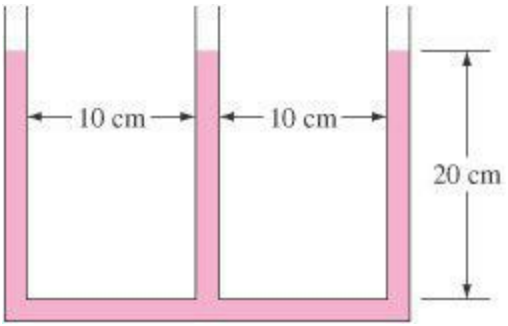

: RUG3UREOPV

- : &RQVLGHUDKROORZ FRQHZLWKDYHQWKROHLQWKHYHUWH[DWWKHVS DORQJ ZLWKDKROORZ F\OLQGHUSHQDWWKHVS ZLWKWKHVDPHEDVHDUHDV WKHFRQH) LQGERWKZLWKZDW WR WKHVS 7KHK\GURVWDWF SDUDGR[LVWKDWERWKFRQWDLQHUVKDYHWKHVDPHIRUFHR WKHERWWP GXHWR WKHZDWU SUHVVXUH DQWRXJK WKHFRQHFRQWDLQV SHUFHQWO ZDWU&DQ\RXH[SODLQWKHSDUDGR["
- : &DQ WKHWPSHUDWXUHHYHUULVHZLWKDOWXGHLQ WKHUHDOWPRVSKHUH' : RXOGQWWKLV FDXVHWKHDLUSUHVXUHWRLQFUHDVHXSZDUG' ([SODLQWKHSK\VLFVRI WKLVVLWXDWQ
- : &RQVLGHUDVXEPHUJHGFXUYHGVXUIDFHWKDWFRQVLWWRI DWZR GLPHQVLRQDOFLFXODUU

RI DUELWDU DQJOH DUELWDU GHSWK DQG DUELWDU RUHQWDWLRQ 6KRZ WKDWWKH UHVXOWDQ
K\GURXDWF SUHVVXUH IRUFH RQWKLV VXUIDFH PXVWSDVV WKURXJK WKH FHQWURLG RI FXUYDWXU
RI WKH DUF

- :) LOOD JODV DSSUR[LPDWH SHUFHQWZLWK ZDWHU DQG DGG DODUJHLFHFXEH 0 DUN WK
ZDWHUOHYHO 7KHLFHFXEH KDYLQJ 6* § WLPHV XS RXWR WKH ZDWHU / HWWKHLF
FXEHPHOWZLWKQHJOLJLEOHHYDSRUDWLRQIURP WKHZDWHUVXUIDFH : LOOWKHZDWHUOHY
EHKLJKHUWKDQ ORZHUWKDQ RUWKHVDPHDVEHRUH'

- : $ VKLS FDUULQJ D ORDG RI VWHHO LV VWDSSHG ZKLOH ORDGLQJ LQ D VPDOO FORVHG ORFN
0 HPEHUV RI WKH FUHZ ZDQWRJHWRXWEXWWKH\ FDQ¶W TXLWUHDFK WKHVRS ZDOOR WK
ORFN $ FUHZ PHPEHUVXJJHVWVWKURZLQJ WKHVWHHORYHUERDUG LQWKHORFN FOLPLQ
WKHVKLSZLOOWKHQULVHDQGWKH\ FDQFOLPERXW: LOOWKLVSODQZRUN'

- : &RQVLGHU D EDOORRQ RI PDVV P IORDWLQJ QHXWUDOO LQ WKH DWPRVSKHUH FDUU\LQJ I
SHUSQEDVHWRP DW0 ! P ' LVFXVVWKHVXEELQWRI WKLVV\VWHP VRGLVXUEDQFHV

- : &RQVLGHUDKHOXP EDOORRQRQDVWULQJ WHG WR WKHVHDRI \RUVWDWRQDU FDU 7K
ZLQGRZVDUHFORVHG VR WKHUHLVQR DLUPRWRQZLWKLQ WKHFDU 7KHFDUEHJLQV W
DFFHOHUDWIRUZDUG : KLFKZD\ ZLOOWKEDORRQOHDQ IRUZDUGRUEDFNZDUG' +LQ
7KHDFFHOHUDWRQVHWXSDKRU]RQWDOSUHVVXUHJUDGLHQWWQWKHDLUZLWKLQWKHFDU

- : 5HSHDWYRXUDQDOVLVRI3URE : WR OHWWKHFDUPRYHDFRQWDQWYHORFLWVDQGJI
DURXQGDFXUYH : LOOWKEDORRQOHDQLQ WRZDUGWKHFHQWUHRFXUYDWXUH RURXW

- : 7KH GHHS VXEPHUVLEOH YHKLFOH $/9,1 LQ WKH FKDOOHQJHU SKRWR ZHJK
DSSUR[LPDWH GEI LQDLU ,WFDUULHV GEP RI VWHOZHJKWVRQWKHVLGHV
$IWUDGHSPLVLRQDQGUHWXUQ WR GEP SLOHVRI VWHODUHOHWRQWKHRFHD
IORRU &DQ\RXH[SODLQ LQWUPVUHOYDQWKLVFKDSWUKRZ WKHVHVWHOZHJKWVDU
XVHG'

(QJLQHHULQJ ([DP 3UREOHPV

-)($ JDJH DWWDFKHG WR D SUHVXU]HG QWRJHQ WDQN UHDGV D JDJH SUHVXUH RI LQ F
PHUFXU ,I DWPRVSKHULF SUHVXUHLV SVLD ZKDWLV WKH DEVROXWH SUHVXUHLQ WK
WDQN'

NBD

NBD

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NBD

NBD

-)(2QDVHDOHYHOWDQGDUGGD\ DSUHVXUHJDJH PRRUHGEHORZ WKHVXUIDFHRI WKHRFHD
6* UHDGVDQDEVROXWHSUHVXUHRI 0 3D +RZ GHHS LVWKHLQWUWKPHQW
P

P

P

P

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-)(,Q)LJ)(LI WKHRLOLQUHJLRQ%KDV6* DQG WKHDEVROXWHSUHVXUHDWSRLQW

LV DW Z KDWLV WKH DEVROXWH SUHVVXUH DW SRLQW %'
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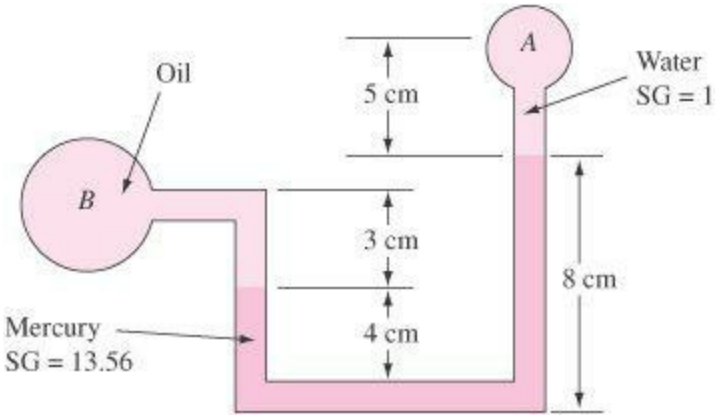

-)(,Q) LJ)(LI WKH RLO LQ UHJLRQ %KDV 6* DQG WKH DEVROXWH SUHVVXUH DW SRLQW
  LV SVLD Z KDW LV WKH DEVROXWH SUHVVXUH DW SRLQW $"
  N3D
  N3D
  N3D
  N3D
  N3D
-)($ VDQNR Z DWU 6* KDV DJDWLQLWY YHUWFDOZ DOO P KLJK DQG P Z LGH 7K
  VWS HGJH R WKH JDWLV P EHOZ WKH VXUDFH : KDW LV WKH K\GURVWDWF IRUFH RQ WK
  JDW'
  N1
  N1
  N1
  N1
  N1
-)(,Q3URE)(KRZ IDUE HOZ WKH VXUDFH LV WKH FHQWURL SUHVVXUH R WKH K\GURVWDW
  IRUFH'
  P
  P
  P
  P
  P
-)($ VRDG P GLDP HWU VSKHUH IORDW DW WKH LQWHUIDFH EHWZHHQ Z DWU 6* DQ
  PHUFXU 6* VXFK WKDW SHUFHQW LV LQ WKH Z DWU : KDW LV WKH VSHFLL
  JUDYLW R WKH VSKHU'

-)($ P GLDPHWUHDOORRQFRQWLQVKHDXP DW N3D DEVRXWDQG f& PRRUHGL
  VHDOHYHOWDQGDUGLU,I WKHJDVFRQWDQWRI KHDXP LV P V . DQGEDOORR
  PDWHULDOZHJKWLVQHJOHFWHG ZKDWLVWKHQHWDWWQJ IRUFHRI WKHEDOORRQ'
  1
  1
  1
  1
  1
-)($ VTXDUHZRRGHQ 6* URG FP E\ FP E\ P ORQJ IORDWYHUWFDOO L
  ZDWHU f& ZKHQ NJ RI VWHD 6* DUHDWDFKHGWRRQHHQG +RZ KLJ
  DERYHWKHZDWHUVXUIDFHGRHVWKHZRRGHQHQGRI WKHURGSURWUGH'
  P
  P
  P
  P
  P
-)($ IORDWLQJ EORG\ ZLOOEHVXEMHFW
  FHQWULFRI JUDYLWLVDERYHLWFHQWULFRI EXR\DQF\
  FHQWULFRI EXR\DQF\ LVEHORZ WKHZDWHUOLQH
  FHQWULFRI EXR\DQF\ LVDERYHLWPHDFHQWU
  PHDFHQWULVDERYHLWFHQWULFRI EXR\DQF\
  PHDFHQWULVDERYHLWFHQWULFRI JUDYLW

SUREOHPV TXHVWLRQV &RPSUHKHQVLYH3UREOHPV

- & 6RPHPDQRPHWHUVDUHFRQVWUXFWHGVLQ) LJ & ZKHUHRQHVLGHLVDOJHUHVHUYRL
  GLPHWHU' DQGWKHRWKHUVLGHLVDVPDOOXEHRI GLPHWHUG RSHQWRWKHDWPRVSKHU
  ,QVXFKDFDVH WKHKHLJKWRI PDQRPHWHUGTXLGRQWKHUHVHUYRLUVLGHGRHVQRWFKDQJ
  DSSUHFLDEO 7KLVKDVWKHDGYDQWDJHWKDWRQO RQHKHLJKWGHG VWREHPHDVXUHGUDWK
  WKDQWZR 7KHPDQRPHWHUGTXLGKDVGHQVLW! P ZKLOHWKHDLUKDVGHQVLW! D ,JQRUHWK
  HIHFWRI VXUIDFHWHQVLRQ: KHQWKHUHVQR SUHVVXUHGLIIHUHQFHDFURVVWKHPDQRPHWW
  WKHHOHYDWRQVRQERWKVLGHVDUHWKHVDPH DVLQGLFDWHG E\ WKHGDVKHGOLQH +HJKWK L
  PHDVXUHGIURP WKH]HURSUHVVXUHOHYHODVVKRZQ D : KHQDKLJKSUHVVXUHLVDSSOH
  WRWKHOHWWLGH WKHPDQRPHWUGTXLGLQWKHOHJUHVHUYRLUJRHVGRZ ZKLOHWKDWW
  WKHWKEHDWWKHULJKWJRHVXS WRFRQVHUYHPDW : ULWHDQH[DFWH[SUHVVLRQIRUS JDJH
  WDNLQJ LQWRDFFRXQWWKHPRYHPHQWRI WKHVXUFHRI WKHUHVHUYRLU<RXUHTXLUHQVKRXOG
  JLYHS JDJH DVDIXQFWLRQRI K !P DQGWKHSK\VLFDOSDUDPHWHUVLQWKHSUREOP K G '
  DQGJUDYLWFRQVWDQWJ E : ULWHDQDSSURSULDWH[SUHVVLRQIRUS JDJH QHJOHFWLQJ WK

FKDQJHLQHOHYDWLRQRI WKHVXUIDFHRI WKHUHVHUYRLUOLTXLG F 6XSSRVHK P LQI
FHUWDLQDSSOLFDWLRQ ,I S$ 3DDQGWKHPDQRPHWULTXLGKDVDGHQVLWRI
NJ P HWHUPDWWKHUDWR' GUHTXLUHGWR NHHSWKHHURURI WKHDSSUR[LPDWRQRI SDU
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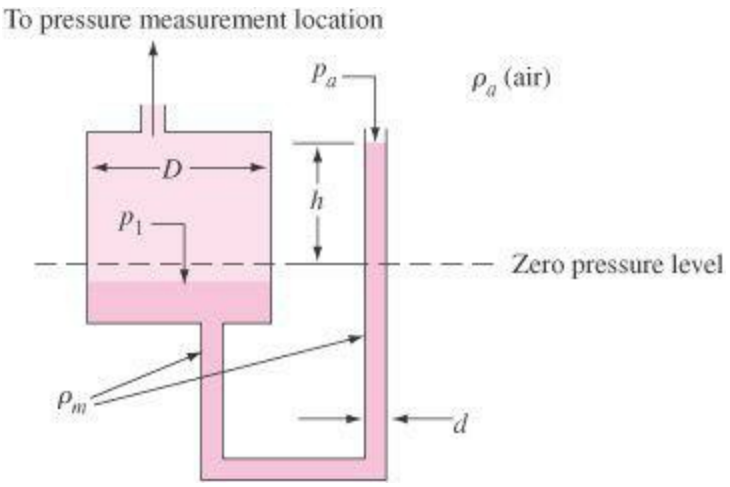

- & $ SUDQWWUKDVDGGHGROLRI VSHFLILF JUDYLW6* WRWKHOHWDJRI WKHPDQRPHWULT
) LJ & 1HYHUWKHOHVV WKH8 WXEHLVWXOXHXODVDSUHVXUHPHDVXULQJGHYLFH ,W
DWWDFKHGWRDSUHVVXUL]HGWDQNDVVKRZQLQWKHILJXUH D) LQGDQH[SUHVVLRQIRUK DVI
IXQFWLRQRI + DQGRWKHUSDUDPHWUVLQWKHSUREOP E) LQGWKHVSHFLDOFDVHRI \RX
UHVXOWLQ D ZKHQS$ S$ F 6XSSRVH+ FP S$LV NG D S$LV NG
KLJKHUWKDQS$ DQG6* &DOFXODWHK LQFP LJQRULQJ VXUIDFHWHQVLRQHIHFWVDQ
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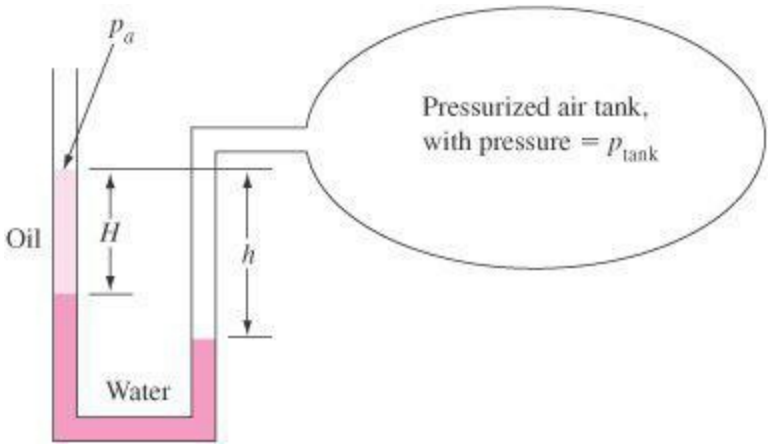
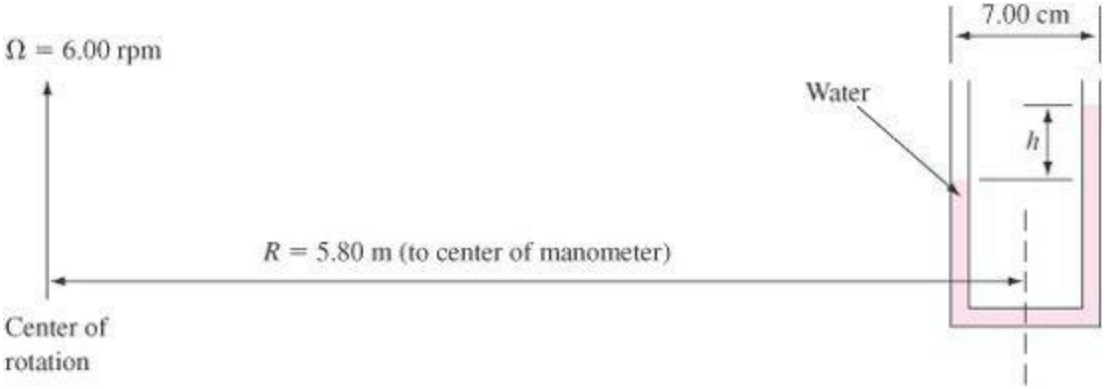


- & 3URHVVRU) ' \QDPLFV UGLQJ WKHPHUV JRURXQG ZLWKKLVVRQ KDVEURXJKWDOQJKL
8 WXEHPDQRPHWU <RX QHYHUNQRZ ZKHQ D PDQRPHWUPLJKWFRPHLQKDQG $V
VKRZQ LQ) LJ & WKHPHUV JRURXQG VSLQV DWFRQVWDQWDQJXODUYHORFLW DQG WK
PDQRPHWU OHUV DUH FP DSDUW7KH PDQRPHWUFHQWUL P IURP WKH D[LV R
URWDWRQ ' HWHUPLQH WKH KHLJKWGLIIHUHQFHK LQ WZR D SSUR[LPDWD E\

DVVXPLQJ ULJLG ERG\ WDQNDWRQ ZLWK D HTXDWRULDO HPSW\ DQG E H[DFWO\ XVLQJ ULJLG ERG\ URWDWRQVWHRU + RZ JRRG LV WKH DSSUR[LPDWRQ"

&



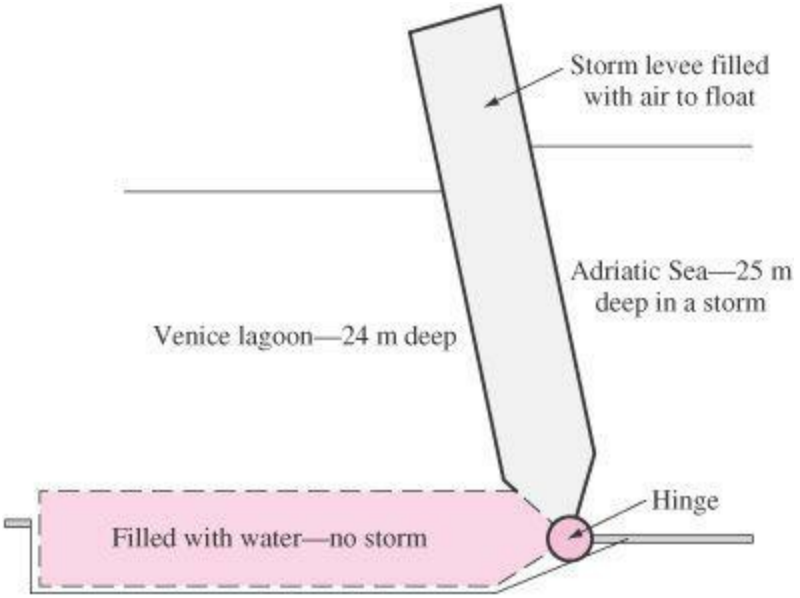
- & $ VXGHQWV QHDV DJDLQ RI FRQH RQR D URWDWLQJ FRQWDLQHU 7KH JDV LV FRQGXFWHG VWLFN DV VLWWLQJ RQ WKH EUP + HZ DQVZHUV NQRZ ZKDWSHUFHQWDJH FRQH KHVKRXOG GULQN EHIRUH WKH UHDEHJLQV VR WKDWQRQH RI LWVSLOOV GXULQJ WKH EL. GURS LQZKLFK WKHURWDWLQJ FRQWDLQHUV J DFFHOHUDWRQDWD *DQGHEHORZ WK KRUL]RQWDO 0 DNH WKH FDOFXODWRQ IRU KLP QHJOHFWLQJ VORVKLQJ DQG DVXPLQJ WKDWWK JDVV/VYHUWFDODVDOWHUV
- & ' UI DGLDEDWF ODSVH UDWH ' $/5 LV GHILQHG DV WKH QHJDWLYH YDOXH RI DWRVSKHU WHPSHUDWXUH JUDGLHQWG7 G] ZKHQ WHPSHUDWXUH DQG SUHVVXUH YDU LQ DQ LVHQWURSL IDVKLRQ $VXPLQJ DLU LV DQ LGHDOJDV ' $/5 íG7 G] ZKHQ7 7 SS D ZKHU H[SRQHQWD Ní N N $S FY LV WKH UDWLR RI VSHFLILF KHDW DQG7 DQGS DUH WKH WHPSHUDWXUH DQG SUHVVXUH DWWHD OHYHO D $VXPLQJ WKDWK\GURVWDW FRQGLWLRQV H[LVWLQ WKH DWPRVSKHUH VKRZ WKDWWKH GU DGLDEDWF ODSVH UDWH LV FRQVWDQ DQG LV JLYHQ E\ ' $/5 J Ní N5 ZKHUH5 LV WKH LGHDOJDV FRQVWDQWRUDLU E &DOFXODWWKH QXPHULFDOYDOXHRI ' $/5 IRUDLULQXQLWVRI f& NP
- & ,Q³VRIW´ OTXLGV ORZ EXON PRGXOXV LWP D\ EH QHFHVDU WR DFFRXQWRU OTXL FRPSUHVVLELOLW\LQK\GURVWDWF FDOFXODWRQV $QDSSUR[LPDWHGQVWUHDWRQZRXGEH-

$$dp \approx \frac{\beta}{\rho}\, d\rho = a^2\, d\rho \qquad \text{or} \qquad p \approx p_0 + a^2(\rho - \rho_0)$$

ZKHUHD LV WKH VSHHGRI VRXQGDQG S ! DUH WKH FRQGLWLRQV DWWKH OTXLG VXUIDFH] 8VH WKLV DSSUR[LPDWRQ WR VKRZ WKDWWKH GHQVLW YDULDWRQ ZLWK GHSWK LQ D VRIW OTXLG LV $\rho = \rho_0 e^{-gz/a^2}$ ZKHUHJ LV WKH DFFHOHUDWRQRI JUDYLW DQG] LV SRVLWLYH XSZ DUG 7KHQ FRQVLGHU YHUWFDOZ DOOR ZLGWKE H[WHQGLQJ IURP WKH VXUIDFH] GRZQWRG=SWK] íK) LQGD DQDOWF H[SUHVVLRQ IRU WKH K\GURVWDWF IRUFH) RQ WKLV ZDOO DQG FRPSDUH LWZ LWK WK LQFRPSUHVVLEOH UHVXOW ! JK E : RXOG WKH FHQWU RI SUHVVXUH EH EHORZ WK LQFRPSUHVVLEOH SRVLWLRQ] íK "
- & 9HQFH ,WDO LV VORZ O\ VLQNLQJ VR QRZ HVSHFLDOO LQZ LQHU SOD] DV DQG ZDOWZ D\ DUH IORRGHG GXULQJ WKRUP V 7KH SURSRVHG SOXWRQ LV WKH IORDWLQJ OHYHO RI) LJ & : KHQ ILOOHG ZLWK DLU LWULVHV WR EORFN RI WKH VHD 7KH OHYHLV P KJK P ZLGH

DQG P GHHS \$VXPHDXQIRUP GHQVLWR NJ P ZKHQ IORDWLQJ)RUWKH F
VHDOHGJRRQGLIIHUHQFHVKRZQ HVWLPDWHWKHDQJOHDWZKLFKWKHOHYHIORDW

&



- & : KDWLVWKHXQFHUWDLQWLQJSUHVVXUHPHDVXUHPHQWDVDQDQJOHRI
VLGHRI DQDLUSODQHPHDVXUHVDORFDOSUHVVXUHRI N3D ZLWKDQXQFHUWDLQWRI N3D
7KHHXWPDWGODSHUDWWKDWGD\ LV . P ZLWKDQXQFHUWRI . P
(IIHFWLYHVHDOHYHOWPSHUDWXUHLV *f*& ZLWKDQXQFHUWRI *f*& (IIHFWLYHVHD
OHYHOSUHVVXUHLV N3D ZLWKDQXQFHUWRI N3D (VWPDWWKHDLUSODQHI
DQWWXGHDQGLWXQFHUWDLQW
- & 7KH \$/9,1 VXEPHUVLEOH YHKLFOH LQ WKH FKDSWURSHQHU SKRWR KDV D SDVVHQJH
FRPSDUWPHQWZKLFKLVDWWDQXP VSKHUHRI LQVLGHGLPHWU LQDQGWKLFNQHVV
LQ ,I WKHYHKLFOHLV VXEPHUJHG WR DGHSWKRI P LQWKHRFHDQ HWPDW D WK
ZDWUSUHVVXUHRXWLGHWKHVSKHUH E WKHPD[LPXP HDWXF VWUHVV LQWKHVSKHUH LQ
GEI LQ DQG F WKHIDFWRURI VDJHWRI WKHWDQXP DOR\ DXPLQXP YDQDGLXP

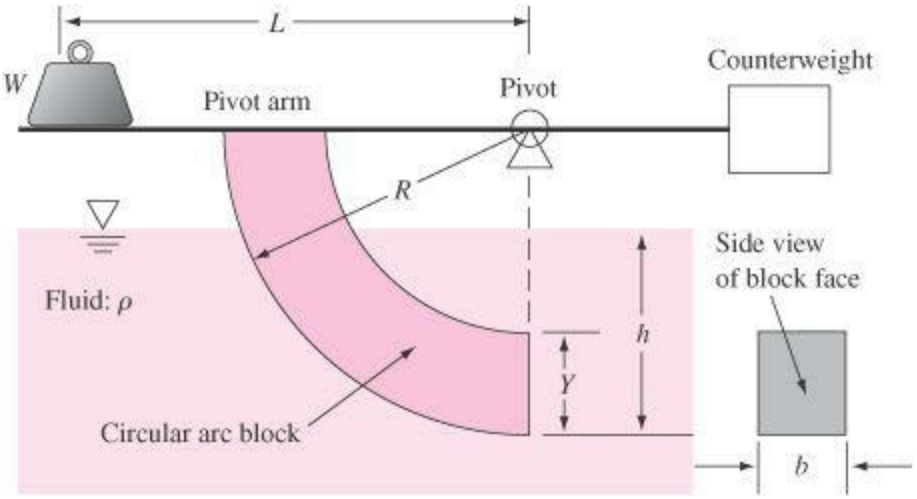
3URMHFW

- ' ,WLVGHVLUHGWRKDYHDERWWRP PRRUHG IORDWQJ VWWP WKDWFUHDWVDQRQDQ-DUFHL
WKHPRRULQJ DQHDVWKHZDWUDHSWKVK EHWZHQ DQG P DV VKRZQ LQ WKHDFFRPSDQL
WDEOH ' HVLJQDEOH\DQWXWWP WKDWZLOOSURYLGHWKLVIRUFHGLVWULEXWRQ 7KHVWWP
VKRXOGEHSUDFWPDO RI LQ[SHQVLYHPDWULDOVDQGVLPSOHFRQWUFWRQ

h, m	F, N	h, m	F, N
6.00	400	7.25	554
6.25	437	7.50	573
6.50	471	7.75	589
6.75	502	8.00	600
7.00	530		

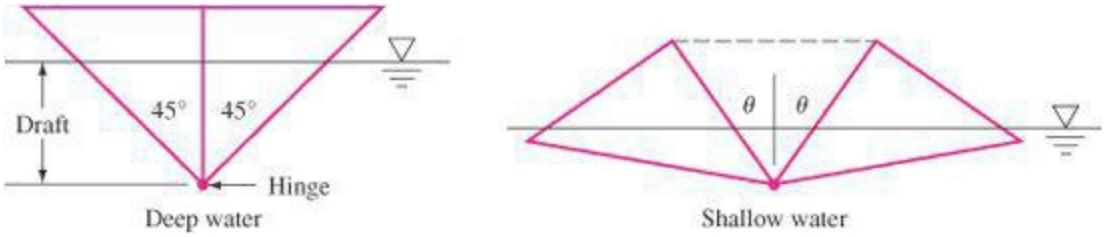
- ' $ ODERUDWRU DSSDUDWXV XVHG LQ VRPH XQLYHUVLWLHV LV VKRZQ LQ) LJ ' 7KH SXUSRV LV WR PHDVXUH WKH K\GURVWDWLF IRUFH RQ WKH IODW DFH RI WKH FLUFXODU DUF EORFN DQ FRPSDUH LWV YDOXH WKHRUHWLFDO YDOXH IRUJLYHQ GHSWK 7KH FRQQHFWHG YHVVHOV DUUDQJH VR WKDW WKH SLYRWHG DUP LV KRUL]RQWDO ZKHQ WKH EORFN LV QRW XEPHUJHG ZKHQ WKH ZHJK : FDQ EH FRUUHFWHG ZLWK WKH K\GURVWDWLF IRUFH ZKHQ WKH VXEPHUJHG DUP LV DJDL EURXJKW WR KRUL]RQWDO) LWWLQJ WKH DSSDUDWXV FRQVLVWV RI D OLQHDU SLQFLSLOH WKH GHULYHD IRUP XOD IRU: DV D IXQFWLRQ RI K LQ WHUP V RI WKH VXVWHP SDUDPHWHUV) LQDOO VXJJHVW WKDW PHDVXUHPHQWV RI < / DQG VR RQ IRUD VXLWDEOH DSSDUDWXV DQG SOR WKHRUHWLFDO YHUVXV K IRU WKHVH YDOXHV

'



- ' 7KH / HDV (QJLQHHULQJ &RPSDQ VH3RSXODU6FLHQFH 1RYHPEHU S KD SURSRVHG D VKLS KXOO ZLWK KLQJHV WKDW DOORZ LWWR RSHQ LQR D IODWWU VKDSH ZKH HQWHULQJ VKDOORZ ZDWHU $ VLPSOLILHG YHUVLRQ VKRZQ) LJ ' ,QGHHS ZDWHU WK KXOO FURVV VHFWLRQ ZRXOG EH WUDQVIHUUHG WR WKH JUDW, QV KDOORZ ZDWHU WKH KLQJH ZRXOG RSHQ DQDORJRXV ZLWK WKH EHKDYLRU RI WKH GUDIW, QFUHDVH ZDWHU GHSWK DV f 7KH GUDIW GQHLQG LFDWHV WKDW WKH HRZ DQ VWUXFWXUH ZRXOG EH FRVHG 0DNH D SDUDPHWULF VWXG RI WKLV FRQFLJXUDWRQ IRU YDULRXV DVXPLQJ DUHDVRQDEOH ZHJKWDQG FHQWURLG JUDYLWDWLRQ 6KRZ KRZ WKH GUDWWK PHDVXHPHQW FKHJKWDQG WKH VKLS¶V VWDELOLW DV WKH KLQJHV DUH RSHQHG &RPPHQWR WKH HIHFWLYHQHVV RI WKLV FRQFHSW

'



5 HIHUHQFHV

8 6 6WDQGDUG\$VWPRVSKHUH * RYHUQPHQW3ULQWLQJ2IILFH: DVKLQJWRQ' &
- \$. QDXVV,QWURGXFWLRQ WR 3K\VLFDO2FHDQRJUDSK\ QG HG : DYHDQG 3UHVV / RQ
* URYH ,/
(& 7XSSHU,QWURGXFWLRQWR1DYDO\$UFKLWHFWXUH WKHG (OVHYLHU1HZ <RUN
' 7 * UHHQZRRG \$GYDQFHG' \QDPLFV &DPEULGJH8QLYHUVLW3UHVV1HZ <RUN
5 ,)OHWFKHU³7KH\$SSDUHQW)LHOG RI * UDYLW LQD 5RWDWLQJ)OXLG 6\VWHP´\$P - 3K\V
YRO -XO SS ±
1DWRQDO &RPPLWWHH IRU)OXLG 0HFKDQLFV)LOPV,QWURGXFWLRQ ([SHULPHQWV LQ)OXL
0HFKDQLFV 0 ,7 3UHVV &DPEULGJH 0\$
- 3 +RPDQ ([SHULPHQWDO0 HWKRGV IRU (QJLQHHUV WKHG 0F* UDZ +LOO1HZ <RU

5 3 %HQGLFW)XQGDPHQWDOVRI 7HPSHUDWXUH 3UHVXUH DQG)ORZ 0HDVXUHPHQW GHG
: LOH\ 1HZ <RUN
7 * %HFNZLWKDQG5 * 0 DUDQJRQL0 HFKDQLFDO0 HDVXUHPHQW WKHG \$GGLVRQ: HVOH\
5HDGLQJ 0\$
- : ' DOO :) 5LOH\ DQG . * 0 F&RQQHOO,QWURGXFWRU IRU (QJLQHHULQ
0 HDVXUHPHQW GHG : LOH\ 1HZ <RUN
(1 * LOEHUW³+RZ 7KLQJV)ORDW \$P 0 DWK 0 RQWKO YRO QR SS ±
5 -)LJORD DQG' (%HDVOH\ 7KHRU DQG' HVLJQ IRU0 HFKDQLFDO0 HDVXUHPHQW V
HG : LOH\ 1HZ <RUN
5 : 0 LOEHU)ORZ 0 HDVXUHPHQW(QJLQHHULQJ +DQGERRN GHG 0F* UDZ +LOO1H
<RUN
/ ' &OD\WRQ (3 (HU1LWH 5 : : DUG DQG5 % : LJJLQV³0 LQLDWXUH &UVWDOO
4 XDUW](OHFWURPHFKDQLFDO6WUXFWXUHV´6HQVRUVDQG\$FWXDWRUV YRO 1RY SS :

\$. LWDL HG / XPLQHVFHQW0 DWHULDOVDQG\$SSOLFDWLRQV-RKQ: LOH\ 1HZ <RUN
% * / LSWDN HG ,QVWUXPHQW(QJLQHHUV¶+DQGERRN 3URFHVV0 HDVXUHPHQWDQG\$QDO\V
VWHG YRO &5& 3UHVV %RFD5DWRQ)/
' 5 * LOOXP ,QGXVWULDO3UHVXUH / HYHODQG' HQVLW0 HDVXUHPHQW,QVWUXPHQW6RFLHWR
\$PHULFD 5HVHDUFK7ULDQJOH3DUN 1&

– \$QLQWURGXFWLRQDSSOLFDWLRQIRUDGUJHHOHPHQWVVHHQLQ) LJ

&KDSWHU
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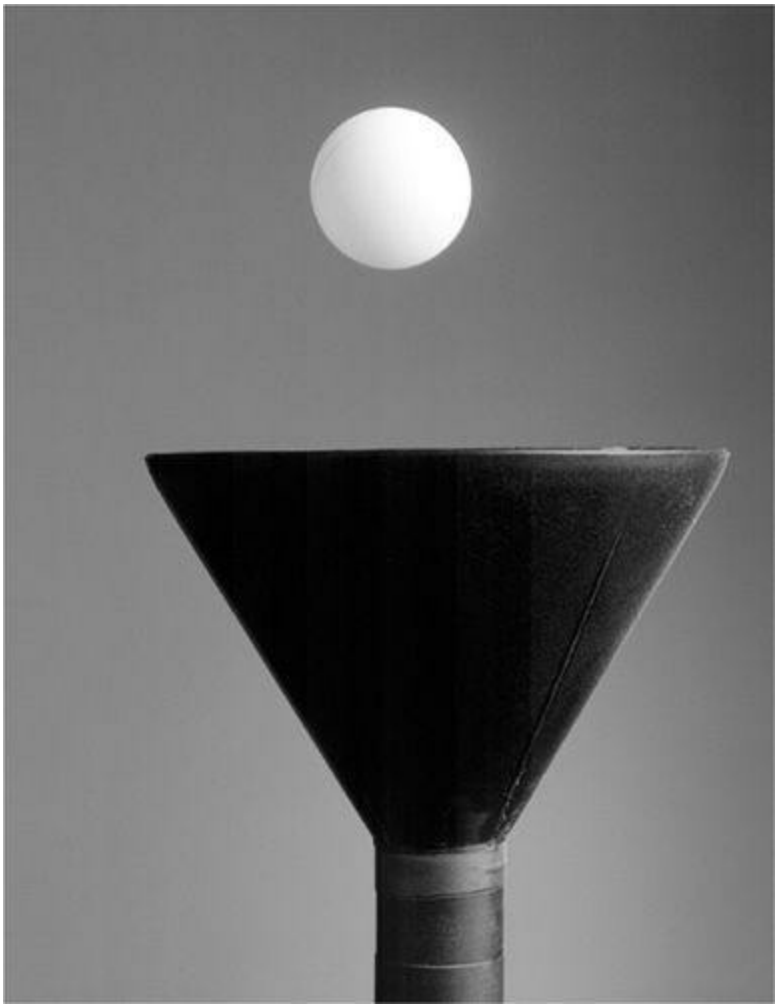

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EDOODQGWKHIRUFHLVWKHEDQGVZHLJKW*Courtesy of Paul Silverman/Fundamental Photographs.)*

0 RWLYDWLRQ

,QDQDO]LQJ IOXLG PRWLRQ ZHPLJKWDNHRQHRI WZR SDWKV VHHNLQJ WR GHVFULEHWKHGHWDLOH
IORZ SDWWHUQDVWHYHU SRLQW[\ ] LQWKHILHOGRU ZRUNLQJ ZLWKDILQLWHUHJLRQ PDNLQJ DEDOOFH
RI IORZ LQYHUVXVIORZ RXWDQGGHWHUPLQLQJ JURVVIORZ HIHFWVXFK DVWKHIRUFHRUWRUTXHRQDERG
RUWKHVRDOHQHJ\ H[FKDQJH 7KHVHFRQGLVWKH³FRQWROYROXPH PHWKRGDQGLVWKHVXEMHFWRI WKL
FKDSWHU7KHILUVWLVWKH³GLIIHUHQWLDO DSSURDFKDQGLVGHYHORSHGLQ&KDS
: HILUVWGHYHORS WKHFRQFHSWRI WKHFRQWROYROXPH LQQHDU WKHVDPHPDQQHUDVRQHGRHVLQ
WKHUPRG\QDPLFVFRXUVH DQGZHILQGWKHUDWHRI FKDQJHRI DQDUEWUDU\ JURVVIOXLG SURSHUW DUHVX
FDOOHG WKH5HQROGVWDQSRUWWKHRUHP : HWKHQ DSSO\ WKLVWKHRUHP LQVHTXHQFH WR PDV CQH
PRPHQWXP DQJXODUPRPHQWXP DQG HQHUJ\ WKXVGHULYLQJ WKHIRXUEDVLF FRQWROYROXPHUHDWRQV

IOXLG PHFKDQLFV 7KH UHDUHPDQ DSSOLFDWLRQV RI FRXUVH 7KH FKDSWHU LQFOXGHV D VSHFLDO FDVH F
IUDFWLRQDWV VKDWWHULQJ WRXJKHQLQJ DQG HQHUJ\ %HQRXOOL HTXDWLRQ 7KH %HQRXOOL HTXDWL
LV D ZRQGHUIXO KLVWRULF UHDGLQJ EXWLWV H[WHPH UHVWULFWLRQV DQG VKRXOG DOZD\V EH YLHZHG ZL
VNHSWLFLVP DQG FDUHIXO DSSOLQJ LWWR DUHDO YLVFRXV IOXLG PRWLRQ

%DVLF 3K\ VLFDO DZ VRI) OXLG 0 HFKDQLFV

,WV WKHQRZ WRUHDO JHWWHUR XV DERXWORZ SUREOHPV 7KH IOXLG VWDWFV DSSOLFDWLRQV RI &KDS
ZHUH PRUH OLNH IXQWKDQ ZRUN DVVDLWLQJ WKLV ZULWHUV VSLQRQ 6WDWFV SUREOHPV EDVLFDOO UHTXLU
RQO WKH GHQVLWV RI WKH IOXLG DQG NQRZOHGJH RI WKH SRVLWLRQ RI WKH IUHH VXUIDFH EXW PRWWOHZ SUREOP
UHTXLUH WKH DQDOVLV RI DQ DUELWUDU VWDWH RI YDULDEOH IOXLG PRWLRQ GHILQHG E\ WKH JHRPHWU WK
ERXQGDU FRQGLWLRQV DQG WKH OZ VRI PHFKDQLFV 7KLV FKDSWUDQG WKH QH[W WZR RXWOLQH WKH WKUHHEDVL
DSSURDFKHV WR WKH DQDOVLV RI DUELWUDU IOXZ SUREOPV

&RQWROYROXP H RUGDUJH VFDOH DQDOVLV &KDS
' LIIHUHQWDO RUVP DOOVFDOH DQDOVLV &KDS
([SHULP HQWDO RUGLP HQVLRQDO DQDOVLV &KDS

7KH WKUHH DSSURDFKHV DUH URXJKO HTXDOLQLP SRUWDQFH &RQWROYROXP HDQDOVLV WKH SUHVHQWWRSL
LV DFFXUDWH IRU DQ IOXZ GLVWULEXWRQ EXWLV RI WKQEDVHG RQ DYHUDJH RU³RQH GLP HQVLRQDÓ SURSHUW
YDOXHV DWWKH ERXQGDULHV ,WDZ D VJLYHV XVHXO³HQJLQHULQ´ HWXP DWV ,QSULQFLSOH WKH GLIIHUHQW
HTXDWRQ DSSURDFK RI &KDS FDQ EH DSSOHG WR DQ SUREOP 2QO DIHZ SUREOPV VXFK DV VWDDJK
SLSH IOZ \LHOG WR H[DFWDQDOWFDO VROXWRQV %XW WKH GLIIHUHQWDO HTXDWRQV FDQ EH PRGHOH
QXPHULFDOO DQG WKH IOXLGVKLQJ ILHOG RI FRP SXWDWRQDOO IOXLG G\QDPLFV &)' > @FDQ QRZ EH XVHG V
JLYH JRRG HVWLP DWV IRU DQ PRVWDQ JHRPHWU) LQDOO WKH GLPHQVLRQDOLQDOVLV RI &KDS DSSOHVV
DQ SUREOP ZKHWKHU DQDOWFDO QXPHULFDO RU H[SHULP HQWDO ,WV SDUWFXODU XVHXOWR UHGXFH WK
FRVWRI H[SHULP HQWDWRQ ' LIIHUHQWDOLQDOVLV RI K\GURG\QDPLFV EHJDQZLWK (XOHUDQG G[$OPEHUW
WKH OZ HTXHJKWHQWK FHQWXU / RUG 5D\OHJK DQG (%XFNLQJKDP SURPHUHG GLPHQVLRQDOLQDOVLV DWWK
HQG RI WKH QLQHWHQWK FHQWXU 7KH FRQWROYROXP HZ DV GHVFUEHG LQ ZRUGV RQ DQ DGKRF RQ FDV
EDVLV E\ ' DQLHO %HUQRXOOL LQ / XGZLJ 3UDQGWO WKH FHOEUDWHG IRXQGHU RI PRGHUQ IOX
PHFKDQLFV) LJ GHYHORSHG WKH FRQWROYROXP HD D VWUDP DWF WRROLQ WKH HDUO V 7K
ZULWHUV WKDFKHUV DW0 , 7 LQWURGXFHG FRQWRO YROXP HQDOVLV LQWR $PHULFDQ WH[WERRNV IR
WKHUPRG\QDPLFVE\ . HQGDOLQ > @DQG IRUIOXLGVE\ +XQVDNHUDG5LJKWWLHQ > @) F
DFRPSOHWHKLWRU RI WKH FRQWROYROXP HVH9LQFHQWV> @

6\ VWHP VYHUVXV &RQWURO9 ROXP HV

$ OWKH OZ VRI PHFKDQLFV DUH ZULWWQ IRUD LX VWHP ZKLFK LV GHILQHG DV DQ DUELWUDU TXDQWLWV
PDWVRI IL[HG LGHQWLW (YHU WKLQJ H[WHUQDO WR WKLV VXVWHP LV GHRWHG E\ WKH WHUP VXUURXQGLQJV DQG WK
VXVWHP LV VHSDUDWHG IURP LWV VXUURXQGLQJV E\ LWVERXQGDULHV 7KH OZ VRI PHFKDQLFV WKHQ VWDWH KD
KDSSHQV ZKHQ WKH UHLVDQLQWHUDFWLRQ EHWZHHQ WKH VXVWHP DQG LWV VXUURXQGLQJV

) LUVW WKH VXVWHP LV D IL[HG TXDQWLW RI PDW GHRWHG E\ P 7KXV WKH PDV RI WKH VXVWHP L
FRQVHUYHG DQG GRHV QRW FKDQJH 7KLV LV D OZ RI PHFKDQLFV DQG KDV D YHU VLPSOH PDWHPDWL
IRUP FDOOHG FRQVHUYDWLRQ RI PDW

RU

$$\frac{dm}{dt} = 0 \qquad (3.1)$$

7KLVLVVR REYLRXVLQVRODPHFKDQLFVSURECOPVWKDWZHRIWQIRUJHWDERXWW,QIXLGPHFKDQLFV ZHPXWWSD\ DORWRI DWWHQWLRQWRPDVVFRQVHUYDWLRQ DQGLWDNHVDOOWKHDQDOVLVWRPDNHDQDO VLVWKHPDWLQWKHV\VWHP ZLOOEHJLQWRDFFHOHUDW+

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{V}}{dt} = \frac{d}{dt}(m\mathbf{V}) \qquad (3.2)$$

,Q(T ZHVDZ WKLVUHODWLRQDSSOLHGWRDGLIIHUHQWDOHPHQWRI YLVFRXVLQFRPSUHVVLEOHIXLG ,QIXLGPHFKDQLFV1HZWRQ¶V VHFRQGODZ LVFDOOHG WKHOLQHDUPRPHQWXP UHODWLRQ1RWWKDWWLVW YHFWRUODZ WKDWPSOLHVWKHWKUHHVFDODUHTXDWLRQV)[PD[ )\ PD\ DQG)] PD]

7KLUG LWKHVXUURXQGLQJVH[HUWDQHWPRPHQW0 DERXWWKHFHQWURI PDVVRI WKHV\VWHP WKHUHZLO EHDURWDWLRQHIIHFW

$$\mathbf{M} = \frac{d\mathbf{H}}{dt} \qquad (3.3)$$

ZKHUH+ UÎ 9 /P LVWKHDQJXODUPRPHQWXP RI WKHV\VWHP DERXWWVFHQWURI PDVV + HUZI FDQ(T WKHDQJXODUPRPHQWXP UHODWLRQ1RWWKDWWLVW DOVR DYHFWRUHTXDWLRQ LQSOLQJ WKUH VFDODUHTXDWLRQVVXFKDV0[G+[GW

)RUDQ DUELWUDU PDVV DQG DUELWUDU PRPHQW+ LV TXLWHFRPSOLFDWHG DQG FRQWUDU QLQH WHUP VHH IRUH[DPSOH 5H ,QHOPHQWDU GQDPLFV ZHFRPPRQO WUHDWRQO D ULJLG ERG URWDWQ DERXWDIL[HG[D[LV IRUZKLFK(T UHGXFHVWR

$$M_x = I_x\frac{d}{dt}(\omega_x) \qquad (3.4)$$

ZKHUH&[LVWKHDQJXODUYHORFLWRI WKHERG DQG,[LVLWPDVPRPHQWRI LQHUWLD DERXWWKH[D[LV 8QRUWXQDWO IOXLG\V\WHPVDUHQRWULJLG DQGUDUH UHGXFHWRVXFK DVLPSOHUHDWLQ DVZHVKDOOVH LQ6HF

)RXUWK LI KHDW/4 LVDGGHGWRWKHV\VWHP RUZRUN/: LVGRQHE\ WKHV\VWHP WKHV\VWHP HQHUJ\ G PXVWFKDQJHDFFRUGLQJWRWKHHQHUJ\ UHDWLRQ RUILUVWODZ RI WKHUPRG\QDPLFV

$$\delta Q - \delta W = dE$$

or

$$\dot{Q} - \dot{W} = \frac{dE}{dt} \qquad (3.5)$$

/LNHPDVFRQVHUYDWLRQ(T WKLVLVDVFDODUUHDWLRQKDYLQJ RQO DVLQJHFRPSRQHQW)LQDOO WKH VHFRQG ODZ RI WKHUPRG\QDPLFV UHDWV HQWURS\ FKDQJHG6 WR KHDWDGGHGG4 DQ

DEVROXWHWHPSHUDWXUH7

$$dS \geq \frac{\delta Q}{T} \qquad (3.6)$$

7KLV LV YDOLG IRU D V\VWHP DQG FDQ EH ZULWWHQ LQ FRQYHQWLRQDO IRUP EXW WKH UHDO PRVW
SUDFWLFDO DSSOLFDWLRQV LQIOLGPHFKDQLFV [FHSWRDQDO]HIORZ GRVGHWLV VHH6HF

$OWKHVHOZ VLQYROYHWKHUPRG\QDPLF SURSHUWLHV DQG WKXV ZHPXWXSSOHPHQWWKHP ZLWK WDU
UHDWRQ/S S! 7 DQGH H! 7 IRUWKH SDUWFXODUIOLG EHLQJ VWXGLHG DV LQ6HF $OWKRXJI
WKHUPRG\QDPLFV LV QRWWKH PDLQ WRSLF RI WKLV ERRN LWLV YHU LPSRUDQWWR WKH JHQHUDOWXG\ RI IOX
PHFKDQLFV 7KHUPRG\QDPLFV LV FUXFLDOWR FRPSUHVLEOHIORZ &KDS 7KHVXEHQWWKRXGUHYLHZ WK
ILUVWODZ DQG WKHVXDMUHDWRQ/ DVGLVFXVHGLQ5HV DQG
7KH SXUSRVH RI WKLV FKDSWHU LV WR SXWRXU EDVLF ODZ V LQWR WKH FRQWROYROXPH IRUP VXLWDEO
IRUDUELWUDU\ UHJLRQ/LQDIORZ

&RQVHUYDWLRQ RI PDVV 6HF
7KHOQHUPRPHQWXP UHDWRQ 6HF
7KHDQJXODUPRPHQWXP UHDWRQ 6HF
7KHHQHUJ\ HTXDWLRQ 6HF

: KHUHYHU QHFHVVDU\ WR FRPSOHWH WKH DQDOVLV ZH DOR LQWRGXFH D VXDM UHDWRQ VXFK DV WK
SHUHFWJDV/DZ

(TXDWLRQ/ WR DSSO WR HLWKHU IOXLG RU VROG V\VWHPV 7KH DUH LGHDO IRU VROG
PHFKDQLFV ZKHUH ZH IRFXV V\VWHP IRUHYHU EHFDXVH LWUHSUHVHQWV WKH SURGXFWZ H DU
GHVLJQLQJ DQG EXLOGLQJ)RU[DPSOH ZHIRFXV DEHDP DVLWGHIOHFWXQGHUDG : HIRFXV DSLWWR
DVLWRVFLOODWRV : HIRFXV DURFNHWV\VWHP DOWKHZD\ WR0 DV/

%XWOXLG V\VWHPVGR QRWGHP DQG WKLVFRQFHQWUDWLRQ ,WLVUDUHWKDWZ HZLK WRIRFXV WK
XOWP DWSDWK RI D VPHFLILF SDUWFOHIRI IOXLG ,QXDGLWLV OLNHO WKDWWKHIOXLG IRUPV WKHHQYLURQPHQ
ZKRVHHIFWRQRXUSURGXFWZ HZLK WR NQRZ)RUWKHWKUHH[DPSOHVJXWWFLWG ZHZLK WR NQRZ WK
ZLQGORDGVRQ WKHEDP WKHIXLG SUHVXUHVRQ WKHSLWRW DQG WKHGUDJ DQG OLWGRDGVRQ WKHURFNH
7KLVUHTXLUHVWKDWWKHEDMF ODZ VEHUZ ULWWQ WR DSSO WR D SHFLILF UHJLRQ LQ WKHQHLJKERUKRRG RI RX
SURGXFW,QRWKHUZ RUGV ZKHUHWKHIXLG SDUWFOHVLQWKHZLQGJR DWWUWKH EHDYHWKHEDP LVRI OXW
LQWHUHWWR D EDP GHVLJQHU 7KH XVHU¶V SRLQW YLHZ XQGHUOLHV WKH QHG IRU WKH FRQWROYROXPH
DQDO VLVRI WKLVFKDSWU

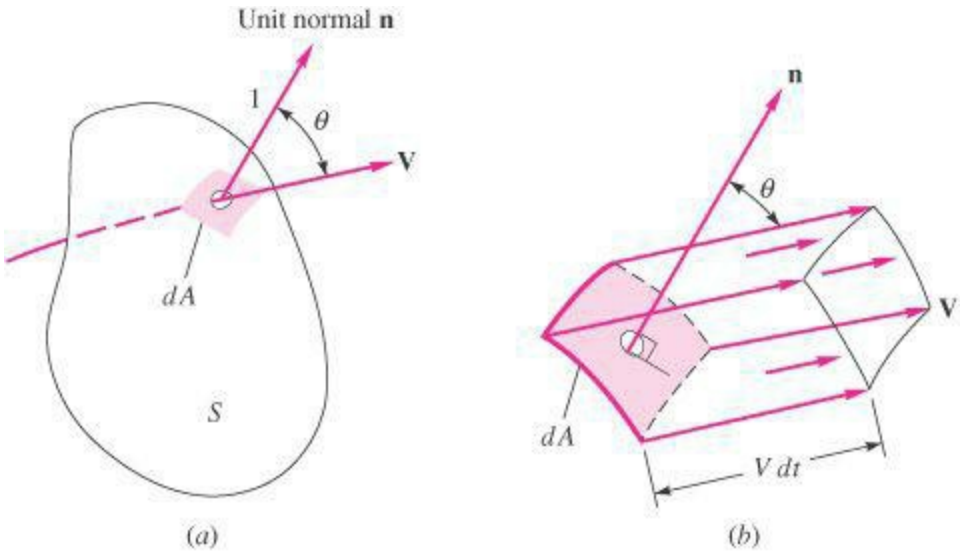
,QDQDO]LQJ D FRQWROYROXPH ZHFRQVHUWKHV\VWHP ODZ V WR DSSO WR D SHFLILF UHJLRQ Z KLF
WKHV\VWHP P D\ RFFXS\ IRURQO DQLQVWDQW7KHV\VWHP SDVVHVRQ DQG RWKHUV\VWHP VFRP HDORQJ EX
QR P DWWU 7KHEDMF ODZ V DUHUHRUP XODWG WR DSSO WR WKLV RHFDOUHJLRQ FDOOHG D FRQWROYROXPH $ C
ZHQHG WR NQRZ LV WKH IORZ ILHOG LQ WKLV UHJLRQ DQG RWQ VLP SOH DVXP SWRQV ZLOOEH DFFXUDW
HQRXJK VXFK DV/XQIRUP LQIOHWDQG RURXOHWWORZ V 7KHIORZ FRQGLWRQV DZ D\ IURP WKHFRQWROYROXPH
DUHWKHQLUHHOYDQW7KHWFKQLTXHIRUP DNLQJ VXFK OREDO]HG DQDO VHV LV WKHVXEMFWRI WKLV FKDSWU

9 ROXP HDQG 0 DV/5 DWHRI) ORZ

$OWKHDQDO VHV LQWKLV FKDSWULQYROYHHYDOXDWLQRI WKHYROXP HIORZ 4 RUP DV/IOW æ SDWLQ
WKURXJK DVXUIDFH LP DJLQDU\ GHLQHGLQWKHIORZ

6XSSRVH WKDW WKH VXUIDFH 6 LQ) LJ DLVD VRUWR LPDJLQDU\ ZLUHPHVK WKURXJK ZKLFK WKH IOXLG SDVVHV ZLWKRXW UHVLVWDQFH +RZ PXFK YROXPH RI IOXLG SDVVHV WKURXJK 6 LQ XQLW WLPH' ,I WKH SLFWXUH 9 YDULHV ZLWK SRVLWLRQ ZH PXVW LQWHJUDWH RYHU WKH HOHPHQWDO VXUIDFHG$ LQ) LJ D \$OR WKH SLFWXUH 9 PD\ SDVV WKURXJK G\$ DW DQ DQJOH RI WKH QRUPDO / HVQ EH GHLQHG DV WKH XQLW YHFWRU QRUPDO WR G\$ 7KHQ WKH DPRXQW RI IOXLG VZHSW WKURXJK G\$ LQ WLPH GW LV WKH YROXPH RI WKH VODQWHG SDUDOOHOHSLSHG LQ) LJ E

$$(V \quad 9\ GW G\$ FRV \quad 9\ ÜQ\ G\$\ GW$$



) LJ 9 ROXPH UDWH RI IORZ WKURXJK DQ DUELWUDU\ VXUIDFH a DQ HOHPHQWDO DUHD dA RQ WKH VXUIDFH b WKH LQFUHPHQWDO YROXPH VZHSW WKURXJK dA HTXDOV $V\ dt\ dA$ FRVθ

7KH LQVWDQWDQHRXV G GW RI WKH VROXPH UDWH RI IORZ 4 WKURXJK WKH VXUIDFH6

$$Q = \int_S (\mathbf{V} \cdot \mathbf{n})\, dA = \int_S V_n\, dA \qquad (3.7)$$

: HFRXOG UHSODFH9 QE\ LWV HTXLYDOHQW9$_Q$ WKH FRPSRQHQW RI 9 QRUPDO WR G\$ EXW WKH XVH RI WK GRWSURGXFW VDOOHV V4 WR KDYH D VLJQ WR GLVWLQJXLVK EHWZHHQ LQIORZ DQG RXWIORZ %\ FRQYHQWLR WKURXJKRXW WKLV ERRN ZH FRQVLGHU Q WR EH WKH RXWZDUG QRUPDO XQLW YHFWRU 7KHUHIRUH9 Q GHQRWHV RXWIORZ LI LWV SRVLWLYH DQG LQIORZ LI QHJDWLYH 7KLV ZLOO EH DQ[WHPH XVHIXO KRXVHNHHSLQJ GHYLFH ZKHQ ZH DUH FRPSXWLQJ YROXPH DQG PDVV IORZ LQWHEDVH FRQWROYROXPHUHDWRV

9RLXPH IORZ FDQ EH PXOWLSOHG E\ GHQVLW\ WR REWDLQ WKH PDVV IORZ æ ,I GHQVLW\ YDULHV RYHU WK VXUIDFH LW P XVWEH SDUW WKH VXUIDFH LQWHJUDO

$$\dot{m} = \int_S \rho(\mathbf{V} \cdot \mathbf{n})\, dA = \int_S \rho V_n\, dA$$

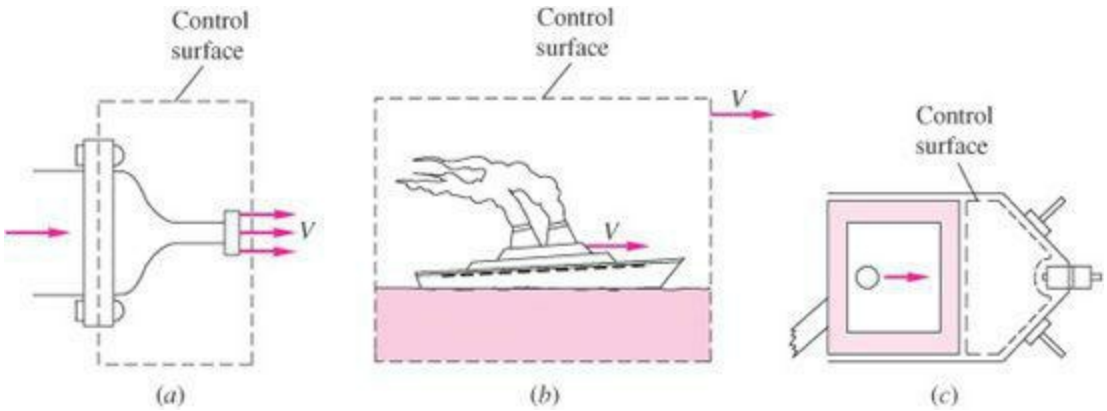
,I GHQVLW\ DQG YHORFLW DUH FRQVWDQW RYHU WKH VXUIDFH6 DVLPSOH[SUHVLRQ UHVXOW

2QH GLPHQVLRQDO SSUR[LPDWLRQ

7KH5 H\ QROGV7 UDQVSRUW7 KHRUHP

7R FRQYHUW D V\VWHP DQDO\VLV WR D FRQWURO YROXPH DQDO\VLV ZH PXVW FRQYHUW RXU DVVHPEODJH RI V\VWHP DSSOLFDWLRQ UDWH WHUPV LQWR V\VWHP UDWH RI FKDQJH WHUPV 7KLV FRQYHUVLRQ FDQ EH DFFRPSOLVKHG XVLQJ WKH5H\QROGV7 UDQVSRUW7 KHRUHP FDQ EH DSSOLHG WR DOO WKH EDVLF ODZV ([DPLQLQJ WKH EDVLF ODZV WR DQ ZH VHH WKDW WKH\ DUH DOO FRQFHUQHG ZLWK WKH WLPH GHULYDWLYH RI IXLG SURSHUWLHV P 9 + DQG(7KHUHRUH ZKDW ZH QHHG LV WR UHODWH WKH WLPH GHULYDWLYH RI D V\VWHP SURSHUW WR WKH UDWH RI FKDQJH RI WKDW SURSHUW ZLWKLQ D FHUWDLQ UHJLRQ

7KH GHVLUHG FRQYHUVLRQ IRUPXOD GLIIHUV VOLJKWO DFFRUGLQJ WR ZKHWKHU WKH FRQWURO YROXPH L[HG PRYLQJ RU GHRUP DEOH) LJXUH LOOXVWUDWHV WKH VHWKUHH FDVHV 7KH IL[HG FRQWURO YROXPH LV) LJ D HQFORVHV D VWDWLRQDU UHJLRQ RI LQWHUHVWWR D QR]]OH GHVLJQHU 7KH FRQWURO VXUIDFH LV DV DEVWDFWRQ HSWDQG GRHV QRWKLQG HU WKH IORZ LQDQ ZD ,WVOHFHV WKURXJK WKH HNHWDDYLQJ WKH QR]]OH HQFORVHV WKH VXUURXQGLQJ DWPRSKHUH DQG VOLFHV WKURXJK WKH IODQJH EROW DQG WKH IXLG ZLWKLQ WKH QR]]OH 7KLV SDUWFXODU FRQWURO YROXPH H[SRVHV WKH VWUXWV LQ WKH IODQJH EROW ZKLFK FRQWULEXWH DSSOLHG IRUFHV LQ WKH P RP HQWXP DQDO\VLV ,Q WKLV VHQVH WKH FRQWURO YROXPH UHVHPEOHV WKH IUHH ERG FRQFHSW ZKLFK LV DSSOLHG WR V\VWHP VLQVRDG P HFKDQLFV DQDO\VHV



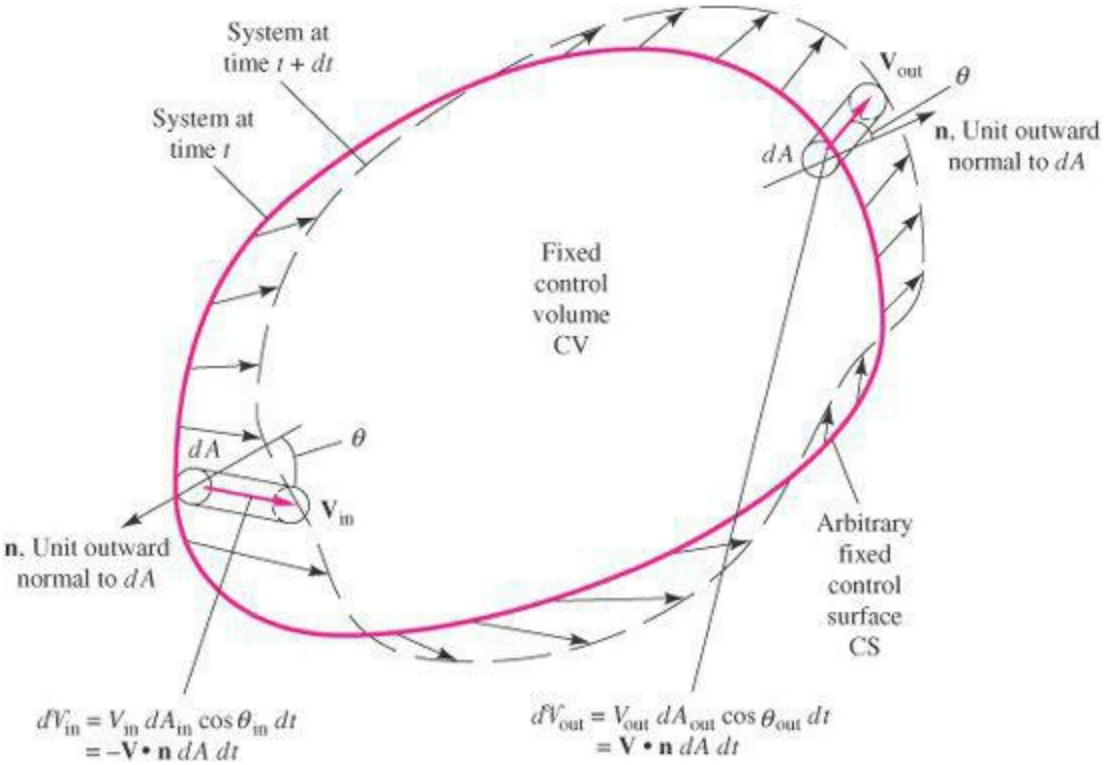
) LJ) L[HG P RYLQJ DQG GH RUP DEOH FRQWURO YROXPHV *a* IL[HG FRQWURO YROXPH IRU QR]]O VWHVV DQDO\VLV *b* FRQWURO YROXPH P RYLQJ DWWKLS VSHHG IRU GUDJ IRUFH DQDO\VLV *c* FRQWURO YROXPH GH RUP LQJ ZLWKLQ A\OLQGHU IRU WDQVLHQWSUHVXUH YDULDWLRQ DQDO\VLV

) LJXUH E LOOXVWUDWHV D P RYLQJ FRQWURO YROXPH + HUH WKH VKLS LV RI LQWHUHVWWR WKH RFHDQ VWKH FRQWURO VXUIDFH FKDVHV WKH VKLS DWKLS VSHHG9 7KH FRQWURO YROXPH LVRI IL[HG YROXPH EXW WKH UHODWLYH P RWLRQ EHWZHHQ ZDWHU DQG VKLS P XVW EH FRQVLGHUHG ,I 9 LV FRQVLGQW WKLV UHODWLYH P RWLRQ LV VWHDG\ IORZ SDWWHUQ ZKLFK VLP SOLILHV WKH DQDO\VLV- ,I 9 LV YDULDEOH WKH UHODWLYH P RWLRQ LV XQVWHDG\ VR WKDW WKH FRP SXWHG UHVXOW DUH WLP H YDULDEOH DQG FHUWDLQ WHUP V HQWHU WKH P RP HQWXP DQDO\VLV WR UHOHFW WKH QRQLQHUWLDO DFFHOHUDWQJ IUDP HR UHHUHQFH

) LJXUH F VKRZV D GH RUP LQJ FRQWURO YROXPH 9DU LQJ UHODWLYH P RWLRQ DW WKH ERXQGDUL EHFRP HV DIFWU DQG WKH UDWH RI FKDQJHR VKDSH RI WKH FRQWURO YROXPH HQW V WHEJLQ E\ GHULYLQJ WKH IL[HG FRQWURO YROXPH FDVH DQG ZH FRQVLGHU WKH RWKHUFDVHV DV DGYDQFHG VRSLFV $ C LQW UHWWQJ KLVWRU RI FRQWURO YROXPH DQDO\VLV LJLYHQE\ 9LQFHQW> @

$ UEL WMDU) L[HG & RQWURO 9 ROXP H

) LJXUH VKRZ VDIL[HG FRQWURO YROXP HZ LWK DQ DUEL WDU IORZ SDWWLQ SDVLQJ WKURXJK 7KH UHDU YDULDEOH VOQYHUV RI LQIORZ DQG RXWIORZ RI IOXLG DOO DERXW WKH FRQWURO VXUIDFH ,Q JHQHUDO HDFK GLIIHUHQWDO DUHDG$ RI VXUIDFH Z LOO KDYH D GLIIHUHQW YHORFLW9 P DNLQJ D GLIIHUHQWDQJOH Z LWK WKH ORFDOQRUP DOWR G$ 6RPH HOHP HQWDO DUHDV Z LOO KDYH LQIORZ YROXP H 9$ FRV LQGW DQG RWKHUV Z LO KDYH RXWIORZ YROXP H 9$ FRV RXWW DV VHHQ LQ) LJ 6RPH VXUIDFHV P LJKW FRUUHVSRQG WR VWUHDP OLQHV f RUVROLG Z DOOV 9 Z LWK QHLWKHU LQIORZ QRU RXWIORZ



) LJ $ QDUEL WDU FRQWURO YROXP HZ LWK DQ DUEL WDU IORZ SDWWLQ

/ HW%EH DQ SURSHUW RI WKH IOXLG HQHUJ\ PRPHQXP HQWKDOS\ HW DQG GHW G%GP EH WKH LQWMQVLYH YDOXH RUWKH DP RXQW RI %SHUXQLW P DVV LQ D P DOOHOHP HQW RI WKH IOXLG 7KH WRWDO DP RXQW %LQ WKH FRQWURO YROXP H VKH VRLG FXUYH LQ) LJ LV WKXV

$$B_{CV} = \int_{CV} \beta\, dm = \int_{CV} \beta \rho\, d\mathcal{V} \qquad \beta = \frac{dB}{dm} \tag{3.8}$$

([DPLQLQJ) LJ Z HVHH WKDW HVRXUFHV RI FKDQJHV LQ%UHDWLQJ WR WKH FRQWROYROXP H

A change within the control volume $\dfrac{d}{dt}\left(\int_{CV} \beta \rho d\mathcal{V}\right)$

Outflow of β from the control volume $\int_{CS} \beta \rho V \cos\theta\, dA_{out}$ (3.9)

Inflow of β to the control volume $\int_{CS} \beta \rho V \cos\theta\, dA_{in}$

7KH QRWDWLRQV &9 DQG &6 UHIHU WR WKH FRQWURO YROXPH DQG FRQWROVXUIDFH UHVSHFWLYHO 1RWH L
) LJ WKDWWKHV\VWHP KDVPRYHGDEWJDLQLQJWKHRXWBOW VOLYHUDQGORVLQJWKHLQBOW VOLYHU,QWK
OLPLWDVGW WKHLQXDQWLWRI %LQWKHV\VWHP LVWKHVXP RI WKHFKDQJHZLWLQ SOXVWK
RXWBOW PLQXVWKHLQBOW

$$\frac{d}{dt}(B_{sys}) = \frac{d}{dt}\left(\int_{CV} \beta\rho\, d\mathcal{V}\right) + \int_{CS} \beta\rho V\cos\theta\, dA_{out} - \int_{CS} \beta\rho V\cos\theta\, dA_{in} \qquad (3.10)$$

7KLV LV WKH5H\QROGV WUDQVSRUWWKHRUHP IRUDQ DUELWUDU\ IL[HG FRQWROYROXPH %\ GHWQJ WK
SURSHUW\%EHPDW PRPHQXP DQJXODUPRPHQXP RUHQHUJ\ ZHFDQUHZULWWDOOWKHEDVLF ODZ VLG
FRQWROYROXPHIRUP 1RWWKDWDOWKURHR WKHLQWUDGV DUHFRQHQGZLWKWKHLQWQVLYHSURSHUW
6LQFHWKHFRQWROYROXPHLVIL[HGLQVSDFH WKHHOPHQWDOYROXPHVG GRQRWYDU ZLWKWLPH VR WKDWWK
WLPHGHULYDWLYHR WKHYROXPHLQWUDOVDQVKHVXQDWHWKU RU! YDUHVZLWKWLPH XQWWDG\ IORZ
(TXDWLRQ H[SUHVVHVWKHEDVLF IRUPXOWDWDVWHP GHULYDWLYHHTXDOVWKHUDWR FKDQJHR
%ZLWKLQWKHFRQWROYROXPHSOXVWKHIOX[R %RXWR WKHFRQWROVXUIDFHPLQXVWKHIOX[R %LQR WK
FRQWROVXUIDFH 7KHTXDQWV% RU PD\ EHDQ YHFWRUVFDODUSURSHUWR WKHIOXLG 7Z RDOWLQDW
IRUPVDUHSRVVLEOHIRUWKHIOX[WHUPV) LUVWZHPD\ QRWHWKDW9 FRV LVWKHFRPSRQHQWR 9 QRUPD
WRWKHDUHDHOPHQWR WKHFRQWROVXUIDFH 7KXVZHFDQZULWH

$$\text{Flux terms} = \int_{CS} \beta\rho V_n\, dA_{out} - \int_{CS} \beta\rho V_n\, dA_{in} = \int_{CS} \beta\, d\dot{m}_{out} - \int_{CS} \beta\, d\dot{m}_{in} \qquad (3.10a)$$

ZKHUHGae S9$_Q$ LVWKHGLIHUHQWLDOPDVIOX[WKURXJKWKHVXUIDFH) RUP D KHSVXVYLVXDOL]I
ZKDWLVEHLQJ FDOFXODWHG

$ VHFRQG DOWLQDWLYHIRUP RIHUVHOHJDQFHDQGFRPSDFWQHVDVDGYDQWDJHV ,I QLVGHILQHGDVWK
RXWZDUGQRUPDOXQLWYHFWRUHYHUZKHUHRQWKHFRQWROVXUIDFH WKHQ9 Q 9$_Q IRURXWBOW DQG9 Q
í 9$_Q IRULQBOW 7KHUHRUHWKHIOX[WHUPVFDQEHUHSUHVHQWHGE\ DVLQJOHILQWUDOLQYROYLQJ 9 QWKD
DFFRXQWVIRUERWKSRVLWLYHRXWBOW DQGQHJDWLYHLQBOW

$$\text{Flux terms} = \int_{CS} \beta\rho(\mathbf{V}\cdot\mathbf{n})\, dA \qquad (3.11)$$

7KHFRPSDFWRUP RI WKH5H\QROGVWUDQVSRUWWKHRUHP LVWKXV

$$\frac{d}{dt}(B_{sys}) = \frac{d}{dt}\left(\int_{CV} \beta\rho\, d\mathcal{V}\right) + \int_{CS} \beta\rho(\mathbf{V}\cdot\mathbf{n})\, dA \qquad (3.12)$$

7KLVLVEHDXWLXOEXWRQO RFFDVLRQDOO XVHXO ZKHQWKHFRRUGLQDWV\VWHP LVIHDOO VXLWHGWR WK
FRQWROYROXPHVHOHFWHG 2WKHUZLVHWKHFRPSXWDWQV DUHDVLHUZKHQWKHIOX[R %RXWV DGGHG DQ
WKHIOX[R %LQLVVXEWUDFWHG DFFRUGLQJ WR RU

7KHWLPHGHULYDWLYHWUDQVIHUVWRTXLWKLQWKHTXLYDOHQWRUP

$$\frac{d}{dt}\left(\int_{CV}\beta\rho\,d\mathcal{V}\right)=\int_{CV}\frac{\partial}{\partial t}(\beta\rho)\,d\mathcal{V} \qquad (3.13)$$

IRUWKHILHGFRQWROYROXPHVLQFHWKHYROXPHHOHPHQWVGRQRWYDU\

&RQWURO9RXPH0RYLQJ DW&RQVWDQW9HORFLW\

,I WKHFRQWROYROXPHLVPRYLQJ XQLIRUP0DWYHORFLW9 $_V$ DVLQ) LJ E DQREVHUYHUILHGWRWKH FRQWROYROXPHZLOOVHHDUHODWLYHYHORFLW9$_r$ RI IOXLGFURVVLQJ WKHFRQWROVXUIDFH GHILQHGE\

$$\mathbf{V}_r=\mathbf{V}-\mathbf{V}_s \qquad (3.14)$$

ZKHUH9 LVWKHIOXLGYHORFLWUHODWLYHWRWKHVDPHFRRUGLQDWHVXVHGLQZKLFKWKHFRQWROYROXPH PRWLRQ9$_V$ LV REVHUYHG 1RWH WKDW(T LV D YHFWRU VXEWUDFWLRQ 7KH IOX[WHUPV ZLOO E SURSRUWLRQDOWR9$_U$ EXWWKH YROXPH LQWHJUDORI(T LV XQFKDQJHG EHFDXVHWKHFRQWROYROXPH PRYHV DV D IL[HG VKDSH ZLWKRXWGHIRUPLQJ 7KH 5HQROGV WDQVSRUWWKHRUHP IRUWKLV FDVH RI XQLIRUP0 PRYLQJ FRQWROYROXPHLV

$$\frac{d}{dt}(B_{sys})=\frac{d}{dt}\left(\int_{CV}\beta\rho\,d\mathcal{V}\right)+\int_{CS}\beta\rho(\mathbf{V}_r\cdot\mathbf{n})\,dA \qquad (3.15)$$

ZKLFKUHGXFHVWR(T LI 9$_V$A

&RQWURO9RXPH RI &RQVWDQW6KDSHEXW9DULDEOH9HORFLW\

,I WKHFRQWROYROXPHPRYHVZLWKDYHORFLW9 $_V$ WWKDWVDWLVILHVWKHQWKHYROXPHHOHPHQW GRQRWFKDQJHZLWKWLPH EXWWKHERXQGDU UHODWLYHYHORFLW9$_U$ 9 UWí 9$_V$ WEHFRPHVDVRPHZKD PRUHFRPSOLFDWHGIXQFWLRQ (TXDWLRQ LVXQFKDQJHGLQIRUP EXWWKHDUHDLQWHJUDOPD\ EHPRU ODERURXVWRHYDOXDWH

$UELWUDU0RYLQJ DQG'HIRUPDEOH&RQWURO9RXPH

7KHPRVWJHQHUDOVLWXDWLRQLVZKHQWKHFRQWROYROXPHLVERWKPRYLQJ DQGGHIRUPLQJ DUELWUDULO DVLOOXVWUDWHGLQ) LJ 7KHIOX[RI YROXPHDFURVVWKHFRQWROVXUIDFHLVDJDLQSURSRUWLRQDOWR WKHUHODWLYHYHORFLW9$_r$ Q DVLQ(T +RZHYHUVLQFHWKHFRQWROVXUIDFHKDVI GHIRUPDWLRQLWVYHORFLW9$_V$ 9 UWí 9$_V$ UWLVRUFDQEHI FRPSOLFDWHGIXQFWLRQ HYHQWKRXJK WKHIOX[LQWHJUDOLVWKHVDPHDVLQ(T 0HDQZKLOH WK YROXPHLQWHJUDOLQ(T PXVWDOZD\ WKHYROXPHHOHPHQWVRGLVRUWZLWKWLPH 7KXVWKHWLP

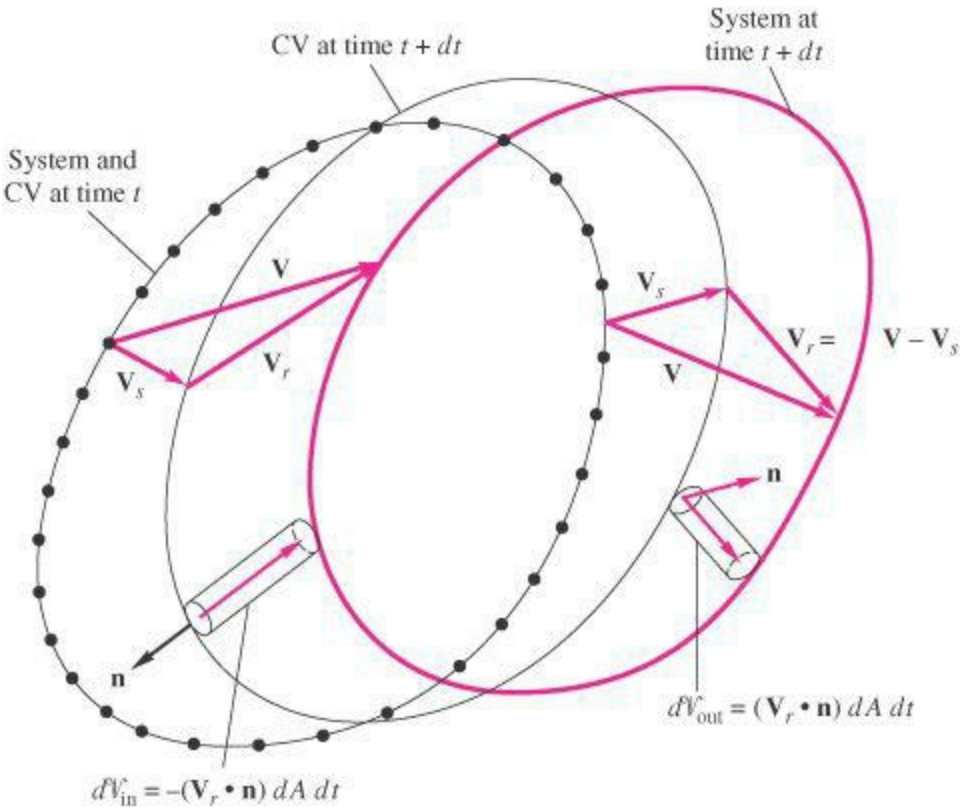
GHULYDWLYH PXVWEH DSSOLHG DIWHU LQWHJUDWLQJ) RU WKH GHRUPLQJ FRQWURO YROXPH WKH WDQVSRU
WKHRUHP WDNHV WKH IRUP

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{CV} \beta\rho\, d\mathcal{V}\right) + \int_{CS} \beta\rho(\mathbf{V}_r \cdot \mathbf{n})\, dA \qquad (3.16)$$

7KLV LV WKH PRVW JHQHUDO FDVH ZKLFK ZH FDQ FRPSDUH ZLWK WKH HTXLYDOHQW RUP IRU D IL[HG FRQWU
YROXPH

$$\frac{d}{dt}(B_{syst}) = \int_{CV} \frac{\partial}{\partial t}(\beta\rho)\, d\mathcal{V} + \int_{CS} \beta\rho(\mathbf{V} \cdot \mathbf{n})\, dA \qquad (3.17)$$

7KH PRYLQJ DQG GHRUPLQJ FRQWURO YROXPH (T FRQWDLQV RQO VWR FRP SOHWRQV 7K
WP H GHULYDWLYH RI WKH ILUVWLQWUDORQ WKH ULJKWPXWEH WDNHQ RXWLGH DQG WKH VHFRQG LQWUDO
LQYROYHV WKH UHDWH YHORFLW9, EHWZHQ WKH IOXLG V\VWHP DQG WKH FRQWROVXUDFH 7KHVH GLIIHUHQF
DQG P DWKHP DWFDOXEWLHV DUH EHWWKRZ QE\ H[DP SOHV



) LJ 5 HDWLYH YHORFLW HIHFWV EHWZHHQ D V\VWHP DQG D FRQWROYROXP H ZKHQ ERWK P RY
DQG GHRUP 7KH V\VWHP ERXQGDULHV P RYH DWYHORFLW 9 DQG WKH FRQWROVXUDFH P RYHV D
YHORFLW 9s

2 QH' LP HQVLRQDO) OX[7HUP \$SSUR[LP DWLRQV

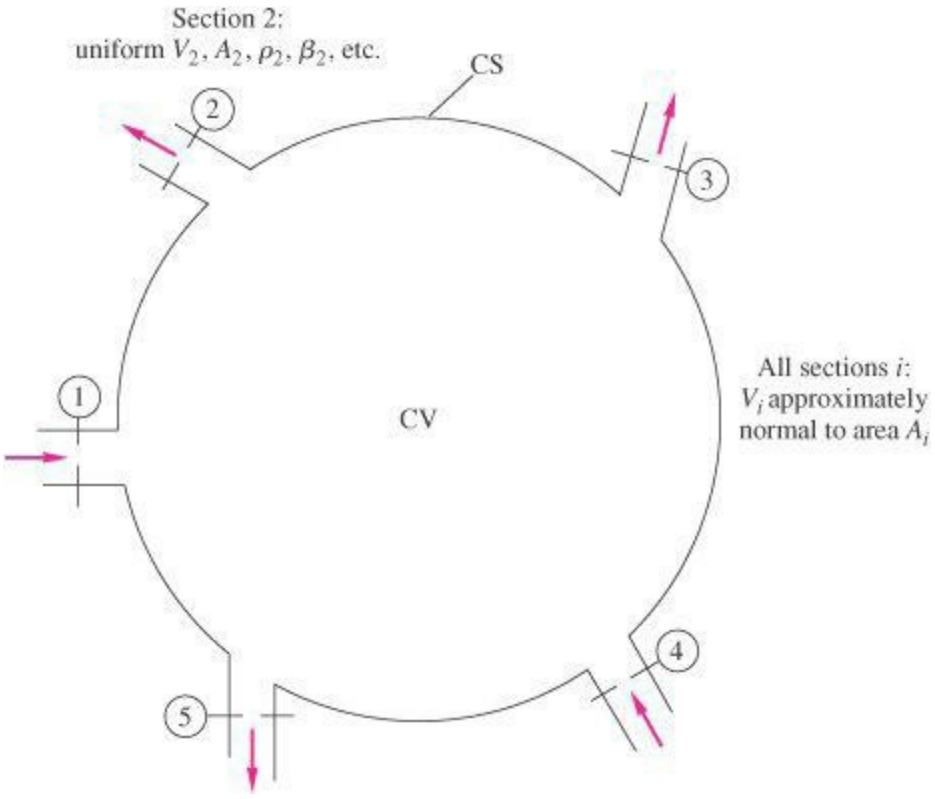
,QP DQ VLXDWRQ WKH IORZ FURVVHV WKH ERXQGDULHV RI WKH FRQWURO VXUIDFH DW LPSRUWDQW LQGH[LWWKDWDWHDSSURSULDWH RQH GLPHQVLRQDO WKDWV IORZ SURSHUWLHV DUH QHDUO XQLIRUP RYHUWKI FURVV VHFWLRQ) RU D IL[HG FRQWROYROXPH WKH VXUIDFH LQWHUVHFWV(T UHGXFHV WR D VXP R SRVLWLYH RXWWDQG QHJDWLYH LQWWSURGXFWWUP VIRUHDFK FURVV VHFWLRQ

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{CV} \beta\, dm\right) + \sum_{outlets} \beta_i \dot{m}_i\,|_{out} - \sum_{inlets} \beta_i \dot{m}_i\,|_{in} \quad \text{where } \dot{m}_i = \rho_i A_i V_i \qquad (3.18)$$

7R WKH ZULWU WKLV LV DQ DWWDFWLYH ZD WR VHWXS D FRQWROYROXPH DQDOVLV ZLWRXWXVLQJ WKH GR SURGXFWQRWDWRQ $QH[DPSOHR PXOWSOHRQHGLPHQVLRQDOIORZ VLWXDWLRQ LV VKRZQLQ) LJ 7KHUHDUHLQG IORZ VDWWHFWLRQV DQG DQGRXWIORZ VDWWHFWLRQV DQG (TXDWLRQ EHFRPHV

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{CV} \beta\, dm\right) + \beta_2(\rho AV)_2 + \beta_3(\rho AV)_3 + \beta_5(\rho AV)_5 - \beta_1(\rho AV)_1 - \beta_4(\rho AV)_4 \qquad (3.19)$$

ZLWKQR FRQWEXWRQIURP DQ RWKHUSRUWRQR WKH FRQWROVXUIDFHEHFDXVHWKHUHLVQR IORZ DFURV WKHERXQGDU



) LJ $ FRQWROYROXPH ZLWK VLPSOLILHGRQH GLPHQVLRQDOLQIWVDQGH[LW

(; $0 3/ (

$ IL[HG FRQWROYROXPH KDV WKUHH RQH GLPHQVLRQDOERXQGDU VHFWLRQV DV VKRZQLQ) LJ (7K IORZ ZLWKLQ WKH FRQWROYROXPH LV VXANDG 7KH IORZ SURSHUWLHV DWHDFK VHFWLRQ DUH WDEXODWHG EHORZ

) LQG WKH UDWH RI FKDQJH RI HQHUJ\ RI WKH V\VWHP WKDW RFFXSLHV WKH FRQWURO YROXPH DW WKLV LQVWDQW

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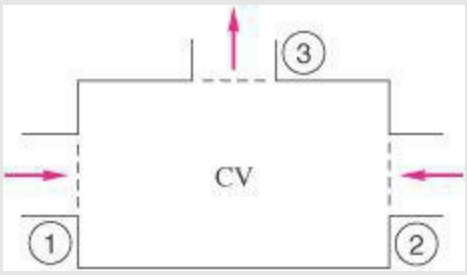


Section	Type	ρ, kg/m^3	V, m/s	A, m^2	e, J/kg
1	Inlet	800	5.0	2.0	300
2	Inlet	800	8.0	3.0	100
3	Outlet	800	17.0	2.0	150

6ROXWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZV WZR LQGHSHQGHQW DQG DQGDVLQJOHRXWOHW CRZ
- $VVXPSWLRQV 6WDG\ IORZ IL[HG FRQWURO YROXPH RQHGLPHQVLRQDO LQGHSHQGHQW[LWORZV
- $SSURDFK $SSO(T ZLWKHQHUJ\ DVWKHSURSHUW ZKHUH% (DQG G(GP H 8VHWKHRQHGLPHQVLRQDOFIOX[DSSUR[LPDWRQDQGWKHQLQWHJUWKHGDWIURP WKHWDEOH
- 6ROXWRQ WXMSV 2XWOHW FRQWUEXWV D SRVLWLYH WUP DQG LQOHW DQG DUH QHJDWLYH 7K DSSURSULDWIRUP RI(T LV

$$\left(\frac{dE}{dt}\right)_{syst} = \frac{d}{dt}\left(\int_{CV} e\rho\, dv\right) + e_3\dot{m}_3 - e_1\dot{m}_1 - e_2\dot{m}_2$$

6LQFHWKHIORZ LVWXDG\ WKHWP HGHULYDWLYHYROXPHLQWUDOWUP LV]HUR ,QWRGXFLQJ ! $9 DVWKHP DWIORZ JURXSLQJ ZHREWLQ

$$\left(\frac{dE}{dt}\right)_{syst} = -e_1\rho_1A_1V_1 - e_2\rho_2A_2V_2 + e_3\rho_3A_3V_3$$

,QWRGXFLQJ WKHQXPHULFDOYDOXHVIURP WKHWEOH ZHKDYH

$$\begin{aligned}\left(\frac{dE}{dt}\right)_{syst} &= -(300\text{ J/kg})(800\text{ kg/m}^3)(2\text{ m}^2)(5\text{ m/s}) - 100(800)(3)(8) + 150(800)(2)(17)\\ &= (-2{,}400{,}000 - 1{,}920{,}000 + 4{,}080{,}000)\text{ J/s}\\ &= -240{,}000\text{ J/s} = -0.24\text{ MJ/s}\end{aligned}$$ *Ans.*

7KXV WKH V\VWHP LV ERXQGHG\ DWWKH UDWH RI 0 - V 0 : 6LQFH ZH KDY DFFRXQWHG IRU DOO IOXLGHQHUJ\ FURVVLQJ WKH ERXQGDU\ ZH FRQFOXGH IURP WKH ILUVW ODZ WKDWWKH PXVWEH KHDWWUDQV WKURXJK WKH FRQWURO VXUIDFH RU WKH V\VWHP PXVWEH GRLQJ ZRUN RQ WK HQYLURQPHQWWKURXJK VRPHGHYLFHORVWWKHXHRI 6, XQLWVOHDGVWRDFRQVLVWU UHVXOWLQ WRXOVSHUVFRQG ZLWRXWDQ FRQYHUWRQ IDFWRUV : H SURPLVHG LQ&KDS WKDWWKL ZRXOGEHWKHFDVH

- &RPPHQW 7KLVSUREOPLQYROYHVHQHUJ\ EXWXSSRVHZHFKHFNWKHEDODQFHRI PDVVOOR 7KHQ% PDVVP DQG GP GP XQLW\$JDLQWKHYROXPHLQWUDOYDQVKHVIRUWWDG\IORZ DQG(T UHGXFHVWR

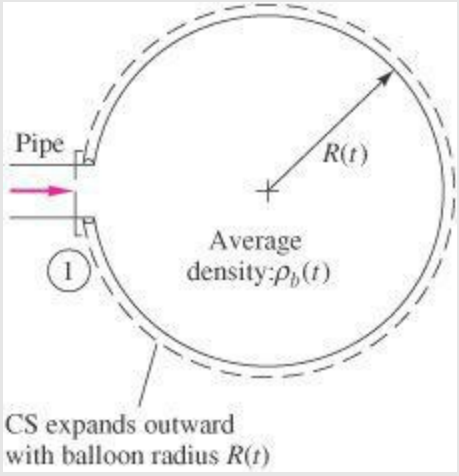
$$\left(\frac{dm}{dt}\right)_{\text{syst}} = \int_{CS} \rho(\mathbf{V}\cdot\mathbf{n})\,dA = -\rho_1 A_1 V_1 - \rho_2 A_2 V_2 + \rho_3 A_3 V_3$$
$$= -(800\ \text{kg/m}^3)(2\ \text{m}^2)(5\ \text{m/s}) - 800(3)(8) + 800(17)(2)$$
$$= (-8000 - 19{,}200 + 27{,}200)\ \text{kg/s} = 0\ \text{kg/s}$$

7KXVWKHV\VWHP PDVVGRHVQRWFKDQJH ZKLFKFRUUHFWV H[SUHVVHVWKHODZ RI FRQVHUYDWRQR V\VWHP PDVV(T

(; $0 3/ (

7KHEDOORRQLQ) LJ (LVEHLQJ ILOOHGWKURXJKVHFWLRQ ZKHUHWKHDUHDLV$ YHORFLWLV9 DQ IOXLGGHQVLWLV! 7KHDYHUDJHGHQVLWZLWKLQWKHEDOORRQLV! E W) LQGDQH[SUHVVLRQIRUWKHUDWH FKDQJHRI V\VWHP PDVZLWKLQWKHEDOORRQDWWKLVLQVWDQW

(



6RXWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZ VRQH LQLWLDO WR H[LWW 7KH FRQWUROYROXPH DQG VXUIDFH H[SDQ VRJHWKHU KHQFH WKH UHDVRQ IRU HYHQWXDO LYHOHFLW9 RQWKHEDOORRQ VXUIDFH
- \$VVXPSWLRQV 8QLIRUP VWDWH IORZ VKH FRQWUROYROXPH PDVV LQFUHDVHV GHRUPDEOH FRQWURO VXUIDFH RQH GLPHQVLRQDOLQDWFRQGLWLRQV
- \$SSURDFK \$SSO(T ZLWK9 RQWKHEDOORRQ VXUIDFHDQG9 DWWKHLQLWW
- 6RXWLRQVWHS 7KHSURSHUWLEHLQJ VXGLHGLVPDW % P DQG GP GP XQLW\$SSO(T 7KHYROXPHLQWUDOLVHYDOXDWHGRQDYHUDJHGHQVLW! E DQG WKHVXUFHLQWUDWHUP LVQHJDWLYH IRUDQLQLW

$$\left(\frac{dm}{dt}\right)_{syst} = \frac{d}{dt}\left(\int_{CV} \rho\, d\mathcal{V}\right) + \int_{CS} \rho(\mathbf{V}_r \cdot \mathbf{n}) dA = \frac{d}{dt}\left(\rho_b \frac{4\pi}{3} R^3\right) - \rho_1 A_1 V_1 \qquad Ans.$$

- &RPPHQWV 7KHUHDWRQJLYHQLVWKHDQZHUWRWKHTXHVWLRQWKDWZDVDVNHG\$FWXDOO\ E\ WK FRQVHUYDWLRQODZ IRUPDW(T GP GW/W DQGWKHDQZHUFRXOGEHUHZULWWHQDV

$$\frac{d}{dt}(\rho_b R^3) = \frac{3}{4\pi}\rho_1 A_1 V_1$$

7KLVLVDILUVWRUGHURGLQDU GLIIHUHQWLDOHTXDWLRQUHODWLQJ JDVGHQVLW DQGEDOORRQUDGLXV , FRXOG IRUP SDUWRI DQ HQJLQHHULQJ DQDOVLV RI EDOORRQ LQIDWLRQ ,WFDQQRWEH VROYHG ZLWKRX IXUWKHUXVHRI PHFKDQLFVDQGWKHUPRG\QDPLFVWRUHDWWKHIRXUQQRZQV! E ! 9 DQG5 7K SUHVXUHDQGWHPSHUDWXUHDQGWKHDLUF SURSHUWLHVRI WKHEDOORRQZRXOGDOVRKDYHWREHEURXJK LQRWKHDQDOVLV

)RUDGYDQFHG VXG\ PDQ PRUHGHWDLOVRI WKHDQDOVLV GHRUPDEOHFRQWROYROXPHVFDQ E IRXQGLQ+DVHQ> @DQG3RWWHUHWDO> @

&RQVHUYDWLRQ RI 0 DVV

7KH5HQROGVWDQVRUWKHRUHP(T RU HXDEOWKHVDUHDWRQEHWZHHQ\WHP UDWH RI FKDQJHDQGFRQWROYROXPHVXUIDFHDQGYROXPHLQWUDOV %XWWHP GHULYDWLYHVDUHUHDWWR WKH EDVLF ODZ VRI PHFKDQLFV(TV WR (OPLQDWQJ \WHP GHULYDWLYHVEHWZHHQWKHWZR JLYH WKH FRQWROYROXPH RULQWUDO IRUPVRI WKH ODZ VRI PHFKDQLFVRI IOXLGV 7KHGXPP\ YDULDEOH% EHFRPHV UHSHFWLYHO PDVV0DQGXPRPHQWXP DQJXODUPRPHQWXP DQGHQHUJ\ )RUFRQVHUYDWLRQ RI PDVV DVGLVFXVVHGLQ([DPSOHV DQG % P DQG GP GP (TXDWLRQ EHFRPHV

$$\left(\frac{dm}{dt}\right)_{syst} = 0 = \frac{d}{dt}\left(\int_{CV} \rho\, d\mathcal{V}\right) + \int_{CS} \rho(\mathbf{V}_r \cdot \mathbf{n})\, dA \qquad (3.20)$$

7KLVLVWKHLQWUDOPDVVFRQVHUYDWLRQODZ IRUDGHRUPDEOHFRQWROYROXPH) RUDIL[HG FRQW

YROXPH Z HKDYH

$$\int_{CV} \frac{\partial \rho}{\partial t}\, dV + \int_{CS} \rho(\mathbf{V} \cdot \mathbf{n})\, dA = 0 \qquad (3.21)$$

,I WKH FRQWURO YROXPH KDV RQO DQXPEHU RI GLPHQVLRQDO LQIOHWV DQG RXWOHWV Z HFDQ Z ULWH

$$\int_{CV} \frac{\partial \rho}{\partial t}\, dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0 \qquad (3.22)$$

2 WKHUVSHFLDOFDVHVRFFXU6XSSRVHWKDWWKHIORZ Z LWKLQWKHFRQWURO YROXPHLVVWHDG\ WKHQ˜! ˜W DQG(T UHGXFHVWR

$$\int_{CS} \rho(\mathbf{V} \cdot \mathbf{n})\, dA = 0 \qquad (3.23)$$

7KLV VLWXDWLRQ WKDWLQ WXUQ IORZ WKH PDVV IORZ V HQWHULQJ DQG GHDYLQJ WKH FRQWURO YROXPH PX\ EDODQFHH[DFWO\– ,I IXUWKHU WKHLQIOHWV DQG RXWOHWV DUHRQH GLPHQVLRQDO Z HKDYHIRUWXQDWHO\ IORZ

$$\sum_i (\rho_i A_i V_i)_{in} = \sum_i (\rho_i A_i V_i)_{out} \qquad (3.24)$$

7KLVVLPSOHDSSUR[LPDWLRQLVZLGHO\ XVHGLQHQJLQHHULQJ DQDOVHV)RUH[DPSOH UHHUULQJ WR) LJ Z HVHHWKDWL WKHIORZ LQWKDWFRQWURO YROXPHLVVWHDG\ WKHWKUHHXQHWP DVV IOX[HVEDODQFHWKH VWR LQOHWV

$$\text{Outflow} = \text{inflow}$$
$$\rho_2 A_2 V_2 + \rho_3 A_3 V_3 + \rho_5 A_5 V_5 = \rho_1 A_1 V_1 + \rho_4 A_4 V_4 \qquad (3.25)$$

7KH TXDQWLW! $9 LV FDOOHG WKH P DV IORZ æ SDVLQJ WKURXJK WKH RQH GLPHQVLRQDO FURVV VHFWLR DQG KDVFRQVLVWHQWXQLWVRI NJRUJUP VSHUVHFRQG RUVOXJVSHUVHFRQG IRU6, RU%* XQLW (TXDWL FDQEHUHZ ULWWQLQWKHVKRUWHUP

$$\dot{m}_2 + \dot{m}_3 + \dot{m}_5 = \dot{m}_1 + \dot{m}_4 \qquad (3.26)$$

DQG LQJHQHUDO WKHVWHDG\ IORZ±P DV FRQVHUYDWLRQ UHDWLRQ FDQEHZ ULWWQDV

$$\sum_i (\dot{m}_i)_{out} = \sum_i (\dot{m}_i)_{in} \qquad (3.27)$$

,I WKH LQOHWV DQG RXWOHWV DUH QRWRQH GLPHQVLRQDO RQH KDV WR FRP SXWæ E\ LQWHJUDWLQJ RYHU WK

VHFWLRQ

$$\dot{m}_{cs} = \int_{cs} \rho(\mathbf{V} \cdot \mathbf{n})\, dA \qquad (3.28)$$

Z KHUH³ FV' VWDQGVIRUFRQVHUYDWLRQ $QLQWXLWLYHQLQ([DPSOH

,QFRP SUHVVLEOH) ORZ

6WUDLJKWIXUWKHUVLPSOLILFDWLRQ LV SRVVLEOH LI WKH IOXLG LV LQFRP SUHVVLEOH Z KLFK Z H PD\ GHILQH D KDYLQJ GHQVLW\ YDULDWLRQV WKDW DUH QHJOLJLEOH LQ WKH PDVV FRQVHUYDWLRQ UHTXLUHPHQW\ $V Z H VDZ L &KDS DOO OTXLGV DUH QHDUO\ LQFRP SUHVVLEOH DQG JDV IORZV FDQEH KDYH DV LI WKH Z HU LQFRP SUHVVLEOH SDUWFXODUO\ LI WKH JDV YHORFLW\ LV OHVV WKDQ DERXW SHUFHQW RI WKH VSHHG RI VRXQG R WKH JDV

$ JDLQ FRQVLGHU WKH IL[HG FRQWRO YROXPH) RU QHDU LQFRP SUHVVLEOH IORZ WKH WHUP %% %W\ VPDOO VR WKH WLPH GHULYDWLYH YROXPH LQWHJUDO LQ(T FDQEH QHJOHFWHG 7KH FRQWLQXLW\ FD WKHQEH UHGXFHG IURP WKH VXUIDFH LQWHJUDO IRU DQLFHVLPSOLILFDWLRQ

$$\frac{d}{dt}\left(\int_{CV} \frac{\partial \rho}{\partial t}\, d\forall\right) + \int_{CS} \rho(\mathbf{V} \cdot \mathbf{n})\, dA = 0 = \int_{CS} \rho(\mathbf{V} \cdot \mathbf{n})\, dA = \rho \int_{CS} (\mathbf{V} \cdot \mathbf{n})\, dA$$

or

$$\int_{CS} (\mathbf{V} \cdot \mathbf{n})\, dA = 0 \qquad (3.29)$$

,I WKHLQODWDQGRXWOHWDUHRQHGLPHQVLRQDO Z HKDYH

$$\sum_i (V_i A_i)_{out} = \sum_i (V_i A_i)_{in} \qquad (3.30)$$

or

$$\sum Q_{out} = \sum Q_{in}$$

Z KHUH4$ L 9$ L LVFDOODHGWKHYROXPHIORZ SDVWLQJ WKURXJK WKHJLYHQFURVV VHFWLRQ

$ JDLQ LI FRQVLWHQWXQLWV DUH XVHG 4 9$ Z LOOKDYH XQLWVR FXELF PHWHUV SHUVHFRQG 6, F FXELF IHHWSHUVHFRQG %* ,I WKHFURVV VHFWLRQLVQRWRQHGLPHQVLRQDO Z HKDYHWRLQWHJUDW

$$Q_{CS} = \int_{CS} (\mathbf{V} \cdot \mathbf{n})\, dA \qquad (3.31)$$

(TXDWLRQ DOORZ VXVWR GHILQHDQDYHUDJHYHORFLW9$ WKDWZKHQPXOWSOLHG E\ WKHVHFWLR DUHD JLYHVWKHFRUUHFWYROXPHIORZ

$$V_{av} = \frac{Q}{A} = \frac{1}{A}\int (\mathbf{V} \cdot \mathbf{n})\, dA \qquad (3.32)$$

WDSSHGDWKHWRS 6XEWUDFWLQJ LQR ZHILQGWKHFKDQJHRI ZDWHUKHLJKW

) RUZDWHU ! ! !Z DQGWKLVUHVXOWHGXFHVWR

$$\frac{dh}{dt} = \frac{A_1V_1 + A_2V_2}{A_t} = \frac{Q_1 + Q_2}{A_t} \tag{3}$$

3DUWE

7KHVZRLQGHWYROXPHIORZ VDUH

$$Q_1 = A_1V_1 = \tfrac{1}{4}\pi(\tfrac{1}{12}\text{ ft})^2(3\text{ ft/s}) = 0.016\text{ ft}^3/\text{s}$$
$$Q_2 = A_2V_2 = \tfrac{1}{4}\pi(\tfrac{3}{12}\text{ ft})^2(2\text{ ft/s}) = 0.098\text{ ft}^3/\text{s}$$

7KHQ IURP (T

$$\frac{dh}{dt} = \frac{(0.016 + 0.098)\text{ ft}^3/\text{s}}{2\text{ ft}^2} = 0.057\text{ ft/s} \qquad \textit{Ans. (b)}$$

6XJJHVWLRQ 5HSHDWWKLVSUREOHP ZLWKWKHVRS RI WKHWDQNRSHQ

$QLOOXVWUDWLRQRI DPDVVEDODQFHZLWKDGHRUPLQJ FRQWROYROXPHKDVDOHDG EHHQJLYHQL ([DPSOH

7KH FRQWROYROXPH PDVV UHDWLRQ/ (T RU DUH IXQGDPHQWDO WR DOOIOXLG IORZ DQDOVHV 7KH\ LQYROYHRQO YHORFLW DQGGHQVLW 9HVRUGLUHFWRQ/DUHRI QR FRQVHTXHQFHH[FHSWWR GHWHUPLQHWKHQRUPDOYHORFLW DWWKHVXUIDFHDQGKHQFHZKHWKHUIORZ LVLQ RURXW$OWKRXJK\RX VSHFLILF DQDOVLVPD\ FRQFHQIRFXVRUPRQWRUHQJ\ \RXPXWDZD\PDNHVXUHWKDWPDWL EDODQFHGDVSDUWRI WKHDQDOVLV RUWKHUZLVHWKHUHXOWZLOOEHXQUHDOWLF DQGSUREDEO LQFRUUHFW: H VKDOOVHHLQWKHH[DPSOHVWKDWRQOZ KRZ PDVFRQVHUYDWRQLVFRQVLGHUHGLQSHUIRUPLQJ DQDOVLVRI RWKHUIORZSURSHUWLHV

7KH/ LQHDU0 RPHQWXP (TXDWLRQ

,Q1HZWRQ/V VHFRQG/DZ (T WKHSURSHUWEHLQJ GLIIHUHQWDWGLV WKHQHDUPRPHQWXP P9 7KHUHRUH RXUGXPP\ YDULDEOHLV% P9 DQG G%GP 9 DQGDSSOLFDWRQ RI WKH5HQROG WDQVSRUWWKHRUHP JLYHVWKHQHDUPRPHQWXP UHDWRQIRUDGHRUDEOHFRQWROYROXPH

$$\frac{d}{dt}(m\mathbf{V})_{sys} = \sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho\, d\forall\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V}_r \cdot \mathbf{n})\, dA \qquad (3.35)$$

7KHIRGBZ LQJ SRLQWFRQFHUQLQJ WKLVUHDWRQVKRXGEHWWRQJO HPSKDVL]HG

7KHWUP 9 LVWKHIOLG YHORFLWUHDWYHWRDQLQHUWDO QRQDFFHOHUDWQJ FRRUGLQDWVXVWP RWKHUZLVH1HZWRQ¶V VHFRQG ODZ PXWEHPRGLILHG WR LQFOXGHQRQLQHUWDOUHDWYHDFFHOHUDWR WUPV VHHWKHHQGRI WKLVVHFWLRQ

7KHWUP) LVWKHYHFWRUVXP RI DOIRUFHVDFWQJ RQWKHVXVWP PDWUDOFRQWLGHUHG DVI IUHHERG WKDWV LWLQFOXGHVVXUDFHIRUFHVRQDOOIOLGVDQGVROGVFXWE\ WKHFRQWUROVXUDFH SOXVDOOERG IRUFHV JUDYLW DQG HOHFWURPDJQHWF DFWQJ RQ WKH PDWHV Z LWKLQ WKH FRQWR YROXPH

7KHHQWUHTXDWRQLVDYHFWRUUHDWRQ ERWKWKHLQWJUDOVDUHYHFWRUVGXHWRWKHWUP 9 LQWK LQWJUDQGV 7KHHTXDWRQWKXVKDVWKUHFRPSRQHQW ,I ZHZDQWRQO VD\ WKH[FRPSRQHQWWK HTXDWRQUHGXFHVWR

$$\sum F_x = \frac{d}{dt}\left(\int_{CV} u\rho\, d\forall\right) + \int_{CS} u\rho(\mathbf{V}_r \cdot \mathbf{n})\, dA \qquad (3.36)$$

DQGVLPLODUO) DQG) ZRXGLQYROYHYDQGZ UHVSHFWLYHO) DLXUHWRDFFRXQWRUWK YHFWRUQDWXUHRI WKHOQHUPRPHQWXP UHDWRQ LVSUREDEO WKHJUHDWHVWVRXUFHRI VWXGHQ HUURULQFRQWROYROXPHDQDOVHV

)RUDIL[HGFRQWROYROXPH WKHUHDWYHYHORFLW9U A9 DQG(T EHFRPHV

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho\, d\forall\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V} \cdot \mathbf{n})\, dA \qquad (3.37)$$

$JDLQZ HVXUHVWKDWWKLVDYHFWRUUHDWRQDQGWKDW9 PXWEHDQLQHUWDOIUDPHYHORFLW0 RVWR WKHPRPHQWXP DQDOVHVLQWKLVWH[WUHFRQFHQHGZLWK(T

2QH' LPHQVLRQDO0 RPHQWXP) OX[

%\ DQDORJ\ ZLWKWKHWUP PDVIORZ XVHGLQ(T WKHVXUFHLQWJUDOLQ(T LVFDOOHG WKHPRPHQWXP IOX[WUP ,I ZHGHQRWHPRPHQWXP E\ 0 WKHQ

$$\dot{\mathbf{M}}_{CS} = \int_{sec} \mathbf{V}\rho(\mathbf{V} \cdot \mathbf{n})\, dA \qquad (3.38)$$

%HFDXVHRI WKHGRWSURGXFWWKHUHVXOWZLOOEHQHJDWLYHIRULQIORZPRPHQWXP IOX[DQGSRVLWLYHIR

RXWOHWWOX[,I WKH FURVV VHFWLRQ LV RQH GLPHQVLRQDO9 DQG! DUH XQLIRUP RYHU WKH DUHD DQG WK
LQWUDVG UHVXOWV

$$\dot{\mathbf{M}}_{\text{seci}} = \mathbf{V}_i(\rho_i V_{ni} A_i) = \dot{m}_i \mathbf{V}_i \tag{3.39}$$

IRUROWWOX[DQG±æ9L IRULQOHWWOX[7KXVLI WKHFRQWROYROXPHKDVRQO RQHGLPHQVLRQDOLQOHW
DQGRXOHW (T UHGXFHVWR

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho\, d\forall\right) + \sum (\dot{m}_i \mathbf{V}_i)_{\text{out}} - \sum (\dot{m}_i \mathbf{V}_i)_{\text{in}} \tag{3.40}$$

7KLVLVDFRPPRQO XVHGDSSUR[LPDWLRQLQHQJLQHHULQJ DQDOVHV ,WLVFUXFLDOWRUHDOL]HWKDWZHDU
GHDOLQJ ZLWKYHFWRUVXPV (TXDWLRQ VDWLVWKDWWKHQHWYHFWRUIRUFHRQDILHGFRQWROYROXPH
HTXDOVWKHUDWHRI FKDQJHRI YHFWRUPRPHQWXP ZLWKLQWKHFRQWROYROXPHSOXVWKHYHFWRUVXP RI RXWO
PRPHQWXP IOX[HVPLQXVWKHYHFWRUVXP RI LQOHWOX[HV

1HW3UHVVXUH)RUFHRQD&ORVHG&RQWURO6XUIDFH

* HQHUDOO\ VSHDNLQJ WKH VXUIDFHIRUFHV RQ D FRQWROYROXPH DUH GXH WR IRUFHV H[SRVHG E
FXWWLQJ WKURXJK VROG ERGLHV WKDWSURWUXGH WKURXJK WKH VXUIDFH DQG IRUFHV GXH WR SUHVVXUH DQ
YLVFRXV VWUHVVHV RI WKH VXUURXQGLQJ IOXLG 7KHFRPSXWDWLRQ RI SUHVVXUHIRFHV UHDOLYHO VLPSOH D
VKRZ QLQ) LJ 5HFDOOIURP &KDS WKDWWKHH[WUDOSUHVVXUHIRFHRQD VXUIDFHLVQRUPDOWR WK
VXUIDFH DQGLQZDUG 6LQFH WKH XQLWYHFWRUQ LV GHILQHG DVRXWZDUG RQH ZD\ WR ZULWH WKH SUHVVXU
IRUFHLV

$$\mathbf{F}_{\text{press}} = \int_{CS} p(-\mathbf{n})\, dA \tag{3.41}$$

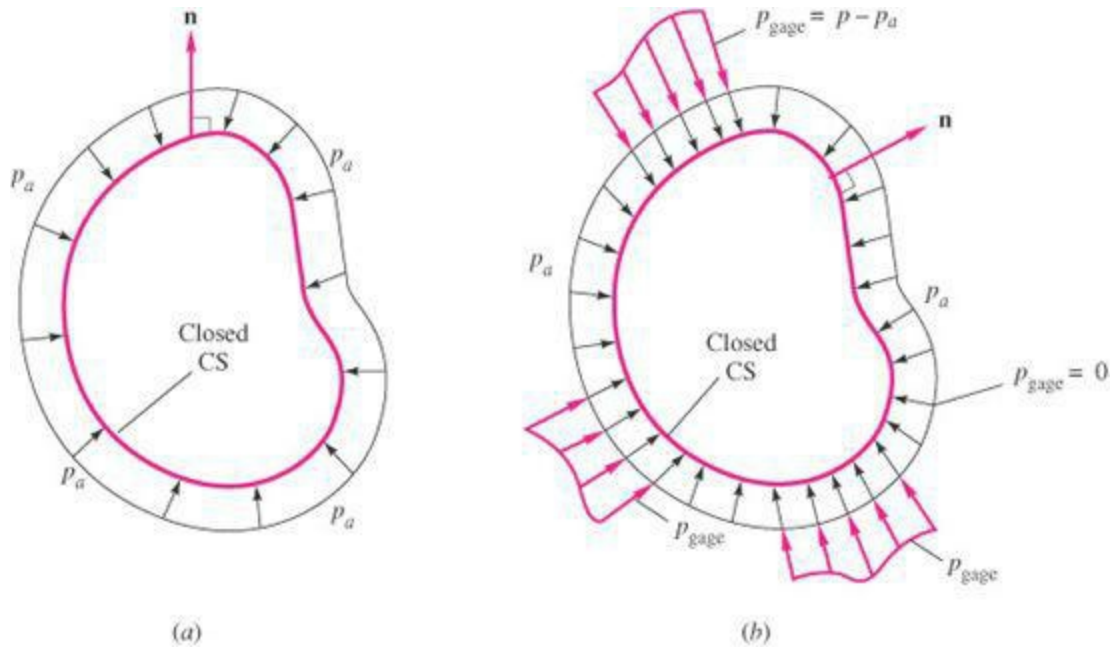
1RZ LI WKHSUHVVXUHKDVDXQLIRUP YDOXHS$ DOODURXQGWKHVXUIDFH DVLQ) LJ D WKHQHWSUHVVXU
IRUFHLV]HUR

$$\mathbf{F}_{\text{UP}} = \int p_a(-\mathbf{n})\, dA = -p_a \int \mathbf{n}\, dA \equiv 0 \tag{3.42}$$

ZKHUH WKH VXEVFULSW3 VWDQGV IRUXQLIRUP SUHVVXUH 7KLV UHVXOWLVGHSHQGHQWRQ WKHVKDSH R
WKHVXUIDFH DVORQJ DV WKHVXUIDFHLVFORVHGDQG DOORXUFRQWROYROXPHVDUHFORVHG 7KXVDVHPLQJ
FRPSOLFDWHGSUHVVXUHIRFHSUREOHP FDQEHVLPSOLILHGE\ VXEWUDFWLQJ DQ FRQVWDQWSUHVVXUH
S$ DQGZRUNLQJ RQO\ ZLWKWKHSUHFHVRI JDJH SUHVVXUHWKDWWHP DV LOOXVWUDWHGLQ) LJ E 6R(T
LVHQWLUHO\ HTXLYDOHQWR

$$\mathbf{F}_{\text{press}} = \int_{CS} (p - p_a)(-\mathbf{n})\, dA = \int_{CS} p_{\text{gage}}(-\mathbf{n})\, dA$$

7KLVWHFKQLTXHLVDOVRXVHIXOLQFRPSXWDWLRQ



) LJ 3UHVVXUH IRUFH FRPSXWDWLRQ E\ VXEWUDFWLQJ D XQLIRUP GLVWULEXWLRQ *a* XQLIRUP SUHVVXUH $\mathbf{F} = -p_a \int \mathbf{n}\, dA \equiv 0;$ *b* QRQXQLIRUP SUHVVXUH $\mathbf{F} = -\int (p - p_a)\mathbf{n}\, dA.$

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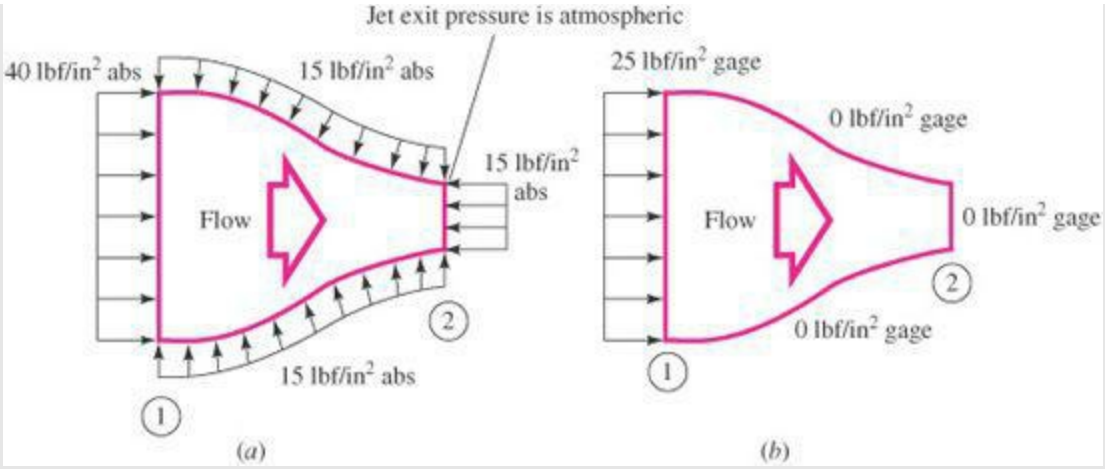
$ FRQWROYROXPHRI DQR]]OHVFRQKDVXUDFHSUHVVXUHVRI GEI LQ DEVROXWHDVWHFWLRQ DQG DWPRVSKHULF SUHVVXUHRI GEI LQ DEVROXWHDVWHFWLRQ DQGRQWKHH[WUDQRXQGHGSDUWRI WKHQR]]OH DVLQ) LJ (D &RPSXWWKHQHWSUHVVXUHIRUFHL ' LQDQG' LQ

6RXWLRQ

- 6\VWHP VNHWFK 7KHFRQWROYROXPHLVWKHRXWLGHRI WKHQR]]OH SOXVWKHFXWHFWLRQV DQG 7KHUHZRXOGDOVR EHVWUHVVLQWKHFXWQR]]OHZDOODWHFWLRQ ZKLFKZHDUHQHJOHFWLQ KHUH 7KHSUHVVXUHVDFWLQJ RQWKHFRQWROYROXPHDUHVKRZQLQ) LJ (D) LJXUH(E VKRZ WKH SUHVVXUHV DWWU GEI LQ KDV EHHQ VXEWUDFWHG IURP DOOVLGHV +HUH ZH FRPSXWH WKH QHW SUHVVXUHIRUFHRQO

(



- $VVXPSWLRQV . QRZ QSUHVVXUHV DVVKRZ Q RQDOOVXUIDFHVRI VKHFRQWROYROXPH
- $SSURDFK 6LQFHVWKUHVXUIDFHVKDYHS GEI LQ VXEWDFWVDPRXQWHYHUZKHUHVR VWKVHVWKUHVLGHVUHGXFHVWR]HUR³JDJHSUHVVXUH IRUFRQYHQLHQFH 7KLVLVDOVRZ DEOHEHFDXVHR (T
- 6RXWRQVWMSV)RUWKHPRGLILHGSUHVVXUHGLVWULEXWLRQ)LJ (E RQO VHFWLRQ LVQHGHG

$$\mathbf{F}_{\text{press}} = p_{\text{gage},1}\,(-\mathbf{n})_1\,A_1 = \left(25\frac{\text{lbf}}{\text{in}^2}\right)\left[-(-\mathbf{i})\right]\left[\frac{\pi}{4}(3\text{ in})^2\right] = 177\mathbf{i}\text{ lbf} \qquad \textit{Ans.}$$

- &RPPHQWV 7KLV ³XQLIRUP VXEWDFWRQ DUWLFH ZKLFK LV HQWLUHO GHJUDG KDV JUHDW VLPSOLILHGWKHFDOFXODWRQRI SUHVVXUHIRFH 1RWH : HZHUHDEOWRR LQRUPDOZKHQPXOWLSO SUHVVXUHLQGEI LQ WPHVDHLQVTXDUHLQFKHV : HDFKLHYHGEI FRUHFW EXWWZRXGEHEHWWH SUDFWHVRFRQYHUWDOGDWRVDQGDUG %* XQWV) XUWKHUQRWH ,QDGGLWRQVR)SUHV WKHUHDU RWKHUIRFHVLQYROYHGLQWKLVIGRZ GXHVRWQWLRQ WUHVHVLQWKHFXQWR]HZDOODQG WKHIXLG ZHJKWQWLGHWKHFRQWROYROXPH

3UHVVXUH&RQGLWRQDWD-HW([LW

)LJXUH(LOOXVWUDWHVDSUHVVXUHERXQGDU FRQGLWRQFRPPRQO XVHGIRUMHWH[LWVURE OHP\ : KHQDIXLGIGRZ GHDYHVDFRQLQHGLQWLDOGXFWDQG H[LWVLQR DQDPELHQW DWPRVSKHUH´ LWIUH VXUFHLVH[SRVHG WR WKDWDWPRVSKHUH 7KHUHRUH WKHMHWLWHO ZLOHXHQWDOO EH DWDWPRVSKHULF SUHVVXUHDOR 7KLVFRQGLWRQZDVXVHGDWHFWQ LQ)LJ (2QO VZRHIFWFRXGPDLQWLQDSUHVXUHGLIHUHQFHEHWZHQWKHDWPRVSKHUHDQGDIUHH[LWMHW 7KHILUWLVXUFHWQWRQ (T ZKFK LVXVDOO QHJOLJEOH 7KHVFRQGHIFWLVDXSHUWRQ NWZKFK FDQ VHSDUDWLWHOIURP DQDWPRVSKHUHZLWH[SDQVLRQRUFRPSUHVVLRQZDYHV &KDS) RUWKHPDWULRI DSSOFDWRQV WKHUHRUHZHVKDOOVHWWKHSUHVXUHLQDQ[WMWDVDWPRVSKULF

(; $0 3/ (

$ IL[HGFRQWROYROXPHRI DWXUEDPWXEHLQWWDG IGRZ KDVDXQLRUP LQOHWGRZ ! $ 9 DQGI

6ROXWLRQ

7KH VXJJHVWHG FRQWROYROXPH LQ) LJ D FXW VKURXJK WKH SODWH VXSSRUWRU H[SRVH WKH GHVLUH IRUFHV 5[DQG 5\ 7KLV FRQWROYROXPH PRYHV DWSHHG 9F DQG WKXV LVIL[HG UHODWLYH WR WKH SODWH DV LQ) LJ E : HPXWWDLW\ ERWKPDVVDQGPRPHQWXP FRQVHUYDWLRQIRUWKHDXXPHGVWDG IORZ SDWWHUQ LQ) LJ E 7KHUHUHVZRRXOWWDQGRQHLQGHWDQG(T DSSOLHVIRUPDVFRQVHUYDWLRQ

: HDVXPHWKDWWKHZDWULVLQFRPSUHVVLEOH! ! !M DQG ZHDUHJLYHQWKDW$ $ ò $N 7KHUHRUH(T UHGXFHVWR

6WUDLJKWVSHDNLQJ WKLVLVDOOWKDWPDVVFRQVHUYDWLRQWHOOVXV + RZHYHUIURP WKHV\PPHWU RI WKHIG HOHFWLRQDQG WKHQHJOHFWRI JUDYLWRQWKHIOXLGWDMHFWRU ZHFRQFOXGHWKDWWKHVZRYHORFLWLHV9 DQ 9 PXWEHHTXDO DQGKHQFH(T EHFRPHV

7KLVHTXDWLRQFDQDOVREHSUHGLFWHG E\ %HUQRXOOL VHTXDWLRQLQ6HFW) RUWKHJLYHQQXPHULFDO YDOXHV ZHKDYH

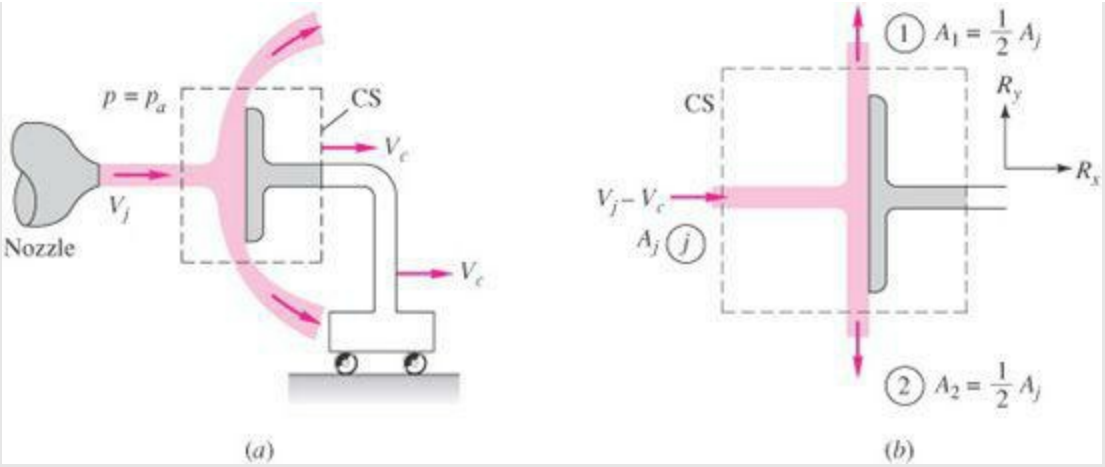
9 9 ± P V

1RZ ZHFDQFRPSXWH5[DQG5\ IURP WKHVZR FRPSRQHQWVRI PRPHQWXP FRQVHUYDWLRQ (TXDWLR DSSOLHVZLWKWKHXQWDUGG\ VWUP]HUR

$$\sum F_x = R_x = \dot{m}_1 u_1 + \dot{m}_2 u_2 - \dot{m}_j u_j \qquad (4)$$

ZKHUHIURP WKHPDVVDQDOVLV $\dot{m}_1 = \dot{m}_2 = \frac{1}{2}\dot{m}_j = \frac{1}{2}\rho_j A_j (V_j - V_c)$ 1RZ FKHFNWKHIORZ GLUHFWRQVDWHDF VHFWRQ X X DQGX M 9M í 9F P V7KXV(T EHFRPHV

$$R_x = -\dot{m}_j u_j = -[\rho_j A_j (V_j - V_c)](V_j - V_c) \qquad (5)$$



) LJ) RUFH RQ D SODWH P RYLQJ DZD\ IURP RQFRPLQJ VWUHDP *a* NHHSLQJ D P RYLQJ SODWH QRUPDO *b* FRQWUROOLQJ P HIIHFWLYHQHVV RI WKH SODWH

) RUWKH JLYHQ QXP HULFDO YDOXHV Z HKDYH

$$R_x = -(1000 \text{ kg/m}^3)(0.0003 \text{ m}^2)(5 \text{ m/s})^2 = -7.5 \text{ (kg} \cdot \text{m)/s}^2 = -7.5 \text{ N} \qquad Ans.$$

7KLV DFWV RQ WKH OHIW VLGH RI WKH SODWH LQ HTXLOLEULXP DQG LV HTXDO DQG RSSRVLWH WR WKH IRUFH RQ WKH ULJKW VLGH RI WKH FRQWURO YROXP H VR WKH QHW 7KH YHUWLFDO IRUFH LV

$$F_y = R_y = \dot{m}_1 v_1 + \dot{m}_2 v_2 - \dot{m}_j v_j$$

Check directions again: $v_1 = V_1$, $v_2 = -V_2$, $v_j = 0$. Thus

$$R_y = \dot{m}_1(V_1) + \dot{m}_2(-V_2) = \tfrac{1}{2}\dot{m}_j(V_1 - V_2) \qquad (6)$$

&KHFN GLUHFWLRQV DJDLQ % 9 % í 9 % 7KXV

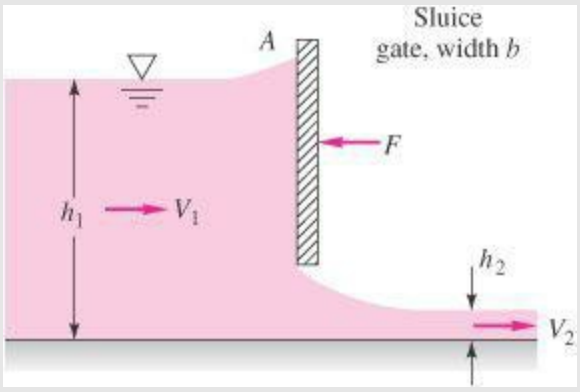
%XWLQFH Z HIRXQG HDUOHU WKDW9 9 WKLV P HDQV WKDW\ DV Z H FRXOG H[SHFW IURP WK \ P P HWU\ RI WKH MHWV GHIOHFWLRQ 7Z R RWKHU UHVXOWV DUH RI LQWHUHVW) LUVW WKH UHODWLYH YHORFLW\ DW Z KL ZDV IRXQG WR EH P V XS IURP (T ,I Z H FRQVLGHU WKLV WR DEVROXW P RWLRQ E\ DGGLQJ RQ WKH FRQVWDQW YHORFLW\9 F P V WR WKH ULJKW Z HILQG WKDW WKH DEVROXW YHORFLW\9 L P V R P V DWDQ DQJOH RI f XSZ DUG DV LQGLFDWHG LQ) LJ D 7KXV WKH DEVROXW MHWV SHHG FKDQJH DIWHU KLWWLQJ WKH SODWH 6HFRQG WKH FRP SXWHG IRUFH5[GRHV QRW FKDQJH Z HDVXP HWKH MHWV GHIOHFWLQJ DOOUDGLDO GLUHFWLRQV DORQJ WKH SODWH VXUIDFH UDWKHU WKDQ XS DQG DZ Q 6LQFH WKH SODWH LV QRUP DOO WKH[D[LV WKHUH Z RXOG VWLOO EH]HUR RXOHW/ P RP HQWXP IOX[Z KHQ(T Z DV UHZ ULWWQ IRU D UDGLDO GHIOHFWRQ FRQGLWLRQ

(; $0 3/ (

7KH VOLFH JDMLQ) LJ (D FRQWROOIORZ LQ SHQFKDQQHOV $WWHFWRQV DQG WKH IORZ LV XQLIRUP DQG WKH SUHVVXUHLVK\GURVWDWF 1HJOHFWLQJ EXWP IUFWLRQ DQG DVVRSHUF SUHVXUH GHULYHI IRUP XOD IRUWKH KRUL]RQWDOIRUFH) UHTXLUHG WR KROG WKH JDM ([SUHVV\RXUILQDOIRUP XOD LQWHUP VRI WK

LQDWYHORFLW9 HDPLQDWQJ9

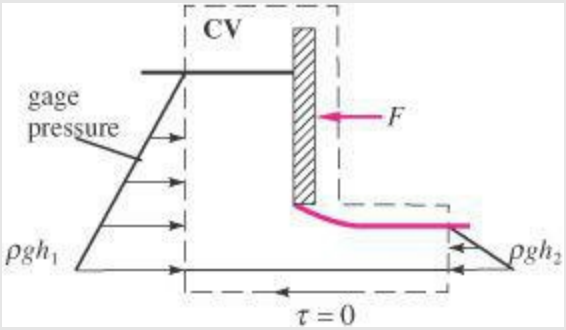
(D



6RXWLRQ

&KRRVH D FRQWURO YROXPH) LJ (E WKDW FXWV WKURXJK NQRZQ UHJLRQV VHFWLRQ DQG VHFWLRQ WKHEWRP DQG WKH DWPRVSKHUH DQG WKDW FXWV DORQJ UHJLRQV ZKHUH XQNQRZQ QLQRUP DWRQLVGHVLUHG WK JDW ZLWK LWV IRUFH)

(E



$VXPH VWDG\ LQFRP SUHVVLEOH IORZ ZLWK QR YDULDWLRQ DFURVV WKH ZLGWKE 7KH LQDWDQG RXWP PDVV IORZ VEDODQFH

æ !9 KE !9 KERU9 9 K K

: HPD\ XVH JDJH SUHVVXUHV IRUFRQVHQHQFHEHFDXVH D XQLRUP DWRVSKHUF SUHVXUH FDXVHV QI IRUFH DV VKRZ QHDUDHULQ) LJ : LWK[SRVLWLYH WR WKH ULJKWHTXDWLWKH QHWKRU]RQDO IRUFHWR WK [GLUHFWLRQ PRPHQWXP FKDQJH

$$\Sigma F_x = -F_{\text{gate}} + \frac{\rho}{2} g h_1 (h_1 b) - \frac{\rho}{2} g h_2 (h_2 b) = \dot{m}(V_2 - V_1)$$

$$\dot{m} = \rho h_1 b V_1$$

6RONHIRU) JDM DQGHOPLQDWH9 XVLQJWKHPDVVIORZ UHODWLRQ7KHGHVLUHGUHVXOWV

$$F_{\text{gate}} = \frac{\rho}{2} g b h_1^2 \left[1 - \left(\frac{h_2}{h_1} \right)^2 \right] - \rho h_1 b V_1^2 \left(\frac{h_1}{h_2} - 1 \right) \qquad \textit{Ans.}$$

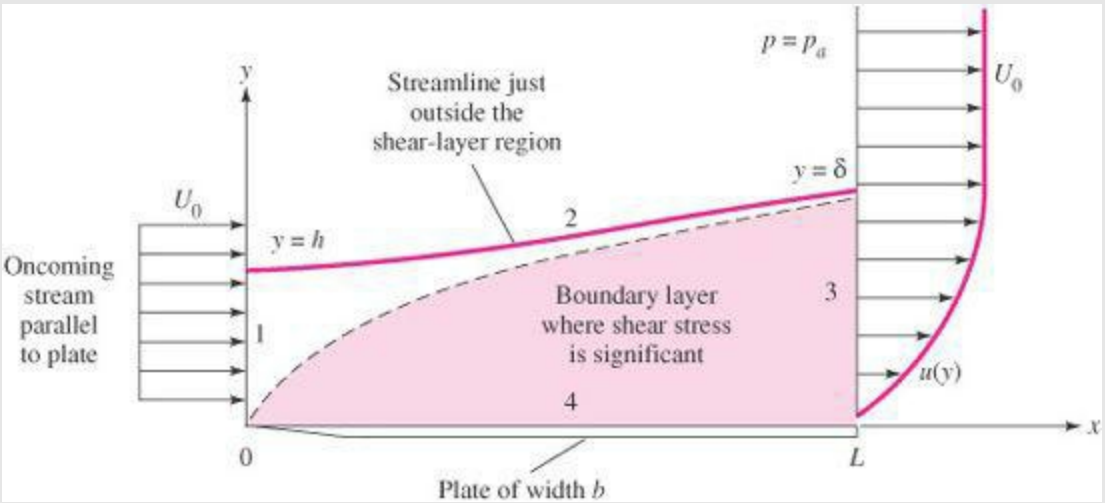
7KLVLVDSRZHUXOUHVXOWURP DUHODWLYHO VLPSOHDQDOVLV / DWU LQ6HF ZHZLOOEHDEOHV
FDOFXODWWKHDFWXDOIORZ UDWIURP WKHZDWUGHSWKVDQGWKHJDWRSHQLQJKHLJKW

(; $0 3/ (

([DPSOH VWDWG DSDWDWDQRUPDOLFFLGHQFHWR DQRQFRPLQJ IORZ ,Q) LJ WKHSDWL
SDUDOOHOWRWKHIORZ 7KHVWHDP LVQRWDWWHXWDEURDGULYHURIUHHVWHDP RI XQIRUP YHORFLW9
8 L 7KHSUHVXUHLVDXXPHGXQIRUP DQGVRLWKDVQR QHWRUFHQWKHSDWW7KHSDWGRHVQRWEORF
WKHIORZ DV LQ) LJ VR WKHRQO HIHFWLV GXH WR ERXQGDU\ VKHDU ZKLFK ZDV QHJOHFWG LQ WK
SUHYLRXVH[DPSOH 7KHQR VOSFRQGLWRQDWWKHZDOOEULQJVWKHIOLG WKUHVWRDKDOWDQG WKHVHORZ C
PRYLQJ SDUWFOHVUHDUG WKHLUQHJKERUV DERYH VR WKDWDWWKHHQG RI WKHSDWW WKHUHLV DVLJQLILFDQ
UHDUGHG VKHDUOD\HU RUERXQGDU\ OD\HU RI WKLFNQHVV\ / 7KHYLVFRXVWUHVWHVDORQJ WKHZDOOFD
VXP VR DILQLWHGUDJ IRUFHRQ WKHSDWW 7KHVHIHFWDUHLOOXVWUDWG LQ) LJ 7KHSUREOP LVV
PDNHDQLQWUDODQDOVLVDQG ILQG WKHGUDJ IRUFH' LQWUP VRI WKHIORZ SURSHUWHV! 8 DQG/ DQ
WKHSDWGLPHQVLRQV// DQGE

6RXWLRQ

/LNHPRVWSUDFWDOFDVHV WKLV SUREOP UHTXLUHV D FRPELQHG PDW DQG PRPHQWXP EDODQFH $
SURSHUVHOHFWLRQRI FRQWROYROXPHLVHVHQWLDO DQGZHVHOHFWWKHIRXUVLGHGUHJLRQIURP VR K VR/ V
/ DQGEDFNVR WKHRULJLQ DV VKRZQLQ) LJ + DGZHFKRVHQVR FXWDFURVVKRU]RQWDO IURP OH
VRULJKWDORQJ WKHKHLJKW K ZHZRXOGKDYHFXWWKURXJK WKHVKDUOD\HUDQGH[SRVHGXQNQRZQVKD
VWUHVVHV ,QWURGZHIRORZ WKHVWUPQHSDWLQJ WKURXJK [\ K ZKLFK LVRXWLGHWKHVKD
OD\HUDQGDOR KDVQR PDWIORZ DFURVVLW7KHIRXUFRQWROYROXPHVLGHVDUHWKXV



)LJ &RQWURO9ROXPH DQDO VLV RI GUDJ IRUFH RQ D IODWSODWH GXH WR ERXQGDU\ VKHDU 7KH FRQWURO9ROXPH LVERXQGHG E\ VHFWLRQV DQG

)URP VR K DRQH GLPHQVLRQDOLQHW9 Q í8
)URP K VR / / DVWHDP0DQH QR VKHDU9 QA
)URP / / VR / DVZR GLPHQVLRQDORXWHW9 Q X\
)URP / VR D VWHDP0DQH MXVWDERYH WKH SODWH VXUIDFH 9 Q VKHDUIRUFH
VXPPLQJ WR WKH GUDJ IRUFH í' LDFWLQJ IURP WKH SODWHRQR WKH UHDUGHGIOXLG

7KH SUHVVXUH LV XQIRUP DQG VR WKHUH LV QR QHWSUHVVXUH IRUFH 6LQFH WKH IORZ LV DVVXPH LQFRPSUHVVLEOH DQG VWHDG\ (T DSSOLHVZLWKQR XQVWHDG\ WHUP DQGIOX[HVRQD DFURVVHFWLRQ DQG

$$\sum F_x = -D = \rho \int_1 u(0, y)(\mathbf{V} \cdot \mathbf{n})\, dA + \rho \int_3 u(L, y)\,(\mathbf{V} \cdot \mathbf{n})\, dA$$
$$= \rho \int_0^h U_0(-U_0) b\, dy + \rho \int_0^\delta u(L, y)[+u(L, y)] b\, dy$$

(YDOXDWLQJ WKHILUVWLQWHJUDODQGUHDUUDQJLQJ JLYH

$$D = \rho U_0^2 bh - \rho b \int_0^\delta u^2 dy \,|_{x=L} \qquad (1)$$

7KLVFRXOGEHFRQVLGHUHGWKHDQVZHUWRWKHSUREOHP EXWWVQRWVXHXOEHFDXVHWKHKHLJKWK LVQR NQRZQ QZLWKUHVSHFWWRWKHVKHDUOD\HUWKLFNQHVV/ 7KLVLVIRXQGE\ DSSOLQJ PDVFRQVHUYDWLRQ VLQFH WKHFRQWROYROXP HIRUP VDWWHDP WEH

$$\rho \int_{CS} (\mathbf{V} \cdot \mathbf{n})\, dA = 0 = \rho \int_0^h (-U_0) b\, dy + \rho \int_0^\delta ub\, dy \,|_{x=L}$$
or
$$U_0 h = \int_0^\delta u\, dy \,|_{x=L} \qquad (2)$$

DWWUDQFHQQJ E DQG! DQGHYDOXDWQJ WKHILUVWLQWHJUDO,QWRGXFHWKLVYDOXHRI K LQWR(T IRUP XFK FOHDQHUUHVXOW

$$D = \rho b \int_0^\delta u(U_0 - u)\, dy \,|_{x=L} \qquad Ans.\ (3)$$

7KLVUHVXOWZDVILUVWGHULYHGE\ 7KHRGRUHYRQ. iUPiQLQ — ,WUHDWVWKHIUFWLRQGUDJ RRQH VLGHRI DIODWSODWHWR WKHLQWHJUDORI WKHPRPHQWXP GHILFLW X8 í X DFURVVWKHWDLQQJ FURV

VHFWRQ RI WKH IORZ SDWWHUQ 6LQFH8 í XYDQLVKHVDV\ LQFUHDVHV WKH LQWUDFKDVDILQLWYDOX(TXDWLRQ LV DQ H[DPSOH RI PRPHQWXP LQWHJUDO WKRU IRUEXQGDU\ FDVHV ZKLFK LV WHDVG LC&KDS

0 RPHQWXP) OX[&RUUHFWLRQ) DFWRU

)RUIORZ LQDGXFWWKHD[LDOYHORFLW\LVXXDOO QRQXQLIRUP DVLQ([DPSOH)RUWKLVFDVHWKVLPSOH PRPHQWXP IOX[FDOFXODWRQ uρ(**V** · **n**) *dA* = *mV* = *ρAV*² LV VRPHZKDWLQ HUURU DQG VKRXOG EH FRUUHFWHGWR !$9 ZKHUH LVWKHGLPHQVLRQOHVVPRPHQWXP IOX[FRUUHFWLRQ IDFWRU •

7KH IDFWRU DFFRXQWV IRUWKH YDULDWLRQ RI X DFURVV WKH GXFWWHFWLRQ 7KDWLV ZH FRPSXWH WKH H[DFWOX[DQG VHWWWHTXDOWR DIOX[EDVHGRQDYHUDJHYHORFLW\LQWKHGXFW

$$\rho \int u^2 dA = \beta \dot{m} V_{av} = \beta \rho A V_{av}^2$$

or

$$\beta = \frac{1}{A} \int \left(\frac{u}{V_{av}} \right)^2 dA \qquad (3.43a)$$

9DOXHVRI FDQEH FRPSXWGEDVHGRQWSLFDOGXFWYHORFLWSURILOHVVLPLODUWRWKRVHLQ(([DPS 7KHUHVXOWDUHDVIROORZV

Laminar flow:

$$u = U_0 \left(1 - \frac{r^2}{R^2} \right) \qquad \beta = \frac{4}{3} \qquad (3.43b)$$

Turbulent flow:

$$u \approx U_0 \left(1 - \frac{r}{R} \right)^m \qquad \frac{1}{9} \le m \le \frac{1}{5}$$

$$\beta = \frac{(1+m)^2(2+m)^2}{2(1+2m)(2+2m)} \qquad (3.43c)$$

7KHWXUEXOHQWFRUUHFWRQIDFWRUVKDYHWKHIROORZLQJ UDQJHRI YDOXHV

Turbulent flow:

m	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$
β	1.037	1.027	1.020	1.016	1.013

7KHVH DUH VR FORVH WR XQLW\ WKDWWKH DUH QRUPDOO QHJOHFWHG 7KH ODPLQDU FRUUHFWRQ LV RIWHQ LPSRUWDQW

7RLOOXVWUDWHWKDWD\SLFDOXVHRI WKHVHFRUUHFWRQIDFWRUV WKHVROXWRQWR(([DPSOH IORUQRQXQLIRUP YHORFLWLHVDWWHFWRQV DQG ZRXOGEHPRGLILHGDV

$$\sum \mathbf{F} = \dot{m}(\beta_2 \mathbf{V}_2 - \beta_1 \mathbf{V}_1) \qquad (3.43d)$$

1 RWH WKDW WKH EDVLF SDUDPHWHUV DQG YHFWRU FKDUDFWHULVWLFV RI WKH UHVXOWDQW QRWFKDQJHG DWDOOE\ WKH FRUUHFWLRQ

/ LQHDU0 RPHQWXP 7LSV

7KH SUHYLRXV H[DPSOHV PDNH LWFOHDU WKDW WKH YHFWRU PRPHQWXP HTXDWLRQ LV PRUH GLIILFXOW KDQGOHG WKDQ WKH VFDODU PDWDQGHQJ\ HTXDWLRQV + HUHDUH VRPH PRPHQWXP WSVV WRUHPHEHU

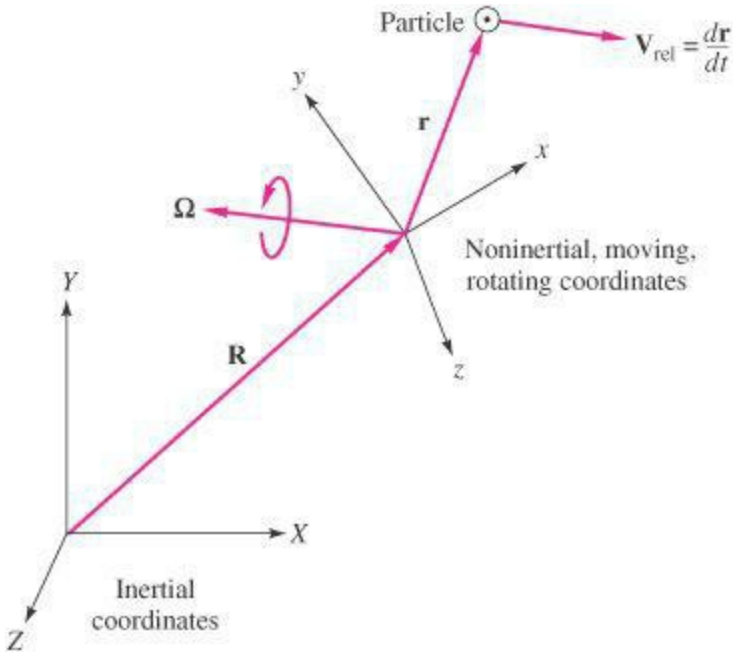
- 7KH PRPHQWXP UHDWLRQ LV DYHFWRU HTXDWLRQ 7KH IRUFHV DQG WKH PRPHQWXP WUPV DU GLUHFWLRQDOO DQG FDQ KDYH WKUHH FRPSRQHQW $ VNHWFK RI WKH VHYHFWRUV ZLOO EH LQGLVSHQVDEOH IR WKH DQDOVLV
- 7KH PRPHQWXP IOX[WUPV VXFK DV ! 9 Q G$ OQNVZR GLIHUHQWWLJQ FRQYHQWLRQV VR VSHFLDO FDUH LV QHHGHG) LUVW WKH YHFWRU FRPSRQHQW 6HFRQG WKHP DWWOBZ WUP ! 9 Q ZLOO KDYH D VLJQ GHSHQGLQJ RQ LV GLUHFWLRQ 6HFRQG WKHP DWWOBZ WUP ! 9 Q ZLOO KDYH DVLJQ í GHSHQGLQJ RQ ZKHWKHULW RXWLQ) RUH[DPSOH LQ) LJ WKH[FRPSRQHQW RI 9 DQG9 X DQG X DUH ERWK SRVLWLYH WKDW V WKH ERWK DFWLRQ WKH HUJKW0 HQZKLOH WKHP DWWOBZ DW LV SRVLWLYH RXWDQG DW L QHJDWLYH LQ
- 7KHRQH GLPHQVLRQDO SSUR[LPDWLRQ (T LV JRRURXV EHFDXVH QRQ XQIRUP YHORFLW GLVWULEXWLRQV UHTXLUH OBERURXV LQWJUDWLRQ DV LQ(T 7KVWKH PRPHQWXP IOX[FRUUHFWLRQ IDFWRUV DUHYHU XVHXOLQDJUDWLRQ HVSHFLDOO IRUSLSHIOBZ
- 7KH DSSOHG IRUFHV) DFWRQDOWKHPDWUDOLQWKHFRQWROYROXPH WKDWV WKH VXUFHV SUHVXUH DQG VK DU VWUHVVHV WKH VRLG VXSSRUW WKDWDUH FXWWKURXJK DQG WKH ZHJKWRI WK LQWHUQDO PDVHV 6WUHVVHV RQ QRQ FRQWROVXUFH SDUW RI WKH LQWHUQDU DUH VHO FDQFHOOQJ DQ VKRXOGEHLJQRUHG
- ,I WKHIOLG[LWVXEVRQFDOO WR DQDFRSKHUH WKHIOLG SUHVXUHWKHUHLVDFRSKLF
- : KHUH SRVWLEOH FKRRVH LQOHWDQG RXOHWXUFHV QRUP DOWR WKHIOBZ VR WKDWSUHVXUHV WK GRP LQDQWRUFHDQG WKHQRUP DOYHOBFLW HTXDOV WKHDFWXDOYHOBFLW

&OHDU Z LWK WKDWP DQ KHSIXOWSV VXEVDQWDOSUDFWHLVQHGGRDFKLHYHPRPHQWXP VNLOO

1 RQLQHUWDO5 HIHUHQFH) UDP H

$ GOSUHYLRXV GHULYDWRQ/DQG H[DPSOHV LQWKLV VHFWRQKDYH DXXP HG WKDWWKH FRRUGLQDWV XWP LQHUWD WKDWV DWUHWRUP RYLQJ DWFRQXDQWYHOBFLW ,QWKLV FDVH WKH UDWH RI FKDQJH RI YHOBFLW HTXDOV WKH DEVROXWDFFHOHUDWRQ RI WKH VXWP DQG1 HZWRQ¶V OBZ DSSOHV GLUHFWO LQWKH IRUP RI (TV DQG

,Q P DQ FDVHV LWLV FRQYHQLHQWWR XVH DQRQLQHUWDO RU DFFHOHUDWQJ FRRUGLQDWV XWP $ H[DPSOHZRXOG EH FRRUGLQDWV IL[HG WR DURFNHWGXULQJ WDNRI $ VFRQG H[DPSOHV DQ IOBZ RQWKH HDUWK¶V VXUFH Z KLFK LV DFFHOHUDWQJ UHODWLYH WR WKH IL[HG WDU/ EHFDXVH RI WKH URWDWRQ RI WKH HDUWK $WRVSKHUF DQG RFHDQ JUDSKLF IOBZ VH[SHULHQFH WKH VR FDOOHG&RULROLV DFFHOHUDWRQ RXWLQHG H[,WLV WSLFDOO OHW WKDQ í J Z KHUHJ LV WKH DFFHOHUDWRQ RI JUDYLW EXWWXDFFXPXODWHG HIHFWRYH GLVWDQFHV RI P DQ NLOPHWHUV FDQ EH GRP LQDQWLQ JHRSK\VLFDO IOBZ V %\ FRQWDVW WKH &RULROL DFFHOHUDWRQLVQHJOJLEOHILQVP DOOVFDOHSUREOPV QHSLSHRUDLURLOIOBZ V



) LJ * HRP HWUL RI IL[HG YHUWXV DFFHOHUDWLQJ FRRUGLQDWHV

6XSSRVH WKDW WKH IOXLG IORZ KDV YHORFLW9 UHODWLYH WR D QRQLQHUWLDO [\] FRRUGLQDWH V\VWHP D VKRZ Q LQ) LJ 7KHQG9 GWZ LOO UHSUHVHQW D QRQLQHUWLDO DFFHOHUDWLRQ WKDWP XVWEH DGGH YHFWRULDOO WR DUHODWLYH DFFHOHUDWLRQD UHODWLYH WR JLYH WKH DEVROXWH DFFHOHUDWLRQD L UHODWLYH WR VRPH LQHUWLDO FRRUGLQDWH V\VWP ; <= DVLQ) LJ 7KXV

$$\mathbf{a}_i = \frac{d\mathbf{V}}{dt} + \mathbf{a}_{rel} \tag{3.44}$$

6LQFH1HZWRQ¶V ODZ DSSOLHV WR WKH DEVROXWH DFFHOHUDWLRQ

$$\sum \mathbf{F} = m\mathbf{a}_i = m\left(\frac{d\mathbf{V}}{dt} + \mathbf{a}_{rel}\right)$$

or

$$\sum \mathbf{F} - m\mathbf{a}_{rel} = m\frac{d\mathbf{V}}{dt} \tag{3.45}$$

7KXV 1HZWRQ¶V ODZ LQ QRQLQHUWLDO FRRUGLQDWHV[\] LV DQDORJRXV WR DGGLQJ PRUH ³IRUFH´ WHUP í PD WR DFFRXQWRUQRQLQHUWLDOHIHFWV ,QWKHPRWWHQHUDOFDVH VNHWKHG LQ) LJ WKH WHUP D FRQVLQVIRXUSDUW WKUHH RI ZKLFK DFFRXQWRUWKHQJXODUYHORFLW WR WKHLQHUWLDOFRRUGLQDWHV %(LQSHFWRQ) LJ WKHDEVROXWGLVSODFHPHQWRI DSDUWFOHLV

$$\mathbf{S}_i = \mathbf{r} + \mathbf{R} \tag{3.46}$$

' LIIHUHQWLDWLQJWZLFH WKHDEVROXWYHORFLW

$$\mathbf{V}_i = \mathbf{V} + \frac{d\mathbf{R}}{dt} + \boldsymbol{\Omega} \times \mathbf{r} \tag{3.47}$$

$ VHFRQGGLIIHUHQWDWRQJLYHVWKHDEVROXWDFFHOHUDWRQ

$$\mathbf{a}_i = \frac{d\mathbf{V}}{dt} + \frac{d^2\mathbf{R}}{dt^2} + \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r} + 2\boldsymbol{\Omega} \times \mathbf{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) \qquad (3.48)$$

%\ FRPSDULVRQZLWK(T ZHVHHWKDWWKHODVWWRXUPVRQWKHULJKWUHSUHVHQWVWKHDGGLWLRQDO UHODWLYHDFFHOHUDWRQ

G5 GWLVWKHDFFHOHUDWRQRI WKHQRQLQHUWLDOULJLQRI FRRUGLQDWHV[\]
G GWî ULVWKHDQJXODUDFFHOHUDWRQHIIHFW
î 9 LVWKH&RULROLVDFFHOHUDWRQ
î î U LVWKHFHQWULSHWDODFFHOHUDWRQ GLUHFWHGIURP WKHSDUWFOHQRUPDOWR WKHD[LVR URWDWRQZLWKPDJQLWXGH÷ / ZKHUH/ LVWKHQRUPDOGLVWDQFHWR WKHD[LV

(TXDWLRQ GLIIHUVIURP (T RQO LQWKHDGGHGLQHUWLDOWHUPRQWKHOHIWKDQGVLGH 7KX VWKHFRQWROYROXPHIRUPXODWLRQRI QQH-DUPRPHQWXP LQQRQLQHUWLDOFRRUGLQDWHV PHUHO DGGV/LQHUWLDO WUPVE\ LQWURGXFLQJ WKHDGGHGUHODWLYHDFFHOHUDWLRQRYHUHDFKGLIIHUHQWLDOPDVLQWKHFRQWROYROXPH

$$\sum \mathbf{F} - \int_{CV} \mathbf{a}_{rel}\, dm = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho\, d\mathcal{V}\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V}_r \cdot \mathbf{n})\, dA \qquad (3.49)$$

where

$$\mathbf{a}_{rel} = \frac{d^2\mathbf{R}}{dt^2} + \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r} + 2\boldsymbol{\Omega} \times \mathbf{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

7KLVLVWKHQRQLQHUWLDODQDORJ RI WKHLQHUWLDOIRUP JLYHQLQ(T 7RDQDO]HVXFKSUREOHPV RQHPXVWXQGHUVWDQG WKHGLVSODFHPHQW9 DQGDQJXODUYHORFLW9 RI WKHQRQLQHUWLDOFRRUGLQDWHV ,I WKHFRQWROYROXPHLVIL[HGLQDPRYLQJIUDPH(T UHGXFHVWR

$$\sum \mathbf{F} - \int_{CV} \mathbf{a}_{rel}\, dm = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho\, d\mathcal{V}\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V} \cdot \mathbf{n})\, dA \qquad (3.50)$$

,QRWKHUZRUGV WKHUJKWKDQGVLGHUHGXFHVWRWKDWRI (T

(; $0 3/ (

$ FDVLF H[DPSOHRI DQ DFFHOHUDWQJ FRQWROYROXPHLV DURFNHWPRYLQJ WKURJKWKS DVLQ) LJ (/ HWWKHLQLWLDOPDVVEH0 DQGDVVXPHDVWDG\ H[KDXVWPDVVIORZ æDQG H[KDXVWYHORFLW9, UHODWLYHWRWKHURFNHWDVVKRZQ ,I WKHIORZ SDWWLQZLWKLQWKHURFNHWPRWRULVWHDG\ DQGDLUGUDJ LQHJOHFWHG GHULYHWKHGLIIHUHQWLDOHTXDWLRQRI YHUWLFDOURFNHWPRWRQ9 WDQGLQWHJUDWXVLQJ WKHLQLWLDFRQGLWLRQ9 DWW

6RXWLRQ

7KH DSSURSULDWH FRQWURO YROXPH LQ) LJ (HQFORVHV WKH URFNHW FXW WKURXJK WKH H[LWWWDQ DFFHOHUDWV XSZ DUG DVWRFNHWSHHG9 W 7KH] PRPHQWXP HTXDWRQ EHFRPHV

$$\sum F_z - \int a_{rel}\, dm = \frac{d}{dt}\left(\int_{CV} w\, d\dot{m}\right) + (\dot{m}w)_e$$

or $$-mg - m\frac{dV}{dt} = 0 + \dot{m}(-V_e) \qquad \text{with} \quad m = m(t) = M_0 - \dot{m}t$$

7KH VWUP D$UHO G9 GVRI WKH URFNHW7KH FRQWROYROXP HLQMUDOYDQVKHV EHFDXVH RI WKH VWDG URFNHWGRZ FRQGLWRQV 6HSDUDWWKH YDULDEOHV DQG LQWUDWH DVXPLQJ 9 DWW

$$\int_0^V dV = \dot{m}\, V_e \int_0^t \frac{dt}{M_0 - \dot{m}t} - g\int_0^t dt \quad \text{or} \quad V(t) = -V_e \ln\left(1 - \frac{\dot{m}t}{M_0}\right) - gt \qquad \textit{Ans.}$$

7KLVLVDFODVVLF DSSUR[LPDWIRUPXODLQURFNHWG QDPLFV 7KHILUVWWHUP LVSRVLWLYHDQG LI WKHIXH PDVWEXUQHGLVDODUJHIUDFWLRQRI LQLWLDOP DVW WKHILQDOURFNHWYHORFLW\FDQ[FHHG9$H

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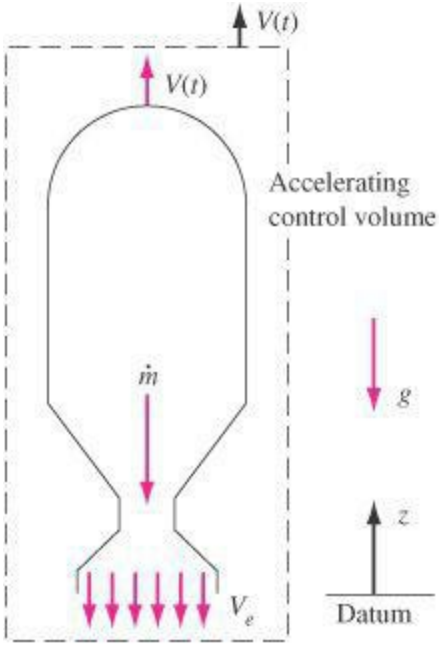


) UIFWRQDPV) ORZ 7KH%HUQRXOOL(TXDWRQ

$ FODVVLF QQHUPRPHQWP DQDOVLVLVDUHDWRQEHWZHHQSUHVVXUH YHORFLW DQGHOHYDWRQLQI IUFWRQDPV IORZ QRZ FDOOHG WKH%HUQRXOOL HTXDWRQ ,WZDVVWDWHG YDJXHO LQZRUGVLQ LQ

$$\frac{p_2 - p_1}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0$$

or

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = \text{const} \qquad (3.54)$$

7KLVLVWKH%HUQRXOOLHTXDWLRQIRUWKDG\QDPLFIORZDORQJDVWUHDPOLQHFRQGLWLRQVSUHYDLOHIIHFWLYHDOVRDOJDVWHPSHUDWXUH

%HUQRXOOL,QWHUSUHWDWLRQDVDQ(QHUJ\5HODWLRQ

7KH%HUQRXOOLUHODWLRQ(T LVDFODVVLFPRPHQXPUHVXOW1HZWRQ¶VODZIRUDIULFWLRQOHVV LQFRPSUHVVLEOHIOXLG ,WPD\ DOVR EH LQWHUSUHWHG KRZHYHU DV DQ LQGHQGHQHUJ\ UHODWLRQ 7KI FKDQJHV IURP WR LQ(T UHSUHVHQWUHYHUVLEOHSUHVXUHZRUN NLQHWLF HQHUJ\ FKDQJH DQ SRWHQWLDOHQHUJ\ FKDQJH 7KHIDFWWKDWWKHVXPRIWKHVHWKUHHWHUPVLVFRQVWDQWPHDQVWKDWWKHUHLV QR HQHUJ\ H[FKDQJHGXHWRYLVFRXVGLVVLSDWLRQ KHDWWUDQVIHURUWKHZRUN6HFWLRQ ZLOOGGGWKHVHHIIHFWVE\ PDNLQJ DFRQWROYROXPHDQDOVLVRI WKH) LUVW/DZ RI7KHUPRG\QDPLFV

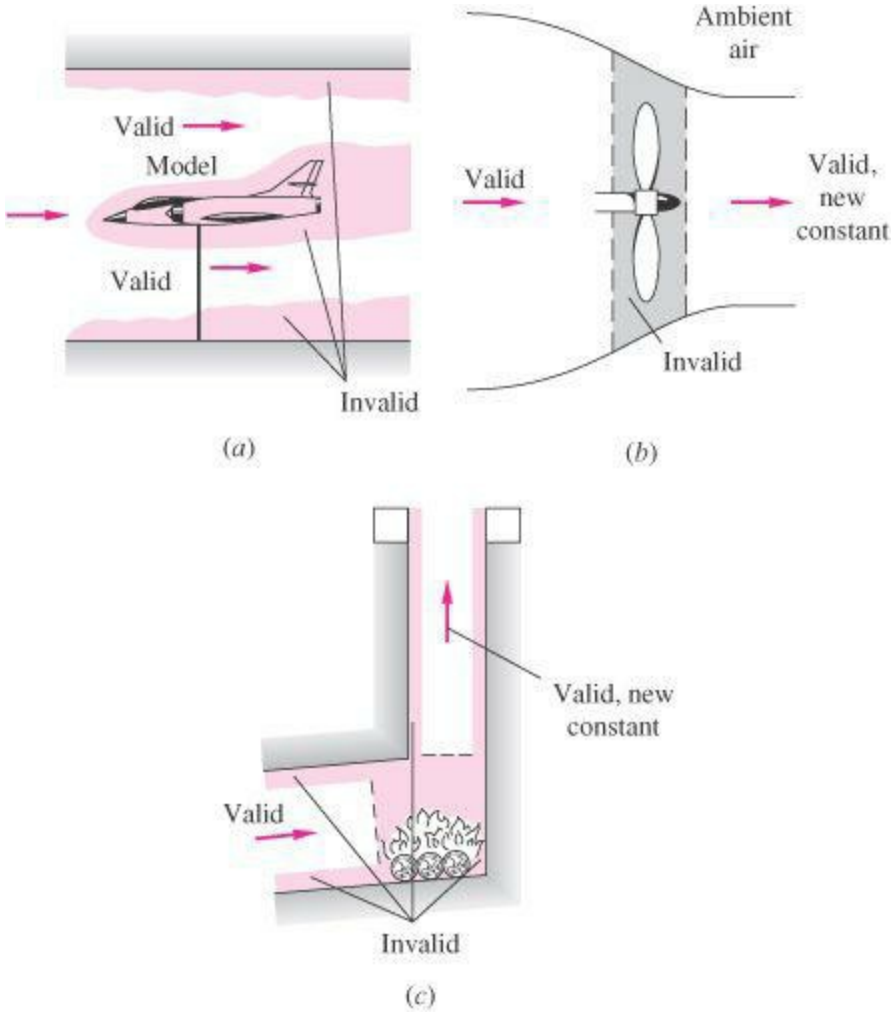
5HVWULFWLRQVRQWKH%HUQRXOOL(TXDWLRQ

7KH%HUQRXOOLHTXDWLRQLVDPRPHQWXP EDVHGIRUFHUHDWLRQDQGZDVGHULYHGXVLQJWKHIROORZLQ
UHVWULFWLYHDVVXPSWLRQV

6WDG\IORZ DFFRPPRQVLXDWLRQ DSSOLFDWLRQWRPRWWOHVLQWKLVWH[W
,QFRPSUHVVLEOHIORZ DSSURSULDWHWR0DFKQXPEHUVWKDQ 7KLVUHVWULFWLR
LVUHPRYHGLQ&KDS E\ DGGLQJIRUFRPSUHVVLELOLW\
)UHHVWUHDPOLQHVIORZ UHVWULFWLYH? VRQGZDOOVDQGPL[LQJLQWRGXFHIULFWLRQHIIHFW
)ORZ DORQJ D VLQJOH VWUHDPOLQH GLIIHUHQWVWUHDPOLQHV PD\ KDYH GLIIHUHQW³%HUQRXOO
FRQVWDQWV Z$R S! 9 J] EXWWKLVLVUDUH ,QPRVWFDVHV DVZHVKDOOSURYHLQ&KDS I
IULFWLRQOHVVIORZ UHJLRQVLVLUURWDWLRQDOWKDWLV FXUO9)RULUURWDWLRQDOIORZ WKH%HUQRXOO
FRQVWDQWVWKHVDPHHYHUZKHUH

7KH%HUQRXOGHULYDWLRQGRHVQRWDFFRXQWIRUSRWLEOHHQHUJ\ H[FKDQJH GXHWR KHDWRUZRUN
7KHVHWKHUPRG\QDPLF HIIHFWV DUH DFFRXQWHG IRULQ WKHVWDG\IORZ HQHUJ\ HTXDWLRQ : HDUHWXU
ZDUGWKDWWKH%HUQRXOOHTXDWLRQPD\ EHPRGLILHGE\ VXFKDQHQHUJ\ H[FKDQJH

)LJXUH LOOXVWUDWHVVRPHSUDFWLFDOPLVWDNHVRQWKHXVHRI%HUQRXOOV HTXDWLRQ)R
WKHZLQGWXQQHOPRGHODWWWR)LJ D WH%HUQRXOHTXDWLRQLVYDOGLQWKHFRUHIORZ RI WKHWXQQ
EXWQRWLQ WKHWXQQHOZDOOERXQGDU\ OD\HUV WKHPRGHOSUHGLFWHERXQGDU\ OD\HUV RUWKHZDNH RI WK
PRGHODORI ZKLFKDUHUHJLRQVZLWKKLJKIULFWLRQ
,QWKHSURSHOOHUIORZ RI)LJ E %HUQRXOOV HTXDWLRQLVYDOGERWKXSWUHDP DQGGRZQWUHDP
EXWZLWKDGLIIHUHQWFRQVWDQWZ S! 9 J] FDXVHGE\ WKHZRUNDGGLWLRQRI WKHSURSHOOHU7KH
%HUQRXOOUHDWLRQ LVQRWYDOGQHDUWKHSURSHOOHUEODGHVRULQWKHKHDFDOYRUWH[QRWWKRZQ
VHH)LJ VKHGGRZQWUHDP RI WKHEODGHGJHV $OVR WKH%HUQRXOFRQVWDQWVDUHKLJKHULQWK
IORZLQJ³VOLSVWUHDP´ WKDQLQWKHDPELHQWDWPRVSKHUHEHFDXVHRI WKHVOLSVWUHDP NLQHWLFHQHUJ\
)RUWKHFKLPQH\ IORZ RI)LJ F (T LVYDOGEHRUH DQGDWU WKHILUH EXWZLWKI
FKDQJHLQ%HUQRXOFRQVWDQWWKDWLV FDXVHG E\ KHDWDGGLWLRQ 7KH%HUQRXOHTXDWLRQ LV QRWYDOG
ZLWKLQWKHILUHWHWRRULQWKHFKLPQH\ ZDOOERXQGDU\ OD\HUV



) LJ , OOXVWUDWRQ RI UHJLRQV RI YDOLGLW\ DQG LQYDOLGLW\ RI WKH %HUQRXOOL HTXDWLRQ *a* VWQQ-PRGHO *b* SURSHOOHU *c* FKLP QH\

- HW/ [LW3 UHVXUH(TXDOV $ VW RVSKHULF 3 UHVXUH

: KHQ D VXEVRQLF MHWWR IOTXLG RUJDV H[LWV IURP D GXFWLQJ RU WKH IUHH DWPRVSKHUH LWLP PHGLDWC VDNHV RQ WKH SUHVXUH RI WKDVDW RVSKHUH 7KLV LV D YHU LP SRUWDQW ERXQGDU FRQGLWRQ LQ VROYLQ %HUQRXOL SUREOPV VLQFH WKH SUHVXUH DWWKDVSRLQW NQRZ Q 7KHLQMUR RI WKHIUHHMWWZ LOODOR E DWRVSKHUF H[FHSWRUVP DOOHIFWGXHVR VXUDFHWQLRQDG WHDPOQHFXYDXH

6WDJQDWRQ 6WDWF DQG ' \QDP LF 3 UHVXUHV

,Q PDQ LQFRP SUHVLEOH IORZ %HUQRXOL DQDOVHV HOHYDWRQ FKDQJHV DUH QHJOJLEOH 7KXV(T UHGXFHVWR DEDOQFHEHWZHQSUHVXUHDQGNLQHWF HQHJ\ : HFDQZULWWKLVDV

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 = p_o = \text{constant}$$

7KHTXDQWWS R LVWKHSUHVXUHDWDQ SRLQWQWKHIUFWRQOHVIORZ ZKHUHWKHYHORFLWLV]HUR ,WL FDOOHG WKHWDJQDWRQ SUHVXUH DQG LVWKHKJKHVWSUHVXUHSRVLEOHLQWKHIORZ LI HOHYDWRQ FKDQJH

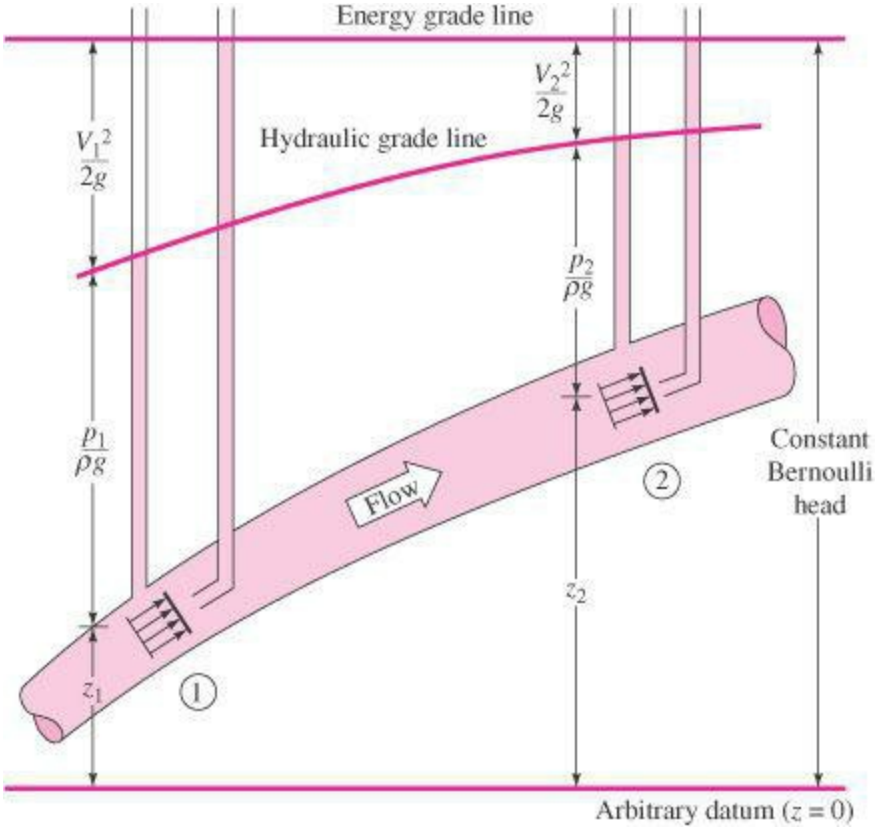
DUHQHJOHFWHG 7KHSDFHZKHUH]HURYHORFLWUHFFXUVLVFDOOHGDVWDJQDWLRQSRLQW)RUH[DPSOHRI
PRYLQJ DLUFUDIWWKH IURQWQRVH DQG WKH ZLQJ OHDGLQJ HGJHV DUH SRLQWV RI KLJKHVWSUHVVXUH 7K
SUHVVXUHVS DQGS DUH FDOOHGVWDJQ SUHVVXUHV LQ WKH PRYLQJ IOXLG 7KH JURXSLQJ !9 KD
GLPHQVLRQV RI SUHVVXUH DQG LV FDOOHG WKHG\QDPLF SUHVVXUH $ SRSXODUHYLHFDOOHG D3LWRWWXEH
VXEH)LJ PHDVXUHV S í S DQGWKHQFDOFXODWV9 IURP WKHG\QDPLF SUHVVXUH

1RWH KRZ HYHUWKDWRQHSDUWFXOU]HURYHORFLWVFRQGLWLRQ QR VOLSIORZ DORQJ DIL[HGZDOOGRH
QRWUHVXOWLQ VDQDQGLQ SUHVVXUH 7KH QR VOLS FRQGLWLRQ LV DIUHFWRQDO HIIHFWDQG WKH %HUQRXO
HTXDWLRQGRHVQRWDSSO

+\GUDXOLF DQG (QHUJ\ * UDGH/ LQHV

$ XVHXOYLVXDOLQWHUSUHWDWLRQRI %HUQRXOOL¶VHTXDWLRQLVWRVNHWFK WZR JUDGHOLQHVRI DIORZ 7KH
HQHUJ\ JUDGHOLQH (* / VKRZV WKHKHLJKWRI WKHWRWDO%HUQRXOL FRQWDQWK] S 9 J ,
IUHHERG\ZLWK QR ZRUN RUKHDWWDQVIHU>(T @WKH (* / KDV FRQWDQWKHLJKW7K
K\GUDXOLF JUDGHOLQH +* / VKRZV WKHKHLJKWFRUUHVSRQGLQJ WR HDFKYDOXH DQG SUHVVXUH KHDG]
SL² WKDWLV WKH(* / PLQXV WKHYHORFLWKHDG9 J 7KH+* / LV WKHKHLJKWZKLFKFDTXLGZRXO
ULVHLQDSLH]RPHWHUWXEH VHH3URE DWWDFKHGWRWKHIORZ ,QDQRSHQFKDQQHOIORZ WKH+* / L
LGHQWLFDOWRWKHIUHHVXUIDFHRI WKHZDWU

)LJXUH LOOXVWUDWHVWKH(* / DQG+* / IRUIUHHERG\IORZ DWWHFWRQV DQG RI DGXFW7K
SLH]RPHWHUWXEHV PHDVXUH WKH VWDWLF SUHVVXUH KHDG] S DQG WKXV RXQGH WKH+* / 7KH SLW
VWDJQDWLRQ YHORFLWVXEHV PHDVXUH WKH VXPRI KHDG] S 9 J ZKLFKFRUHVSRQGVWR WKH(* / ,
WKLVSDUWFXODUFDVHWKH(* / LVFRQVWDQWDQG WKH+* / ULVHVGXHWRDGURSLQYHORFLW



)LJ +\GUDXOLF DQGHQHUJ\ JUDGHOLQHVIRUIUHHERG\IORZ LQDGXFW

,QPRUHJHQHUDOIORZ FRQGLWLRQV WKH(* / ZLOOGURS VORZO GXHWRIULFWLRQORVVHVDQGZLOOGUR
VKDUSO GXHWRDVXEVWDQWLDOORVV DYDOYHRUEVWUXFWLRQ RUGXHWRZRUNH[WUDFWLRQ WRDVXUEOQH 7K
(* / FDQ ULVHRQO LI WKHUH LV ZRUN DGGLWLRQ DV IURP D SXPS RUSURSHOOHU 7KH+ * / JHQHUDO
IRORZ VWKHEHKDYLRURI WKH(* / ZLWKUHVSHFWRORVVHVRUZRUNWUDQVIHUDQGLWLVHVDQGRIDOOYL WK
YHORFLWVGHFUHDVHVDQGRULQFUHDVHV

$V PHQWLRQHG EHRUH QR FRQVHUYDWLRQ IDFWRU DUH QHHGHG LQ FRPSXWDWLRQV ZLWK WKH %HUQRXO
HTXDWLRQ LI FRQVLVWHQWV6, RU%* XQLWDUHXVHG DV WKHIRORZ LQJ H[DPSOHVZLOOVKRZ

,QDOO%HUQRXOL WSH SUREOPV LQ WKLV WH[W WZ HFRQVLVWHQW WDN H SRLQW XSWUHDP DQG SRLQW
GRZ QWUHDP

(; $0 3/ (

) LQG D UHDVRQ EHWZHHQ QR]]OH GLVFKDUJH YHORFLW9 DQG WDQN IUHH VXUDFH HKLJKWK DV LQ) LJ
($VXPHVWDQG IULFWLRQOHVV IORZ

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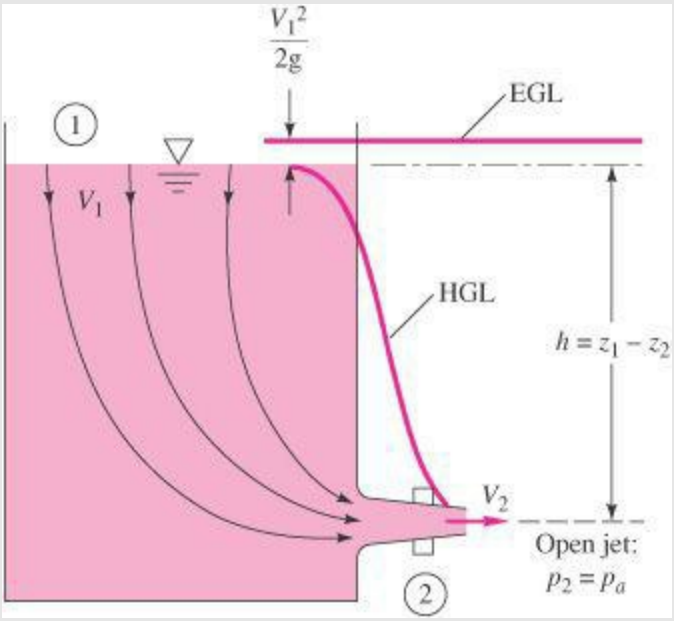

6RO XWLRQ

$VPHQWLRQHG ZHDZ DVFKRRVHSRLQW XSWUHDP DQGSRLQW GRZQWUHDP 7U WKFKRRVHSRLQ
DQG ZKHUHPD[LPXP LQRUP DWRQ LV NQRZ QRUGHWLUHG +HUH ZH VHOHFWSRLQW DV WKH WDQN IUH
VXUDFH ZKHUHHOHYDWRQDQGSUHVVXUHDUHNQRZ Q DQGSRLQW DV WKHQR]]OHH[LWZ KHUHDJDLQSUHVXU
DQGHOHYDWRQDUHNQRZ Q 7KHVZRXQNQRZ QVDUH9 DQG9

0 DVFRQVHUYDWRQLVXVXDOO DYLWOSDUWR %HUQRXOL DQDO VHV ,I $ LVWKHVDQFURVVVHFWRQDQ
$ WKHQR]]OHDUHD WKLVLVDSSUR[LPDWH DRQH GLP HQVLRQDOFRQWUROYROXPHDQG WKH FRQWDQWV

$$A_1V_1 = A_2V_2 \qquad (1)$$

%HUQRXOOL¶V HTXDWLRQ JLYHV

$$\frac{p_1}{\rho} + \tfrac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \tfrac{1}{2}V_2^2 + gz_2$$

%XWWLQFK¶V FDQG DUH ERWK H[SRVHG WR DWPRVSKHULF SUHVVXUH S S$ WKH SUHVVXUH VWUP VFDQFHOOLQJ

$$V_2^2 - V_1^2 = 2g(z_1 - z_2) = 2gh \qquad (2)$$

(OLPLQDWLQJ 9 EHWZHHQ(TV DQG Z HREWDLQWKHGHVLUHGUHVXOW

$$V_2^2 = \frac{2gh}{1 - A_2^2/A_1^2} \qquad Ans.\ (3)$$

* HQHUDOO WKH QR]]OH DUHD $ LV YHU PXFK VP DOOHU WKDQ WKH WDQN DUHD $ VR WKDW WKH UDWLR $ $ LV GRXEO QHJOLJLEOH DQG DQ DFFXUDWH DSSUR[LPDWLRQ IRU WKH HQGHVLUHG UHVXOW LV

$$V_2 \approx (2gh)^{1/2} \qquad Ans.\ (4)$$

7KLV IRUPXOD GLVFRYHUHG E\ (YDQJHOLVWD 7RUULFHOOL LQ WDXJKW VWXGHQWV WKDW WKH GLVFKDUJH YHORFLW HTXDOV WKH VSHHG WKDW D IUHHO\ SDUWLFOH Z RXOG DWWDLQ LI LWHOO IUHHO IURP SRLQW WR SRLQW , RWKHUZRUGV WKH SRWHQWLDO HQHUJ\ RI WKH VXUFH IOXLG LV HQWLUHO FRQYHUWHG WR NLQHWF HQHUJ\ RI HIOX[Z KLFK LV FRQVLVWHQW Z LWK WKH HQHUJ\ FRQVHUYDWLRQ DQG WKH IDFWKDWQR QHWSUHVXUH Z RUN LV GRQH 1 RV WKDW(T LVLQGHSHQGHQWRI WKH IOXLG GHQVLW DFKDUDFWHULVWLF RI JUDYLW GULYHQIORZ V

([FHSWRUWKHZDOOERXQGDU OD\HUV WKH VXUFHP OQHVIURP WR DOOEHKDYHLQWKHVDPHZD DQ Z HFDQ DVXPH WKDWWKH %HUQRXOOI RQXDQWK LV WKH VDPH IRUDOOWKH FRUHIORZ + RZHYHU WKH RXW IORZ LV OLNHO WR EH QRQXQIRP QRWRQHGLPHQVLRQDO VR WKDWWKH DYHUDJH YHORFLW LV RQ DSSUR[LPDWH HTXDO WR 7RUULFHOOI¶V UHVXOW7KH HQJLQHHUZLOWKHQ DGXVWWKH IRUPXOD WR LQFOXGH GLP HQVLRQOHVV GLVFKDUJHFRHILFLHQW $G

$$(V_2)_{av} = \frac{Q}{A_2} = c_d(2gh)^{1/2} \qquad (5)$$

$ VGLVFXVVHGLQ6HF WKHGLVFKDUJHFRHILFLHQWV DQR]]OHYDULHVIURP DERXW WR DV IXQFWLRQRI GLP HQVLRQOHVV IOXLG FRQGLWLRQV DQG QR]]OHVKDSH

6XUIDFH9 HORFLW &RQGLWLRQ IRUD/ DUJH7DQN

0 DQ %HUQRXOOL DQG DOR VWDG\ IORZ HQHUJ\ SUREOHPV LQYROYH OTXLGIORZ IURP D ODUJH VDQN R UHVHUYRLU DVLQ([DPSOH ,I WKH RXWIOW LVVP DOOFRP SDUHG WR WKH YROXP HRI WKH VDQN WKH VXUFH

RI WKH WDQN KDUGO PRYHV 7KHUHRUH WKHVH SURELOPV DUH DQDO]HG DVXPLQJ]HUR YHORFLW DW WKH WDQ
VXUIDFH 7KH SUHVVXUH DW WKH WRS RI WKH WDQN RUUHVSRQGV WR WKH DWPRVSKHULF

%HFDXVH SURFHHGLQJ ZLWK PRUH H[DPSOHV ZH VKRXOG QRWH FDUHIXOO WKDW D VROXWLRQ E\ %HUQRXOOL
HTXDWLRQ GRHV QRW UHTXLUH D VHFRQG FRQVWUDLQW HTXDWLRQ DV H[SODLQHG LQ VHFWLRQ RI VZR SRLQW
DQG DORQJ D JLYHQ VWUHDPOLQH 7KH FRQVWDQW LV YDOLG DORQJ WKH VWUHDPOLQH IRU IULFWLRQOHVV IORZ ZLWK
KHDWWDQVIHUUVKDWZ RUN DQG DFRQVWDQW GHQVLW RU VSHFLILF ZHLJKW

$ FDXWLFDO%HUQRXOL DSSOLFDWLRQ LV WKH IDPLOLDU SURHWV RI VSKROLQJ DIXLG IURP RQH FRQWLQ
WR DQRWKHU 1 R SXPS LV LQYROYHG D K\GURVWDF SUHVXUH GLIHUHQFH SURYLGHV WKH PRWLYH IRUFH : K
DQDO]H WKLV LQWHIRODZ LQJ H[DPSOH

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&RQVLGHU WKH ZDWUVSKRQ VKRZ QLQ) LJ ($VXPLQJ WKDW%HUQRXOL¶V HTXDWLRQ LV YDOG D
ILQGDQH[SUHVLRQ IRU WKH YHORFLW9 H[LWLQJ WKH VSKRQ WXEH E ,I WKH WXEH LV FP LQ GLDPHWHU DQG]
FP] í FP] FP DQG] FP HWUP DWWKH IORZ UDWH LQ FP V

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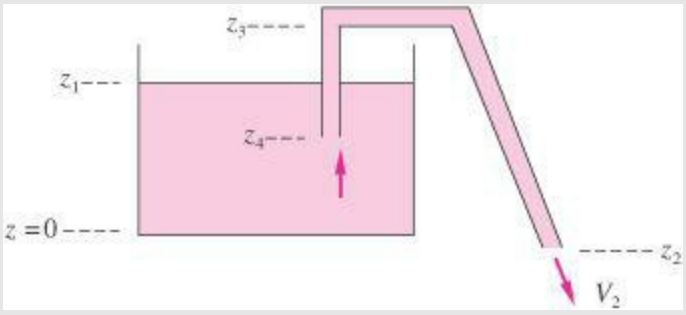

6RO XWLRQ

- $VVXPSWLRQV) UIFWLRQOHVV VWDG\ LQFRPSUHVLEOH IORZ : ULWH%HUQRXOL¶V HTXDWLRQ VDWLVI IURP ZKHUH LQRUP DWLRQ LV NQRZQ WKH VXUIDFH] DQG SURFHHGLQJ WR ZKHUH LQRUP DWLRQ LV GHVLUHG WKH WXEH H[LW]

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

1 RWH WKDW WKH YHORFLW LV DSSUR[LPDWH]HUR DW DQG D WUHDP OLQH JRHV IURP] WR] 1 RZ IXUWKHUPRUH S DQG S DUH ERWK DWPRVSKHULF S $W DQG WKH HRUH FDQFHO D 6RO YH IRU WKH H[L YHORFLW IURP WKH XEH

$$V_2 = \sqrt{2g(z_1 - z_2)} \qquad \textit{Ans. (a)}$$

7KH YHORFLW[LQ WKH VSKRQLQFUHDVHV DV WKH WXEH[LWV GRZ HUHG EHORZ WKH WDQN VXUIDF-
7KHUH LV QR VSKRQ HIHFW WKH H[LWV DW RUDERYH WKH WDQN VXUIDFH 1RWH WKDW V DQG] GR QR
GLUHFW HQWHU WKH DQDO VLV + RZ HYHU] WKRXGQRWE WRR KJK E FDXVH WKH SUHVXUH WKHUH ZLOOE
GRZ HUWKDQ DWPRVSKHUF DQG WKH QTXLGPLJKWYDSRU] H E)RUWKH JLYHQ QXPHULFDOLQRUPDWRQ
ZHQHGRQ] DQG] DQGFDOFXDWH LQ6, XQLW

$$V_2 = \sqrt{2(9.81\ \text{m/s}^2)[0.6\ \text{m} - (-0.25)\ \text{m}]} = 4.08\ \text{m/s}$$

$$Q = V_2 A_2 = (4.08\ \text{m/s})(\pi/4)(0.01\ \text{m})^2 = 321\ \text{E} - 6\ \text{m}^3/\text{s} = 321\ \text{cm}^3/\text{s} \qquad \textit{Ans. (b)}$$

- &RPPHQW 1RWH WKDW WKLV UHVXOW LV LQGHSHQGHQW RI WKH GHQVLW\ RI WKH IOXLG $ VDQH[HUFLVH \R
PD\ FKHFN WKDW IRU ZDWHU NJ P S LV 3DE-HORZ DWPRVSKHUF SUHVXUH

,Q&KDS ZHZ LOOPRGLI\ WKLVH[DPSOHWRLQFOXGHIULFWLRQHIHFWV

(; $0 3/ (

$ FRQWUDFWLRQLQDSLSHZ LOOFDXVHWKHYHORFLW\WR ULVHDQGWKHSUHVXUHWRIDOODWWHFWLRQ LQWK
WKURDW7KHSUHVXUHGLIIHUHQFHLVDPHDVXUHR WKHIORZ UDWWKURXJKWKHSLSH 7KHVPRRWKO QHFNHG
GRZ QV\VWHP VKRZ QLQ) LJ (LVFDOHGDY-HQXULWXEH) LQGDQH[SUHVLRQIRUWKHPDVIOX[LQWK
WXEHDVDIXQFWLRQR WKHSUHVXUHFKDQJH

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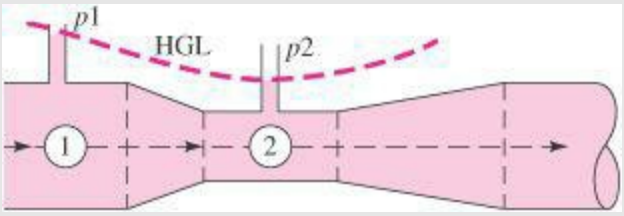

6RXWLRQ

%HUQRXOOL¶VHTXDWLRQLVDSSOLHGWRKRGDORQJ WKHFHQWUXWXUHDP GLQH

$$\frac{p_1}{\rho} + \tfrac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \tfrac{1}{2}V_2^2 + gz_2$$

,I WKHWXEHLVKRUL]RQWDO]] DQGZHFDQVROYHIRU9

$$V_2^2 - V_1^2 = \frac{2\,\Delta p}{\rho} \qquad \Delta p = p_1 - p_2 \tag{1}$$

: HUHDWWKHYHORFLWLHVIURP WKHLQFRPSUHVVLEOHFRQWLQXLWHDWLRQ

$$A_1V_1 = A_2V_2$$

or
$$V_1 = \beta^2 V_2 \qquad \beta = \frac{D_2}{D_1} \tag{2}$$

&RPELQLQJ DQG ZHREWDLQDIRUPXODIRUWKHYHORFLWLQWKHWKURDW

$$V_2 = \left[\frac{2\,\Delta p}{\rho(1-\beta^4)}\right]^{1/2} \tag{3}$$

7KHPDVVIOX[LVJLYHQE\

$$\dot{m} = \rho A_2 V_2 = A_2\left(\frac{2\rho\,\Delta p}{1-\beta^4}\right)^{1/2} \tag{4}$$

7KLVLVWKHLGHDOIUFWLRQDWVPDVVIOX[,QSUDFWLFH ZHPHDVXUHaeDFWXDO FGaeGLVFKDUJHFRHILFLHQWF
GLPHQVLRQOHVVGLVFKDUJHFRHILFLHQWF

(; $0 3/ (

$ FP ILUH KRVH ZLWK D FP QR]]OH GLVFKDUJHV P PLQ WR WKH DWPRVSKHUH $VVXPLQ
IUFWLRQOHVVIORZ ILQG WKHIRUFH) % H[HUWHGE\ WKHIOXLGHERWWRKROG WKHQR]]OHRQWKHKRVH

6RXWLRQ

: HXVH%HUQRXOOLVHTXDWLRQDQGFRQWLQXLWWRILQGWKHSUHVXUHS XSWUHDP RI WKHQR]]OH DQGWKH
ZHXVHDFRQWROYROXPHPRPHQWXP DQDOVLVWRFRPSXWWKHERWWRUFH DVLQ) LJ (

(

1 RWKHUWKDQWKHVRIWZDUHPRGHOWHTXDOLWKHVXPPDUL RQRI PRGHOWRI DOOLDSSOLHG IRUHVDERXWSRLQW 5HFDOO KRZHYHU WKDWWKLV OLZ QNH 1HZWRQ¶V OLZ DVXPHV WKDWWKH SDUWFOHYHORFLW9 LU UHODWLYH WR DQLQHUWLDO FRRUGLQDWH VWVWHP ,I QRW WKH PRPHQW DERXWSRLQW RI WKH UHODWLYH DFFHOHUDWRQWUPVD LQ(T PXWWDOR EHLQFOXGHG

ZKHUH WKH IRXUWUPV FRQWUWXLQJ D DUHJLYHQLQ(T 7KXV WKHPRWJHQHUDFHRI WK DQJXODU PRPHQWXP WKHRUHP LV IRU D GHRUPDEOH FRQWROYROXPH DVRFLDWHG ZLWK D QRQLQHUWL FRRUGLQDWVWVWHP : HFRPELQH(TV DQG WRREWLQ

)RUDQRQGHRUPDEOHLQHUWLDOFRQWROYROXPH WKLVUHGXFHVWR

)XUWKHU LI WKHUH DUH RQO RQHGLPHQVLRQDOLQHWV DQG H[LWV WKH DQJXODU PRPHQWXP IOX[WUP HYDOXDWHG RQWKHFRQWROVXUDFHEHFRPH

$OWKRXJK DWKLV VWDJH WKH DQJXODU PRPHQWXP WKHRUHP FDQEH FRQVLGHUHG DVXSSOHPHQWDU WRSLF L KDV GLUHFWDSSOLFDWRQ WR PDQ LPSRUWDQWVWHP IOLG IORZ SUREOHPV LQYROYLQJ WRUTXHV RU PRPHQW $ SDUWFXODUO LPSRUWDQWFDVH LV WKH DQDOVLV RI URWDWLQJ IOLG IORZ GHYLFHV XVXDOO FDOOH WXUERPDFKLQHV &KDS

(; $0 3/ (

$VVKRZ QLQ) LJ (D DSLSHEHQGLVXSSRUWHG DWSRLQW$ DQG FRQQHFWHG WR DIORZ VWVWHP EO[LEOHFRXSOQJV DWWHFWRQV DQG 7KHIOLGLVLQFRPSUHVLEOH DQG DP EHQWSUHVXUHS LV]HU D) LQGDQH[SUHVLRQIRUWKHWRUTXH7 WKDWP XWWEHUHLWHG E\ WKHVXSSRUWDW$ LQWUP VRI WKHIORZ SURSHUWLVDWWHFWRQV DQG DQGWKHGLVWDQFHVK DQGK E &RPSXWWKLV WRUTXHLI ' ' LQ S GBEI LQ JDJH S GBEI LQ JDJH 9 IWW K LQ K LQ DQG! VOXJVIW

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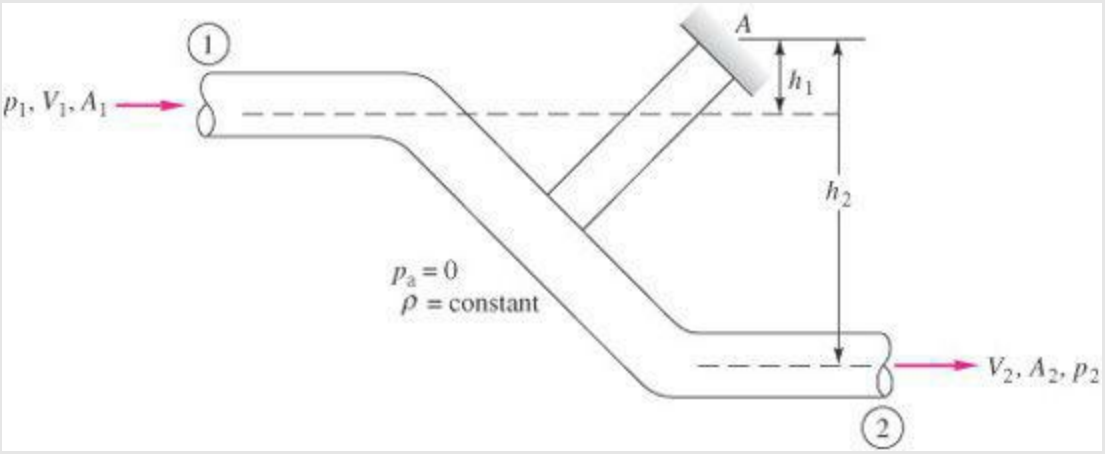


6RO XWLRQ

3DUW D

7KH FRQWURO YROXPH FKRVHQ LQ) LJ (EF XW WKURXJK VHFWLRQV DQG DQG WKURXJK WKH VXSSRUW $ ZKHUH WKH WRUTXH 7$ LV GHVLUHG 7KH IOH[LEOH FRXSOLQJ VHSDUDWHV WKH UHLVQRWRUTX DW WKH UVHFWLRQ RU DQG VR WKH FXW WKH UH[SRVHQRP RP HQW) RU WKH DQJXODU PRP HQWXP VWUP VU 9 UVKRXOG EH WDNHQ IURP SRLQW$ VR VHFWLRQV DQG 1RW WKDW WKH JDJH SUHVVXUH IRUFHV S $ DQG S $ ERWK KDYH PRP HQW DERXW$ (TXDWLRQ ZLWK RQH GLP HQVLRQDO FIX[WUP VEHFRP HV

$$\sum \mathbf{M}_A = \mathbf{T}_A + \mathbf{r}_1 \times (-p_1 A_1 \mathbf{n}_1) + \mathbf{r}_2 \times (-p_2 A_2 \mathbf{n}_2)$$
$$= (\mathbf{r}_2 \times \mathbf{V}_2)(+\dot{m}_{out}) + (\mathbf{r}_1 \times \mathbf{V}_1)(-\dot{m}_{in}) \quad (1)$$

) LJXUH(F VKRZ VWKDWDOWKHFURVVSURGXFWDUHDVRFLDWHGZLWKHWKHUU VLQ K RUU VL K WKHSHUSHQGLFXODUGLVWDQFHVIURP SRLQW$ WR WKHSLSHD[HVDW DQG 5HPHPEHUWKDWæ $_{IQ}$ æ $_{RXW}$IURP WKH WKDG IORZ FRQWUROLWU UHDWRQ ,QWUP V RI FRXQWUFRFNZLVHPRP HQWV (T WKH EHFRP HV

$$T_A + p_1 A_1 h_1 - p_2 A_2 h_2 = \dot{m}(h_2 V_2 - h_1 V_1) \quad (2)$$

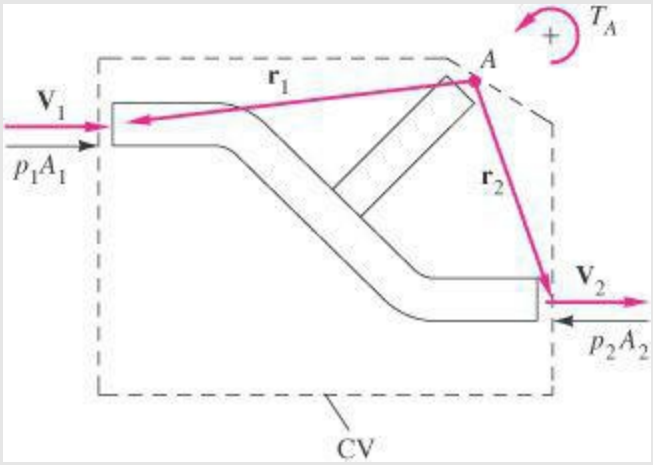
5HZ UWQJ WKLV ZHILQG WKHGHVLUHG WRUTXH WR EH

$$T_A = h_2(p_2 A_2 + \dot{m}V_2) - h_1(p_1 A_1 + \dot{m}V_1) \quad \textit{Ans. (a)} \ (3)$$

FRXQWUFRFNZLVH 7KH TXDQWLWHVS DQG S DUH JDJH SUHVVXUHV 1RW WKDWKLV UHVXOW LV LQGHSHQGHQ RI WKH VKDSH RI WKH SLSH EHQG DQG YDULHV RQO ZLWK WKH SURSHUWLHV DW WHFWRQV DQG DQG WKH GLVWDQFH

K DQGK—

(E



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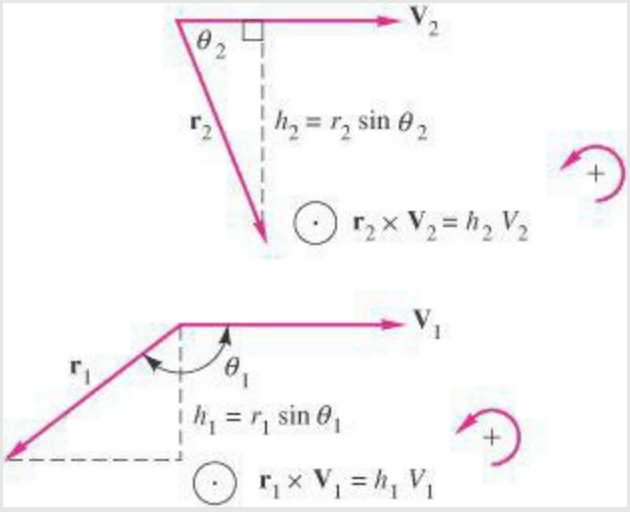


3DUWE

) RUWKHQXPHULFDOH[DPSOHFRQVLGHUHGDERYH%* XQLWV

$$D_1 = D_2 = 3\text{ in} = 0.25\text{ ft} \quad p_1 = 100\frac{\text{lbf}}{\text{in}^2} = 14{,}400\frac{\text{lbf}}{\text{ft}^2} \quad p_2 = 80\frac{\text{lbf}}{\text{in}^2} = 11{,}520\frac{\text{lbf}}{\text{ft}^2}$$

$$h_1 = 2\text{ in} = \frac{2}{12}\text{ ft} \quad h_2 = 10\text{ in} = \frac{10}{12}\text{ ft} \quad \rho = 1.94\frac{\text{slug}}{\text{ft}^3}$$

7KHLQEWDQGH[LWDUHDVDUHWKHVDPH$ $ CE IW IW 6LQFHWKHGHQLWLV
FRQWDQWZHFRQFOXGHIURP PDWFRQWHUYDWLRQ!$ 9 !$ 9 WKDW9 9 IW 7KHPDWIGRZ
LV

$$\dot{m} = \rho A_1 V_1 = \left(1.94 \frac{\text{slug}}{\text{ft}^3}\right)(0.0491 \text{ ft}^2)\left(40 \frac{\text{ft}}{\text{s}}\right) = 3.81 \frac{\text{slug}}{\text{s}}$$

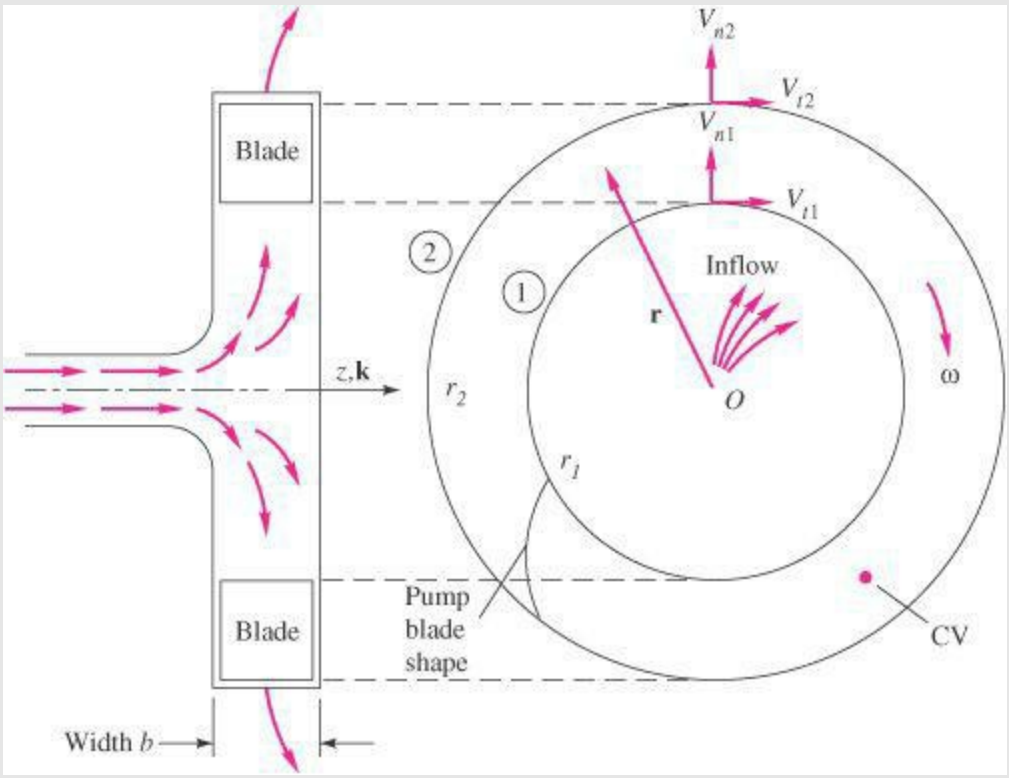
- (YDOXDWRQRI WKHWRUTXH 7KHGDWDFDQQRZ EHVXEVWLWXWHGLQWR(T

$$T_A = \left(\frac{10}{12}\text{ ft}\right)\left[\left(11{,}520 \frac{\text{lbf}}{\text{ft}^2}\right)(0.0491 \text{ ft}^2) + \left(3.81 \frac{\text{slug}}{\text{s}}\right)\left(40 \frac{\text{ft}}{\text{s}}\right)\right]$$
$$- \left(\frac{2}{12}\text{ ft}\right)\left[\left(14{,}400 \frac{\text{lbf}}{\text{ft}^2}\right)(0.0491 \text{ ft}^2) + \left(3.81 \frac{\text{slug}}{\text{s}}\right)\left(40 \frac{\text{ft}}{\text{s}}\right)\right]$$
$$= 598 \text{ ft} \cdot \text{lbf} - 143 \text{ ft} \cdot \text{lbf} = 455 \text{ ft} \cdot \text{lbf counterclockwise} \qquad \textit{Ans. (b)}$$

- &RPPHQWV 7KHXVHRI VWDQGDUG%* XQLWVLVFUXFLDOZKHQFRPELQLQJ GLVWLPLODUWHUPV VXF DVSUHVVXUHWHUPHVDUHDDQGPDVVIORZ WHUPHVYHORFLW LQRUSURSHUDGGLWLYHXQLWVIRUDQHUFD VROXWLRQ

(; $0 3/ (

) LJXUH VKRZ VDVFKHPDWF RI DFHQWULXJDOSXPS 7KHIOLGHQWHUVD[LDOO DQGSDVVHVWKURXJ VKHSXPSEODGHV ZKLFKURWDWHDVDQJXODUYHORFLW& VKHYHORFLWRI VKHIOLGLVFKDQJHGIURP 9 V9 DQGLWSUHVVXUHIURP S VS D) LQGDQH[SUHVLRQIRUWKHWRUTXH7R VKDWPXWWEHDSSOLHGWR VKHP EODGHVWR PDLQWDLQWKLVIORZ E 7KHSRZHUVXSSOLHGWR VKHSXPS ZRXOGEH3 &7R 7R LOOXVWDW QHUFDO VXSSRVHU P U P DQGE P /HWWKHSXPSURWDWHDW UPLQDQ GHOYHUZDWUDW P VZLWKDGHQVLWRI NJ P &RPSXWWKHWRUTXHDQGSRZHUVXSSOHG



) LJ 6FKHPDWLF RI DVLPSOLILHG FHQWULXJDO SXPS

6RXWLRQ

3DUW D

7KH FRQWURO YROXPH LV FKRVHQ WR EH WKH DQQXODU UHJLRQ EHWZHHQ VFUROOV DQG ZKHUH WKH IORZ SDVVHV WKURXJK WKH SXPS EODGHV VHH) LJ 7KH IORZ LV VWDGH DQG DXWRPHGLQFRPSUHVLEOH 7K FRQWLEXWLRQ RI SUHVVXUH WR WKH WRUTXH DERXW [LV2 LV]HUR VLQFH WKH SUHVVXUH IRUFHV DW DQG DF UDGLDOO WKURXJK2 (TXDWLRQ EHFRPHV

$$\sum \mathbf{M}_o = \mathbf{T}_o = (\mathbf{r}_2 \times \mathbf{V}_2)\dot{m}_{\text{out}} - (\mathbf{r}_1 \times \mathbf{V}_1)\dot{m}_{\text{in}} \qquad (1)$$

ZKHUH WKH IORZ FRQWURO VXUIDFH

$$\text{æ}_{\text{IQ}} \; !\, 9_{\text{Q}} \; \text{CEJE} \quad \text{æ}_{\text{RXW}} \; !\, 9_{\text{Q}} \; \text{CEJE} \; !\, 4$$

7KH FURVVSURGXFWî 9 LVIRXQG WR EH FORFNZLVH DERXW2 DVEHIRUH WKH VFUROOV

$$\text{U} \quad 9 \quad \text{U}9_{\text{W}} \, \text{VLQ} \quad f \, \text{N} \quad \text{U}9_{\text{W}} k \quad \text{FORFNZLVH}$$

$$\text{U} \quad 9 \quad \text{U}9_{\text{W}} \text{N} \, \text{FORFNZLVH}$$

(TXDWLRQ WKXV EHFRPHV WKH GHVLUHG IRUP XOD IRU WRUTXH

$$T_o = \rho Q(r_2 V_{t2} - r_1 V_{t1})k \qquad \text{clockwise} \qquad \textit{Ans. (a)} \quad (2a)$$

7KLV UHDWLRQ LV FDOOHG(XOHUTV WXUELQH IRUPXOD ,Q DQ LGHDOL]HG SXPS WKH LQOHWDQG RXWOHW VDQJHQWLDOYHORFLWLHV ZRXOG PDWK WKH EODGH URWDWLRQDO VSHHGV9W &U DQG9W &U 7KHQ WKH IORUPXOD IRUWRUTXHVXSSOLHGEHFRPHV

$$T_o = \rho Q\omega(r_2^2 - r_1^2) \qquad \text{clockwise} \qquad (2b)$$

3DUW E

&RQYHUW& WR CE UDGV 7KHQRUPDOYHORFLWLHVDUHQRWQHHGHGKHUHEXWRQOBZ IURP WKHIORZ UDWH

$$V_{n1} = \frac{Q}{2\pi r_1 b} = \frac{2.5\ \text{m}^3/\text{s}}{2\pi(0.2\ \text{m})(0.15\ \text{m})} = 13.3\ \text{m/s}$$

$$V_{n2} = \frac{Q}{2\pi r_2 b} = \frac{2.5}{2\pi(0.5)(0.15)} = 5.3\ \text{m/s}$$

) RUWKHIGHDQ]HGLQOHWDQGRXWOHWWDQJHQWLDOYHORFLWLHVTXDOWKHSSHHG

$$V_{t1} = \omega r_1 = (62.8\ \text{rad/s})(0.2\ \text{m}) = 12.6\ \text{m/s}$$
$$V_{t2} = \omega r_2 = 62.8(0.5) = 31.4\ \text{m/s}$$

(TXDWLRQ D SUHGLFWVWKHUHTXLUHGWRUTXHWRE H

$$T_o = (1000\ \text{kg/m}^3)(2.5\ \text{m}^3/\text{s})[(0.5\ \text{m})(31.4\ \text{m/s}) - (0.2\ \text{m})(12.6\ \text{m/s})]$$
$$= 33{,}000\ (\text{kg}\cdot\text{m}^2)/\text{s}^2 = 33{,}000\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

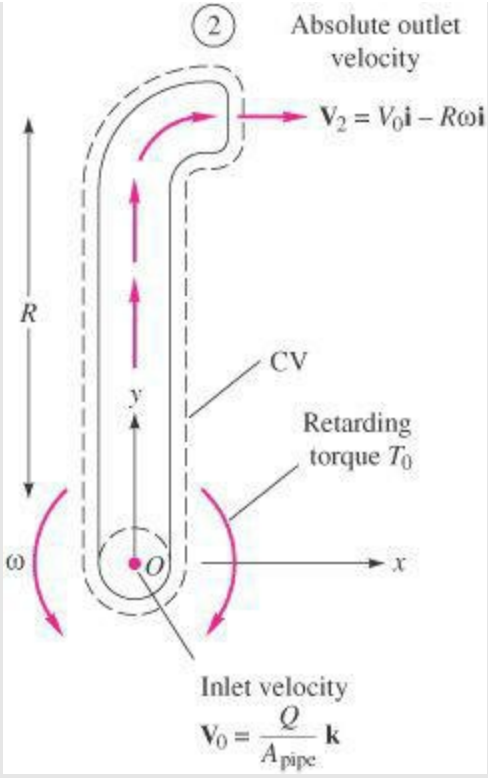
7KHSRZHUUHTXLUHGLV

$$P = \omega T_o = (62.8\ \text{rad/s})(33{,}000\ \text{N}\cdot\text{m}) = 2{,}070{,}000\ (\text{N}\cdot\text{m})/\text{s}$$
$$= 2.07\ \text{MW}\ (2780\ \text{hp}) \qquad \textit{Ans.}$$

,QDFWXDOSUDFWLFHWKHVDQJHQWLDOYHORFLWLHVDUHFRQVLGHUDEO GHVWKDQWKHLPSHOOHUWSSHGV DQG WKHGHVLJQSRZHUUHTXLUHPHQWIRUWKLVSXPSPD\ EHRQO 0: RUOHVV

(; $0 3/ (

) LJXUH VKRZVDOOZQVSULQNOHUDUP YLHZHGIURP DERYH 7KHDUP URWDWVDERXW DWFRQVWDQ DQJXODUYHORFLW& 7KHYROXPHIOX[HQWHULQJ WKHDUP DW LV4 DQG WKHIOXLGLVIRRP SUHVVLEOH 7KHU LVDUHGXFLQJ WRUTXHDW GXHWRE HDULQJ IULFWRQ RI DPRXQW7R N) LQGDQH[SUHVVLRQIRUWKHURWDWR & LQWHUPVRI WKHDUP DQGIOXLG SURSHUWLHV



) LJ 9 LHZ IURP DERYH RI DVLQJOH DUP RI DURWDWLQJ ODZQ VSULQNOHU

6RXWLRQ

7KH HQWUDQW YHORFLWLV9 N ZKHUH9 4 $_{SSH}$ (TXDWLRQ DSSOLHV WR WKH FRQWURO YROXP VHOHFWHG LQ) LJ RQO LI 9 LV WKH DEVROXWH YHORFLW UHODWLYH WR DQ LQHUWLDO IUDPH 7KXV WKH H[L YHORFLWV DWH[WFLRQ LV

9 9 L± 5&L

(TXDWLRQ WKH QSUHGLFWWKDW RUXWDG IORZ

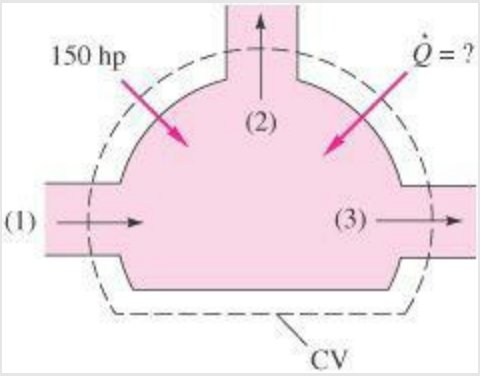
$$\sum \mathbf{M}_o = -T_o\mathbf{k} = (\mathbf{r}_2 \times \mathbf{V}_2)\dot{m}_{\text{out}} - (\mathbf{r}_1 \times \mathbf{V}_1)\dot{m}_{\text{in}} \qquad (1)$$

ZKHUH IURP FRQWQXLW $a_{RXW} a_{LQ}$!4 7KH FURVSURGXFWZ LWKUHHUHQFHWRSRLQW2 DUH

$$\mathbf{r}_2 \times \mathbf{V}_2 = R\mathbf{j} \times (V_0 - R\omega)\mathbf{i} = (R^2\omega - RV_0)\mathbf{k}$$
$$\mathbf{r}_1 \times \mathbf{V}_1 = 0\mathbf{j} \times V_0\mathbf{k} = 0$$

(TXDWLRQ WKXVEHFRPHV

(



: RUNLVSURYLGHGWRWKHPDFKLQHDWWKHUDWHRI KS) LQGWKHSUHVVXUHS LQGE LQ DEVROXWHDQ WKHKHDWWDQVIHU $VVXPHWKDWWLVDSHUFHQWJDVZLWK5 DQGF$S IWGE VOXJ *f5

6RODWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZV LQDW QHJDWLYH IOX[DQG RXWHW DQG SRVLWLY IOX[HV
- $VVXPSWLRQV 6WDG\ IORZ RQHGLPHQVLRQDOLQDW DQG RXWHW LGHDOJDV QHJOLJLEOH VKHD ZRUN 7KH IORZ LVQRWLQFRPSUHVLEOH 1RWH WKDW4 • 4 4 EHFDXVH WKH GHQVLWLHV DU GLIIHUHQW
- $SSURDFK (YDOXDWH WKH YHORFLWLHV DQG GHQVLWLHV DQG HQWKDOSLHV DQG VXEVWLWXWH LQR(T 8VH%* XQLWIRUDOOSURSHUWLHV LQFOXGLQJ WKHSUHVVXUHV : LW4L JLYHQ ZHHYDOXDW9L 4L $L

$$V_1 = \frac{Q_1}{A_1} = \frac{100\ \text{ft}^3/\text{s}}{0.4\ \text{ft}^2} = 250\,\frac{\text{ft}}{\text{s}} \qquad V_2 = \frac{40\ \text{ft}^3/\text{s}}{1.0\ \text{ft}^2} = 40\,\frac{\text{ft}}{\text{s}} \qquad V_3 = \frac{50\ \text{ft}^3/\text{s}}{0.25\ \text{ft}^2} = 200\,\frac{\text{ft}}{\text{s}}$$

7KHGHQVLWLHVDWWHFWLRQV DQG IRGRZ IURP WKHLGHDOJDVGZ

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(20 \times 144)\ \text{lbf/ft}^2}{[1716\ \text{ft-lbf/(slug°R)}][(70 + 460)\text{°R}]} = 0.00317\,\frac{\text{slug}}{\text{ft}^3}$$

$$\rho_2 = \frac{(30 \times 144)}{(1716)(100 + 460)} = 0.00450\,\frac{\text{slug}}{\text{ft}^3}$$

+RZ HYHU S LVXQNQRZ Q VRKRZ GRZ HILQG! " 8VHWKHVWDG\ IGRZ FRQWUROXPH WRGHWHUPLQH

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \qquad \text{or} \qquad \rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3 \tag{1}$$

$$\left(0.00317\,\frac{\text{slug}}{\text{ft}^3}\right)\left(100\,\frac{\text{ft}^3}{\text{s}}\right) = 0.00450(40) + \rho_3(50) \quad \text{solve for } \rho_3 = 0.00274\,\frac{\text{slug}}{\text{ft}^3}$$

. QRZ LQJ ! HQDEOHV XV WR ILQG S IURP WKH LGHDO JDV ODZ

$$p_3 = \rho_3 R T_3 = \left(0.00274\,\frac{\text{slug}}{\text{ft}^3}\right)\left(1716\,\frac{\text{ft-lbf}}{\text{slug }^\circ\text{R}}\right)(200 + 460^\circ\text{R}) = 3100\,\frac{\text{lbf}}{\text{ft}^2} = 21.5\,\frac{\text{lbf}}{\text{in}^2} \quad \textit{Ans.}$$

-) LQDOVROXWRQ VWHSV) RUDQLGHDOJDV VLPSOH DSSUR[LPDWHQHVVDSSOLHVDVK F S7 L 7KHVKD ZRUN LVQHJDWLYH LQRR WKH FRQWURO YROXPH DQG YLVFRXV ZRUN LV QHJOHFWHG IRUWKLV VROG ZDO PDFKLQH

$$\dot{W}_v \approx 0 \qquad \dot{W}_s = (-150\text{ hp})\left(550\,\frac{\text{ft-lbf}}{\text{s-hp}}\right) = -82{,}500\,\frac{\text{ft-lbf}}{\text{s}} \quad \text{(work } \textit{on}\text{ the system)}$$

) RU VWDG\ IGRZ WKH YROXPH LQWHJUDO LQ(T YDQLVKHV DQG WKH HQHUJ\ HTXDWLR EHFRPHV

$$\dot{Q} - \dot{W}_s = -\dot{m}_1(c_p T_1 + \tfrac{1}{2}V_1^2 + gz_1) + \dot{m}_2(c_p T_2 + \tfrac{1}{2}V_2^2 + gz_2) + \dot{m}_3(c_p T_3 + \tfrac{1}{2}V_3^2 + gz_3) \tag{2}$$

) URP RXUFRQWURO YROXPH DQDO\VLV(T DERYH WKHP DVV IGRZ VDUH

$$\dot{m}_1 = \rho_1 Q_1 = (0.00317)(100) = 0.317\,\frac{\text{slug}}{\text{s}} \qquad \dot{m}_2 = \rho_2 Q_2 = 0.180\,\frac{\text{slug}}{\text{s}}$$

$$\dot{m}_3 = \rho_3 Q_3 = 0.137\,\frac{\text{slug}}{\text{s}}$$

,WLV LQWXLWLYH WR VHSDUDWH WKH IOX[WHUPV LQ WKH HQHUJ\ HTXDWLR IRU H[DPLQDWLR

$$\begin{aligned}
\text{Enthalpy flux} &= c_p(-\dot{m}_1 T_1 + \dot{m}_2 T_2 + \dot{m}_3 T_3) \\
&= (6003)[(-0.317)(530) + (0.180)(560) + (0.137)(660)] \\
&= -1{,}009{,}000 + 605{,}000 + 543{,}000 \approx +139{,}000 \text{ ft-lbf/s} \\
\text{Kinetic energy flux} &= \tfrac{1}{2}(-\dot{m}_1 V_1^2 + \dot{m}_2 V_2^2 + \dot{m}_3 V_3^2) \\
&= \tfrac{1}{2}[-0.317(250)^2 + (0.180)(40)^2 + (0.137)(200)^2] \\
&= -9900 + 140 + 2740 \approx -7000 \text{ ft-lbf/s} \\
\text{Potential energy flux} &= g(-\dot{m}_1 z_1 + \dot{m}_2 z_2 + \dot{m}_3 z_3) \\
&= (32.2)[-0.317(1.0) + 0.180(4.0) + 0.137(1.5)] \\
&= -10 + 23 + 7 \approx +20 \text{ ft-lbf/s}
\end{aligned}$$

(TXDWLRQ PD\ QRZ EH HYDOXDWHG IRU WKH KHDW WUDQVIHU

$$\dot{Q} - (-82{,}500) = 139{,}000 - 7{,}000 + 20$$

or
$$\dot{Q} \approx \left(+49{,}520\,\frac{\text{ft-lbf}}{\text{s}}\right)\left(\frac{1\ \text{Btu}}{778.2\ \text{ft-lbf}}\right) = +64\,\frac{\text{Btu}}{\text{s}} \qquad \textit{Ans.}$$

- &RPPHQWV 7KH KHDW WUDQVIHU LV SRVLWLYH Z KLFK PHDQV LQWR WKH FRQWROYROXPH ,WLV WSLFD RI JDVIORZ V WKDW SRWHQWLDO HQHUJ\ IOX[LV QHJOLJLEOH HQHUJ\ IOX[LV GRPLQDQW DQG NLQHWLF HQHUJ\ IOX[LV VP DOOXQDWV WKH YHORFLWLHV DUH YHU\ KLJK WKDWV KLJK VXEVRQLF RU VXSHUVRQLF

7KH6WHDG\ )ORZ (QHUJ\ (TXDWLRQ

)RUVWHDG\ IORZ ZLWKRQHLQOHWDQGRQHRXWOHWERXQGDU\VXUIDFHODEHOHGDQGRQHHQHUJ\WUDQVIHU
WKURXJKHDFKEUDQFKRQXVHGLQPDQ\HQJLQHHULQJDQDOVHV/HWVHFWLRQEHWKHLQOHWDQGVHFWLRQ WK
RXWOHW7KHQ

$$\dot{Q} - \dot{W}_s - \dot{W}_v = -\dot{m}_1(\hat{h}_1 + \tfrac{1}{2}V_1^2 + gz_1) + \dot{m}_2(\hat{h}_2 + \tfrac{1}{2}V_2^2 + gz_2) \qquad (3.69)$$

%XWIURP FRQWLQXLW\ æ æ æZHFDQUHDUUDQJH DVIROORZV

$$\hat{h}_1 + \tfrac{1}{2}V_1^2 + gz_1 = (\hat{h}_2 + \tfrac{1}{2}V_2^2 + gz_2) - q + w_s + w_v \qquad (3.70)$$

ZKHUH$q = \dot{Q}/\dot{m} = dQ/dm$ WKHKHDWWUDQVIHUUHGWRWKHIOXLGSHUXQLWPDVV6LPLODUO\ Z$_V$: $_V$æ
G $_V$ GP DQGZ$_Y$ %$_Y$ æ G $_Y$GP (TXDWLRQ LVDJHQHUDOIRUP RI WKHVWHDG\ IORZ HQHUJ\
HTXDWLRQ ZKLFK VWDWHV WKDWWKH XSWUHDP VWDJQDWLRQ HQWKDOS\ $H_1 = (h + \tfrac{1}{2}V^2 + gz)_1$ GLIIHUV IURP WK
GRZQVWUHDP YDOXH+ RQO\ LI WKHUHLVKHDWWUDQVIHU WKDWZRUN RUYLVFRXVZRUNDVWKHIOXLGSDVVH
EHWZHHQVHFWLRQV DQG 5HFDOOWKDWTLVSRVLWLYHLI KHDWLVDGGHGWRWKHFRQWUROYROXPHDQGWKDWZ
DQGZ$_%$ DUHSRVLWLYHLI ZRUNLVGRQHE\ WKHIOXLGRQWKHVXUURXQGLQJV

(DFKWHUP LQ(TXDWLRQ KDVWKHGLPHQVLRQV RI HQHUJ\ SHUXQLWPDVV RUYHORFLW\VTXDUHG ZKLFK
LVDIRUP FRPPRQO\ XVHGE\ PHFKDQLFDOHQJLQHHUV ,I ZHGLYLGHWKURXJKE\ J HDFKWHUP EHFRPHVI
OHQJWK RUKHDG ZKLFK LVDIRUP SUHIHUUHGE\ FLYLOHQJLQHHUV 7KHWDGLVRQDOV\PEROIRUKHDG LVK
ZKLFK ZHGR QRWZLVK WR FRQIXVH ZLWKHQWKDOS\ 7KHUHRUH ZHXVHLQWHUQDOHQHUJ\ LQUHZULWLQJ WK
KHDGIRUP RI WKHHQHUJ\ UHDWLRQ

$$\frac{p_1}{\gamma} + \frac{\hat{u}_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\hat{u}_2}{g} + \frac{V_2^2}{2g} + z_2 - h_q + h_s + h_v \qquad (3.71)$$

ZKHUHK$_T$ TJ K$_V$ Z$_V$J DQGK$_%$ Z$_X$ J DUHWKHKHDGIRUPVRI WKHKHDWDGGHG VKDIWZRUNGRQ
DQG YLVFRXVZRUNGRQH UHVSHFWLYHO\ 7KHWHUP S LVFDOOHGSUHVXUHKHDG DQGWKHWHUP 9 J L
GHQRWVDVYHORFLW\KHDG

) ULFWLRQDQG6KDIW RUNLQ/ RZ6SHHG) ORZ

$ FRPPRQDSSOLFDWLRQRI WKHVWHDG\ IORZ HQHUJ\ HTXDWLRQLVIRUIORZ VSHHG LQFRPSUHVVLEOH IORZ
WKURXJKDSLSHRUGXFW$ SXPS RUWXUELQHPD\ EHLQFOXGHGLQWKHSLSHV\VWHP 7KHSLSHDQGPDFKLQ
ZDOOVDUHVROG VRWKHYHORFLW\ZRUNLV]HUR (TXDWLRQ PD\ EHZULWWHQDV

$$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right) + \frac{\hat{u}_2 - \hat{u}_1 - q}{g} \qquad (3.72)$$

(YHU WKLV LQWHJUDWHG HQJLQHHULQJ RUKHDG 7KH WHUP VLQSDUHQWKHVHV DUH WKH XSWUHDP DQG GRZQWUHDP YDOXHV RI WKH XVHXORUDYDLODEOHKHDG RU WKH IOXLG GHQVLW\ K 7KH ODVW WHUP RQ WKH ULJKW LV WKH GLIIHUHQFH K í K ZKLFK FDQ LQFOXGH SXPS KHDG LQSXW WXUELQH KHDG H[WDFWLRQ DQG WKH IULFWLRQ KHDG ORVV K DOZD\V SRVLWLYH 7KXV LQ LQFRPSUHVVLEOH IORZ ZLWKRXW HQHUJ DQG RQH RXWHWZ HP D\ ZULWH

$$\left(\frac{p}{\gamma}+\frac{V^2}{2g}+z\right)_{\text{in}}=\left(\frac{p}{\gamma}+\frac{V^2}{2g}+z\right)_{\text{out}}+h_{\text{friction}}-h_{\text{pump}}+h_{\text{turbine}} \qquad (3.73)$$

0 RVWRI RXULQWHUHVWHG SUREOHPV ZLOOEH VROYHG ZLWK WKH DLG RI (T 7KH K WHUPV DUH DO SRVLWLYH WKDWLV IULFWLRQ ORVV LV DOZD\V SRVLWLYH LQ UHDO YLVFRXV IORZV D SXPS DGGV HQHUJ LQFUHDVHV WKH HLWKDQG VLGH DQG D WXUELQH H[WDFWV HQHUJ\ IURP WKH IOXLG ,I K DQG RUK DU LQFOXGHG WKH SXPS DQG RU WXUELQH PXVW OHEHWZHHQ SRLQW DQG ,Q&KDSV DQG ZH VKDO GHYHORS PHWKRGV RI FRUUHODWLQJ ORVVHV ZLWK IORZ SDUDPHWHUV LQ SLSHV YDOYHV ILWWLQJV DQG RWKHU LQWHUQDO ORZ GHYLFHV

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* DVROLQHDW f& LV SXPSHG WKURXJK D VPRRWK FP GLDPHWHU SLSH NP ORQJ DWD IORZ UDWH P K JDOPLQ 7KH LQGHWLV IHGE\ DSXPS DWDQ DEVROXWH SUHVVXUH RI DW 7KH H[WLV D VDQGDUG DWPRVSKHULF SUHVVXUH DQG LV P KJKHU (VWLPDWH WKH IULFWLRQDOK HDG ORVV K DQG FRPSDU LWWR WKH YHORFLW\KHDG RI WKH IORZ 9 J 7KHVH QXPEHUV DUH TXLWH UHDOLVWLF IRU OLTXLG IORZ WKURXJ ORQJ SLSHOLQHV

6RXWLRQ

- 3URSHUWLHV) URP 7DEOH$ IRU JDVROLQH f& ! NJ P RU 1 P
- $VVXPSWLRQV 6WDG\ IORZ 1RVKDIWZRUN WKXVK K ,I] WKHQ] P
- $SSURDFK) LQG WKH YHORFLW\ DQG WKH YHORFLW\KHDG 7KHVH DUH QHHGHG IRU FRPSDULVRQ 7KHYDOXDWWKH IULFWLRQORVV IURP (T
- 6RXWLRQ VWHSV 6LQFH WKH SLSH GLDPHWHU LV FRQVWDQW WKH DYHUDJH YHORFLW\ LV WKH VDPH HYHUZKHUH

$$V_{\text{in}}=V_{\text{out}}=\frac{Q}{A}=\frac{Q}{(\pi/4)D^2}=\frac{(75\ \text{m}^3/\text{h})/(3600\ \text{s/h})}{(\pi/4)(0.12\ \text{m})^2}\approx 1.84\ \frac{\text{m}}{\text{s}}$$

$$\text{Velocity head}=\frac{V^2}{2g}=\frac{(1.84\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)}\approx 0.173\ \text{m}$$

6XEWUDFWLQJ(T DQG VROYH IRU WKH IULFWLRQ KHDG ORVV 8VH SDFDO IRU WKH SUHVVXUH DQG QRWH WKDW WKH YHORFLWK HDG VFDQFHOEHFDXVH RI WKH FRQWLQXLW\ HTXDWLRQ

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} = \frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} + h_f$$

$$\frac{(24)(101{,}350\ \text{N/m}^2)}{6670\ \text{N/m}^3} + 0.173\ \text{m} + 0\ \text{m} = \frac{101{,}350\ \text{N/m}^2}{6670\ \text{N/m}^3} + 0.173\ \text{m} + 150\ \text{m} + h_f$$

or $$h_f = 364.7 - 15.2 - 150 \approx 199\ \text{m} \qquad \textit{Ans.}$$

7KH IULFWLRQ KHDG LV ODUJH UHODWLYH WR WKH HOHPHQWDU\ FKDQJH û] DQG WKH SXPS PXVW GLYH WKH IORZ DJDLQVW WKDW FKDQJHV KHQFH WKH KLJK LQHWSUHVXUH 7KHUHIRUH IULFWLRQ\ YHORFLWK HDGV

$$\frac{h_f}{V^2/(2g)} \approx \frac{199\ \text{m}}{0.173\ \text{m}} \approx 1150 \qquad \textit{Ans.}$$

- &RPPHQWV 7KLV KLJK UDWLR LV WSLFDO RI ORQJ SLSHOLQHV 1RWH WKDW Z HGLG QRW P DNH GLUHFWXV RI WKH P SLSH OHQJWK Z KRVH HIHFWLV KLGGHQ Z LWKLQ K, ,Q&KDS Z H FDQ VWDWH WKL SUREOP LQDP RUHGLUHFWDVKLRQ * LY-QWKHIORZ UDWH IXLG DQG SLSHVI]H Z KDWLQHWSUHVXUHL QHHGHG' 2XUFRUHDVRQVIRUK, Z LOOOHDGVRWKHHWWP DWS$QW§ DW DVXDWGKUH

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$ LU>5 FS IW GEI VOXJ *f5 @IORZ V VWADGLO DVVKRZQ LQ) LJ (VKURXJK I VXUELQHWKDVSURGXFHV KS)RUWKHLQHWDQGH[WFRQGLWRQVVKRZQ HWP DW D WKHH[WY-DFLW9 DQG E WKHKEAWDQVHUHG4 LQ%ØXK

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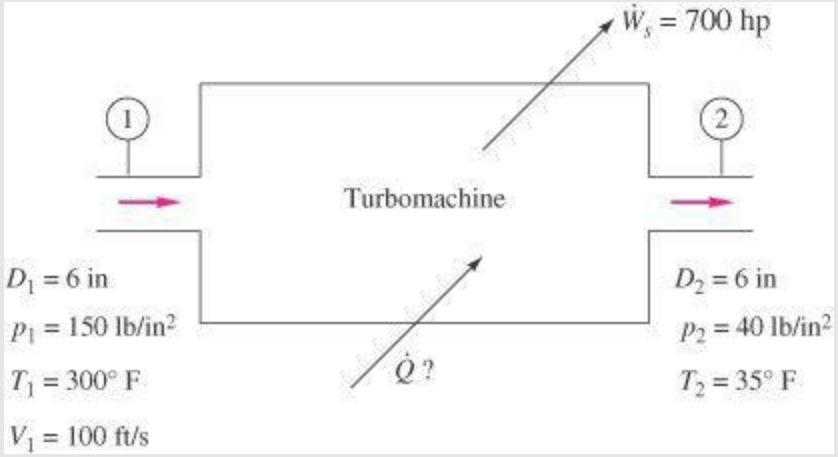

6ROXWLRQ

3DUWD

7KHLQLWLDOGHQVLWLHVFDQEHIRXQGIURP WKHSHUIHFWJDVODZ

$$\rho_1 = \frac{p_1}{RT_1} = \frac{150(144)}{1716(460 + 300)} = 0.0166 \text{ slug/ft}^3$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{40(144)}{1716(460 + 35)} = 0.00679 \text{ slug/ft}^3$$

7KHPDVVIORZ LVGHWHUPLQHGE\ WKHLQLWLDOFRQGLWLRQV

$$\dot{m} = \rho_1 A_1 V_1 = (0.0166)\frac{\pi}{4}\left(\frac{6}{12}\right)^2(100) = 0.325 \text{ slug/s}$$

. QRZ LQJ PDVVIORZ ZHFRPSXWHWKHH[LWYHORFLW

$$\dot{m} = 0.325 = \rho_2 A_2 V_2 = (0.00679)\frac{\pi}{4}\left(\frac{6}{12}\right)^2 V_2$$

or $$V_2 = 244 \text{ ft/s} \qquad \textit{Ans. (a)}$$

3DUWE

7KHVWDG\ IORZ HQHUJ\ HTXDWLRQ DSSOLHVZ LWK%Y]] DQG© F S7

$$\dot{Q} - \dot{W}_s = \dot{m}(c_p T_2 + \tfrac{1}{2}V_2^2 - c_p T_1 - \tfrac{1}{2}V_1^2)$$

&RQVHUWWKHWXUELQHZ RUNWRIRRWSRXQGVIRUHHSHUVHFRQGZLWKWHFRQVHUVLRQIDFWRU KS I
GEI V 7KHWXUELQHZ RUN%Y LVSRVLWLYH

$$\dot{Q} - 700(550) = 0.325[6003(495) + \tfrac{1}{2}(244)^2 - 6003(760) - \tfrac{1}{2}(100)^2]$$
$$= -510{,}000 \text{ ft} \cdot \text{lbf/s}$$

or $$\dot{Q} = -125{,}000 \text{ ft} \cdot \text{lbf/s}$$

&RQVHUWWKLVWR%WXLWK WHUPDOXQLWDVIRGORZ V

$$\dot{Q} = (-125{,}000 \text{ ft} \cdot \text{lbf/s})\frac{3600 \text{ s/h}}{778.2 \text{ ft} \cdot \text{lbf/Btu}}$$
$$= -578{,}000 \text{ Btu/h} \qquad \textit{Ans. (b)}$$

. LQHWLF(QHUJ\ &RUUHFWLRQ) DFWRU

2 IWHQWKHIORZ HQHUJLQJ RUOHDYLQJ DSRUWLVQRWXQLIRUP LQHDFKFDVH WKHYHORFLW PD\ YDU\ RYHUWKHFURVVVHFWLRQ DVLQ) LJ (,QWKLVFDVHWKHNLQHWLF HQHUJ\ WUP LQ(T IRU JLYHQ SRUWWKRXGG EH PRGLILHG E\ D GLPHQVLRQOHVV FRUUHFWLRQ IDFWRU. VR WKDWWKH LQWJUDOFDQ EH SURSRUWLRQDOWR WKHVTXDUH RI WKHDYHUDJHYHORFLWVWKURXJKWKHSRUW

$$\int_{\text{port}} (\tfrac{1}{2}V^2)\rho(\mathbf{V}\cdot\mathbf{n})\,dA \equiv \alpha(\tfrac{1}{2}V_{\text{av}}^2)\dot{m}$$

where $$V_{\text{av}} = \frac{1}{A}\int u\,dA \qquad \text{for incompressible flow}$$

,I WKH GHQVLW LV DOVR YDULDEOH WKH LQWJUDWRQ LV YHU FXPEHUVRPH ZH VKDOO RWWUHDWWKLV FRPSOLFDWRQ %\ GHWQJ X EH WKH YHORFLW VQRUPDOWR WKH SRUWWKH ILUWHTXDWRQ DERYH EHFRPHV IR LQFRPSUHVLEOHIORZ

$$\tfrac{1}{2}\rho\int u^3 dA = \tfrac{1}{2}\rho\alpha V_{\text{av}}^3 A$$

or $$\alpha = \frac{1}{A}\int\left(\frac{u}{V_{\text{av}}}\right)^3 dA \qquad (3.74)$$

7KHWWUP . LVWKHNLQHWLF HQHUJ\ FRUUHFWLRQ IDFWRU KDYLQJ DYDOXHRI DERXW IRUIXOO GHYHORSHG ODPLQDUSLSHIORZ DQG IURP VR IRUWXUEXOHQWSLSHIORZ 7KHFRPSOHWLQFRPSUHVLEOHVWDG IORZ HQHUJ\ HTXDWRQ LQFOXGLQJ SXPSV WXUELQHV DQG ORVVHV ZRXOGJHQHUDOL]HWR

$$\left(\frac{p}{\rho g} + \frac{\alpha}{2g}V^2 + z\right)_{\text{in}} = \left(\frac{p}{\rho g} + \frac{\alpha}{2g}V^2 + z\right)_{\text{out}} + h_{\text{turbine}} - h_{\text{pump}} + h_{\text{friction}} \qquad (3.75)$$

ZKHUHWKHKHDGWWUP VRQWKHULJKWK$W K$ K DUHDOOQXPHULFDOO SRVLWLYH $ODDGGLWLYHWWUP VLQ(T KDYHGLPHQVLRQV RI OHQJWK^/` ,QSUREOPV LQYROYLQJ WXUEXOHQWSLSHIORZ LWLV FRPPRQ DWXP HWKDW § 7R FRPSXWQXPHULFDOYDOXHV ZHFDQXVHWKHHDSSUR[LPDWRQVWR EHGLVFXWHG LQ&KDS / DPLQDUIORZ

Laminar flow: $$u = U_0\left[1 - \left(\frac{r}{R}\right)^2\right]$$

from which $$V_{\text{av}} = 0.5U_0$$

and $$\alpha = 2.0 \qquad (3.76)$$

Turbulent flow: $u \approx U_0\left(1 - \frac{r}{R}\right)^m \qquad m \approx \frac{1}{7}$

from which, in Example 3.4,

$$V_{av} = \frac{2U_0}{(1 + m)(2 + m)}$$

Substituting into Eq. (3.74) gives

$$\alpha = \frac{(1 + m)^3(2 + m)^3}{4(1 + 3m)(2 + 3m)} \tag{3.77}$$

DQG QXPHULFDOYDOXHVDUHDVIROORZV

Turbulent flow:	m	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$
	α	1.106	1.077	1.058	1.046	1.037

7KHVHYDOXHVDUHRQO VOLJKWO GLIIHUHQWURP XQLWDQGDUHRIWQQHJOHFWGLQHPHQDU VXEVHQ IORZ DQDOVHV +RZHYHU. VKRXOGQHYHUEHQHJOHFWGLQDPLQDUIORZ

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$ K\GURHOHFWULF SRZHU SODQW) LJ (VDNHV LQ P V RI ZDWHU WKURXJK LWV WXUELQH DQ GLVFKDUJHV LVWRRWKHDWRVSKHUHDW P V 7KHKHDGORVWLQWKHWXUELQHDQGSHQVWRFNV LVK P \$VXPLQJWXUEXOHQWORZ . § HVWLPDWHWKHSRZHULQ0: H[WUDFWHGE\WKHWXUELQH

6RXWLRQ

: H QHJOHFWYLVFRXV ZRUN DQG KHDWWDQVIHU DQG VDNH VHFWLRQ DWWKH UHVHUYRLU VXUIDFH) LJ (ZKHUH9 § S S$W DQG] P 6HFWLRQ LVDWWKHWXUELQHRXWOHW

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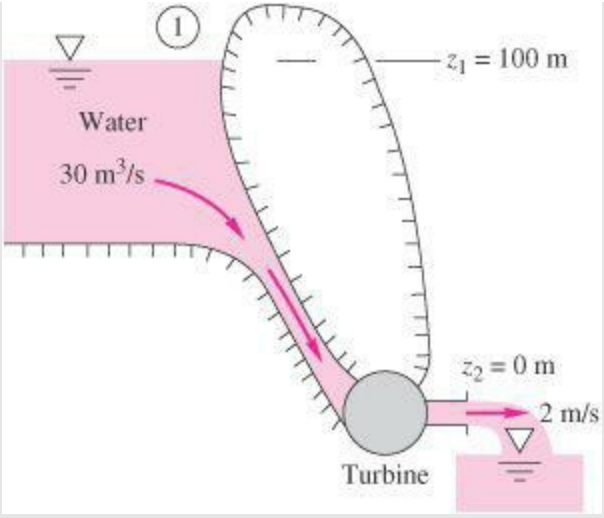

7KH VWDQGDUG IORZ HQHUJ\ HTXDWLRQ EHFRPHV LQKHDGIRUP

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_t + h_f$$

$$\frac{p_a}{\gamma} + \frac{1.06(0)^2}{2(9.81)} + 100\text{ m} = \frac{p_a}{\gamma} + \frac{1.06(2.0\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0\text{ m} + h_t + 20\text{ m}$$

7KH SUHVVXUH WHUPV FDQFHO DQG ZH PD\ VROYH IRU WKH WXUELQH KHDG ZKLFK LV SRVLWLYH

$$K_W \quad \pm \quad \pm \quad \S \quad P$$

7KH WXUELQH H[WUDFWV DERXW SHUFHQW RI WKH P KHDG DYDLODEOH IURP WKH GDP 7KH VRXUSRZHU H[WUDFWV GPD\ EH HYDOXDWHG IURP WKH ZDWHU PDVV IORZ

$$P = \dot{m} w_s = (\rho Q)(g h_t) = (998\text{ kg/m}^3)(30\text{ m}^3/\text{s})(9.81\text{ m/s}^2)(79.8\text{ m})$$
$$= 23.4\text{ E6 kg} \cdot \text{m}^2/\text{s}^3 = 23.4\text{ E6 N} \cdot \text{m/s} = 23.4\text{ MW} \qquad \textit{Ans.}$$

7KH WXUELQH GULYHV DQ HOHFWULF JHQHUDWRU WKDW SURGXFHV RI DERXW SHUFHQW RI WKH QSRZHU JHQHUDWHG E\ WKLV K\GURHOHFWULF SODQW V DERXW 0 :

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7KH SXPS LQ) LJ (GHOLYHUV ZDWHU IW DW IWV VR DP DFKLQHDWH FRQ ZKLFK L IWKJKHU WKDQ WKH UHVHUYRLU VXUIDFH 7KH ORVVHV EHWZHHQ DQG DUH JLYHQ E\ K, . 9 J ZKHUH. § LV DGLPHQVLRQOHVV ORVV FRHIILFLHQW VHH6HF 7DNH. §) LQG WKH KRUVHSRZHU UHTXLUHG IRU WKH SXPS LI LWV SHUFHQW HILFLHQW

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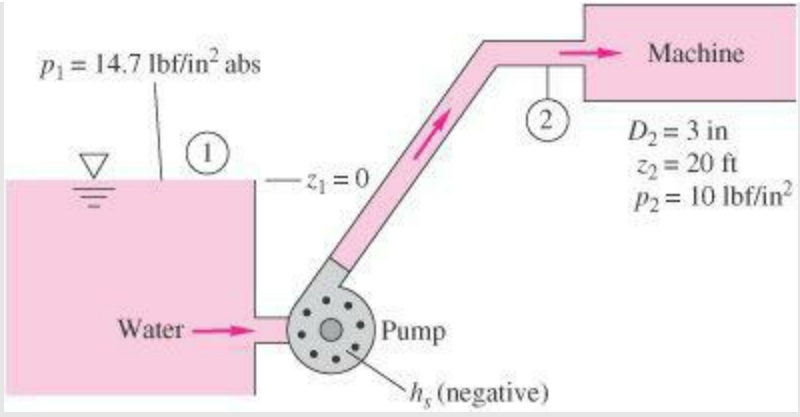


6RO XWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZV WKH SURSHUWLHV DQG IRUPXODWLRQV DQG
- $VVXPSWLRQV 6WDWH IRXU QHJOLJLEOH YLVFRXV RUN ODJHUHVHUYRLU 9 §
- $SSURDFK) LQGLQG WKH YHORFLW9 DWKH H[WKHQDSSO WKH WADG\ IORZ HQHUJ\ HTXDWRQ
- 6RO XWRQ WMSV 8VH%* XQLW S GEI IWDQGS GEI IW

) LQG9 IURP WKH NQRZQ IORZ UDWHDQG WKH SLSHGLDPHWU

$$V_2 = \frac{Q}{A_2} = \frac{1.5 \text{ ft}^3/\text{s}}{(\pi/4)(3/12 \text{ ft})^2} = 30.6 \text{ ft/s}$$

7KH WADG\ IORZ HQHUJ\ HTXDWRQ ZLWK D SXPS QR WXUELQH SOXV] § DQG9 § EHFRPHV

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 - h_p + h_f, \quad h_f = K\frac{V_2^2}{2g}$$

or

$$h_p = \frac{p_2 - p_1}{\gamma} + z_2 + (\alpha_2 + K)\frac{V_2^2}{2g}$$

- &RPPHQW7KHSXPS PXWEDODQFHIRXUGLIIHUHQWHIHFW WKHSUHVWXUHFKDQJH WKHHOHYDWR FKDQJH WKHH[WWWXLQHWF HQHUJ\ DQG WKHIULFWRQORVVHV
-) LQDOVROXWRQ) RUWKHJLYHQGDW Z HFDQHYDOXDWWKHUHTXLUHGSXPSKHDG

$$h_p = \frac{(1440 - 2117)\text{ lbf/ft}^2}{62.4 \text{ lbf/ft}^3} + 20 + (1.07 + 7.5)\frac{(30.6 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = -11 + 20 + 124 = 133 \text{ ft}$$

: LWK WKH SXPS KHDG NQRZQ WKH GHOLYHUHG SXPS SRZHU LV FRPSXWHG VLPLODUWR WKH WXUELQH L

([DPSOH

$$P_{\text{pump}} = \dot{m}w_s = \gamma Q h_p = \left(62.4\,\frac{\text{lbf}}{\text{ft}^3}\right)\left(1.5\,\frac{\text{ft}^3}{\text{s}}\right)(133\text{ ft})$$

$$= 12450\,\frac{\text{ft}-\text{lbf}}{\text{s}} = \frac{12{,}450\text{ ft-lbf/s}}{550\text{ ft-lbf/(s}-\text{hp)}} = 22.6\text{ hp}$$

,I WKH SXPS LV SHUFHQWHILFLHQWWKHQ ZH GLYLGH E\ WKH HILFLHQF\ WR ILQG WKH LQSXWSRZH UHTXLUHG

$$P_{\text{input}} = \frac{P_{\text{pump}}}{\text{efficiency}} = \frac{22.6\text{ hp}}{0.80} = 28.3\text{ hp} \qquad \textit{Ans.}$$

- &RPPHQW 7KH LQFOXVLRQ RI WKH NLQHWLF HQHUJ\ FRUUHFWLRQ IDFWRU. LQ WKLV FDVH PDGH L GLIIHUHQFH RI DERXW SHUFHQWLQ WKH UHVXOW7KH IULFWLRQ ORVV QRWWKH H[WWWZ DV WKH GRPLQDQ SDUDPHWHU

6XPPDU\

7KLV FKDSWHUKDV DQDO]HG WKH IRXUEDVLF HTXDWLRQV RI IOXLG PHFKDQLFV FRQVHUYDWLRQ RI PDV QDUHDUPRPHQWXP DQJXODUPRPHQWXP DQG HQHUJ\ 7KH HTXDWLRQV ZHUH DWWDFNHG³LQ WKI ODUJH² WKDWLV DSSOLHG WR ZKROH UHJLRQV RI D IORZ $ V VXFK WKH WSLFDO DQDO VLV ZLOOLQYROYH D DSSUR[LPDWRQ RI WKH IORZ ILHOG ZLWKLQ WKH UHJLRQ JLYLQJ VRPHZKDWFUXGH EXWDZ D V LQWWKFWLYH TXDQWDWLYH UHVXOW + RZ HYHU WKHEDVLF FRQWROYROXPH UHDWRQV DUH UJRURXV DQG FRUUHFWDQG ZLG JLYHH[DFWUHVXOWV LDSSOLHG WR WKH H[DFWORZ ILHOG

7KHUH DUH ZR PDLQ SRLQWV WR D FRQWROYROXPH DQDO VLV 7KH ILUWWLV WKH VHOHFWLRQ RI D SURSH FOHYHU ZRUNDEOH FRQWROYROXPH 7KHUHLV QR VXEVWXWXWIRUH[SHULHQFH EXWWKH IRQRZLQJ JXLGHOLQH DSSO 7KH FRQWROYROXPH VKRXOG FXWWKURXJK WKH SODFH ZKHUH WKH LQRUP DWRQ RU VROXWRQ LV GHVLUHG ,WWKRXOG FXWWKURXJK SODFHVZKHUHP D[LPXP LQRUP DWRQ LV DOHDG NQRZ Q ,I WKHP RP HQWXP HTXDWR LVWR EH XVHG LWWKRXOG QRWFXWWKURXJK VROG ZDOVXQDW QHFHVDU\ VLQFH WKLV ZLOOH[SRV SRVVLEOH XQNQRZ Q VWUHVVHV DQG IRUFHV DQG P RP HQW WKDWP DNH WKH VROXWRQ IRU WKH GHVLUHG IRUF GLIILFXOWRULP SRVVLEOH) LQDOO HYHU DWWP SWKRXOG EH P DGHWR SODFH WKH FRQWROYROXPH LQD IUDP R UHHUHQFH ZKHUH WKH IORZ LV VWDG RUTXDL VWDG VLQFH WKH VWDG IORP XQDWRQ LV P XFK VLP SOHUW HYDOXDW

7KHVHFRQG P DLQ SRLQWWR D FRQWROYROXPH DQDO VLV LV WKH UHGXFWLRQ RI WKH DQDO VLV WR D FDVH WKD DSSOHV WR WKH SUREOP DWKDQG 7KH H[DPSOHV LQWKLV FKDSWUJLYHRQ DQ LQWURGXFWLRQ RU PRUH H[DPSOHV WR EHFRP IRUDSSURSULDWH LP SOD\LQJ DVXP SWRQ V <RX ZLOOQH G WR VROYH RU PRUH H[DPSOHV WR EHFRP WKO H[SHUHQFHG LQ VLP SOD\LQJ WKH SUREOP MXWHQRXJK DQG QR PRUH ,Q WKH P HDQWP H LWZ RXOG E ZLVH IRUWKH EHJLQQHU WR DGRSWD YHU JHQHUDO IRUP RI WKH FRQWROYROXPH FRQVHUYDWQ ODZ V DQG WKH P DN HD VHUHV RI VLP SOLILFDWRQ V WR DFKLHYH WKH ILQDO DQG V 6WDUWQJ Z LWK WKH JHQHUDO IRUP RQH FD DVNDVHUHV RI TXHVWRQV

,VWKHFRQWROYROXPHQRQGHRUPLQJRUQRQDFFHOHUDWLQJ"
,VWKHIORZ ILHOGVWDGG" &QZHFKDQJHVRDVWDGG IORZ IUDPH'
&QIULFWLRQEHQHJOHFWHG'
,VWKHIOXLGLQFRPSUHVVLEOH' ,IQRWVLVWKHSHUHFWJDVODZ DSSOLFDEOH'
$UHJUDYLWURURWKHUEG IRUFHVQHJOJLEOH'
,VWKHUHKDWWDQVHU VKDWZRUN RUYLVFRXZRUN'
$UHWKHILQDWDQGRXWWWORZ VDSSUR[LPDWH RQHGLPHQVLRQDO"
,VDVVRVSKHUF SUHVVXUHLPSRUWDQWWKHDQDOVLV' ,VWKHSUHVXUHK\GURVWDWF RQDQ SRUWRQ
RI WKHFRQWROVXUIDFH'
$UH WKHUH UHVHUYRLU FRQGLWLRQV WKDWFKDQJH VR VORZ 0 WKDWWKH YHORFLW DQG WPH UDWV F
FKDQJHFDQEHQHJOHFWHG'

,QWKLV ZD\ E\ DSSURYLQJ RU UHMHFWQJ D QXWRI EDVLF VLP SOLFDWRQV QH WKHVH RQH FDQ DYRL
SXOOQJ %HUQRXOOL¶V HTXDWRQ RI WKH VKHD ZKHQLWGRHVQRWDSSO

3UREOHPV TXHVWLRQV3UREOHPV

0 RVWRI WKHSUREOHPVKHUHQDUHIDLUO VWUDLJKWRUZ DUG 0 RUHGLIILFXOWRURSHQHGHGDVLJQPHQ
DUHODEHOHGZLWKDQDXWULN 3UREOHPVODEHOHGZLWKQ((6LFRQEES ZLOOEHQHLWURP WKHXVHRI WK
(QJLQHHULQJ (TXDWRQ6RROYHU ((6 ZKLFKILJXUHVZLWKDFRPSXWHULFRQ PD\ UHTXLUHWKHXVHRI I
FRPSXWU 7KH VWDQGDUG HQGRI FKDSWU3UREOPV 3 WR3 FDWJRU]HG LQ WKH SUREOP OD
KHUH DUH IROOWHG E\ ZRUG3UREPV : WR: IXQGDPHQWDO RI HQJLQHHULQJ)(H[DP
3UREPV) (WR) (FRPSUHHQVLYH3UREPV& WR& DQGGHVLJQSURMHFW

3UREOP ' LVWULEXWRQ

Section	Topic	Problems
3.1	Basic physical laws; volume flow	P3.1–P3.5
3.2	The Reynolds transport theorem	P3.6–P3.9
3.3	Conservation of mass	P3.10–P3.38
3.4	The linear momentum equation	P3.39–P3.109
3.5	The Bernoulli equation	P3.110–P3.148
3.6	The angular momentum theorem	P3.149–P3.164
3.7	The energy equation	P3.165–P3.185

%DVLF SK\VLFDOODZV YROXP HIORZ

- 3 ' LVFXV1HZWRQ¶VVHFRQGODZ WKHQHWXP RPHQWXP UHDWRQ LQWKHVHWKUHHIRUPV

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum \mathbf{F} = \frac{d}{dt}(m\mathbf{V})$$

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{\text{system}} \mathbf{V}\rho\, d\forall\right)$$

$UH WKH\ DOO HTXDOO YDOG' $UH WKH\ HTXLYDOHQW $UH VRPH IRUPV EHWWU IRU IOXL
PHFKDQLFV DVRSSRVHG WR VROGPHFKDQLFV"

- 3 &RQVLGHU WKH DQJXODU PRPHQWXP UHDWLRQ LQ WKH IRUP

$$\sum \mathbf{M}_O = \frac{d}{dt}\left[\int_{\text{system}} (\mathbf{r} \times \mathbf{V})\rho\, d\mathcal{V}\right]$$

: KDW GRHV UPHDQ LQ WKLV UHODWLRQ' ,V WKLV UHODWLRQ YDOG LQ ERWK VROG DQG IOXL
PHFKDQLFV" ,VLWUHDWHG WR WKH OLQHDUPRPHQWXP HTXDWLRQ 3URE " ,QZKDWPDQQHU'

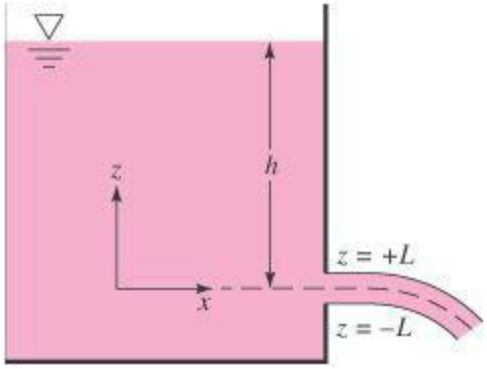
- 3)RUWKDG\ GRZ 5H\QROGV QXPEHU ODPLQDU IORZ WKURXJK DORQJ WKH VH3URE
WKH DLOYHOFLWV GLVWULEXWLRQ LV JLYHQ E\ X & 5 í U ZKHUH 5 LV WKH WXEH UDGLX
DQGU" 5 ,QWURGXFH U WR ILQG WKH WRWDO YROXPHIORZ 4 WKURXJK WKH WXEH

- 3 $ ILUH KRVH HDV D LQLWLDO GLDPHWUDQG ZDWULV IORZLQJ DW JDOPLQ 7KHIORZ
H[LWV WKURXJK DQR]]OH FRQQHFWRQ ZLWK D GLDPHWUDQG' Q) RUWKDG\ IORZ ZKDWWKRXG
' Q EH LQLQFKHV WR FUHDWH DQ DYHUDJH H[LWYHORFLWR P V"

- 3 : DWUDW *& IORZ V WKURXJK D LQGLDPHWUDQ VPRRWKSLSH DWDKJK 5H\QROGV QXPEH
IRU ZKLFK WKH YHORFLW SURILOH LV DSSUR[LPDWHG E\ X 8R \ 5 ZKHUH8R LV WK
FHQWHUOLQH YHORFLW 5 LV WKH SLSH UDGLXV DQG\ LV WKH GLVWDQFH PHDVXUHG IURP WKH ZD
WR ZDUG WKH FHQWHUOLQH ,I WKH FHQWHUOLQH YHORFLW LV IWV HVWLPDWH WKH YROXPHIORZ
UDWH LQ JDOORQV SHU PLQXWH

7KH5H\QROGV WUDQVSRUW WKHRUHP

- 3 : KHQ D JUDYLW GULYHQ IUHH VXUIDFH IORZ LQ) LJ 3 DL
DSSUR[LPDWH IRUPXOD IRU WKH H[LWYHORFLW GLVWULEXWLRQ L\ $u \approx \sqrt{2g(h-z)}$ ZKHUH K LV WKH GHSW
RI WKH IUHH FHQWHUOLQH 1HDU WKH VORW WKH IUHH VXUIDFH FROODSVHV WR GLPHQVLRQDO DQG R
WKLFNQHVV / DV VKRZ Q) LQG DJHQHUDO H[SUHVVLRQ IRU WKH VROXPHIORZ 4 LVXLQ
IURP WKH VORW WKHQ DVVXPH WKH OPLWDWLRQ \RXUUHVXOW / ≪ K

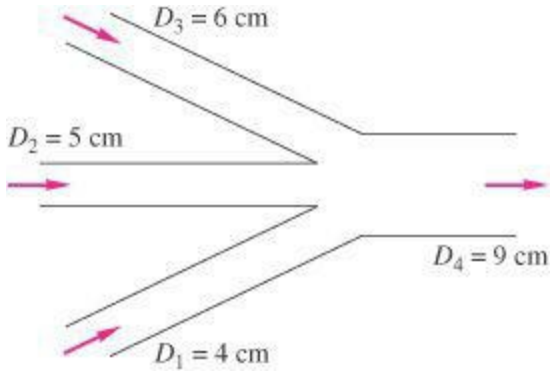


3

- 3 $ VSKHULFDO DQN RI GLDPHWU FP LV HDNLQJ DLU WKURXJK D PP GLDPHWUKROHLQ
VLGH 7KH DLU H[WWKH KROH DW P V DQG DGHQVLWR NJ P $VVXPLQJ XQLRUP
PL[LQJ D ILQG DIRUP XQLIRUWKHUDWRI FKDQJHRI DYHUDJHGHQVLWLQWKHVDQN DQG E
FDOFXODWH DQ PHUFDOYDOXHIRU G GWLQWKHVDQNIRUWKHJLYHQGDW

- 3 7KUHH SLSHV VWDGLO GHOLYHU ZDWUDW *& WR D ODUJH H[WSLSH LQ) LJ 3 7K
YHORFLW9 P V DQG WKH H[WORZ UDW4 P K) LQG D 9 E 9 DQG F

9 LI LWLV NQRZQ WKDW WKH FUHDVLQJ 4 E\ SHUFHQW ZRXOG LQFUHDVH 4 E\ SHUFHQW

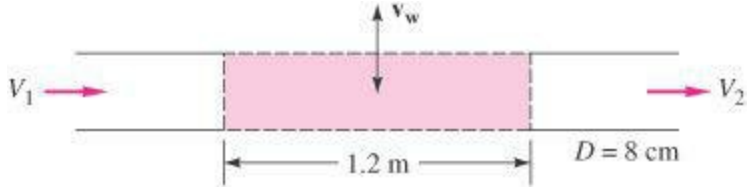


3

- 3 $ GDERUDWRU VWXGHQW FROOHFWV VHDZDWHU IURP VDOLQH 6DQG GHQVLW! : DWHU HQWHUV WKH VDQNDWFRQGLWRQV 6 ! $ 9 DQG LV DVVXPHG WR PL[LPPHGLDWHO LQWKH VDQN 7DQ ZDWHU GHDYHV WKURXJK DQ RXWOHW$ DVVXPHG WR EH 9 ,I VDOWLV D³FRQVHUYDWLYH SURSHUW QHLWKHU FUHDWHG QRU GHVWUR HG XVH WKH 5H\QROGV WUDQVSRUWWKHRUHP WR ILQG D H[SUHVVLRQ IRUWKH UDWH RI FKDQJH RI VDOW PDVV0 $VDOW ZLWKLQ WKH VDQN

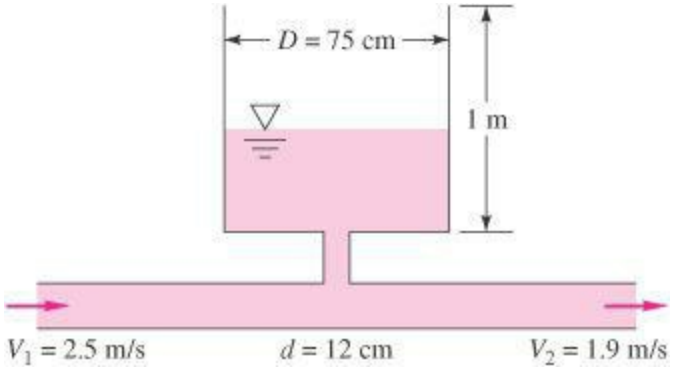
&RQVHUYDWLRQ RI PDVV

- 3 : DWHU IORZLQJ WKURXJK DQ FP GLDPHWU SLSH HQWHUV D SRURXV VHFWLRQ DV LQ) LJ 3 ZKLFK DOORZV D XQLIRUP UDGLDO YHORFLW Y WKURXJK WKH ZDOO VXUIDFHV IRUI GLVWDQFH R P ,I WKH HQWDQFH DYHUDJH YHORFLW9 LV P V ILQG WKH H[L YHORFLW9 LI D Y FP VRXWR WKH SLSH ZDOOV RU E Y FP VLQWR WK SLSH F : KDWVDOXHRI Y ZLOOPDNH9 P V"



3

- 3 7KHLQGXVWULDOQR DYDFXXP FOHDQHUVDUHXVDQJH LQE\ LQ 7KHEORZHULVDEO VR SURYLGHVXFWLRQDW FXELF IHHWSHUPLQXW D : KDWLVWKHDYHUDJHYHORFLW D WKHLQGXWLQP V" E ,I FRQGLWRQVDUHVDOHYHOXDQGDU ZKDWLVWKHPDVIORZ R DLU LQNJ V"
- 3 7KHSLSHIORZ LQ) LJ 3 ILOOVDFYOQGULFDOXJHWDQDVVKRZQ $WWLPHW WK ZDWHUGHSWKLQWKHWDQNLV FP (VWLPDWWKHWLPHUHTXLUHGWRILOOWKHUHPDLQGHU WKHWDQN

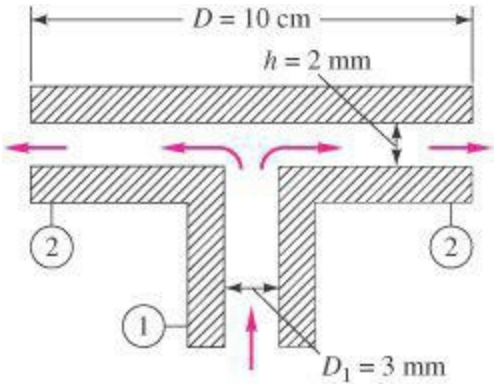


3

- 3 7KHFRQGXFWLYLW LQ) LJ 3 LV FP LQ GLDPHWU DQG KDV D FRQFH

3

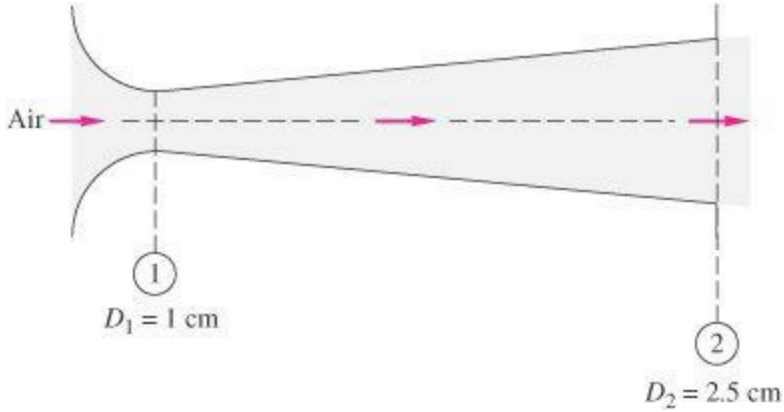
- 3 2LO 6* HQWHUV DW WHFWRQ LQ) LJ 3 DW D ZHJKW RI 1 K W OXEULFDWH D WKLQ WEHDULQJ 7KH WRS RI WKH FRUURXJK WKH QDUURZ FOHDUDQFH EHWZHHQ WKH SODWH & RPSXWH D WKH RXOHW YHORFLW\ LQ P/ VDQG E WKH DYHUDJH RXOHW YHORFLW\ V



3

- 3)RU WKH WZR SRUWDQN LQ) LJ (GHWHUPLQH WKH VPHE EXWDWXPH9 IW DQG WKDW9 LV XQNQRZQ ,I WKH ZDWUVXUFHLVULQJ DW UDWR LQV D GHWHUPLQH WKH DYHUDJH YHORFLW\ DW WHFWRQ E ,V WKH IORZ DW WHFWRQ LQRURXW
- 3 7KH FRQYHUJLQJ±GLYHUJLQJ QR]]OH VKRZQ LQ) LJ 3 H[SDQGV DQG DFFHOHUDWHV GU DLUWR VXSHUVRQLF VSHHGV DW WKH H[LW ZKHUH S N3D DQG7 . \$W WKH WKURDW S N3D 7 . DQG9 P V) RUW WKDG FRPSUHVVLEOH IORZ RI D LGHDO JDV HVWLPDW D WKH PDVV IORZ LQNJ K E WKH YHORFLW9 DQG F WKH0 DFK QXPEHU0 D

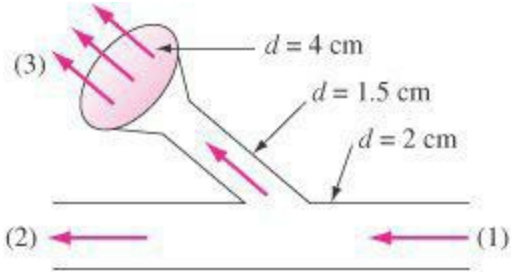
- 3 7KH K\SRGHUPLF QHHGOH LQ) LJ 3 FRQVLVW D OTXLG VHUXP 6* ,I WK VHUXP LV WR EH LQMHFWHG WKDGLO DW FP V KRZ IDVWLQ LQV VKRXOG WKH SXQJHUEH DGYDQFHG D LI OHDNDJH LQ WKH SXQJHUFOHDUDQFH LV QHJOHFWHG DQG E LI OHDNDJH LV SHUFHQW WR WKH QHHGOH IORZ"



3

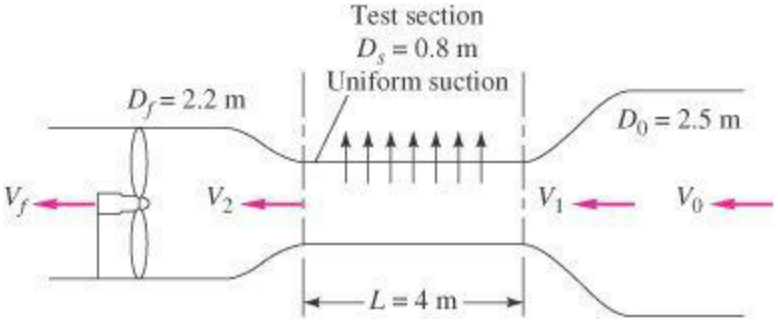
3

- 3 : DWUDW *f*& IORZ V VWDGLO VKURXJK WKH SLSLQJ MXQFWLRQ LQ) LJ 3 HQWULQ VHFWLRQ DW JDOOLQ 7KH DYHUDJH YHORFLW DWWHFWLRQ LV P V $ SRUWLRQ R WKH IORZ LV GLYHUWHG WKURXJK WKH VKRZHUKHDG Z KLFK FRQVLVV KROHV RI PP GLDPHWU $VVXPLQJ XQLRUP VKRZHU IORZ HTXLP DW WKH H[LWYHORFLW IURP WK VKRZHUKHDG MWV



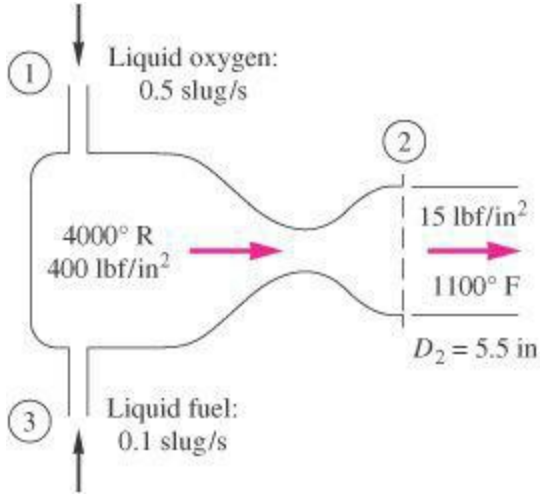
3

- 3 ,QVRPH ZLQG WXQQHOV WKH WHVWLQJ VHFWLRQ LV SHUPDQHQW VR VXFK RXWOLGDQG SURYLGH WKLQYLVFRXV ERXQGDU OD HU 7KH WHVWLQJ VHFWLRQ Z DOOLQ) LJ 3 FRQVLVV KROHV RI PP GLDPHWU HDFK SHUVTXDUH P HWUR Z DOODUHD 7KH VXFWLRQ YHORFLW WKURXJ HDFK KROH LV9 $ P V DQG WKH WHVWLQJ HQGDQFH YHORFLW LV9 P V $VVXPLQJ LQFRPSUHVLEOH WHDG IORZ RI DLUDW *f*& FRPSXW D 9 E 9 DQ F 9 LQP V



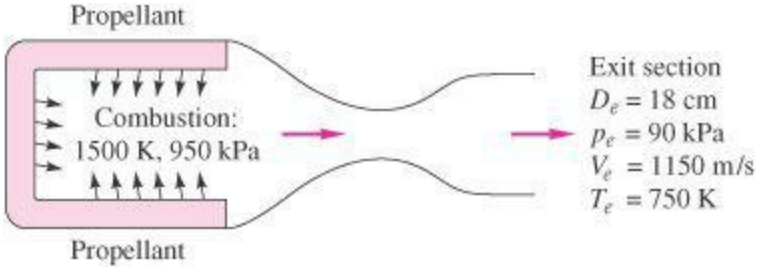
3

- 3 $ URFNHWPRWRU LV RSHUDWQJ WKDG DV VKRZQ LQ) LJ 3 7KH SURGXFW FRPEXWLRQ IORZLQJ RXWKH H[KDXVWQR]]OH DSSUR[LPDW D SHUFHWJDV Z LWK PROHFXODUZHJKWR) RUWKHJLYHQFRQGLWRQVFDOFXODW9 LQIW



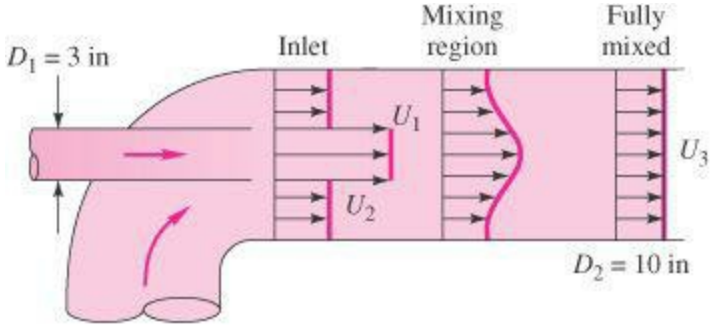
3

- 3 ,QFRQWDVWRWKHOTXLGURFNHWQ) LJ 3 WKHVROG SURSHOODQWURFNHWQ) LJ 3 LVVHOI FRQWDLQHGDQGKDVQRHQWDFHGXFW 8VLQJ DFRQWROYROXPHDQDOVLVIRUWK FRQGLWLRQVVKRZQLQ) LJ 3 FRPSXWHWKHUDWH RI PDVV IORZ RI WKH SURSHOODQ DWXPLQJ WKDWWKHH[LWJDVKDVDPROHFXODUZHLJKWRI



3

- 3 7KH NHWVXPS LQ) LJ 3 LQMHFWZDWHUDW8 P VWKURXJK D LQSLSH DQ HQWHUVDVHFRQGDU IORZ RI ZDWHU8 P VLQWKH DQQXODUUHJLRQ DURXQG WK VPDOO SLSH 7KH VWR IORZV EHFRPH IXOO PL[HG GRZQWUHDP ZKHUH8 L DSSUR[LPDWHO FRQVWDQW) RUWKDG\ LQFRPSUHVVLEOHIORZ FRPSXWH8 LQP V

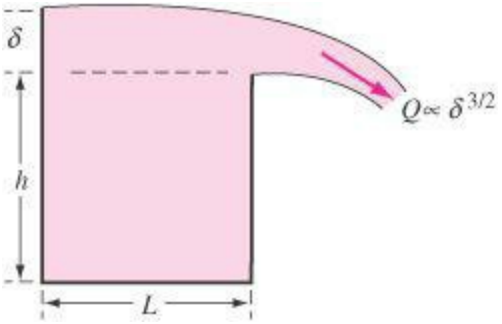


3

- 3 ,I WKHUHFWDQJXODUWDQNIXOORI ZDWHULQ) LJ 3 KDVLWVULJKWKDQG ZDOOORZHUHGE\ DQDPRXQW' DVVKRZQ ZDWHUZLOOIORZ RXWDVLWZRXOGRYHUDZHLUORUGDP ,Q3URE 3 ZHGHGXFHGWKDWWKHRXWORZ 4 ZRXOGEHJLYHQE\

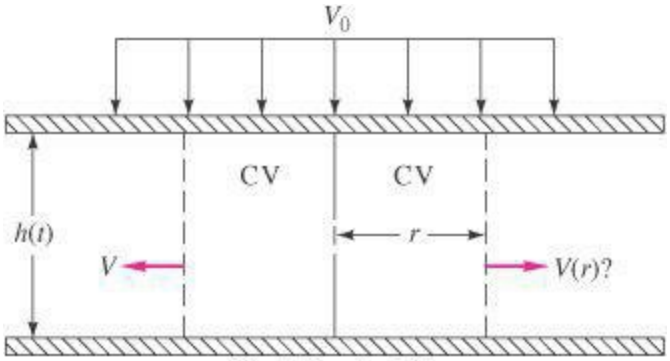
4 &EJ /

ZKHUHE LV WKH WDQN ZLGWKLQR WKH SDSHU J LV WKH DFFHOHUDWRQ RI JUDYLW DQG& LV I GLPHQVLRQOHVVFRQWDQW$ VXPHWKDWWKH ZDWHUVXUIDFHLVKRUL]RQWDO QRWWOUJKW FXUYHG D LQWKHILJXUH / HWWKHILQLWDOH[FHVZDWHUGHSWKEH/R ' HULYHDIRUPXODIRUWKHWPHUHTXLUH WRUHGXFHWKHH[FHVZDWHUGHSWKWR D /R DQG E]HUR



3

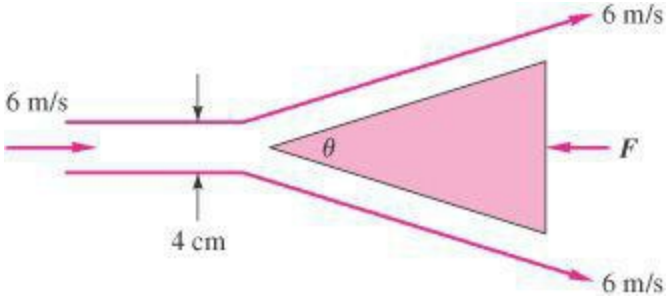
- 3 $ QLQFRPSUHVVLEOHIOXLGLQ) LJ 3 LVEHLQJ VTXHH]HGRXWDUGEHWHHQWZR ODUJH FLUFXODUGLVNVE\ WKHXQLRUP GRZQZDUGPRWRQ9 RI WKHXSSHUGLVN \$VVXPLQJRQ-GLPHQVLRQDOUDGLDORXWZDUG XVHWKHFRQWURO YROXPHVKRZQWRGHULYHDQH[SUHVVLRQ IRU9 U



3 Fixed circular disk

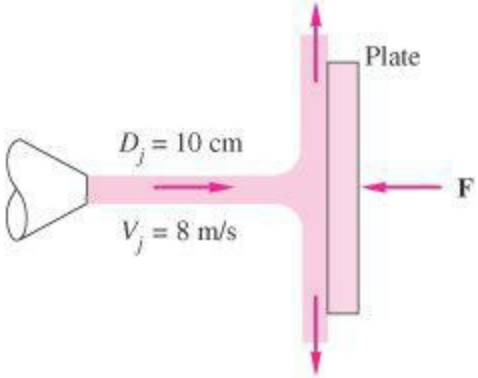
7KHQDQHUPRPHQWXP HTXDWLRQ

- 3 $ ZHGJHVSDLQWDVKHHWRI *f& ZDWHU DVVKRZQLQ) LJ 3 %RWKZHGJHDQGVKHHW DUHYHU\ORQJ LQWRWKHSDSHU ,I WKHIRUFHUHTXLUHGWRKROGWKHZHGJHVDWDQJ LV) 1 SHUPHWHURI GHSWKLQWRWKHSDSHU ZKDWLVWKHDQJOH RI WKHZHGJH"



3

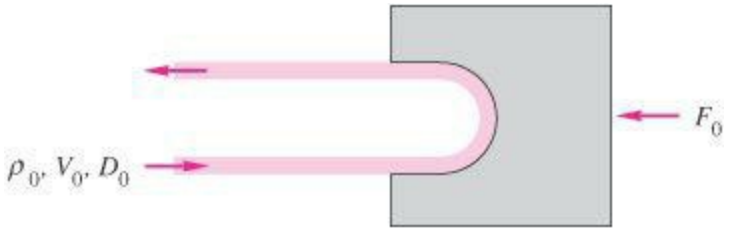
- 3 7KH ZDWHU MHWLQ) LJ 3 VWULNHV QRUPDOWR D IL[HG SODWH 1HJOHFWJUDYLW DQ IULFWLRQ DQGFRP SXWHWKHIRUFH) LQQHZWRQVUHTXLUHGWRKROGWKHSODWHIL[HG



3

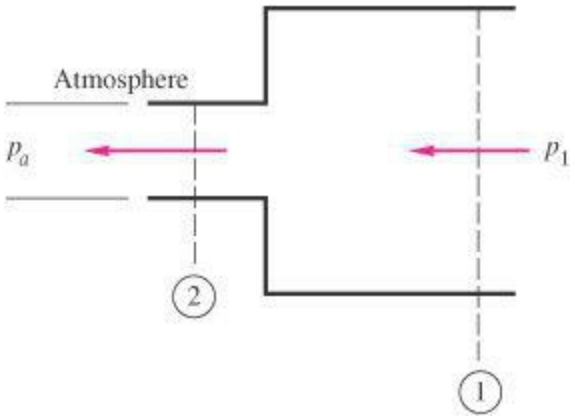
- 3 ,Q) LJ 3 WKHYDQHVXUQVWKHZDWHUMHWWRPSOHWHO DURXQG) LQGDQH[SUHVVLRQIR

WKHPD[LPXP MHWYHORFLW9 LI WKHPD[LPXP SRVVLEOHVXSSRUWRUFHLV)



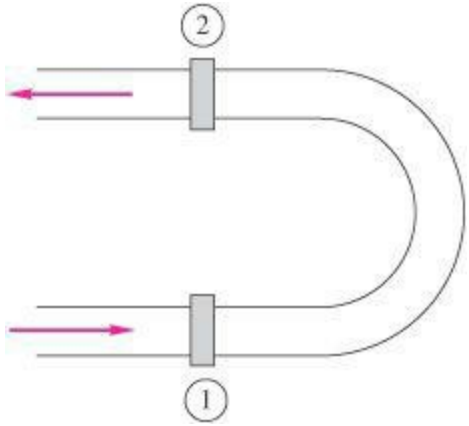
3

- 3 $ QDTXLGRI GHQVLW! IORZ VWKURXJK WKH VXGGHQFRQWUDFWLRQ) LJ 3 DQG H[LWV WKH DIWRSSHUH $VXPH XQLRUP FRQGLWLRQV S 9 ' DWWHFWLRQ DQG S 9 ' DWWHFWLRQ) LQG DQ H[SUHVVLRQ IRU WKH IRUFH) H[HUWHG E\ WKH IOXLG RQ WK FRQWUDFWLRQ



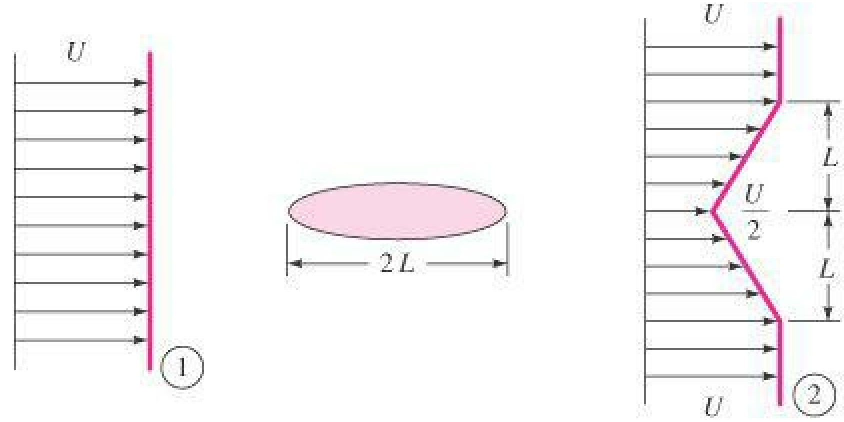
3

- 3 : DWUDW *& IORZ VWKURXJKD FP GLDPHWUSLSHWKDWKDVD *YHUWFDOEHQG D LQ) LJ 3 7KHVRDOOHQJWKRI SLSHEHZHHQIODQJHV DQG LV FP : KHQWK ZHLJKWORZ UDWH LV 1 V S N3D DQGS N3D 1HJOHFWQJ SLS ZHLJKWGHWHUPLQHWKHVRDOIRUFHWKDWWKHIODQJHVPXWZLWKVWDQGIRUWKLVIORZ



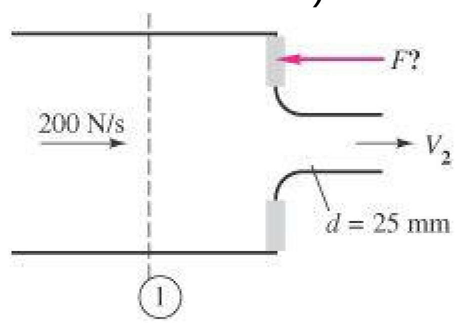
3

- 3 : KHQ D XQLRUP VWHDP IORZ V SDVVDQ LPPHUVHG WKLFN FA OQGHU D EURDG ORZ YHORFLW ZDNH LV FUHDWHG GRZQVWHDP LGHDOL]HG DV D 9 VKDSH LQ) LJ 3 3UHVVXUHVS DQGS DUH DSSUR[LPDWH HTXDO ,I WKH IORZ LV VWR GLPHQVLRQDOO LQFRPSUHVVLEOH ZLWK ZLGWK E LQ R WKH SDSHU GHULYH DIRUPXOD IRUWKHGUDJ IRUFH) RQ WKHFA OQGHU 5HZULWH \RXUHVXOWLQ WKHIRUP RI DGLPHQVLRQOHVVGUDJ FRHILFLHQ EDVHGRQERG\ OHQJWK&) !8 E/



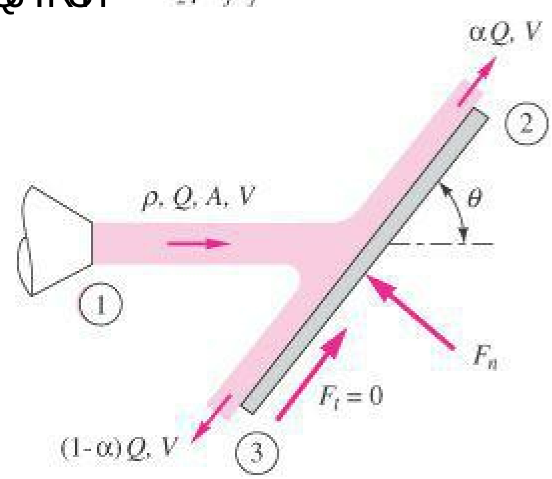
3

- 3 $ FP GLDPHWHUSLSH FRQWDLQLQJ ZDWHUIORZLQJ DW 1 V LV FDSSHGE\ DQRULILFH SODWH DVLQ) LJ 3 7KHH[LWMHWLV PP LQGLDPHWHU 7KHSUHVVXUHLQWKHSLSHD VHFWLRQ LV N3D JDJH &DOFXODWHWKHIRUFH) UHTXLUHGWRKROGWKHRULILFHSODWH



3

- 3 : KHQDMHWVWULNHVDQLQFOLQHGIL[HGSODWH DVLQ) LJ 3 LWEUHDNVLQWRWZRMHWVDW DQG RI HTXDOYHORFLW\9 9MHW EXWXQHTXDOIOX[HV.4 DW DQG í .4 DWVHFWLRQ . EHLQJ D IUDFWLRQ 7KH UHDVRQ LV WKDW IRU IULFWLRQOHVV IORZ WKH IOXLG FDQ H[HUW Q WDQJHQWLDO IRUFH)W RQ WKH SODWH 7KH FRQGLWLRQ)W HQDEOHV XV WR VROYH IRU. 3HUIRUP WKLV DQDO\VLV DQG ILQG. DV D IXQFWLRQ RI WKH SODWH DQJOH : K\ GRHVQ¶W WKH DQVZHU GHSHQG RQ WKH SURSHUWLHV RI WKH MHW
- 3 $ OLTXLG MHW RI YHORFLW\9 DQG GLDPHWHU' VWULNHV D IL[HG KROORZ FRQH DV LQ) LJ 3 DQGGHIOHFWVEDFNDVDFRQLFDOVKHHWDWWKHVDPHYHORFLW\)LQGWKHFRQHDQJOH IRUZKLFKWKHUHVWUDLQLQJIRUFH $F = \frac{3}{2}\rho A_j V_j^2$.



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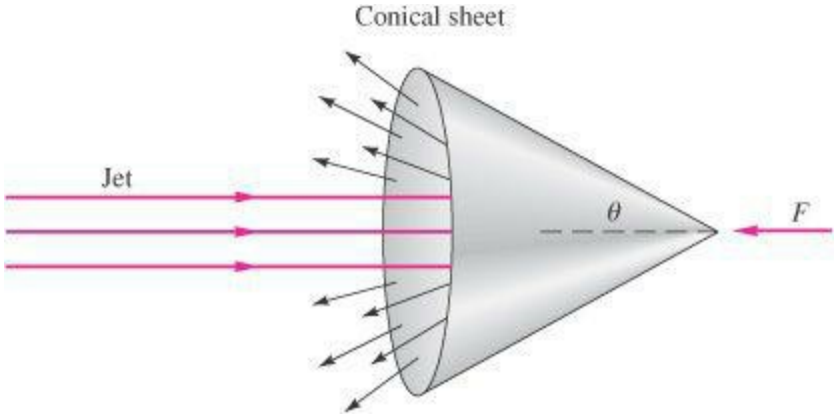
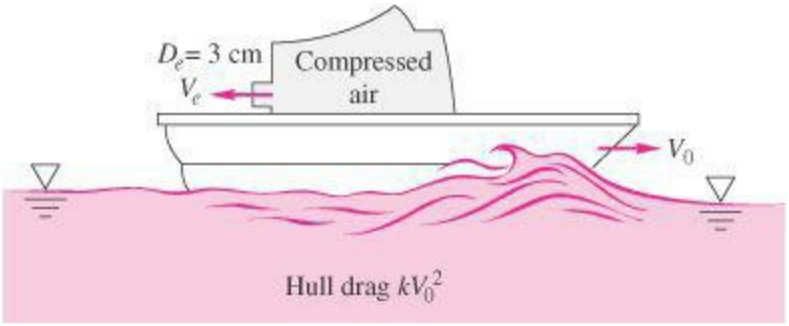


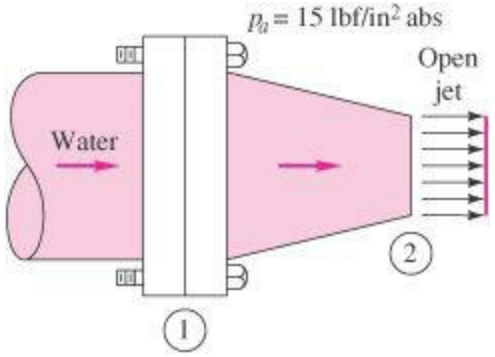
3

- 3 EES 7KH VPDOO ERDW LQ) LJ 3 LV GULYHQ DW D VWHDG VSHHG 9 E\ D MHW RI FRPSUHVVHG DLU LVVXLQJ IURP D FP GLDPHWHU KROH DW 9H P V -HW[L FRQGLWLRQV DUH S H DW DQG 7H f& $ LUGUDJ LV QHJOLJLEOH DQG WKH KXOO GUDJ LV kV02, ZKHUH N§ 1 V P (VWLPDWH WKH ERDW VSHHG 9 LQ P V



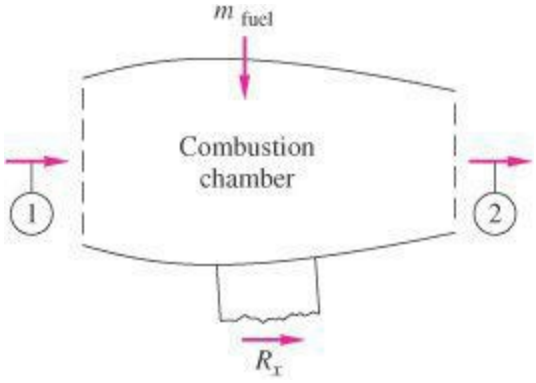
3

- 3 7KH KRUL]RQWDO QR]]OH LQ) LJ 3 KDV' LQ DQG' LQ ZLWK LQOHW SUHVVXUH S GE LQ DEVROXWH DQG 9 IWV) RUZDWHU DW f& FRPSXWH WKH KRUL]RQWDO IRUFH SURYLGHG E\ WKH IDQJH EROWV WR KROG WKH QR]]OH LQ[HG



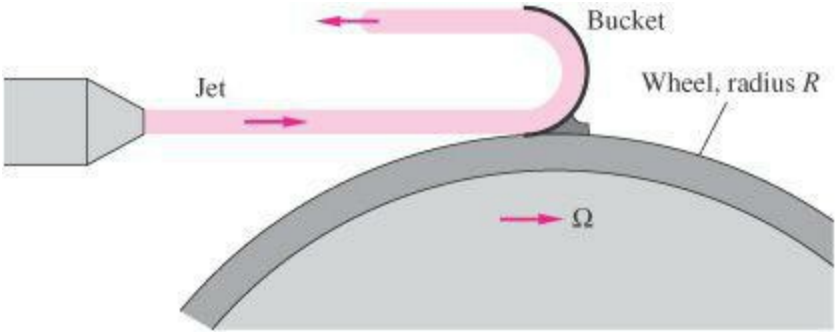
3

- 3 7KH MHW HQJLQH RQ D WWXUERIDQ LQ) LJ 3 DGPLWV DLU DW DWH FWLRQ ZKHUH$ P DQG 9 P V 7KH IXHO WR DLU UDWLR LV 7KH DLUFUDI VHFWLRQ DW DWRVSKHULF SUHVVXUH DQG KJKHU WKDQ SHUDXUH ZKHUH 9 P V DQG $ P &RPSXWH WKH KRUL]RQWDO WWXUERIDQ5[QHHGHG WR KROG WKLV HQJLQH IL[HG



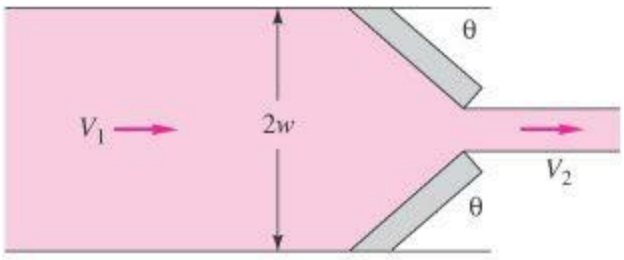
3

- 3 $ CTXLGMHWRI YHORFLW9 DQGDUHD$ VWUHDPOLQHVDUHJHQHUDWHGE\WKHPRWLRQRIDVPDOOHOHPHQWRIIOXLGDORQJWKHVWUHDPOLQHV& DVLQ) LJ 3 ' HULYHDQH[SUHVVLRQIRUWKHSRZHU3 GHOLYHUHGWRWKLVZKHHODVWKLVLQWKDQVDIXQFWLRQRI WKHVWHDP SDUDPHWHUV $ ZKDWDQJXODUYHORFLWLVWKHPD[LPXP SRZHUGHOLYHUHG' +RZ ZRXOG\RXUDQDOVLVGLIIHULI WKHUHZHUHPDQ PDQ EXFNHWVRQWKHZKHHOVRWKDWWKHIMHWZDVFRQWLQXDOO\VWULNLQJDWWRSHDWRQHEXFNHW

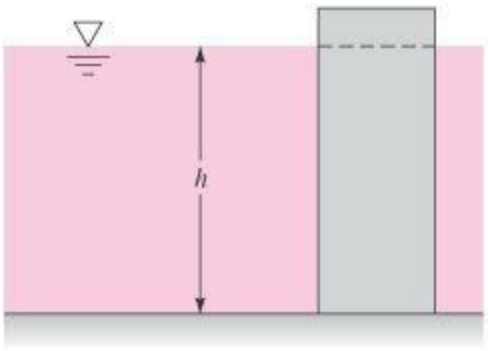


3

- 3 7KHYHUWLFDOJDWHLQDZDWHUFKDQQHOLVSDUWLDOO RSHQ DVLQ) LJ 3 $VXPLQJ QI FKDQJHLQZDWHUOHYHODQGKGURVWDWF SUHVVXUHGLVWULEXWLRQ GHULYHDQH[SUHVVLR IRUWKHVWHDPZLVHIRUFH)[RQRQHKDOI RI WKHJDWHDV DIXQFWLRQRI ! K Z 9 $SSO\RXUHVXOWWR WKHFDVHRI ZDWUDW *& 9 P V K P Z P DQG *



Top view



Side view

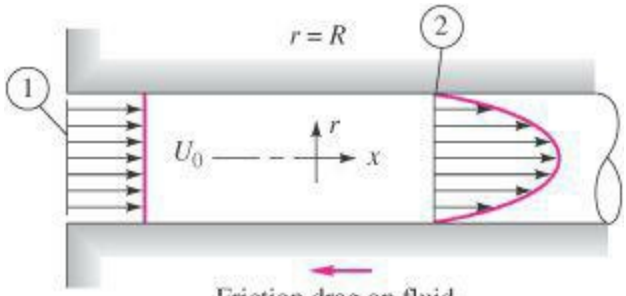
3

- 3 &RQVLGHUIRRPSUHVVLEOHIORZ LQWKHHQWUDQFHRI DFLUFXODUWXEH DVLQ) LJ 3

7KH LQDWWOH VXQIRUP X 8 7KH IORZ DWWHFWRQ LVGHYHORSHG SLSH IORZ) LQG WKH ZDOO GUDJ IRUFH) DVDIXQFWLRQRI S S ! 8 5 LI WKH IORZ DWWHFWRQ LV

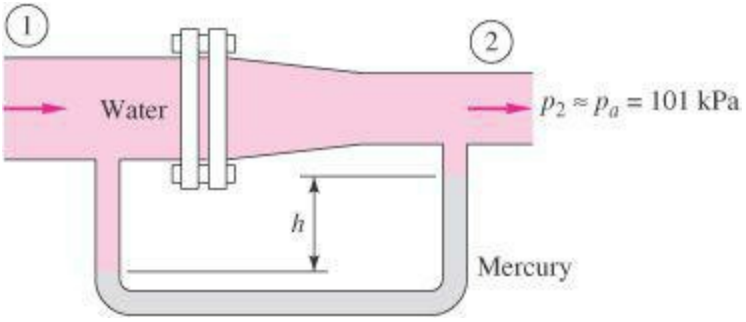
$$(a)\ \text{Laminar: } u_2 = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$

$$(b)\ \text{Turbulent: } u_2 \approx u_{\max}\left(1 - \frac{r}{R}\right)^{1/7}$$



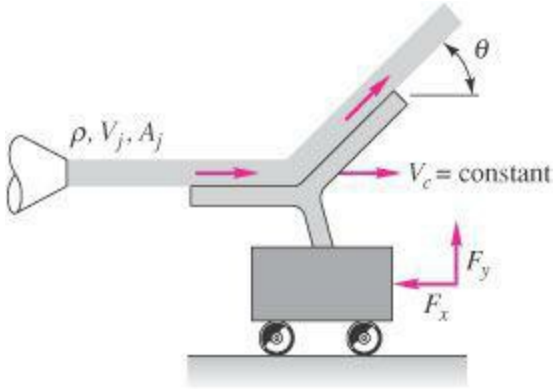
3

- 3) RUWKHSLSHIORZ UHGXFLQJ VHFWLRQRI) LJ 3 ' FP ' FP DQGS DW $ CDIOXLGV DUHDW *f& ,I 9 P VDQG WKHPDQRPHWHUUHDGLQJ LVK FP HWXP DWWKHVRDOIRUFHUHVLVWHG\ WKHIOXLGHERWV



3

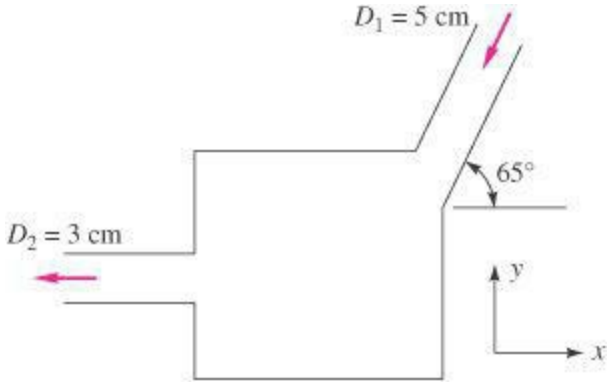
- 3 ,Q) LJ 3 WKHMHWVWUHDPHVDYDQHWKDWPRYHVWKHULJKWDWFRQVWDQWYHORFLW9F RQI IULFWLRQOHVVFDUW&RPSXWH D WKHIRUFH)[UHTXLUHGWR UHVWDLQWKHFDUWDQG E WK SRZHU3 GHOLYHUHGWRWKHFDUW$OVRILQGWKHFDUWYHORFLWIRUZKLFK
- F WKHIRUFH)[LVDPD[LPXP DQG G WKHSRZHU3 LVDPD[LPXP



3

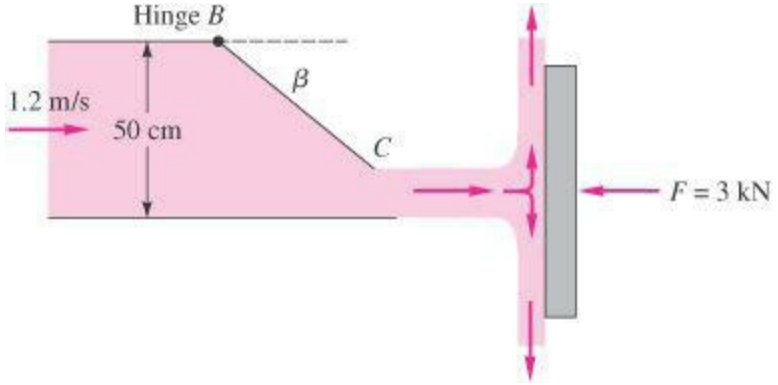
- 3 : DWUDW *f& IORZ VWDGLO WKURXJK WKHER[LQ) LJ 3 HQWHULQJ WDQQ DW P V &DOFXODWH WKH D KRUL]RQWDODQG E YHUWLFDOIRUFHV UHTXLUHG WR KROG WKHER[

VWDWLRQDU DJDLQVWWKHIORZ PRPHQWXP



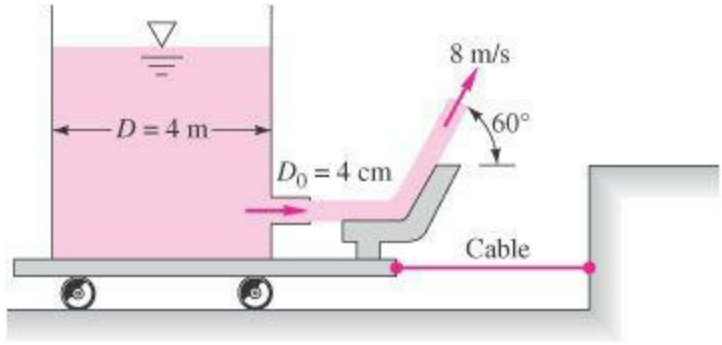
3

- 3 : DWUHDGRZ VWKURXJK WKH GXFWLQ) LJ 3 ZKLFK LV FP ZLGH DQG P GHHS LQ WKH SDSHU * DW%& FRPSDUHG FORVHV WKH GXFWZKHQ f $VXPLQJ RQH-GLPHQVLRQDOIORZ IRUZKDWDQJOH ZLOOWKHIRUFHRI WKHH[LWWMWRQWKHSODWHEH NL"



3

- 3 7KHZDWUWDQNLQ) LJ 3 VWDQGVRQDIUFWRQOHVVFDUWDQGIHHGVDWWRI GLDPHWH FP DQGYHORFLW P V ZKLFK LVGHOHFWHG f E\ DYDQH &RPSXWWKHWQWRQL WKHVXSSRUWQJ FDEOH

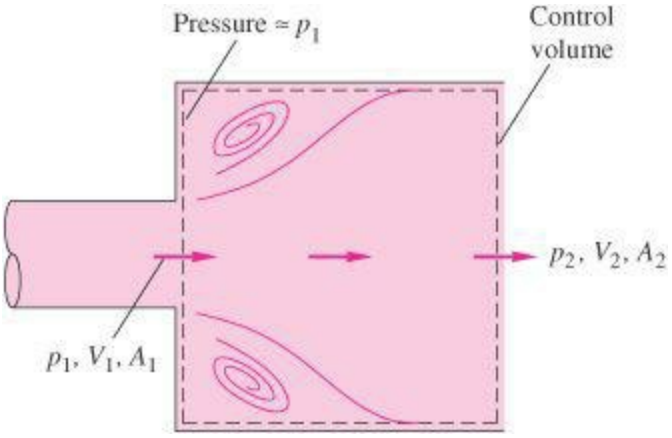


3

- 3 : KHQDSLSHIORZ VXGGHQO H[SDQGVIURP $ WR$ DVLQ) LJ 3 ORZ VSHHG ORZ IUFWRQ HGGLHV DSSHDU LQ WKH FRUQHUV DQG WKH IORZ JUDGXDOO H[SDQGV WR$ GRZ QWUHDP 8VLQJ WKH VXJJHWWHG FRQWROYROXP HIRULQFRP SUHVWLEOH VWDG\ IORZ DQG DVXPLQJ WKDWS § S RQ WKH FRUQHU DQQXODU UHJLRQ DV VKRZ Q VKRZ WKDWWKH GRZ QWUHDP SUHVWXUHLVJLYHQE\

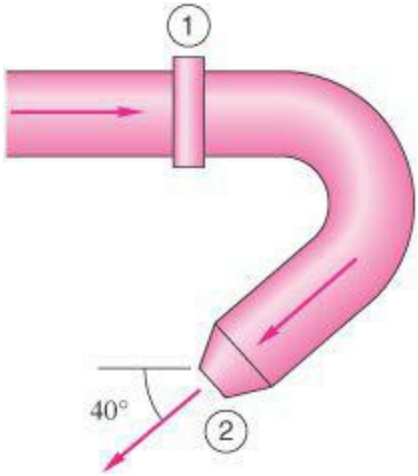
$$p_2 = p_1 + \rho V_1^2 \frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right)$$

1 HJOHFWZ DOOUFWRQ



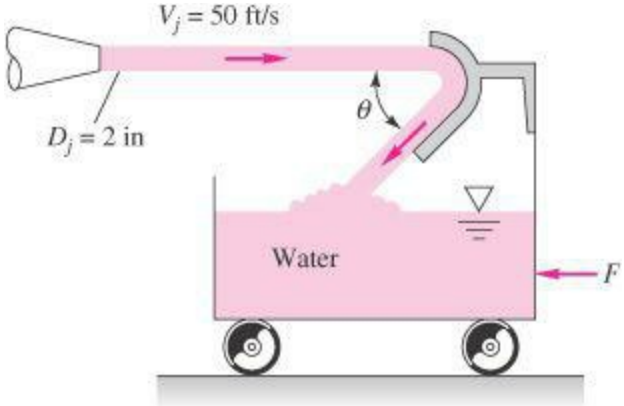
3

- 3 : DAUDW *f*& IGRZ VWKURXJK WKH HGERZ LQ) LJ 3 DQG H[LWV WR WKH DWPRVSKHUH 7KH SLSH GLDPHWHUV' FP ZKLOH' FP $VVXPH UDWHG 1 V WKH SUHVVXUHS DW JDJH 1 HJOHFWQJ WKH ZHJKW RI ZDWHU DQG HGERZ HWXP DW WKH IRUFHRQ WKH IDQJHERWV DW WHFWRQ



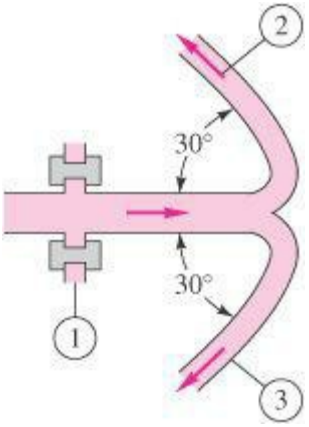
3

- 3 $ *f*& ZDUMWWXUNHV DYDQHPRXQG RQ DVDQN ZLWK IUFWLRQOHVV ZKHHOV DV LO) LJ 3 7KH NWWXLQV DQG IDOV LQR WKH VDQN ZLWRXW SLOOQJ RXW,I *f* HYDOXDWH WKH KRUL]RQWDO IRUFH) UHTXLUHG WR KROG WKH VDQN VWDWRQDU

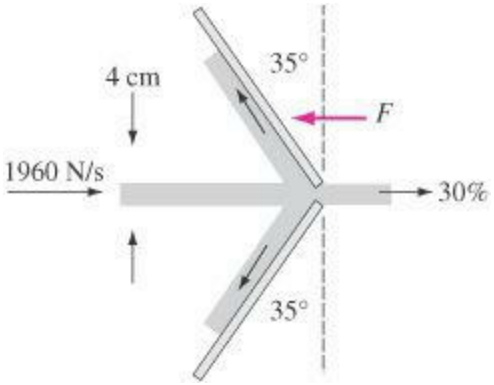


3

- 3 : DAUDW *f*& H[LWV WR WKH VDQGDUG VHD OHYHO DW RSKHUH WKURXJK WKH VSOWQR[]OHL) LJ 3 ' XFWDUHDV DUH$ P DQG$ $ P ,I S N3 DEVROXW DQG WKH IGRZ UDWLV4 4 P K FRP SXWH WKH IRUFHRQ WKH IDQJI ERWV DW WHFWRQ

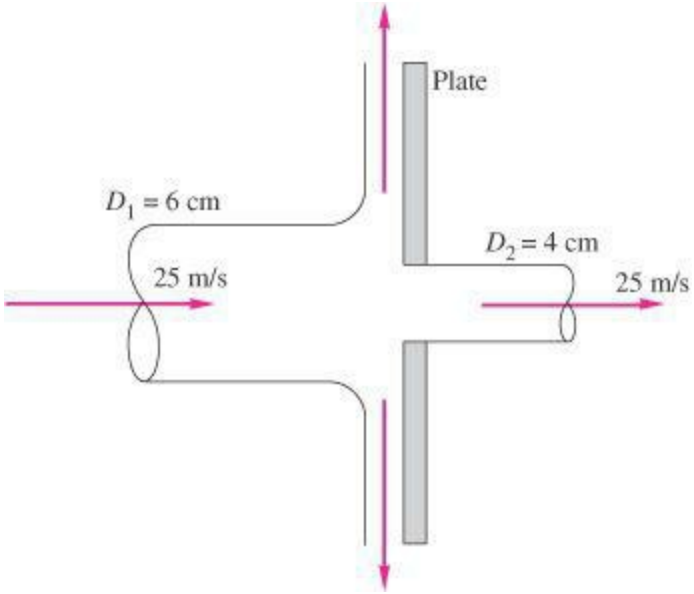


- 3 $ VWHDG\ VWDWH GLPHQVLRQDO DQDOVLV RI D VLQJOH EUDQFK SLSH 1 V VXEMHFW WR GLPHQVLRQDO DQDOVLV 3 3UHVVXUH DQG YHORFLW\ DUH FRQVLGHUHG HYHU\ZKHUH 7KLV SHUFHQW RI WKH MHWSDVVHV WKURXJK WKH VORW 7KH UHVWSOD\ PPHWFDOO DORQJ WKH EUDQFK &DOFXODWH WKH HRUFRQDO IRUFH) QHHGHG WR KROG WKH EUDQFHUSHUXQLWGHSWKLFQHVVLQ WKH SDSHU



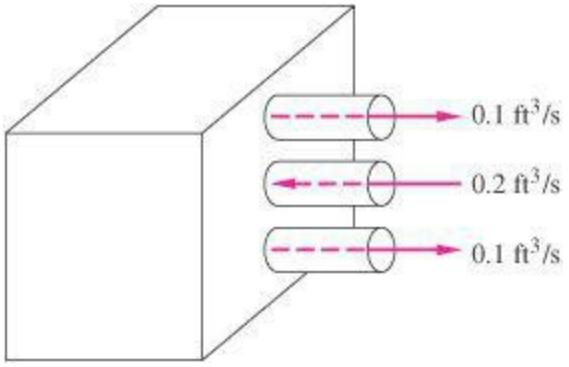
3

- 3 7KH FP GLDPHWHU *f*& ZDWHUMHW) LJ 3 VXEMHFWV D SODWH FRQGLQJ DKRGHR FP GLDPHWHU 3DUW RI WKH MHWSDVVHV WKURXJK WKH KROH DQG SDUWLV GHIOHFWHG ' HWHUPLQH WKH KRUL]RQWDO IRUFH UHTXLUHG WR KROG WKH SODWH



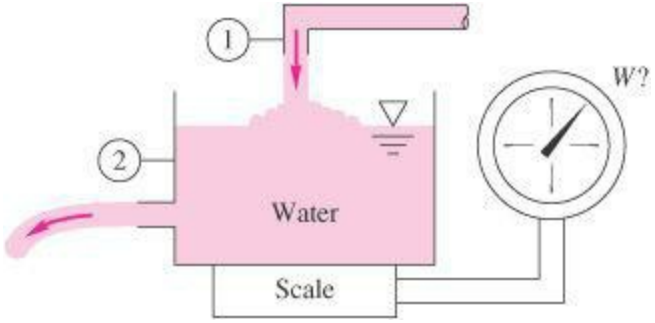
3

- 3 7KH EHQ) LJ 3 KDV WKUHH LQKROHV RQ WKH ULJKW VLGH 7KH YROXPHIORZ VR *f*& ZDWHUVKRZ QDUHVWHDG\ EXW WKH GHDLOV RI WKH LQWHUIDFH RQ WKH EHQ &RPSXWH WKH IRUFH LI DQ WKDW WKLV ZDWHUIORZ FDXVHV RQ WKH EHQ



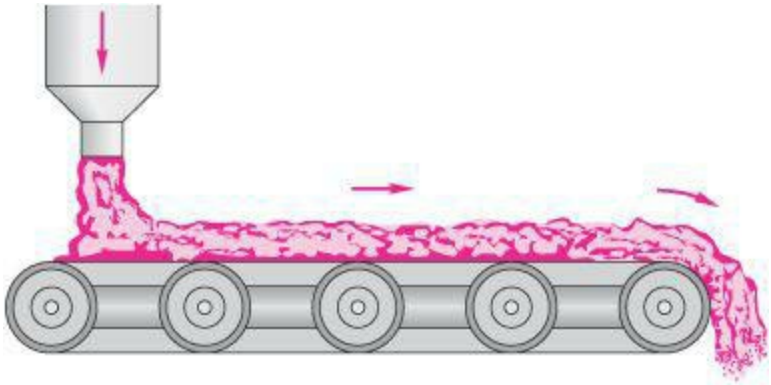
3

- 3 7KH VDQN LQ) LJ 3 ZHLJKV 1 HPSW DQG FRQWDLQV / RI ZDWHU *
  3LSHV DQG KDYH HTXDO GLDPHWHUV RI FP DQG HTXDO WDG YROXPH IORZ VR
  P K: KDWWKRXGWKHVFDOHUHDGLQJ: EHLQ1"



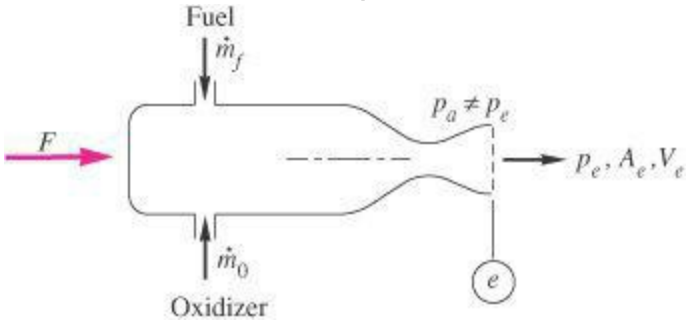
3

- 3 * UDYHOLVGXPSHGIURP DKRSSHU DWDUDWHRI 1 V RQWR DPRYLQJ EHOWDVLQ) LJ
  3 7KHJUDYHOWHQSDWHVRI WKHHQGRI WKHEHOW7KHGULYHZKHHOVDUH FP L
  GLDPHWHUDQGURWDWFORFNZLVHDW UPLQ1HJOHFWQJ VWWP IULFWRQDQGDLUGUDJ
  HWLP DWWKHSRZHUHTXLUHGWRGULYHWKLVEHOW



3

- 3 7KHURFNHWLQ) LJ 3 KDV D VXSHUVRQF H[KDXVWDQG WKH H[LWSUHVVXUHS$ LV QF
  QHFHVVDULO HTXDOWRS' 6KRZ WKDWWKHIRUFH) UHTXLUHGWRKROGWKLVURFNHWRQWKHWHV
  VWDQGLV$F = ρeAeVe² + Ae(pe − pa). ,VWKLVIRUFH) ZKDWZ HWHUP WKHWKUXVWRI WKHURFNHW

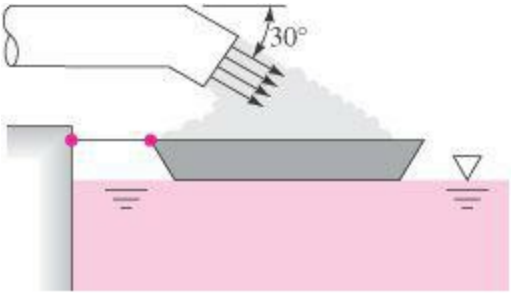


3

- 3 $ XQLRUP UHFWDQJXODUSODWH FP ORQJ DQG FP GHHS LQWR WKHSDSHU KDQJVLQDL
  IURP DKLQJHDWWDWWRS WKH FP VLGH ,WV VXUIFHWHPSHUDWXUE\ DKRUL]RQWDO
  FP GLDPHWHUMHWRI ZDWHUPRYLQJ DW P V ,I WKHJDWHKDVDPDWRI NJ HWLP DL

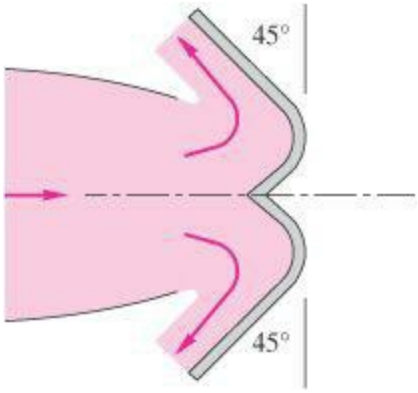
VKHDQJOHDVZKLFKVKHSDVVHVOLKDQJIURP VKHYHUWLFDO

- 3 7KHGUHGJHULQ) LJ 3 LVORDGLQJ VDQG 6* RQRXDEDUJH 7KHVDQGOHDYH-
VKHGUHGJHUSLSHDW IWVZLVKDZHJKWOX[RI GEI V (VWPDWWKHWRQVLRQRQVK
PRRULQJOLQHFDXVHGE\ VKLSORDGLQJ SURFHVV



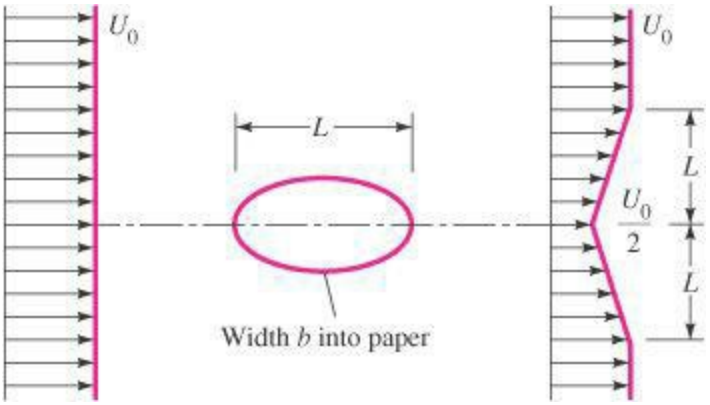
3

- 3 6XSSRVHWKDWDGHOHFWRULVGHSOR\HGDWKHH[LWRI VKHNWWHQLQHRI 3URE 3 D
VKRZQ LQ) LJ 3 : KDWZLOOWKH UHDFWRQ5[RQ VKHWWWXDQGEHQRZ" ,V WKL
UHDFWRQ VXIILFLHQWRVHUYHDVDEUDNLQJ IRUFHGXULQJ DLUSODQHODQGLQJ"



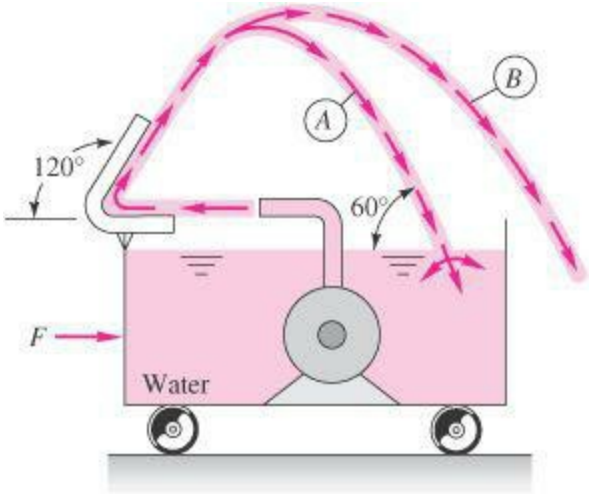
3

- 3 : KHQLPPHUVHGLQ DXQIRUP VWUHDP DWKLFNHVSWDOR\QGHUFUHDWV DEURD
GRZQVWUHDP ZDNH DVLGHDOL]HG LQ) LJ 3 7KH SUHVXUH DWWKH XSVWUHDP DQ
GRZQVWUHDP VHFWLRQV DUHDSSUR[LPDWHO HTXDO DQGWKHIOXLGLVZDWUDW *& ,I8
P VDQG/ FP HVWLPDWWKHGUDJ IRUFHRQWKHF\OLQGHUSHUXQLWZLGWKLQ
WKHSDSHU$OORFFRPSXWWKHGLPHQVLRQOHVVGUDJ FRHILFLHQW&) !8 E/



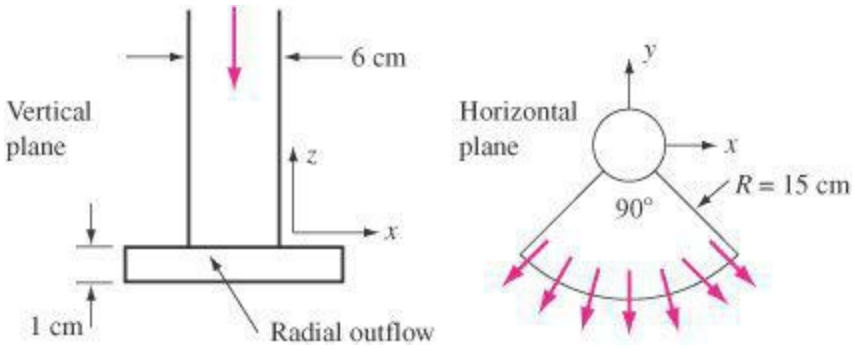
3

- 3 $ SXPSLQDVDQNRI ZDWUDW *& GLUHFWDWDWDW IWWDQG JDOOLQDJDLQWW
YDQH DVVKRZQLQ) LJ 3 &RPSXWWKHIRUFH) WKROGWKHFDWWXDWRQU LI WK
WWWRORZV D SDW$RU E SDW%7KHVDQNKROGV JDORI ZDWUDWKLVLQWDQW



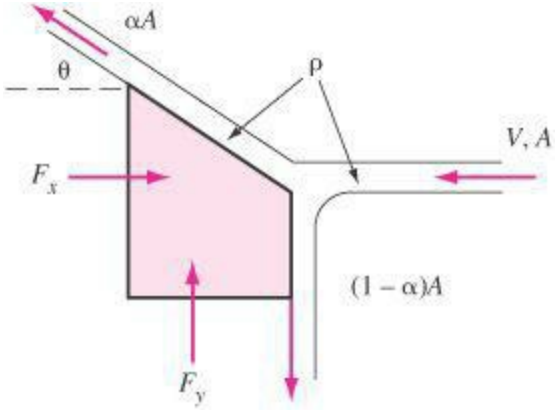
3

- 3 : DWUDW *f*& IORZ VGRZ QWKURXJK DYHUWFDO FP GLDPHWUXEHDW JDOPLQ D LQ) LJ 3 7KH IORZ VKHQVLQV KRUL]RQWDO DQG H[LWV WKURXJK D *f* UDGLDOGXF VHJPHQW FP WKLFN DVVKRZQ ,I WKH UDGLDORXWIORZ LV XQIRUP DQG VWDGG HVWLP DW WKH IRUFHV)[ )\ )] UHTXLUHG WR VXSSRUW WKLV VWWHP DJDLQVW OLG PRPHQWXP FKDQJHV



3

- 3 $ MHWVRI OLTXLG RI GHQVLW! DQG DUHD$ VWULNHV DEDFNDQG VSODWLQRV WR MHW DVLG) LJ 3 $VVXPH WKH VDPH YHORFLW9 IRU DOO WKUHH MHWV 7KH XSSHU MHW[LWV DWD DQJOH DQG DUHD. $ 7KH ORZHU MHWV VXQGHG *f GRZQZDUG 1HJOHFWLQJ IXO ZHLJKW D GHULYH DQ H[SUHVVLRQ IRU WKH IRUFHV)[ )\ UHTXLUHG WR VXSSRUW WKH EDFN DJDLQVW OLG PRPHQWXP FKDQJHV E 6KRZ WKDW\ RQO LI . F) LQG WK YDOXHVRI . DQG IRUZKLFKERWK)[DQG)\ DUH]HUR

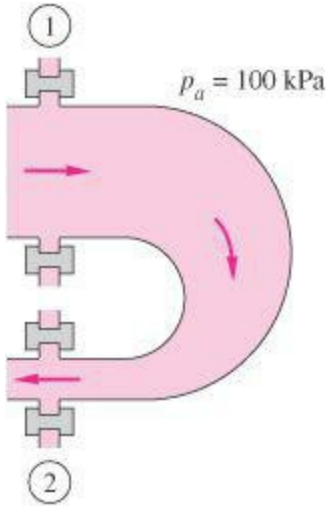


3

- 3 $ VZR GLPHQVLRQDOVKHHWRI ZDWU FP WKLFN DQG PRYLQJ DW P V WULNHV DI[HG ZDOOLQFOLQHG DW *f ZLWK UHVSHFWWR WKH MHWVGLUHFWLRQ $VVXPLQJ IULFWLRQOHVV IORZ ILQG D WKHORUP DOIRUFHRQWKHZDOOSHUPHWUR GHSWK DQG ILQG WKHZLGWKRI WK VKHHWGHOHFWHG E XSWUHDP DQG F GRZQWUHDP DORQJ WKHZDOO
- 3 : DWUDW *f& IORZ VVWHDGLO WKURXJK DUHGXFLQJ SLSHEHQG DVLQ) LJ 3 . QRZ (

FRQGLWLRQV DUH S NBD ' FP 9 P V S NBD DQG '
FP 1 HJOHFWLQJ EHQG DQG ZDWHU HJKVHWWP DWWKH WRS DQG IRUFH WKDW P XVW EH UHVLVWH
E\ WKH IODQJHEROWV

- 3 $ ICXLG MHWVR GLDPHWU' HQWHUV DFDFDGHR PRYLQJ EODGHV DWDE VRXWH YHORFLW9
DQG DQJOH DQG LWVHDYHV DWDE VRXWH YHORFLW9 DQG DQJOH DV LQ) LJ 3
7KH EODGHV PRYH DWYHORFLWX ' HULYH D IRUPXOD IRU WKH SRZHU3 GHOLYHUHG WR WK
EODGHV DV D IXQFWLRQ RI WKH VHSDUDWHWV



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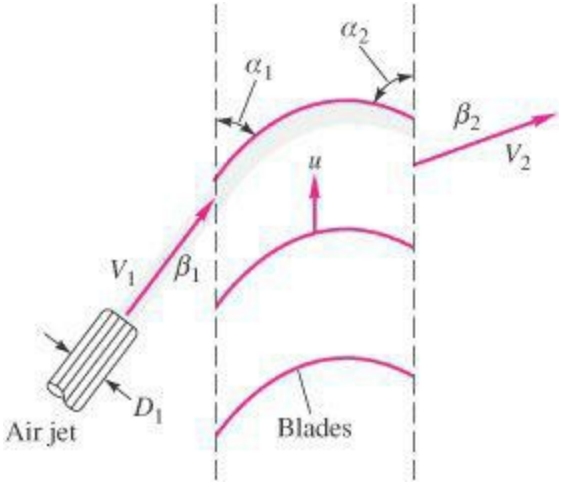
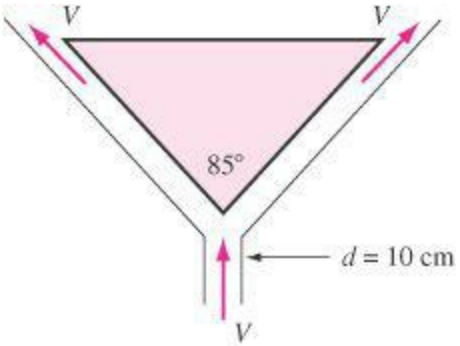


3

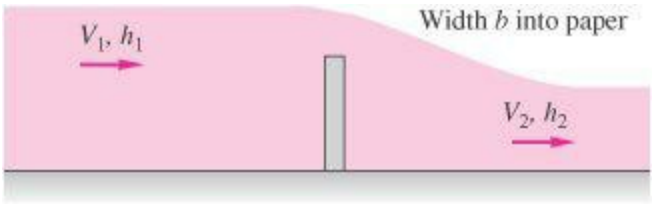
- 3 $ LUDW *& DQG DW HQWHUV WKH ERWWRP RI DQ *FRQFDOICRZ PHWUGXFWDWDP DV
ICRZ RI NJ V DV VKRZ QLQ) LJ 3 ,WLV DEOH WR VXSSRUW D FHQWUHG FRQFD
ERG\ E\ WWDG DQQXODU ICRZ DURXQG WKH FRQH DV VKRZ Q 7KH DLU YHORFLW DWWK
XSSHUHGJHRI WKHERG\ HTXDOV WKH HQWUDQW YHORFLW (VWP DWWKHZ HJKWRI WKHERG\
LQQHZ WRQV



3

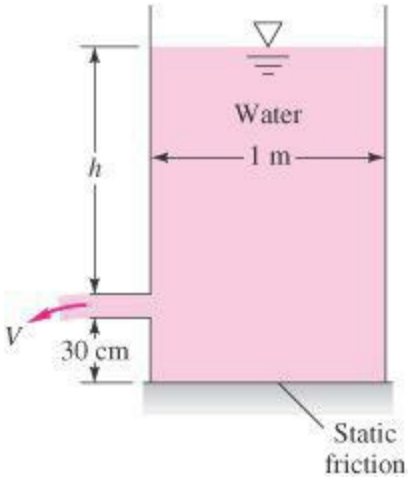
- 3 $ ULYHUR ZLGWKE DQG GHSWKK SDWHV RYHUD VXEP HUJHG REVWDFOH RU³GURZQH
ZHLU LQ) LJ 3 HPHUJLQJ DWD QHZ ICRZ FRQGLWLRQ 9 K 1HJOHF

DWPRVSKHULF SUHVVXUH DQG DVXPH WKDWWKH ZDWHU SUHVVXUH LV K\GURVWDWLF DWERWK VHFWLRQV DQG ' HULYH DQ H[SUHVVLRQ IRU WKH IRUFH H[HUWHG E\ WKH ULYHU RQ WKH REVWDFOH LQ WHUPV RI 9 K K E ! DQGJ 1 HJOHFWZDWHU IULFWLRQ RQ WKH ULYHUERWWRP



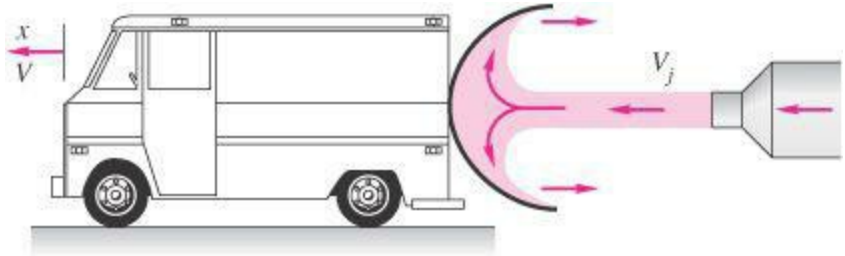
3

- 3 EES 7RUULFHOOLV[VLGHDQ]DWRQ HIOX[IURP DKROHLQWKHVLGHRI DWDQNLV$V VKRZQ LQ) LJ 3 7KH FA\OLQGULFDOWDQN ZHJKV 1 ZKHQ HPSW DQ FRQWDLQV ZDWHU DW f& 7KH WDQN ERWWRP LV RQ YHU VPRRWK LFH VWDWF IULFWLRQ FRHILFLHQW § 7KH KROHGLDPHWULV FP)RUZKDWZDWUGHSWKK ZLOOWKH WDQNMXVWEHJLQWRPRYHWKHULJKW



3

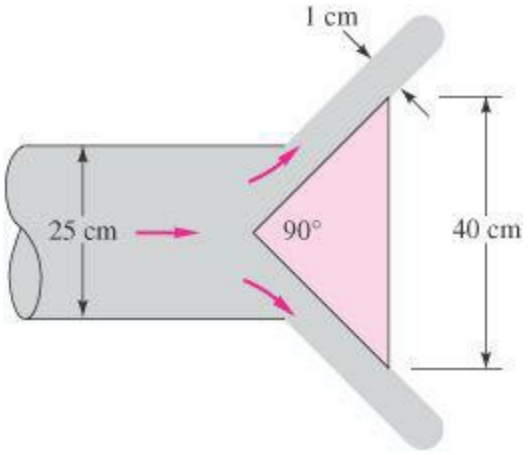
- 3 7KHPRGHOFDULQ) LJ 3 ZHJKV 1 DQGLVWREHDFFHOHUDWHGIURP UHVWE\ D FP GLDPHWUZDWHUNHWWPRYLQJ DW P V 1HJOHFWQJ DLUGUDJ DQG ZKHHOIULFWLRQ HVWLPDWWKHYHORFLWRI WKHFDUDWWULWKDVPRYHGIRUZDUG P



3

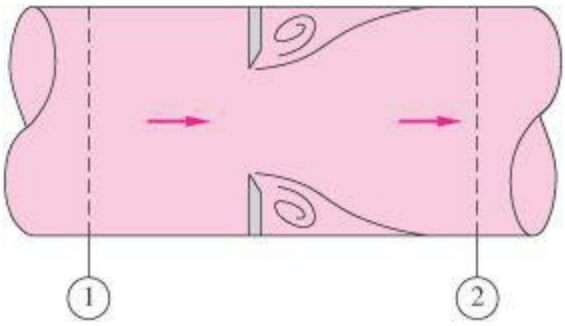
- 3 * DVROLQH DW f& LV IORZLQJ DWP P V LQ D FP GLDPHWUSLSH ZKHQ L HQFRXQWHUV D P OHQJWK RI XQLIRUP UDGLDOZDOOVFWLRQ $WWKH HQG RI WKLV VXFWLRQ UHJLRQ WKH DYHUDJH IOXLG YHORFLW KDV GURSSHG WR9 P V ,I S NJ HWLPDWS LI WKHZDOOIULFWLRQORVVHVDUHQHJOHFWG

- 3 $LUDW f& DQG DWP IORZ VLQD FP GLDPHWUGXFWDW P V DVLQ) LJ 3 7KHH[LWVFKRNHGE\ D f FRQH DVVKRZQ (VWLPDWWKHIRUFH HV RQWKHFRQH



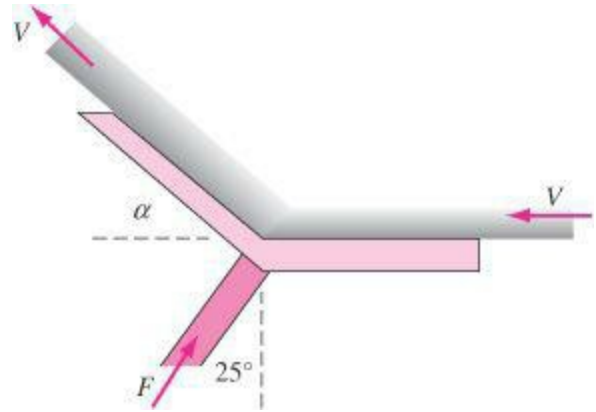
3

- 3 7KH WKLQ SODWH RULILFH LQ) LJ 3 FDXVHV D ODUJH SUHVVXUH GURS) RU *& ZDW IORZ DW JDOPLQ ZLWK SLSH' FP DQG RULILFH G FP S í S § N3D ,I WKH ZDOO WKLFNQHVV LV QHJOLJLEOH HVWLPDWH WKH IRUFH RI WKH ZDWHU RQ WKH RULILFH SODW



3

- 3) RU WKH ZDWHU IORZ SXPS RI 3UREOHP 3 DGG WKH IROORZLQJ GDWD S S GHWHUPLQH DQG WKH GLVDQFH EHWZHHQ VHFWLRQV DQG LV ,I WKH DYHUDJH ZDOO VKHDU VWUHVV EHWZHHQ VHFWLRQV DQG LV GHWHUPLQH WKH SUHVVXUH S : K\ LV LWKLJKHU WKDQ S "
- 3 $ YDQHVXLQV DZDWHUWLJKWQDQGJOH. DV VKRZQ LQ) LJ 3 1HJOHFWIULFWLRQ RQ WKH YDQH ZDOOV D : KDWLV WKH DQJOH. IRU WKH VXSSRUWV RFH WR EH LQ SXU FRPSUHVVLRQ' E &DOFXODWH WKLV FRPSUHVVLRQ IRUFH LI WKH ZDWHU YHORFLW LV IW DQG WKH MHWWURXVVHFWLRQV LQ



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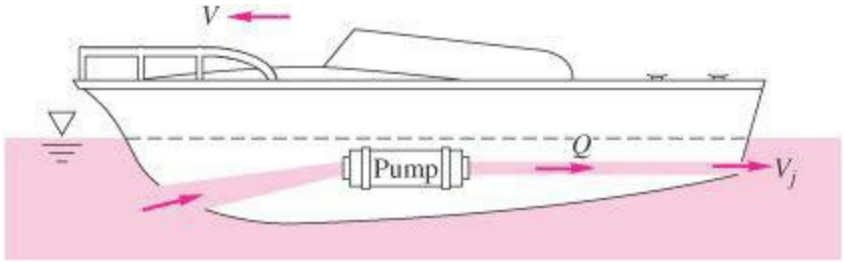
- 3 7KH EDWQ) LJ 3 LV MHWSURSHOOHG E\ DSXPS WKDW GHYHORSV DYROXPHWULF UDW 4 DQG HMHFWV ZDWHU URXWKH VWULQDWY HORFLW9 ,I WKH EDWGUDJ IRUFH LV) N9 ZKHUHNLV DFRQVWDQWGHYHORS DIRUPXODIRUWKH VWHDG\ IRUZDUGVSHG9 RI WKH EDW



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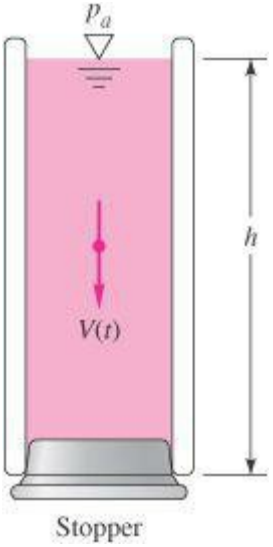
- 3 &RQVLGHU)LJ 3 DVDJHQHUDOSUREOHP IRUDQDDVLVRI DPL[LQJ HVFARUSXPS , DOOFRQGLWLRQV S ! 9 DUHNQRZQDWWHFWRQV DQG DQGLI WKHZDOIUFWRQLV QHJOLJLEOHGHULYHIRUPXODVIRUHWMDWQJ D 9 DQG E S
- 3 $VVKRZQLQ)LJ 3 DQDTXLGFROXPQRI KHJKWK LVFRQILQHGLQDYHUWFDOWXEHR FURVVVHFWLRQDO DUHD$ E\ D WARSSHU $WW WKH WARSSHULV VXGGHQO UHPRYHG H[SRVLQJ WKHERWWRP RI WKHDTXLGWRDWPRVSKHULF SUHVVXUH



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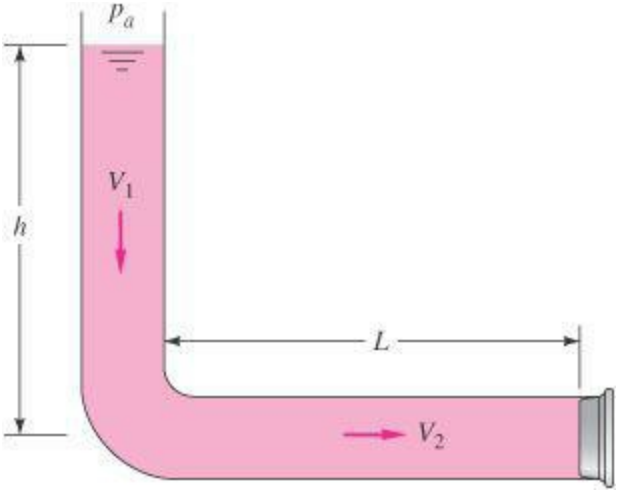
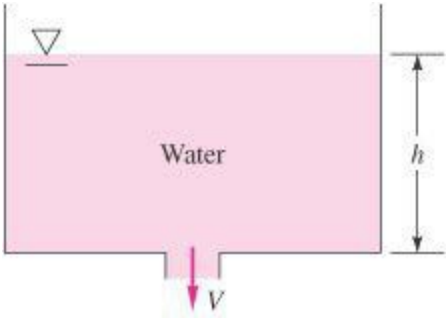
8VLQJ DFRQWROYROXPHDQDOVLVRI PDVVDQGYHUWFDOPRPHQWXP GHULYHWKHGLIIHUHQWL HTXDWRQ IRU WKH GRZQZDUG PRWRQ9 W RI WKH DTXLG $WXPH RQHGLPHQVLRQDO LQFRPSUHVVLEOH IUFWLRQOHVVIORZ

- 3 ([WQG3URE 3 WR LQFXGH D DQHU DPLDU DYHUDJH ZDOVKHDU VWUHVVDQFHRI WKHIRUP 2 § F9 ZKHUHF LV D FRQVWDQW) LQG WKHGLIIHUHQWLDOHTXDWR IRUG9 GWDQG WKHQVROYHIRU9 W DWXPLQJ IRUVPSOFLW WKDWWKHZDOODUHDUHDLQ FRQVWDQW
- 3 $ PRUHLQYROYHGYHUVRQRI 3URE 3 LVWKHHOBRZ VKDSHGWXEHLQ)LJ 3 ZLWK FRQVWDQW FURVVVHFWLRQDO DUHD$ DQG GLDPHWU' ≪ K / $VXPI LQFRPSUHVVLEOH IORZ QHJOHFWUFWLRQ DQG GHULYHDGLIIHUHQWLDOHTXDWRQIRUG9 G ZKHQWKHWXSSHULVRSHQHG +LQW&RPELQHWZR FRQVWROYROXPHV RQHIRUHDFK OH RI WKHWXEH



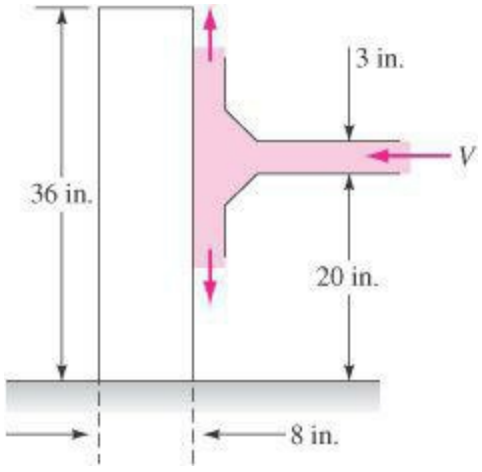
3

- 3 $ FFRUGLQJ WR 7RUULFHOOL¶V WKHRUHP WKH YHORFLW\ RI DIOXLG GUDLQLQJ IURP D KROH LQ D VDQNLV9 § JK ZKHUH K LV WKH GHSWK RI ZDWHU DERYH WKH KROH DV LQ) LJ 3 / HW WKH KROH KDYH DUHD $R DQG WKH FRQGLWLRQ KDYH FURVV VHFWLRQ DUHD $E ! ! $R ' HULYH DIRUPXOD IRU WKH WLPH UHTXLUHG WR GUDLQ WKH VDQN FRPSOHWHO IURP DQLQLWLDO GHSWK K R



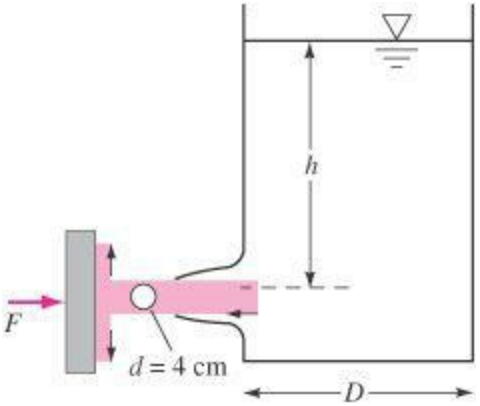
3

- 3 $ ZDWHU WLQLQGLPHUVUXQHV DFRQFUHWH 6* VDE ZKLFK UHVWV IUHO R DOHYHOOLQJ RU ,I WKH VDE LV IWZ LGHQWLFDO WKH SDSHU FDQ EH XVHG WR ZLOO UXWEHJLQ WR SWKH VDE RYHU



3

- 3 $ VDOZDWHU DQG GLVFKDUJHV WKURXJK DZHOOURXQGHG RULILFH DV LQ) LJ 3 8VH WK 7RUULFHOOL IRUPXOD RI 3UREOHP VR HVWLPDWH WKH[LWYHORFLW D ,I DVWKLV LQLWLDO WKH IRUFH) UHTXLUHG WR KROG WKH SODWH 1 ZKDW LV WKH GHSWK' E ,I WKH VDQ VXUIDFH LV GURSSLQJ DW WKH UDWH RI FP V ZKDW LV WKH VDQNGLPHU' "



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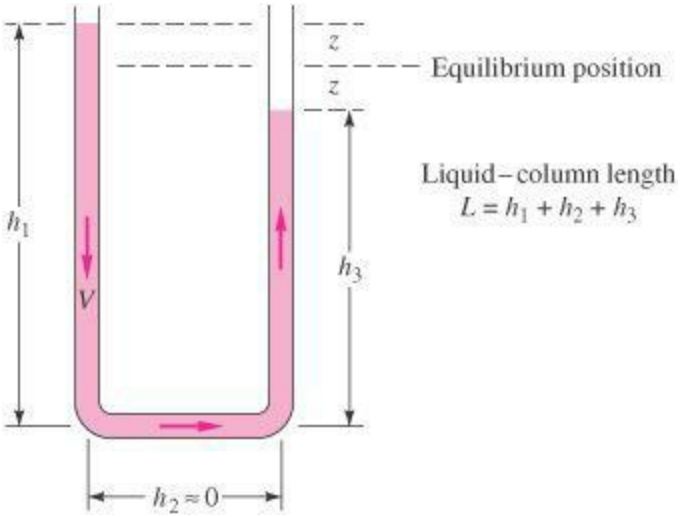
- 3 ([WHQG3URE 3 VR WKH FDVH RI WKH OLTXLG PRWLRQ LQ DIUFWLRQOHVV 8 WXEH ZKRV OLTXLG FROXPQ LVGLVSODFHG DGLVWDQFH = XSZ DUG DQG WKHQUHOHDVHG DVLQ) LJ 3



3

1HJOHFWWKH VKRUWKRUL]RQWDOHJ DQG FRPELQH FRQWROYROXPH DQDOVHV IRUWKH OHWDQ ULJKWOHJV WR GHULYH DVLQJOH GLIIHUHQWLDO HTXDWLRQ IRU 9 WR WKH OLTXLG FROXPQ

- 3 ([WHQG3URE 3 VR LQFOXGH D OLQHDU ODPLQDU DYHUDJH ZDOO VKHDU VWUHVVIRUP 2§ 9 ' ZKHUH LV WKH IOXLG YLVFRVLW) LQG WKH GLIIHUHQWLDO HTXDWLRQ IRUG9 GWDQG WKHQ VROYH IRU 9 W DWXP LQ DQLQWDOGLVSODFHPHQW] 9 DWW 7KHUHVXOWWKRXOGEHDGDPSHGRVFLOODWRQWKQGLQJ WRZ DUG]
- 3 $V DQ H[WHQVLRQ RI ([DPSOH OHWWKH SODWH DQG LWV FDUW VHH) LJ D EH XQUHVWUDLQHG KRUL]RQWDOO ZLWK IUFWLRQOHVV ZKHOV ' HULYH D WKH HTXDWLRQ R PRWLRQ IRUFDUWYHORFLW9 WDQG E DIRUP XODIRUWKH WLPHUHTXLUHG IRUWKH FDUWV DFFHOHUDWIURP UHVWWR SHUFHQWRI WKH WHUYHORFLW DXWXPLQ WKH WHUFRQVWQX VWUNH WKH SODWKRUL]RQWDOO F &RPSXWQXPHULFDOYDOXHVIRUSDUWE XVLQJ WK FRQGLWLRQVRI ([DPSOH DQGDFDUWPDVRI NJ
- 3 /HWWKHURFNHWRI) LJ (VWDUWDVY ZLWKFRQVWDQW[LWYHORFLW DQG[LWPDV IORZ DQG UHVHYHUWFDOO ZLWK]HUR GUDJ D 6KRZ WKDWDV ORQJ DV IXHOEXUQLQ FRQVWQW VWHYHUWFDOKHLJKW6 WUHDFKHGLVJLYHQE\

$$S = \frac{V_e M_o}{\dot{m}} [\zeta \ln\zeta - \zeta + 1], \text{ where } \zeta = 1 - \frac{\dot{m}t}{M_o}$$

E $SSO WKLV WR WKH FDVH9$_H$ P VDQG0$_R$ NJ WR ILQG WKH KHLJKWUHDFKH DWWUDEXUQRI VHFRQGV ZKHQWKHILQDOURFNHWPDVLV NJ

- 3 6XSSRVH WKDWWKH VRQG SURSHOODQWURFNHWRI 3URE 3 LV EXLOWVLQR DPLWWHR

GLDPHWU FP DQG OHQJWK P 7KHV\VWHP ZHJKV 1 ZKLFK LQFOXGHV 1
RI SURSHOODQW1 HJHFWDLUGUDJ ,I WKH PLWLOHLV ILUHG YHUWLFDOO IURP UHVWDWH
OHYHO HVWLPDWH D LWV YHORFLW DQGKHJKWDWXHOEXUQRXWDQG E WKHPD[LPXP KHJK
LWZ LOODWWLQ

- 3 : DWUDW *& IORZ VVWHDGLO WKURXJK WKH VDQNLQ) LJ 3 . QRZ QFRQGLWRQV DU
' FP 9 P V DQG' FP $ ULJKWZDUGIRUFH) 1 LVUHTXLUHGV
NHHSWKHVDQNIIL[HG D : KDWLV WKHYHORFLWOHDYLQJ VFDWRQ " E ,I WKHVDQNFURV
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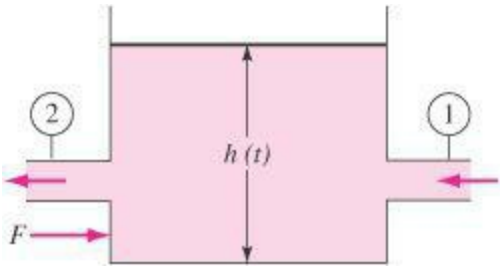

3

- 3 $ VFDQRWWQEHVHQLDNWKHQLNZKHQWKHIDFHWLV UKQQLQJ DKJK VSHH
FKDQQHOIORZ 9 K PD³XPS´ VRDORZ VSHHG ORZ HQHUJ\ FRQGLWRQ 9 K D
LQ) LJ 3 7KHSUHVXUHDWVHFWLRQV DQG LVDSSUR[LPDWD KVGURVWDWF DQ
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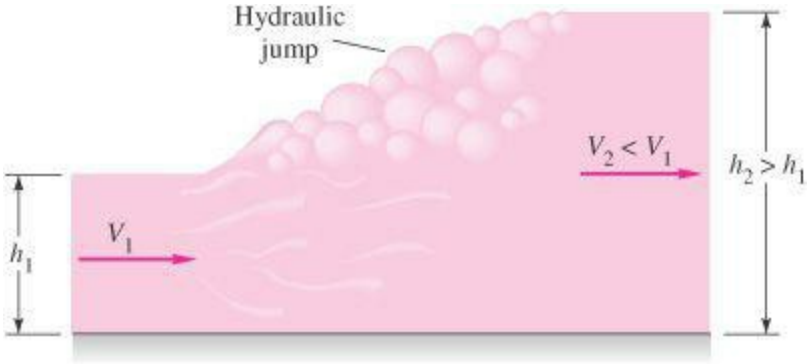
9 LQWHUPVRI K 9



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- 3 6XSSRVHWKDWWKHVRQG SURSHOODQWURFNHWRI 3UE 3 LVPRXQWHGRQD N
FDUWRSURSHOOLWXS DORQJ VORSHRI ƒ 7KHURFNHWPRWRUZHLJKV 1 ZKLF
LQFOXGHV 1 RI SURSHOODQW,I WKHFDUWVWDUWV IURP UHVWZKHQWKHURFNHWLVILUHG
DQGLI DLUGUDJ DQGZKHHOIULFWLRQDUHQHJOHFWHG HWHP DWWKHPD[LPXP GLVWDQF
WKDWWKHFDUZLOOWDYHOXSWKHKLOO

- 3 $ URFNHWVDWWDFKHGWRDULJLGKRUL]RQWDOURGKLQJHGDWWKHRULJLQDVLQ) LJ 3
,WLQLWLDOPDVVLV0 DQGLWH[SURSHOOHVDUHæDQG9$ UHODWLYHWRWKHURFNHW6H
XS WKHGLIIHUHQWLDOHTXDWLRQIRUURFNHWPRWLRQ DQGVROYHIRUWKHDQJXODUYHORFLW
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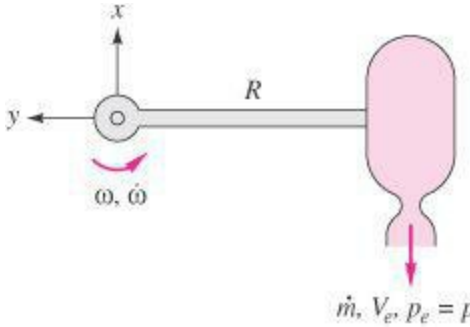

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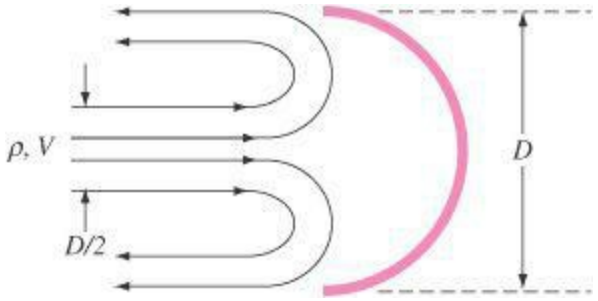
- 3 ([WHQG3URE 3 VR WKH FDVH ZKHUH WKH URFNHW KDV DQ HDUOLHU GUDJ IRUFH) F9 ZKHUH F LV D FRQVWDQW $VVXPLQJ QR EXUQRXW RI& W DQG ILQG WKH WHUPLQDO DQJXODU YHORFLW\ WKDW LV WKH ILQDO PRWRQ ZKHQ WKH DQJXODU DFFHOHUDWRQ LV]HUR $SSO WR WKH FDVH0 NJ 5 P æ NJ V 9$_H$ P V DQG F 1 VP WR ILQG WKH DQJXODU YHORFLW DW WU VR I EXUQLQJ
- 3 $FWXDOO LIRZ SDWWDS SDUDFKXWH FUHDWHV DYDULDEOH GLVWULEXWRQ RI YHORFLWLHV DQ GLUHFWRQV / HWXV PRGHOWKLV DV D FLUFXODU DLUIRLO GHP HWUKDO WKH SDUDFKXW GHP HWU ZKLFK LV WKUQHGFRP SOHWHO DURXQG E\ WKH SDUDFKXW DVLQ) LJ 3 D) LQG WKH IRUFH) UHTXLUHG WR VXSSRUW WKH FKXW E ([SUHVV WKLV IRUFH DV I GLPHQVLRQOHVVGUDJ FRHILFLHQW&) > !9 CE ' @DQG FRPSDUH ZLW 7DEOH



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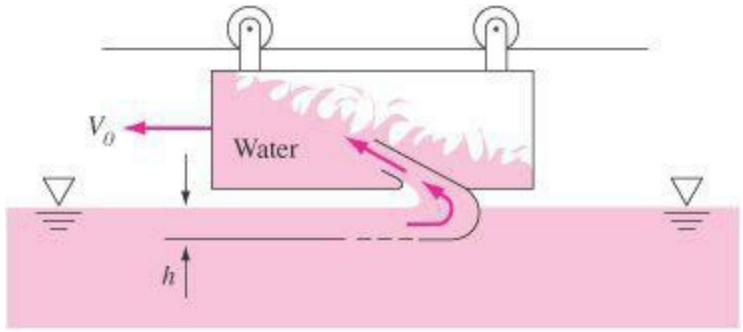
- 3 7KH FDUWQ) LJ 3 PRYHV DWFRQVWDQW YHORFLW9 P V DQG WDNHV RQ ZDW ZLWK D VFRRS FP ZLGH WKDW GLSV K FP LQR D SRQG 1HJOHFWDLUGUDJ DQ ZKHHOLFRQ (VWLP DWH WKH IRUFH UHTXLUHG WR NHHS WKH FDUW PRYLQJ



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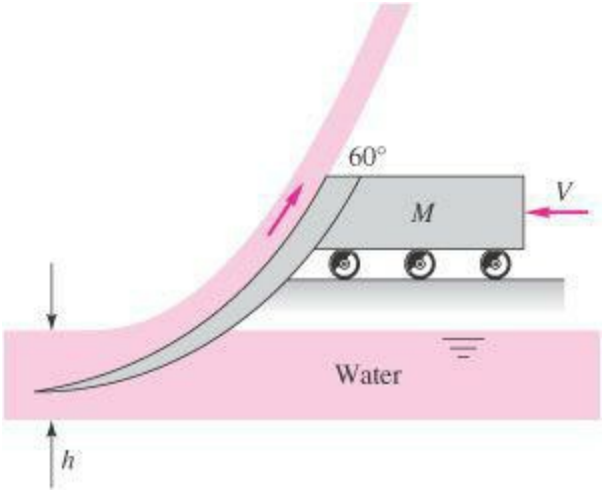
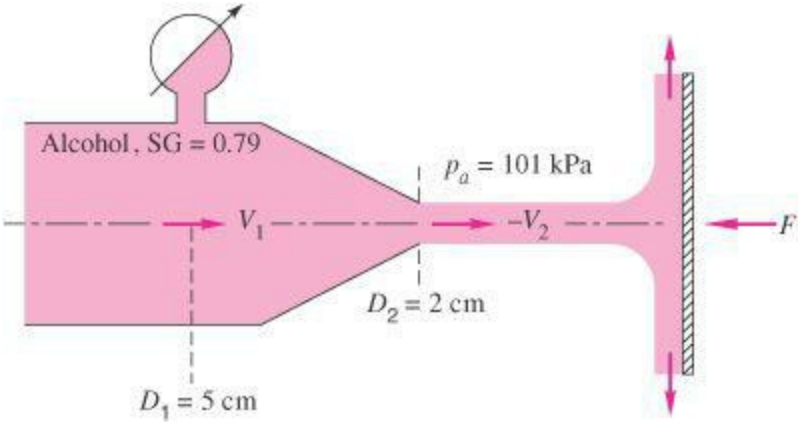
- 3 $ URFNHW OHG RI PDVV0 LV WR EH GHFHOHUDWHG E\ D VFRRS DV LQ) LJ 3 ZKLH KDV ZLGWK E LQR WKH SDSHU DQG GLSV LQR WKH ZDWHU DW DQ DQXSZDUG DQJOH f 7KH URFNHW WKUXVW V7 WR WKH OHW/ HWXV WKH LQLWLDO YHORFLW EH9 DQG QHJOHFWDLUGUDJ DQG ZKHHOLFRQ) LQG DQH[SUHVLRQ IRU9 WR WKH VOHG IRU D 7 DQG E ILQLWM7 •
- 3) RU WKH EORFN DQG FDUHUIORZ LQ) LJ GHWWKH H[WYHORFLW SURILOH DW/ / VLPXODWH WKLV EHQWXEHQW IORZ X § 8 \ / D) LQG D UHDVRQE HWZHHQK DQG/ E) LQG DQH[SUHVLRQ IRU WKH GUDJ IRUFH) RQ WKH SODWE HWZHHQ DQG/



3

7KH%HUQRXOOL(TXDWLRQ

- 3 5HSHDW3ROVE 3 E\ DVVXPLQJ WKDWS LVXQNQRZQ DQG XVLQJ %HUQRXOOL¶V HTXDWLR ZLWK QR ORVVHV &RPSXWH WKH QHZ EROWRUFH IRUWKLV DVVXPSWLRQ : KDWLV WKH KHD ORVV EHWZHHQ DQG IRUWKHGDWDRI3ROVE 3 "
- 3 ([WHQG WKH VSKRQDQDOVLVRI([DPSOH DVIROORZV / HWS DW DQG ORWWK IOXLG EHKRWZDWUDW *& / HWV] DQG] EHWKHVDPH ZLWK] XQNQRZQ)LQG WK YDOXHRI] IRUZKLFKWKHZDWUPLJKWEHJLQWRYDSRUL]H
- 3 $ MHWRI DOFRKROVWUNHVWKH YHUWFDOSODWH LQ)LJ 3 $ IRUFH) § 1 L UHTXLUHG WR KROG WKH SODWH VWDWRQDU\ $VVXPLQJ WKHUH DUH QR ORVVHV LQ WKH QR]]OH HWUP DW D WKHP DVIORZ UDWHRI DOFRKRODQG E WKHDEVROXWSUHVXUHDWH[LW



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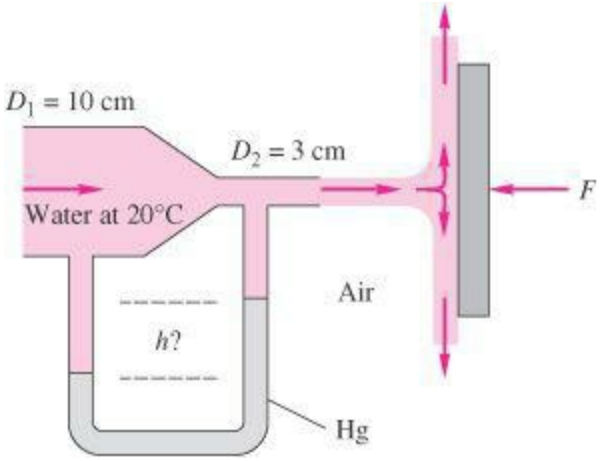
- 3 $QDLUSODQHLVIOLQJDW PLKDW P WDQGDUGDOWWKH$VLVWSLFDOWKHDL YHORFLWUHODWLYHWRWKHXSSHUVXUFHRI WKHZLQJ QHDUWPD[LPXP WKLFNHVV LV SHUFHQWKLJKHUWKDQWKH SODQH¶V YHORFLW 8VLQJ %HUQRXOOL¶V HTXDWRQ FDOFXODWH WK DEVROXW SUHVXUH DWWKLV SRLQWRQ WKH ZLQJ 1HJOHFWHOHYDWRQ FKDQJHV DQ FRPSUHVLELOLW
- 3 : DWUIORZ VWKURXJK DFLUFXODUQR]]OH H[LWLQWR WKHDLUDV DWWWDQG WXUNHVDSODW DV VKRZQ LQ)LJ 3 7KHIRUFH UHTXLUHG WR KROG WKH SODWH VWDG LV 1 $VXPLQJ WXDG IUFWLRQOHVV RQHGLPHQVLRQDOIORZ HVXP DW D WKHYHORFLWVD VHFWRQV DQG DQG E WKHPHUFXU PDQRPHWUHDGLQJ K



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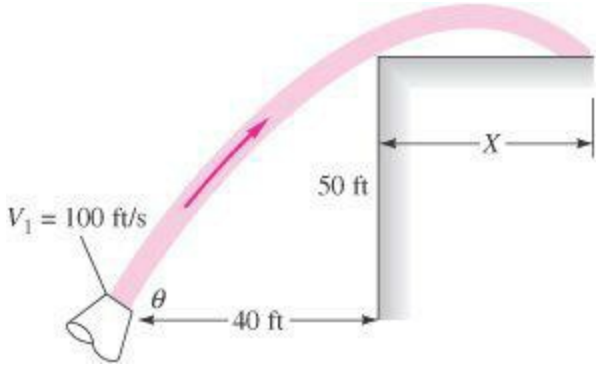
- 3 $ IUHH OLTXLG MHWDV LQ) LJ 3 KDV FRQWDQWDP EHQWSUHVVXUH DQG VP D GRXHV KHOFH IURP %HUQRXOOL[V HTXDWLRQ] 9 J LV FRQWDQWDORQJ WKH MHW) RUWK ILUHQR]]OHLQWKHILJXUH ZKDWDUH D WKHPLQLPXP DQG E WKHPD[LPXP YDOXHVRI IRUZKLFK WKHZDWHUMHWZLOOFOHDUWKHFRQHURI WKHEXLOGLQJ")RUZKLFK FDVHZLO WKHMHWVWHOFLWEHKJKHUZKHQLWVWULNHVWKHURRI RI WKHEXLOGLQJ"



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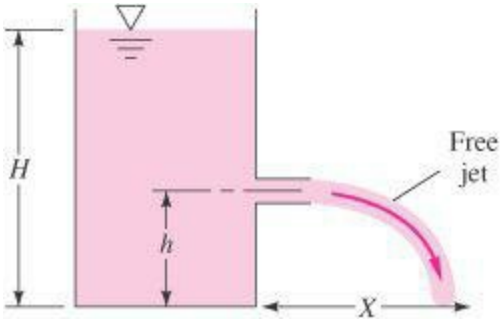
- 3)RUWKHFRQGLQHURI) LJ 3 XVH%HUQRXOOL[V HTXDWLRQ WR GHULYH DIRUPXOD IR WKH GLVWDQFH; ZKHUH WKHIUHHMHWOHDYLQJ KRU]RQWDO ZLOOVWULNH WKHIORRU DV I IXQFWLRQ RI K DQG+)RU ZKDWUDWLRK + ZLOO, EH PD[LPXP" 6NHWFK WKH WKUH WDMHFWRULHVIRUK + DQG



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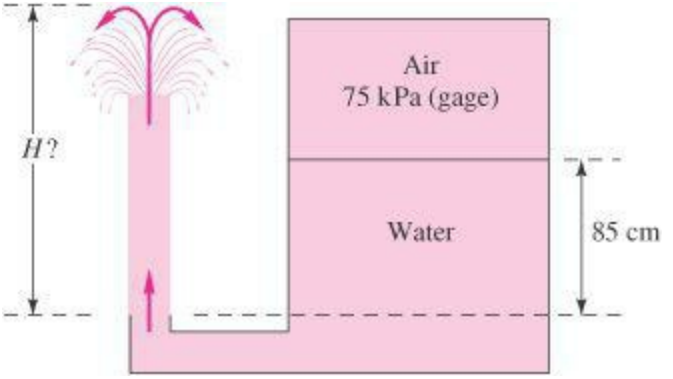
- 3 : DWUDW *& LQ WKH SUHVVXUL]HG WDQN RI) LJ 3 IORZ V RXWDQG FUHDWV I YHUWLFDOMHWDV VKRZQ $VVXPLQJ VWDG IULFWLRQOHVV IORZ GHWHUPLQH WKH KHLJKW+ W ZKLFK WKHMHWUHV



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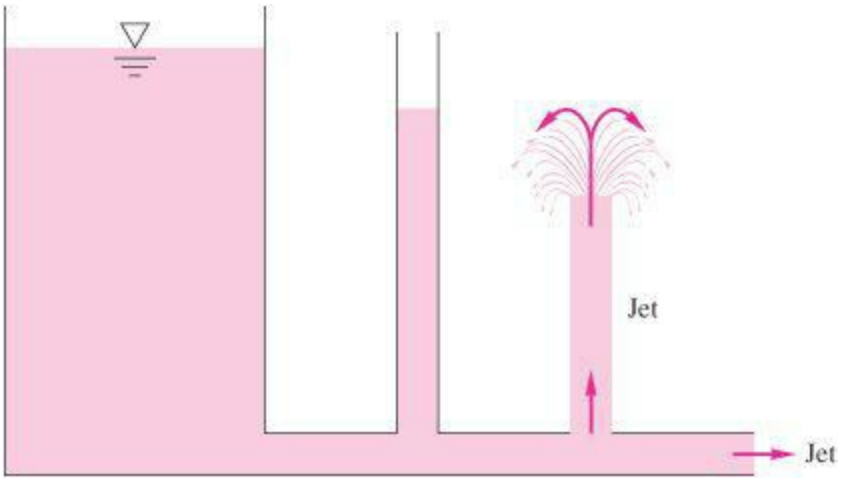
- 3 %HQRXOOL¶V WKHRUHP+\GURG\QDPLFDFRQVLQ/PDQ H[FHOHQWN HIRZ SDWWHUQ/ UHDVRQDEOH VR KLV IULFWLRQOHVV UHDVRQ 2QH KRZ HYHU UHGXFHV KHU DV/) LJ 3 VHHPV SK\VLFDOO PLVOHDGLQJ &DQ\RX H[SODLQ ZKDWPLJKWEH ZURQJ ZLWK HILJXUH'



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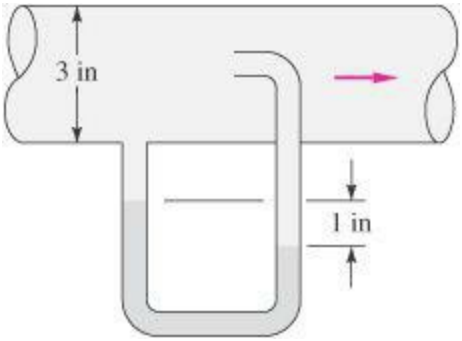
- 3 $ ORQJ IL[HG WXEH ZLWK DURXQG-GQRVH DQJHG ZLWK QRQFRPLQJ IORZ FDQEHXVH VR PHDVXUH YHORFLW 0 HDVXUHPHQW DUH PDGH HR WKH SUHVVXUH DW WKH IURQW DQG DKRDILQWKHVLGHR WKHWEHIXUKHUDORQJ ZKHUH WKH SUHVXUHQHDU HTXDO WXWDP SUHVXUH

D 0 DNH D VNHWK RI WKLV GHYLFH DQG VKRZ KRZ WKH YHORFLW LV FDOFXODWG E)RU SDUWFXODU VHDOHYHOLUORZ WKHGLIHUHQFHEHWZHQQRVHSUHVXUHDQGVLGHSUHVXUHLV GEH LQ : KDWLV WKHDLUYHORFLW LQPLK"

- 3 7KHPDQRPHWUIOXLG LQ) LJ 3 LV PHUFXU (VWPDWWKHYROXPHIORZ LQWK WXEHLI WKHIORZ LQJ IOXLGLV D JDVROLQHDQG E QLWURJHQ DW *f& DQG DW



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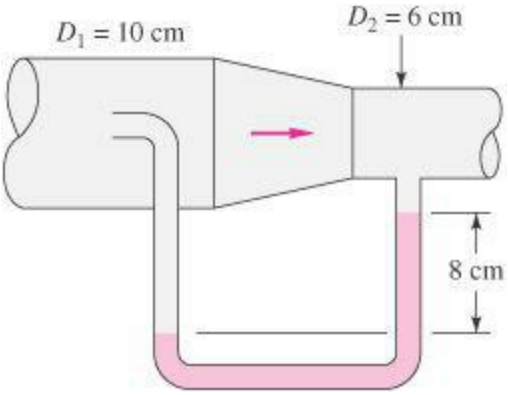
- 3 ,Q) LJ 3 WKHIORZ LQJ IOXLGLV&2 DW *f& 1 HJOHFWORVVHV ,IS N3DDQ WKHPDQRPHWUIOXLGLV0 HUFP UHGRLO 6* HWWP DW D S DQG E WKHJD IORZ UDWHLQP K



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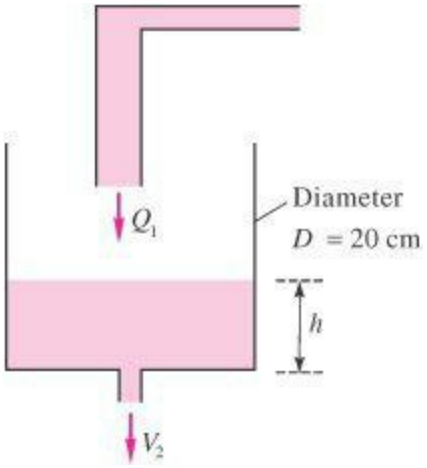
- 3 7KHFA\OQGULFDOZDAUDQNLQ) LJ 3 LVEHLQJ ILOOHGDWDYROXPHIORZ 4 JDOPLQ ZKLOHWKHZDAUDOR GULQVIURP DERWRP KROHRI GLDPHWUG PP \$ WPHW K) LQGDQG SORWWKHYDULDWRQK WDQGWKHHYHQWDOPD[LPXP ZDA GHSWKK_{PD[} \$VXPHWKDW%HUQRXOOL|VWADG IORZ HTXDWRQLVYDOG



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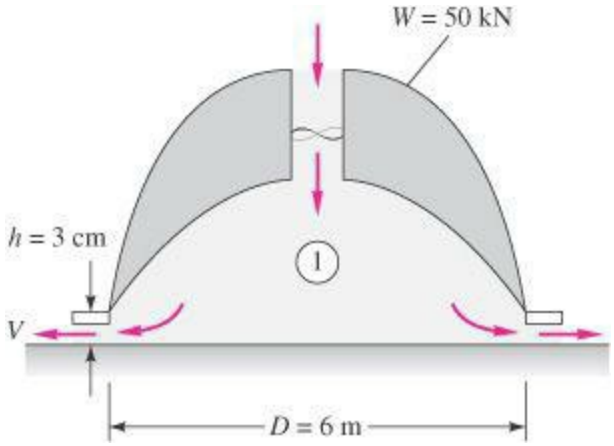
- 3 7KHDLUFXVKLRQYHKLFOHILQ) LJ 3 EULQJVLQVHDOHYHOWDQGDUDLUKURXJKDID DQGGLVFKDUJHVLWDWKLJKYHORFLWVKURXJKDQDQXOUVNLWRI FP FOHDUDQFH ,I WK YHKLFOHZHLJKV N1 HXWPDA D WKHUHTXLUHGDLUORZ UDWDQG E WKHIDQSRZH LQN



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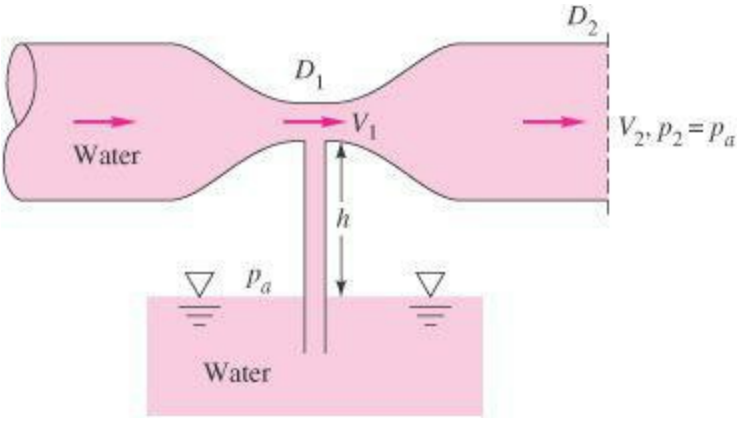
- 3 \$ QHFNGGRZQVHFWRQLQDSLSHIORZ FDOOHG DYHQXUL GHYHORSV DORZ WKURD SUHVXUHWKDWFDQDVSLUDWHIOXLGXSZDUGIURP DUHVHUYRLU DVLQ) LJ 3 8VLQ %HUQRXOOL|VHTXDWRQZLWKQRORVVHV GHULYHDQH[SUHVLRQIRUWKHYHORFLW9 WKDWW IXWWXILFLHQWWREULQJ UHVHUYRLUIOXLGLQRWKHWKURDW



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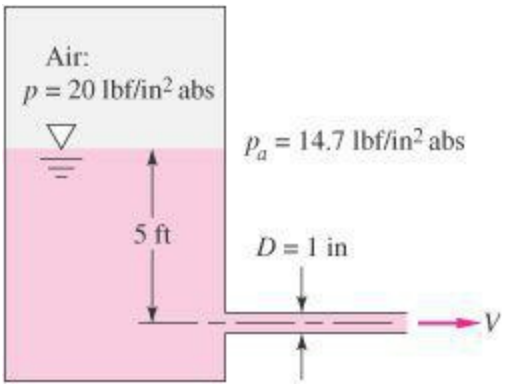
- 3 6XSSRVH\RX DUH GHVLJQLQJ DQ DLUKRFNH\ WDEOH 7KH WDEOH LV î IWLQ DUH Z LWK LQGLYLGXDO WKROHV VSDFHG HYHU LQFK LQ D UHFWDQJXODU JULG SDWWHUQ KROHV WRWDO 7KH UHTXLUHG PDVV IORZ UDWH LV <RX NQE LV WR VHOHFW DQ DSSURSULDWH EORZHU WKDW LV FDSDEOH RI SURYLGLQJ (WKP DW WK YROXPHWULF IORZ UDWH LQ IWP LQ DQG SUHVVXUH ULVH LQ E LQ UHTXLUHG RI WK EORZHU +LQW$VVXPH WKDW WKH DLU LV VXJQDQWLQ WKH OLJHYROXPHRI WKHPDQIRO XQGHUWKHWDEOHVXUIDFH DQG QHJOHFWDQ IULFWLRQDOORVHV

- 3 7KHQ) LJ 3 LVN HURVHQHDW *f& (WKP DW WKH IORZ UDW IRU D QR ORVHV DQG E SLSHORVHVK§ 9 J



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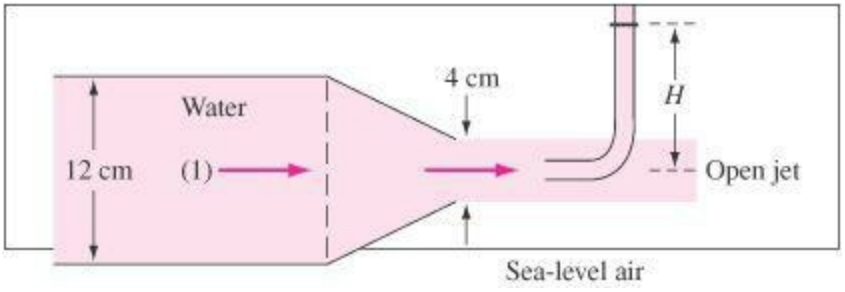
- 3 ,Q) LJ 3 WKHRSHQMATWR Z DWUDW *f& H[LWV DQR]]OHLQR VHDOHYHODLUDQ VWUNHVDVWDQGDUGQWEHDVVKRZQ



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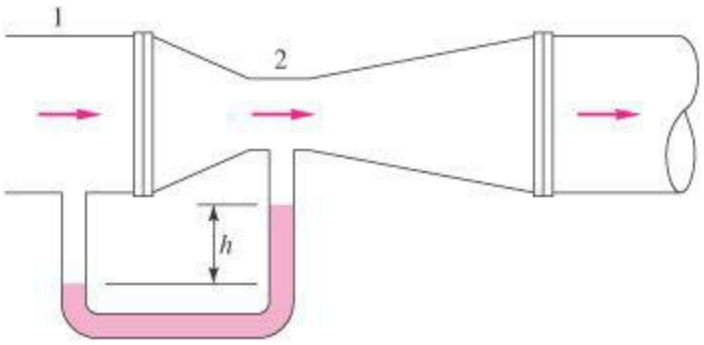
,I WKH SUHVVXUH DWWKH FHQWHUOLQH DWVHFWRQ LV N3D DQG ORVHV DUH QHJOHFWHWP D WKHP DVVIORZ LQNJ VDQG E WKHKHJKW+ RI WKHOXLGLQWKHVWDQGDUQWEH

- 3 $ YHQWXULPHWU VKRZQLQ) LJ 3 LVDFDUHXOO GHVLJQHGFRQWUDFWRQZKRV SUHVVXUH GLIIHUHQFH LV D PHDVXUH RI WKH IORZ UDW LQ D SLSH 8VLQJ %HUQRXOOL HTXDWRQ IRUVWDGG LQFRPSUHVVLEOH IORZ ZLWKQR ORVVHV VKRZ WKDWWKH IORZ UDW4 LVUHODWHGWR WKHPDQRPHWUHDGLQJ KE\

$$Q = \frac{A_2}{\sqrt{1 - (D_2/D_1)^4}} \sqrt{\frac{2gh(\rho_M - \rho)}{\rho}}$$

ZKHUH! $_0$ LVWKHGHQVLWRI WKHPDQRPHWHUIOXLG



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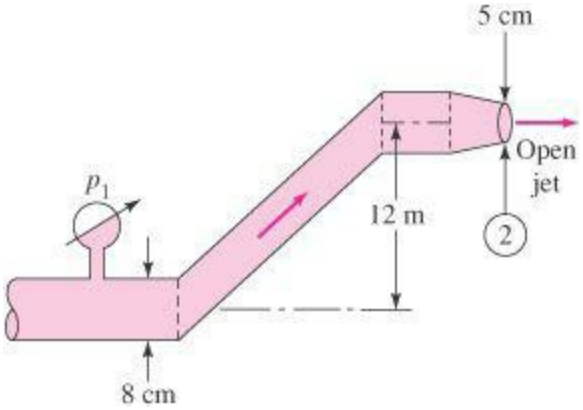
- 3 $QRSHQFLUFXLWZLQGWXQQHOGUDZVLQVHDOHYHODLUDQGDFFHOHUDWHV L WKURXJK DFRQWUDFWLRQ LQRD P E\ P WHVWVHFWLRQ $ GLIIHUHQWLDOPDQRPHWHUPHDVXUHVDSUHVVXUHGLIIHUHQFHRI PP RI ZDWHUEHWZHHQWKHLQWLGHDQGRXWLGH(VWLPDWH WKHWHVWVHFWLRQYHORFLWLQPLKDQG EWKHDEVROXWSUHVVXUHQWKHIURQWRVHFWLRQPGLQWKHWHVWVHFWLRQ
- 3 ,Q) LJ 3 WKHIOXLGLVJDVROLQHDW *f*& DWDZHJKWOX[RI 1 V$VXPLQ QRGRVVHVHWLPDWWKHJDJHSUHVXUHDWVHFWLRQ



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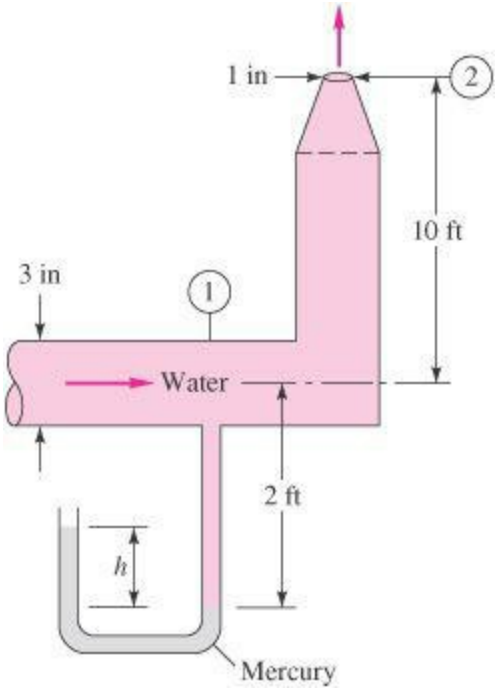
- 3 ,Q) LJ 3 EHWKIOXLGVDUHDW *f*& ,I9 IWVDQGDRVVHVDUHQHJOHFWHG ZKDWKRXGWKHPDQRPHWHUGLJKIWEH'



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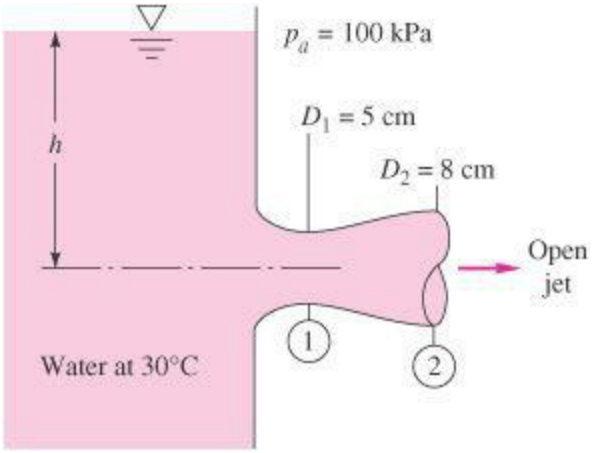
- 3 ([WHQGWKHVSKRQDQDOVLV R[([DPSOH WR DFFRXQWRUIUHFWLRQLQWKHWXEH D

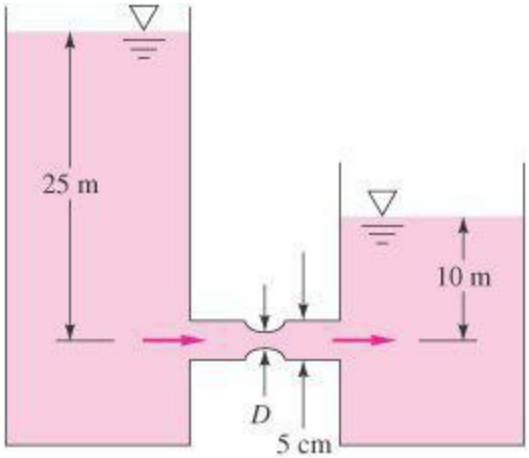
IRGOWZ V / HWWKH IULFWLRQ KHDG ORVV LQ WKH WXEH EH FRUUHODWHG DV 9WXEH J
ZKLFK DSSUR[LPDWHV WKH EXON QXPEHU &DOFXODWH WKH H[LWYHORFLWL
P VDQG WKH YROXPHIORZ UDWH LQ P V DQG FRPSDUH WR ([DPSOH

- 3 ,I ORVVHV DUH QHJOHFWHG LQ)LJ 3 IRU ZKDW ZDWHU YHORFLW ZLOO WKH IORZ EH JUHDWHU IRU P YDSRUFDYLWLHV DW WKH WKURDW WR WKH QR]]OH'



3

- 3)RU WKH *f& ZDWHU IORZ LQ)LJ 3 HVWLPDWH WKH YROXPHIORZ WKURXJK WK SLSH DXXPLQJ QR ORVVHV WKHQ H[SODLQ ZKDW LV ZURQJ ZLWK WKLV VHHPLQJO LQFRQVLVWHQF ,I WKH DFWXDOIORZ UDWH LV4 P K FRPSXWH D WKH KHDG ORVV LQIWDQG E WKH FRQWUDFWLRQ FRHIILFLHQW' WKDWFDXVHVFDYLWDWLRQ DXXPLQJ WKDW WKH WKURDWGLYLGHV WKH KHDG ORVV HTXDOO DQG WKDWKDQJLQJ WKH FRQWUDFWLRQ FRHIILFLHQW QR DGGLWLRQDOORVVHV



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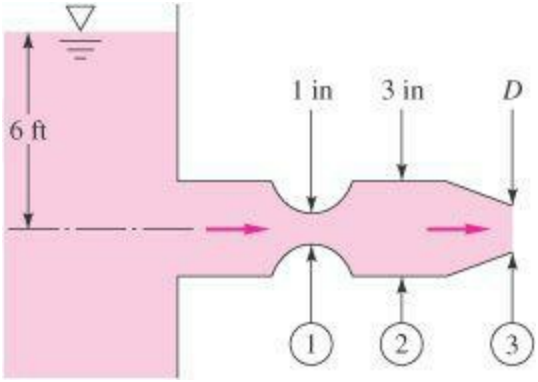
- 3 7KH *f& ZDWHU IORZ RI)LJ 3 GLVFKDUJHV WR VHD OHYHO WKURXJK D GUDZRVSKHU 1HJOHFWLQJ ORVVHV IRU ZKDWQR]]OH GLDPHWHU ZLOOFDYLWDWLRQ EHJLQ WR RFFXU' 7R DYRLGFDYLWDWLRQ VKRXOG\RX LQFUHDVHRUGHFUHDVH' IURP WKLVFULWLFDOYDOXH'



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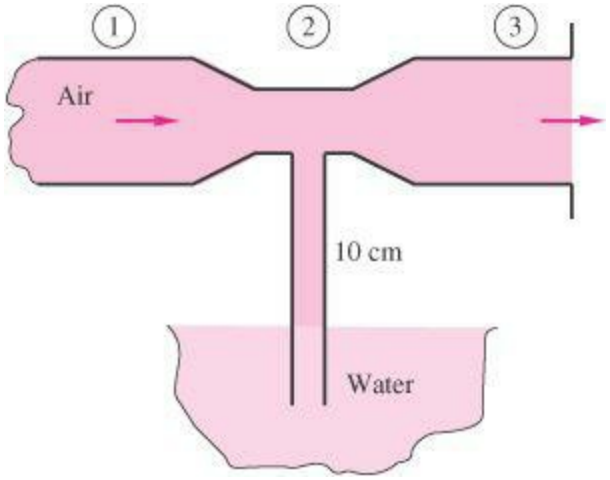
- 3 $LU DXXPHG IULFWLRQOHVV IORZ V WKURXJK D WXEH H[WQJ WR VHD OHYHO DW RSSH

' LDPHWHUV DW DQG DUH FP ZKLOH' FP : KDWP DVV IORZ RI DLUIV UHTXLUH VR VXFN ZDWHUXS FP LQWR VHFWLRQ RI)LJ 3 "



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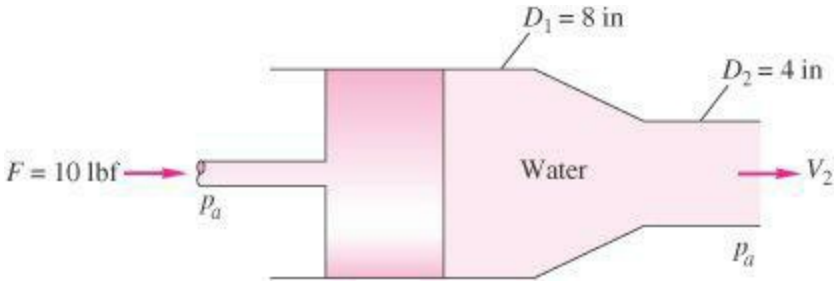
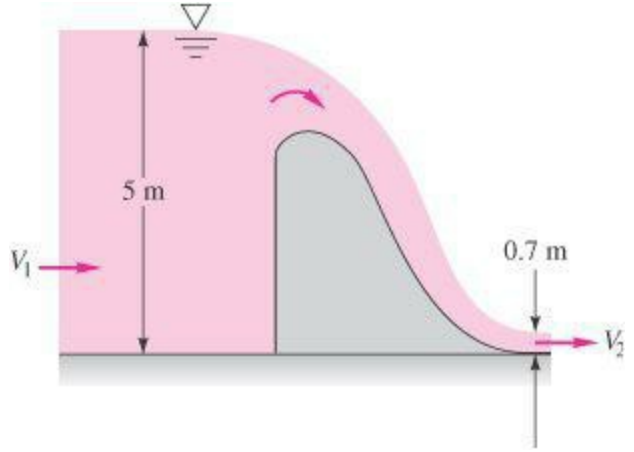
- 3 ,Q)LJ 3 WKH SLVWRQGULYHV ZDWHUDW *& 1HJOHFWLQJ IRVVHV HVWLP DWH WKH H[I YHORFLWV9 IW ,I ' LVIXUKHUFRQWUIWG ZKDWV WKH PD[LPXP SRVLEOH YDOXHR 9 "



3

- 3)RU WKH VXLFH JDWH IORZ RI ([DPSOH XVH %HUQRXOOLV HTXDWRQ DORQJ WK VXUIDFH WR HVWP DWH WKH IORZ UDW4 DV DIXQFWRQ RI WKH WZR ZDWUGHSWKV $VVXPI FRQVWDQWZ LGWKE
- 3 ,QWKHVSLOOZD\ IORZ RI)LJ 3 WKHIORZ LVDVXPHGXQLRUP DQGK\GURVWDF VHFWLRQV DQG ,I ORVVHVDUHQHJOHFWG FRPSXW D 9 DQG E WKHIRUFHSHUXQ ZLGWKRI WKHZDWURQWKHVSLOOZD\



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- 3) RUWKHZDWUFKDQQHOIORZ RI)LJ 3 K P + P DQG9 P \ 1HJOHFWLQJ ORVVHV DQG DVVXPLQJ XQLRUP IORZ DWWHFWLRQV DQG ILQG WK GRZQWUHDP GHSWKK DQG VKRZ WKDWWZR UHDOWLF VROXWLRQV/DUHSRVLEOH

VR NHHS WKH DUP V IURP URWDWLQJ DQG E WKH URWDWRQ UDWH UPLQ LI WKHUH LV QR UHWDUGLQJ WRUTXH

3

- 3 ,Q3URE 3 ILQG WKH WRUTXHFDXVHG DURXQG IODQJH LI WKHFHQWUSRLQWRI H[LW LV P GLUHFWO EHORZ WKHIODQJHFHQWU
- 3 7KHZ\HINRLQWLQ)LJ 3 VSOLWWKHSLSHIORZ LQWR HTXDOPRXQWV4 ZKLFH H[LWDV VKRZQ DGLVWDQFH5 IURP WKH D[LV 1HJOHFWJUDYLWDQG IULFWLRQ)LQG D H[SUHVVLRQIRUWKHWRUTXH7 DERXWWKH[D[LVUHTXLUHGWRNHSWKHV\VWP URWDWLQJ DQJXODUYHORFLW9

3

- 3 0 RGLI\ ([DPSOH VR WKDWWKH DUP VDUW IURP UHVWDQG VSLQV XS WR LW ILQD URWDWRQVSHG 7KHPRPHQWRI LQHUWLD RI WKH DUP DERXW LV, 1HJOHFWQJ DLUGUDJ ILQGG& GWDQG LQWUDUWWR GHWUPLQHWKHDQJXODUYHORFLW& W DVXPLQJ & DV
- 3 7KHWKUHH DZ QVPLQNDURI)LJ 3 UHFHLYHV f& ZDWUWKURXJK WKHFHQWU DW P K ,I FRODUIULFWRQLVQHJOLJLEOH ZKDWLV WKHWXUDG URWDWRQUDWLQUPL IRU D f DQG E f"

3

- 3 : DWUDW f& IORZ VDW JDOPLQWKURXJK WKH LQGLPHWUGRXEOHSLSHEHQ RI)LJ 3 7KHSUHVXUHV DUHS GEI LQ DQGS GEI LQ &RPSXWWK WRUTXH7 DWSRQW%QHFHVDU WRNHSWKHSLSHIRP URWDWQJ
- 3 7KHFHQWULXJDOSXPS RI)LJ 3 KDV DIORZ UDW4 DQGH[WWKHLPSHOHUDWD DQJOH UHODWLYH WR WKH EODGHV DV VKRZQ 7KH IOXLG HQWUV D[LDOO DWWFHWRQ $VXPLQJ LQFRPSUHVLEOHIORZ DWWKDIWDQJXODUYHORFLW& GHULYHDIRUPXODIRUWK

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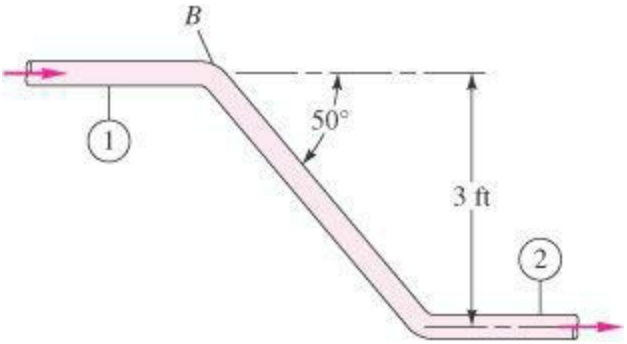

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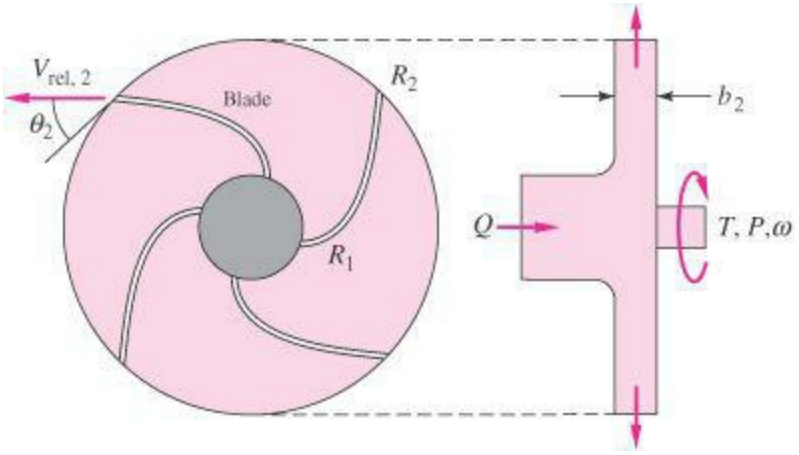

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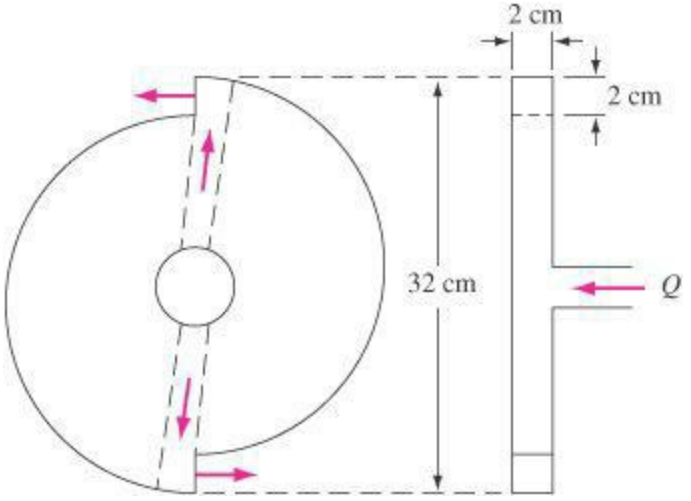
- 3 $ VLPSOHVXUERPDFKLQHLVFRQWUDFWGIURP DGLVNZLWKWZRLQWHUQDOGXFWVWKDWH[L VDQJHQWDOO WKURXJKVTXDUHKROHV DVLQ)LJ 3 : DWUDW f& HQWHUVQRUPDOO WKHGLVNDWWKHFHQWUDVKRZQ 7KHGLVNPXVWGULYH DW UPLQ DVPDOOGHYLFH ZKRVHUHDGLQJ WRTXHLV 1 P : KDWLV WKHSURPDWIORZ RI ZDWU LQ NJ V"



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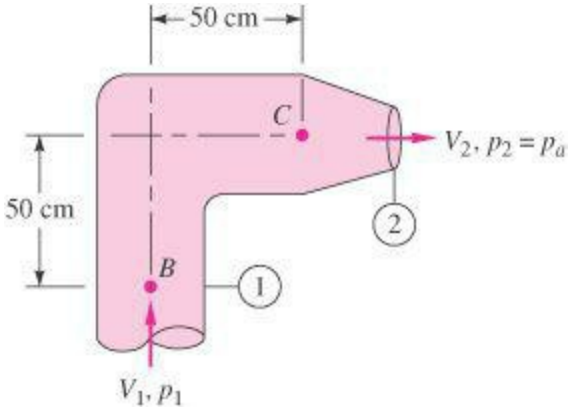
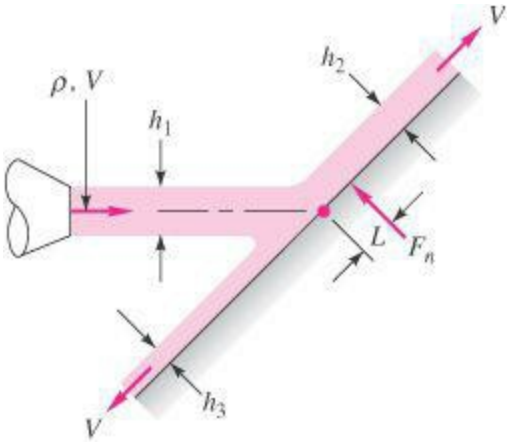
- 3 5HYHUVHWKHIORZ LQ)LJ 3 VR WKDWWKH VXWWP RSHUDWV DV D UDGLDOLQIORZ WXUELQH $VVXPLQJ WKDWWKHRXWORZ LQWR VHFWLRQ KDVQR VDQJHQWDOYHORFLW GHULYH DQH[SUHVLRQIRUWKHSRZHU3 H[WUDFWHGE\ WKHWXUELQH
- 3 5HYLVLWWKH WXUELQH FDVFDGH VXWWP RI3URE 3 DQG GHULYH DIRUPXODIRUWK SRZHU3 GHOLYHUHG XVLQJ WKHDQJXODUPRPHQWXP WKHRUHP RI(T
- 3 $ FHQWULXJDOSXPS LPSHOOHUGHOLYHUV JDOOLQRI ZDWUDW f& ZLWKDVKDU URWDWRQUDWRI UPLQ 1HJOHFWORVVHV ,I U LQ U LQ E E LQ 9W IWV DQG9W IWV FRPSXWWKHDEVROXWYHORFLWLHV D 9 DQG E 9 DQG F WKHKRUVHSRZHUHTXLUHG G &RPSDUHZLWKWKHLGHDOKRUVHSRZHUHTXLUHG
- 3 7KHSLSHEHQGRI)LJ 3 KDV' FP DQG' FP : KHQZDWUDW f&

ICRZ V WKURXJK WKH SLSH DW JDOPLQ S NBD JDJH &RPSXWH WKH VRUTX
UHTXLUHG DVSRQVH%WR KROG WKH EHQG VDIHO



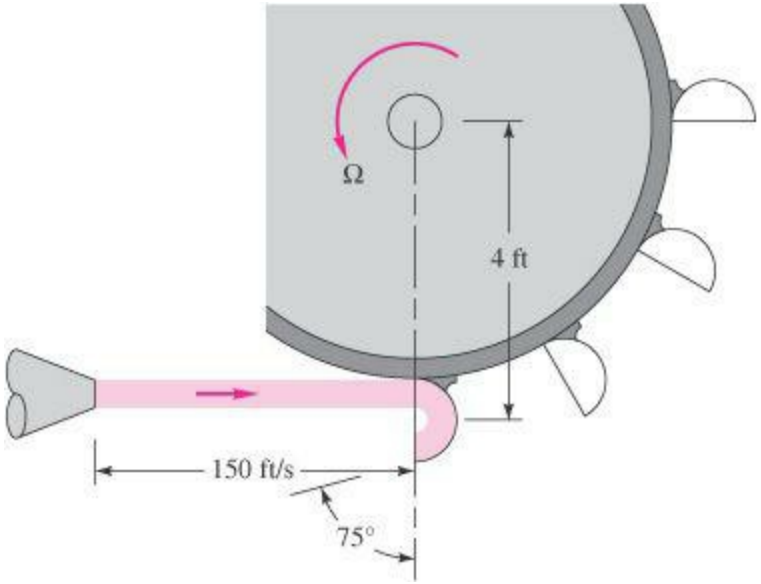
3

- 3 ([WKG3URE 3 WR WKH SUREOHP RI FRPSXWLQJ WKH FHQWURI SUHVXUH/ RI WK QRUPDO IDFH) Q DV LQ) LJ 3 $ WKH FHQWURI SUHVXUH QR PRPHQWV DU UHTXLUHG WR KROG WKH SODWH DW WHVW 1 HJOHFWUDWLRQ ([SUHVV\ RXUUHVXOWLQ WHUP RI WKH VKHHWWKLFNQHVV K DQG WKH DQJOH EHWZHHQ WKH SODWH DQG WKH RQFRPLQJ MHW



3

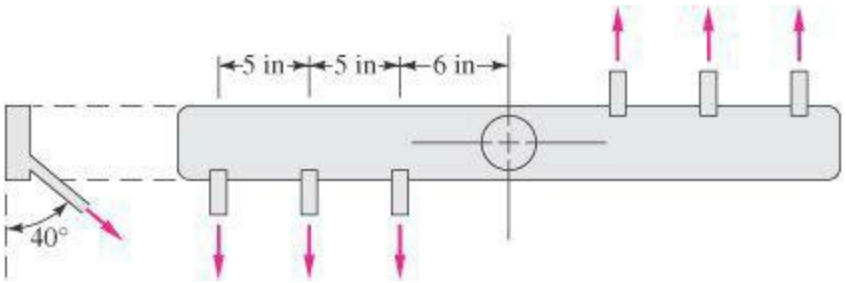
- 3 7KH ZDWHU ZKHHOLQ) LJ 3 LV EHLQJ GULYHQ DW UPLQ E\ D IW/ MHW RI ZDWHU DW *f*& 7KH MHWV GLDPHWHU LV LQ $VVXPLQJ QR ORVVHV ZKDWLV WK KRUVHSRZHU GHYHORSHG E\ WKH ZKHHO") RUZKDWSHHG 9 UPLQZLOOWKH KRUVHSRZH GHYHORSHG EHDPD[LPXP" $VVXPHWKDWWKHUHDUHPDQ EXFNHWVRQWKHZDWHUZKHO



3

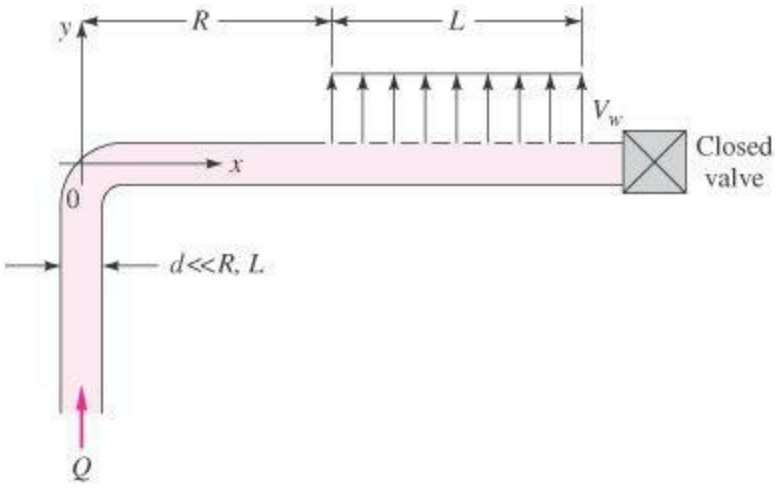
- 3 $ URWDWQJ GLVKZDVKHUDUP GHOLYHUV DW *f*& VR VL[QR]]OHV DV LQ) LJ 3 7K VRDOORZ UDWLV JDOPLQ (DFKQR]]OHKDV DGLDPHWURI LQ ,I WKHQR]]OHIORZ

DUHHTXDOODQGIULFWLRQOLVQHJOHFWHG HWHUPDWWKHWWDGK URWDWRQUDWHRI WKHDUP L
UPLQ



3

- 3 $ CTXLGRI GHQVLW! IORZ VWKURXJKD ƒ EHQGDVKRZQLQ) LJ 3 DQGLWXH
YHUWFDOO IURP DXQIRUP SRURXVVHFWRQRI GHQWK/ 1HJOHFWQJ SLSHDQGCTXL
ZHJKWGHULYHDQH[SUHVLRQIRUWKHVRUTXH0 DVSRLQW UHTXLUHGWRKROGWKHSLSH
WDWRQDU



3

7KHHQHUJ\ HTXDWRQ

- 3 7KHUHLV D WDGG LVRWKHUPDOIORZ RI ZDWUDW ƒ& WKURXJK WKH GHYLFHLQ) LJ
3 +HDWWDQVHU JUDYLW DQG WPSHUDWXHHIHFWDUHQHJOLJLEOH. QRZQGDU
DUH' FP 4 P K S NBD' FP 4 P K S
NBD' FP DQGS NBD &RPSXWWKHUDWRI VKDWZRUNGRQHIRUWK
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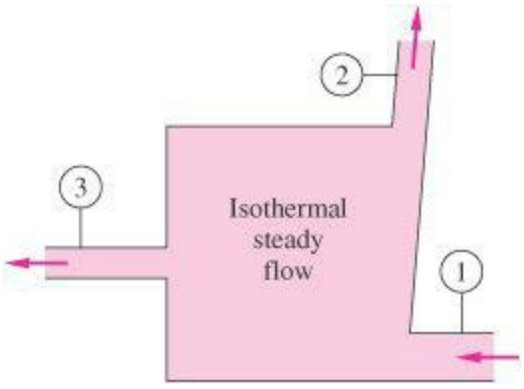
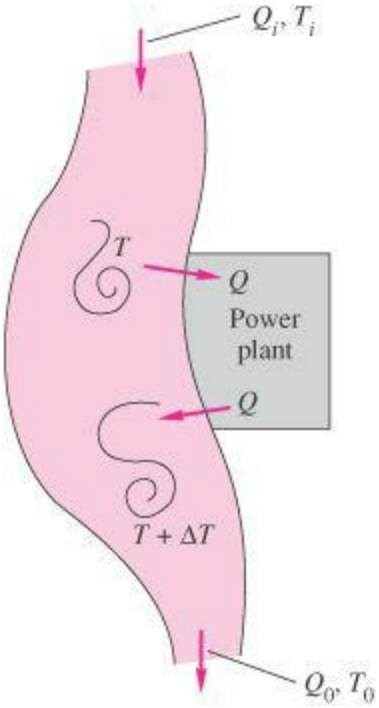


3

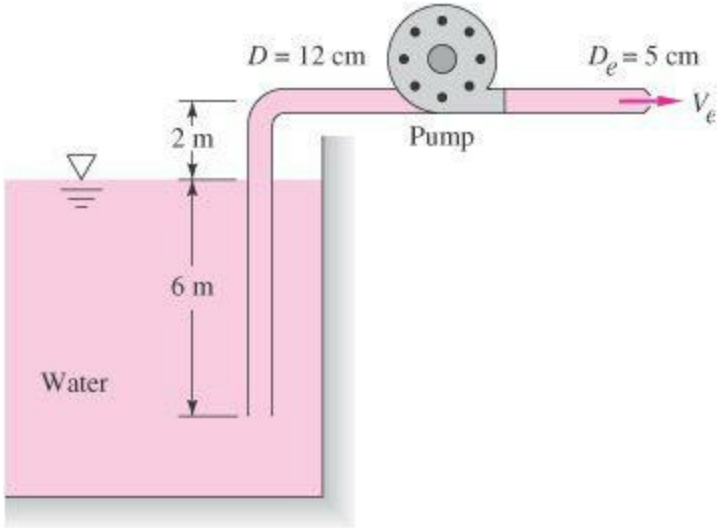
- 3 $ SRZHUSODQWRQDULYHUDVLQ) LJ 3 PXWWHDPLQDW 0: RI ZDWWKHDWW
WKHULYHU7KHULYHUFRQGLWLRQVXSWWHDP DUH4$L P VDQG7$L ƒ& 7KHULYH
LV P ZLGH DQG P GHHS ,I KHDWORVVHV WR WKH DWPRVSKHUH DQG JURXQG DU
QHJOLJLEOH HWWDPDWWKHGRZQWUHDP ULYHUFRQGLWLRQV 4 7

- 3) RUWKHFRQGLWLRQVRI3URE 3 LI WKHSRZHUSODQWLVWRKHDWWKHQHDUE\ULYH ZDWUE\ QR PRUHWKDQ f& ZKDWWKRXGEHWKHPLQPXP IORZ UDWH4 LQP V WKURXJK WKHSODQWKHDW[FKDQJHU + RZ ZLOOWKHYDOXHRI4 DIIHFWWKHGRZQWUHDP FRQGLWLRQV 4 7 "
- 3 0 XQQDPDK) DOOVLQWKH&RXPELD5LYHU* RUJHKDVDVKHUGURSRI IW8VLQJ W VWDQGDUG IORZ HQHUJ\ HTXDWLRQ HVWLPDWWKHZDWUWHPSHUDXUHFKDQJHLQf) FDXVHGE WKLVGURS



3

- 3 : KHQWKHSXPSLQ) LJ 3 GUDZV P KRI ZDWUDW f& IURP WKHUHVHUYRL WKH VWDOO IUFWLRQ KHDGORVV P 7KH IORZ GLVFKDUJHV WKURXJK D QR]]OH WR WK DWPRVSKHUH (VWLPDWWKHSXPSSRZHULQN GHOLYHUHGWRWKHZDWU



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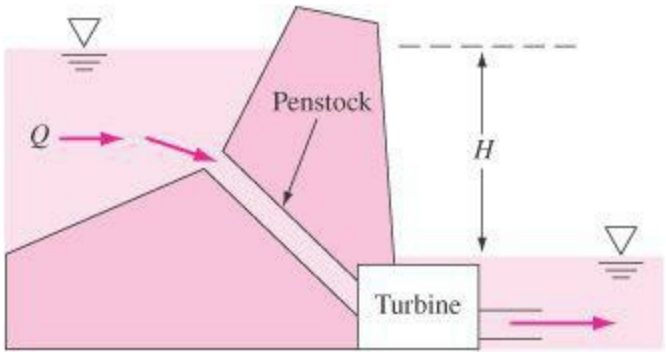
- 3 $ VWDP WXUELQHRSHUDWVWKURXJKRXW WKHIROORZLQJ FRQGLWLRQV $WWKHLQOHWS 0 3D7 f& DQG9 P V $WWKHRXWOHWS N3D7 f& DQG9 P V D ,I ZHQHJOHFWKHOHYDWLRQFKDQJHVDQGKHDWORVVHU KRZ PXFK ZRUN LV GHOLYHUHGWRWKHWXUELQHE0DGH VLN- NJ" E ,I WKHPDVVIORZ LV NJ V KRZ PXF WRWDOSRZHULVGHOLYHUHG' F ,VWKHWXUELQH ZHWDVLVWHDY-HVWKH[LW
- 3 &RQVLGHUDWXUELQH[WUDFWLQJ HQHUJ\ IURP DSHQVWRFNLQDGDP DVLQ) LJ 3) RUWXUEXOHQWSLSHIORZ &KDS WKHIULFWLRQKHDGORVVLVDSSUR[LPDWHO K &4

Z KHUH WKH FRQWUDFW& GHSHQGV RQ SHQWRFNLPHQWV DQG WKH SURSHUWLHV RI ZDWHU 6KRZ WKDW IRU DJLYHQ SHQWRFNJHRPHWU DQG YDULDEOH UHVHUYRLU 4 WKH PD[LPXP WXUELQH SRZHU SRWLEOH LQ WKLV FDVH LV3PD[!J+4 DQG RFFXUV ZKHQ WKH IORZ UDWH LV $Q = $H/(3C)



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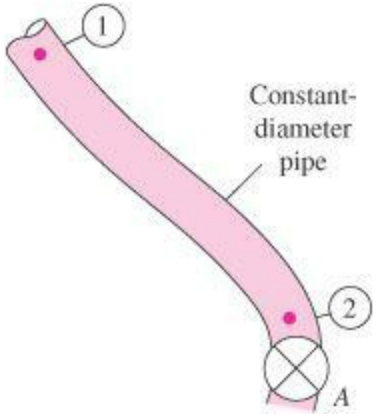
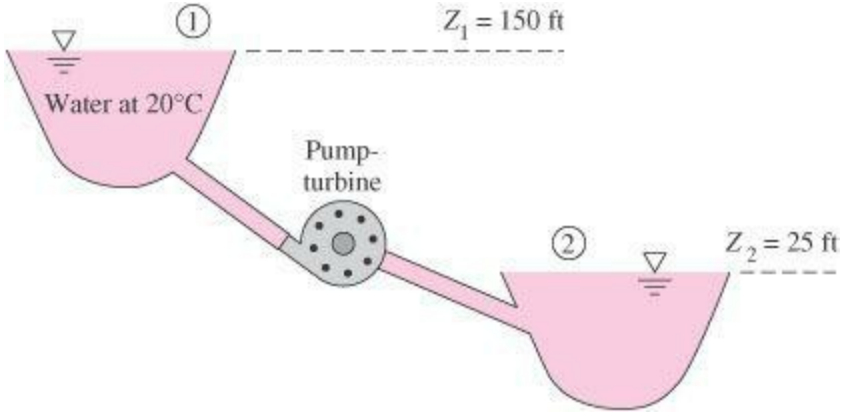
- 3 7KH ORQJ SLSHLQH) LJ 3 LV ILOOHG ZLWK ZDWHU DW f& : KHQ YDOYH $ LV FORVHG S í S N3D : KHQ WKH YDOYH LV RSHQ DQG ZDWHU IORZV DW P K S í S N3D : KDW LV WKH IULFWLRQ KHDG ORVV EHWZHHQ DQG LQ P IRU WKH IORZ LQ FRQGLWLRQ'



3

- 3 $ LQGLDP HWU SLSHOLQH FDUULHV RLO 6* DW PLQRQ EDUHOV SHU GD EEOGD\ EEO 8 6 JDO 7KH IULFWLRQ KHDG ORVV IW IWR SLSH ,W SODQQHG WR SODFH SXPSLQJ VWDWLRQV HYHU PL DORQJ WKH SLSH (VWLP DW WK KRUVHSRZHU WKDW PXVW EH GHOLYHUHG WR WKH RLOE\ HDFK SXPS
- 3 7KH SXPS VXUELQH VVWHP LQ) LJ 3 GUDZ V ZDWU IURP WKH XSSHU UHVHUYRLU WKH GD\ WP H WR SURGXFH SRZHU IRU D FLW $WQJKWLWSXPSV ZDWU IURP ORZHU XSSHU UHVHUYRLV WR UHWXUH WKH VLXDWLRQ) RU D GHVLJQ IORZ UDWH RI JDOP LQL HLWK HUGLUHFWLRQ WKH IULFWLRQ KHDG ORVV IW(VWLP DW WKH SRZHU LQ N D H[WUDFWHG E\ WKH WXUELQH DQG E GHOLYHUHG E\ WKH SXPS



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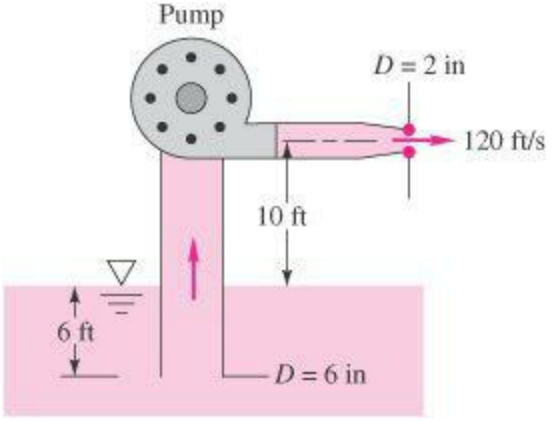
- 3 : DW DW f& LV GHOLYHUHG IURP RQH UHVHUYRLU WR DQRWKHU WKURXJK D ORQJ FP

GLDPHWUSLSH 7KHORZHUUHVHUYRLUKDVDVXUIDFHHOHYDWRQ] P 7KHIULFWR
ORVVLQWKHSLSHLVFRUUHODWHGE\WKHIRUPXODK$Y§ 9 J ZKHUH9 LVWK
DYHUDJHYHORFLW\LQWKHSLSH ,IWKHVWDWH\IORZ UDWHWKURXJKWKHSLSHLV JDOORQ
SHUPLQXWHWKHQGHWHUPLQHWKHVXUIDFHHOHYDWRQRIWKHKLJKHUUHVHUYRLU

- 3 $ ILUHERDWGUDZVVHDZDWU 6* IURP DVXEPHUJHGSLSHDQGGLVFKDUJHVL
WKURXJK D QR]]OH DV LQ) LJ 3 7KH VRXOKEDG ORVV LV IW,I WKH SXPP
HILFLHQF\ LV SHUFHQWZKDWKRUVHSRZHUPRWRULVUHTXLUHGWRGULYHLW



3

- 3 $ GHYLFH IRU PHDVXULQJ ODTXLG YLVFRVLWV LV VKRZQ LQ) LJ 3 : LWK WK
SDUDPHWHUV ! / + G NQRZQ WKH IORZ UDWH4 LV PHDVXUHG DQG WKH YLVFRVLW
FDOLEXODWHG DVXPLQJ DODPLQDUIORZ SLSHORVVIURP &KDS K / 9 ! JG
+HDWWDQVHU DQG DOORWKHUORVVHV DUH QHJOLJLEOH D ' HULYH D IRUPXOD IRU WK
YLVFRVLW RI WKHIOXLG E &DOFXODWH IRUWKHFDVHG PP ! NJ P /
FP + FP DQG4 FP K F : KDWLV\RXJXHVWRI WKHIOXLGLQSD
E" G 9HUI\ WKDWWKH5H\QROGVQXPEHU5H LVOHVVWKDQ ODPLQDUSLSHIORZ
- 3 7KHKRU]RQWDOSXPPSLQ) LJ 3 GLVFKDUJHV *& ZDWDUDW P K 1HJOHFWLQ
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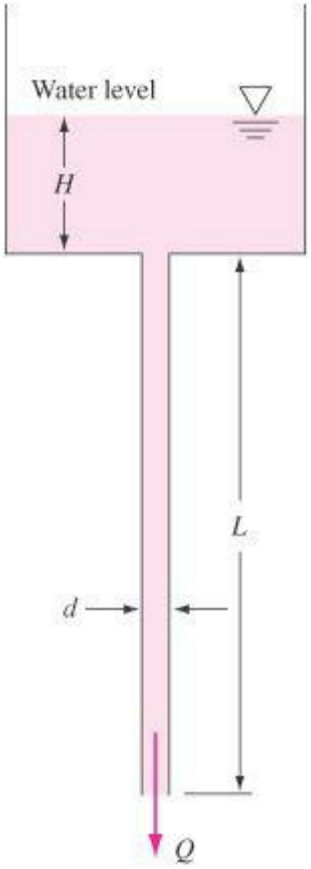

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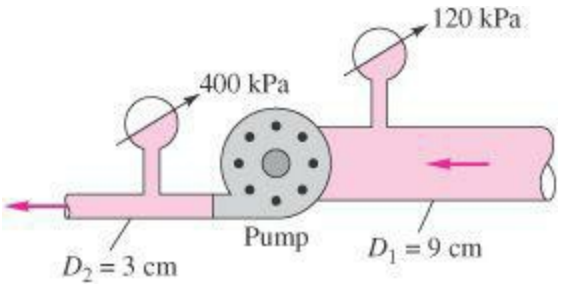
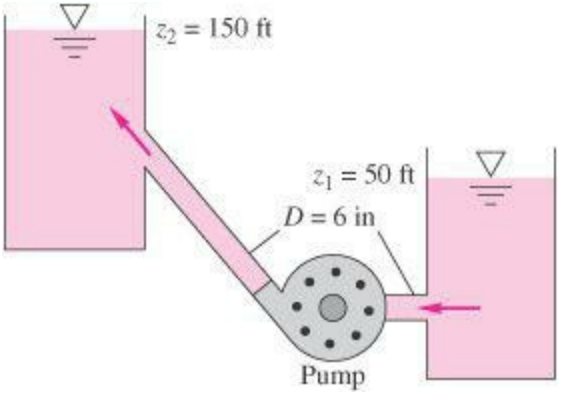


3

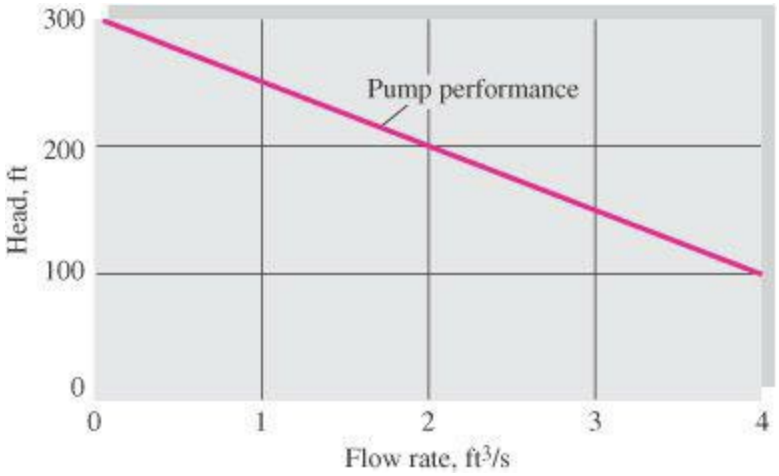
- 3 6\WDP HQWUV DKRUL]RQWDO WXUELQH DW GEI LQ DEVROXWH f& DQG IVW DQG LV GLVFKDUJHG DW IVW DQG f& VDWXUDWHG FRQGHQVDWH 7KH PDVV IORZ LV GEP V DQG WKH KHDG ORVVHV DUH %DVHG RQ WKLV ,I KHDG ORVVHV DUH QHJOLJLEOH KRZ PXFK KRUVHSRZHU GRHV WKH WXUELQH GHYHORS'
- 3 : DWUDW f& LV SXPSHG DW JDOP LQIURP WKH ORZHU WR WKH XSSHU UHVHUYRLU D LQ) LJ 3 3LSH IULFWLRQ ORVVHV DUH DSSUR[LPDWHG E\ K§ 9 J Z KHUH 9 LV WKH DYHUDJH YHORFLW LQ WKH SLSH ,I WKH SXPS LV SHUFHQW HILFLHQW Z KDW KRUVHSRZHU LVQHGHG WR GULYH LW



3

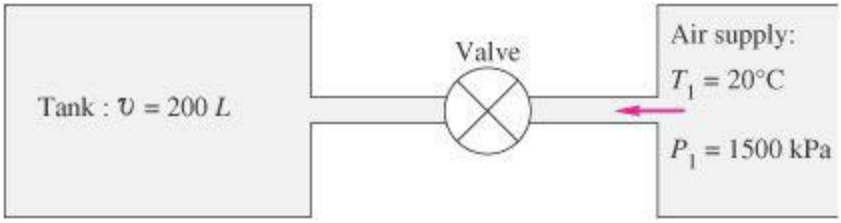
- 3 $ WWSLFDO SXPS KDV DKHDG WKDW RUDJLYHQ VKDWURDWRQUDW YDULHV Z LWK WKH IORZ UDW UHVXOWQJ LQ DSXPS SHUFRUPDQFH FXUYH DV LQ) LJ 3 6XSSRVH WKDW WKH

SXPS LV SHUFHQWHILFLHQWDQGLVXVHGIRUWKHVWXP LQ3URE (WDPDW D
WKHIORZ UDWH LQJDOPLQ DQG E WKHKRUVHSRZHUHGHGXRGULYHWKHSXPS



3

- 3 7KHLQVXODWHGWDQNLQ) LJ 3 LVWREHILOOHGIURP DKLJK SUHVVXUH DLUVXSSO ,QLWLDOFRQGLWLRQV LQ WKH WDQN DUH7 *f& DQGS N3D : KHQ WKH YDOYH L RSHQHG WKHLQLWLDOPDVVIORZ UDWHLQWR WKHWDQNLV NJ V $VVXPLQJ DQLGHDOJDV HVWLPDWHWKHLQLWLDOUDWHRI WPSHUDWXUHULVHRI WKHDLULQWKHWDQN



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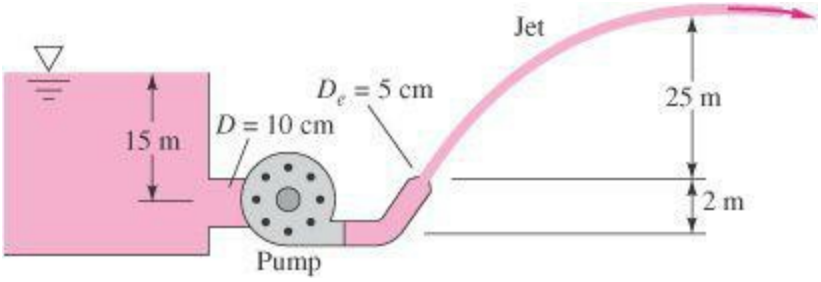
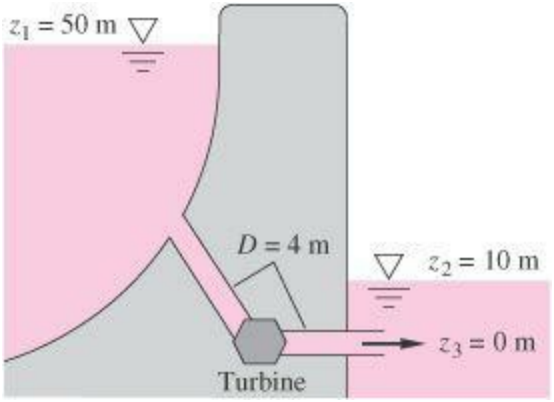
- 3 7KHSXPS LQ) LJ 3 FUHDWHVD *f& ZDWHUMHWWRULQHQWHUWDLQHGWRWDYHOHDPDQLPXP KRUL]RQWDO GLVWDQFH 6\VWHP IULFWLRQ KHDG ORVVHV DUH P 7KH MHWPDWHULDO E DSSUR[LPDWHG E\ WKH WDWHFWRU RI IULFWLRQOHVV SDUWFOHV : KDWSRZHU PXWWEH GHOLYHUHGE\ WKHSXPS'



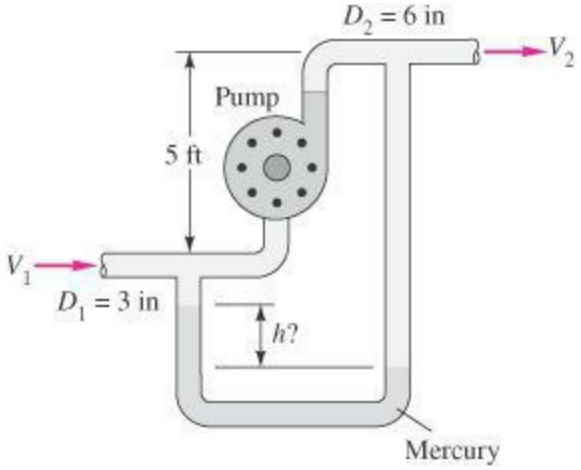
3

- 3 EES 7KHOLQHWXUELQHLQ) LJ 3 GLVHUWWKHULVHUIORZ XQGHUDGDP DVVKRZQ 6\VWHP IULFWLRQORVVHV DUHK, 9 J ZKHUH9 LVWKHDYHUDJHYHORFLW\LQWKH VXSSO SLSH) RUZKDWOLVHUIORZ UDWHLQP VZLOOWKHSRZHUHGHOLYHUHGEH 0 : : KLFKRI WKHVZR SRVVLEOHVROXWLRQVKDVDEHWWU³FRQYHUVLRQHIILFLHQF\´"



3

- 3 . HURVLQHDW *f*& IORZ VWKURXJK WKH SXPS SLQ) LJ 3 DW IWV + HDG ORVV EHWZHHQ DQG DUH IWDQG WKH SXPS GHOLYHUV KS WR WKH IORZ : KDW KRXOG WK PHUFXU PDQRPHWHU UHDGLQJ K IWEH'



3

: RUG3UREOHPV

- : ' HULYH D FRQYHQLHQW IRUP RI WKH VHFRQG ODZ RI WKHUPRG\QDPLFV 6XJJHVWLRQ SUDFWLFDO XVHIRU\RXUHDGHUQLQDQFH]LQJ UHDOLVWLF ORGLFRZ V
- : 6XSSRVH WKDW WKH GHVLUHG WR HWXP DW YROXPH IORZ 4 LQD SLSHE\ PHDVXULQJ WKH D[LD YHORFLWX U DWSHFLILF SRLQW) RUFRVWUHDQVRQ WKUHHPHDVXULQJ SRLQWDUHV EHXVHG : KDWDUH WKHEHWWDGLVHOFWRQVIRUWKHVHWKUHSRLQW'
- : &RQVLGHUZDWIORZ LQJ E\ JUDYLW WKURXJK D VKRUWSLSH FRQHFWQJ WZR UHVHUYRL ZKRVH VXUIDFH HOHYDWLRQ GLIHU E\ DQ DPRXQW] : K\ GRHV WKH LQFRP SUHVVLEO IUFWRQDW %HUQRXOOL HTXDWRQ HDG WR DQ DEVXUGLW ZKHQ WKH IORZ UDW WKURXJK WK SLSH LV FRP SXWG' ' RHV WKH SDUDGR[KDYH VRPHWKLQJ WR GR ZLWK WKH HQJWK RI WK VKRUWSLSH' ' RHV WKH SDUDGR[GLVDSSHDUL ZHURXQG WKH HQJDQFH DQG H[WHGJHV R WKHSLSH'
- : 8VH WKH VWDG\ IORZ HQHUJ\ HTXDWRQ WR DQDO]H IORZ WKURXJK D ZDWUIDXFHWZKRV VXSSO SUHVXUH LVS : KDWSK\VFDO PHFKDQLVP FDXVHV WKH IORZ WR YDU FRQVLQRXV IURP]HUR WR P D[LPXP DVZHRSHQ WKHIDXFHWYDOYH'
- : &RQVLGHUDORQJ VHZHUSLSH KDOIXOOR ZDWU VORSLQJ GRZQ DUGDWQJH \$QRLQ &Kp]\ LQ GHWHUPLQHG WKDWWKH DYHUDJH YHORFLW RI VXFK DQ RSHQFKDQQHOIORZ VKRXOG E\ $V \approx C\sqrt{R\tan\theta}$ ZKHUH 5 LV WKH SLSH UDGLXV DQG& LV D FRQVWDQW+RZ GRH

VKLV IDP RXV IRUP XOD UHDVRQDEOH DPRXQW RI WKH WUDG\ IORZ HQHUJ\ HTXDWLRQ DSSOLHG WR D OHQJWK/
RI WKH FKDQQHO"

- : 3XWD VDEOH WHQQLV EDOO LQ D IXQQHO DQG DWWDFK WKH VPDOOHQG RI WKH IXQQHO WR DQ DL
VXSSO\ <RX SUREDEO\ ZRQ¡W WEH DEOH WR EORZ WKH EDOOHLWKHU XS RUGRZ QRXWR WKH
IXQQHO([SODLQZK\

- : +RZ GRHV DVLSKRQ ZRUN" $UH WKHUH DQ\ OPLWDWLRQV VXFK DV KRZ KLJK RUKRZ ORZ
FDQ\RX VLSKRQ ZDWHU D\ IURP DWDQN" $OVR KRZ ID3 FRXOG\RX XVH DIOH[LEO
WXEH WR VLSKRQ ZDWHU IURP DWDQN WR DSRLQW IWDZ D\"

3UREOHPV TXHVWLRQV) XQGDPHQWDOV RI (QJLQHHULQJ ([DP 3UREOHPV

-)(,Q)LJ)(ZDWHU[LWIURP DQR]OHLQR DW RSKHUF SUHVXUH N3D ,I WK
IORZ UDWH LV JDOOLQ ZKDWLV WKH DYHUDJH YHORFLWDWHFWRQ "
P V
P V
P V
P V
P V

-)(,Q)LJ)(ZDWHU[LWIURP DQR]OHLQR DW RSKHUF SUHVXUH N3D ,I WK
IORZ UDWH LV JDOOLQ DQG IULFWLQQ LV QHJOHFWHG ZKDWLV WKH JDJH SUHVXUH D
VHFWLRQ "
N3D
N3D
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N3D

-)(,Q)LJ)(ZDWHU[LWIURP DQR]OHLQR DW RSKHUF SUHVXUH N3D ,I WK
H[LWV HORFLW LV9 P VDQG IULFWLQQ LV QHJOHFWHG ZKDWLV WKH D[LDO IDQJH IRUF
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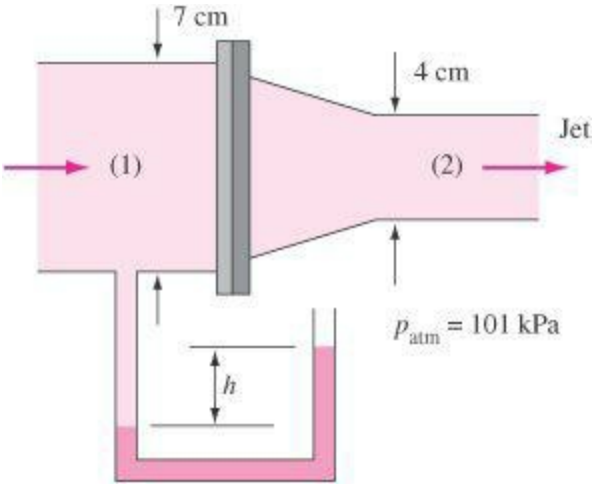

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-)(,Q)LJ)(ZDWUH[LWIURP DQR]OHLQRWDWRVSKHULFSUHVVXUHR NBD ,I WK PDQRPHWUICXLGKDVDSHFLILFJUDYLWR DQGK FP ZLWKIUHVWRQQHJOHFWHG ZKDWLVWKHDYHUDJHYHORFLWDWWHFWRQ "
  P V
  P V
  P V
  P V
  P V
-)($ MHWRI ZDWU FP LQGLDPHWUXULNHVQRUPDOWRDSODWHDVLQ)LJ)(,I WKHIRUF UHTXLUHGWRKROGWKHSODWLV 1 ZKDWLVWKHMHWYHORFLW"
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  P V
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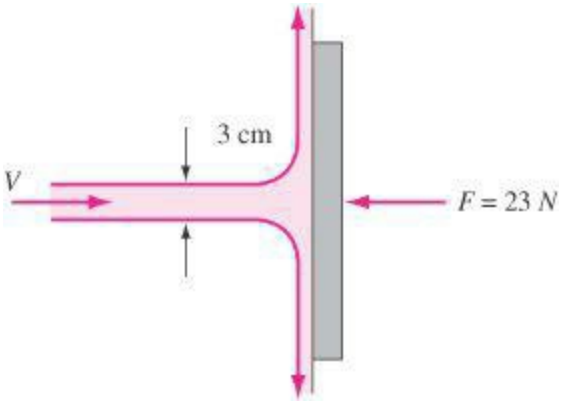

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-)($ IUHERDWSXPS GHOLYHUVZDWUWRDYHUWFDOQR]OHZLWKD GLPHWUDWR DVLQ)LJ)(,I IUFWLRQLVQHJOHFWHGDQGWKHIORZ UDWHLV JDOPLQ KRZ KLJKZLQ WKHRXWOHWZDWUMHWLV'
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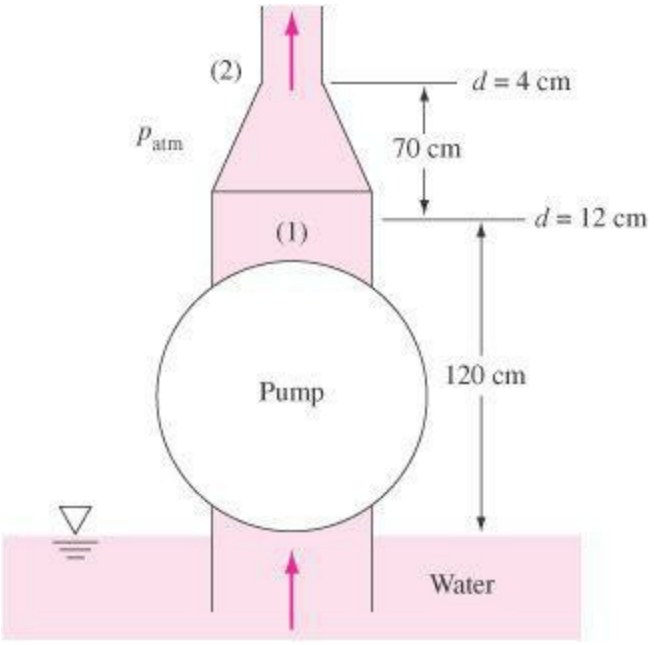

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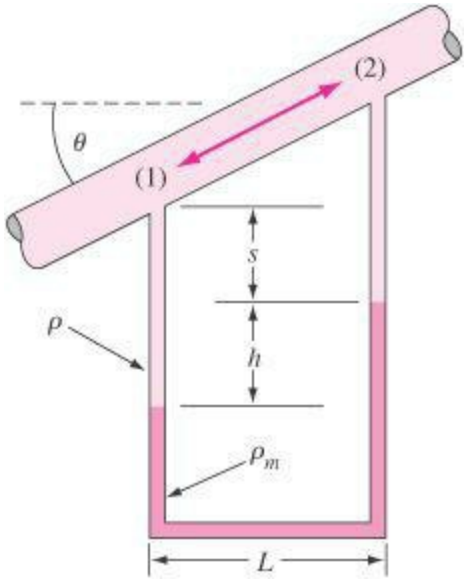
)LJ)(,I IUFWLRQLVQHJOHFWHG GDQG WKH SXPS LQFUHDVHV WKH SUHVVXUH DWWHFWLRQ
  WR NBD JDJH ZKDWZLOOEHWKHUHXQGQJIORZ UDWH"
  JDOPLQ
  JDOPLQ
  JDOPLQ
  JDOPLQ
  JDOPLQ
-)($ ILUHERDWSXPS GHOLYHUV ZDWHUWR DYHUWFDOQR]]OH ZLWK D GLDPHWUDWR DV/C
)LJ)(,I GXFWDQG QR]]OH IUFWLRQ DUHQHJOHFWHG GQG WKH SXPS SURYLGHV I
  RI KHDGWR WKHIORZ ZKDWZLOOEHWKHRXWOHWRZ UDWH"
  JDOPLQ
  JDOPLQ
  JDOPLQ
  JDOPLQ
  JDOPLQ
-)(: DWUIORZLQJ LQDVPRRWK FP GLDPHWUSLSHHQWHUVDYHQXLFRQWFWRQZLWKD
  WKURDWGLDPHWURI FP 8SWUHDP SUHVVXUHLV NBD ,I FDYLWDWRQRFFXUVLQWK
  WKURDWDWDIORZ UDWHRI JDOPLQ ZKDWLV WKHHXWP DWG IOXLG YDSRUSUHVVXU
  DVXPLQJ LGHDOIUFWLRQOHVVIORZ"
  NBD
  NBD
  NBD
  NBD
  NBD
-)(: DWUIORZLQJ LQDVPRRWK FP GLDPHWUSLSHHQWHUVDYHQXLFRQWFWRQZLWKD
  WKURDWGLDPHWURI FP 8SWUHDP SUHVVXUHLV NBD ,I WKHSUHVVXUHLQWKHWKURD
  LV NBD ZKDWLVWKHIORZ UDWH DVXPLQJ LGHDOIUFWLRQOHVVIORZ"
  JDOPLQ
  JDOPLQ
  JDOPLQ

JDOPLQ
JDOPLQ

&RPSUHKHQVLYH3UREOHPV

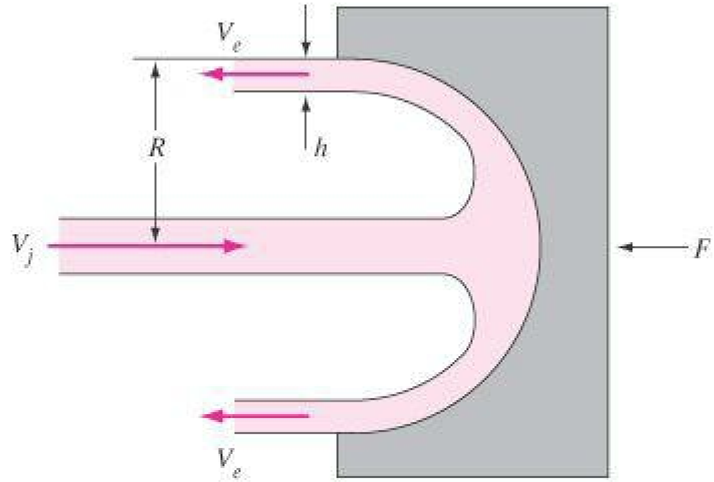
- & ,QDFHUWDLQLQGXVWULDOSURFHVV RLORI GHQVLW! IGRZ VWKURXJK WKHLQFOLQHG SLSHLQ) LJ & $ 8 WKEHPDQRPHWU ZLWK IXLG GHQVLW!P PHDVXUHV WKH SUHVXUH GLIIHUHQFH EHWZHHQ SRLQWV DQG DV VKRZQ 7KH SLSH IGRZ LV VWDG\ VR WKDW WKH IXLG VLQWK PDQRPHWU UHDGLQJ D) LQG DQ DQDO\WF H[SUHVVLRQ IRUS í S LQWUP VRI WK VWWP SDUDPHWUV E ' LVFXVV WKHFRQGLWRQV RQK QHFHVVDU IRUWKHUHWR EHQR IGRZ LQWKHSLSH F : KDWDERXWGRZ XS IURP WR " G : KDWDERXWGRZ GRZQ IURP W "



&

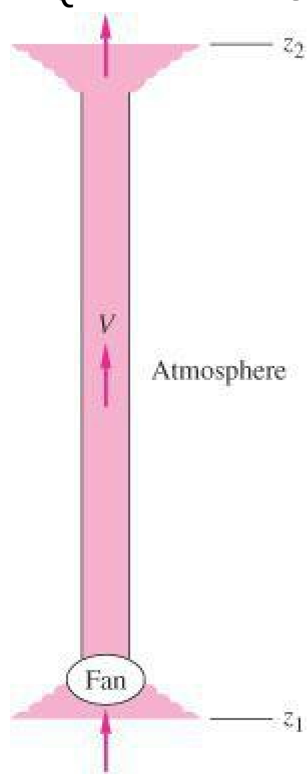
- & $ ULJLGWDQNRI YROXPH P LVLQLWDOO ILOHGZ WKDLUDW *& DQGS NBD $WWP HW DYDFXXP SXPS LVWXLQHGRQDQGHYDFXDWHDLUDWD FRQWDQWYROXP HIGRZ UDW4 / PLQ UHJDUGHVVRI WKHSUHVXUH $WXP HDQLGHDOJDV DQGDQLVRWKHUP SURFHVV D 6HWXS DGLIIHUHQWDOHTXDWRQIRUWKLVIGRZ E 6RQYHWKLVHTXDWRQIRUWD DIXQFWRQRI 4 S S F &RPSXWWKHWP HLQPLQXWVRSXPS WKHWDQNGRZQWR NBD +LQW<RXUDQZHUVKRXOGGHEHWZHHQ DQG PLQ

- & 6XSSRVHWKHVDPH WXEDG ZDWUMWDV LQ3URE 3 MWWYHORFLW P V DQG MW GLDP HWU FP LPSLQJHVLQWXEDGRQDFXSFDYLWDVKRZQLQ) LJ & 7KHZDWUL WXLQG *DQGH[LW GXHWRIUFWRQ DWRZHUYHORFLW 9H P V /RRNLQJIRP WK OHWWKHH[LWWWVDFLFXDUDQQXOXVRI RXWUDGLXV5 DQGWKFNHVV K IGRZ LQJ WRZ DJ WKHYLHZ 7KHFXSKDVDUDGLXVRI FXUYDWXUHR FP) LQG D WKHWKFNHVV K RI WK H[LWWWVDQG E WKHIRUFH) UHTXLUHGWRKROGWKHFXSSHGREMFWLQSODFH F &RPSDU SDUWE WR3URE ZKHUH) § 1 DQGJLYHDSK\VFDO[SODQDWRQDVWR ZK\ ) KDVFKDQJHG



&

- & 7KH DLUIORZ XQGHUQHDWK DQ DLUKRFNH\ SXFN LV YHU\ FRPSOH[HVSHFLDOO\ VLQFH WKH DL MHWV IURP WKH DLUKRFNH\ WDEOH LPSLQJH RQ WKH XQGHUVLGH RI WKH SXFN DW YDULRXV SRLQW QRQV\PPHWULFDOO\ $ UHDVRQDEOH DSSUR[LPDWLRQ LV WKDW DW DQ JLYHQ WLPH WKH JDJH SUHVVXUH RQ WKH ERWWRP RI WKH SXFN LV KDOIZD\ EHWZHHQ]HUR DWPRVSKHULF SUHVVXUH DQG WKH VWDJQDWLRQ SUHVVXUH RI WKH LPSLQJLQJ MHWV 6WDJQDWLRQ SUHVVXUH LV GHILQHG DV $p_0 = \frac{1}{2}\rho V_{jet}^2$ D)LQG WKH MHW YHORFLW\ 9_{MHW} UHTXLUHG WR VXSSRUW DQ DLU KRFNH\ SXFN R ZHLJKW DQG GLDPHWHU G *LYH \RXU DQVZHU LQ WHUPV RI : G DQG WKH GHQVLW\ ! RI WKH DLU E)RU : OEI DQG G LQ HVWLPDWH WKH UHTXLUHG MHW YHORFLW\ LQ IW V



&

- & 1HJOHFWLQJ IULFWLRQ VRPHWLPHV OHDGV WR RGG UHVXOWV <RX DUH DVNHG WR DQDO\]H DQG GLVFXVV WKH IROORZLQJ H[DPSOH LQ)LJ & $ IDQ EORZV DLU WKURXJK D GXFW IURF VHFWLRQ WR VHFWLRQ DV VKRZQ $VVXPH FRQVWDQW DLU GHQVLW\ ! 1HJOHFWLQJ IULFWLRQD ORVVHV ILQG D UHODWLRQ EHWZHHQ WKH UHTXLUHG IDQ KHDG K_S DQG WKH IORZ UDWH DQG WKH HOHYDWLRQ FKDQJH 7KHQ H[SODLQ ZKDW PD\ EH DQ XQH[SHFWHG UHVXOW

' HVLJQ 3URMHFWV

- ' / HWKVJHQHUDO]H3UREV 3 DQG3 LQZKLFKDSXPSSHUROPDQFHFXUY ZDVXVHGWRGHWHUPLQHWKHIORZ UDWHEHWZHHQUHVHUYRLUV 7KHSDUDPSXPSLQ) LJ 3 LV RQH RI D IDPLO RI SXPSV RI VLPLODU VKDSH ZKRVH GLPHQVLRQOHV SHUROPDQFHLVDVIROORZV

+HDG

$$\phi \approx 6.04 - 161\zeta \qquad \phi = \frac{gh}{n^2 D_p^2} \quad \text{and} \quad \zeta = \frac{Q}{nD_p^3}$$

(IILFLHQF\

$$\eta \approx 70\zeta - 91{,}500\zeta^3 \qquad \eta = \frac{\text{power to water}}{\text{power input}}$$

ZKHUHK$ LVWKHSXPS KHDG IWQ LVWKHVKDWURWDWLRQDO UV DQG' $ LVWKHLPSHOOH GLDPHWU IW7KHUDQJHRI YDOLGLW LV 7KHSXPS R) LJ 3 KDG' $ IW GLDPHWUDQGURWDWG DWQ UV UPLQ7KHVROXWLRQ WR3URE 3 QDPHO 4 IWV DQGK$ IWFRUHVSRQGV WRË RU SHUFHQW DQG SRZHUWR WK ZDWU !J4K$ IW OEI V K$ 3OHDVHFKHFNWKHVHQXPHULFDOYDOXHVEHRUHEHJLQQLQ WKLVSURMHFW

1RZ UHYLVLW3URE 3 DQVHOHFWDGRZ FRWSXPS WKDWURWDWV DWD UDWQR VOWRZHUWKD UPLQDQG GHOLYHUV QR OHVW WKDQ IWV RI ZDWU \$VXPH WKDWWKH FRWWR WKH SXPS LV OLQHDU SURSRUWLRQDOWR WKHSRZHULQSXWUHTXLUHG &RPPHQWRQDQ OPLWDWLRQVWR\RXUHVXOWV

5HIHUHQFHV

' 7 * UHHQZRRG \$GYDQFHG' \QDPLFV &DPEULGJH8QLYHUVLW3UHVV 1HZ <RUN

7 YRQ. iUPiQ 7KH: LQGDQG%HRQG / LWWOH %URZQ %RVWRQ

- 3 +ROPDQ +DW7UDQVIHU WKHG 0F* UDZ +LOO1HZ <RUN

\$ * +DQVHQ) OXLG0HFKDQLFV : LOH\ 1HZ <RUN

0 & 3RWWHU' & : LJJHUWDQG0 +RQG]R0HFKDQLFVFR) OXLGV %URRNV&ROH &KLFDJR

5 (6RQQWDJ & %RUJQDNNH DQG* - 9DQ: \OHQ) XQGDPHQWDOVRI 7KHUPRG\QDPLFV WHG -RKQ: LOH\ 1HZ <RUN

< \$ &HQJHO DQG 0 \$ %RODV 7KHUPRG\QDPLFV \$Q (QJLQHHULQJ \$SSURDFK WK HG 0F* UDZ +LOO1HZ <RUN

- ' \$QGHUVRQ &RPSXWDWLRQDO) OXLG' \QDPLFV 7KH%DVLFVZLWK\$SSOLFDWLRQV0F* UDZ +LOO1HZ <RUN

: * 9LQFHQW³&RQVWUR9RXPH\$QDOVLV \$ ' LIIHUHQFHLQ7KLQNLQJ EHWZHHQ(QJLQHHULQ DQG3K\VLFV´7HFKQRORJ\ DQG&XOWXUH YRO QR SS ±

- . HQDQ 7KHUPRG\QDPLFV : LOH\ 1HZ <RUN

- +XQDNHUDQG % 5LJKWVLUH(QJLQHHULQJ $SSOLFDWLRQV R) OXLG0HFKDQLFV 0F* UDZ +LOO1HZ <RUN

– : HDUHQHJOHFWLQJ QXFOHDUUHDFWRUV ZKHUHPDWHULDOVFDQEHFKDQJHGWRHQHUJ\

– : HDUHQHJOHFWLQJ UHDOLYLWF HIHFWV ZKHUH1HZ\RQJVODZ PXWWEHPRGLILHG

– $ ZLGGXQQHOXVHVDIL[HGPRGHOWRVLPXODWHIORZ RYHUDERG\ PRYLQJ WKURXJKDIOLG $ WRZ WDQNXVHVDPRYLQJ PRGHOWRVLPXODWHWKHVPHVLXDWRQ

– 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVVRI FRQWLQXLW

– 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVVRI FRQWLQXLW

– 7KURXJKRXWWKLVVHFWLRQZHDUHQHJOHFWLQJ VRXUFHVRUVLQNVRI PDVVWKDWPLJKWEHHPEHGGHGL WKHFRQWRO YROXPH (TXDWLRQV DQG FDQUHDGLO EHPRGLILHGWRDGGVRXUFHDQGVLQNWHUP\ EXWKLVLVUDUHO QHFHVDU

– %HZDUHGWKDWWKHUHLVVXEVHFWLQVSHFLI\LQJ LQFRPSUHVVLEOHW 2FHDQRJUDSKHUVFRQVLG D SHUFHQWGHQVLWYDULDWLRQYHU VLJQLILFDQWZKLHHDURG\QDPLFVWPD\ QHJOHFWGHQVLWYDULDWLRQ LQ KJKO FRPSUHVLEOH HYHQ KSHUVRQF JDV IORZ V <RU WDN LV WR IXWXUH WKH LQFRPSUHVLEG DSSURI LPDWRQZKHQ\RXPDNHLW

– &DQ\RX SURYH WKLV" ,WV DFRQVHTXHQFHRI * DXVV¶V WKHRUHP IURP YHFWRUDQDO\VLV

– 6\PPHWU FDQ EH D SRZHUXO WRROL XVHG SURPHO7U WR HDQ PRUH DERXWWKH XVHV DQ PLXVHVRI \PPHWU FRQGLWRQV

— 7KHJHQHUDODQDO\VLVRI VXFK ZDOOKDUSUREOPV FDQBHGERXQGDU\ OD\HUWKHRU LVWHDWG LW 6HF

— 7KHDXWRELRJUDSK\ RI WKLVJUHDWZHQWHUWKFHQWXU HQJLQHHUDQG WDFKHU> @LVUHFRPPHQGHGIR LWWKLVWRUFDOOG VFLHQWLF LQJKW

— 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVVRI FRQWLQXLW

— $ FRP SOHWH GLVFXVVLRQ RI WKH VHQVLWLYLWLHV RQ LQWHOOHFWXDO FRRUGLQDWLRQ VLJQLILFDQFH LQ5H SS ±

— 7KLV VHFWLRQ PD\ EH RPLWWHG ZLWKRXW RVV RI FRQWLQXLW\

— ,QFUHDVH WKH SLSHEHQG VKDSH SURFEDO DIHFWV WKH SUHVXUH FKDQJH IURP S WR S

— 7KLV VHFWLRQ VKRXOG EH UHDG IRU LQIRUPDWLRQ DQG HQULFKPHQW HYHQ LI \RX ODFN IRUPD EDFNJURXQG LQWHUPRGXODWLRQ PDWHULDOV

— 7KH HQHUJ\ HTXDWLRQ IRU D GHRUPDEOH FRQWRO YROXPH LV UDWKHU FRPSOLFDWHG DQG LV QF GLVFXVVHG KHUH 6HH 5HV DQG IRU IXUWKHU GHWDLOV

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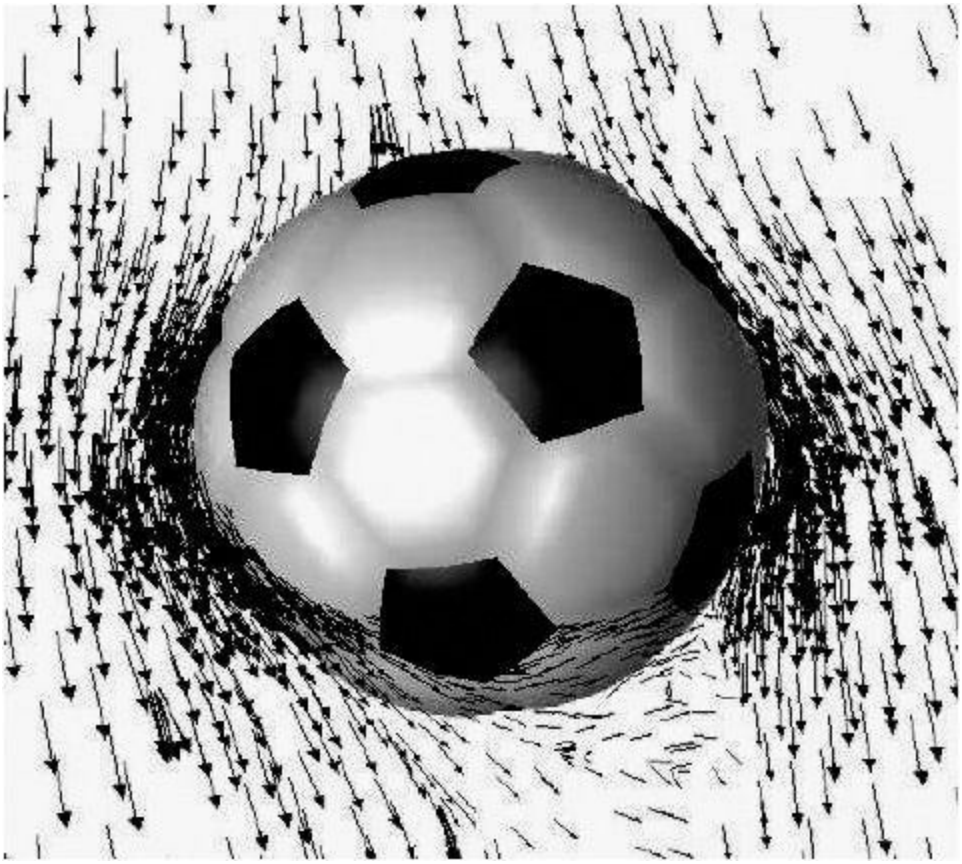


7KHGLIIHUHQWLDOHTXDWLRQVRI WKLVFKDSWHUPD\ EHPRGHOHGE\ &RPSXWDWLRQDO) OXLG' \QDPLF &)' IRUDOPRVWDQ\ IORZ SUREOHP 7KHH[DPSOHDERYHVKRZVWKHFRPSXWHGYHORFLW\ YHFWRU IRUIORZ SDWWHUQ FXUYLQJ VSLQQLQJ VRFFHUEDOO7KLV&)' SURMHFWKHDGHGE\ ' U. HLWK+DQQDF)/8(17,QF ZDVLQVSLUHGE\ DPLUDFXORXVIUHHNLFNE\ %UD]LOLDQ5REHUWR&DUORVDWWKH: RU &XS &DUORV¶NLFN VDLGHGKJK DQGZLGHWRWKHULJKWDQGWKHQFXUYHGGRZQRYHUWKHGHIHQGHU KHDGVLQWRWKHQHW7KHFXUYHGWUDMHFWRU\ LVFDOOHGWKH***Magnus effect*** +HLQULFK0DJQXV 7KHYHORFLWLHVLQWKHILJXUHDUHXVHGWRFRPSXWHWKHVXUIDFHSUHVVXUHVDQGVKHDUVWUHVVHV ZKLFK DUHWKHQVXPPHGWR\LHOGWKHWDQJHQWLDORFHQWKHEDOO***(Image Courtesy of ANSYS. Inc.)***

0 RWLYDWLRQ

,QDQDO]LQJ IOXLG PRWLRQ ZHPLJKWDVNHRQHRI VZR SDWKV VHHNLQJ DQHZWPDWH JURVVHIIHFW PDWIORZ LQGXFHG IRUFH HQHUJ\ FKDQJH RYHUDILQLWHUHJLRQRUFRQWURO YROXPHRU VHHNLQJ WK SRLQWE\ SRLQWGHWDLOVRI DIORZ SDWWHUQE\ DQDO]LQJ DQLQILQLWHVLPDOUHJLRQRI WKHIORZ 7KHIRUPH RUJURVV DYHUDJHYLHZ SRLQWZDVWKHVXEMHFWRI &KDS 7KLVFKDSWHUWHDWWKHVHFRQGLQRXUWRR RI WKFKQLTXHVIRUDQDO]LQJ IOXLG PRWLRQ VPDOOVFDOH RUGLIIHUHQWDO DQDO\VLV 7KDWLV ZHDSSO\ RXUIRXUEDVLF FRQVHUYDWLRQ ODZ VWR DQ LQILQLWHVLPDO DOO VPDOOFRQWURO YROXPHRU DOWHUQDWLYHO\ VWR DQ LQILQLWHVLPDO IOXLG VWUHDP ,QHLWKHUFDVHWKHUHVXOW\LHOG VWKH EDVLF GLIIHUHQWLDO HTXDWLRQV RI IOXLG PRWLRQ $SSURSULDWHERXQGDU\ FRQGLWLRQV DUH DOV

GHYHORSHG

,QWKHUPRWWEDVLF IRUP WKHVHGLIIHUHQWDOHTXDWRQVRI PRWRQDUH TXLWHGLIILFXOWWRVROYH DQ YHU OLWWOHLVNQRZQDERXWKHUJHQHUDOPDWKHPDWLFDOSURSHUWLHV + RZHYHUFHUWDLQWKLQJVFDQEHGRQ WKDWKDYHJUHDWHGXFDWRQDOYDOXH) LUVWDVVKRZQLQ&KDS WKHHTXDWRQV HYHQLI XQVROYHG UHYHD WKHEDVLF GLPHQVLRQDOWSDUDPHWHUVWKDWJRYHUQIOXLG PRWRQ 6HFRQG DVVKRZQLQ&KDS DJUHD QXPEHURI XVHIXOVROXWRQVFDQEHIRXQGLI RQHPDNHVWR VLPSOLI\LQJDVVXPSWLRQV VWDG\ IORZ DQG LQFRPSUHVVLEOHIORZ $ WKLUGDQGUDWKHUGUDVWLF VLPSOLILFDWLRQ IURFWRQOHVVIORZ PDNHVRX RQG IUHG WKH %HUQRXOOL HTXDWRQ YDOG DQG \LHOGV D ZLGH YDULHWV RI LGHDOLQHG RUSHUHFWOXLG SRWLEOHVROXWRQV 7KHVHILGHDOL]HGIORZ VDUHWWHDWHGLQ&KDS DQGZHPXWEHFDUHXOWRDVFHUWDL ZKHWKHUVXFK VROXWRQV DUHILQIDFWDOOXLGPRWRQ) LQDOO HYHQWK GIIILFXOWJHQHUDO GIIHUHQWLDO HTXDWRQV QRZ \LHOG WR WKH DSSUR[LPDWQJ WHFKQLTXH NQRZQ D FRPSXWDWRQDOIXLG G\QDPLFV &)' ZKHUHE\ WKHGHULYDWLYHV DUH VLPXODWGE\ DJHEUDLF UHODWRQ EHWZHHQ D ILQLWQXPEHURI JULG SRLQWV LQ WKHIORZ ILHOG ZKLFK DUH WKHQ VROYHG RQ D FRPSXWHU 5HUHQFH LVDQ[DPSOHRI DWK[WERRNGHYRWGHQWLHO WRQXPHUFDOQDO VLVRI IOXLGPRWRQ

7KH$FFHOHUDWRQ) LHOGRI D) OXLG

,Q6HF ZHHVWDEOLVKHGWKHFDUWHVLDQ YHFWRUIRUP RI DYHORFLWILHOGWKDWYDULHVLQVSDFHDQ WLPH

$$\mathbf{V}(\mathbf{r}, t) = \mathbf{i}u(x, y, z, t) + \mathbf{j}v(x, y, z, t) + \mathbf{k}w(x, y, z, t) \qquad (1.4)$$

7KLVLVWKHPRWWPSRUWDQWYDULDEOHLQIOXLGPHFKDQLFV . QRZ OHGJHRI WKHYHORFLWYHFWRUILHOGL QHDUO HTXLYDOHQWWR\RQOLQJ DIOXLG IORZ SUREOP 2XUFRRUGLQDWV DUH IL[HG LQ VSDFH DQG ZH REVHUYHWKHIOXLGDVLWSDVVHVE\² DVLI ZHKDG\FUHEHDVHWRI FRRUGLQDWOQHVRQDJODWZLQGRZ LQ D ZLQG WXQQHO 7KLV LV WKHHXOHULDQ IUDPHRI UHIHUHQFH DVRSSRVHG WR WKH DJUDQJLDQ IUDPH ZKLFK IROORZV WKHPRYLQJ SRVLWRQRI LQGLYLGXDOSDUWFOHV

7R ZULWH1HZWRQ¶V VHFRQG ODZ IRUDQ LQILQLWHVLPDO IOXLG \XVWP ZHQHHG WR FDOFXODWH WKH DFFHOHUDWRQ YHFWRUILHOGD RI WKH IOXLG 7KXV ZH FRPSXWH WKH WRWDOWLP HGHULYDWLYH RI WKH YHORFLW YHFWRU

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \mathbf{i}\frac{du}{dt} + \mathbf{j}\frac{dv}{dt} + \mathbf{k}\frac{dw}{dt}$$

6LQFHHDFK VFDODUFRPSRQHQWX % Z LVDIXQFWRQRI WKHIRXUYDULDEOHV [\ ] W ZHXVHWKH FKDLQUXOHWRREWLQHDFK VFDODUWP HGHULYDWLYH) RUH[DPSOH

$$\frac{du(x, y, z, t)}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

%XWE\ GHILQLWRQ G[GWV\VWHORFDOYHORFLWFRPSRQHQWX DQGG\ GW % DQGG] GW Z 7KHWRWDO WP HGHULYDWLYH RI X PD\ WKXVEH ZULWWQ DVIRQORZV ZLWK[DFWO VLPLODU[SUHVLRQVIRUWKHWP GHULYDWLYHVRI %DQGZ

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial u}{\partial t} + (\mathbf{V}\cdot\nabla)u$$
$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{\partial v}{\partial t} + (\mathbf{V}\cdot\nabla)v \qquad (4.1)$$
$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{\partial w}{\partial t} + (\mathbf{V}\cdot\nabla)w$$

6XPPLQJ WKHVH LQWR D YHFWRU ZH REWDLQ WKH VRDOOHG FRQYHFWLYH DFFHOHUDWLRQ

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \underbrace{\frac{\partial \mathbf{V}}{\partial t}}_{\text{Local}} + \underbrace{\left(u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}\right)}_{\text{Convective}} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} \qquad (4.2)$$

7KH WHUP Á9 ÁWLV FDOOHG WKH ORFDO DFFHOHUDWLRQ ZKLFK YDQLVKHV LI WKH IORZ LV VWHDG² WKDWLV LQGHSHQGHQWRI WLPH 7KH WHUP VLQ SDUHQWKHVHV DUH FDOOHG WKH FRQYHFWLYH DFFHOHUDWLRQ ZKLFK DULVHV ZKHQ WKH SDUWFOH PRYHV WKURXJK UHJLRQV RI VSDWLDOO YDULQJ YHORFLW DV LQ D QR]]OH R GLIIXVHU) ORZ VWHDWUHDPOLQHV ³VWHDG´ PD\ KDYH FXUYHDFFHOHUDWLRQ/GXHWRWKHFRQYHFWLYHWHUPV 1RWHRXUXVHRI WKHFRPSDFWQRWDWLRQ9 DQG WKHJUDGLHQWRSHUDWRU∇

$$u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} = \mathbf{V}\cdot\nabla \qquad \text{where} \qquad \nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

7KH WRWDO WLPH GHULYDWLYH? VRPHWLPHV FDOOHG WKH VXEVWDQWLDO RUPDWHULDO GHULYDWLYH? FRQH PD\ EHDSSOLHG WR DQ YDULDEOH VXFK DV WKH SUHVVXUH

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \frac{\partial p}{\partial t} + (\mathbf{V}\cdot\nabla)p \qquad (4.3)$$

: KHUHY HU FRQYHFWLYH HIIHFWV RFFXU LQ WKH EDVLF ODZ V LQYROYLQJ PDVV PRPHQWXP RU HQHUJ\ WK EDVLF GLIIHUHQWLDO HTXDWRQV EHFRPH QRQOLQHDU DQG DUH XVXDOO PRUH FRPSOLFDWHG WR VROYH QRQOLQHDU FRQYHFWLYH FKDQJHV

: H HPSKDVL]H WKDW WKLV VWHDG WLPH GHULYDWLYH IRQORZ V D SDUWFOH RI IL[HG LGHQWLW PDNLQJ L FRQYHQLHQW RU H[SUHVVLQJ ODZ V RI SDUWFOH PHFKDQLFV LQ WKH HXOHUDQ IXLG ILHOG GHVFULSWRQ 7K RSHUDWRUGGWLV VRPHWLPHV DXWJQG D VSHFLDO\PERO VXFK DV' ' WD/ D IXUKHU UHPLQGHU WKDW FRQYLQV IRXUWHUV DQG IRQORZ V D IL[HG SDUWFOH

$ VDQRWKHU UHPLQGHU RI WKH VSHFLDOQDXUH RI GGWVRP HZ ULWHV JLYH LWWKH QDPH VXEVWDQWLDO PDWHULDO GHULYDWLYH

(; $0 3/ (

* LYHQWKHHXOHUDQYHORFLW YHFWRUILHOG

$$\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

ILQG WKH WRWDO DFFHOHUDWLRQ RI D SDUWFOH

6RXWLRQ

- $VXPSWLRQV * LYHQWKHHNQRZQXQLWDG YHORFLWFRPSRQHQWV X W% [] DQGZ W
- $SSURDFK &DUU RXWDOOWKHUHTXLUHG GHULYDWLYHVZLWKUHVSHFWWR [\ ] W VXEVWLWXWLQ WKHVRDOOFFHOHUDWLRQYHFWRU (T DQGFRQFWUPV
- 6RXWLRQVWHS) LUWZ RUNRXWWKHORFDOFFHOHUDWLRQÁ9 ÁW

$$\frac{\partial \mathbf{V}}{\partial t} = \mathbf{i}\frac{\partial u}{\partial t} + \mathbf{j}\frac{\partial v}{\partial t} + \mathbf{k}\frac{\partial w}{\partial t} = \mathbf{i}\frac{\partial}{\partial t}(3t) + \mathbf{j}\frac{\partial}{\partial t}(xz) + \mathbf{k}\frac{\partial}{\partial t}(ty^2) = 3\mathbf{i} + 0\mathbf{j} + y^2\,\mathbf{k}$$

- 6RXWLRQVWHS ,QDVLPLODUPDQQHUWKHFRQYHFWLYHDFFHOHUDWLRQWUPV IURP (T DUH

$$u\frac{\partial \mathbf{V}}{\partial x} = (3t)\frac{\partial}{\partial x}(3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}) = (3t)(0\mathbf{i} + z\mathbf{j} + 0\mathbf{k}) = 3tz\,\mathbf{j}$$

$$v\frac{\partial \mathbf{V}}{\partial y} = (xz)\frac{\partial}{\partial y}(3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}) = (xz)(0\mathbf{i} + 0\mathbf{j} + 2ty\mathbf{k}) = 2txyz\,\mathbf{k}$$

$$w\frac{\partial \mathbf{V}}{\partial z} = (ty^2)\frac{\partial}{\partial z}(3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}) = (ty^2)(0\mathbf{i} + x\mathbf{j} + 0\mathbf{k}) = txy^2\,\mathbf{j}$$

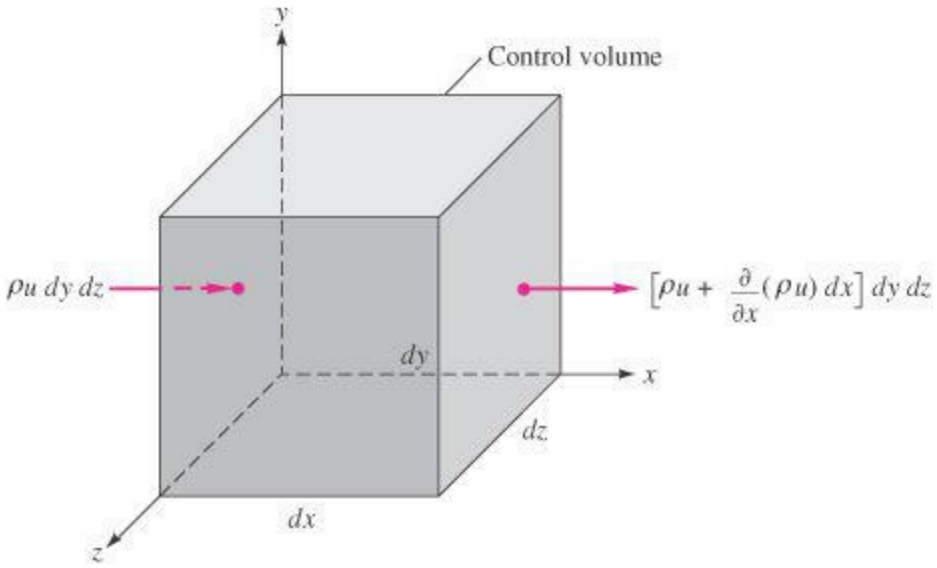
- 6RXWLRQVWHS &RPELQH DOOIRXUWHUPV DERYH LQWR WKH VLQJOH ³WRWDO RU ³XEVWDQWLDO GHULYDWLYH

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} = (3\mathbf{i} + y^2\mathbf{k}) + 3tz\mathbf{j} + 2txyz\mathbf{k} + txy^2\mathbf{j}$$

$$= 3\mathbf{i} + (3tz + txy^2)\mathbf{j} + (y^2 + 2txyz)\mathbf{k} \qquad Ans.$$

- &RPPHQWV $VXPLQJ WKDW9 LV YDQG HYHUZ KHUH DV JLYHQ WKLV WRWDO DFFHOHUDWLRQ YHFWR G9 GWDSSOLHVWR DOOSRVLWLRQV DQGWLPHVZ LWKLQWKHIORZ ILHOG

7KH' LIIHUHQWLDO(TXDWLRQ RI 0 DVV&RQVHUYDWLRQ



) LJ (OHP HQWDU FDUWHVLDQ IL[HG FRQWURO YROXP H VKRZLQJ WKH LQIORZ DQG RXWIORZ DW IDFH VR WKH x IDFHV

&RQVHUYDWLRQ RI PDVV RI WKH FDOOHG WKH FRQWLQXLW UHODWLRQ VWDWHV WKDW WKH IOXLG PDVV FDQQ FKDQJH : H DSSO\ WKLV FRQFHSWXDO YHU VPDOO UHJLRQ $OO WKH EDVLF GLIIHUHQWLDO HTXDWLRQV FDQ EH GHULYHG E\ FRQVLGHULQJ HLWKHU DQ HOHPHQWDU FRQWURO YROXP H RU DQ HOHPHQWDU VWWHP : H FKRRVH D LQILQLWHVLPDO IIL[HG FRQWURO YROXP H G[G\ G] DV LQ) LJ DQG XVH RXU EDVLF FRQWURO YROXP I UHODWLRQV IURP &KDS 7KH IOXR WKURXJK HDFK VLGH RI WKH HOHP HQWV DSSUR[LPDWH RQH GLP HQVLRQDO DQG VR WKH DSSURSULDWH PDVV FRQVHUYDWLRQ UHODWLRQ WR XVH KHUHLV

$$\int_{CV} \frac{\partial \rho}{\partial t}\, d\mathcal{V} + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0 \qquad (3.22)$$

7KH HOP HQWLV VR VP DOO WKDW WKH YROXP HLQWUDO LPS UHGXFH V WR DGLIIHUHQWLDO WUP

$$\int_{CV} \frac{\partial \rho}{\partial t}\, d\mathcal{V} \approx \frac{\partial \rho}{\partial t}\, dx\, dy\, dz$$

7KH P DW IGRZ WUP VRFFXUR QDOV[IDFHV WKUH HLQIWDQG WKUH RXWIW : H PDNH XVH RI WKH ILHO RUFRQWUROXXP FRQVHSWUP &KDS ZKHUHDOOIXLG SURSHUWV DUH FRQVLGHUHG WR EH XQIRUP 0 YDU LQ IXQFWLRQV RI WP HDQG SRVLWLRQ VXFK DV! ! [\ ] W 7KV LI 7 LV WKH WMP SHUDXUH RQ WKH HIWDFH R WKH HOP HQWLQ) LJ WKH ULJKWDFH ZLOOKDYH D VOJKW GLIIHUHQWWMP SHUDXUH 7 ~7 ~[G[ ) F P DW FRQVHUYDWLRQ LI ! X LV NQRZ QRQ WKH HIWDFH WKH YDOXH RI WKLV SURGXFWRQ WKH ULJKWDFH LV! X ~! X! [G[

) LJXUH VKRZ VRQO WKH P DW IGRZ VRQ WKH[RUHIWDQG ULJKWDFHV 7KH IGRZ VRQ WKH\ ERWWF DQG WS DQG WKH] EDFN DQG IURQWIDFHV KDYH EH QRP LWWG VR DYRLG FOXWWULQJ XS WKH GUDZ LQJ : H FDQ DWWDOWKH VHVL[IGRZ VDV IROORZV

,QWRGXFH WKH VHWMP VLQR(T DQG Z HKDYH

Face	Inlet mass flow	Outlet mass flow
x	$\rho u\, dy\, dz$	$\left[\rho u + \frac{\partial}{\partial x}(\rho u)\, dx\right] dy\, dz$
y	$\rho v\, dx\, dz$	$\left[\rho v + \frac{\partial}{\partial y}(\rho v)\, dy\right] dx\, dz$
z	$\rho w\, dx\, dy$	$\left[\rho w + \frac{\partial}{\partial z}(\rho w)\, dz\right] dx\, dy$

7KH HOHPHQWDU\ FRQWURO PHFKDQLVPV GHDOW ZLWK DOORZLQJ D SDUWLFOH HTXDWLRQ LQYROYLQJ WK GHULYDWLYHV RI GHQVLW\ DQG YHORFLW\

$$\frac{\partial \rho}{\partial t}\,dx\,dy\,dz + \frac{\partial}{\partial x}(\rho u)\,dx\,dy\,dz + \frac{\partial}{\partial y}(\rho v)\,dx\,dy\,dz + \frac{\partial}{\partial z}(\rho w)\,dx\,dy\,dz = 0$$

7KLV LV WKH GHVLUHG UHVXOW FRQVHUYDWLRQ RI PDVV IRUDQ LQILQLWHVLPDO FRQWURO YROXPH ,WV RIWV FDOOHG WKH HTXDWLRQ RI FRQWLQXLW\ EHFDXVH LWUHTXLUHV QR DVVXPSWLRQV H[FHSWWKDWWKH GHQVLW\ DQ YHORFLW\ DUH FRQWLQXXP IXQFWLRQV 7KDWLV WKH IORZ PD\ EH HLWKHU WUDQG RU XQWUDQG YLVFRXV R IULFWLRQDWV FRP SUHVVLEOH RULQFRP SUHVVLEOH + RZ HYHU WKH HTXDWLRQ GRHV QRWDOORZ IRUDQ VRXUF RUVLQNVLQJXODULWHVZ LWKLQWKHHOHP HQW

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0} \qquad (4.4)$$

7KHYHFWRUJUDGLHQWRSHUDWRU

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

HQDEOHVXVWRUHZ ULWWKHHTXDWLRQRI FRQWLQXLWLQDFRP SDFWRUP QRWKDWWKHSVP XFKLQILQGLQ DVRXWRQ 7KHGDWKUHHWUP VRI (T DUHHTXLYDOHQWWRWKHGLYHUJHQFHRI WKHYHFWRU! 9

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \equiv \nabla \cdot (\rho \mathbf{V}) \qquad (4.5)$$

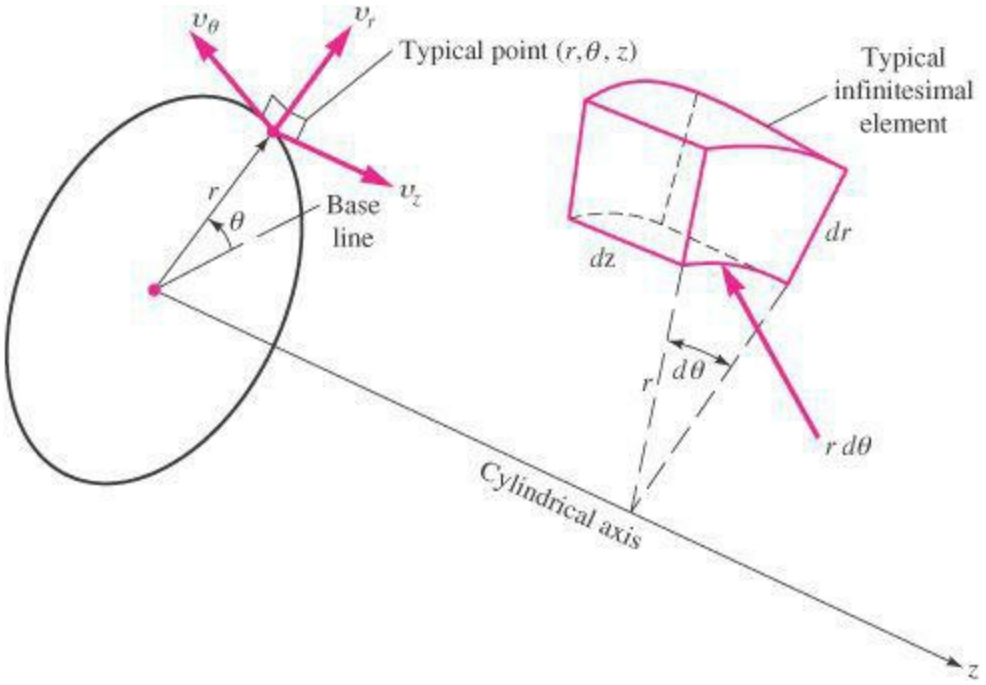
VR WKDWWKHFRP SDFWRUP RI WKHFRQWLQXLWUHDWRQLV

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \qquad (4.6)$$

,Q WKLV YHFWRU IRUP WKH HTXDWRQ LV VLPLO TXLWH JHQHUDO DQG FDQ UHDGLO EH IRQYHUWHG WR RWKHFRRUGLQDWVVWHPV

&\ OLQGULFDO3RODU&RRUGLQDWHV

7KH PRWWRP PRQDGWLQDLYHWRWKHFDUWHVLDQVWHP LVWKHFWOQGULFDOSRODUFRRUGLQDWHVWHP
VHWWKHGLQ) LJ $QDUELWUDU SRLQW3 LVGHILQHGE\ DGLVWDQFH] DORQJ WKHD[LV DUDGLDOGLVWDQFHU
IURP WKHD[LV DQG D URWDWRQ DQJOH DERXWWKHD[LV 7KHWKUHLQGHSHQGHQWRUWKRJRQDOYDULDEOHV
FRPSRQHQWVDUHDQD[LDOYHORFLW\% DUDGLDOYHORFLW\%U DQGDFLUFXPIHUHQWLDOYHORFLW\% ZKLFK LV
SRVLWLYHFRXQWHUFORFNZLVH? WKDWLV LQWKHGLUHFWLRQ RI LQFUHDVLQJ ,QJHQHUDO DOOFRPSRQHQWV D
ZHOODVSUHVVXUHDQGGHQVLW\DQGRWKHUILXLG SURSHUWLHV DUHFRQWLQXRXVIXQFWLRQVRI U] DQGW



) LJ ' HILQLWLRQVNHWFKIRUWKHFWOQGULFDOFRRUGLQDWHVWHP

7KHGLYHUJHQFHRI DQ YHFWRUIXQFWLRQ$ U] WLVIRXQGE\ PDNLQJ WKHVWDQGDUGRUPDWRQRF
FRRUGLQDWHV

$$r = (x^2 + y^2)^{1/2} \qquad \theta = \tan^{-1}\frac{y}{x} \qquad z = z \tag{4.7}$$

DQGWKHUHVXOWLVJLYHQKHUHZLWKRXWSURRI

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(A_\theta) + \frac{\partial}{\partial z}(A_z) \tag{4.8}$$

7KHJHQHUDOFRQWLQXLW\HTXDWLRQ LQF\OLQGULFDOSRODUFRRUGLQDWHVLVWKXV

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \tag{4.9}$$

7KHUHDUHRWKHURUWKRJRQDOFXUYLOLQHDUFRRUGLQDWHVV QRWDEO\VSKHULFDOSRODUFRRUGLQDWHV
ZKLFK RFFDVLRQDOO PHULWXVH LQ DIOXLG PHFKDQLFV SUREOHP : HVKDOOQRWXVHWKHVHV\VWHPVKHU
H[FHSWLQ3URE 3

7KHUHDUHDOR RWKHUZD\VRGHULYHWKHEDVLF FRQWLQXLW\HTXDWLRQ WKDWDUHLQWHUHVWLQJ DQG
LQWKFWLYH 2QH H[DPSOHLV WKH XVH RI WKH GLYHUJHQFH WKHRUHP $VN\RXULQWUXFWRUDERXWWKHP
DQGOLQDUYHDSSURDFKHV

6WDG\ &RP SUHVVLEOH) ORZ

,I WKH IORZ LV WUDG\ ÁÁW DQG DOO SURSHUWLHV DUH IXQFWLRQV RI SRVLWLRQ RQO (TXDWLRQ UHGXFHV WR

$$\text{Cartesian:} \qquad \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\text{Cylindrical:} \qquad \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (4.10)$$

6LQFH GHQVLW\ DQG YHORFLW\ DUH ERWK YDULDEOHV WKHVH DUH VWLOO QRQOLQHDU UHODWLRQV IRU LGHDO EH[DQG EHUQRXOOL SHFLDOFDVHV RXWLQHG

,QFRP SUHVVLEOH) ORZ

$ VSHFLDOFDVHWKDWZDUUDQWVJUHDWVLPSOLILFDWLRQLVLQFRP SUHVVLEOHIORZ ZKHUHWKHGHQVLW\FKDQJH DUHQHJOLJLEOH 7KHQ% %W UHJDUGOHVV RI ZKHWKHUWKHIORZ LV WUDG\ RUXQWUDG\ DQG WKHGHQVLW\FD EHWUHDWHGDVDSSHGRXWWKHGLYHUJHQFHLQ(T DQGGLYLGHGRXW7KHUHVXOWV

$$\nabla \cdot \mathbf{V} = 0 \qquad (4.11)$$

LVYDOLGIRUWUDG\ RUXQWUDG\ LQFRP SUHVVLEOHIORZ 7KHWZR FRRUGLQDWHIRUP VDUH

$$\text{Cartesian:} \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (4.12a)$$

$$\text{Cylindrical:} \qquad \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0 \qquad (4.12b)$$

7KHVH DUHOQHDUGLIIHUHQWLDOHTXDWLRQV DQG DZLGHYDULHWR VROXWLRQV DUH NQRZ Q DV GLVFXVVHG LQ&KDSV WR 6LQFHQR DXWKRULQJXWKFRUFDQUHLWWDZLGHYDULHWR VROXWLRQV LWWRGRZ VWKDW JUHDWGHDORI WLP HLV VSHQWWXG LQJ LQFRP SUHVVLEOHIORZ V) RUWKDWPD WKLVVH[DFW ZKDWWKRXGEH GRQH EHFDXVH PRVWSUDFWDO HQJLQHHULQJ IORZ V DUH DSSUR[LPDWH LQFRP SUHVVLEOH WKH FKLH[FHSWRQEHLQJ WKHKJK VSHHGJDVIORZ VWUHDPGLQ&KDS

: KHQ LV D JLYHQ IORZ DSSUR[LPDWH LQFRP SUHVVLEOH" : HFDQ GHULYH D QFH FULWHULRQ E\ XVLQ VRP H GHQVLW DSSUR[LPDWLRQV ,Q HVWHQFH Z H Z LVK WR VOS WKH GHQVLW RXWR VOS WKH GLYHUJHQFH LQ(T DQG DSSUR[LPDWH WKH\SLFDOWUP VXFK DV

$$\frac{\partial}{\partial x}(\rho u) \approx \rho \frac{\partial u}{\partial x} \qquad (4.13)$$

7KLVLVHTXLYDOHQWWR WKHVWRQJ LQHTXDOLW

$$\text{or} \qquad \left| u\frac{\partial \rho}{\partial x} \right| \ll \left| \rho \frac{\partial u}{\partial x} \right| \qquad \left| \frac{\delta \rho}{\rho} \right| \ll \left| \frac{\delta V}{V} \right| \qquad (4.14)$$

$ VVKRZ QLQ(T WKHSUHVXUHFKDQJHLV DSSUR[LPDWHO SURSRUWLRQDOWR WKHGHQVLW FKDQJH

DQG WKH VTXDUH RI WKH VSHHG RI VRXQG D RI WKH IOXLG

$$\delta p \approx a^2 \, \delta \rho \qquad (4.15)$$

0 HDQZKLOH LI HOHYDWRQ FKDQJHV DUH QHJOLJLEOH WKH SUHVVXUH LV UHDVRQDEO\ FRQVWDQW DQG WKH YHORFLW\ FKDQJHV %HUQRXOOL¶V HTXDWLRQ

$$\delta p \approx -\rho V \, \delta V \qquad (4.16)$$

&RPELQLQJ (TV VR Z HREWDLQ DQ [SUHVVLRQ IRU LQFRPSUHVVLEOH IORZ

$$\frac{V^2}{a^2} = \mathrm{Ma}^2 \ll 1 \qquad (4.17)$$

Z KHUH 0 D 9 D LV WKH GLPHQVLRQOHVV0 DFK QXPEHU RI WKH IORZ +RZ VPDOO LV VPDOO 7K FRPPRQO DFFHSWHG OLPLWV

$$\mathrm{Ma} \leq 0.3 \qquad (4.18)$$

) RU DLUDVWURQDXWLFDO FRQGLWLRQV DI IORZ FDQ WKXV EH FRQVLGHUHG LQFRPSUHVVLEOH LI WKH YHORFLW\ VWD WKDQ DERXW P V IWV 7KLV HQFRPSDVVHV D ZLGH YDULHW\ RI DLUIORZ V DXWRPRELOH DQG WUDP PRWLRQV OLJKW DLUFUDIW ODQGLQJ DQG WDNHRII RI KLJK VSHHG DLUFUDIW PRVW SLSH IORZ V DQ VXUERPDFKLQHU\ DVPRGHUDWH URDWRQDOVSHHGV) XUWKHU LWV FOHDU WKDWDOP RVWDOO OTXLG IORZ V DU LQFRPSUHVVLEOH VLQFH IORZ YHORFLWLHV DUH VPDOODQG WKHVSHHG RI VRXQG LV YHU ODUJH

%HFDXVH DWWHPSWLQJ WR DQDO]H WKH FRQWLQXLW\ HTXDWLRQ Z HVKDOO SURFHHG Z LWK WKH GHULYDWLRQ RI WK PRPHQWXP DQG HQHUJ\ HTXDWLRQV VR WKDW Z H FDQ DQDO]H WKH P DVDJURXS \$ YHU FOHYHUGHYLFHFDOOHG VWHDP IXQFWLRQ FDQ RIWQ PDNH VKRUWZ RUN RI WKH FRQWLQXLW\ HTXDWLRQ EXW Z HVKDOOVDYH LWXQWL 6HF

2 QHIXUWKHUPRUNLV DSSURSULDWH 7KHFRQWLQXLW\ HTXDWLRQLV DZ D VLPSOHXQGHUGHWHUPLQHG IRUP RI DUDWLRQDOODQDO\VLV RI DIORZ SDWWHUQ \$Q QHZ GLVFRYHUHG PRPHQWXP RUHQHUJ\ ³VROXWLRQ ZLOOXOWLPDWH IDLOZKHQ VXEVHTXHQW WR FULWLFDO DQDO\VLV LW WUDQVSLUHV QRWDOOR VDWLV\ WKH FRQWLQXLW\ HTXDWLRQ

(; \$0 3/ (

8QGHUZKDWFRQGLWLRQVGRHVWKHYHORFLWLHOG

$$9 \quad D[\quad E\ \quad F] L \quad D[\quad E\ \quad F] M \quad D[\quad E\ \quad F] N$$

Z KHUHD E HW FRQVWUHSUHVHQWDQLQFRPSUHVVLEOHIORZ WKDWFRQVHUYHVPDW"

6RO XWLRQ

5HFDOOQJ WKDV9 XL %M ZN Z H VHH WKDWX D[E\ F] HW 6XEVWLWXWLQJ LQR(T D IRULQFRPSUHVVLEOHFRQWLQXLW Z HREWDLQ

$$\frac{\partial}{\partial x}(a_1x + b_1y + c_1z) + \frac{\partial}{\partial y}(a_2x + b_2y + c_2z) + \frac{\partial}{\partial z}(a_3x + b_3y + c_3z) = 0$$

RU

D E F

$ WOHDVWZR RI FRQVWDQWV D E DQG F PXVWKDYH RSSRVLWH VLJQV &RQVLGHU LPSRVH Q
UHVWULFWLRQV ZKDWHYHU RQ FRQVWDQWV E F D F D DQG E ZKLFK GR QRW FRQWUDGLFW XVH WR D YROXPH L
LQFUHDVHRUGHFUHDVHRI DGLIIHUHQWLDOHOHPHQW

(; $0 3/ (

$ QLQFRPSUHVVLEOHYHORFLWILHOGLVJLYHQE\

X D[\ XQNQRZQ QZ E

ZKHUHD DQG E DUH FRQVWDQWV : KDWP XVWWKH IRUP RI WKHYHORFLWFRPSRQHQW%EH"

6RXWLRQ

$JDLQ(T D DSSOLHV

$$\frac{\partial}{\partial x}(ax^2 - ay^2) + \frac{\partial v}{\partial y} + \frac{\partial b}{\partial z} = 0$$

or
$$\frac{\partial v}{\partial y} = -2ax \qquad (1)$$

7KLVLVHDVLO LQWJUDWHG SDUWLDOO ZLWKUHVSHFWR\

[\ ] W D[\ I [ ] W

7KLV LV WKH RQO SRVLEOH IRUP IRU%WKDWWDLWLHV WKH LQFRPSUHVVLEOH FRQWLQXLW HTXDWLRQ 7K
IXQFWLRQ RI LQWJUDWLRQ I LVHQWLUHO DUELWUDU VLQFHLWYDQLVKHVZKHQ%LVGLIIHUHQWLDWHGZLWKUHVSHFWW
_

(; $0 3/ (

$ FHQWULXJDOLP SHOOHUURI FP GLDP HWUL XVHG WR SXPS SK\GURJHQDW *& DQG DW SUHVXUH
(VWLP DWH WKH PD[LPXP DOORZ DEOH LP SHOOHUURWDWLRQDO VSHHG WR DYRLG FRP SUHVVLELOLW HIHFWV DWWK
EODGHWSV

6RXWLRQ

- $VXPSWLRQV 7KHPD[LPXP IOXLGYHORFLWLVDSSUR[LPDWHOHTXDOWRWKHLPSHOOHUVSVSHHG

$$v_{max} \approx v_{max} \text{ where } v_{max} = \text{ P}$$

- $SSURDFK) LQGWKHVSHHGRI VRXQGRI K\GURJHQDQGPDNHVXUHWKDWv_{max} LVPXFKOHVV
- 3URSHUWLHV) URP 7DEOH$ IRUK\GURJHQ 5 P V í . DQGN) URP (T DW *f*& . FRPSXWHWKHVSHHGRI VRXQG

$$a_{H_2} = \sqrt{kRT} = \sqrt{1.41[4124\ \text{m}^2/(\text{s}^2 - \text{K})](288\ \text{K})} \approx 1294\ \text{m/s}$$

-) LQDOVROXWLQ 8VHRXUUKHRI WKXPE (T WR HVWLPDWHWKHPD[LPXP LPSHOOHVSHHG

$$V = \Omega r_{max} \le 0.3a \quad \text{or} \quad \Omega(0.2\ \text{m}) \le 0.3(1294\ \text{m/s})$$

$$\text{Solve for} \quad \Omega \le 1940 \frac{\text{rad}}{\text{s}} \approx 18{,}500 \frac{\text{rev}}{\text{min}} \qquad \textit{Ans.}$$

- &RPPHQWV 7KLVLVDKJKUDWHFDXVHWKHVSHHGRI VRXQGRI K\GURJHQ DQJKWDV LVQHDU IRXUWLPHVJUHDWHUWKDQWKDWRI DLU$QLPSHOOHUPRYLQJ DWWKLVVSHHGLQDLUZRXOGFUHDWHVKRFN ZDYHV

7KH' LIIHUHQWLDO(TXDWLRQ RI / LQHDU0 RPHQWXP

7KLVVHFWLRQXVHVDQHOHPHQWDOYROXPHWRGHULYH1HZWRQ¶V/DZ IRUDPRYLQJ IOXLG $QDOWHUQDWLYH DSSURDFK ZKLFKWKHUHDGHUPLJKWSXUVXH ZRXOGEHDIRUPDOPHWKRGRQHOHPHQWDOPRYLQJ SDUWLFOH +DYLQJGRQHLWRQFHLQ6HF IRUPDWVFRQVHUYDWLRQ ZHFDQPRYHDORQJ DQDXWRPDWLFVWHS: H XVHWKHVDPHHOHPHQWDOFRQWURO YROXPHDVLQ) LJ IRUZKLFKWKHDSSURSULDWHIRUP RI WKHOQHDU PRPHQWXP UHODWLRQLV

$$\sum \mathbf{F} = \frac{\partial}{\partial t}\left(\int_{CV} \mathbf{V}\rho\, d\mathcal{V}\right) + \sum (\dot{m}_i \mathbf{V}_i)_{out} - \sum (\dot{m}_i \mathbf{V}_i)_{in} \qquad (3.40)$$

$JDLQWKHOHPHQWVVRVPDOOWKDWWKHYROXPHLQWHJUDOLPSO UHGXFHVWRDGHULYDWLYHWLPH

$$\frac{\partial}{\partial t}(\mathbf{V}\rho\, d\mathcal{V}) \approx \frac{\partial}{\partial t}(\rho \mathbf{V})\, dx\, dy\, dz \qquad (4.19)$$

7KHPRPHQWXP IOX[HVRFFXURQDOOIDFHV WKUHHLQWHJUDQGWKUHHRXWWR 5HHUULQJ DJDLQWR) LJ ZHFDQIRUP DWDEOHRI PRPHQWXP IOX[HVE\ H[DPLQDWLRQ\ ZLWKWKHGLVFXVVLRQWKDWOHGXS WRWK

HTXDWRQIRUQHWPDVIOX[

Faces	Inlet momentum flux	Outlet momentum flux
x	$\rho u\mathbf{V}\,dy\,dz$	$\left[\rho u\mathbf{V} + \frac{\partial}{\partial x}(\rho u\mathbf{V})\,dx\right] dy\,dz$
y	$\rho v\mathbf{V}\,dx\,dz$	$\left[\rho v\mathbf{V} + \frac{\partial}{\partial y}(\rho v\mathbf{V})\,dy\right] dx\,dz$
z	$\rho w\mathbf{V}\,dx\,dy$	$\left[\rho w\mathbf{V} + \frac{\partial}{\partial z}(\rho w\mathbf{V})\,dz\right] dx\,dy$

,QWRGXFHWKHVHWHUPVDQG(T LQWR(T DQGJHWWKLVLQWRWUHGLDWUHXOV

$$\sum \mathbf{F} = dx\,dy\,dz\left[\frac{\partial}{\partial t}(\rho\mathbf{V}) + \frac{\partial}{\partial x}(\rho u\mathbf{V}) + \frac{\partial}{\partial y}(\rho v\mathbf{V}) + \frac{\partial}{\partial z}(\rho w\mathbf{V})\right] \qquad (4.20)$$

1 RWHWKDWWKLVLVDYHFWRUUHODWLRQ $ VLPSOLILFDWLRQRFFXUVLI ZHVSOLWKS WKHWUP LQEUDFNHWVD IRGOBRZ V

$$\begin{aligned}&\frac{\partial}{\partial t}(\rho\mathbf{V}) + \frac{\partial}{\partial x}(\rho u\mathbf{V}) + \frac{\partial}{\partial y}(\rho v\mathbf{V}) + \frac{\partial}{\partial z}(\rho w\mathbf{V})\\ &\quad = \mathbf{V}\left[\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{V})\right] + \rho\left(\frac{\partial\mathbf{V}}{\partial t} + u\frac{\partial\mathbf{V}}{\partial x} + v\frac{\partial\mathbf{V}}{\partial y} + w\frac{\partial\mathbf{V}}{\partial z}\right)\end{aligned} \qquad (4.21)$$

7KHWUP LQEUDFNHWRQWKHULJKWKDQGVLGHLVVHQWREHWKHHTXDLWRQRI FRQWLQXLW(T ZKLFKYDQLVKHVLGHQWLFDOO 7KHORQJWUP LQSDUHQWKHVHVRQWKHULJKWKDQGVLGHLVVHQIURP (T WREHWKHWRWDOFFHOHUDWLRQRI DSDUWFOHWKDWLQXDQGQRXV RFFXSLHVWKHFRQWROYROXPH

$$\frac{\partial\mathbf{V}}{\partial t} + u\frac{\partial\mathbf{V}}{\partial x} + v\frac{\partial\mathbf{V}}{\partial y} + w\frac{\partial\mathbf{V}}{\partial z} = \frac{d\mathbf{V}}{dt} \qquad (4.2)$$

7KXVZHKDYHQRZ UHGXFHG(T WR

$$\sum \mathbf{F} = \rho\frac{d\mathbf{V}}{dt}\,dx\,dy\,dz \qquad (4.22)$$

,WPLJKWEHJRRGIRU\RXWRWRSDQGUHWQRZ DQGWKLQNDERXWZKDWZHKDYHMXWGRQH : KDWLVWK UHODWLRQEHWZHHQ(TV DQG IRUDQLQLQWLPDOFRQWROYROXPH' &RXOGZHKDYHEHJXQ WKHDQDOLVDV(T "

(TXDWLRQ SRLQWVRXWKDWWKHQHWRUFHRQWKHFRQWROYROXPHPXWEHRI GLIIHUHQWDOV[I DQG SURSRUWLRQDOWR WKHHOHPHQWYROXPH 7KHVHIRUFHVDUHRI VZR WSHVERG IRUFHV DQGVXUIDF IRUFHV %RG IRUFHVDUHGXHWR[WUQDOILHOGV JUDYLW PDJQHWLP HOHFWULF SRWHQWLDO WKDWDFWRQWKH HQWLUHPDWZLWKLQWKHHOHPHQW7KHRQO ERG IRUFHZHVKDOOFRQVLGHULQWKLV ERRN LV JUDYLW 7K JUDYLWIRUFHRQWKHGLIIHUHQWLDOPDW! G[G\ G] ZLWKLQWKHFRQWROYROXPHLV

$$d\mathbf{F}_{\text{grav}} = \rho\mathbf{g}\,dx\,dy\,dz \qquad (4.23)$$

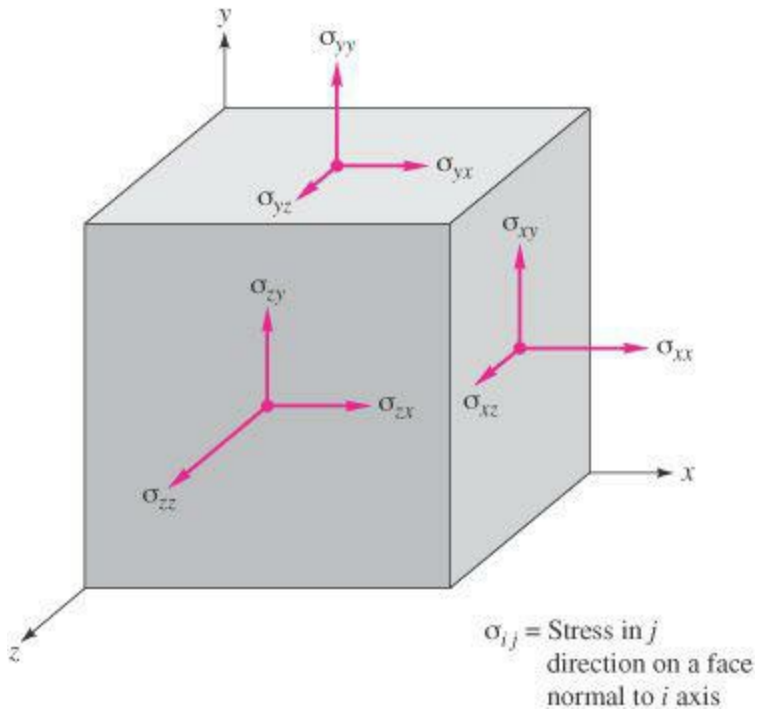
ZKHUHJ PD\ LQJHQHUDOKDYH DQDUELWUDU RUHQWDWRQ ZLWK UHVSHFWWR WKHFRRUGLQDWHV\VWP ,(PDQ DSSOLFDWLRQV VXFK DV%HUQRXOOL[VHTXDWLRQ ZHWDNH] ³XS´ DQGJ íJN

7KHVXUIDFHIRUFHVDUHGXHWRWKHVWUHVVRQWKHVLGHVRI WKHFRQWROYXUFH 7KHVHVWUHVVHVDU WKH VXP RI K\GURVWDWF SUHVXUH SOXV YLVFRXV VWUHVVHV2tM WKDWDULVH IURP PRWRQ ZLWK YHORFLW

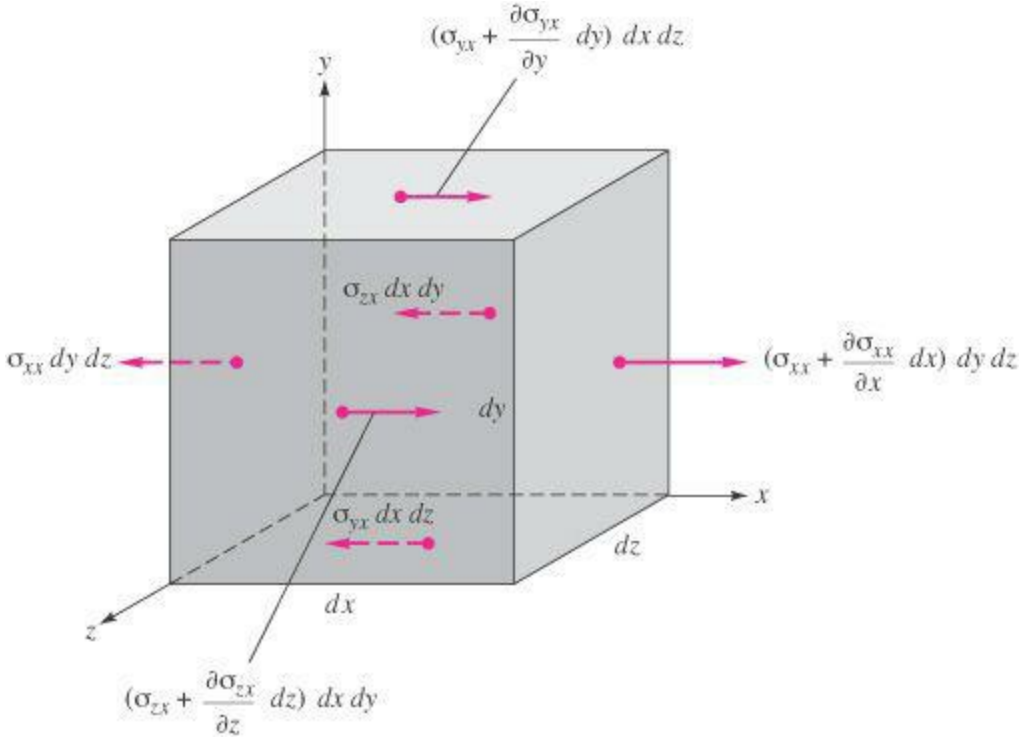
JUDGLHQWV

$$\sigma_{ij} = \begin{vmatrix} -p + \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & -p + \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & -p + \tau_{zz} \end{vmatrix} \qquad (4.24)$$

7KH VXEVFULSWQRWDWLRQ IRU VWUHVVHV LV JLYHQ LQ) LJ 8QLNH YHORFLW9 ZKLFK LV D VKHFRP SRQHQWYHFWRU VWUHVV1LM DQG2LM DQG VWUDLQUDWH'LJ DUHQLQH FRP SRQHQWWQVRUV DQG UHTXLUHVZR VXEVFULSWVWR GHILQHDFK FRP SRQHQW) RU IXUWKHUVWXG RI WQVRUDQDOVLV VHH5HV RU



) LJ 1RVDWLRQ IRU VWUHVVHV



) LJ (OHP HQWDO FDUWHVLDQ IL[HG FRQWURO YROXPH VKRZLQJ WKH VXUIDFH IRUFHV LQ WKH[GLUHFWLRQ RQO

,WV QRWKHVHVXUIDFHVEXWKHLUJUDGLHQWV RUGLIIHUHQFHV WKDWFDXVHDQHWRUFHRQWKHGLIIHUHQWD FRQWURO VXUIDFH 7KLV LV VHHQ E\ UHHUULQJ WR) LJ ZKLFK VKRZV RQO WKH[GLUHFWLRG VWUHVVHV W DYROLG FOXWWHULQJ XS WKHGUDZLQJ) RUH[DPSOH WKHOHIWZDUGIRUFH1[[G\ G] RQWKHOHWDFHLVEDODQFH E\ WKHULJKWZDUGIRUFH1[[G\ G] RQWKHULJKWDFH OHDYLQJ RQO WKHQHWULJKWZDUGIRUFH ~1[[~ G[G G] G[G\ G] RQWKHULJKWDFH 7KHVDPHWKLQJ KDSSHQV RQ WKHRWKHUIRXUFHV VR WKDWWKH QHWXUDF IRUFHLQWKH[GLUHFWLRQLVJLYHQE\

$$dF_{x,\text{surf}} = \left[\frac{\partial}{\partial x}(\sigma_{xx}) + \frac{\partial}{\partial y}(\sigma_{yx}) + \frac{\partial}{\partial z}(\sigma_{zx})\right] dx\, dy\, dz \qquad (4.25)$$

: HVHHWKDWWKLVIRUFHLVSURSRUWLRQDOWRWKHHOHPHQWYROXPH 1RWHWKDWWKHVWUHVVWUPVDUHVDNH IURP WKHWAS URZ RI WKHDUUD LQ(T 6SOLWWLQJ WKLVURZ LQWR SUHVXUHSOXVYLVFRXVWUHVV ZI FDQUHZ ULWH(T DV

$$\frac{dF_x}{d\mathcal{V}} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) \qquad (4.26)$$

ZKHUHG G[G\ G] ,QH[DFWO VLPLODUP DQQHU ZHFDQGHULYHWKH\ DQG] IRUFHVSHUXQLWYROXPI RQWKHFRQWURO VXUIDFH

$$\frac{dF_y}{d\mathcal{V}} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial y}(\tau_{yy}) + \frac{\partial}{\partial z}(\tau_{zy})$$
$$\frac{dF_z}{d\mathcal{V}} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + \frac{\partial}{\partial z}(\tau_{zz}) \qquad (4.27)$$

1RZ ZH PXOWLSO(TV DQG E\ L M DQGN UHVSHFWLYHO DQG DGG WR REWDLQ D H[SUHVVLRQIRUWKHQHWYHFWRUVXUIDFHIRUFH

$$\left(\frac{d\mathbf{F}}{d\mathcal{V}}\right)_{\text{surf}} = -\nabla p + \left(\frac{d\mathbf{F}}{d\mathcal{V}}\right)_{\text{viscous}} \qquad (4.28)$$

ZKHUHWKHYLVFRXVIRUFHKDVDWKUHHFRPSRQHQWV

$$\left(\frac{d\mathbf{F}}{d\mathcal{V}}\right)_{\text{viscous}} = \mathbf{i}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \mathbf{j}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \mathbf{k}\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) \qquad (4.29)$$

6LQFH HDFK WHUP LQSDUHQWKHVHVLQ UHSUHVHQWV WKHGLYHUJHQFHRI DVXLWDEOHFRPSRQHQWYHFWR DFWLQJ RQWKH[\ DQG] IDFHV UHSHFWLYHO(T LVVRPHWLPHVH[SUHVVHGLQGLYHUJHQFHIRUP

$$\left(\frac{d\mathbf{F}}{d\mathcal{V}}\right)_{\text{viscous}} = \nabla \cdot \boldsymbol{\tau}_{ij} \tag{4.30}$$

ZKHUH

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix} \tag{4.31}$$

LV WKH YLVFRXV VWUHVV WHQVRU DFWLQJ RQ WKH HOHPHQW 7KH VXUIDFH IRUFHV WKDW WKH SUHVXUH JUDGLHQW YHFWRU DQG WKH GLYHUJHQFH RI WKH YLVFRXV VWUHVV WHQVRU 6XEVWLWXWLQJ LQWR(T DQG XVLQJ LQ(T ZHKDYHWKHEDVLF GLIIHUHQWLDOPRPHQWXP HTXDWLRQIRUDQLQFRPSUHVVLEOHPHGLXP

$$\rho\mathbf{g} - \nabla p + \nabla \cdot \boldsymbol{\tau}_{ij} = \rho\frac{d\mathbf{V}}{dt} \tag{4.32}$$

ZKHUH

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \tag{4.33}$$

: HFDQDOVR H[SUHVV(T LQZRUGV

Gravity force per unit volume + pressure force per unit volume
+ viscous force per unit volume = density × acceleration (4.34)

(TXDWLRQ LV VR EUIH DQG FRPSDFWKDWLW LQKHUHQWFRP SOH[LW LV DOPRVWLQYLVLEOH ,WLV YHFWRUHTXDWLRQ HDFK RI ZKRVHFRPSRQHQWHTXDWLRQVFRQWDLQV / HWXVWKHUHRUHZULWHRX WKHFRPSRQHQWHTXDWLRQVLQIXOOWRLOOXVWUDWWKHPDWKHPDWDOGLIILFXOWLHVLQKHUHQWLQWKHPRPHQWF HTXDWLRQ

$$\begin{aligned}
\rho g_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} &= \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \\
\rho g_y - \frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} &= \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) \\
\rho g_z - \frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} &= \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)
\end{aligned} \tag{4.35}$$

7KLV LV WKH GLIIHUHQWLDOPRPHQWXP HTXDWLRQ LQ LWV IXOOJHRU DQG LWLV YDOLG IRUDQ IOXLG LQ DQ JHQHUDOPRWRQ SDWWHUQOLUIOXLGV EHLQJ FKDUDFWHUL]HG E\ SDWWHUQOLUYLVFRXV VWUHVV VWUHVV 1RWWKDWWKH OLWWKUHH 3FRQYHFWLYH WHUPV RQ WKH ULJKWKDQG VLGH RI HDFK FRPSRQHQWHTXDWLRQ LQ DU QRQOLQHDU ZKLFK FRPSOLFDWHV WKHJHQHUDOPDWKHPDWDOGLIILFXOWLHV

,QYLVFLG)ORZ (XOHUV(TXDWLRQ

(TXDWLRQ LV QRZ UHDG\ IRU XVH XQOLNH WKH YLVFRXV VWUHVVHV LQ WKLV FDVH WKHVH FRPSRQHQWV 7KHVH SDUWLFXODU VWUHVV FRPSRQHQWV DUH IXQFWLRQV RI 2 DQG IRU ZKLFK (T UHGXFHV WR

$$\rho \mathbf{g} - \nabla p = \rho \frac{d\mathbf{V}}{dt} \tag{4.36}$$

7KLV LV (XOHUfV HTXDWLRQ IRU LQYLVFLG IORZ : H VKRZ LQ6HF WKDW (XOHUfV HTXDWLRQ FDQ EH LQWHUSUHWHG DORQJ D VWUHDPOLQH VR \LHOGV WKH IULFWLRQOHVV %HUQRXOOL HTXDWLRQ RU 7K FRPSOHWH DQDO\VLV RI LQYLVFLG IORZ ILHOGV XVLQJ FRQWURO VROXWLRQ DQG WKH %HUQRXOOL UHDWLRQ LV JLYHQ L &KDS

1HZWRQLDQ)OXLG 1DYLHU6WRNHV (TXDWLRQV

)RU D QHZWRQLDQ IOXLG DV GLVFXVVHG LQ6HF WKH YLVFRXV VWUHVVHV DUH SURSRUWLRQDO WR WK HOHPHQWV WUDLQ UDWHV DQG WKH FRHILFLHQW RI YLVFRVLW\ )RU LQFRPSUHVVLEOH IORZ WKH JHQHUDO] DWLRQ R (T WR WKUHH GLPHQVLRQV YLVFRXV IORZ LV

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x} \qquad \tau_{yy} = 2\mu \frac{\partial v}{\partial y} \qquad \tau_{zz} = 2\mu \frac{\partial w}{\partial z}$$
$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \tag{4.37}$$
$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

ZKHUH LV WKH YLVFRVLW\ FRHILFLHQW6XEVWLWXWLRQ LQR(T JLYHV WKH GLIIHUHQWLDO PRPHQWXP HTXDWLRQ IRU D QHZWRQLDQ IOXLG ZLWK FRQVWDQW SURSHUWLHV DQG YLVFRVLW\

$$\begin{aligned}
\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) &= \rho \frac{du}{dt} \\
\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) &= \rho \frac{dv}{dt} \\
\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) &= \rho \frac{dw}{dt}
\end{aligned} \tag{4.38}$$

7KHVH DUH WKH LQFRPSUHVVLEOH IORZ 1DYLHU6WRNHV HTXDWLRQV QDPHG DIWU& / 0 + 1DYL ± DQG 6LU* HRUJH* 6WRNHV ± ZKR DUH FUHGLWHG ZLWK WKH LQGHULYDWLRQ 7KH DUH VHFRQG RUGHU RQOLQHDU SDUWLDO GLIIHUHQWLDO HTXDWLRQV DQG DUH TXLWH IRUPLGDEOH EHFDXVH WKHLU VROXWLRQV KDYH EHHQ IRXQG RQO D YDULHW\ RI LQWHUHVWLQJ YLVFRXV IORZ SUREOHPV VRPH RI ZKLFK DUH GLVFXVVHG LQ6HF DQG LQ&KDS VHH DOR 5HV DQG)RUFRPSUHVVLEOH IORZ VHH(T RI 5H (TXDWLRQV KDYH IRXU XQNQRZQV S X % DQG Z 7KH VKRXOG EH FRPELQHG ZLWK WK LQFRPSUHVVLEOH FRQWLQXLW UHDWLRQ >(TV @WR IRUP IRXU HTXDWLRQV LQ WKH IRXU XQNQRZQV : K VKDOO GLVFXVV WKLV DJDLQ LQ6HF ZKLFK SUHVHQW WKH DSSURSULDWH ERXQGDU FRQGLWLRQV IRU WKHVH HTXDWLRQV

(YHQ WKRXJK WKH 1DYLHU6WRNHV HTXDWLRQV KDYH RQO D OLPLWHG QXPEHU RI NQRZQ DQDO\WL VROXWLRQV WKH UHPHQDEHU WKDW ILQH JULGGHG FRPSXWHUPRGHOLQJ > @7KH ILHOG RI &)' LV PDWKLC

IDWWZ LWK PDQ FRPPHUFLDOVRIWZDUHVRROVDYDLODEOH ,WLVSRVVLEOHQRZ WRDFKLHYHDSSUR[LPDWH EX
UHDOWLPH &)' UHVXOWVIRUDZLGHYDULHWRI FRPSOH[WZR DQGWKUHHGLPHQVLRQDOYLVFRXVIORZV

(; $0 3/ (

7DNHWKHYHORFLWYILHOGRI ([DPSOH ZLWKE IRUDQJHEUDLF FRQYHQLHQFH

X D[\ D[\ Z

DQG GHWHUPLQH XQGHUZKDWFRQGLWLRQV LWLV D VROXWLRQ WR WKH 1DYLHU6WRNHV PRPHQWXP HTXDWLRQ
$VXPLQJ WKDWWKHVHFRQGLWLRQVDUHPHWGHWHUPLQHWKHUHVXOWLQJ SUHVVXUHGLVWULEXWLRQZKHQ] L
3XS J[J\ J] í J

6RXWLRQ

- $VXPSWLRQV &RQVLGHUWKHQVLWDQGYLVFRVLW VWDG IORZ XDQG%LQGHSHQGHQWWR WLPH
- $SSURDFK 6XEVWLWXWH WKH NQRZQ X % Z LQWR (TV DQG VROYH IRU WKH SUHVXU JUDGLHQW ,I DXQLTXHSUHVXUHIXQFWLRQS[\ ] FDQWKHQEHIRXQG WKHJLYHQVROXWLRQLVH[DFW
- 6RXWLRQVWHS 6XEVWLWXWH X % Z LQWR(TV LQVHTXHQFH

$$\rho(0) - \frac{\partial p}{\partial x} + \mu(2a - 2a + 0) = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = 2a^2\rho(x^3 + xy^2)$$
$$\rho(0) - \frac{\partial p}{\partial y} + \mu(0 + 0 + 0) = \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = 2a^2\rho(x^2y + y^3)$$
$$\rho(-g) - \frac{\partial p}{\partial z} + \mu(0 + 0 + 0) = \rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}\right) = 0$$

5HDUUDQJHDQGVROYHIRUWKHWKUHHSUHVVXUHJUDGLHQWV

$$\frac{\partial p}{\partial x} = -2a^2\rho(x^3 + xy^2) \qquad \frac{\partial p}{\partial y} = -2a^2\rho(x^2y + y^3) \qquad \frac{\partial p}{\partial z} = -\rho g \qquad (1)$$

- &RPPHQWV 7KHYHUWFDOSUHVVXUHJUDGLHQWLVK\GURVWDWF &RXOG\RXKDYHSUHGLFWHGWKLVE\ QRWLQJ LQ(TV WKDWZ " +RZ HYHU WKHSUHVXUHLVYHORFLWG HSHQGHQWWQWKH[\ SODQH
- 6RXWLRQVWHS 7RGHWHUPLQHL WKH[DQG\ JUDGLHQWVRI SUHVXUHLQ(T DUHFRPSDUHGWR HYDOXDWWKHPL[HGGHULYDWLYH %S %[Á\ WKDWV FURVVGLIIHUHQWDWWKHVHWZRHTXDWRQV

$$\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) = \frac{\partial}{\partial y}[-2a^2\rho(x^3 + xy^2)] = -4a^2\rho xy$$
$$\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial y}\right) = \frac{\partial}{\partial x}[-2a^2\rho(x^2y + y^3)] = -4a^2\rho xy$$

- &RPPHQWV 6LQFHWKHVHDUHHTXDOWKHJLYHQYHORFLWGLVWULEXWLRQVLQGHHGDQH[DFWVROXWLR RI WKH1DYLHU6WRNHVHTXDWRQV

- 6RONWRQ VWHS 7R ILQG WKH SUHVVXUH LQWHJUDWH(TV FRQVWDQG FRPSDUH 6WDWH WKH %&%p 7KH SURFHGXUH UHTXLUHV FDUH ,QWHJUDWHSDUWLDOO ZLWK UHVSHFWWR[KROGLQJ \ DQG] FRQVWDQW

$$p = \int \frac{\partial p}{\partial x}\,dx|_{y,z} = \int -2a^2\rho(x^3 + xy^2)\,dx|_{y,z} = -2a^2\rho\left(\frac{x^4}{4} + \frac{x^2y^2}{2}\right) + f_1(y,z) \qquad (2)$$

1RWH WKDWWKH ³FRQVWDQW´ RI LQWHJUDWLRQ LV DIXQFWLRQ RI WKH YDULDEOHV WKDWZHUH QR LQWHJUDWHG 1RZ GLIIHUHQWLDWH(TV ZLWKUHVSHFWWR\ DQGFRPSDUHZLWKÁSÁ\ IURP(TV

$$\frac{\partial p}{\partial y}|_{(2)} = -2a^2\rho\,x^2y + \frac{\partial f_1}{\partial y} = \frac{\partial p}{\partial y}|_{(1)} = -2a^2\rho(x^2y + y^3)$$

$$\text{Compare: } \frac{\partial f_1}{\partial y} = -2a^2\rho\,y^3 \quad \text{or} \quad f_1 = \int \frac{\partial f_1}{\partial y}\,dy|_z = -2a^2\rho\,\frac{y^4}{4} + f_2(z)$$

$$\text{Collect terms: So far } \quad p = -2a^2\rho\left(\frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{y^4}{4}\right) + f_2(z) \qquad (3)$$

7KLVWPHWKH³FRQVWDQW´RI LQWHJUDWLRQ LVDIXQFWLRQRI] RQO WKHYDULDEOHRWLQWHJUDWHG 1RZ GLIIHUHQWLDWH(TV ZLWKUHVSHFWWR] DQGFRPSDUHZLWK%&%p IURP(TV

$$\frac{\partial p}{\partial z}|_{(3)} = \frac{df_2}{dz} = \frac{\partial p}{\partial z}|_{(1)} = -\rho g \qquad \text{or} \qquad f_2 = -\rho g z + C \qquad (4)$$

ZKHUH& LV D FRQVWDQW7KLV FRPSOHWHV RXUWKUHH LQWHJUDWLRQV &RPELQH(TV DQG WK REWDLQWKH IXOO[SUHVVLRQ IRUWKHSUHVVXUHGLVWULEXWLRQLQWKLV IORZ

$$p(x,y,z) = -\rho g z - \tfrac{1}{2}a^2\rho(x^4 + y^4 + 2x^2y^2) + C \qquad Ans.\ (5)$$

7KLV LV WKH GHVLUHG VROXWLRQ ' R\RX UHFRJQL]H LW1RWKQDW\RX JR EDFNWR WKHEHJLQQLQ DQGVTXDUHWKHYHORFLWFRPSRQHQW

$$u^2 + v^2 + w^2 = V^2 = a^2(x^4 + y^4 + 2x^2y^2) \qquad (6)$$

&RPSDULQJ ZLWK(TV ZHFDQUHZULWWKHSUHVVXUHGLVWULEXWLRQDV

$$p + \tfrac{1}{2}\rho V^2 + pgz = C \qquad (7)$$

- &RPPHQW 7KLV LV %HUQRXOOL¶V HTXDWLRQ 7KDWLV QR DFFLGHQWEHFDXVHWKH YHORFLW GLVWULEXWLRQJLYHQLQWKLV SUREOHP LVRQHRI DIDPLO RI IORZ V WKDWDUH VROXWLRQV WR WKH 1DYLH 6WRNHV HTXDWLRQV DQG WKDWWDLW\ %HUQRXOOL¶V LQFRPSUHVVLEOH HTXDWLRQ HYHU\ZKHUH LQ WKH IORZ ILHOG 7KH\ DUH FDOOHG LUURWDWLRQDOIORZV IRU ZKLFK FXUO9 î 9 7KLV VXEMHFWL GLVFXVVHGDJDLQLQ6HF

7KH' LIIHUHQWLDO(TXDWLRQ RI $QJXODU0 RPHQWXP

+DYLQJ QRZ EH+Q WKURXJK WKH VDPH DSSURDFK IRU ERWK PDW DQG QQ+DUPRQLF ZH FDQ JI UDSLGO WKURXJK D GHULYDWLRQ RI WKH GLIIHUHQWLDO DQJXODU PRPHQWXP UHODWLRQ 7KH DSSURSULDWH IRUP F WKH LQWHJUDO DQJXODU PRPHQWXP HTXDWLRQ IRU D IL[HG FRQWURO YROXPH LV

$$\sum \mathbf{M}_o = \frac{\partial}{\partial t}\left[\int_{CV} (\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V}\right] + \int_{CS} (\mathbf{r} \times \mathbf{V})\rho(\mathbf{V} \cdot \mathbf{n}) \, dA \qquad (3.59)$$

: H VKDOO FRQVLGHU RXUVHOYHV WR DQ D[LV WKURXJK 2 WKDW LV SDUDOOHO WR WKH] D[LV DQG SDVV WKURXJI WKH FHQWURLG RI WKH HOHPHQWDO FRQWROYROXPH 7KLV LV VKRZ QLQ) LJ / HW EH WKH DQJOH RI URWDWR DERXW 2 RI WKH IOXLG Z LWKLQ WKH FRQWROYROXPH 7KH RQO VWUHVVHV WKDW KDYH PRPHQWV DERXW2 DUH WK VKHDU VWUHVVHV2\ DQG2[: H FDQ HYDOXDWH WKH PRPHQWV DERXW2 DQG WKH DQJXODU PRPHQWXP WHUP DERXW2 $ GRWR DQJHEUDLVLQYROYHG DQG Z HJLYH K+UHRQ WKHUHVXOW

$$\left[\tau_{xy} - \tau_{yx} + \frac{1}{2}\frac{\partial}{\partial x}(\tau_{xy})\,dx - \frac{1}{2}\frac{\partial}{\partial y}(\tau_{yx})\,dy\right] dx\,dy\,dz = \frac{1}{12}\rho(dx\,dy\,dz)(dx^2 + dy^2)\frac{d^2\theta}{dt^2} \qquad (4.39)$$

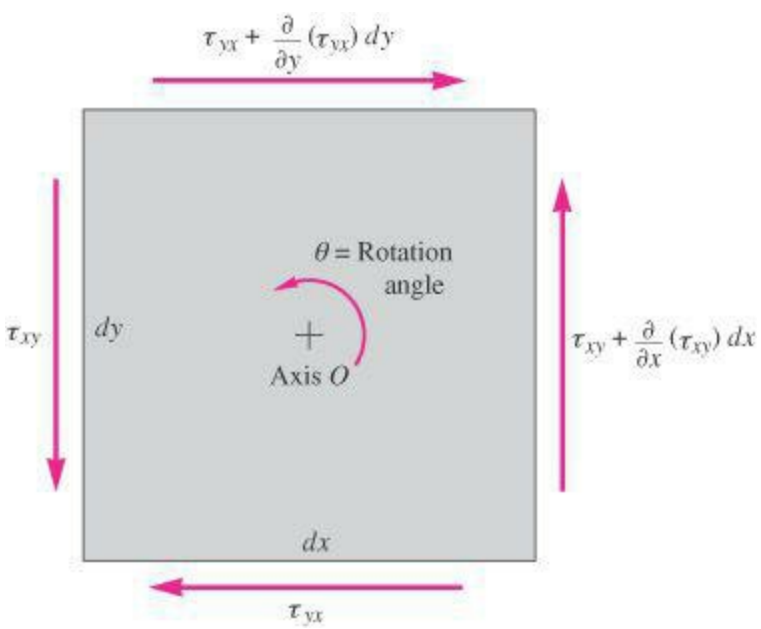
$ VXPLQJ WKDW WKH DQJXODU DFFHOHUDWLRQ G GW LV QRW LQILQLWH Z H FDQ QHJOHFW DOO KLJK+UGH GLIIHUHQWLDO WHUPV Z KLFK OHDYHV DILQLWH DQG LQWHUPHGLDWH UHVXOW

$$\tau_{xy} \approx \tau_{yx} \qquad (4.40)$$

+DG Z H VXPPHG PRPHQWV DERXW D[HV SDUDOOHO WR \ RU[Z H Z RXOG KDYH REWDLQHG H[DFW DQDORJRXV UHVXOWV

$$\tau_{xz} \approx \tau_{zx} \qquad \tau_{yz} \approx \tau_{zy} \qquad (4.41)$$

7KHUH LVQR GLIIHUHQWLDO DQJXODU PRPHQWXP HTXDWLRQ $SSOLFDWLRQ RI WKH LQWHJUDO WKHRUHP WR GLIIHUHQWLDO HOHPHQWJLYHV WKH UHVXOW Z HOONQRZ QWR VXSHUSRVH RI VXUIDQDO VLVRUVXUIQJWKRI PDWULDO WKDWWKH VKHDU VWUHVVHV DUH V\PPHWULF 2LM 2MI 7KLV LV WKH RQO UHVXOWRI WKLV VHFWLRQ 7KHUH LV QI GLIIHUHQWLDOHTXDWLRQ WR UHPHPEHU Z KLFK OHDYHV URRP LQ\RXUEUDLQ IRUWKHQH[W WRSLF WKH GLIIHUHQWLDO HQHUJ\ HTXDWLRQ



) LJ (OHP HQWDOFDUWHVLDQ IL[HG FRQWUROYROXP H VKRZLQJ VKHDU VWUHVVHV WKDWP D\ FDXVH I QHWDQJXODUDFFHOHUDWLRQ DERXWD[LV*

7KH' LIIHUHQWLDO(TXDWLRQ RI (QHUJ\

: HDUHQRZ VR XVHG WR WKLV WSH RI GHULYDWLRQ WKDWZ HFDQ UDFHWKURXJK WKH HQHUJ\ HTXDWLRQ DW EHZ LOOHULQJ SDFH 7KHDSSURSULDWHJUDQGUHDWRQ IRUWKHIL[HGFRQWROYROXP HR) LJ LV

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left(\int_{CV} e\rho\, d\mathcal{V}\right) + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho(\mathbf{V}\cdot\mathbf{n})\, dA \qquad (3.66)$$

ZKHUHy V EHFDXVHWKHUHFDQEHQR LQLQWHUDFWLRQ SURWKGLQJ LQWR WKHFRQWROYROXP H %0 DQDORJ\ ZLWK(T WKHULJKWKDQGVLGHEHFRP HV IRUWKLVWQ HOHP HQW

$$\dot{Q} - \dot{W}_v = \left[\frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x}(\rho u\zeta) + \frac{\partial}{\partial y}(\rho v\zeta) + \frac{\partial}{\partial z}(\rho w\zeta)\right] dx\, dy\, dz$$

ZKHUH H S! : KHQZ HXVHWKHFRQWQXLWHTXDWRQE\ DQDORJ\ ZLWK(T WKLVEHFRP HV

$$\dot{Q} - \dot{W}_v = \left(\rho\frac{de}{dt} + \mathbf{V}\cdot\boldsymbol{\nabla}p + p\boldsymbol{\nabla}\cdot\mathbf{V}\right) dx\, dy\, dz \qquad (4.42)$$

7KHUP DO&RQGXFWLYLW) RXUHU¶V/ DZ

7R HYDOXDWÌ ZHQHJHFWUDGLDWRQ DQG FRQLGHURQ) KHDWFRQGXFWRQ WKURXJK WKH VLGHV RI WK HOHP HQW([SHULP HQWVIRUERWKIOLGVDQGVROGV VKRZ WKDWWKHYHFWRUKHDWIOX[HUSHUXQLWDUHDq LV SURSRUWRQDOWR WKHYHFWRUJUDGLHQWRI WHPSHUDWXUH ² 7 7KLVSURSRUWRQDOLW) RXUHU¶V/DZ

RI FRQGXFWRQ ZKLFK LV DQDORJRXV WR 1HZWRQ¶V YLVFRVLW\ OD

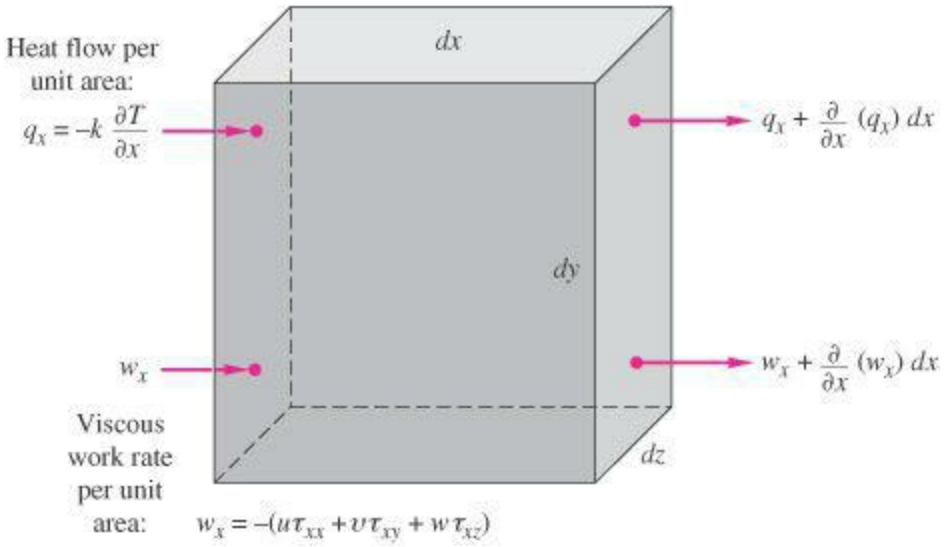
$$\mathbf{q} = -k\nabla T$$

$$\text{or:} \quad q_x = -k\frac{\partial T}{\partial x}, \quad q_y = -k\frac{\partial T}{\partial y}, \quad q_z = -k\frac{\partial T}{\partial z} \qquad (4.43)$$

ZKHUH N LV FDOOHG WKH WKHUPDO FRQGXFWLYLW\ DIXLG SURSHUW\ WKDW YDULHV ZLGH SUHVXUH LQPXFK WKH VDPH ZD\ DV YLVFRVLW\ 7KHPLQXVVLJQVDQGWLHVWKHFRQYHQWLRQWKDWKHDWIOX[LV SRVLWLYHLQWKHGLUHFWLRQRI GHFUHDVLQJ WHPSHUDWXUH) RXULHU¶V OD LVGLPHQVLRQDOO FRQVLVWHQWDQGI KDV6, XQLWVRI NRXOHVSHU VHF PHWUHKHLQ DQGFDQEHFRUUHODWHG ZLWK7 LQPXFK WKH VDPH ZD\ D (TV DQG IRUJDVHVDQGOLTXLGV UHVSHFWLYHO

) LJXUH VKRZVWKHKHDWORZ SDWWLQJ WKURXJKWKH[IDFHV WKH\ DQG] KHDWORZ VEHLQJ RPLWWH IRUFODULW\ : HFDQQDWWKHVHVL[KHDWOX[WUPV

Faces	Inlet heat flux	Outlet heat flux
x	$q_x\,dy\,dz$	$\left[q_x + \frac{\partial}{\partial x}(q_x)\,dx\right] dy\,dz$
y	$q_y\,dx\,dz$	$\left[q_y + \frac{\partial}{\partial y}(q_y)\,dy\right] dx\,dz$
z	$q_z\,dx\,dy$	$\left[q_z + \frac{\partial}{\partial z}(q_z)\,dz\right] dx\,dy$



) LJ (HPHQWDU\ PDQFDUWHVLDQ FRQWUROYROXPH VKRZLQJ KHDWORZ DQG YLVFRXV ZRUN UDWHWUP LQWKHx GLUHFWLRQ

%\ DGGLQJ WKHLQIOHWWHUPV DQG VXEWUDFWLQJ WKHRXWOHWWHUPV ZHREWDLQ WKHQHWKHDWDGGHG WR WK HOHPHQW

$$\dot{Q} = -\left[\frac{\partial}{\partial x}(q_x) + \frac{\partial}{\partial y}(q_y) + \frac{\partial}{\partial z}(q_z)\right] dx\,dy\,dz = -\nabla \cdot \mathbf{q}\,dx\,dy\,dz \qquad (4.44)$$

$ VH[SHFWHG WKHKHDWOX[LV SURSRUWLRQDOWR WKHHOHPHQWYROXPH ,QWURGXFLQJ) RXULHU¶V OD IURP (T ZHKDYH

$$\dot{Q} = \nabla \cdot (k \nabla T)\, dx\, dy\, dz \qquad (4.45)$$

7KH UDWH RI ZRUN GRQH E\ YLVFRXV VWUHVVHV HTXDOV WKH SURGXFW RI WKH VWUHVV FRPSRQHQW LQ FRUUHVSRQGLQJ YHORFLW\ FRPSRQHQW DQG WKH DUHD RI WKH HOHPHQW)LJXUH VKRZV WKH ZRUN UDWH RQ WKH OHIW[IDFH LV

$$\dot{W}_{v,LF} = w_x\, dy\, dz \qquad \text{where } w_x = -(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) \qquad (4.46)$$

ZKHUH WKH VXEVFULSW) VWDQGV IRU OHIW DQG D VOLJKWO GLIIHUHQW ZRUN RQ WKH ULJKW IDFH GXH WR WKH JUDGLHQW LQ ZRUN UDWH DFURVV WKH HOHPHQW 7KHVH ZRUN LQSXWV FRQWULEXWH LQGLYLGXDOO WKH VPHPEHUV WKH KHDW[LQ WKH SUHVVXUH WHUP ZLWK T[UHSODFLQJ T[DQG VR RQ $IWHU RXU WHWUDKHGUDO VXEGLYLVLRQ LQWR WHWUDKHGUD WKH QHWYLVFRXV ZRUN UDWH EHFRPHV

$$\begin{aligned}\dot{W}_v = -\Big[&\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{yx} + v\tau_{yy} + w\tau_{yz}) \\ &+ \frac{\partial}{\partial z}(u\tau_{zx} + v\tau_{zy} + w\tau_{zz})\Big]dx\, dy\, dz \\ = -\nabla \cdot (\mathbf{V} \cdot \boldsymbol{\tau}_{ij})\, &dx\, dy\, dz \end{aligned} \qquad (4.47)$$

: H QRZ VXEVWLWXWH(TV DQG LQWR(T VR REWDLQ RQH IRUP RI WKH GLIIHUHQWLDO HQHUJ\ HTXDWLRQ

$$\begin{aligned}&\rho\frac{de}{dt} + \mathbf{V} \cdot \nabla p + p\nabla \cdot \mathbf{V} = \nabla \cdot (k\,\nabla T) + \nabla \cdot (\mathbf{V} \cdot \boldsymbol{\tau}_{ij}) \\ &\text{where } e = \hat{u} + \tfrac{1}{2}V^2 + gz\end{aligned} \qquad (4.48)$$

$ PRUH XVHIXO IRUP LV REWDLQHG LI ZH VSOLW XS WKH YLVFRXV ZRUN WHUP

$$\nabla \cdot (\mathbf{V} \cdot \boldsymbol{\tau}_{ij}) \equiv \mathbf{V} \cdot (\nabla \cdot \boldsymbol{\tau}_{ij}) + \Phi \qquad (4.49)$$

ZKHUH) LV VKRUWIRU WKH YLVFRXV GLVVLSDWLRQ IXQFWLRQ) RU D QHZWRQLDQ LQFRPSUHVVLEOH YLVFRXV IOXLG WKLV IXQFWLRQ KDV WKH IRUP

$$\begin{aligned}\Phi = \mu\Big[&2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 \\ &+ \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\Big]\end{aligned} \qquad (4.50)$$

6LQFH DOO WHUPV DUH TXDGUDWLF YLVFRXV GLVVLSDWLRQ LV DOZD\V SRVLWLYH VR WKDW D YLVFRXV IORZ DOZD\V VWRGV VR RPH LWV DYDLODEOH HQHUJ\ GXH WR GLVVLSDWLRQ LQ DFFRUGDQFH ZLWK WKH VHFRQG ODZ R WKHUPRG\QDPLFV 1RZ VXEVWLWXWH(T LQWR(T XVLQJ WKH OLQHDU PRPHQWXP HTXDWLRQ WR HOLPLQDWH Á W, 7KLV ZLOO FDXVH WKH NLQHWF DQG SRWHQWLDO HQHUJLHV VR FDQFHO OHDYLQJ D PRU FXUWDLOHG IRUP RI WKH JHQHUDOLIHUHQWLDOHQHUJ\ HTXDWLRQ

$$\rho \frac{d\hat{u}}{dt} + p(\nabla \cdot \mathbf{V}) = \nabla \cdot (k \nabla T) + \Phi \qquad (4.51)$$

7KLV HTXDWLRQ LV YDOLG IRU D QHZWRQLDQ IOXLG XQGHU YHU JHQHUDO FRQGLWLRQV RI XQVWHDG FRPSUHVVLEOH YLVFRXV KHDWFRQGXFWLQJ IORZ H[FHSWWKDWLWQHJOHFWV UDGLDWLRQ KHDWWDQVIHU DQ LQWHUQDOVRXUFHVRI KHDWWKDWPLJKWRFFXUGXULQJ DFKHPLFDORUQXFOHDUUHDFWLRQ

(TXDWLRQ LV WRR GLIILFXOWWR DQDO]H H[FHSWRQ D GLJLWDOFRPSXWHU> @,WLV FXVWRPDU\ WR PDNHWKHIROORZLQJ DSSUR[LPDWLRQV

$$d\hat{u} \approx c_v\, dT \quad c_v, \mu, k, \rho \approx \text{const} \qquad (4.52)$$

(TXDWLRQ VKHQVDNHVWKHVLPSOHUIRUP IRU Á9

$$\rho c_v \frac{dT}{dt} = k\nabla^2 T + \Phi \qquad (4.53)$$

ZKLFK LQYROYHV WKH SHUPDXUH7 DVWKHVROHSULP DU\ YDULDEOHSOXV YHORFLW\DVD VHFRQGDU\ YDULDEOH WKURXJKWKHWRWDOGHULYDWLYHRSHUDWRU

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \qquad (4.54)$$

$ JUHDWPDQ LQWHUHVWLQJ VROXWLRQV WR(T DUH NQRZQ IRU YDULRXV IORZ FRQGLWLRQV DQ H[WHQGHGWHDWPHQWDUH JLYHQLQDGYDQFHGERRNVRQYLVFRXVIORZ > @DQGERRNVRQKHDWWUDQVIHU> @

2QH ZHOONQRZQ VSHFLDOFDVH RI(T FFFXUV ZKHQ WKH IOXLG LV DWUHVWDQGKDV QHJOLJLEOH YHORFLW\ ZKHUHWKHGLVWULSDWLRQ) DQGFRQYHFWLYHWUDQVIHUPVEHFRPHQHJOLJLEOH

$$\rho c_p \frac{\partial T}{\partial t} = k\nabla^2 T \qquad (4.55)$$

7KH FKDQJH IURP F% VWF S LV FRUUHFWDQG MXVWLILHG E\ WKH IDFWKDWZKHQ SUHVVXUH WUDQV DU QHJOHFWHG IURP DJDVIORZ HQHUJ\ HTXDWLRQ> @ZKDWHPDLQVLVSSUR[LPDWH DQHQWKDOS\ FKDQJH QRWDQ LQWHUQDOHQHUJ\ FKDQJH 7KLV LV FDOOHG WKHKHDWFRQGXFWLQ HTXDWLRQ LQ DSSOLHG PDWKHPDWLF DQG LV YDOLG IRUVROGV DQG IOXLGV DWUHVW7KHVROXWLRQ WR(T IRUYDULRXV FRQGLWLRQV LV DOOJI SDUWV FRXUVHV DQGERRNVRQKHDWWUDQVIHU

7KLVFRPSOHWHV WKHGHULYDWLRQ RI WKHEDVLF GLIIHUHQWLDOHTXDWLRQV RI IOXLGPRWLRQ

%RXQGDU\ &RQGLWLRQV IRU WKH%DVLF (TXDWLRQV

7KHUHDUHWKUHHEDVLF GLIIHUHQWLDOHTXDWLRQV RI IOXLGPRWLRQ MXVWGHULYHG / HWXV VXPPDUL]HWKHP KHUH

$$\text{Continuity:} \qquad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \qquad (4.56)$$

Momentum: $$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla p + \nabla \cdot \boldsymbol{\tau}_{ij} \tag{4.57}$$

Energy: $$\rho \frac{d\hat{u}}{dt} + p(\nabla \cdot \mathbf{V}) = \nabla \cdot (k \nabla T) + \Phi \tag{4.58}$$

ZKHUH) LV JLYHQ E\ (T ,QJHQHUDO WKH GHQVLW LV YDULDEOH VR WKDW WKH VHW RI HTXDWLRQ FRQVLVWV RI 9 S € DQG7 7KHUHRUH ZH QHHG WR DGG LWRQDO UHDWLRQV WR FRPSOHWH WK VWHP RI HTXDWLRQV 7KHVH DUH SURYLGHG E\ GDWD RU DJHEUDLF H[SUHVVLRQV IRU WKH WKHUPRG\QDPLF SURSHUWLHV

$$\rho = \rho(p, T) \qquad \hat{u} = \hat{u}(p, T) \tag{4.59}$$

) RUH[DPSOH IRU D SHUHFWJDV ZLWK FRQVWDQW VSHFLILF KHDW ZH FRPSOHWH WKH VWHP ZLWK

$$\rho = \frac{p}{RT} \qquad \hat{u} = \int c_v \, dT \approx c_v T + \text{const} \tag{4.60}$$

,WLV VKRZQ LQ DGYDQFHG ERRNV> @WKDW WKLV VWHP RI HTXDWLRQV VR LV ZHOOSRVH DQG FDQ EH VROYHG DQDOWLFDOO RU QXPHULFDOO VXEMHFW WR WKH SURSHU ERXQGDU FRQGLWLRQV

: KDWDUH WKH SURSHU ERXQGDU FRQGLWLRQV") LUVWL WKH IORZ LV XQLWRUP\ WKHUHP XUWEHDQLQWD FRQGLWLRQ RU ULQWDOVSDWDOGLVWULEXWLRQ QRZ QIRUHDFK YDULDEOH

At $t = 0$: $$\rho, V, p, \hat{u}, T = \text{known } f(x, y, z) \tag{4.61}$$

7KHUHDWU IRUDOOWP HVWR EH DQDO]HG ZH PXVWNQRZ VRPHWKLQJ DERXWWKH YDULDEOHV DW+DF ERXQGDU HQFORVLQJ WKH IORZ

) LJXUH LOOXVWUDWHV WKH WKUHH PRWFRPPRQ WSHV RI ERXQGDULHV HQFRXQWHG LQ IOXLG IORZ DQDOVLV D VROG ZDOO DQLQDHWRURXWWDQGDOTXLG±JDV LQWHUIDFH

) LUWMRUD VROG LP SHUP HDEOHZDOO WKHUHLV QR VOS DQG QR WP SHUDWXUHP XP S LQD YLVFRXV KHDW FRQGXFWLQJ IOXLG

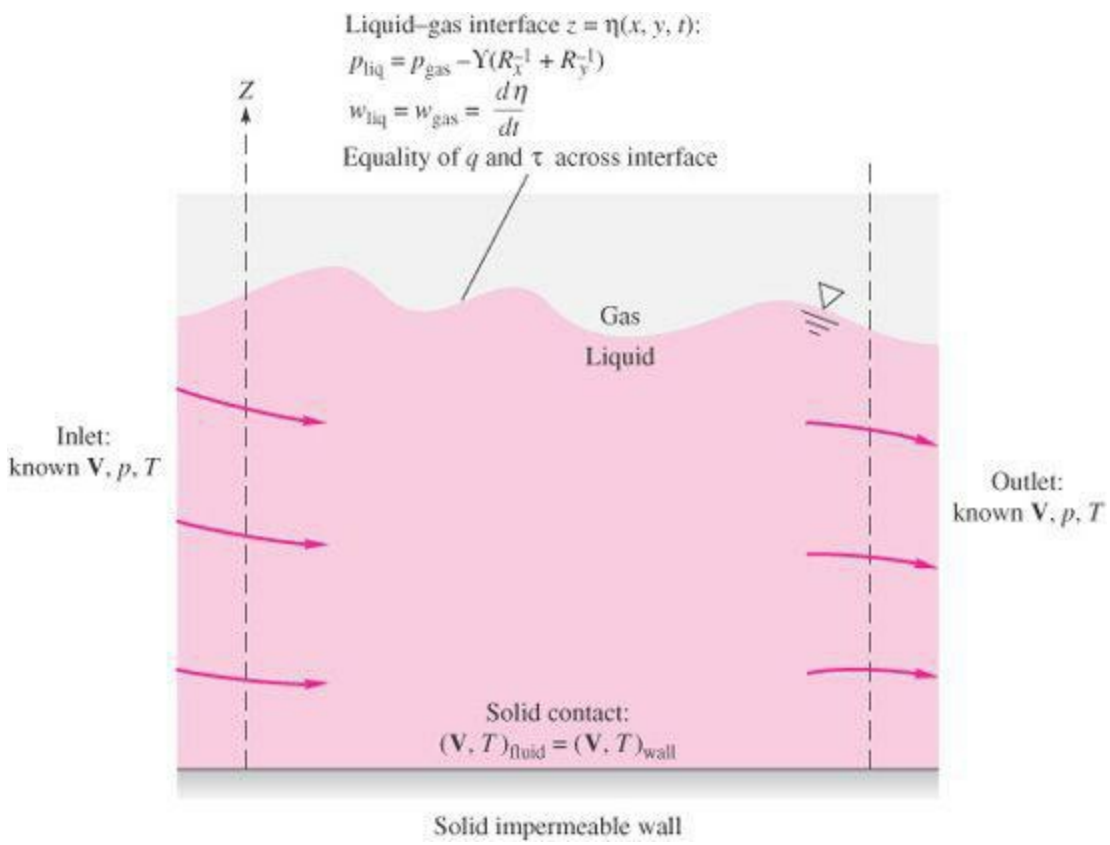
Solid wall: $$\mathbf{V}_{\text{fluid}} = \mathbf{V}_{\text{wall}} \qquad T_{\text{fluid}} = T_{\text{wall}} \tag{4.62}$$

7KHRQG H[FHSWRQV(T RFFXUV LQ DQH[WHP UDUHLHG JDV IORZ ZKHUH VOSSDJHFDQEH SUHVHQW> @

6HFRQG DWDQ LQDHWRURXWWWFWRQ RI WKH IORZ WKH FRP SOHWHG LVWULEXWRQ RI YHORFLW SUHVXUH DQG WP SHUDWXUHP XWEHNQRZ QIRUDOOWP HV

Inlet or outlet: $$\text{Known } \mathbf{V}, p, T \tag{4.63}$$

7KHVH LQDHWDQG RXWWWFWRQV FDQ EH DQG RIWQ DUH DW ' VLP XODWQJ D ERG LPP HUVHG LQ D LQILQLWH[SDQVHR IOXLG



) LJ 7\SLFDOERXQGDU\ FRQGLWLRQVLQDYLVFRXVKHDWFRQGXFWLQJ IOXLGIORZ DQDO\VLV

) LQDO\VHWKHPRVWFRP SOH[FRQGLWLRQVRFFXUDWDQDTXLG±JDVLQWHUIDFH RUIUHHVXUIDFH DVVHHWKH LQ) LJ / HWXVGHQRWHWKHLQWHUIDFHE\

Interface: $z = \eta(x, y, t)$ (4.64)

7KHQWKHUHPXVWEH HTXDOLW\ RI YHUWLFDO YHORFLW\ DFURVV WKH LQWHUIDFH \R WKDWQR KROHV DSSHD EHWZHHQDTXLGDQGJDV

$$w_{liq} = w_{gas} = \frac{d\eta}{dt} = \frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} \qquad (4.65)$$

7KLVLVFDOOHGWKHNLQHP DWF ERXQGDU\ FRQGLWLRQ

7KHUH PXVWEH P HFKDQLFDO HTXLOLEULXP DFURVV WKH LQWHUIDFH 7KH YLVFRXV VKHDU VWUHVVHV PX\ EDODQFH

$$(\tau_{zy})_{liq} = (\tau_{zy})_{gas} \qquad (\tau_{zx})_{liq} = (\tau_{zx})_{gas} \qquad (4.66)$$

1 HJOHFWLQJ WKH YLVFRXV QRUP DO VWUHVVHV WKH SUHVXUHV P XWEDODQFH DWWKH LQWHUIDFH H[FHSWIR VXUIDFHWHQVLRQ HIHFWV

$$p_{liq} = p_{gas} - \Upsilon(R_x^{-1} + R_y^{-1}) \qquad (4.67)$$

ZKLFK LV HTXLYDOHQWWR(T 7KH UDGLL RI FXUYDWXUH FDQ EH ZULWWQ LQ WHUP V RI WKH IUH VXUIDFHSRVLWLRQ

$$R_x^{-1} + R_y^{-1} = \frac{\partial}{\partial x}\left[\frac{\partial\eta/\partial x}{\sqrt{1 + (\partial\eta/\partial x)^2 + (\partial\eta/\partial y)^2}}\right] + \frac{\partial}{\partial y}\left[\frac{\partial\eta/\partial y}{\sqrt{1 + (\partial\eta/\partial x)^2 + (\partial\eta/\partial y)^2}}\right] \tag{4.68}$$

) LQDOO VWHS GHWHUPLQH WKH VROXWLRQ RI WKH LQWHUIDFH VLQFH QR KDWFDQ EH VWRUHG LQ WKH LQLWLDO GDWD WKLQLVWHS

$$\begin{aligned} &\text{At infinity:} && \psi = U_\infty y + \text{const} \\ &\text{At the body:} && \psi = \text{const} \end{aligned} \tag{4.90}$$

1 HJOHFWLQJ UDGLDWLRQ WKLV LV HTXLYDOHQWV

$$\left(k\frac{\partial T}{\partial z}\right)_{\text{liq}} = \left(k\frac{\partial T}{\partial z}\right)_{\text{gas}} \tag{4.70}$$

7KLV LV DV PXFK GHWDLO DV ZH ZLVK WR JLYH DWWKLV GHYHORI H[SRVLWLRQ) XUWKHUDQG HYHQ PRU FRPSOLFDWHG GHWDLOVRQIOXLGIORZ EHKDYLRUV DQGLWRQVDUHJLYHQLQ5HV DQG

6LP SOLILHG) UHH6XUIDFH&RQGLWLRQV

,QWKHLQWRGXFWRU DQDO VHVJLYHQLQWKLVERRN VXFK DVRSHQFKDQQHOIORZ VLQ&KDS ZHVKDO EDFN DZD IURP WKH H[DFWFRQGLWLRQV WR DQG DVXPH WKDWWKH XSSHU IOXLG LV D 3DWRVSKHUH WKDWP HUHO H[HUWV SUHVVXUH RQ WKH ORZHU IOXLG ZLWK VKHDU DQG KHDWFRQGXFWRU QHJOHFWLQJLEOH : HDOVR QHJOHFWQRQOLQHDUWUP VLQYROYLQJ WKH VORSHVRI WKHIUHHVXUIDFH : HWKHQKDYHD PXFK VLPSOHUDQGOLQHDUVHWRI FRQGLWLRQV DWWKH VXUIDFH

$$p_{\text{liq}} \approx p_{\text{gas}} - \Upsilon\left(\frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial y^2}\right) \qquad w_{\text{liq}} \approx \frac{\partial\eta}{\partial t}$$
$$\left(\frac{\partial V}{\partial z}\right)_{\text{liq}} \approx 0 \qquad \left(\frac{\partial T}{\partial z}\right)_{\text{liq}} \approx 0 \tag{4.71}$$

,QP DQ FDVHV VXFK DVRSHQFKDQQHOIORZ ZHFDQDOR QHJOHFWVXUIDFHWHQVLRQ VR WKDW

$$p_{\text{liq}} \approx p_{\text{atm}} \tag{4.72}$$

7KHVHDUHWKHWSHVRI DSSUR[LPDWRQVWKDWZLOOEHXVHGLQ&KDS 7KHQRQGLPHQVLRQDOIRUPVRI WKHVHFRQGLWLRQVZLOODOR EHXVHXOLQ&KDS

,QFRP SUHVVLEOH) ORZ Z LWK &RQVWDQW3 URSHUWLHV

) ORZ Z LWKFRQVWDQW DQGNLVDEDWLF VLP SOLILFDWRQWKDWZLOOEHXVHG IRUH[DPSOH WKURXJKRX &KDS 7KHEDVLF HTXDWRQVRI PRWRQ WR UHGXFHWR

$$\text{Continuity:} \qquad \nabla \cdot \mathbf{V} = 0 \tag{4.73}$$

$$\text{Momentum:} \qquad \rho\frac{d\mathbf{V}}{dt} = \rho\mathbf{g} - \nabla p + \mu\nabla^2\mathbf{V} \tag{4.74}$$

Energy: $$\rho c_p \frac{dT}{dt} = k\nabla^2 T + \Phi \qquad (4.75)$$

6LQFH! LV FRQVWDQW WKURXJKRXW WKH ÀRZ Q V S 9 DQG 7 7KH V\VWHP LV FRPSOHWH 1RWRQ WKDW WKH V\VWHP VSDQV DSDUW &RQVLGHU DQ HOHPHQW DUHD RI WKH SUHVVXUH DQG YHORFLW\ ÀHOGV IRU WKH SUHVVXUH DQG YHORFLW\ ÀHOGV DUH

Solid surface: $$\mathbf{V} = \mathbf{V}_{\text{wall}} \qquad (4.76)$$

Inlet or outlet: $$\text{Known } \mathbf{V}, p \qquad (4.77)$$

Free surface: $$p \approx p_a \qquad w \approx \frac{\partial \eta}{\partial t} \qquad (4.78)$$

/ DWU XVXDOO LQDQRVKHUFRXVH Z HFDQVROYHIRUWKHWHPSHUDWXUHGLVWULEXWLRQIURP (T Z KLFK GHSHQGVRQYHORFLW9 WKURXJKWKHGLVSDWLRQ) DQGWKHWRWDOWHGHULYDWLYHRSHUDWRUGGW &KDSWHU DWXPHVLQYLVFLG ICRZ WKURXJKRXWIRU Z KLFK WKH YLVFRVLW 7KH PRPHQWF HTXDWLRQ UHGXFHVWR

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla p \qquad (4.79)$$

7KLV LV(XOHUTV HTXDWLRQ LWFDQ EH LQWHJUDWHG DORQJ D VWUHDPOLQH WR REWDLQ %HUQRXOOTV HTXDWLR VHH6HF %\ QHJOHFWLQJ YLVFRVLW Z HKDYH ORVWWKH VHFRQG RUGHUGHULYDWLYHRI 9 LQ(T WKHUHRUH Z H PXVWUHDI RQH ERXQGDU FRQGLWLRQ RQ YHORFLW 7KH RQO PDWKHPDWFDO VHQVLEC FRQGLWLRQ WR GUPS LVWKH RQ VOLS FRQGLWLRQ DWWKH Z DOO: HOHWWKH ICRZ VOS SDUDOOHOWR WKH Z DOOEXWG QRWDOOR LWWR ICRZ LQR WKH Z DOO 7KH SURPHULQYLVFLG FRQGLWLRQ LV WKDWWKH QRUPDOYHORFLWV PX\ PDWK DWQ VRQG VXUFH

Inviscid flow: $$(V_n)_{\text{fluid}} = (V_n)_{\text{wall}} \qquad (4.80)$$

,QPRWWFDVHVWKH Z DOOLVIL[HG WKHUHRUHWKHSURPHULQYLVFLG ICRZ FRQGLWLRQLV

$$V_n = 0 \qquad (4.81)$$

7KHUHLVQR FRQGLWLRQ Z KDWVY HURQ WKH WDQJHQWLDOYHORFLW FRP SRQHQWDWWKH Z DOOLQLQYLVFLG ICRZ 7KHWDQJHQWLDOYHORFLW Z LOOEHSDUWR WKHVROXWQ WR DQLQYLVFLG ICRZ DQDO\VLV VHH&KDS

(; $0 3/ (

) RUWWDG\ LQFRP SUHVVLEOHIDP LQDU ICRZ WKURXJK D ORQJ WXEH WKHYHORFLW GLVWULEXWQLVJLYHQE\

$$v_z = U\left(1 - \frac{r^2}{R^2}\right) \qquad v_r = v_\theta = 0$$

Z KHUH8 LVWKHPD[LPXP RUFHQWUDOOQH YHORFLW DQG5 LVWKHWXEHUDGLXV ,I WKH Z DOOWMP SHUDWU LVFRQVDQWDW7Z DQGWKHWMP SHUDWXUH7 7 U RQO ILQG7 U IRUWKLVICRZ

6RO XWLRQ

: LWK7 7 U (T UHGXFHVIRUWWDG\ ICRZ WR

$$\rho c_p v_r \frac{dT}{dr} = \frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \mu\left(\frac{dv_z}{dr}\right)^2 \qquad (1)$$

%XWLQFH%ULIRUWKLVIORZ WKHFRQYHFWLYHWHUP RQWKHOHIWYDQLVKHV ,QWRGXFH%LQR(T WR
REWDLQ

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\mu\left(\frac{dv_z}{dr}\right)^2 = -\frac{4U^2\mu r^2}{R^4} \tag{2}$$

0 XOWLSO\ WKURXJKE\ UNDQG LQWHJUDWLQJ RQFH

$$r\frac{dT}{dr} = -\frac{\mu U^2 r^4}{kR^4} + C_1 \tag{3}$$

' LYLGH WKURXJKE\ UDQG LQWHJUDWLQJ RQFH DJDLQ

$$T = -\frac{\mu U^2 r^4}{4kR^4} + C_1 \ln r + C_2 \tag{4}$$

1RZ ZH DUH LQSRVLWLRQ WR DSSO\ RXU ERXQGDU\ FRQGLWLRQV WR HYDOXDWH& DQG&
) LUVW WLQFH WKH RJDULWKP RI]HUR LVí ' WKH WHPSHUDWXUH DW Z LOO EH LQ LQWXQDWH

$$C_1 = 0 \tag{5}$$

7KXV ZH HOLPLQDWH WKH SRVVLELOLW\ RI DQ RJDULWKPLF VLQJXODULW\ 7KH VPHWKLQJ ZLOO KDSSHQ LI Z
DSSO\ WKH\ PPHWU FRQGLWLRQ G7 GU DW U VR(T 7KH FRQVWDQW& LV WKHQ IRXQG E\ WKH Z DO
WHPSHUDWXUH FRQGLWLRQ DW U 5

$$T = T_w = -\frac{\mu U^2}{4k} + C_2$$

RU

$$C_2 = T_w + \frac{\mu U^2}{4k} \tag{6}$$

7KH FRUUHFWVRXUQLV WKXV

$$T(r) = T_w + \frac{\mu U^2}{4k}\left(1 - \frac{r^4}{R^4}\right) \qquad \textit{Ans. (7)}$$

Z KLFK LV D IRXUWK RUGHU SDUDEROLF GLVWULEXWLRQ Z LWK D PD[LPXP YDOXH7 7_Z 8 N DW WK
FHQWHUOLQH

7KH6WHDP) XQFWLRQ

: HKDYH VHQLQ6HF WKDW WHYHQL WKH WHPSHUDWXUH LV XQFRXSOHG IURP RXU V\VWHP RI HTXDWLRQ
RI PRWLRQ Z H PXVW VROYH WKH FRQWLQXLW\ DQG PRPHQWXP HTXDWLRQV VLPXOWDQHRXVO\ IRU SUHVVXUH DQ

YHORFLW 7KHVWUHDP IXQFWLRQ$ LVDFOHYHUGHYLFHWKDWDOORZVXVWRVDWLV\ WKHFRQWLQXLWHTXDWLRQDQG WKHQ VROYH WKH PRPHQWXP HTXDWLRQ GLUHFWO\ IRU WKH VLQJOH YDULDEOH$ / LQHV RI FRQVWDQW$ DU VWUHDPOLQHVRI WKHIORZ

7KH VWUHDP IXQFWLRQ LGHD ZRUNV RQO LI WKH FRQWLQXLW HTXDWLRQ FDQ EH UHGXFHG WR WZR VWUPV ,QJHQHUDO ZHKDYHIRXUVWUPV

Cartesian: $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (4.82a)$$

Cylindrical: $$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (4.82b)$$

) LUVWOHWXVHOP LQDWWXQWDG IORZ ZKLFK LVDSHFXODUDQGXQUHDOLVWLF DSSOLFDWLRQ RI WKHVWUHDP IXQFWLRQ LGHD 5HGXFH HLWKHU RI (TV VR DQ VZR VWUPV 7KH PRWLFRP PRQ DSSOLFDWLRQ L LQFRP SUHVVLEOHIORZ LQWKH[\ SODQH

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (4.83)$$

7KLVHTXDWLRQLVVDWLVILHGLGHQWLFDOO LI DIXQFWLRQ$ [\ LVGHILQHGVXFKWKDW(T EHFRPHV

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) \equiv 0 \qquad (4.84)$$

&RPSDULVRQRI DQG VKRZ VWKDWWKLVQHZ IXQFWLRQ$ PXWWEHGHILQHGVXFKWKDW

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad (4.85)$$

RU

$$\mathbf{V} = \mathbf{i}\frac{\partial \psi}{\partial y} - \mathbf{j}\frac{\partial \psi}{\partial x}$$

,VWKLVGHULWLPDWH <HV LWLVMXVWDPDWKHPDWFDOWULFNRI UHSODFLQJ VZR YDULDEOHV X DQG% E\ I VLQJOHKLJKHURUGHUIXQFWLRQ$ 7KHYRUWLFLW RUFXUO9 LVDQLQWHUHVWLQJ IXQFWLRQ

$$\text{curl } \mathbf{V} = -\mathbf{k}\nabla^2\psi \qquad \text{where} \qquad \nabla^2\psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \qquad (4.86)$$

7KXV LI ZH WDNH WKH FXUORI WKH PRPHQWXP HTXDWLRQ DQG XWLOL]H(T ZH REWDLQ I VLQJOHHTXDWLRQIRU$IRULQFRP SUHVVLEOHIORZ

$$\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi) = \nu\nabla^2(\nabla^2\psi) \qquad (4.87)$$

ZKHUH ! LVWKHNLQHPDWF YLVFRVLW 7KLVLVSDUWO DYLFWRU DQG SDUWO DGHHDW(T L VFDODUDQG KDVRQO RQH YDULDEOH $ EXWLWKRZ FRQWLQXIORXWKRUGHUGHULYDWLYHV DQG SUREDEO ZLO UHTXLUH FRP SXWDWLDQDO VLV 7KHUH ZLOOEH IRXUERXQGDU FRQGLWLRQV UHTXLUHG RQ$) RUH[DPSOH IR

LL χ_U LVJUHDWHU b IORZ WKURXJK HOHPHQWL χ_L LVJUHDWHU

%RWK WKH VWUHDP IXQFWLRQ DQG WKH YHORFLW SRWHQWLDO Z HUH LQYHQWHG E\ WKH) UHQFK PDWKHPDWLFD -RVHSK/ RXLV/ DJUDQJHDQG SXEOLVKHG LQKLV WUHDWLVH RQ IOXLG PHFKDQLFV

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,I DVWUHDP IXQFWLRQ H[LWV IRU WKH YHORFLW ILHOG RI ([DPSOH

X D[\ D[\ Z

ILQG LWS OR WV WDQG LQWHUSUHW

6RXWLRQ

- $VXPSWLRQV ,QFRPSUHVVLEOH VZR GLPHQVLRQDO IORZ
- $SSURDFK 8VH WKH GHILQLWLRQ RI VWUHDP IXQFWLRQ G HULYDWLYHV (TV WR ILQG $ [\
- 6RXWLRQ VWHS 1 RWH WKDW WKLV YHORFLW GLVWULEXWLRQ Z DV DOR H[DPLQHG LQ([DPSOH , VDWLVILHV FRQWLQXLW(T EXW GRHV QRW VDWLVI WKH Z LVH $ Z LOO QRW H[LVW

7KXV Z HDUH FHUWDLQ WKDW D VWUHDP IXQFWLRQ H[LVW

- 6RXWLRQ VWHS 7R ILQG $ Z ULWH RXW(TV DQG LQWUDWH

$$u = \frac{\partial \psi}{\partial y} = ax^2 - ay^2 \qquad (1)$$

$$v = -\frac{\partial \psi}{\partial x} = -2axy \qquad (2)$$

DQG Z RUN IURP HLWKHU RQH WR Z DUG WKH RWKHU , QWUDWH SDUWLDOO

$$\psi = ax^2y - \frac{ay^3}{3} + f(x) \qquad (3)$$

' LIHUHQWLDWH Z LWK UHVSHFW WR [DQG FRPSDUH Z LWK

$$\frac{\partial \psi}{\partial x} = 2axy + f'(x) = 2axy \qquad (4)$$

7KH UHRUHI• [RUI FRQWDQW7KH FRPSOHWH VWUHDP IXQFWLRQ LV WKXV IRXQG

$$\psi = a\left(x^2y - \frac{y^3}{3}\right) + C \qquad Ans.\ (5)$$

7R SORWWKLV VHW& IRUFRQYHQLHQFHDQGSORWWKHIXQFWLRQ

$$3x^2y - y^3 = \frac{3\psi}{a} \tag{6}$$

IRU FRQVWDQWYDOXHV RI $ 7KH UHVXOWLV VKRZQ LQ) LJ (D WR EH VL[f ZHGJHV R
FLUFXODWLQJ PRWLRQ HDFK ZLWKLGHQWLFDO SDWWHUQV 2QFHWKHVWUHDPOLQH
DUHODEHOHG WKHIORZ GLUHFWLRQVIRUOBOZ IURP WKHVLJQFRQYHQWLRQRI) LJ +RZ FDQWKHIORZ
EHLQWHUSUHWHG' 6LQFHWKHUHLVVOLSDORQJ DOOWUHPOQHV QRWUHPOQHFDQWKO UHSUHVHQWD VROL
VXUIDFHLQDYLVFRXVIORZ +RZHYHUWKHIORZ FRXOGUHSUHVHQWHLPSLQJHPHQWRI WKUHHLQFRPLQ
WUHPVDW DQG f 7KLVZRXOGEHDUDWHUXQHDOLVWF\HW[DFWVROXWLRQWRWKH1DYLH
6WRNHVHTXDWLRQV DVZHVKRZHGLQ([DPSOH

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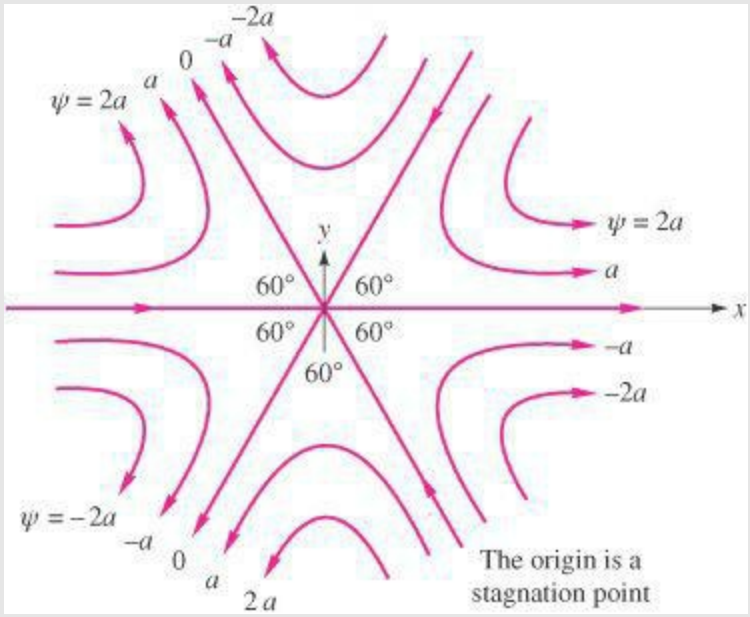

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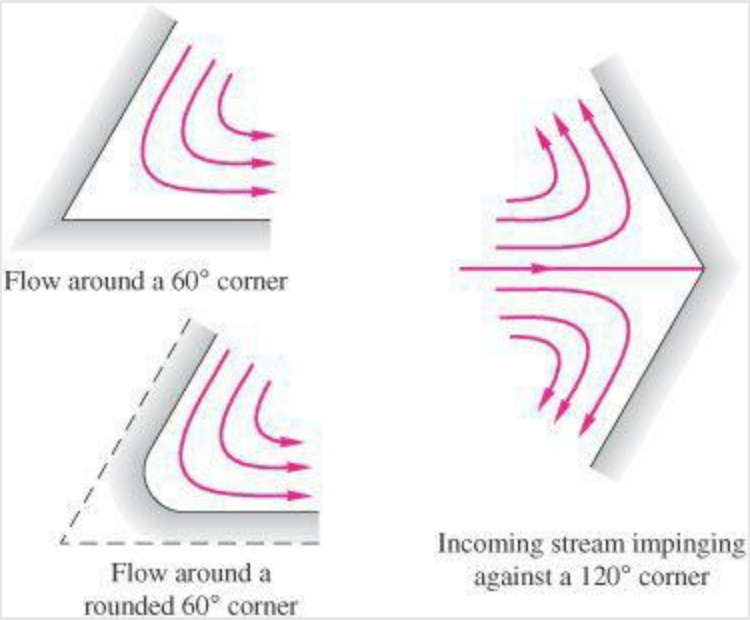


%\ DQGRZ LQJ WKH IORZ VR VDS DV DIUFWLRQDO WDSSUR[LPDWLRQ ZHFRXOG GHWDQ JLYHQ WHPSHUDWXH DERG\ VKDSH 6RPHH[DPSOHV DUH VKRZQ LQ) LJ (E

$ VWHDP IXQFWLRQ DOVR H[LVWV LQ DY DUHWVRI RWKHUSK\VLFDOVLWXDWLRQV ZKHUHRQO VWR FRRUGLQDWH DUHQHGHGVRGHILQHWKHIORZ 7KUHH[DPSOHV DUHLOOXVWUDWHGKHUH

6WUDG\ 3QDQH&RP SUHVVLEOH) ORZ

6XSSRVHQRZ WKDWWKHGHQVLW LV YDULDEOHEXWWKDWZ VR WKDWWKHIORZ LVLQWKH[\ SODQH 7KH WKHHTXDWRQ RI FRQWLQXLWEHFRPHV

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad (4.96)$$

: HVHHWKDWWKLV LVLQH[DFWO WKHVDPHIRUP DV(T 7KHUHDFRP SUHVVLEOHIORZ VWUHDP IXQFWLRQFDQEHGHILQHGVXFKWKDW

$$\rho u = \frac{\partial \psi}{\partial y} \qquad \rho v = -\frac{\partial \psi}{\partial x} \qquad (4.97)$$

$ JDLQQQHVRI FRQWDQW$ DUHVWUHDPOLQHVRI WKHIORZ EXWWKHFKDQJHLQ$ LVQRZ HTXDOWR WKHP DV IORZ QRWWKHYROXPHIORZ

GÀ ! 9 Q G$ G

$$\dot{m}_{1\to 2} = \int_1^2 \rho(\mathbf{V}\cdot\mathbf{n})\,dA = \psi_2 - \psi_1 \qquad (4.98)$$

RU

7KHVLJQFRQYHQWLRQRQIORZ GLUHFWLRQLVWKHVDPHDVLQ) LJ 7KLVSDUWFXODUVWUHDP IXQFWLRQ FRP ELQHVGHQVLW ZLWK YHORFLW DQG PXWEHVXEVWDQWLDOO\ QRWRQO PRPHQWXP EXWDOVR WKHHQHUJ DQG VWDWHRIV DQG ZLWKSUHVVXUHDQGWHPSHUDWXHV FRPSDQLRQYDULDEOHV 7KVVWK FRP SUHVVLEOHVWUHDP IXQFWLRQVRVDJUHDWYLFWRU DQGIXUWKHUDWXPSWLRQV PXWEHP DGHWR HIFWLQWDQFH 5H &KDS

,QFRP SUHVVLEOH3QDQH) ORZ LQ3RODU&RRUGLQDWHV

6XSSRVHWKDWWKHP SRUDQWFRRUGLQDWHV DUHUDQG ZLWK%] DQGWKDWWKHGHQVLW VFRQVWDQ 7KHQ(T E UHGXFHVWR

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) = 0 \qquad (4.99)$$

$IWHUPXOWLSO LQJ WKURXJKE\ U ZHVHHWKDWWKLVLVWKHDQDORJRXVIRUP R(T

$$\frac{\partial}{\partial r}\left(\frac{\partial \psi}{\partial \theta}\right) + \frac{\partial}{\partial \theta}\left(-\frac{\partial \psi}{\partial r}\right) = 0 \qquad (4.100)$$

%\ FRP SDULVRQ RI DQG ZHGHGXFH WKH IRUP RI WKH LQFRP SUHVVLEOH SRODU FRRUGLQDW VWUHDP IXQFWLRQ

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = -\frac{\partial \psi}{\partial r} \qquad (4.101)$$

2QH DJDLQ QQH VR FRQYDQW$ DUH VWUHDPOLQHV DQG WKH FKDQJHLQ$ LV WKH YROXPHIORZ 4 : % % 7KH VLJQ FRQYHQWLRQ LV WKH VDPH DV LQ) LJ 7KLV WSH RI VWUHDP IXQFWLRQ LV YHU XVHXO DQDO]LQJ IORZ VZLWFK\QQH VRUWHV VRXUFHV DQG VLQNV &KDS

,QFRP SUHVVLEOH$[LV\PPHWULF) ORZ

$V D ILQDO H[DPSOH VXSSRVH WKDWWKH IORZ LV WKUHHGLPHQVLRQDO %U L%] L EXWZLWK QI FLUFXPIHUHQWLDOYDULDWLRQV % ÁÁ VHH) LJ IRUGHILQLWLRQ RI FRRUGLQDWV 6XFK DIORZ LV WUHGD[LV\PPHWULF DQG WKH IORZ SDWWHUQLV WKH VDPHZKHQYHZHGRQDQ PHULGLRQDOSODQHWKURXJ WKH D[LV RI UHYROXWLRQ) RULQFRP SUHVVLEOHIORZ (T E EHFRPHV

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial}{\partial z}(v_z) = 0 \qquad (4.102)$$

7KLVGRHVQ]WVHP VRZRUN &DQ]WZHJHWWLGRI WKHRQHURXWLGH' %XWZKHQZHUHDO]HWKDWWUDQG DUHLQGHSHQGHQWYDULDEOHV (T FDQEHUHZULWWHQDV

$$\frac{\partial}{\partial r}(r v_r) + \frac{\partial}{\partial z}(r v_z) = 0 \qquad (4.103)$$

%\ DQDORJ\ ZLWK(T WKLVKDV WKH IRUP

$$\frac{\partial}{\partial r}\left(-\frac{\partial \psi}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \psi}{\partial r}\right) = 0 \qquad (4.104)$$

%\ FRP SDULQJ DQG ZHGHGXFH WKH IRUP RI DQLQFRP SUHVVLEOHD[LV\PPHWULF VWUHDP IXQFWLRQ$ U]

$$v_r = -\frac{1}{r}\frac{\partial \psi}{\partial z} \qquad v_z = \frac{1}{r}\frac{\partial \psi}{\partial r} \qquad (4.105)$$

+ HUH DJDLQ QQH VR FRQYDQW$ DUH VWUHDPOLQHV EXWWKHUH LV D IDFWRU CE LQ WKH YROXPH IORZ $Q_{1\to 2} = \psi_2 - \psi_1$. 7KHVLJQFRQYHQWLRQRQIORZ LVWKH VDPH DV LQ) LJ

(; $0 3/ (

,QYHVWLJDWHWKH VWUHDP IXQFWLRQLQSRODUFRRUGLQDWV

$$\psi = U \sin\theta\left(r - \frac{R^2}{r}\right) \qquad (1)$$

ZKHUH8 DQG5 DUH FRQVWDQWV DYDLODEOH IURP DQDO\WLFDO RHVSHFWLYHO 3ORWWKH VWUHDPOLQHV : KD
GRHVWKHIORZ UHSUHVHQW,VLVDUHDOWLF VROXWLRQWRWKHEDVLF HTXDWLRQ"

6ROXWLRQ

7KH VWUHDPOLQHV DUH OLQHV RI FRQVWDQW$ ZKLFK KDV XQLWV RI VTXDUH PHWHUV SHU VHFRQG 1RWWKD
$ 85 LVGLPHQVLRQOHVV 5HZULWH(T LQGLPHQVLRQOHVVIRUP

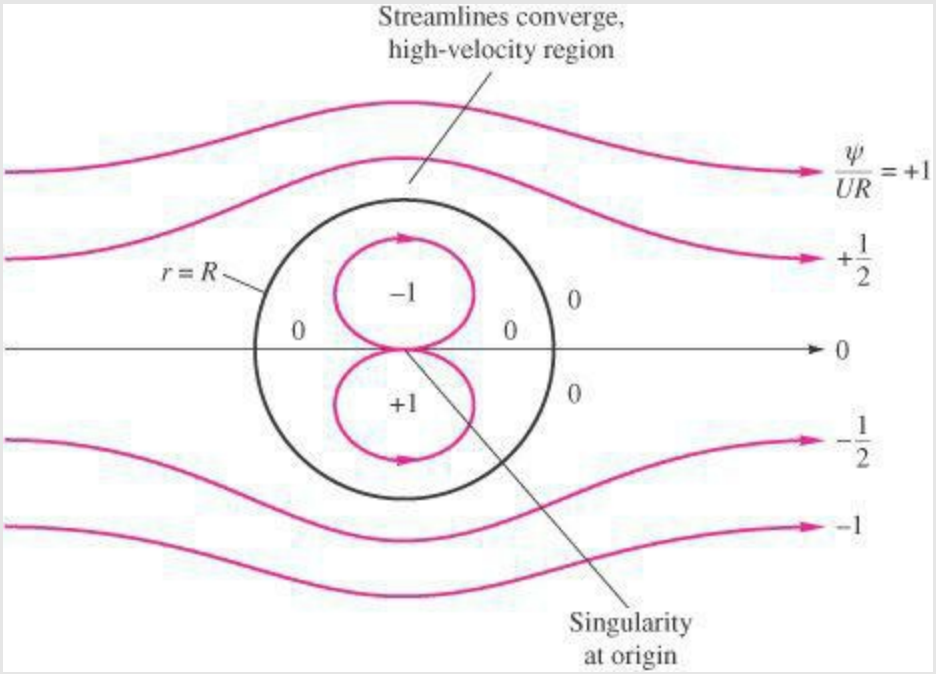
$$\frac{\psi}{UR} = \sin\theta\left(\eta - \frac{1}{\eta}\right) \qquad \eta = \frac{r}{R} \tag{2}$$

2I SDUWFXODULQWHUHVWVWKHVSHFLDOOLQH$) URP(T RU WKLVRFFXUVZKHQ D R
f DQG E U 5 &DVH D LVWKH[D[LV DQGFDVH E LV DFLUFOHRI UDGLXV5 EHWKRI ZKLFK DU
SDUWWGLQ) LJ (
) RUDQ RWKHUQRQ HURYDOXHRI $LWLVHDVLHVWWRSLFNDYDOXHRI UDQGVROYHIRU

$$\sin\theta = \frac{\psi/(UR)}{r/R - R/r} \tag{3}$$

,QJHQHUDO WKHUH ZLOOEH WZR VROXWLRQV IRU EHFDXVH RI WKH VYPPHWU DERXWWKH\ D[LV) R
H[DPSOH WDNH$ 85

(



Guess r/R	3.0	2.5	2.0	1.8	1.7	1.618
Compute θ	22° 158°	28° 152°	42° 138°	53° 127°	64° 116°	90°

7KLV QH LV SORWWG LQ) LJ (DQG SDWVRYHUWKH FLUFOHU 5 %HFDXVH XO WKRXJK EHFDXV

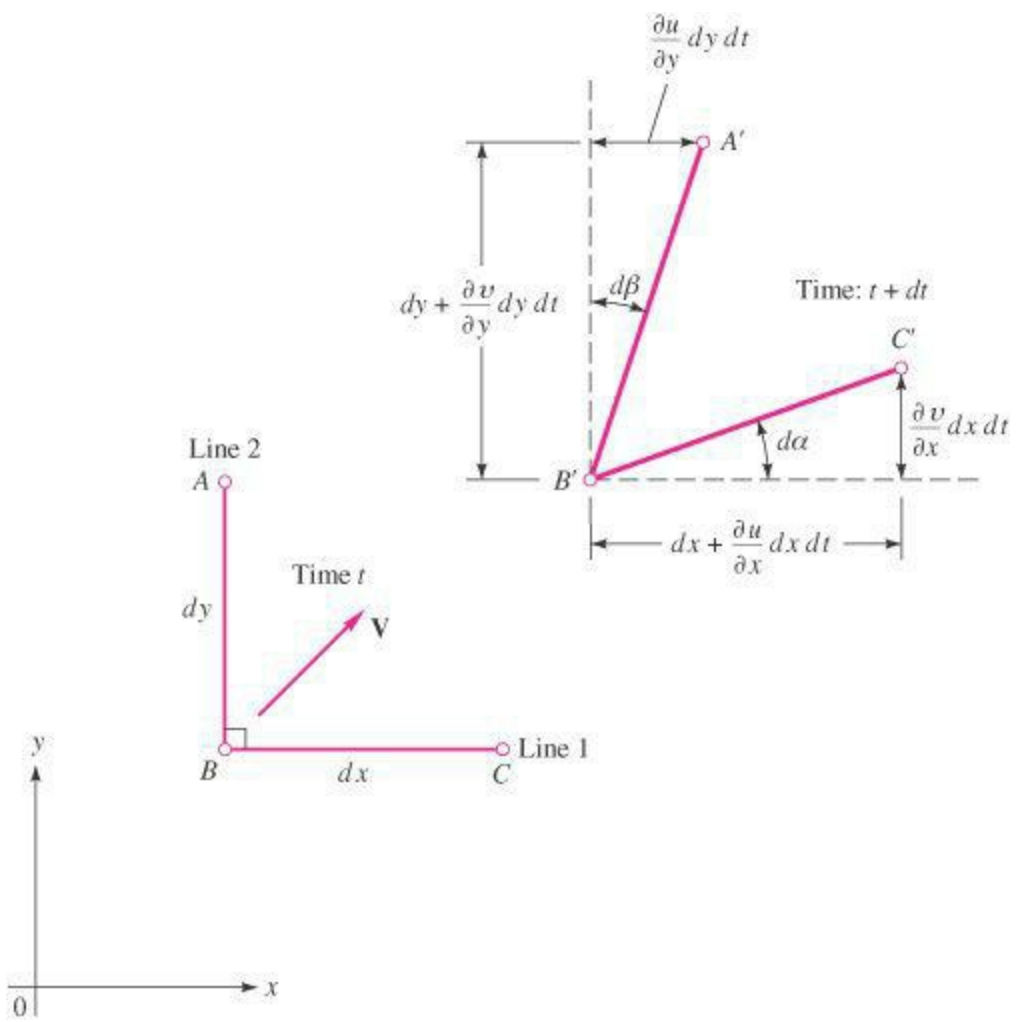
WKHUHLVDVHFRQGFXUYHIRU$ 85 IRUVPDOOU 5EHORZ WKH[D[LV
7KLVVHFRQGFXUYHSORWVDVDFRVHGFXUYHLQVLGHWKHFLUFOHU 5 7KHUHLVDVLQJXODULWRI LQILQLW
YHORFLWDQGLQGHWHUPLQDWHIORZ GLUHFWLRQDWWKHRULJLQ)LJXUH(VKRZVWKHIXOOSDWWHUQ

Guess r/R	0.618	0.6	0.5	0.4	0.3	0.2	0.1
Compute θ	$-90°$	$-70°$ $-110°$	$-42°$ $-138°$	$-28°$ $-152°$	$-19°$ $-161°$	$-12°$ $-168°$	$-6°$ $-174°$

7KHJLYHQVWHDP IXQFWLRQ(T LVDQH[DFWDQGFORVHGIRUP VROXWLRQWRWKHPRPHQWXP HTXDWLR
IRUIULFWLRQOHVVIORZ 2XUGHWKHFLUFOHU 5LWUHSUHVHQWVWZRGLPHQVLRQDOLQYLVFLGIORZ RI I
XQLRUP VWHDP SDWWDFLFXODUA\QQG6HF ,QGHWKHFLUFOHLWUHSUHVHQWVDUDWKHUXQUHDOLVWL
WDSSHGFLUFXODUQJPRWLRQRI ZKDWLVFDOOHGDQHGRXEOHW

9RUWLFLW DQG,UURWDWLRQDOLW

7KHDVXPSWLRQRI]HURIOLGDQJXODUYHORFLW RULUURWDWLRQDOLW LVDYHU XVHXOVLPSOLFDWLRQ
+HUHZHVKRZ WKDWDQJXODUYHORFLWLVDVRFLDWHGZLWKWKHFXUORI WKHORFDOYHORFLWYHFWRU
7KHGLIIHUHQWDOUHJLPHVIRUGHRUPDWLRQRI DIOXLGHOHPHQWFDQEHGHULYHGE\ H[DPLQLQJ)LJ
7ZRIOXLGQHV$%DQG%& LQLWLDOO SHUSHQGLFXODUDWWPHWPRYHDQGGHRUP VRWKDWDW G
WKH KDYHVOJKWO GLIHUHQWQHQJWK$•% DQG%&• DQG DUHVOJKWO RI WKHSHUSHQGLFXODUE\ DQJOHVG
DQGG 6XFKGHRUPDWRQFFXUVNQHPDWFDOO EHFDXVH$ % DQG& KDYHVOJKWO GLIHUHQWYHORFLW
ZKHQWKHYHORFLWILHOG9 KDVVSDWDOJUDGLHQW $OWKHVHGLIIHUHQWDOFKDQJHVLQWKHPRWRQRI $ %
DQG& DUHQRWAGLQ)LJ



) LJ $ QJXODU YHORFLW DQG VWDLQ UDWH RI WZR LQWHUVHFWLQJ OLQHV GHILQHG LQ WKH xy SODQH

: H GHILQH WKH DQJXODU YHORFLW &] DERXW WKH] D[LV DV WKH DYHUDJH UDWH RI FRXQWHUFORFNZLVH URWDWLRQ RI WKH WZR OLQHV

$$\omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} - \frac{d\beta}{dt}\right) \tag{4.106}$$

%XW URP) LJ G DQG G DUH DFKLHYHG UHODWLYH GHULYDWLYHV LQ WKH OLPLWV VP DO GW

$$\begin{aligned} d\alpha &= \lim_{dt\to 0}\left[\tan^{-1}\frac{(\partial v/\partial x)\,dx\,dt}{dx + (\partial u/\partial x)\,dx\,dt}\right] = \frac{\partial v}{\partial x}\,dt \\ d\beta &= \lim_{dt\to 0}\left[\tan^{-1}\frac{(\partial u/\partial y)\,dy\,dt}{dy + (\partial v/\partial y)\,dy\,dt}\right] = \frac{\partial u}{\partial y}\,dt \end{aligned} \tag{4.107}$$

&RPELQLQJ (TV DQG JLYHV WKH GHVLUHG UHVXOW

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{4.108}$$

,QH[DFW VLPLODUP DQQHUZ HGHWUP LQHWKH RWKHUZ RUDWV

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \tag{4.109}$$

7KHYHFWRU& LY[MY\ NY] LVVRPHWLPHVFDOOHGWKHFXUORI WKHYHORFLWYHFWRU

$$\omega = \frac{1}{2}(\text{curl } \mathbf{V}) = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \tag{4.110}$$

6LQFHWKHIDFWRUR LVDQQRLQJ PDQ ZRUNHUVSUHHUWRXVHDYHFWRUWZLFHDVODUJH FDOOHGWKH YRUWFLW

$$\zeta = 2\omega = \text{curl } \mathbf{V} \tag{4.111}$$

0 DQ IORZ VKDYHQHJOLJLEOHRU]HURYRUWFLWDQGDUHFDOOHGLUURWDWRQDO

$$\text{curl } \mathbf{V} \equiv 0 \tag{4.112}$$

7KHQ[WVHFWLRQH[SDQGVRQWKLVLGHD 6XFK IORZ VFDQEHLQFRPSUHVVLEOHRUFRPSUHVVLEOH WKDG RUXQVWDG\

: H PD\ DOR QRWH WKDW WKH YLJ GHPRQVWDWHV WKHVKHDU VWUDLQ UDWH RI WKH HOHPHQWZKLFK L GHILQHGDVWKHUDWHRI FKDQJHRI WKHLQWHUQDO SHUSHQGLFXODUQHVV

$$\dot{\varepsilon}_{xy} = \frac{d\alpha}{dt} + \frac{d\beta}{dt} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{4.113}$$

: KHQPXOWSOHGE\ YLVFRVLW WKLVHTXDOVWKHVKHDUVWUHVV2[\ LQDQHZWRQLDQ IXLG DVGLVFXVVH HDUOHU LQ(TV $SSHQGL[' OWV VWUDLQ UDWH DQG YRUWFLW FRPSRQHQWV LQ F\OLQGULFD FRRUGLQDWHV

) UHFWRQOHVV, UURWDWRQDO) ORZV

: KHQ D IORZ LV ERWK IUHHRIOHVV DQG LUURWDWRQDO SOHDVDQWWKLQJV KDSSHQ) LUVWWKH P RP HQWXP HTXDWRQ UHGXFHVWR(XOHU¶V HTXDWRQ

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla p \tag{4.114}$$

6HFRQG WKHUH LV D JUHDWLPSOLFDWRQ LQ WKH DFFHOHUDWRQ WHUP 5HFDOO IURP 6HF WKDW DFFHOHUDWRQKDVWZR WHUPV

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} \tag{4.2}$$

$ EHDXWLXOYHFWRULGHQWLW[LWWIRUWKHVHFRQGWHUP >@

$$(\mathbf{V}\cdot\nabla)\mathbf{V} \equiv \nabla(\tfrac{1}{2}V^2) + \boldsymbol{\zeta}\times\mathbf{V} \qquad (4.115)$$

Z KHUH FXUO9 IURP (T LV WKH OLQH YRUWLFLW

1 RZ FRQVLGHU DQG GLYLGH E\ ! DQG UHDUUDQJH RQ WKH OHIW KDQG VLGH ' RVWK HQWUHHTXDWLRQ LQWR DQ DUELWUDU YHFWRU GLVSODFHPHQW

$$\left[\frac{\partial\mathbf{V}}{\partial t} + \nabla\left(\frac{1}{2}V^2\right) + \boldsymbol{\zeta}\times\mathbf{V} + \frac{1}{\rho}\nabla p - \mathbf{g}\right]\cdot d\mathbf{r} = 0 \qquad (4.116)$$

1 RWLQJ Z RUN VUJKVXQDWZ HFDQJHWLG RI WKH WKLG WUP : HZ DQW

$$(\boldsymbol{\zeta}\times\mathbf{V})\cdot(d\mathbf{r}) \equiv 0 \qquad (4.117)$$

7KLV Z LOO EH WKH XQGHUYDUR XV FRQGLWRQV

9 LV]HUR WUYLDO QR IORZ KV GURVDWFV
LV]HUR LUURVDWRQDO IORZ
GULVSHUVHQGLFXODUR î 9 WKLVLVUDWKHUVSHFLDO]HGDQGUDUH
GULVDUDGHOR9 Z HLQWUDWDORQJ D VWUHDPOLQH VHH6HF

&RQGLWRQ LV WKH FRPPRQ DVXPSWRQ ,I Z H LQWUDW DORQJ D VWUHDPOLQH LQ IUFWRQOHV FRPSUHVLEOH IORZ DQG VDN HIRUFRYHQLHQFH J í JN (T UHGXFHVWR

$$\frac{\partial\mathbf{V}}{\partial t}\cdot d\mathbf{r} + d\left(\frac{1}{2}V^2\right) + \frac{dp}{\rho} + g\,dz = 0 \qquad (4.118)$$

([FHSWRUWKH ILUVWWUP WKHVH DUH H[DFWGLIHUHQWDOV ,QWUDWH EHWZHHQ DQ VWR SRLQWV DQG DORQJ WKH VWUHDPOLQH

$$\int_1^2 \frac{\partial V}{\partial t}\,ds + \int_1^2 \frac{dp}{\rho} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0 \qquad (4.119)$$

Z KHUHGV LV WKH DUF OHQWK DORQJ WKH VWUHDPOLQH (TXDWRQ LV %HUQRXOOL[V HTXDWRQ IR IUFWRQOHV XQVWDG IORZ DORQJ D VWUHDPOLQH DQG LV LGHQWFDO WR(T) RU LQFRPSUHVLEO VWDG IORZ LVWHGXFHVWR

$$\frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{constant along streamline} \qquad (4.120)$$

7KH FRQVWDQWV P D\ YDU IURP VWUHDPOLQH WR VWUHDPOLQH XQGHUW WKH IORZ LV DOVR LUURVDWRQDO DVXPSWRQ) RULUURVDWRQDO IORZ WKHRI IHQGLQJ WUP (T YDQLVKHVUHJDUGOHVV RI WK GUHFWRQ RI GU DQG(T WKHQKRGVDOORYHUWKH IORZ ILHOGZLWKWKHVDPHFRQVWDQW

9 HORFLW 3RWHQWLDO

,UURVDWRQDOW JLYHVULVHWR DVFDODUIXQFWRQË VP LODUDQG FRPSOHPHQDU WR WKH VWUHDP IXQFWRQ$) URP DWKHRUHP LQ YHFWRUDQDOVLV > @D YHFWRU Z LWK]HUR FXUOP XWEH WKH JUDGLHQWRI D VFDOD

IXQFWLRQ

$$\text{If} \quad \nabla \times \mathbf{V} \equiv 0 \quad \text{then} \quad \mathbf{V} = \nabla\phi \tag{4.121}$$

ZKHUHË Ë[\ ] WLVFDOOHGWKHYHORFLW\SRWHQWLDOIXQFWLRQ . QRZOHWJHRÈ WKXVLPPHGLDWH JLYHVWKHYHORFLW\FRPSRQHQWV

$$u = \frac{\partial\phi}{\partial x} \qquad v = \frac{\partial\phi}{\partial y} \qquad w = \frac{\partial\phi}{\partial z} \tag{4.122}$$

/ LQHVRI FRQVWDQWÈ DUHFDOOHGWKHSRWHQWLDOOLQHVRI WKHIORZ

1 RWH WKDWÈ XQGHU WKH VWHDP IXQFWLRQ LV IXOO WKUHHGLPHQVLRQDO DQG QRWOLPLWHG WR WZR FRRUGLQDWHV ,WUHGXFHV D YHORFLW\ SUREOHP ZLWK WKUHH XQNQRZQV % DQGZ WR D VLQJOH XQNQRZQ SRWHQWLDOÈ PDQ H[DPSOHV DUH JLYHQLQ&KDS 7KHYHORFLW\SRWHQWLDOOLQHVWKHXQLWDG %HUQRXOOLHTXDWLRQ EHFDXVHU Ë H[LWV ZHREWDLQ

$$\frac{\partial \mathbf{V}}{\partial t} \cdot d\mathbf{r} = \frac{\partial}{\partial t}(\nabla\phi) \cdot d\mathbf{r} = d\left(\frac{\partial\phi}{\partial t}\right) \tag{4.123}$$

DORQJ DQ DUELWDU GLUHFWLRQ (TXDWLRQ WKHQEHFRPHVDUHDWLRQEHWZHHQË DQGS

$$\frac{\partial\phi}{\partial t} + \int \frac{dp}{\rho} + \frac{1}{2}|\nabla\phi|^2 + gz = \text{const} \tag{4.124}$$

7KLV LV WKH XQVWDG\ LUURWDWLRQDO %HUQRXOOL HTXDWLRQ ,WLV YHU LPSRUWDQW LQ WKH DQDO\VLV F DFFHOHUDWLQJ IORZ ILHOGV VHH5HV DQG EXWWKHRQO DSSOLFDWLRQLQWKLV WH[WZLOOEHLQ6HF IRUXVDG\ IORZ

2 UWKRJRQDOLW\ RI 6WUHDP ODQHV DQG 3RWHQWLDO LQHV

,I DIORZ LV ERWK LUURWDWLRQDO DQG GHVFULEHG E\ RQO WZR FRRUGLQDWHV $ DQGË ERWK H[WDQG WKH VWUHDP ODQHV DQG SRWHQWLDO ODQHV DUH HYHU Z KHUH P XWXDOO SHUSHQGLFXODU[FHSWDWD VWDJQDWLRQ SRLQ) RU H[DPSOH IRU ORFRP SUHVVLEOHIORZ LQWKH[\ SODQH Z HZ RXGKDYH

$$u = \frac{\partial\psi}{\partial y} = \frac{\partial\phi}{\partial x} \tag{4.125}$$

$$v = -\frac{\partial\psi}{\partial x} = \frac{\partial\phi}{\partial y} \tag{4.126}$$

&DQ\RX WKLQN E\ LQVSHFWLRQ QRWRQO WKDWWKHVH UHODWLRQV LPSO RUWKRJRQDOLW\ EXWDOVR WKDWË DQGS VDWLV\ / DSODFH¶V HTXDWLRQ" $ OOHRI FRQVWDQWË Z RXGEHVXFK WKDWWKHFKDQJHLQË LV]HUR

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy = 0 = u\,dx + v\,dy \tag{4.127}$$

6RQYLQJ ZHKDYH

$$\left(\frac{dy}{dx}\right)_{\phi=\text{const}} = -\frac{u}{v} = -\frac{1}{(dy/dx)_{\psi=\text{const}}} \qquad (4.128)$$

(TXDWLRQ LV WKH PDWKHPDWLFDO FRQGLWLRQ WKDW OLQHV RI FRQVWDQW DQG$ EH PXWXDOO\ RUWKRJRQDO ,WP D\ QRWEHWKHDWDVWQDWRQSRLQWZKHUHERWX DQG%DU]HUR VRWKDWWKHUHDWR LV (T LVLQGHWHUPLQDWH

* HQHUDWLRQ RI 5RWDWLRQDOLW\

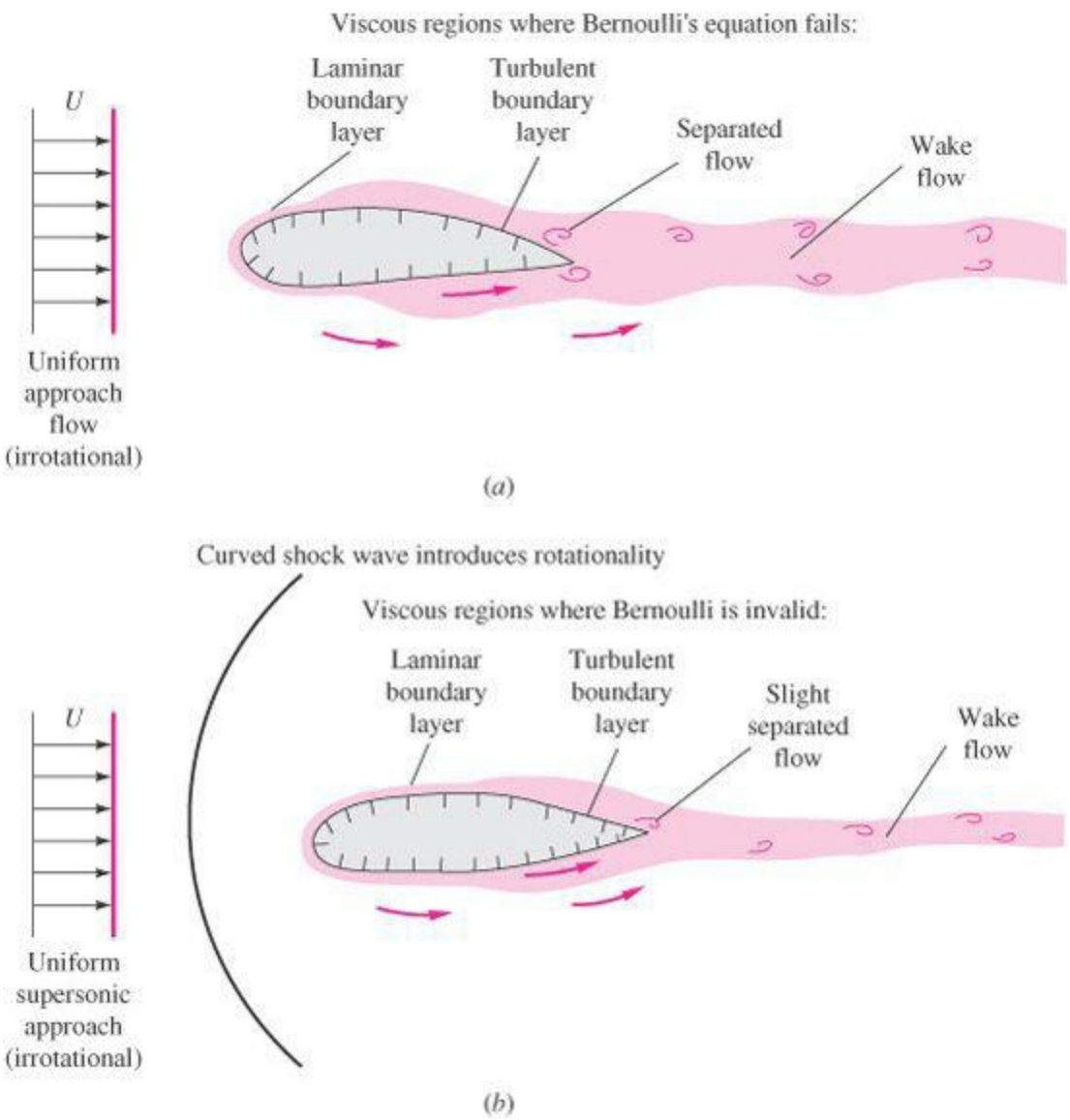
7KLVLVWKHVHFRQGWHZHKDYHGLVFXVVHG%HUQRXOOLVHTXDWLRQXQGHUGLIIHUHQWFLUFXPVWDQFHV WK ILUWZDV LQ6HF 6XFK UHLRUHPHQWLV XVHXO VLQFH WKLV LV SUREDEO WKH PRWZLGHO XVH HTXDWLRQLQIOXLGPHFKDQFV ,WUHTXLUHVIUFWLRQOHVIORZ ZLWKQR VKDWZRUNRUKHDWWDQVHUEHWZHH VHFWLRQV DQG 7KHIORZ PD\ RUPD\ QRWEH LUURWDWLRQDO WKH ODWWUEHLQJ DQHDVLHUFRQGLWLRQ DOORZ LQJ DXQLYHUVDO%HUQRXOOLFRQWDQW

7KHRQO UHPDLQLQJ TXHVWLRQLV WKLV : KHQLV DIORZ LUURWDWLRQDO" ,QRWKHUZRUGV ZKHQGRHVI IORZ KDYH QHJOLJLEOH DQJXODU YHORFLW\" 7KH H[DFWDQVZHU RI IXLG URWDWLRQDOLW XQGHU DUEWUDU FRQGLWLRQVLVDVRSLF IRUDGYDQFHG VWXG IRUH[DPSOH 5H 6HF 5H 6HF DQG5H 6HF : HVKDOOLPSO VWDWWKRVHUHVXOWKHUHZLWKRXWSURRI

$ IXLGIORZ WKDWLVLQLWLDOO LUURWDWLRQDOPD\ EHFRPHURWDWLRQDOL

7KHUHDUHVLJQLILFDQWYLVFRXVIRUFHVLQGXFHGE\ MHW Z DNHV RUVRQGERXQGDULHV ,QWKLVFDV %HUQRXOOLVHTXDWLRQZLOOQRWEHYDOGLQVXFKYLVFRXVUHJLRQV

7KHUHDUHQWPS\ JUDGLHQWVFDXVHGE\ FXUYHGVKRFNZDYHV VHH) LJ E



) LJ 7\SLFDO IORZ SDWWHUQV LOOXVWUDWLQJ YLVFRXV UHJLRQV SDWFKHG RQWR QHDUO\ IUHHVWUHDP UHJLRQV *a* ORZ VXEVRQLF IORZ SDVW D ERG\ *U* ! *a* IUHHVWUHDP LUURWDWLRQDO SRWHQWLDO IORZ RXWVLGH WKH ERXQGDU\ OD\HU %HUQRXOOL HTXDWLRQ YDOLG *b* VXSHUVRQLF IORZ SDVW D ERG\ *U* ! *a* IUHHVWUHDP URWDWLRQDO IORZ RXWVLGH WKH ERXQGDU\ OD\HU %HUQRXOOL HTXDWLRQ YDOLG SRWHQWLDO IORZ LQYDOLG

7KHUHDUHGHQVLW\JUDGLHQWVFDXVHGE\ VWDWLFDWLRQ XQHYHQKHDWLQJ UDWKHUWKDQE\ SUHVVXUH JUDGLHQWV

7KHUH DUH VLJQLILFDQWQRQLQHUWLDO HIHFWV VXFK DV WKH HDUWK¶V URWDWLRQ WKH &RULROLV DFFHOHUDWLRQ

,QFDVHV WR %HUQRXOOL¶V HTXDWLRQ VXEVRQLF IORZ DURXQG D VXUHDP OLQH LI IUHHVWUHDP LV QHJOLJLEOH : I VKDOORZ VWUDLJKW FDVHV DQG LQWHUQDO ERXQGDU\ OD\HU ZLOOEHWWHDWHGEULHO\ LQ&KDS RQJDVG\QDPLFV 3UDQGWL ZHDUHFRQFHUQHG ZLWKFDVH ZKHUHURWDWLRQLVLQGXFHGE\ YLVFRXV VWUHVVHV 7KLVRFFXUVRQG VXUIDFHV ZKHUHWKHQR VOLSFRQGLWLRQFUHDWHV DERXQGDU\ OD\HUWKURXJK ZKLFK VWHDF YHORFLW\GURSV WR]HUR DQGLQMHVW DQG ZDNHV ZKHUHVWUHDPVRI GLIIHUHQWYHORFLWLHVPHHWLQDUHJLR RI KLJK VKHDU

,QWLQDOOIORZV VXFK DV SLSHV DQG GXFWV DUHP RVWO\ YLVFRXV DQGWKH ZDOOOD\HU JURZ WKURXJKWKHFRUHRI WKHGXFW%HUQRXOOL¶V HTXDWLRQGRHVQRWKROGLQVXFKIORZ VXQDEVWUDFWHGIRUYLVFRX

GRXVHV

([WHQGLQJ IRZ V VXFK DV DERG LPPHUVHG LQ DWWHP DUHSDUW YLVFRXV DQG SDUW LQYLVFLG WK
WZR UHJLRQV EHLQJ SDUWKHG WRJHWKHUDWWKHHGJH RI WKH VKDUS DUHURUERXQGDU\ OD\HU 7Z R H[DPSOH
DUH VKRZ Q LQ) LJ) LJXUH D VKRZ V DGR VSHHG VXEVRQLF IORZ SDWWD ERG 7KH DSSURDFK
VWHDP LV LURRDWRQDO WKDWLV WKH FXUORI D FRQWDQWV]HUR EXWYLVFRXV WHWHV FUHDWD URDWRQDO
VKHDUDU\HUEHWLGHDGGRZQWHP RI WKHERG * HQHUDO VSHDNLQJ VHH&KDS WKH VKHDUDU\HUL
OPLQDU RU VPRRWK QHDU WKH IURQWRI WKH ERG DQG WXUEXOHQWRUGLVRUGHUO WRZDUG WKH UHDU $
VHSDUDWHG RUGHDZDWU UHJLRQ XVXDOO RFFXU QHDU WKH WDLOLQJ HGJH IRORZHG E\ DQ XQWDG
WXUEXOHQWZDNH[WHQGLQJ IDUGRZQWHP 6RPH VRUWRI OPLQDURUWXUEXOHQWYLVFRXV WKRU PXWE
DSSOHG WR WKHVH YLVFRXV UHJLRQV WKH DUH WKHQSDWKHGRQR WKHRXWHUIORZ ZKLFK LV IUHHWRQHWV DQ
LUURWDWRQDO,I WKH WXWHP 0 DFK QXPEHULV OHVV WKDQ DERXW ZHFDQFRPELQH(T ZLWK WK
LQFRP SUHVVLEOHFRQWLQXLW HTXDWRQ

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RU

$$\nabla^2\phi = 0 = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} \qquad (4.129)$$

7KLV LV / DSODFH¶V HTXDWRQ LQ WKUHH GLPHQVLRQV WKHUH EHLQJ QR UHVWULQWRQ WKH QXPEHU R
FRRUGLQDWH V\VWHPV\RXFDQXVH &KDS ZLOOEHFRQFHUQHGZLWKVROYLQJ (T IR
SUDFWLFDOHQJLQHHULQJ SUREOHPV LWKRGVLQWKHQHXHUJRQ) LJ DRXWLGHWKHVKHDUDU\HU
) LJXUH EVKRZ VDVXSHUVRQLF IORZ SDWWDURXQG QRVHGERG $ FXUYHGVKRFNZDYHJHQHUDWHG
IRUP V LQIURQWDQG WKHIORZ GRZQWHP LVURWDWRQDOGXHWR QRQUHS\ JUDGLHQW FDVH : HFDQXV
(XOHU¶V HTXDWRQ LQWKLVIUHFWRQWV UHJRQ EXWQRVSRWHQWLDOWKRU 7KHVKHDUDU\HUVKDYHWK
VDP HJHQHUDFKDUDFWHULVWLF) LJ DH[FHSWWKDWWKHVHSDUDWRQ]RQHLVVOJKWURUWHQDEVHQWDQG WK
Z DNH LV XVXDOO WKLQQHU 7KHRU R VHSDUDWHG IORZ LV SUHVHQW TXDOLWDWLYH EXWZH FDQ PDN
TXDQWLWDWLYH HVWLPDWHV R OPLQDUDQG WXUEXOHQWERXQGDU\ OD\HUVDQG Z DNHV

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,I DYHORFLW SRWHQWLDOH[LVWV IRUWKHYHORFLW ILHOGRI ([DPSOH

X D[\ D[\ Z

ILQGLWSORWWDQG FRPSDUHZ LWK([DPSOH

6RQXWLRQ

6LQFHZ WKHFXUORI 9 KDVRQO RQH] FRPSRQHQWDQG Z HP XWWKRZ WKDWWLV]HUR

$$(\nabla \times \mathbf{V})_z = 2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}(-2axy) - \frac{\partial}{\partial y}(ax^2 - ay^2)$$
$$= -2ay + 2ay = 0 \quad \text{checks} \qquad \textit{Ans.}$$

7KHIORZ LVLQGHHGLUURWDWLRQDO\$ YHORFLW\SRWHQWLDOH[LWV
7RILQGË [\ VHW

$$u = \frac{\partial \phi}{\partial x} = ax^2 - ay^2 \tag{1}$$

$$v = \frac{\partial \phi}{\partial y} = -2axy \tag{2}$$

,QWUDWH

$$\phi = \frac{ax^3}{3} - axy^2 + f(y) \tag{3}$$

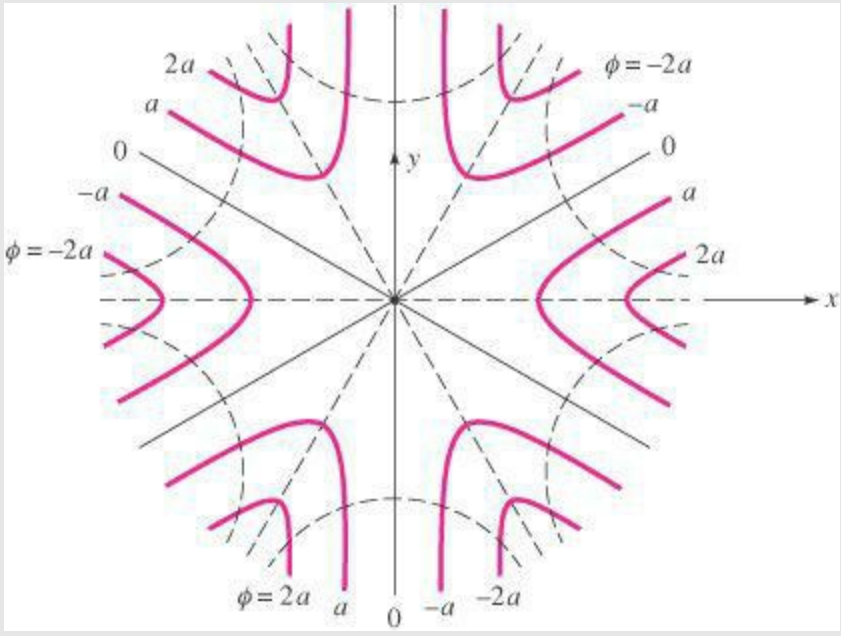
' LIIHUHQWLDWH DQGFRPSDUHZLWK

$$\frac{\partial \phi}{\partial y} = -2axy + f'(y) = -2axy \tag{4}$$

7KHUHRUHI• RUI FRQVWDQW7KHYHORFLW\SRWHQWLDOLV

$$\phi = \frac{ax^3}{3} - axy^2 + C \qquad \textit{Ans.}$$

(



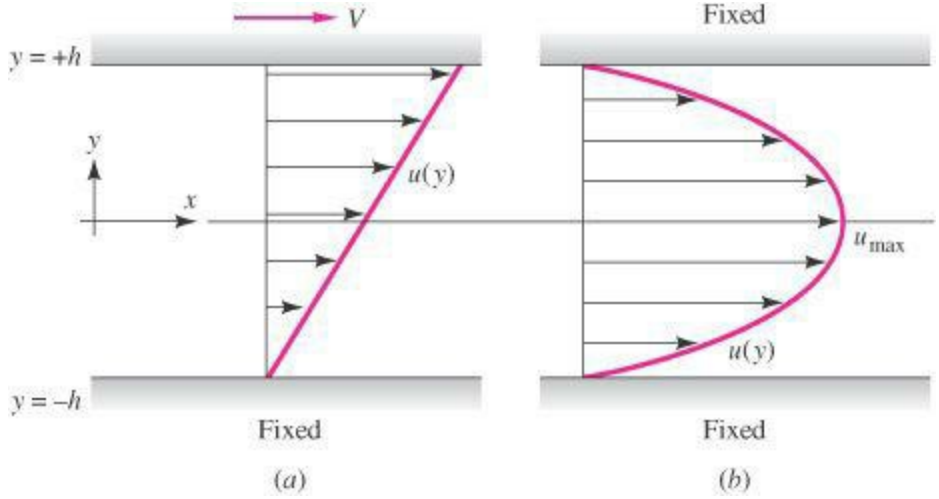
/ HWWQJ & Z HFDQSORWWKHË OQHVLQWKHVDPHIDVKLRQDVLQ([DPSOH 7KHUHVXOWLVVKRZ

LQ) LJ (QR DUURZ VRQË) RUWKLV SDUWFXODU SUREOHP WKHË OQHV IRUP WKH VDP H SDWWHUQ DV WKH OQHV RI ([DPSOH Z KLFK DUH VKRZ QKHUH DV GDVKHG OQHV EXW DUH GLVSODFHG ƒ 7KHË DQG $ OQH DUH HYHU Z KHUH SHUSHQGLFXODU [FHSWDWWKH RULJLQ D WDQQDWRQ SRLQW Z KHUH WKH DUH ƒ DSDUW: H H[SHFWHG WR EH DWWKH WDQQDWRQ SRLQW DQG WKHUH LV QR JHQHUDO UXOH IRU GHWHUPLQLQJ WKH EHKDYLRUR WKH OQHV DWWKDW SRLQW

6RP H, OOXVWUDWLYH, QFRP SUHVVLEOH9 LVFRXV) ORZV

,QYLVFLG IORZV GR QRW DOZ\V WKHQR VOLS FRQGLWLRQ 7KH ³VOLS´ DWWKHZ DOO EXW GR QRW IORZ WKURXJ WKH ZDOO 7R OHDUN DW XQG YLVFRXV QR VOLS FRQGLWLRQ Z H PXVW DWWDFN WKH FRP SOHW 1DYLHU6WRN HTXDWLRQ DQG WKHUHXOW XVXDOO QRWDWDOLUDWLRQDO QRUGHVDYHOFLWSRWQWDO[LW: H OHRN KHUH DW WKUHH FDVHV IORZ EHZHHQ SDUDOOHOSODWHV GXH WR D PRYLQJ XSSHU ZDOO IORZ EHZHHQ SDUDOOHOSODWHV GXH WR SUHVXUH JUDGLHQWDQG IORZ EHZHHQ FRQFHQWULF F\OQGHUZ KHQWK LQQHU RQH URWDWV 2 WKH UFDVHV Z LOO EH JLYHQ DV SUREOHP DWLJQPHQWV RU FRQVLGHUHG LQ&KDS ([WQVLYH VROXWLRQV IRU YLVFRXV IORZ V DUH GLVFXVVHG LQ 5HV DQG $ OOIORZ V LQ WKLV VHFWLRQ DU YLVFRXV DQG URWDWLRQDO

&RXHWWH) ORZ EHWZHHQ D) L[HG DQG D0 RYLQJ 3ODWH



) LJ ,QFRP SUHVVLEOH YLVFRXV IORZ EHWZHHQ SDUDOOHOSODWHV *a* QR SUHVXUH JUDGLHQW XSSHUSODWH PRYLQJ *b* SUHVXUH JUDGLHQW‰ % Z LWK ERWK SODWHV IL[HG

&RQVLGHUWR GLP HQVLRQDOLQFRP SUHVVLEOH SODQH %‰ YLVFRXV IORZ EHWZHHQ SDUDOOHOSODWHV DGLDWDQFH K DSDUWDV VKRZ QLQ) LJ : HDVXPH WKDWWKH SODWHV DUH YHU Z LGH DQG YHU ORQJ VR WKDWWKH IORZ LV HVHQWLDOO D[LDO XÁ EXW% Z 7KH SUHVHQFDVHLV) LJ D Z KHUH WKH XSSH SODWH PRYHV DW YHORFLW9 EXWWKH UHLVQR SUHVXUH JUDGLHQW1 HJOHFWUDYLWH IHFW : HOHDUQ IURP WK FRQWLQXLW\ HTXDWLRQ WKDW

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 = \frac{\partial u}{\partial x} + 0 + 0 \qquad \text{or} \qquad u = u(y) \text{ only}$$

7KXVWKHUHLVDVLQJOHQRQHURD[LDOYHORFLWFRPSRQHQWWKDWYDULHVRQO DFURVVWKHFKDQQHO 7K
IORZ LVVDLGWREHIXOO GHYHORSHG IDUGRZQVWUHDP RI WKHHQWUDQFH 6XEVWLWXWLX X\ LQWR WKH[
FRPSRQHQWRI WKH1DYLHU6WRNHVPRPHQWXP HTXDWLRQ IRUWZR GLPHQVLRQDO [\ IORZ

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

or
$$\rho(0+0)=0+0+\mu\left(0+\frac{d^2u}{dy^2}\right) \qquad (4.130)$$

0 RVWRI WKHWHUPVGURSRXWDQGWKHPRPHQWXP HTXDWLRQUHGXFHVWRVLPSOH

$$\frac{d^2u}{dy^2}=0 \quad \text{or} \quad u=C_1y+C_2$$

7KHVHWZRFRQGLWLRQVDUHIRXQGE\ DSSO\LQJWKHQR VOLS FRQGLWLRQDWWKHXSSHUDQGORZHUSODWHV

At $y=+h$: $u=V=C_1h+C_2$

At $y=-h$: $u=0=C_1(-h)+C_2$

or $C_1=\dfrac{V}{2h}$ and $C_2=\dfrac{V}{2}$

7KHUHRUHWKHVROXWLRQIRUWKLVFDVH D IORZ EHWZHHQSODWHVZLWKDPRYLQJ XSSHUZDOLV

$$u=\frac{V}{2h}y+\frac{V}{2} \qquad -h\le y\le +h \qquad (4.131)$$

7KLVLV&RXHWWHIORZ GXHWRDPRYLQJ ZDOO DQGDUYHORFLWSURILOHZLWKQR VOS DWHDFK ZDOO D
DQDFLSDWHG DQG VNHWFKHG LQ) LJ D 1RWWKDWWKH RUJLQKDV EHHQ SODFHG LQ WKH FHQWUH RI WK
FKDQQHOIRUFRQYHQLHQFHLQFDVH E ZKLFKIORZV
: KDWZHKDYHMXVWSUHVHQWHGLV DULJRURXVGHULYDWLRQRI WKHPRUHLQRUP DOO GLVFXVVHGIORZ F
) LJ ZKHUH\ DQGKZHUHGHILQHGGLIIHUHQWO

) ORZ ' XHWR3UHVVXUH* UDGLHQWEHWZHHQ7ZR) L[HG3ODWHV

&DVH E LVVNHWFKHGLQ) LJ E %RWKSODWHVDUHIL[HG 9 EXWWKHSUHVVXUHYDULHVLQWKH[
GLUHFWLRQ ,I% Z WKHFRQGXLWHTXDWLRQEHGVWRWKHVLPHFRQFOXVLRQDVFDVH D² QDPHO WKD
X X\ RQO 7KH[PRPHQWXP HTXDWLRQ FKDQJHVRQO EHFDXVHWKHSUHVVXUHLVYDULDEOH

$$\mu\frac{d^2u}{dy^2}=\frac{\partial p}{\partial x} \qquad (4.132)$$

$OR VLQFH% Z DQGJUDYLWVLQHJHFWG WKH\ DQG] PRPHQWXP HTXDWLRQVEHGWR

$$\frac{\partial p}{\partial y} = 0 \quad \text{and} \quad \frac{\partial p}{\partial z} = 0 \quad \text{or} \quad p = p(x) \text{ only}$$

7KXV WKH SUHVXUH JUDGLHQW(T LV WKH VDPH RQ JUDGLHQW

$$\mu \frac{d^2u}{dy^2} = \frac{dp}{dx} = \text{const} < 0 \tag{4.133}$$

: K\ GLG ZH DGG WKH IDFWRU GS G[LV FRQVLGHU 5HFDOO D XVHIXO FRQFOXVLRQ IURP WKH WKHRU F VHSDUDWRQ RI YDULDEOHV ,I WZR TXDQWLWLHV DUH HTXDO RQH YDULHV RQO ZLWK\ DQG WKH RWKHU YDULH RQO ZLWK[WKHQ WKH\ PXVW EHRWKHTXDO WKH VDPH FRQVWDQW2 WKHUZ LVH WKH\ ZRXOG RVEHLQG HSHQGHQW HDFK RWKHU

: K\ GLG ZH VXDW WKDWWKH FRQVWDQW LVQHJDWLYH' 3K\VLFDOO WKH SUHVXUH PXVW GHFUHDVH LQ WKH IORZ GLUHFWLRQ LQ RUGHU RGULYH WKH IORZ DJDLQVW WKH IULFWLRQ Z DOO7KH UHVXOW V X\ PXVWKDYH QHJDWLYH FXUYDWXUH HYHUZ KHUH DVDQDFSDWGDQG VNHWKHGLQ) LJ E

7KH VROXWLRQ\(T LV DFFRPSOLVKHG E\ GRXEOH LQWHJUDWLRQ

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2$$

7KHFRQVWDQWV DUH IRXQG IURP WKH QR VOLS FRQGLWLRQ DWWDFK Z DOO

$$\text{At } y = \pm h: \quad u = 0 \quad \text{or} \quad C_1 = 0 \quad \text{and} \quad C_2 = -\frac{dp}{dx}\frac{h^2}{2\mu}$$

7KXV WKH VROXWLRQ\ RFDVH E IORZ LQDFKDQQHOGXHWR SUHVXUHJUDGLHQWLV

$$u = -\frac{dp}{dx}\frac{h^2}{2\mu}\left(1 - \frac{y^2}{h^2}\right) \tag{4.134}$$

7KH IORZ IRUPV D3RLVHXLOOH SDUDEROD RI FRQVWDQWQHJDWLYH FXUYDWXUH 7KH PD[LPXP YHORFLW RFFXUV DWWKH FHQWHUOLQH\

$$u_{\max} = -\frac{dp}{dx}\frac{h^2}{2\mu} \tag{4.135}$$

2WKHU OLPLQJU IORZ SDUDPHWHUV DUH FRPSXWHG LQWKH IROORZLQJ H[DPSOH

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) RUFDVH E LQ) LJ E IORZ EHWZHHQSDUDOOHOSODWHV GXHWR WKH SUHVXUHJUDGLHQWFRPSXW D WKH ZDOOVKHDUVWHVV E WKHVXUHP IXQFWLRQ F WKHYRUWLFLW G WKHYHORFLW SRWHQWLDO DQG H WK DYHUDJHYHORFLW

6RXWLRQ

$ OO SDUDPHWHUV FDQ EH FRPSXWHG IURP WKH EDVLF VROXWLRQ (T E\ PDWKHPDWLFDO PDQLSXODWLRQ

3DUWD

7KHZDOOVKHDUIRUFHIURP WKHGHILQLWLRQRI DQHZWRQLDQIOXLG (T

$$\tau_w = \tau_{xy\,\text{wall}} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\Big|_{y=\pm h} = \mu\frac{\partial}{\partial y}\left[\left(-\frac{dp}{dx}\right)\left(\frac{h^2}{2\mu}\right)\left(1 - \frac{y^2}{h^2}\right)\right]\Big|_{y=\pm h}$$

$$= \pm\frac{dp}{dx}h = \mp\frac{2\mu u_{\max}}{h} \qquad \textit{Ans. (a)}$$

7KHZDOOVKHDUKDVWKHVDPHPDJQLWXGHDWHDFKZDOO EXWE\ RXUVLJQFRQYHQWLRQ) LJ WK XSSHUZDOKDVQHJDWLYHVKHDUVWUHVV

3DUWE

6LQFHWKHIORZ LVSDUDOOHO VWHDG\ DQGLQFRPSUHVVLEOH DVWHDP IXQFWLRQH[LVWV

$$u = \frac{\partial\psi}{\partial y} = u_{\max}\left(1 - \frac{y^2}{h^2}\right) \qquad v = -\frac{\partial\psi}{\partial x} = 0$$

,QWHJUDWLQJ DQG VHWWLQJ $ DWWKHFHQWHUOLQH IRUFRQYHQLHQFH Z HREWDLQ

$$\psi = u_{\max}\left(y - \frac{y^3}{3h^2}\right) \qquad \textit{Ans. (b)}$$

$ WKHZDOOV \ “ KDQG$ “ X PDK UHVSHFWLYHO

3DUWF

,QSDUHQWHIORZ WKHUHLVRQO DVLQJOHQRQ HURYRUWLFLW FRPSRQHQW

$$\zeta_z = (\text{curl } \mathbf{V})_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{2u_{\max}}{h^2}y \qquad \textit{Ans. (c)}$$

7KH YRUWLFLW LV KLJKHVWDWWKH Z DOO DQG LV SRVLWLYH FRXQWHUFORFNZLVH LQ WKH XSSHU KDO DQ QHJDWLYH FORFNZLVH LQ WKHORZHU KDOI RI WKHIOXG 9LVFRXVIORZ V DUH WWSLFDOO IXOO RI YRUWLFLW DQ DUHQRWZDWDOLUURUDWRQDO

3DUWG

) URP SDUWF VKH YRUWFLW LV ILQLWH 7KHUHRUH VKH IORZ LV QRWLURWDWLRQDO DQG VKH YHORFLW
SRWHQWLDOFRHIILFLHQW[LWV$QV G

3DUWH

7KH DYHUDJH YHORFLW\ LV GHILQHG DV $ ZKHUH 4 UKG$ RYHUWKH FURVV VHFWLRQ) RURX
SDUWFXODUGLVWULEXWLRQX \ IURP (T ZHREWDLQ

$$V_{av} = \frac{1}{A}\int u\,dA = \frac{1}{b(2h)}\int_{-h}^{+h} u_{max}\left(1 - \frac{y^2}{h^2}\right) b\,dy = \frac{2}{3}u_{max} \qquad Ans.\ (e)$$

,Q SODQH 3RLVHXLOOH IORZ EHWZHHQ SDUDOOHO SODWHV VKH DYHUDJH YHORFLW\ LV WZR WKLUGV RI WK
PD[LPXP RUFHQWHUOLQH YDOXH 7KLV UHVXOW FRXOG DOVR KDYH EHHQ REWDLQHG IURP WKH VWUHDP IXQFWLR
GHULYHG LQ SDUW E) URP (T

$$Q_{channel} = \psi_{upper} - \psi_{lower} = \frac{2u_{max}h}{3} - \left(-\frac{2u_{max}h}{3}\right) = \frac{4}{3}u_{max}h \text{ per unit width}$$

ZKHQFH9 DY 4 $E X PD[K K X PD[WKHVDPHUHVXOWV
7KLVH[DPSOHLOOXVWUDWHVDVWDWHPHQWPDGHHDUOLHU . QRZOHGJHRI WKHYHORFLWYHFWRU9 >DVLQ(T
@LVHVVHQWLDOO WKHVROXWLRQ WR DIOXLGPHFKDQLFVSUREOP VLQFHDOORWKHUIORZ SURSHUWLHVFD
WKHQEHFDOFXODWHG

) XOO ' HYHORSHG/ DP LQDU3LSH) ORZ

3HUKDSVWKHPRVWXVHXO[DFWRXWRQRI WKH1DYLHU6WRNHVHTXDWLRQVIRULQFRPSUHVVLEOHIORZ
LQDVWUDLJKWFLUFXODUSLSHRI UDGLXV5 ILUVWVWXGLHG H[SHULPHQWDOO E\ * + DJHQLQ DQG- /
3RLVHXLOOHLQ %\IXOO GHYHORSHG ZHPHDQ WKDWWKH UHJLRQ VWXGLHG LV IDUHQRXJK IURP WK
HQWUDQFHWKDWWKHIORZ LVSXUHO D[LDO% Á ZKLOH%\DQG% DUH]HUR : HQHJOHFWWUDYLWDQGDOV
DVXPHD[LDOV\PPHWU² WKDWWV ÁÁ 7KHHTXDWLRQ RI FRQWLQXLW\LQF\OLQGULFDOFRRUGLQDWV(T
E UHGXFHVWR

$$\frac{\partial}{\partial z}(v_z) = 0 \qquad \text{or} \qquad v_z = v_z(r) \qquad \text{only}$$

7KHIORZ SURFHHGVWUDLJKWGRZQWKHSLSHZLWKRXWDGGLWLRQDOPRWLRQ 7KHUPRPHQWXP HTXDWLRQ L
F\OLQGULFDOFRRUGLQDWV (T ' VLPSOLILHVWR %S%U RUS S] RQO 7KH] PRPHQWXP
HTXDWLRQLQF\OLQGULFDOFRRUGLQDWV (T ' UHGXFHVWR

$$\rho v_z\frac{\partial v_z}{\partial z} = -\frac{dp}{dz} + \mu\nabla^2 v_z = -\frac{dp}{dz} + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

7KH FRQYHFWLYH DFFHOHUDWLRQ WHUP RQ WKH OHIW LV EHFDXVH RI WKH SUHVHQFH RI WKH JLYHQ FRQVWDQW HTXDWLRQ 7KXV WKH PRPHQWXP HTXDWLRQ PD\ EH UHDUUDQJHG DV IROORZV

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \frac{dp}{dz} = \text{const} < 0 \qquad (4.136)$$

7KLV LV H[DFWO\ WKH VLWXDWLRQ WKDW RFFXUHG IRU IORZ EHWZHHQ IODWSODWHV LQ(T $ JDLQ WK 3VHSDUDWLRQ FRQVWDQWV QHJDWLYH DQG SLSH IORZ ZLOO RRN PXFK OLNH WKH SODWH IORZ LQ) LJ E (TXDWLRQ LVOLQHDU DQG P\ EHLQJ UDWKHU ZLWK WKH UHVXOWV

$$v_z = \frac{dp}{dz}\frac{r^2}{4\mu} + C_1\ln(r) + C_2$$

ZKHUH & DQG & DUH FRQVWDQWV 7KH ERXQGDU\ FRQGLWLRQV DUH QR VOLS DW WKH ZDOO DQG ILQLWH YHORFLW DW WKH FHQWHOLQH

$$\text{No slip at } r = R\text{:}\quad v_z = 0 = \frac{dp}{dz}\frac{R^2}{4\mu} + C_1\ln(R) + C_2$$
$$\text{Finite velocity at } r = 0\text{:}\quad v_z = \text{finite} = 0 + C_1\ln(0) + C_2$$

7R DYRLG D ORJDULWKPLF VLQJXODULW WKH FHQWHOLQH FRQGLWLRQ UHTXLUHV WKDW& 7KHQ IURP C VOS & íGS G] 5 7KH ILQDO DQG IDPRXV VROXWLRQ IRU IXOO GHYHORSHG+DJHQ3RLVHXLOO IORZ LV

$$\boxed{v_z = \left(-\frac{dp}{dz}\right)\frac{1}{4\mu}(R^2 - r^2)} \qquad (4.137)$$

7KH YHORFLW SURILOH LV DSDUEROLG ZLWK D PD[LPXP DWWKH FHQWHOLQH - XWWDV LQ([DPSOH NQRZ OHGJHRI WKH YHORFLW GLVWULEXWLRQ HQDEOHV RWKHUSDUDPHWHUV WR EHFDOFXODWHG

$$V_{\max} = v_z(r=0) = \left(-\frac{dp}{dz}\right)\frac{R^2}{4\mu}$$

$$V_{\text{avg}} = \frac{1}{A}\int v_z\,dA = \frac{1}{\pi R^2}\int_0^R V_{\max}\left(1-\frac{r^2}{R^2}\right)2\pi r\,dr = \frac{V_{\max}}{2} = \left(-\frac{dp}{dz}\right)\frac{R^2}{8\mu}$$

$$Q = \int v_z\,dA = \int_0^R V_{\max}\left(1-\frac{r^2}{R^2}\right)2\pi r\,dr = \pi R^2 V_{\text{avg}} = \frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right) = \frac{\pi R^4\Delta p}{8\mu L}$$

$$\tau_{\text{wall}} = \mu\left|\frac{\partial v_z}{\partial r}\right|_{r=R} = \frac{4\mu V_{\text{avg}}}{R} = \frac{R}{2}\left(-\frac{dp}{dz}\right) = \frac{R}{2}\frac{\Delta p}{L} \qquad (4.138)$$

1RWH WKDW ZH KDYH VXEVWLWXWHG WKH HTXDOLW íGS G] ûS/ ZKHUH ûS LV WKH SUHVVXUH GURS DORQ WKH HQWLUH OHQJWK/ RI WKH SLSH

7KH VHIRUPXODV DUH YDOLG DV ORQJ DV WKH IORZ LV ODPLQDU WKDW LV ZKHQ WKH GLPHQVLRQOHV

5H\QROGVQXPEHUR WKHIORZ 5H ! 9_{DVJ} 5 LVOHVVWKDQDERXW 1RWDOOR WKDWWKHIORUPXOD GRQRWGHSHQGRQGHQVLW WKHUHDVRQEHLQJWKDWWKHFRQYHFWLYHDFFHOHUDWLRQRI WKLVIORZ LV]HUR

(; $0 3/ (

6$(: RLODW *& IORZVDW P KWKURXJKDKRU]RQWDOSLSHZLWKG FP DQG/ P)LQG D WKHDYHUDJHYHORFLW E WKH5H\QROGVQXPEHU F WKHSUHVVXUHGURS DQG G WKHSRZH UHTXLUHG

6RXWLRQ

- $VXPSWLRQV /DPLQDUWKDG +DJHQ3RLVHXLOOHSLSHIORZ
- $SSURDFK 7KHIORUPXODVR(TV DUHDSSURSULDWIRUWKLVSUREOP 1RWWKDW6 P
- 3URSHUWYDOXHV)URP7DEOH$ IRU6$(: RLO! NJ P DQG NJ P V
- 6RXWLRQVWHSV 7KHDYHUDJHYHORFLWVIORZVHDVLO IURP WKHIORZ UDWHDQGWKHSLSHDUHD

$$V_{avg} = \frac{Q}{\pi R^2} = \frac{(1.1/3600)\ \text{m}^3/\text{s}}{\pi(0.01\ \text{m})^2} = 0.973\frac{\text{m}}{\text{s}} \qquad \textit{Ans. (a)}$$

: HKDG WR FRQYHUW4 WR P V 7KH GLPHWU 5H\QROGV QXPEHUIRORZ VIURP WKHDYHUDJ YHORFLW

$$\text{Re}_d = \frac{\rho V_{avg} d}{\mu} = \frac{(870\ \text{kg/m}^3)(0.973\ \text{m/s})(0.02\ \text{m})}{0.104\ \text{kg/(m-s)}} = 163 \qquad \textit{Ans. (b)}$$

7KLV LV OHVV WKDQ WKH ³WDQVLWLRQ YDOXHV R VR WKHIORZ LV LQGHGODPLQDU DQG WK IRUPXODVDUHYDOG 7KHSUHVVXUHGURS LVFRPSXWGIURP WKHWKLGR(TV

$$Q = \frac{1.1}{3600}\frac{\text{m}^3}{\text{s}} = \frac{\pi R^4 \Delta p}{8\mu L} = \frac{\pi(0.01\ \text{m})^4 \Delta p}{8(0.104\ \text{kg/(m-s)})(12\ \text{m})} \quad \text{solve for } \Delta p = 97{,}100\ \text{Pa} \qquad \textit{Ans. (c)}$$

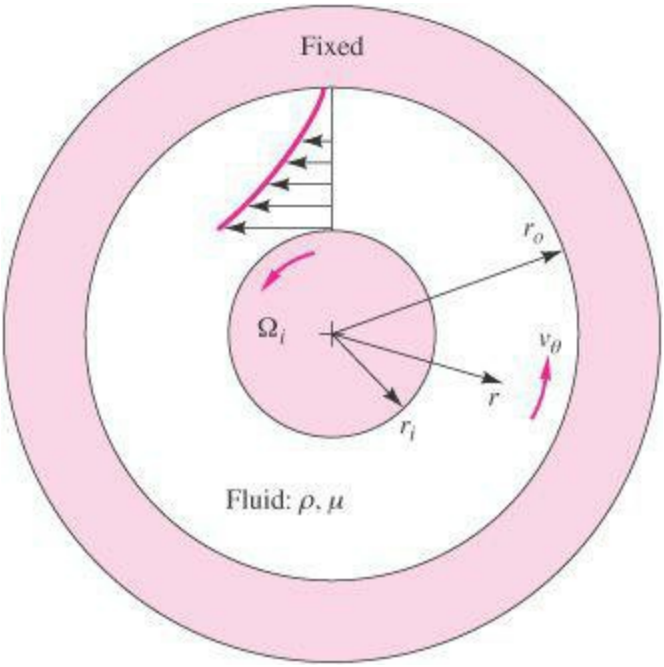
: KHQ XVLQJ 6, XQLWV WKH DQVZHU UHVXOWV LQ SDVFDOV QR FRQYHUVLRQ IDFWRUV DUH QHHGHG)LQDOO WKHSRZHUUHTXLUHGLVWKHSURGXFWRI IORZ UDWHDQGSUHVVXUHGURS

$$\text{Power} = Q\Delta p = \left(\frac{1.1}{3600}\ \text{m}^3/\text{s}\right)(97{,}100\ \text{N/m}^2) = 29.7\frac{\text{N-m}}{\text{s}} = 29.7\ \text{W} \qquad \textit{Ans. (d)}$$

- &RPPHQWV 3LSH IORZ SUREOPV DUH VWDLJKWRUZDUG DQGEUDF H[HUFLVHV LI WKH GDWD DU FRPSDWEOH 1RWDGJDLQWKDW6, XQLWVFDQEHXVHGLQWKHIORUPXODVZLWKRXWFRQYHUVLRQIDFWRUV

) ORZ EHWZHHQ / RQJ &RQFHQWULF &\OLQGHUV

&RQVLGHUDIOXLGRI FRQVWDQW! EHWZHHQWZR FRQFHQWULF F\OLQGHUV DVLQ) LJ 7KHUHLVQ DQLDOPRWRQRUHQGHIHFW% %0% / HWWKHLQQHUF\OLQGHUURWDWHDWDQJXODUYHORFLW9 VXEL VXE / HWWKHRXWHUF\OLQGEHIL[HG 7KHUHLVFLUFXODUV\PPHWU VRWKHYHORFLWGRHVQRWYDU ZLWK DQ YDULHVRQO ZLWKU



) LJ &RRUGLQDWHV\VWHP IRULQFRPSUHVVLEOHYLVFRXVIORZ EHWZHHQDIL[HGRXWHUF\OLQ-DQGDVWHDGLO URWDWLQJ LQQHUF\OLQGHU

7KHFRQWLQXLWHTXDWLRQIRUWKLVSUREOHP LV(T E ZLWK%

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0 = \frac{1}{r}\frac{d}{dr}(rv_r) \quad \text{or} \quad rv_r = \text{const}$$

1 RWHWKDW% GRHVQRWYDU ZLWK 6LQFH%0 DWERWKWKHLQQHUDQGRXWHUF\OLQGHU LWIROORZV WKDW%0 HYHUZKHUH DQG WKH PRWRQ FDQ RQO EH SXUHO FLUFXPIHUHQWLDO% % U 7KH PRPHQWXP HTXDWLRQ ' EHFRPHV

$$\rho(\mathbf{V}\cdot\nabla)v_\theta + \frac{\rho v_r v_\theta}{r} = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2}\right)$$

) RUWKHFRQGLWLRQVRI WKHSUHVHQWSUREOHP DOOWHUPVDUH]HURH[FHSWWKHODVW7KHUHRUHWKHEDVL GLIIHUHQWLDOHTXDWLRQIRUIORZ EHWZHHQURWDWLQJ F\OLQGHUVLV

$$\nabla^2 v_\theta = \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_\theta}{dr}\right) = \frac{v_\theta}{r^2} \qquad (4.139)$$

7KLVLVDOQHDUVHFRQGRUGHURUGLQDU GLIHUHQWLDOHTXDWLRQZLWKVHSDUDEOH

$$v_\theta = C_1 r + \frac{C_2}{r}$$

7KHFRQVWDQWVDUHIRXQGE\ VKHQR VOLSFRQGLWLRQDWWKHLQQHUDQGRXWHU\OLQGHUV

$$\text{Outer, at } r = r_o\text{:} \qquad v_\theta = 0 = C_1 r_o + \frac{C_2}{r_o}$$

$$\text{Inner, at } r = r_i\text{:} \qquad v_\theta = \Omega_i r_i = C_1 r_i + \frac{C_2}{r_i}$$

7KHILQDOVROXWLRQIRUWKHYHORFLW GLVWULEXWLRQLV

$$\text{Rotating inner cylinder:} \qquad v_\theta = \Omega_i r_i \frac{r_o/r - r/r_o}{r_o/r_i - r_i/r_o} \tag{4.140}$$

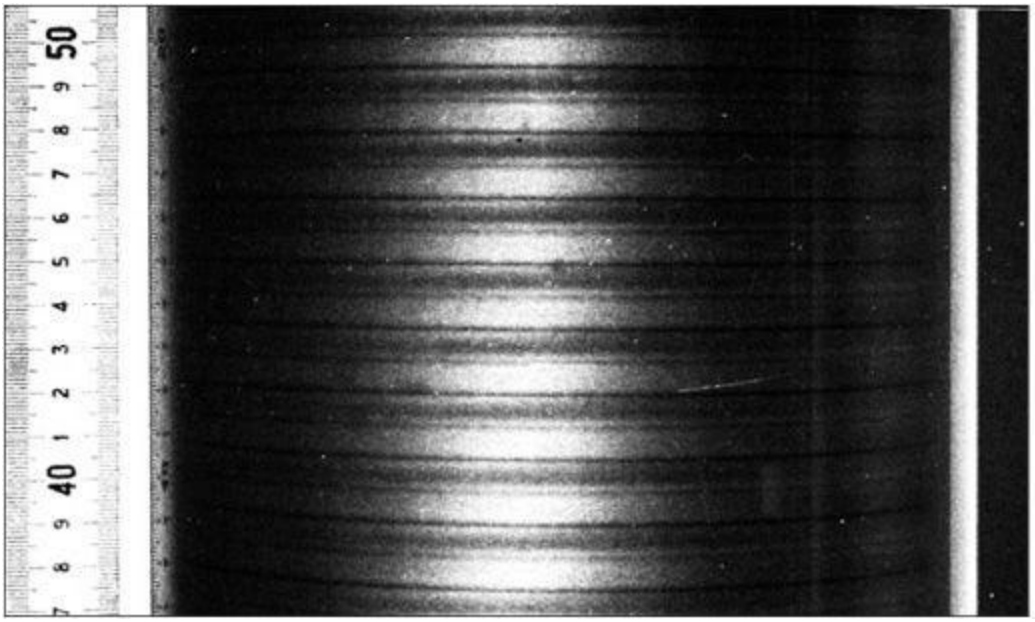
7KHYHORFLW SURILOHFORVHO UHVHPEOHVWKHVNHWFKLQ) LJ 9DULDWLRQVRI WKLVFDVH VXFKDV/D URWDWLQJ RXWHU\OLQGHU DUHJLYHQLQWKHSUREOHP DVWLJQPHQW

,QVWDELOLWLHV RI 5RWDWLQJ ,QQHU

7KH FODVVLF &RXHWWH IORZ VROXWLRQ RI (T GHVFULEHV D SK\VLFDOO VDWXU\LQJ FRQFDY-VZR GLPHQVLRQDO OLPLQDUIORZ YHORFLW SURILOHDVLQ) LJ 7KHVROXWLQVPDWKHPDWDOO H[DFIRUDQLQFRPSUHVVLEOHIOXLG +RZHYHU LWEHFRPHVXQXVXDOEHDWUHDWLQYHO ORZ UDWLRI URWDWLRQRI WKLQQHUF\OLQGHU DVVKRZQ LQDFODVVLF SDSHUE\ * , 7D\ORU> @$ WDFLWDVVXPSWLRQ ZKHQ QRZ FDOOHG WKHGLPHQVLRQOHVV7D\ORUQXPEHU GHQRWHG7D

$$\mathrm{Ta}_{\mathrm{crit}} = \frac{r_i (r_o - r_i)^3 \Omega_i^2}{\nu^2} \approx 1700 \tag{4.141}$$

WKHSDQHIORZ RI) LJ YDQVKHVDQGLVUHSODFHGE\ DODPLQDUWKUHHGLPHQVLRQDOIORZ SDWWHUQ FRQVLVWLQJ RI URZV RI QHDUO VTXDUH DOWHUQDWLQJ VRUURGDOYRUWHFHV $QH[SHULPHQWDOGHPRQVWUDWRQ RI VRUURGDO7D\ORUYRUWHFHV LVVKRZQLQ) LJ D PHDVXUHGDV7D§ 7D$FULW\E\ . RVFKPLHGHU> ($WKLJKHU 7D\ORU QXPEHUV WKH YRUWHFHV DOVR GHYHORS D FLUFXPIHUHQWLDO SHULRGLFLW EXWDUH VWLO ODPLQDU DVLOOXVWUDWHGLQ) LJ E $WWZOOKJKHU7D WXUEXOHQFHHQVXHV 7KLVLQWHUHVWLQJ LQVWDELOLW UHPLQGV XV WKDWWKH 1DYLHU6WRNHV HTXDWLRQV EHLQJ QRQOLQHDU GR DGPLWWR PXOWSOH QRQXQLTXH ODPLQDUVROXWLRQVLQDGGLWLRQWR WKHXVXDOLQVWDELOLWLHVDVRFFDVLRQDOZLWKWXUEXOHQFHDQGFKDRWF G\QDPLV\VWHPV



(*a*)

(*b*)

) LJ ([SHULP HQWDOYHULILFDWLRQ RI WKH LQWDELOLW\ RI IORZ EHWZHHQ D IL[HG RXWHU DQG I
URWDWLQJ LQQHU F\OLQGHU *a* 7RUROGDO7D\ORUYRUWHV[LWDW WLP HV WKH FULWLFDOVSHHG *b* D
WLP HV WKH FULWLFDOVSHHG WKH YRUWHV DUHGRXEO SHUIGLF *Courtesy of Cambridge University*
Press—E.L. Koschmieder, "Turbulent Taylor Vortex Flow," Journal of Fluid Mechanics, vol.
93. pt. 3, 1979, pp. 515–527. 7KLVLQWDELOLW\ GRHVQRWRFFXUL RQO WKHRXWHUF\OLQGHUURWDWHV

6XPPDU\

7KLV FKDSWHU FRP SOHPHQW&KDS E\ XVLQJ DQ LQILQLWHVLP DOFRQWROYROXPH WR GHULYH WKH EDVL
SDUDDOGLIHUHQWLDOHTXDWRQV RI PDV PRPHQWXP DQG HQHUJ\ IRUDIOLG 7KHVHHTXDWRQV VRJHWK-
ZLWKWKHUPRG\QDPLF VWDWHUHODWRQVIRUWKHIOLGDQGDSSURSULDWHERXQGDU FRQGLWLRQV LQSULQFLSOHFD
EHVROYHG IRUWKHFRP SOHWHIORZ ILHOGLQDQ JLYHQIOLG PHFKDQLFV SUREOP ([FHSWRU&KDS L
PRWWR WKHSUREOPVWR EHVWXGLHGKHUHDQLFRP SUHVLEOHIOLGZLWKFRQVDQWYLVFRVLWVLVDWXPHG

,Q DGGLWLRQ WR GHULYLQJ WKH EDVLF HTXDWLRQV RI PDVV PRPHQWXP DQG HQHUJ\ WKLV FKDSWH LQWURGXFHG VRPH VXSSOHPHQWDU\ LGHDV² WKH VWUHDP IXQFWLRQ YRUWLFLW\ LUURWDWLRQDOLW\ DQG WKH YHORFLW SRWHQWLDO² ZKLFK ZLOO EH XVHIXO LQ FRPLQJ FKDSWHUV HVSHFLDOO &KDS 7HPSHUDWXUH DQG GHQVLW YDULDWLRQV ZLOO EH HQFRXQWHUHG &KDS ZKHUHFRPSUHVVLELOLW LV VXEGLHG

7KLV FKDSWHU HQGHG E\ GLVFXVVLQJ DIHZ FODVVLFDO VROXWLRQV IRU ODPLQDU YLVFRXV IORZV &RXHW IORZ GXH WR PRYLQJ ZDOO 3RLVHXLOOH GXFW IORZ GXH WR SUHVVXUH JUDGLHQW DQG IORZ EHWZHHQ URWDWLQ F\OLQGHUV : KROH ERRN> ± @GLVFXVVFODVVLFDOSSURDFKHV WR IOLG PHFKDQLFV DQG RVKH VW[W> ± @H[WHQGV WKH VHVXGLHV WR WKH UHDO RI FRQWLQXXP PHFKDQLFV 7KLV GRHV QRW PHDQ WKDW SUREOPV FDQ EH VROYHG DQDO WFDOO 7KH QHZ ILHOG RI FRPSXWDWLRQDO IOLG G\QDPLFV> @WKRZ VJUHD SURPLVHV RI DFKLHYLQJ DSSUR[LPDWH VROXWLRQV WR DZLGH YDULHWL RI IORZ SUREOPV ,QDGGLWLRQ ZKHQ WKH JHRPHW DQG ERXQGDU FRQGLWLRQV DUH WKO FRPSOH[H[SHULPHQWDWLRQ &KDS LV D SUHFXUH DQDO\WLFDO

SUREOPV TXHVWLRQV3UREOPV

0 RVW RI WKH SUREOPV KHUHQDUH IDLU VWUDLJKWIRUZ DUG 0 RUHGLIILFXOWURSHQHGHGDVLJQPHQ DUH ODEHOHG ZLWK DQ DVWHULVN 3UREOPV ODEHOHG ZLWK DQ((6 LFRQ ZLOOEH QHLWURP WKH XVH RI WK (QJLQHHULQJ (TXDWLRQ6RIYH ((6 ZKLFKSUREOPVZLWKDFRPSXWHULFRQ PD\ UHTXLUHWKHXVHR DFRPSXWU 7KH VWDQGDUG HQGRI FKDSWUSUREOPV3 WR3 FDWHJRUL]HG LQWKH SUREOP OD KHUH DUH IROORZHG E\ ZRUGSUREOPV: WR: IXQGDPHQWDOV RI HQJLQHULQJ H[DP SUREOPV)(WR)(DQG FRPSUHKHQVLYHSUREOPV& DQG&

Problem Distribution

Section	Topic	Problems
4.1	The acceleration of a fluid	P4.1–P4.8
4.2	The continuity equation	P4.9–P4.25
4.3	Linear momentum: Navier-Stokes	P4.26–P4.38
4.4	Angular momentum: couple stresses	P4.39
4.5	The differential energy equation	P4.40–P4.41
4.6	Boundary conditions	P4.42–P4.46
4.7	Stream function	P4.47–P4.55
4.8 and 4.9	Velocity potential, vorticity	P4.56–P4.67
4.7 and 4.9	Stream function and velocity potential	P4.68–P4.78
4.10	Incompressible viscous flows	P4.79–P4.96
4.10	Slip flows	P4.97–P4.98

7KHDFFHOHUDWRQ RI DIOXLG

- 3 $QLGHDOL]HGYHORFLWILHOGLVJLYHQE\ WKHIRUPXOD

  9 \[L WY M []N

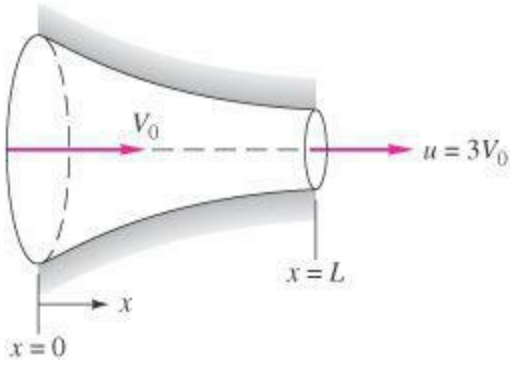
  ,VWKLVIORZ ILHOGVWDG RUXQWDG\" ,VLWZR RUWKUHGLPHQVLRQDO" $VWKHSRLQW[\ ] í FRPSXW D WKHDFFHOHUDWRQ YHFWRUDQG E DQ XQWYHFWRUQRUP DOWR WK DFFHOHUDWRQ

- 3) GRZ WKURXJK WKH FRQYHUJLQJ QR]]OH) LJ 3 FDQ EH DSSUR[LPDWHG E\ WKH RQH-GLPHQVLRQDO YHORFLWY GLVWULEXWLRQ

$$u \approx V_0\left(1 + \frac{2x}{L}\right) \quad v \approx 0 \quad w \approx 0$$

D) LQG D JHQHUDOH[SUHVVLRQ IRU WKH IOXLG DFFHOHUDWLRQ LQ WKH QR]]OH E) RU WK VSHFLILF FDVH9 IVW DQG/ LQ FRPSXWH WKH DFFHOHUDWLRQ LQJ¶V DW WKH HQWUDQFH DQ DW WKH H[LW

3



- 3 $ VZR GLPHQVLRQDO YHORFLWY ILHOGV JLYHQ E\

9 [\ [L [\ \ M

LQ DUELWUDU XQLWV $W [\ FRPSXWH D WKH DFFHOHUDWLRQV D[DQG D\ E WK YHORFLWY FRPSRQHQWV RQ WKH GLUHFWLRQ *f* F WKH GLUHFWLRQ RI PD[LPXP YHORFLWY DQG G WKH GLUHFWLRQ RI PD[LPXP DFFHOHUDWLRQ

- 3 $ VLPSOH IORZ PRGHO IRU DZR GLPHQVLRQDO FRQYHUJLQJ QR]]OH LV WKH GLVWULEXWLRQ

$$u = U_0\left(1 + \frac{x}{L}\right) \quad v = -U_0\frac{y}{L} \quad w = 0$$

D 6NHWFK DIHZ VWUHDPOLQHV LQ WKH UHJLRQ [/ DQG \ / XVLQJ WKH PHWKRG RI 6HFWLRQ E) LQG H[SUHVVLRQV IRU WKH KRUL]RQWDO DQG YHUWLFDO DFFHOHUDWLRQV F : KHUH LV WKH OLQH WUDYHUVHG E\ D SDUWLFOH RQ WKH DFFHOHUDWLRQ DQG LWV QXPHULFDO YDOXH"

- 3 7KH YHORFLWY ILHOG QHDU D VWDJQDWLRQ SRLQW V HH ([DPSOH PD\ EH ZULWWHQ LQ WKH IRUP

$$u = U_0\left(1 + \frac{x}{L}\right) \quad v = -U_0\frac{y}{L} \quad w = 0$$

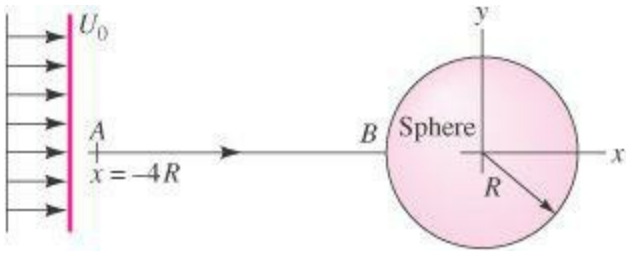
D 6KRZ WKDW WKH DFFHOHUDWLRQ YHFWRU LV SXUHO UDGLDO E) RU WKH SDUWFXODU FDVH/ P LI WKH DFFHOHUDWLRQ DW [\ P P LV P V ZKDW LV WKH YDOXH RI 8 "

- 3 $QLQFRPSUHVVLEOHSODQHIORZ KDVWKHYHORFLWFRPSRQHQWX \ [Z D) LQGWKHDFFHOHUDWLRQFRPSRQHQW E ' HWHUPLQHLWKHYHFWRUDFFHOHUDWLRQLVUDGLDO F : KLFKZD\RVWHDPOLQHVRIWKLVIORZ DUHVWUDLJKWOLQHV"
- 3 &RQVLGHUDVSKHUHRI UDGLXV5 LPPHUVHGLQDXQLIRUP VWHDP8 DVVKRZQLQ) LJ 3 $FFRUGLQJ WRWKHWKHRU RI&KDS WKHIOXLG YHORFLW\ DORQJ VWUHDPOLQH$%LV JLYHQE\

$$\mathbf{V} = u\mathbf{i} = U_0\left(1 + \frac{R^3}{x^3}\right)\mathbf{i}$$

) LQG D WKHSRVLWLRQRI PD[LPXP IOXLGDFFHOHUDWLRQDORQJ $%DQG E WKHWLPHUHTXLUH IRUDIOXLGSDUWFOHWRWDYHOIURP $ WR%

3

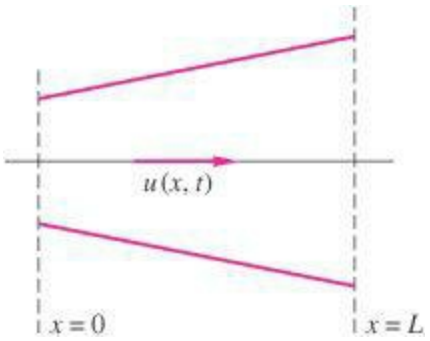


- 3 : KHQDYDOYHLVRSHQHG IOXLGIORZ VLQWKHH[SDQVLRQGXFWRI) LJ 3 DFFRUGLQJ W WKHDSSUR[LPDWRQ

$$\mathbf{V} = i U\left(1 - \frac{x}{2L}\right)\tanh\frac{Ut}{L}$$

) LQG D WKHIOXLGDFFHOHUDWLRQDW[W / / 8 DQG E WKHWLPHIRUZKLFKWKHIOXL DFFHOHUDWLRQ DW[/ LV]HUR : K\ GRHV WKHIOXLGDFFHOHUDWLRQ EHFRPHQHJDWLYH DWV FRQGLWLRQ E"

3



7KHFRQWLQXLW\HTXDWLRQ

- 3 $Q LGHDO]HG LQFRPSUHVLEOH IORZ KDV WKH SURSHUWLHV GHQVLW\ DQG YHORFLW\ GLVWULEXWLRQ

9 [\ L I \ M]\ N

) LQG WKH DSSURSULDWH IRUP RI WKH IXQFWLRQ I \ WKDW VDWLVILHV WKH FRQWLQXLW\ HTXDWLRQ

- 3 &RQVLGHU WKH VLPSOH LQFRPSUHVLEOH SDWWHUQ 8 9 DQG Z ZKHUH 8 DQG 9 DUH FRQVWDQWV D &RQYHUW WKHVH YHORFLWLHV LQWR SRODU FRRUGLQDWHV FRPSRQHQWV U DQG + LQW 0 DNH D VNHWFK RI WKH YHORFLW\ FRPSRQHQWV E ' HWHUPLQH ZKHWKHU WKH VHQHZ FRPSRQHQWV VDWLVI\ WKH FRQWLQXLW\ HTXDWLRQ LQ SRODU FRRUGLQDWHV
- 3 ' HULYH(T E IRU F\OLQGULFDO FRRUGLQDWHV E\ FRQVLGHULQJ WKH IOX[RI D LQFRPSUHVLEOH IOXLG DQG RXWR WKH HOHPHQWDO FRQWROYROXPH) LJ
- 3 6SKHULFDO SRODU FRRUGLQDWHV U Ë DUH GHILQHG LQ) LJ 3 7KH FDUWHVLDQ VDQG VSKHULFDO SRODU FRRUGLQDWHV DUH

[UVLQ FRV¥

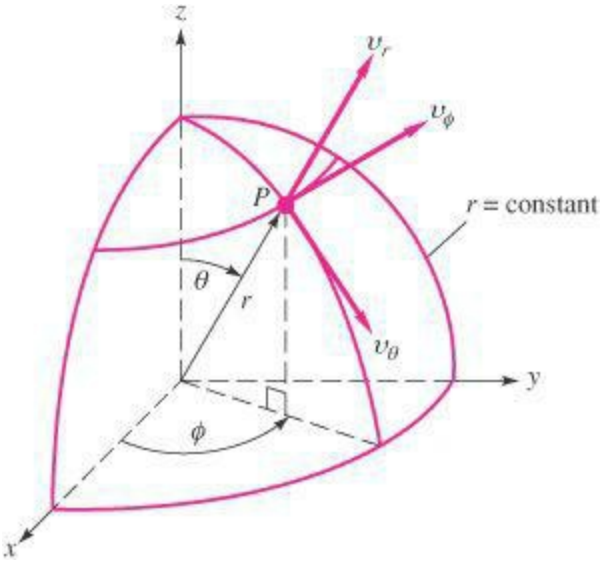
\ UVLQ VLQ¥

[UFRV

' RQRWWKRZ WKDWWKH FDUWHVLDQ LQFRPSUHVLEOH FRQWLQXLW\ HTXDWLRQ≥(T D@FDQ EH WDQVIRUPHG WR WKH VSKHULFDO SRODU IRUP

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(v_\phi) = 0$$

- : KDWLV WKH PRVW JHQHUDO IRUP RI %U Z KHQ WKH IORZ LV SXUHO UDGLDO WKDWLV % DQG %H DU]HUR'



- 3) RUDQLQFRPSUHVVLEOHSODQHIORZ LQSRODUFRRUGLQDWHV ZHDUHJLYHQ

  U FRV¥ U VLQ¥

) LQGWKHDSSURSULDWHIRUP RI FLUFXPIHUHQWLDOYHORFLWIRUZKLFKFRQWQXLWLVVDWLVILHG
- 3) RULQFRPSUHVVLEOHSRODUFRRUGLQDWHIORZ ZKDWLVWKHPRVWHQHUDOIRUP RI DSXUHO FLUFXODWRU PRWLRQ % % U WDQG%ƫ WKDWVDWLVILHVFRQWQXLW"
- 3 : KDWLV WKHPRVWHQHUDOIRUP RI DSXUHO UDGLDOSRODUFRRUGLQDWLQFRPSUHVVLEO IORZ SDWWHUQ %ƫ %ƫU WDQG% WKDWVDWLVILHVFRQWQXLW"
- 3 &RQVLGHUWKHSRODUFRRUGLQDWHYHORFLWGLVWULEXWLRQ

$$v_r = \frac{C}{r} \qquad v_\theta = \frac{K}{r} \qquad v_z = 0$$

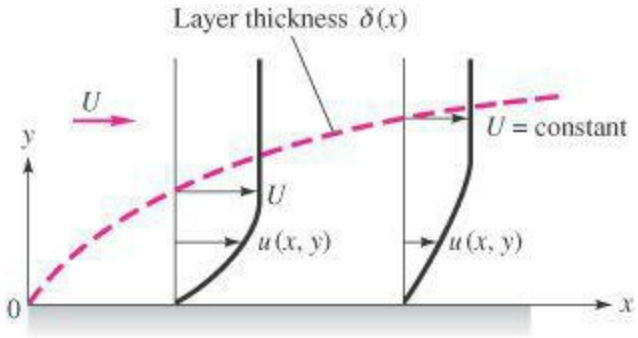
ZKHUH& DQG. DUHFRQVWDQWV D ' HWHUPLQHLWKHHTXDWLRQRI FRQWQXLWLVVDWLVILHG E %\ VNHWFKLQJ VRPH YHORFLW YHFWRUGLDFUDPV SORWD VLQJOH VWUHDPOLQH IRU& . : KD PLJKWWKLVIORZ ILHOGVLPXODWH"

- 3 $Q H[FHOOHQWDSSUR[LPDWLRQ IRU WKH VZR GLPHQVLRQDO LQFRPSUHVVLEOH ODPLQD EHWZHHQDU OD\HURQWKHIODWXUFHLQ) LJ 3 LV

$$u \approx U\left(2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\right) \quad \text{for } y \le \delta$$

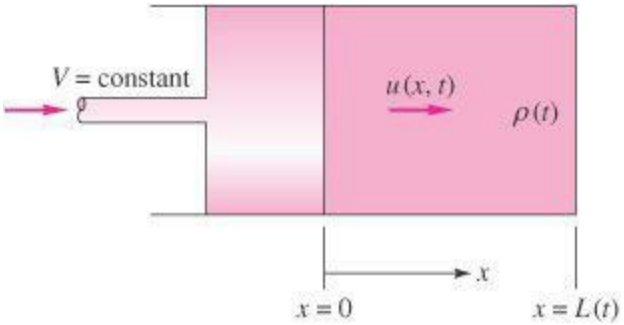
$$\text{where } \delta = Cx^{1/2},\ C = \text{const}$$

D $VVXPLQJ D QR VOLS FRQGLWLRQ DWWKH ZDOO ILQG DQ H[SUHVVLRQ IRU WKH YHORFLW FRPSRQHQW%[\ IRU\ ” / E 7KHQILQGWKHPD[LPXP YDOXHRI %DWWKH WDLOHQ[P IRUWKHSDUWFXODUFDVHRI DLUIORZ ZKHQ8 P VDQG/ FP



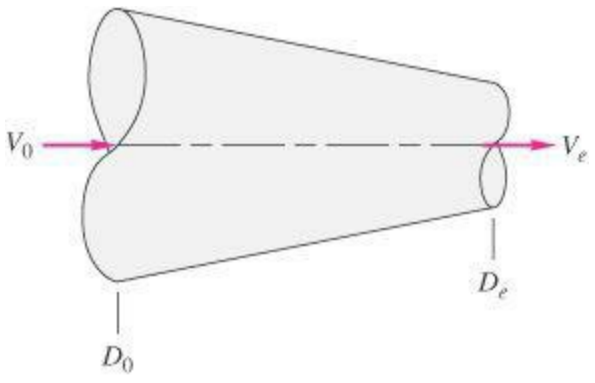
- 3 $ SLWRQ FRP SUHVVHV JDV LQ D F\OLQGHU E\ PRYLQJ DW FRQVWDQW VSHHG 9 DV LQ) LJ 3 / HW WKH JDV GHQVLW\ DQG GHQVLW\ DW EH! DQG/ UHSHFWLYHO\ / HW WKH JDV YHORFLW\ YDU\ OLQHDUO\ IURP X 9 DW WKH SLVWRQ IDFH WR X DW/ / ,I WKH JDV GHQVLW\ YDULHV RQO\ ZLWK WLPH ILQG DQH[SUHVVLRQ IRU! W

3



- 3 $ QLQFRP SUHVVLEOH IOXLG KDV WKH F\OLQGULFDO FRP SRQHQW% &U %θ . 5 í U %θ ZKHUH& DQG. DUH FRQVWDQWV DQG U ” 5] ” / ' RHV WKLV IOXLG VDWLV\ FRQWLQXLW\ : KDW PLJKW\RX SUHVXPH SK\VLFDOO\"
- 3 $ VZRGLPHQVLRQDOLQFRP SUHVVLEOH YHORFLW\ ILHOG KDV X . í HÍD\ IRU [” / DQG ” \ ” ' : KDW V WKH PRVW JHQHUDO IRUP RI %[\ IRU ZKLFK FRQWLQXLW\ LV VDWLVILHG DQG% % DW " : KDW DUH WKH SURSHUGLPHQVLRQV IRU FRQVWDQW. DQG D'
- 3 $ LU IORZ V XQGHU WKDG\ DSSUR[LPDWH RQH GLPHQVLRQDO FRQGLWLRQ V WKURXJK WK FRQFDOQR]]OHLQ) LJ 3 ,I WKH VSHHG RI VRXQG LV DSSUR[LPDWH P V ZKDW LV WKH PLQLPXP QR]]OH GLPHWHU U UDLR' H' IRU ZKLFK ZH FDQ VDIHO\ QHJOHFW FRPSUHVVLELOLW\ HIIHFWV 9 D P VDQG E P V"

3



- 3 ,QDQDLVPPHWULF IORZ QRWKLQJ YDULHVZLWK WKHRQG QRQHUR YHORFLWLHVDUH% DQG% VHH) LJ ,I WKHIORZ LVWUDG DQGLQFRPSUHVVLEOHDQG% %] ZKHUH% LVFRQVWDQWLQGWKHPRWJHQHUDOIRUP RI %ZKLFKVDWLVILHVFRQWLQXLW
- 3 $ VDQNYROXPH FRQWLQJJDVDWFRQGLWRQV ! S 7 $WWLPHW LWLVSXQFWXUH E\ DVPDOOKROHRI DUHD$ $FFRUGLQJ WR WKHWKHRU RI &KDS WKHPDWIORZ RXWR VXFK DKROHLV DSSUR[LPDWHO SURSRUWLRQDOWR$ DQG WR WKHVDQ SUHVXUH ,I WKHVDQ WPSHUDWXUH LV DWXPHG FRQWDQWDQG WKH JDV LV LGHDO ILQG DQ H[SUHVVLRQ IRU WK YDULDWLRQRI GHQVLWZLWKLQWKHVDQ
- 3)RU ODPLQDU IORZ EHWZHHQ SDUDOOHO SODWHV VHH) LJ E WKHIORZ LV WZR GLPHQVLRQDO • LI WKHZDOOVDUHSRURX\$ VSHFLDOFDVHVROXWLRQLVX \$ %[K \ ZKHUH\$ DQG%DUHFRQVWDQW D) LGDJHQHUDOIRUPXODIRUYHORFLW%LI % D \ E : KDWLVWKHYDOXHRI WKHFRQVWDQW%LI % %DW K'
- 3 $QLQFRPSUHVVLEOHIORZ LQSRODUFRRUGLQDWHVLVJLYHQE\

$$v_r = K\cos\theta\left(1 - \frac{b}{r^2}\right)$$

$$v_\theta = -K\sin\theta\left(1 + \frac{b}{r^2}\right)$$

' RHV WKLV ILHOG VDWLV\ FRQWLQXLW') RU FRQVLVWHQF\ ZKDWVKRXOG WKH GLPHQVLRQV RI FRQVWDQWV. DQGE EH' 6NHWFK WKHVXUFHZKHUH% DQGLQWHUSUHW

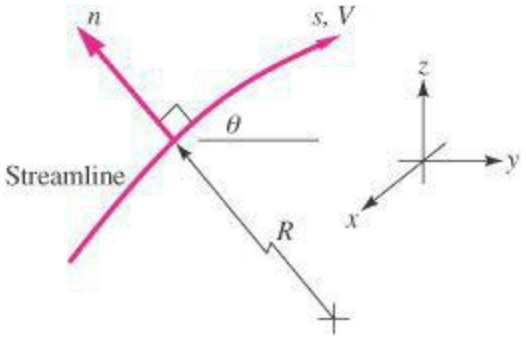
/ LQHDUP RP HQXP 1DYLHU6WRNHV

- 3 &XUYLOLQHDU RUWKRJRQDO FRRUGLQDWHVDUHGHILQHG) LJ 3 ZKHUHQLVQRUPDO WR WKH VWUHDPOLQH LQ WKH SODQH RI WKH UDGLXV RI FXUYDWXUH5 (XOHU¶V IUHFWLRQOHVV PRPHQWXP HTXDWLRQ LQVWUHDPOLQHFRRUGLQDWHVEHFRPHV

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + g_s \qquad (1)$$

$$-V\frac{\partial \theta}{\partial t} - \frac{V^2}{R} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + g_n \qquad (2)$$

- 6KRZ WKDWWKHLQWHJUDORI (T ZLWKUHVSHFWWR VLVQRQHRWKHUWKDQRXURGIUHQ %HUQRXOOL¶VHTXDWLRQ



- 3 $ IULFWLRQDO IORZ LQ FRPSUHVVLEOH VWDG\ IORZ ILHOGV LV JLYHQ E\

9 [\L \ M

LQ DUELWDU\ XQLW / HWWKH GHQLW EH! FRQWDQWV DQG QHJDWLYH) LQG D H[SUHVVLRQ IRU WKH SUHVXUH JUDGLHQW WKH[GLUHFWLRQ

- 3 &RQVLGHU WKH LQFRPSUHVVLEOH IORZ ILHOGV RI 3UREOHP 3 ZLWK YHORFLW FRPSRQHQWV \ % [Z 1HJOHFW JUDYLW DQG DVVXPH FRQVWDQW YLVFRVLW D ' HWHUPLQ ZKHWKHU WKLV IORZ VDWLVILHV WKH1DYLHU6WRNHV HTXDWLRQV E ,I VR ILQG WKH SUHVXUH GLVWULEXWLRQ S[\ LI WKH SUHVXUH DW WKH RULJLQ LV S R
- 3 &RQVLGHU D VWDG\ VZR GLPHQVLRQDO LQFRPSUHVVLEOH IORZ RI D QHZWRQLDQ IOXLG L ZKLFK WKH YHORFLW ILHOG LV NQRZQ X í [\ % \ í [Z D ' RHV WKLV IORZ VDWLV\ FRQVHUYDWLRQ RI PDVV" E) LQG WKH SUHVXUH ILHOG S[\ LI WKH SUHVXUH D WKH SRLQW[\ LV HTXDO WR S D
- 3)RU WKH YHORFLW GLVWULEXWLRQ RI 3UREOHP 3 GHWHUPLQH L D WKH HTXDWLRQ RI FRQWLQXLW DQG E WKH 1DYLHU6WRNHV HTXDWLRQ DUH VDWLVILHG F ,I WKH DWWULEXWH ILQG WK SUHVXUH GLVWULEXWLRQ S[\ ZKHQ WKH SUHVXUH DW WKH RULJLQ HTXDOV S R
- 3 $ FFRUGLQJ WR SRWHQWLDO WKHRU &KDS IRU WKH IORZ DSSURDFKLQJ DURXQG HG VZR GLPHQVLRQDO ERG DV LQ) LJ 3 WKH YHORFLW DSSURDFKLQJ WKH VXUIDFH QRQ JLYHQ E\ X 8 í D [ZKHUH D LV WKH RVH UDGLXV DQG 8 LV WKH YHORFLW ID XSVWUHDP &RPSXWH WKH YDOXH DQG SRVLWLRQ RI WKH PD[LPXP YLVFRXV QRUPDO VWUH DORQJ WKLV VXUIDFH

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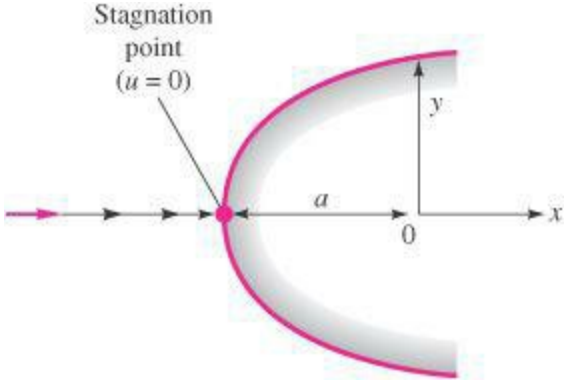

,V WKLV DOR WKH SRVLWLRQ RI PD[LPXP IOXLG GHFHOHUDWLRQ (YDOXDWH WKH PD[LPXP YLVFRX

QRUPDOWHVWLWKHIOLGLV6$(RLODW f& ZLWK8 P VDQGD FP

- 3 7KHDQZHUWR3URE 3 LV% IURQ ' RQRWUHYHDOWKLVWR\RXUIULHQGVLWKH DUHVWLOOZRUNLQJRQ3URE 3 6KRZ WKDWWKLVIORZ ILHOGLVDQH[DFWRXWRQWRWK 1DYLHU6WRNHV HTXDWLRQV IRUURQD VWR VSHFLDOFDVHV RI WKH IXQFWLRQ I U 1HJOHFWJUDYLW ,QWLSUHWKHVHWRRFDVHVSK\VLFDOO
- 3 &RQVLGHULQFRPSUHVLEOHIORZ DWD YROXPHUDWH4 WRZDUGDGUDLQDWWKH YHUWH[RI I f ZHGJH RI ZLGWKE DVLQ) LJ 3 1HJOHFWJUDYLW DQG IULFWLRQ DQG DVVXPH SXUHO UDGLDOLQIORZ D) LQG DQ H[SUHVVLRQ IRU%U E 6KRZ WKDWWKH YLVFRX WHUP LQWKHUPRPHQWXP HTXDWLRQV]HUR F) LQG WKHSUHVVXUHGLVWULEXWLRQS U LI S R DWU 5

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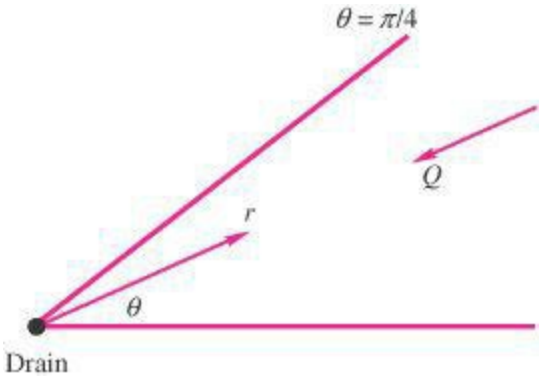

- 3 $ SURSRVHG WKUHHGLPHQVLRQDOLQFRPSUHVLEOHIORZ ILHOG KDV WKH IRORZLQJ YHFWR IRUP

9 .[L .\M .]N

D ' HWHUPLQHLI WKLVILHOGLVDYDOGVROXWLRQWRFRQWLQXLW DQG1DYLHU6WRNHV E ,I J íJN ILQGWKHSUHVVXUHILHOGS[\ ] F ,VWKHIORZ LURWDWLRQDO
- 3)URP WKH 1DYLHU6WRNHV HTXDWLRQV IRU LQFRPSUHVLEOHIORZ LQ SRODUFRRUGLQDW $SS ' IRU FYOLQGULFDO FRRUGLQDWV ILQG WKH PRVWJHQHUDO FDVH RI SXUHO FLUFXODWLQJ PRWLRQ% U %U %I IRUIORZ ZLWK QR VOLS EHWZHHQ WZR IL[H FRQFHQWULF F\OLQGHUV DVLQ) LJ 3

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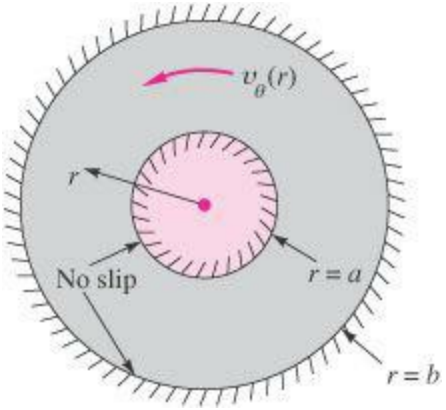

- 3 $ FRQWDQWWKLFNQHVV ILOP RI YLVFRXV OLTXLG IORZV LQ ODPLQDUP RWRQ GRZQ D SODQ LQFOLQHG DWDQJOH DV LQ) LJ 3 7KH YHORFLWSURILOHLV

X &\ K \ ! Z

) LQG WKH FRQWDQW& LQWHUPV RI WKH VSHFLILF ZHLJKWDQG YLVFRVLWDQG WKH DQJOH) LQ WKH YROXPHIOX[4 SHUXQLWZLGWKLQWHUPV RI WKHVHSDUDPHWHUV

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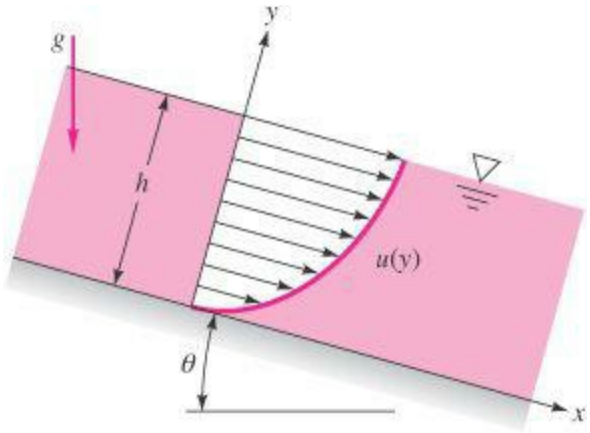

- 3 $ YLVFRXV OLTXLG RI FRQWDQW DQG IDOOV GXH WR JUDYLW EHWZHHQ WZR SODWHV I GLVWDQFH K DSDUW DV LQ) LJ 3 7KH IORZ LV IXOO GHYHORSHG ZLWK D VLQJO YHORFLWFRPSRQHQW Z [7KHUHDUHQR DSSOLHG SUHVVXUHJUDGLHQWV RQO JUDYLW 6ROYHWKH1DYLHU6WRNHVHTXDWRQIRUWKHYHORFLWSURILOHEHWZHHQWKHSODWHV

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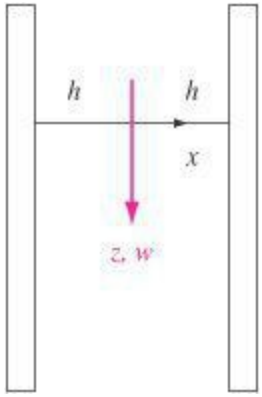


- 3 6KRZ WKDWWKHLQFRPSUHVVLEOHIORZ GLVWULEXWLRQ LQF\OLQGULFDOFRRUGLQDWHV

$Y_U \quad Y \quad \&U_Q Y_]$

ZKHUH& LVDFRQVWDQW D VDWLVILHVWKH1DYLHU6WRNHVHTXDWLRQIRUURQD VWRYDOXHVRI Q 1HJOHFWJUDYLW E . QRZ LQJ WKDWS SURQD ILQGWKHSUHVVXUHGLVWULEXWLRQIRUHDFKFDV DVXPLQJ WKDWWKHSUHVVXUHDWU 5 LVS : KDWPLJKWWKHVHZRFDVHVUHSUHVHQW

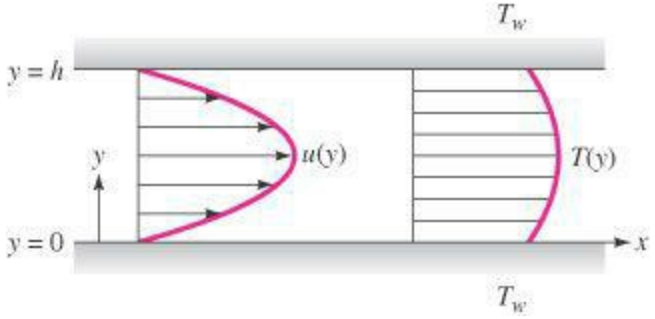
$QJXODUPRPHQWXP FRXSOHVWUHVVHV

- 3 5HFRQVLGHUWKHDQJXODUPRPHQWXP EDODQFHRI) LJ E\ DGGLQJ DFRQHQWDWGERG FRXSOH&] DERXWWKH] D[LV> @' HWHUPLQHDUHDWRQEHWZHHQWKHERG\ FRXSOHDQ VKHDUVWUHVVIRUHTXLOLEULXP : KDWDUHWKHSURSHUGLPHQVIRU&] " %RG\ FRXSOH DUHLPSRUWDQWQFRQWQXXVPHGLDZLWKPLFURVWUXFWXUH VXFKDVJUDQXODUPDWULDO

7KHGLIIHUHQWLDOHQHUJ\ HTXDWLRQ

- 3)RUSUHVVXUHGULYHQ ODPLQDUIORZ EHWZHHQSDUDOOHOSODWHV VHH) LJ E WK YHORFLWFRPSRQHQWDUHX 8 ±\ K % DQGZ ZKHUH8 LVWKHFHQWHUOLQH YHORFLW,QWKHVSHFLDOFDVH ([ILQGWKHPD[LPXPYHORFLW7 \ IRUDFRQVWDQ ZDOOWHPSHUDWXUH7Z
- 3 $VPHQWLRQHGLQ6HF VWKHYHORFLWSURILOHIRUODPLQDUIORZ EHWZHHQVWR SODWHV DVLQ) LJ 3 LV

3



$$u = \frac{4u_{\max} y(h-y)}{h^2} \qquad v = w = 0$$

,I WKH ZDOO WHPSHUDWXUHV DUH GLIIHUHQW IURP WKH XSVWUHDP IORZ HQHUJ\ HTXDWR
WR VROYH IRU WKH WHPSHUDWXUH GLVWULEXWLRQ 7 \ EHWZHHQ WKH ZDOO IRUPXODWLRQ

%RXQGDU\ FRQGLWLRQV

- 3 6XSSRVH ZH ZLVK WR DQDO]H WKH URWDWLQJ SDUW IXOF\ OQGHUR) LJ DV D VSLQ
  XS SUREOHP VWDUWLQJ IURP UHVWDQG FRQVLQ XQLOVROG ERG\ URWDWRQLV DFKLHYHG
  : KDWDUH WKH DSSURSULDWH ERXQGDU\ DQG LQLWLDO FRQGLWRQV IRUWKLV SUREOHP"
- 3)RU WKH GUDLQLQJ OTXLG ILOP R) LJ 3 ZKDWDUH WKH DSSURSULDWH ERXQGDU\
  FRQGLWRQV D DWWKH ERWWRP \ DQG E DWWKH VXUIDFH\ K"
- 3 6XSSRVH WKDW ZH ZLVK WR DQDO]H WKH VXGGHQSLSHH[SDQVLRQ IORZ R) LJ 3 XVLQ
  WKH IXOO FRQVHUYDWLRQ DQG 1DYLHU6WRNHV HTXDWRQV : KDWDUH WKH SURSHU ERXQGDU
  FRQGLWRQV WR KDQGOH WKLV SUREOHP"
- 3)RUWKH VROLFH JDWH SUREOHP R([DPSOH DWWDOWKH ERXQGDU\ FRQGLWRQV QHGH
  WR VROYH WKLV IORZ H[DFWO E\ VD\ &RPSXWDWRQDO) OXLG' \QDPLFV
- 3)OXLG IURP DOJHUHV-HUVRLUDWWP SHUDXUH7 IORZ VLQR DFLFXODUSLSH R UDGLXV5
  7KH SLSH ZDOO DUH ZRXQG ZLWK DQHODFWLF UHVLVWDQFH FRLOWKDWGHOLYHUV KHDWR WK
  IOXLG DWD UDWH T_Z HQHUJ\ SHUXQLWZ DOODUHD ,I ZH ZLVK WR DQDO]H WKLV SUREOHP E\
  XVLQJ WKH IXOO FRQVHUYDWLRQ 1DYLHU6WRNHV DQG HQHUJ\ HTXDWRQV ZKDWDUH WKH SURSH-
  ERXQGDU\ FRQGLWRQV IRUWKH DQDO\VLV"

6WHDP IXQFWLRQ

- 3 $ VZR GLPHQVLRQDO LQFRPSUHVVLEOH IORZ LVJLYHQ E\ WKH YHORFLWLILHOG9 \L [N
  LQ DUELWDU\ XQLWV ' RHV WKLV IORZ VDWLV\ FRQVHUWY" ,I VR ILQG WKH VWUHDP IXQFWLR
  $[\ DQG SORWV DIHZ VWUHDPOLQHV ZLWK DUURZV
- 3 &RQVLGHU WKH IORZLQJ VZR GLPHQVLRQDO LQFRPSUHVVLEOH IORZ ZKLFK FOHDUO
  VDWLVILHV FRQVHUYDWLRQ

X 8 FRQVDQW! 9 FRQVDQW

)LQG WKH VWUHDP IXQFWLRQ$ U R WKLV IORZ XVLQJ SRODU FRRUGLQDWHV

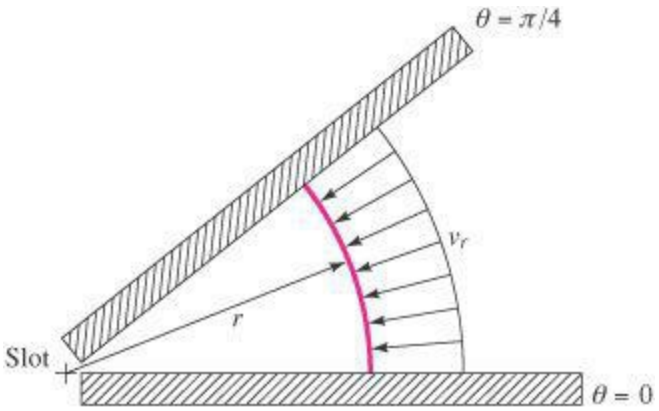
- 3 ,QYHVWLJDWH WKH VWUHDP IXQFWLRQ$. [í \ . FRQVDQW3ORW WKH VWUHDPOLQHV L
  WKH IXOO[\ SODQH ILQG DQ VXLWDEOH SRLQW DQG LQWURGXFH ZKDWWKH IORZ FRXO
  UHSUHVHQW
- 3 ,Q * HRJH6WRNHV R 1DYLHU6WRNHV IDPH \RXHG WKH SURSHUW R WWDO
  LQFRPSUHVVLEOH IORZ 5H\QROGV QXPEHU IORZ SDWWD VSKHUH XVLQJ VSKHULFDO SROD
  FRRUGLQDWHV U >5H SDJH @,QWKHVH FRRUGLQDWHV WKH HTXDWRQ R FRQVHUYDWL
  LV

$$\frac{\partial}{\partial r}(r^2 v_r \sin\theta) + \frac{\partial}{\partial \theta}(r v_\theta \sin\theta) = 0$$

D ' RHVDVWUHDP IXQFWLRQ[LVWRUWKHVHFRRUGLQDWH" E ,I VR ILQGLWIRUP

- 3 7KH YHORFLW SURILOH IRU SUHVXUHGULYHQ ODPLQDU IORZ EHWZHHQ SDUDOOHO SODWHV VH) LJ E KDV WKH IRUP X & K±\ ZKHUH& LV D FRQVWDQW D ' HWHUPLQH LI VWUHDP IXQFWLRQ[LVW E ,I VR ILQGDIRUPXODIRUWKHVWUHDP IXQFWLRQ
- 3 $ VZRGLPHQVLRQDO LQFRPSUHVVLEOH IUFWLRQDOHVV IOXLG LV JXLGHG E\ ZHGJH VKDSHG ZDOOV LQWR D VPDOO VORWDWWKH RULJLQ DVLQ) LJ 3 7KH ZLGWK LQWR WKH SDSHU LVE DQG WKH YROXPHWULF IORZ UDWH LV4 \$VDQ JLYHQ GLVWDQFH IURP WKH VORWWKH IORZ LV UDGLDOLQZDUG ZLWK FRQVWDQW YHORFLW) LQG DQ H[SUHVVLRQ IRUWKH SRODU FRRUGLQDWH VWUHDP IXQFWLRQ RI WKLV IORZ

3



- 3)RU WKH IXOO GHYHORSHG ODPLQDU SLSH IORZ VROXWLRQ R(T ILQG WK D[LXPPHWULF VWUHDP IXQFWLRQ\$U] 8VH WKLV UHVXOWWR GHWHUPLQH WKH DYHUDJH YHORFLW9 4 \$LQWKHSLSHDVDUDWR RI X PD[
- 3 \$QLQFRPSUHVVLEOHVWUHDP IXQFWLRQLVGHILQHGE\

$$\psi(x, y) = \frac{U}{L^2}(3x^2y - y^3)$$

ZKHUH8 DQG/ DUH SRVLWLYH FRQVWDQWV : KHUHLQWKLVFKDSWUDUHWKHVWUHDPOQHVRI WKLV IORZ SORWWHG' 8VH WKLV VWUHDP IXQFWLRQ WR ILQG WKH YROXPHIORZ 4 SDWLQJ WKURXJK WKH UHFWDQJXODUVXUFHZKRVHFRUQHUVDUHGHILQHGE\ [\ ] / / E / E DQG / 6KRZ WKHGLUHFWLRQRI4

- 3)RUWKHLQFRPSUHVVLEOHSODQHIORZ RI 3URE 3 ZLWKYHORFLWFRPSRQHQWX \ % [Z GHWHUPLQH D LI DVWUHDP IXQFWLRQ[LVW E ,I VR GHWHUPLQHWKHIRUP RI WKHVWUHDP IXQFWLRQ DQG F SORWVDIHZ UHSUHVHQWDWLYHVWUHDPOLQHV

9 HORFLW SRWHQWLDO YRUWLFLW

- 3 ,QHXWDWWKHYHORFLW SRWHQWLDÖ . [\ . FRQVWDQW6NHWFK WKHSRWHQWLDOOLQHV L WKH IXOO[\ SODQH ILQG DQ VWUHDPIXQFWLRQ SRLQW DQG VNHWFK LQE\ H\HWKRUWRJRQD

VWUHDPOLQHV : KDWFRXOGWKHIORZ UHSUHVHQW

- 3 $ VZLPPLQJSRROLQFRPSUHVVLEOHIORZ ILHOGLVGHILQHGE\ WKHYHORFLWFRPSRQHQW

$$u = 2V\left(\frac{x}{L} - \frac{y}{L}\right) \qquad v = -2V\frac{y}{L}$$

ZKHUH9 DQG/ DUH FRQVWDQWV ,I WKH H[LWWILQG WKH VWUHDP IXQFWLRQ DQG YHORFLV SRWHQWLDO

- 3 6KRZ WKDWWKHILQFRPSUHVVLEOHYHORFLWSRWHQWLDOLQSODQHSRODUFRRUGLQDWHVË U L VXFKWKDW

$$v_r = \frac{\partial \phi}{\partial r} \qquad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

)LQDOO VKRZ WKDWË DVGHILQHGKHUHVDWLVILHV/ DSODFH¶VHTXDWLRQLQSRODUFRRUGLQDWHVIR LQFRPSUHVVLEOHIORZ

- 3 &RQVLGHUWKHVZLPPLQJSRROLQFRPSUHVVLEOHYHORFLWSRWHQWLDOË [\ [í \ D ,VLWWUXHWKDWË DQG LI VR ZKDWGRHVWKLVPHDQ' E ,I LWH[LWWILQGWKHVWUHDP IXQFWLRQ$[\ RI WKLV IORZ F)LQGWKHHTXDWLRQ RI WKHVWUHDPOLQHWKDWSDVV WKURXJK [\

- 3 /LTXLG GULQV IURP D VPDOOKROHLQ D WDQN DV VKRZQ LQ)LJ 3 VXFK WKDWWK YHORFLWLVILHOGVHWXSLVJLYHQE\ %U§ %] § % . 5 UZKHUH] + LVWKHGHSW RI WKHZDWHUIURP WKHKROH ,VWKLVIORZ SDWWHUQURWDWLRQDORUUURWDWLRQDO') LQGWK GHSWK]& RI WKHZDWHUDWUDWKHUDGLXV 5

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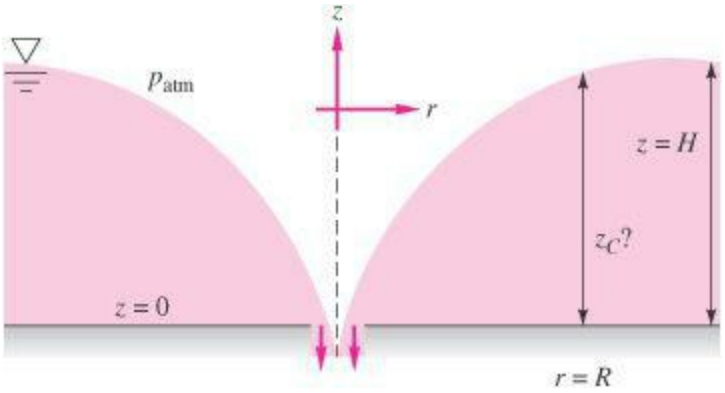


- 3)RUWKHLQFRPSUHVVLEOHSODQHIORZ RI3URE 3 ZLWKYHORFLWFRPSRQHQWX \ % [Z GHWHUPLQH D LI DYHORFLWSRWHQWLDO[LWW E ,I VR GHWHUPLQHWKHIRUP RI WKHYHORFLWSRWHQWLDO DQG F SORWDIHZ UHSUHVHQWDWLYHSRWHQWLDOOLQHV

- 3 6KRZ WKDWWKHOQHDU&RXHWWHIORZ EHWZHHQSODWHVLQ)LJ KDVDVWUHDP IXQFWLRQEH QRYHORFLWSRWHQWLDO: K\ LVWKLVVR'

- 3)LQG WKHVZRGLPHQVLRQDOYHORFLWSRWHQWLDOË U IRUWKHSRODUFRRUGLQDWHIORZ SDWWHUQ%U 4 U% . UZKHUH4 DQG. DUHFRQVWDQWV

- 3 6KRZ WKDWWKHYHORFLWSRWHQWLDOË U] LQD[LVPPHWULF FRQGXFWLRQIORZ VH

) LJ LVGHLQHGVXFKWKDW

$$v_r = \frac{\partial \phi}{\partial r} \quad v_z = \frac{\partial \phi}{\partial z}$$

) XUWKHUVKRZ WKDWRULQFRPSUHVVLEOHIORZ WKLVSRWHQWLDOLVDOVRLV/ DSODFH¶VHTXDWLRQL
U] FRRUGLQDWHV

- 3 $ VWUHDPOLQHORDOLQFRPSUHVVLEOHIORZ LVGHILQHGE\

$$u = -\frac{Ky}{x^2 + y^2} \quad v = \frac{Kx}{x^2 + y^2}$$

ZKHUH. FRQVWDQW,V WKLVIORZ LUURWDWLRQDO' ,I VR ILQGLWVYHORFLW\SRWHQWLDO VNHWFK
IHZ SRWHQWLDOOLQHV DQGLQWHUSUHWWKHIORZ SDWWHUQ

- 3 $ SDQHOSRODUFRRUGLQDWHYHORFLW\SRWHQWLDOLVGHILQHGE\

$$\phi = \frac{K \cos \theta}{r} \quad K = \text{const}$$

) LQGWKHVWUHDP IXQFWLRQIRUWKLVIORZ VNHWFK VRPHVWUHDPOLQHV DQGSRWHQWLDOOLQHV DQ
LQWHUSUHWWKHIORZ SDWWHUQ

- 3 $ VWUHDP IXQFWLRQIRUDSDQHO LURWDWLRQDO SRODUFRRUGLQDWHIORZ LV

& . OQU& DQG. FRQVW

) LQGWKHYHORFLW\SRWHQWLDOIRUWKLVIORZ 6NHWFK VRPHVWUHDPOLQHV DQGSRWHQWLDOOLQHV
DQGLQWHUSUHWWKHIORZ SDWWHUQ

6WUHDP IXQFWLRQ *and* YHORFLW\ SRWHQWLDO

- 3) RUWKH YHORFLW\ GLVWULEXWLRQ RI 3URE 3 D GHWHUPLQH LI D YHORFLW\ SRWHQWL
H[LWV DQG E LI LWGRHV ILQGDQH[SUHVVLRQIRUË[\ DQG VNHWFK WKHSRWHQWLDOOLQ
ZKLFKSDVVHVWKURXJKWKHSRLQW[\ / /
- 3 $ VWDG\ VWDWH LQFRPSUHVVLEOH IORZ KDV WKH IROORZLQJ SRODUFRRUGLQDWH YHORFLW
SRWHQWLDO

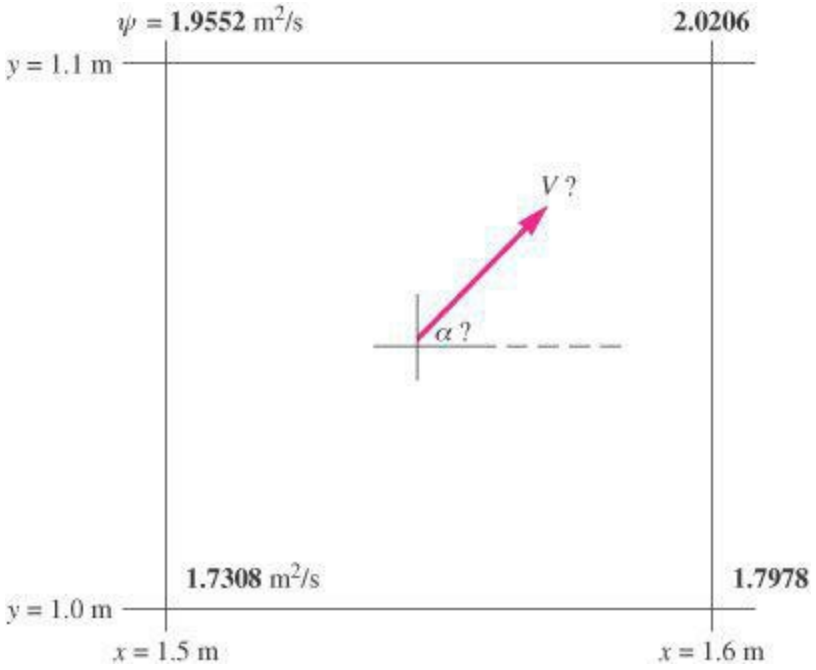
¥ & UFRV . OQU

ZKHUH& DQG. DUH FRQVWDQWV ' HWHUPLQH WKH VWUHDP IXQFWLRQ$ U IRUWKLV IORZ) R
H[WD FUHGLW OHW& EH D YHORFLW VFDOH8 DQG OHW 8/ VNHWFK ZKDWWKH IORZ PLJK
UHSUHVHQW

- 3 $ &)' PRGHORI VWDG\ VWDWH LQFRPSUHVVLEOHIORZ KDV SULQFLSDO RXWVI
YDOXHVRI VWUHDP IXQFWLRQ$[\ LQP V DWHDFK RI WKHIRXUFRUQHUVRI D VPDOO

FP E\ FP FHOO DV VKRZQ LQ) LJ 3 8VH WKH VHQXPEHUV WR HVWLPDWH WKH
UHVXOWDQW YHORFLW\ DW WKH FHQWURLG RI WKH FHOO DQG LWV DQJOH. ZLWK UHVSHFW WR WKH[D[LV

3



- 3 &RQVLGHU WKH IORZ LQJ VWR GLPHQVLRQDO IXQFWLRQ I [\

I $[%[\ &[' ZKHUH $!

D 8QGHU ZKDW FRQGLWLRQV LI DQ RQ $ % & ' FDQ WKLV IXQFWLRQ EH DVVRFLDWHG SODQH-
IORZ YHORFLW\ SRWHQWLDO" E ,I \RX ILQG DË [\ WR VDWLV\ SDUW D RI ILQG WKH DVVRFLDWHG
VWUHDP IXQFWLRQ $[\ LI DQ IRUW KLV IORZ

- 3 : DWHU IORZV WKURXJK DVWR GLPHQVLRQDO QR ZLQJ ZHGJH DW JDOP LQ SHUPHWHU
RI ZLGWK LQ RR WKH SDSHU) LJ 3 ,I WKLV LQZDUG IORZ LV SXUHO UDGLDO ILQG D
H[SUHVVLRQ LQ6, XQLW IRU D WKH VWUHDP IXQFWLRQ DQG E WKH YHORFLW\ SRWHQWLDO R
WKH IORZ $VVXPH RQH GLPHQVLRQDO IORZ 7KH LQFOXGHG DQJOH RI WKH ZHGJH LV *f*

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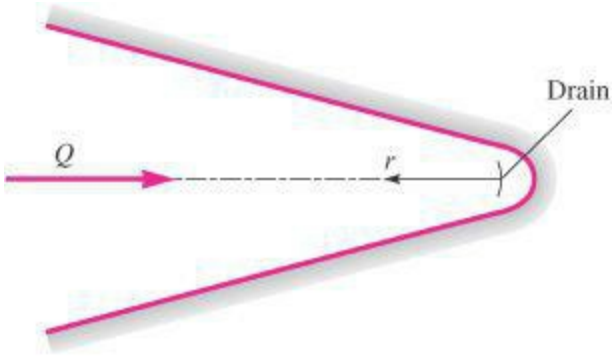

- 3 $ &) ' PRGHO RI VWDG VWR GLPHQVLRQDO LQFRPSUHVVLEOH IORZ KDV SULQFLSDO RXWW
YDOXHV RI YHORFLW\ SRWHQWLDO Ë [\ LQP V DW HDFK RI WKH IRXU FRUQHUV RI D VPDO

FP E\ FP FHOO DV VKRZ QLQ) LJ 3 8VH WKH VH QXPEHUV WR HVWLP DWH WK
UHVXOWDQW YHORFLW\ LQ WKH FHQWUH RI WKH FHOO DQG LWV DQJOH. ZLWK UHVSHFW WR WKH[D[LV

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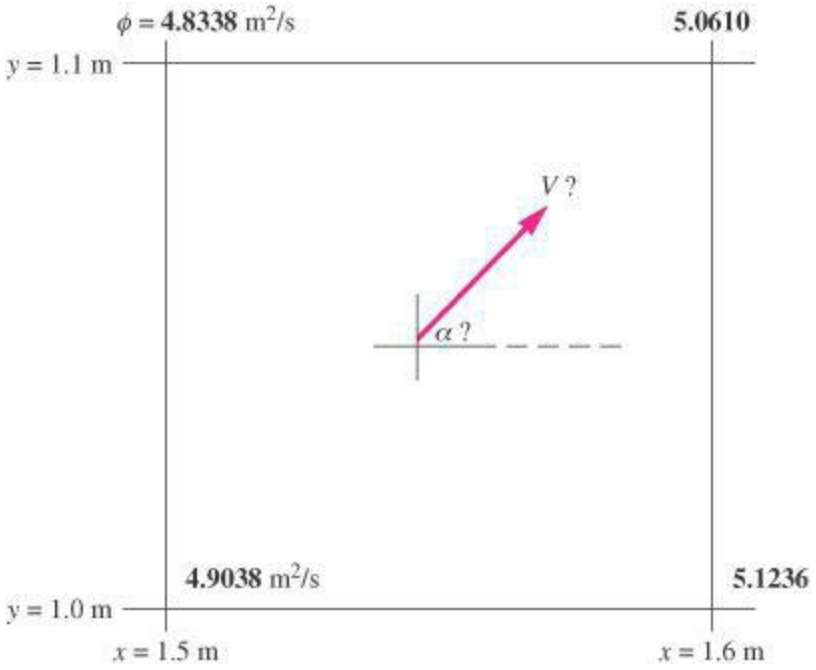

- 3 &RQVLGHU WKH VZR GLPHQVLRQDO LQFRPSUHVVLEOH SRODUFRRUGLQDWH YHORFLW\ SRWHQWLDO

¥ %UFRV %/

ZKHUH %LV D FRQVWDQW DQG / LV D FRQVWDQW WHQJWK VFDOH D : KDW DUH WKH GLPHQVLRQV R
%' E / RFDWH WKH RQO VWDJQDWLRQ SRLQW LQ WKLV IORZ ILHOG F 3URYH WKDW D VWUHDP IXQFWLR
H[LVWV DQG WKH ILQG WKH IXQFWLRQ $ U

- 3 * LYHQ WKH IROORZLQJ VWDG D[LV\PPHWULF VWUHDP IXQFWLRQ

$$\psi = \frac{B}{2}\left(r^2 - \frac{r^4}{2R^2}\right) \quad \text{where } B \text{ and } R \text{ are constants}$$

YDOG LQ WKH UHJLRQ U 5 DQG] / D : KDW DUH WKH GLPHQVLRQV R WKH FRQVWDQ
%' E 6KRZ ZKHWKHU WKLV IORZ SRVVHVVHV DY HORFLW\ SRWHQWLDO DQG LI VR ILQG LW F : KD
PLJKW WKLV IORZ UHSUHVHQW+LQW([DPLQH WKH D[LDO YHORFLW\ ]

- 3 $ VZR GLPHQVLRQDO LQFRPSUHVVLEOH IORZ KDV WKH YHORFLW\ SRWHQWLDO

Ë . [\ &,Q [\

ZKHUH . DQG & DUH FRQVWDQWV ,Q WKLV GLVFXVVLRQ DYRLG WKH RUJLQ ZKLFK LV DVLQJXODUL
LQ LQLWLDO YHORFLW\ D) LQG WKH VRHVWDQGDUG SRLQW WKLV IORZ ZKLFK LV VRPHZ KHUH L
WKH XSSHUKDO SODQH E 3URYH WKDW D VWUHDP IXQFWLRQ H[LVW DQG WKH ILQG $ [\ XVLQJ WK
KLQWKDWHG[D [DWÓ [D

- 3 2XWLGH DQ LQQHU LQMHFWLRQ FLUFOH RI UDGLXV5 D WURSLFDO VWRUP FDQ EH

VLPXODWHG E\ DSRODUFRRUGLQDWH YHORFLWV SRWHQWLDOË U 8R5 ZKHUH8R LV WK
ZLQG YHORFLWDWDGLXV5 D ' HWHUPLQHWKH YHORFLWV FRPSRQHQWV RXWLGHU 5 E
,I DV PL WKH YHORFLWV PLK DQG WKH SUHVXUH N3D FDOFXODWH WK
YHORFLWV DQG SUHVXUHDWU PL

- 3 $Q LQFRPSUHVLEOH LURWDWRQDO VZRGLPHQVLRQDO IORZ KDV WKH IROORZLQJ VWUHDP
IXQFWLRQLQSRODUFRRUGLQDWHV

$ U VLQ Q ZKHUH$ DQGQDUHFRQVWDQWV

)LQGDQH[SUHVVLRQIRUWKHYHORFLWVSRWHQWLDORI WKLVIORZ

,QFRPSUHVLEOHYLVFRXVIORZV

- 3 6WDUW WKHFRPELQHGHIFWRI WKHVZRRYLVFRXVIORZVLQ)LJ 7KDWLV ILQGX\
ZKHQ WKH XSSHU SODWH PRYHV DWVSHHG9 DQG WKHUH LV DOR D FRQVDQWSUHVXU
JUDGLHQWGS G[,VVXSHUSRVLWLRQSRVLEOH' ,I VR H[SODLQZK\ 3ORWUHPHQGDWY
YHORFLWV SURILOHVIRU D]HUR E SRVLWLYH DQG F QHJDWLYH SUHVXUHJUDGLHQWIR
WKHVDPHXSSHUZDOVSHHG9
- 3 2LO RI GHQVLW! DQG YLVFRVLW GUDLQV WKURXJK D YHUWLFDO SODW DV VK
)LJ 3 $IWUDGHYHORSPHQWHUHQFHWKHSRS RI WKHSODW WKHRLOILP ZLOOEHFRPI
LQGHSHQGHQWRI] DQG RI FRQVWDQWVK / $VXPH WKDWZ Z [RQO DQG WKDWWK
DWRVSKHUH RIHUV QR VKHDU UHVLVWDQFH WR WKH VXUIDFH RI WKH ILOP D 6RYH WKH 1DYL
6WRNHVHTXDWRQIRUZ [DQG VNHWFK LWVDSSURSULPDWH 6XSSRVHWKDWWLOP WKLFNQHV
/ DQG WKH VORSH RI WKH YHORFLW SURILOH DWWKH ZDOO>ÁZ Á[@DO DUHPHDVXUHG ZLWK D ODVHU
' RSSOHUDQHPRPHWU &KDS)LQGDQH[SUHVVLRQIRUROLVFRVLW DVDIXQFWLRQRI ! /
J >ÁZ Á[@DO

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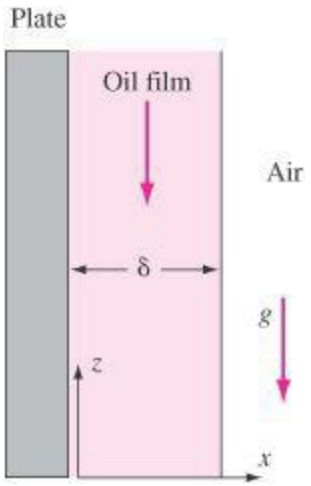

- 3 0 RGLI\ WKH DQDO\VLV RI)LJ WR ILQG WKH YHORFLWX ZKHQ WKH LQQHUF\OQGHUL
IL[HG DQG WKH RXWHUF\OQGHU URWDWHV DWDQJXODU YHORFLW9 0D\ WKLV VROXWLRQ E
DGDSWHG WR(T WR UHSUHVHQW WKH IORZ FDXVHG ZKHQ ERWK LQQHU DQG RXW

F\OQGHUV URWDW ([SODLQ\RXUFRQFOXVLRQ

- 3 $ VROG FLUFXODU F\OQGHU RI UDGLXV 5 URWDWV DW DQJXODU YHORFLW 9 LQ D YLVFRX LQFRP SUHVVLEOH OLGXLG WKDW LV DW UHVW DW UHPRWH GLVWDQFHV IURP WKH F\OQGHU DV LQ) LJ 3 0 DN VLPSOLI\LQJ DVVXPSWLRQV DQG GHULYH WKH JRYHUQLQJ GLIIHUHQWLDO HTXDWLRQ DQG ERXQGDU\ FRQGLWLRQV IRU WKH YHORFLW ILHOG % LQ WKH OLG ' RQRWRWH YHXQDWV\R DUH REVHUYHG Z LWK WKLV SUREOHP : KDWLV WKH VWDQG\ VWDWH OHZ ILHOG IRU WKLV SUREOHP"

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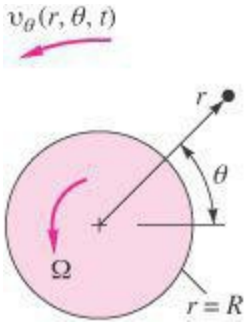
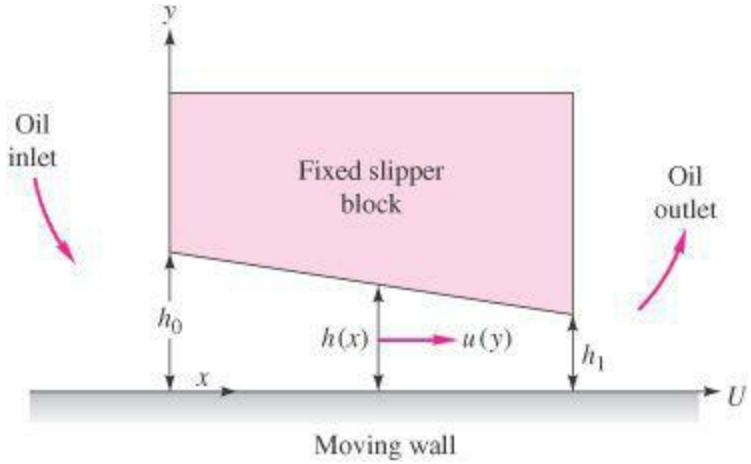


- 3 7KH IORZ SDWWHUQ LQ EHDULQJ OXEULFDWRQ FDQ EH LOOXVWUDWHG E\ ) LJ 3 Z KHUH L YLVFRXV RLO ! LV IRUFHG LQWR WKH JDS K[EHWZHHQ DIL[HG VOLSSHU EORFN DQG DZ PRYLQJ DWY HORFLW8 ,I WKH JDS LV WKLQ K≪ / LWFDQ EH VKRZQ WKDW WKH SUHVVXUH DQ YHORFLW GLVWULEXWLRQV DUH RI WKH IRUP S S[X X\ % Z 1HJOHFWLQJ JUDYLW UHGXFH WKH 1DYLHU6WRNHV HTXDWLRQV WR D VLQJOH GLIIHUHQWLDO HTXDWLRQ IRU X \ : KDWDUH WKH SURSHUERXQGDU\ FRQGLWLRQV" ,QWUDWHDQG VKRZ WKDW

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - yh) + U\left(1 - \frac{y}{h}\right)$$

Z KHUHK K[PD\ EHDQDUELWUDU VORZ0 YDULQJ JDS Z LGWK) RUIXUWKHULQRUP DWR RQOXEULFDWRQWKHRU VHH5H

3



- 3 &RQVLGHU D YLVFRXV ILOP RI OLTXLG GUDLQLQJ XQGHUP0 GRZ QWKH VLGH RI DYHUWFDOUR

VWHWVEHWKHVDPHDWERWKFRQGHUVXUIDFHV"

- 3 ,WLVGHVLUHGWRSXPSHWKDQRODW *f*& WKURXJK P RI VWDLJKWPRRWKWXELQJ XQGHUODPLQDUIORZ FRQGLWLRQV 5H! 9G 7KHDYDLODEOHSUHVVXUHGURSLV NBD D : KDWLV WKH PD[LPXP SRVVLEOHPDVV IORZ LQ NJ K" E : KDWLV WKDSSURSULDWHGLDPHWU"

- 3 &RQVLGHUZRGLPHQVLRQDOLQFRPSUHVVLEOHVWDG\ &RXHWWIORZ IORZ EHWZHHQWZ LQILQLWHSDUDOOHOSODWHVZLWKWKHXSSHUSODWPRYLQJDWFRQVWDQWVSHHGDQGWKHORZH SODWHVWDWLRQDU\ DVILJ D /HWWKHIOXLGEHQRQHZWRQLDQ ZLWKLWVYLVFRX VWUHVVJLYHQE\

ZKHUHD DQGF DUH FRQVWDQWV RI WKH IOXLG 0DNHDOOWKH VLPHDVXPSWRQV DV LQ WK GHULYDWLRQ RI (T D) LQG WKH YHORFLW SURILOHX \ E +RZ GRHV WKH YHORFLV SURILOHIRUWKLVFDVHFRPSDUHWRWKDW DQHZWRQLDQIOXLG'

- 3 $ VDQNRI DUHD$ LVGULOOLQJ LQDPLQDUIORZ WKURXJKDSLSHRI GLDPHWU' DQGOHQJV / DV VKRZQ ILJ 3 1HJOHFWLQJ WKH H[WWWWLQHWF HQHUJ\ DQG DVVXPLQJ WK SLSHIORZ LVGULYHQE\ WKHK\GURVWDWF SUHVXUHDWWWHQWDQFH GHULYHDIRUPXODIR WKHVDQNOHYHOK WLI LWLQLWLDOOHYHOLVK

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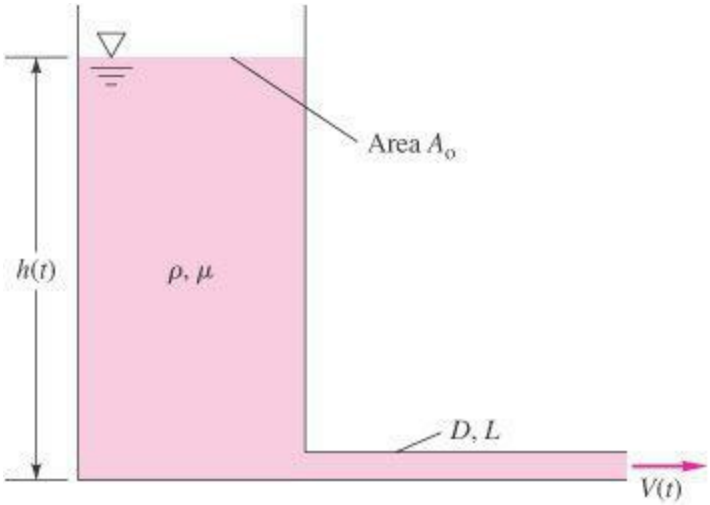

- 3 $ QXPEHURI VWDLJKW FP ORQJ PLFURXEHVRI GLDPHWUGDUHEXQGOHGWRJHWKHULQ D³KRQH\FRPE´ ZKRVHWRWDOFURVVVHFWLRQDODUHDLV P 7KHSUHVVXUHGURS IURP HQWDQFHWR H[LWLV NBD ,WLVGHVLUHGWKDWWKHWRWDOYROXPHIORZ UDWHWKURXJKWKHPLFURXEHGLDPHWU' E +RZ PDQ PLFURXEHVDUHLQWKHEXQGOH' F :KDWLV WKH5HQROGV QXPEHURI HDF

PLFURXEH'

- 3 $ ORQJ VROLG FA\OLQGHU URWDWHV VWHDGLO LQ D YHU YLVFRXV IOXLG DV LQ) LJ 3 $VVXPLQJ ODPLQDU IORZ VROYH WKH 1DYLHU6WRNHV HTXDWLRQ LQ SRODU FRRUGLQDWHV W GHWHUPLQH WKH UHVXOWLQJ YHORFLW GLVWULEXWLRQ 7KH IOXLG LV DVVXPHG IURP WK FA\OLQGHU >+LQW WKH FA\OLQGHU RHVQRW OQGXFH DQ UDGLDO PRWLRQ@

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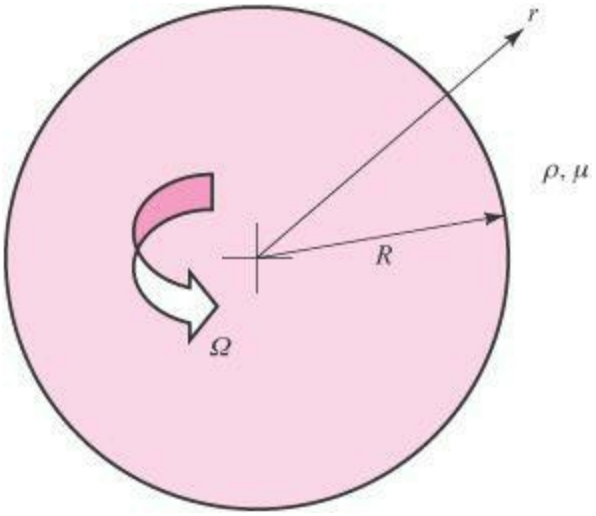

- 3 7Z RLPPLVFLEOHOLTXLGV RI HTXDO WKLFNQHVV K DUH EHLQJ VKHDUHG EHWZHHQ DIL[HG DQ DPRYLQJ SODWH DV LQ) LJ 3 * UDYLWLVQHJOHFWHG DQG WKHUHLVRYDULDWLRQ Z LW[ ) LQG DQH[SUHVVLRQ IRU D WKHYHORFLW DVWKHLQWHUFHDQG E WKHVKHDUVWUHVV LQ HDFK IOXLG \$VVXPHVWDG\ ODPLQDU IORZ

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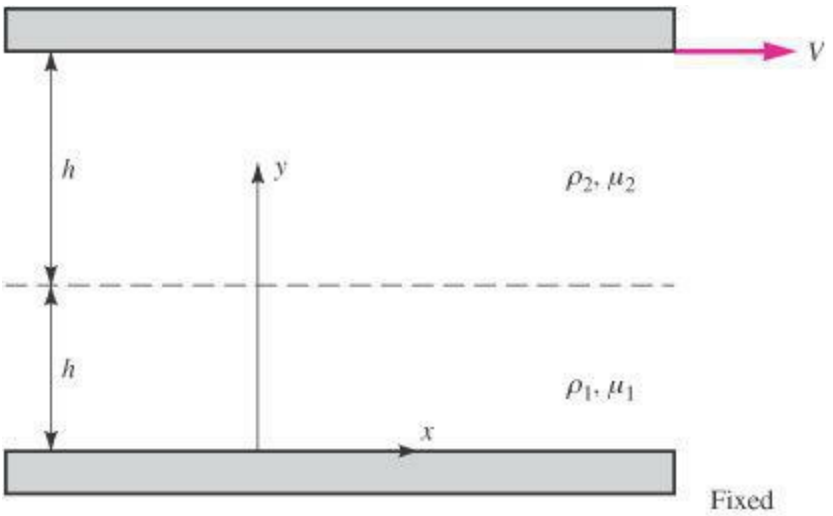

- 3 5HFRQVLGHU3URE 3 DQG FDOFXODWH D WKHLQQHUVKHDUVWUHVV DQG E WKHSRZHU UHTXLUHG LI WKHH[DFWODPLQDU IORUPXOD (T LV XVHG F ' HWHUPLQ ZKHWKHUWKLV IORZ SDWWHUQ LV VWDEOH + LQW 7KH VKHDUVWUHVV LQ U FRRUGLQDWHV LV QRWQHSDQHIORZ

6OLS IORZV

- 3)RUWKHSUHVXUHJUDGLHQWORZ EHWZHHQWZRSDUDOOHOSODQHV)LJ E UHDGDO]I IRUWKHFDVHRIVOSIORZ DWERWKZDOOV 8VHWKHVLPSOHVOSFRQGLWLRQX=DOO GXG\ =DOO ZKHUHOLVWKHPHDQIUHHSDWKRI WKHIOXLG D 6NHWFKWKHH[SHFWHG YHORFLWSURILOH E)LQGDQH[SUHVVLRQIRUWKHVKHDUVWUHVDWWDFKZDOO F)LQ WKHYROXPHIORZ EHWZHHQWKHSODWHV
- 3)RUWKHSUHVXUHJUDGLHQWORZ LQDFLUFXODUWEH LQ6HFW UHDGDO]HIRUWKHFD RI VOSIORZ DWWKHZDOO8VHWKHVLPSOHVOSFRQGLWLRQ%=DOO OGY GU=DOO ZKHUH LVWKHPHDQIUHHSDWKRI WKHIOXLG D 6NHWFKWKHH[SHFWHGYHORFLWSURILOH E)LQ DQH[SUHVVLRQIRUWKHVKHDUVWUHVDWWKHZDOO F)LQGWKHYROXPHIORZ WKURXJKWK WXEH

SURECOPVTXHVWLRQV: RUG3URECOPV

- : 7KHVROODFHFHOHUDWRQRI DIOXLGSDUWFOHLVJLYHQE\(T LQWKHHXOHUDQVWWAP ZKHUH9 LV DNQRZQIXQFWLRQ RI VSDFH DQG WPH ([SODLQKRZ ZH PLJKWHYDOXDW SDUWFOH DFFHOHUDWRQ LQWKH OJUDQJLDQIUDPH ZKHUHSDUWFOHSRVLWRQU LV DNQRZ IXQFWLRQ RI WPH DQG LQLWDO SRVLWRQ U IFQU W &DQ\RX JLYH DQ LQWXWDLYH H[DPSOH'
- : ,VLWWUXHWKDWWKHFRQWLQXLWUHDWRQ(T LVYDOGIRUERWKYLVFRXVDQGLQYLVFLG QHZWRQLDQDQGQRQQHZWRQLDQ FRPSUHVVLEOHDQGLQFRPSUHVVLEOHIORZ" ,I VR DUHWKHU DQ\ OPLWDWLRQVRQWKLVHTXDWLRQ'
- : &RQVLGHUD&' FRPSDFWGLVF URDWLQJ DWDQJXODUYHORFLW9 ' RHVLWKDYHYRUWFLW LQWKHVHQVHRI WKLVFKDSWU' ,I VR KRZ PXFK YRUWFLW'
- : +RZ PXFK DFFHOHUDWRQFDQIOXLGVHQGXUH' $UHIOXLGVOLNHDXWRQDXW ZKR IHHOWKH J LV VHYHUH' 3HUKDSVXVHWKHIORZ SDWWHUQRI(DPSOH DWU 5 WR PDNHVRPH HWWP DWWRI IOXLGDFFHOHUDWRQPDJQLWXGHV
- : 6WDWWKHFRQGLWLRQV WKHUHUHPRUHWKDQRQH XQGHUZKLFKWKHDQDOVLVRI WMPSHUDWXU GLVWULEXWLRQLQDIORZ ILHOGFDQEHFRPSOHWHO XQFRXSOHG VR WKDWDVHSDUDWHDQDOVLV IRUYHORFLWV DQG SUHVXUHLVSRVVLEOH &DQZHGR WKLVIRUERWKODPLQDUDQG WXUEXOHQ IORZ"
- : &RQVLGHUDTXLGIORZ RYHUDGDP RUZHU+RZ PLJKWWKHERXQGDU FRQGLWLRQVDQGWK IORZ SDWWHUQFKDQJHZKHQZHFRPSDUHZDWUIORZ RYHUDOJH SURRWSHWR6$(RLOIORZ RYHUDWQ VFDOHPRGH"
- : :KDWLVWKHGLIIHUHQFHEHWZHHQWKHVWHDP IXQFWRQ$DQGRXUPHWKRGRI ILQGLQJ WK VWHDPIQFWRQ6HF " 2UDUHWKH\HVWHQDDOWKHVDPH'
- : 8QGHUZKDWFRQGLWLRQVGRERWKWKHVWHDP IXQFWRQ$DQGWKHYHORFLWSRWHQWLDOÉH[L IRUDIORZ ILHOG': KHQGRHVRQHH[LWEXWQRWWKHRWKHU'
- : +RZ PLJKWWKH UHPDUNDEOH WKUHGLPHQVLRQDO7D\ORU LQWDEHORI)LJ EH SUHGLFWHG' ' LVFXVV DJHQHUDOSURFHGXUHIRUH[PLQLQJ WKHVWDELOLW RI DJLYHQIORZ SDWWHUQ
- : &RQVLGHUDQLUURWDWRQDO LQFRPSUHVVLEOH D[LVPPHWUF %% IORZ LQ U] FRRUGLQDWV' RHVDVWHP IXQFWRQH[LW'V,I VR GRHVLWVDWLV\ / DSODFH¶VHTXDWRQ $UHQHV RI FRQWDQW$HTXDO WR WKHIORZ VWHP IXQFWRQ" ' RHV D YHORFLW SRWHQWL H[LW'V,I VR GRHVLWVDWLV\ / DSODFH¶VHTXDWRQ'$UHQHV RI FRQWDQWÉHYHUZKU

SHUSHQGLFXODUWRWKH$QQDOV"

SUREOHPVTXHVWLRQV)XQGDPHQWDOVRI(QJLQHHULQJ([DP3UREOHPV

7KLVFKDSWHULVQRWDIDYRULWHRI WKHSHRSOHZKR SUHSDUHWKH)(([DP 3UREDE0 QRWDVLQJ
SUREOHP IURP WKLVFKDSWHUZLOODSSHDURQWKHH[DP EXWL VRPHGLG WKH PLJKWEHONHWKHVH

-)(* LYHQWKHVWDWH LQFRPSUHVVLEOHYHORFLWGLVWULEXWLRQ9 [L &\M N ZKHUH&
  LVDFRQVWDQWLI FRQVHUYDWLRQRI PDVVLVVDWLVILHG WKHYDOXHRI & VKRXOGEH

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-)(* LYHQWKHVWDWH YHORFLWGLVWULEXWLRQ9 [L M &\N ZKHUH& LVDFRQVWDQWL
  WKHIORZ LVLUURWDWRQDO WKHYDOXHRI & VKRXOGEH

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-)(* LYHQWKHVWDWH LQFRPSUHVVLEOHYHORFLWGLVWULEXWLRQ9 [L &\M N ZKHUH&
  LVDFRQVWDQWWKHVKHDUVWUHVV2[\ DWWKHSRLQW[\ ] LVJLYHQE\

  [&\

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-)(* LYHQWKHVWDWH LQFRPSUHVVLEOHYHORFLWGLVWULEXWLRQX $[% %a DQGZ &[\
  ZKHUH $ %& DUHFRQVWDQW7KLVIORZ VDWLVILHVWKHHTXDWRQRI FRQWLQXLWL $HTXDO
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-)( )RUWKHYHORFLWVILHOGLQ3UREOHP)(WKHFRQYHFWLYHDFFHOHUDWRQLQWKH[GLUHFWLR
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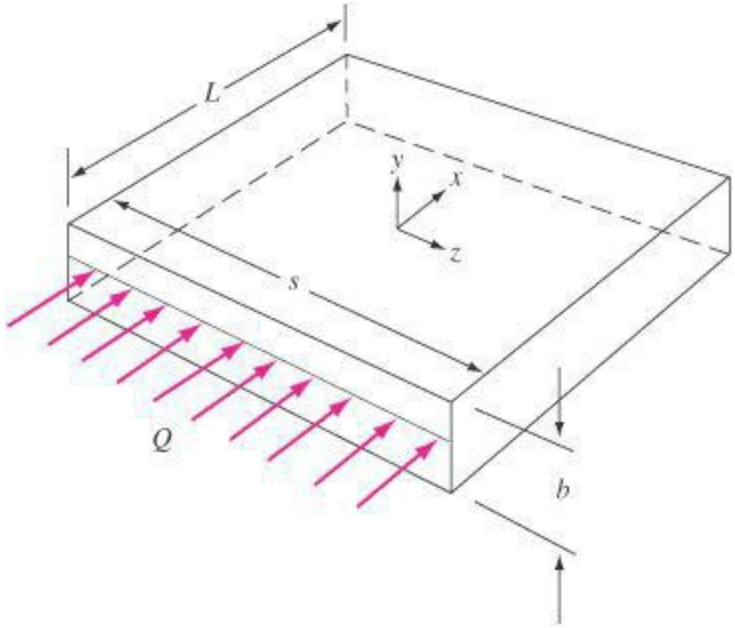
-)(,I IRUFHPLQDUIORZ LQD VPRRWKWXEHJKWXEH WKHWXEHGLDPHWHUDQG OHQJWKER

GRXEOH ZKLOHHYHUYKLQJ HOVHUHPDLQVWKHVDPH WKHYROXPHIORZ UDWH ZLOOLQFUHDVHE
DIDFWRUR

SUREOHPV TXHVWLRQV&RPSUHKHQVLYH3UREOHPV

- & ,Q D FHUWDLQ PHGLFDO DSSOLFDWLRQ ZDWHU DWURRP WPSHUDWXUH DQG SUHVVXUH IORZ
WKURXJKDUHFWDQJXODUFKDQQHORI OHQJWK/ FP ZLGWKV FP DQGJDSWKLFNQHVV
E PP DVLQ) LJ & 7KHYROXPHIORZ UDWHLVVLQXVRLGDOZLWKDPSOLWXGH
P/ VDQGIUHTXHQF\ I +] LH 4 VLQ GEW D &DOFXODWHWKHPD[LPXP
5H\QROGVQXPEHU 5H 9E% EDVHGRQPD[LPXP DYHUDJHYHORFLWDQGJDSWKLFNQHVV
&KDQQHOIORZ QXPEHUVUHPDLQVODPLQDUIRU5HOHVVWKDQDERXW ,I 5HLVJUHDW
WKDQDERXW WKHIORZ ZLOOEHWXUEXOHQW,VWKLVIORZ ODPLQDURUWXUEXOHQW E ,
WKLVSUREOHP WKHIUHTXHQF\ LVORZ HQRXJK WKDWDDQ JLYHQ WPH WKHIORZ FDQ EH
VROYHG DV LI LWZHUH VWDGH DWWKH JLYHQIORZ UDWH 7KLV LV FDOOHG DTXDVL VWDGH
DVXPSWLRQ $WDQ DUELWUDU LQWDQWR WPH ILQG DQ H[SUHVVLRQ IRU VWHDG\ZL
YHORFLWX DV DIXQFWLRQ R\ GS G[DQGE ZKHUHGS G[LV WKH SUHVVXUH JUDGLHQ
UHTXLUHG WR SXVK WKHIORZ WKURXJK WKHFKDQQHOD7YROXPHIORZ UDW4 ,Q DGGLWLRQ
HWWP DW WKHPD[LPXP PDJQLWXGHRI YHORFLWFRPSRQHQWX F $WDQ LQWDQWR WPH
ILQG DUHDWRQKLS EHWZHQ YROXPHIORZ UDW4 DQG SUHVXUH JUDGLHQWGS G[<RX
DQZHU VKRXOG EH JLYHQ DV DQ H[SUHVVLRQ IRU4 DV D IXQFWLRQ R GS G[V E DQ
YLVFRVLW G (VWP DW WKHZDOOVKDUWUHVV2 DVDIXQFWLRQRI I E V DQG WPI
W H)LQDOO IRUWKHQXPEHUVJLYHQLQWKHSUREOHP VWDWPHQWWWP DW WKHDPSOLWXG
R WKHZDOOVKDUWUHVV LQ1 P
- & $ EHOWPRYHVXSZDUGDWYHORFLW9 GUDJJLQJ DILOP R YLVFRXVOLTXLG R WKLFNQHVVK
DVLQ) LJ & 1HDUWKHEHOWKHILOP PRYHVXSZDUGGXHWRQRVOS $WWRXWUHGJH
WKHILOP PRYHVGRZQZDUGGXHWRJUDYLW $WXPLQJ WKDWWKHRQO QRQHUR YHORFLWLV
%[ZLWK]HUR VKHDUWUHVVDWWKHRXWHUILOP HGJH GHULYHDIRUPXODIRU D %[E
WKHDYHUDJHYHORFLW9DYJ LQWKHILOP DQG F WKHYHORFLW9F IRUZKLFKWKHUHLVQR QH
IORZ HLWKHUXSRUGRZQ G 6NHWFK%[IRUFDVH F

&



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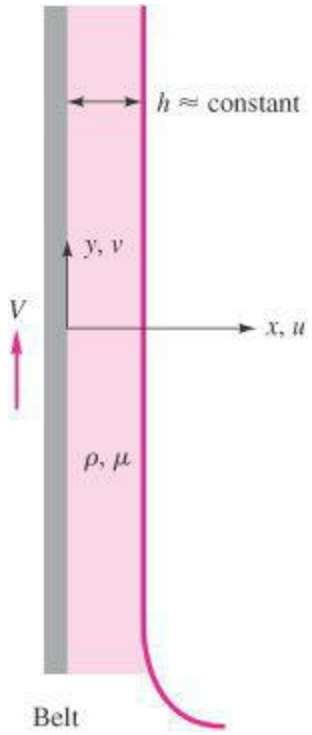


5 HIHUHQFHV

- ' $QGHUVRQ &RPSXWDWLRQDO) OXLG' \QDPLFV 7KH %DVLFV ZLWK $SSOLFDWLRQV 0 F* UDZ +LOO 1HZ <RUN

& (%UHQQHQ) XQGDPHQWDOV RI 0 XOWLSKDVH) ORZ &DPEULGJH 8QLYHUVLW\ 3UHVV 1HZ <RUN 6HHDOVR 85/

KWWS FDQWHKERRN FDQWHKGX PXOWLSKDVH!

6 0 6HQDWH &5& +DQGERRN RI 7DEOHV IRU 0DWKHPDWLFV VHFRQG &5& 3UHVV ,QF &OHYHODQG 2+

+ 6FKDFKWQJHVDO %RXQGDU\ /DYHU7KHRUHP 6SULQJHU 1HZ <RUN

) 0 : KLWH 9LFWRU) OXLG) ORZ GHG 0F* UDZ +LOO 1HZ <RUN

- 1 5HGG\ $Q,QWURGXFWRQ&RQWLQXXP 0HFKDQLFV &DPEULGJH 8QLYHUVLW\3UHVV 1HZ <RUN

- 3 +RPDQ +HW7UDQVIHU VHHG 0F* UDZ +LOO 1HZ <RUN

: 0 . DVDQG0 (&UDZIRUG&RQYHFWLYH+HDWDQG0DVV7UDQVIHU VHHG 0F* UDZ +LOO 1HZ <RUN

* . %DWKHORU$Q ,QWURGXFWRQ) OXLG ' \QDPLFV &DPEULGJH 8QLYHUVLW\ 3UHVV &DPEULGJH (QJODQG

/ 3UDQGWDQG 2 * 7LHWMHQV) XQGDPHQWDOV RI +\GURDQG $HURPHFKDQLFV ' RYHU 1HZ <RUN

5 $UV 9HFWRUV 7HQVRUV DQG WKH%DVLF (TXDWLRQV RI) OXLG0 HFKDQLFV ' RYHU 1HZ <RUN

2 * RQDOH] DQG $ 0 6WDUW$) LUVW&RXUVH LQ &RQWLQXXP 0HFKDQLFV &DPEULGJH 8QLYHUVLW\3UHVV 1HZ <RUN

' $ ' DQLHORQ 9HFWRUV DQG 7HQVRUV LQ (QJLQHHULQJ DQG 3K\VLFV G HG : HVWYLHZ 3HUVHXV 3UHVV %RXOGHU &2

5 , 7DQQHU (QJLQHHULQJ 5KHRORJ\ GHG 2[IRUG8QLYHUVLW\3UHVV 1HZ <RUN

+ / DPE +\GURG\QDPLFV WKHG ' RYHU 1HZ <RUN

0 0 . KRQVDUL DQG (5 %RRVHU$SSOHG 7UEERJ\ %HDULQJ ' HVLJQ DQG / XEULFDWLRQ : LOH\ 1HZ <RUN

* , 7D\ORU³6VDELOLW\ RI D 9LVFRXV / LTXLG &RQWLQLQHG EHWZHHQ 7Z R 5RWDWLQJ &\OLQGHUV 3KLORV 7UDQV 5R\ 6RF / RQGRQ6HU $ YRO SS ±

(/ . RVFKPLHGHU³7XUEXOHQW7D\ORU9RUWH[ )ORZ´ -) OXLG0 HFK YRO SW SS ±

0 7 1DU7 . 6HQJXSWD DQG 8 6 &KDXKDQ³) ORZ 3DVW5RWDWLQJ &\OLQGHUV DW+L 5H\QROGV1XPEHUV8VLQJ +LJKHU2UGHU8SZLQG6FKHPH´&RPSXWHUV DQG) OXLGV YRO QR SS ±

0 &RQWDQFHDX DQG& 0HQDUG³,QVWDELOLWHV RI 5RWDWRQ RQ WKH 1HDU: DNH ' HYHORSPHQW EHKLQG DQ,PSXOVLYHO 6WDUWHG &LUFXODU&\OLQGHU -) OXLG 0 HFKDQLFV YRO SS ±

– 2QH FDVH ZKHUH(T PLJKWQHG VSHFLDOFDUH LVWZR SKDVH IORZ ZKHUH WKH GHQVLW\ GLVFRQWLQXRXVEHWZHHQWKHSKDVHV)RUIXUWKHUGHWDLOVRQWKLVFDVH VHH5H IRUH[DPSOH

– 6HH IRUH[DPSOH 5H S

– $Q H[FHSWLRQ RFFXUV LQ JHRSK\VLFDO IORZV ZKHUH D GHQVLW\ FKDQJH LV LPSRVHG WKHUP DOO R PHFKDQLFDO UDWKHUWKDQ E\ WKH IORZ FRQGLWLRQV WKHPVHOYHV $Q H[DPSOH IS IUHVK ZDWUD\ DW UR ZDUP DLUD\HGXSRQFRG DLULQWKH DIWRSSKHUH : HVD\ WKDWWKH IXLG LV VWUDWLFG DQ ZHPXWDFFRXQWRUYHUWFDOGHQVLW\FKDQJHVLQ(T HYHQLI WKHYHORFLW\ DQG

– 7KLVLVDYHU UHDOWFIORZ WKDWLPXODWVWKHWKLQQLQJ RI DQLQYLVFLGIOXLG WKURXJKD ƒDQJH
VHH([DPSOHV DQG

– : KHQ FRPSUHVVLEOHOW LV VLJQLFDQW DGGLWRQDO VPDOO WUPV DULVH FRQWULQJ WKH HOPHQ
YROXPHH[SDQVLRQUDWHDQGDVFRQGFRHILFLHQWR YLVFRVLW VHH5HV DQG IRUGHWDLOV

– : HDUH QHJOHFWQJ WKH SRVLEOHW RI DILQLWHFRXSOHEHLQJ DSSOHG WR WKH HOPHQWE\ VRPH
SRZHUXOH[WHQGDORFHILHG 6HH IRUH[DPSOH 5H

– 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVWRI FRQWLQXLW

–)RUIXUWKHUGHWDLOV VHH HJ 5H S

–)RUWKLVVXVWHP ZKDWDUHWKHWKHUPRG\QDPLFHTXLYDOHQWV"(T "

— 6LQFH WKHSHUDWXUH LV HQWHUHO XQFRXSOHG E\ WKLV DVXPSWLRQ ZH PD\ QHYHUJHWDURXQG V
VROYLQJ IRULWKHUHQGPD\ DVN\RX WRZDLWXQWO\RX VDNHDFRXWHRQKDWWDQWHU

— 6HH6HFWLRQ

— 6HH6HFWLRQ

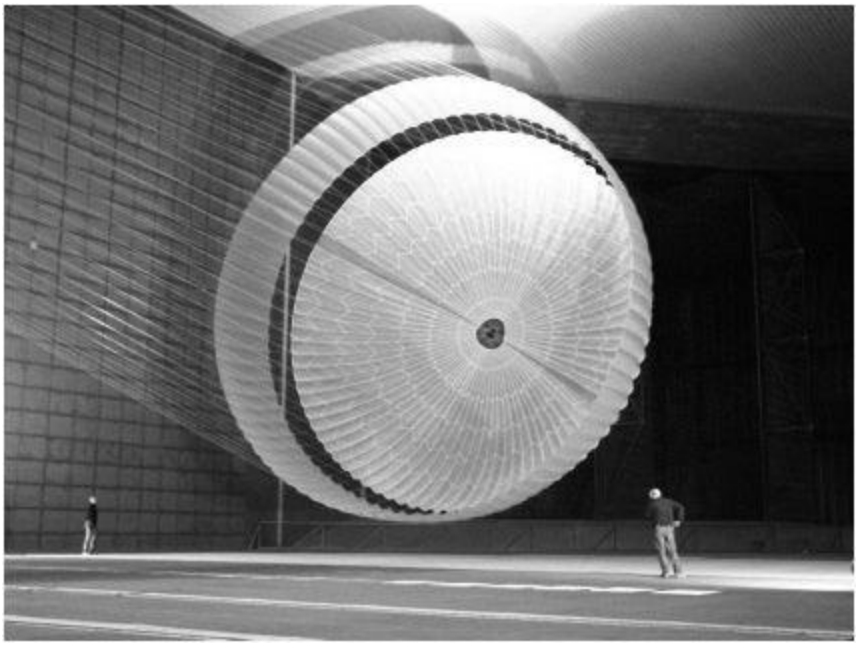
— (TXDWLRQV DQG DUHFDOHG WKH&DXFK\ 5LHPDQQ HTXDWLRQVDQG DUH WXGLHG L
FRPSOH[YDULDEOHWKRU

— 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVWRI FRQWLQXLW

— 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVWRI FRQWLQXLW &\OLQGHU) ORZ

— 1 DPHG DWU0 &RXHWW ZKRVH SLRQHULQJ SDSHULQ HWDE0WKHGURDWQJ F\OLQGHUV DV
PHWKRG WXGLHGWRGD\ IRUPHDVXULQJ WKHYLVFRVLWRI IOXLGV

&KDSWHU ' LP HQVLRQDO$QDO\VLVDQG6LP LODULW\



7ZR HQJLQHHUV DUH GZDUHG E\ D IXOOVFDOH SDUDFKXWH EHLQJ WHVWHG LQ DWWKH 1\$6\$ \$PHV5HVHDUFK &HQWHULQ0RIIHWW)LHOG &DOLIRUQLD 7KHZLGHVWXQQHO IWKLJKDQG IWZLG LVWKHODUJHVWXFKIDFLOVLQWKHZRUOG 7KHSDUDFKXWH IWLQGLDPHWHULVLQWHQGHGWRKHSODQ DQDO\WFDOURYLQJ ODERUDWRU\ RQ0DUV ,WKDV VXVSHQVRQOLQHVDQGLVWKHODUJHVW GLVNJDS EDQGWSH SDUDFKXWH> @HYHUEXLOW,WZDVGHVLJQHGE\ 3LRQHHU\$HURVSDFH RI 6RXWK: LQGVRU &RQHFWLFXWDQGFDQZLWKVWDQGRDGVLQH[FHVVRI OE

3KRWRFRXUWHV\ RI 1\$6\$ -3/ &DOWHFK

0 RWLYDWLRQ

,QWKLVFKDSWHUZHGLVFXVVWKHSDQQLQJ SUHVHQWDWLRQ DQGLQWURGXFWRU\ H[SHULPHQWDOGDWD : H VKDOOWW VR FRQYLQFH\RX WKDWXFK GDWD DUHEHVWSUHVHQWHGLQGLPHQVLRQOHVVIRUP ([SHULPHQWVWKD PLJKWHXOWLQWDEOHVRI RXSXWRUHYHQXHOSHYROXPHVRI WDEOHV PLJKWEHUHGXFHGWRDVLQJOHVHW FXUYHV RUHYHQDVLQJOHFXUYH ZKHQVXLWDEO QRQGLPHQVLRQDO]HG 7KHWHFKQLTXHIORUGRLQJWKLV L GLPHQVLRQDOQDO\VLV ,WV DOR HIFWLYHLQWKRUHWFDOWXGLHV

&KDSWHU SUHVHQWVGDWD VFDOHFRQWROYROXPHEDOOQHVRI PDV PRPHQWXP DQGHQHUJ\ ZKFH GWRJREDOUHVXOW PDV IORZ IRUFH VRUTXH VRDOZRUNGRQH RUKDWDQVHU&KDSWHU SUHVHQW LQLQWWLP DOEDOOQHV WKDWGG WR WKH EDVF SDWDOGIIHUHQWDOHTXDWRQV RI IOLG IORZ DQG VRP SDUWFXODUVROXWQV IRUERWKLQYLVFGDQGYLVFRXV ODPLQDU IORZ 7KHVHVWDJKWDQDOWFDOWFKQLTXH DUH OPLWG WR VLPSOH JHRPHWLHV DQG XQIRUP ERXQGDU FRQGLWRQV 2QO D IUFWRQ RI HQJLQHULQ IORZ SUREOPVFDQEHVROYHG E\ GLUHFWDQDOWFDOIRUPXODV

0 RWSUDFWFDOIOLG IORZ SUREOPV DUH WR FRP SOH[ERWK JHRPHWFDO DQG SK\VFDO WR EH VROYHG DQDOWFDOO 7KH PXWEH WWWG E\ H[SHULPHQWRU DSSUR[LPDWG E\ FRPSXWDWQDOIOXG GQDPLFV &)' > @7KHVH UHVXOW DUH WWSLFDO UHSRWWG DV H[SHULPHQWDORUQXPHULFDOGDWSRLQ

DQG VPRRWKHG FXUYHV 6XFK GDWD KDYH PXFK PRUH JHQHUDOLW LI WKH DUH H[SUHVVHG LQ FRP SDF
HFRQRPLF IRUP 7KLVLVWKHPRWLYDWRQIRUGLPHQVLRQDODQDOVLV 7KHWHFKQLTXHILVDPDLQWRO RI IOX
PHFKDQLFV DQG LV DOVR ZLGHO XVHG LQ DOOHQJLQHULQJ ILHOGV SXVWKHSK\VLFDO EHRRUDO PHGLFD
DQG VRFLDOVFLHQFHV 7KHSUHVHQWFKDSWUVKRZ VKRZ GLPHQVLRQDODQDOVLVPSURYHVWKHSUHVHQWDWR
RI ERWKGDWDDQG WKHRU

,QWURGXFWLRQ

%DVLFDOO GLPHQVLRQDO DQDOVLV LV D PHWKRG IRU UHGXFLQJ WKH QXPEHU DQG FRPSOH[LW R
H[SUHVVLRQVD YDULDEOHV WKDWDIHFWD JLYHQ SK\VLFDO SKHQRPHQRQ E\ XVLQJ D VRUWRI FRPSDFWQ
WKFKQLTXH ,I DSKHQRPHQRQG HSHQGVRQQ GLPHQVLRQDOYDULDEOHV GLPHQVLRQDODQDOVLVZLOOUHGXFHWKH
SUREOP WR RQ NGLPHQVLRQDWV YDULDEOHV ZKHUH WKH UHGXFWLRQQí N RU GHSHQGLQJ R
WKHSUREOP FRPSOH[LW * HQHUDOO Qí NHTXDOVWKHQXPEHURI GLIIHUHQWGLPHQVLRQV VRPHLPHVFDOOH
EDVLF RUSULPDU RUIXQGDPHQWDOGLPHQVLRQV WKDWURYHUQWKHSUREOP ,QIOXLG PHFKDQLFV WKHIRX
EDVLF GLPHQVLRQVDUHXVXDOO WDNHQWREHPDVV0 HQJWK/ WPH7 DQG WPSHUDWXUH, RUDQ0 / 7,
V\VWP IRUVKRUW$OWKRXJKWKHDYHO RQHXVHVDQ)/7, V\VWP ZLWKIRUFH) UHSODFLQJ PDV

$OWKRXJK LWVSXUSRVHLVWRUHGXFH YDULDEOHV DQG JURXS WKHP LQGLPHQVLRQOHVW IRUP GLPHQVLRQ
DQDOVLV KDVVHYHUDOVLGHEHQHILWV 7KHILUVWLV HQRUPRXV VDYLQJV LQ WPH DQG PRQH 6XSSRVHRQ
NQHZ WKDWWKHIRUFH) RQDSDUWFXODUERG\ VKDSHLPPHUVHGLQDXWHDP RI IOXLG GHSHQGHGRQ RQWK
ERG\ GHQJWK/ VWHDP YHORFLW9 IOXLG GHQVLW! DQGIOXLG YLVFRVLW WKDWV

$$F = f(L, V, \rho, \mu) \tag{5.1}$$

6XSSRVHIXUWKHUWKDWWKH JHRP HW DQG IORZ FRQGLWRQV DUH VR FRP SOFDWG WKDWRXU LQWUL
WKHRUHV &KDS DQG GLIIHUHQWDOHTXDWRQV &KDS IDLOWR\LHOGWKHVROXWRQIRUWKHIRUFH 7KHQZ
PXVWILQGWKHIXQFWRQI / 9 ! H[SUHVVHGRUQXPHULFDO

* HQHUDOO VSHDNLQJ LWWDNHVDERXW SRLQWVWRGHILQHDFXUYH 7RILQGWKHIIHFWRI ERG\ GHQJWKL
(T ZHKDYHWRUHQWKH H[SHULPHQWRU GHQJWK/) RUHDFK/ ZHQHG YDOXHVRI 9
YDOXHVRI ! DQG YDOXHVRI PDNLQJ DJUDGWRWDORI RU H[SHULPHQWV $W SH
H[SHULPHQW ZHOO\RX VHHZKWZHDUHJHWWQJ LQWR +RZHYHU WKLVGLPHQVLRQDODQDOVLV ZHFD
LPPHGLDWO UHGXFH(T WRWKHHTXLYDOHQWRUP

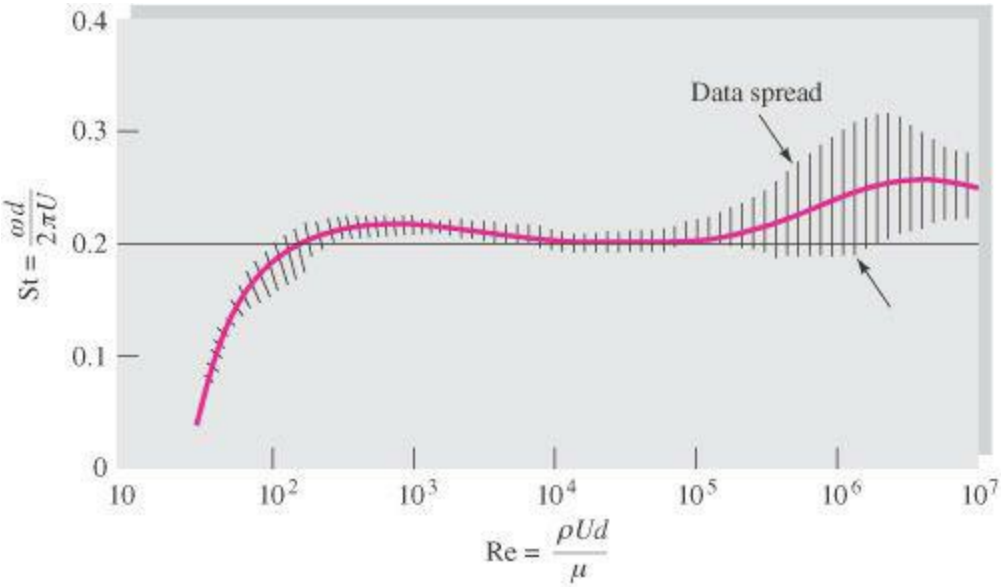
$$\frac{F}{\rho V^2 L^2} = g\left(\frac{\rho V L}{\mu}\right) \qquad C_F = g(\mathrm{Re}) \tag{5.2}$$

RU

7KDWLV WKH GLPHQVLRQDWVIRUFH FRHILFLHQW) !9 / LV D IXQFWRQ RQO RI WKH GLPHQVLRQDW
5H\QROGVQXPEHU!9/ : HVKDOOHDUQH[DFWO KRZ WRPDNHWKLVUHGXFWRQLQ6HFV DQG
1RWWKDW(T LVMXWDQH[DPSOH QRWWKH IXQGWRU RI IRUFHVFDXVHGE\ IOXLGIORZV 6RPH
IOXLGIORFHVKDYHDYHU ZHDNRUQHJOJLEOH5H\QROGVQXPEUGHSHQGHQFHLQZLGHUHJRQV) LJ D
2WKHUJURXSVPD\ DOVREHLPSRUWDQW7KHIRUFHFRHILFLHQWPD\ GHSHQG LQKLJK VSHGJDVIORZ RQWK
0DFKQXPEU0D 9 DZKHUHDLVWKHVSHGRI VRXQG ,QIUHXUDFHIORZV VXFKDVWKLS GUDJ &)

PD\ GHSHQG XSRQ) URXGHQXPEHU) U 9 J/ ZKHUH J LV WKH DFFHOHUDWLRQ RI JUDYLW ,QWKLEXOH
IORZ IRUFHPD\ GHSHQG XSRQWKHURXJKQHVVUDWLR ë/ ZKHUHë LVWKHURXJKQHVVKHLJKWRI WKHVXUIDFH

(*a*)



(*b*)

) LJ 9RUWH[VKHGGLQJ IURP DFLUFXODUF\OLQGHU *a* YRUWH[VWUHHWEHKLQG DFLUFXODUF\OLQGH
(Courtesy of U.S. Navy); b ***H[SHULPHQWDOVKHGGLQJ IUHTXHQFLHV(data from Refs. 25 and 26)***

7KHIXQFWLRQJ LVGLIIHUHQWPDWHULDOWDEOH IURP WKHRULJLQDOIXQFWLRQI EXWWKRQDOLQVDOOWKHVDPH
LQRUPDWRQ 1RWKLQJ LVRWWLQDGLPHQVLRQDOLQDOLV\ $QGWKLQNRI WKHVDYLQJV: HFDQHXDEOWKJE\
UXQQLQJ WKH H[SHULPHQWRURQOD YDOXHVRI WKHVLQJOHYDULDEOHFDOOHG WKH5H\QROGVQXPEHU: HGF
QRWKDYHWRYDU / 9 ! RU VHSDUDWHO EXWRQO WKHJURXSLQJ !9/ 7KLVZHGRPHUHO E\ YDULQ
YHORFLW9 LQ VD\ DZLQGWXQQHORUGURSWWWRUZDWUFKDQQHODQG WKHUHLV QR QHG WR EXLOG
GLIHUHQWERGLHVRUILQG GLIHUHQWOXLGVZLWK GHQVLWLHV DQG YLVFRVLWLHV 7KHFRWVLQRZ

DERXW PDVEHOHVV

$ VHFRQG VLGH EHQHILW RI GLPHQVLRQDOLQDOVLV VKDWLWKHSVRXUWKLQNLQJ DQG SDQQLQJ IRUD H[SHULPHQWRUWKHRU ,WXJJHVWVGLPHQVLRQDVZDVRI ZULWQJ HTXDWRQVEHRUHZHVHQGPRQH RI FRPSXWHU DQDOVLV WR ILQG VROXWRQV ,WVXJJHVWV YDULDEOHV WKDWFDQ EH GLVFDUGHG VRPHWLPH GLPHQVLRQDOLQDOVLZLOOLPPHGLDWH UHMFWYDULDEOHV DQG DWRWKHUWLPHVLWUXSVWKP RI WKH VLGH ZKHUHDIHZ VLPSOHWHWVZLOOKRZ WKHP WREHXQLPSRUWDQW) LQDOO GLPHQVLRQDOLQDOVLZLO RIWQJLYHDJUHDWGHDORI LQVLJKWQRWKHIRUP RI WKHSK\VFDOUHDWRQVLS ZHDUHWULQJ WRVXG

$ WKLUGEHQHILWVWKDWGLPHQVLRQDOLQDOVLV SURYLGHVVFDOQJ ODZVWKDWFDQFRQYHUWGDWIURP I FKHDS VPDOOPRGHOWR GHVLJQ LQRUP DWRQ IRUDQ H[SHQVLYH ODUJHSURWRWSH : H GR QRWEXLGG L PLOOLRQGROODUDLSODQHDQGVHHZKHWKHULWKDVHQRXJKOLWRUH : HPHDVXHWKHOLWRQDVPDOOPRGH DQGXVHDVFDOQJ ODZ WRSUHGLFWWKHOLWRQWKHIXOOVFDOHSURWRWSHDLSODQH 7KHUHDUHUKHVZHVKO H[SODLQIRUILQGLQJVFDOQJ ODZV : KHQWKHVFDOQJ ODZ LVYDOG ZHVD\ WKDWDFRQGLWRQRI VLPLODULW H[LWVEHWZHHQWKHPRGHODQGWKHSURWRWSH ,QWKHVLPSOHFDVHRI (T VLPLODULWLVDFKLHYHGL WKH5H\QROGVQXPEHULWKHVDPHIRUWKHPRGHODQGSURWRWSHEHFDXVHWKHIXQFWLRQJ WKHQUHTXLUHVWK IRUFHFRHILFLHQWWREHWKHVDPHDOR

$$\text{If} \quad \mathrm{Re}_m = \mathrm{Re}_p \quad \text{then} \quad C_{Fm} = C_{Fp} \tag{5.3}$$

ZKHUH VXEVFULSWP DQGS PHDQPRGHODQG SURWRWSH UHVSHFWLYHO) URP WKHGHILQLWLRQ RI IRUFH FRHILFLHQWWKLVPHDQVWKDW

$$\frac{F_p}{F_m} = \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 \tag{5.4}$$

IRUGDWVDNHQZKHUH!S9S/S S !P9P/P P (TXDWRQ LVDVFDOQJ ODZ ,I \RXPHDVXHWK PRGHOIRUFHDWWKHPRGHO5H\QROGVQXPEHUWKHSURWRWSHIRUFHDWWKHVDPH5H\QROGVQXPEHUHTXDO WKHPRGHOIRUFHWLPHVWKHGHQVLWUDWLRVTXDUHGWLPHVWKHOHQJWKUDWLRVTXDUHG : HVKDOOJLYHPRUHH[DPSOHVODWU

' R\RXXQGHUVWDQGWKHVHLQWRGXFWRU H[SODQDWLRQV" %HFDXVHXOHDUQLQJ GLPHQVLRQDOLQDOVLVL QLHOHDUQLQJ WRSOD\ WHQQLV 7KHUHDUHOHYHOVRI WKHJDPH : HFDQHVWDEOLVKVRPHJURXQGUXOHVDQGGH VRPHIDLO JRRGZRUNLQWKLVEUHFKDSWU EXWGLPHQVLRQDOLQDOVLVLQWKHEURDGYLHZ KDVPDQ VXEWOHWLHV DQG QXDQFHV WKDWRQO WLPH SUDFWLFH DQG PDXULW HQDEOH \RX WR PDXWU $OWKRXJ GLPHQVLRQDOLQDOVLVKDVDIILP SK\VFDOLQGPDWKHPDWFDOIRXQGDWLRQ FRQVLGHUDEOHDWWDQGWKLOODU QHHGHGWRXVHLWHIFLHQWO

(; $0 3/ (

$ FRSHSRGLVDZDUUFUXWDFHDQDSSUR[LPDWH PP LQGLDPHWU : HZDQWWR NQRZ WKHGUD IRUFHRQWKHFRSHSRGZKHQLWPRYHVWKURXJ0 LQIUHKZDWU$ VFDOHPRGHO WLPHVODUJHULVPDGH DQG WWWWG LQ JOFHULQ DV9 FP V 7KH PHDXUHG GUDJ RQ WKH PRGHOLV 1) RU VLPLOD FRQGLWRQV ZKDWDUH WKH YHORFLW DQG GUDJ RI WKH DFWXDOFRSHSRG LQ ZDWU $ VWXPHWKDW(T DSSOHVDQG WKHWLPSHUDWXUHLV *&

6RXWLRQ

- 3URSHUWLHV) URP 7DEOH$ WKH GHQVLWLHV DQG YLVFRVLWLHV DW f& DUH

Water (prototype):	$\mu_p = 0.001$ kg/(m-s)	$\rho_p = 998$ kg/m^3
Glycerin (model):	$\mu_m = 1.5$ kg/(m-s)	$\rho_m = 1263$ kg/m^3

- $VVXPSWLRQV (TXDWLRQ LV DSSURSULDWH DQG VLPLODULWLHV DFKLHYHG WKDWLV WKH PRGHO DQG SURWRWSHKDYHWKHVDPH5H1ROGVQXPEHUDQG WKHUHIRUH WKHVDPHIRUFHFRHILFLHQW
- $SSURDFK 7KH GHQJWK VFDOHV DUH/ P PP DQG/ S PP &DOFXODWH WKH 5H1ROGV QXPEHUDQG IRUFHFRHILFLHQWRI WKHPRGHODQG VHWWKH HTXDOWRSURWRWSHYDOXHV

$$\mathrm{Re}_m = \frac{\rho_m V_m L_m}{\mu_m} = \frac{(1263\ \text{kg/m}^3)(0.3\ \text{m/s})(0.1\ \text{m})}{1.5\ \text{kg/(m-s)}} = 25.3 = \mathrm{Re}_p = \frac{(998\ \text{kg/m}^3)V_p(0.001\ \text{m})}{0.001\ \text{kg/(m-s)}}$$

Solve for $V_p = 0.0253$ m/s = 2.53 cm/s *Ans.*

,QQHPDQQHU XVLQJ WKHSURWRWSHYHORFLWLMXVWRXQG HTXDWWKHIRUFHFRHILFLHQW

$$C_{Fm} = \frac{F_m}{\rho_m V_m^2 L_m^2} = \frac{1.3\ \text{N}}{(1263\ \text{kg/m}^3)(0.3\ \text{m/s})^2(0.1\ \text{m})^2} = 1.14$$

$$= C_{Fp} = \frac{F_p}{(998\ \text{kg/m}^3)(0.0253\ \text{m/s})^2(0.001\ \text{m})^2}$$

Solve for F_p = 7.3E-7N *Ans.*

- &RPPHQWV $VVXPLQJ ZHPRGHOHG WKH5H1ROGVQXPEHUFRUUHFWO WKHPRGHOWWWLV DYHV JRRGLGHD DVLWZRXOGREYLRXVO EHGLIILFXOWRPHDVXUHVXFKDWLQ FRSHRGGUDJ IRUFH

+LVWRULFDOO WKH ILUVWSHUVRQ WR ZULWH[WQLYHO DERXWXQLW DQG GLPHQVLRQDO UHDVRQLQJ LQ SK\VLFDO UHDVRQV ZDV (XOHUL (XOHU¶V LGHDV ZHUH IDUDKHDG RI KLV WLPH DV ZHUH WKRVH R -RVHSK) RXULHU ZKRVH ERRN$QDO7KHRU RI +HDWRXWQHG ZKDWLV QRZ FDOOHG WK SULQFLSOH RI GLPHQVLRQDOKRPRJHQHLW DQG HYHQ GHYHORSHG VRPH VLPLODULW UXOHV IRU KHDWIORZ 7KHUH ZHUH QR IXUWKHU VLJQLILFDQWDGYDQFHV XQWLO/ RUG 5D\HLJK¶V ERRN LQ 7KHRU RI 6RXQ ZKLFK SURSRVHG D³PHWKRG RI GLPHQVLRQ´ DQG JDYH VHYHUDO[DPSOHV RI GLPHQVLRQDODQDOVLV 7K ILQDO EUHDNWKURXJK WKDWHVWDEOLVKHG WKH PHWKRG DV ZH NQRZ LWWRGD\ LV JHQHUDOO FUHGLWHG WR (%XFNLQJKDP LQ > @ZKRVH SDSHUURXWLQHG ZKDWLV QRZ FDOOHG WKH%XFNLQJKDP 3L7KHRUHP IR GHVFULELQJ GLPHQVLRQDOV\VWHPDWLV VHH6HF + RZHYHU LWLV QRZ NQRZQWKDWD) UHQFKPDQ $ 9DVFK LQ DQG D 5XVLDQ ' 5LDERXFKLQVN LQ KDG LQGHSHQGHQWO SXEOLVKHG SDSHU UHSRUWLQJ UHVXOWV HTXLYDOHQWWR WKH SL WKHRUHP) RORZLQJ %XFNLQJKDP¶V SDSHU 3 : %ULGJPD SXEOLVKHG DFODVVLF ERRNLQ > @RXWOLQLQJ WKH JHQHUDOWKHRU RI GLPHQVLRQDODQDOVLV ' LPHQVLRQDODQDOVLV LV VR YDOXDEOH DQG VXEWOH Z LWK ERWK VNLOODQG DUWLQRYHG WKDWLWKD VSDZQHG DZLGHYDULHWRI WRSLFV7KHZULWUL DZ DUHRI PRUHWKDQ ERRNVRQWK VXEMHFWRI ZKLFK KLV HQJLQHHULQJ IDYRULWHV DUH OLWWG KHUH > ± @' LPHQVLRQDO DQDOVLV LV QR FRQLQHG WR IOXLG PHFKDQLFV RUHYHQWRHQJLQHHULQJ 6SHFLDOL]HG ERRNVKDYH EHHQ SXEOLVKHG RQ

DSSOLFDWLRQ RI GLP HQVLRQDODQDO VLV WR PHWRRJ\ > @DWPRSK\VLFV> @HFRQRPLFV> @FKHPLVWU
> @K\GURRJ\ > @PHGLFDWLRQ/> @FOLQLFDOPHGLFLQH> @FKHPLFDOSURFHVVLQJSODQW> @
VRFLDOVFLHQFHV> @ELRPHGLFDOVFLHQFHV> @SKDUPDF\ > @IUDFWDOJHRPHWU > @DQGHYHQW
JUDZ WKRI SODQW> @&OHDU WKLVLVD VXEMHFWZHOZ RUWKOHDUQLQJ IRUPDQ FDUHUSDWK

7KHSULQFLSOHRI GLP HQVLRQDOKRP RJHQHLW\

,QPDNLQJ WKHUHPDUNDEOHIXP SIURP WKHILYH YDULDEOH(T WR WKHWZR YDULDEOH(T ZI
ZHUH H[SORLWLQJ D UKHWKDVLV DOP RVWD VHO HYLGHQWL[LRP LQ SK\VLFV 7KLV UKHSULQFLSOHR
GLP HQVLRQDOKRP RJHQHLW\ 3' + FDQEHVWDWHGDVIROORZV

$OWKHHTXDWRQVWKDWDUH GHULYHG IURP WKHWKRU RI PHFKDQLFV DUH RI WKLVIRUP) RUH[DPSO
FRQVLGHUWKHUHDWRQWKDWW[SUHVVHVWKHGLVSODFHPHQWRI DIDOOLQJ ERG\

$$S = S_0 + V_0 t + \tfrac{1}{2} g t^2 \qquad (5.5)$$

(DFKWHUP LQWKLVHTXDWRQLV DGLVSODFHPHQWRUOHQJWK DQGKDVGLP HQVLRQV^/` 7KHHTXDWRQL
GLP HQVLRQDOO KRP RJHQHRXV 1 RWH DOR WKDWDQ FRQVWDQWQWHWR XQLW FDQ EH XVHG WR FDOFXODWH
UHVXOWV

,I DQHTXDWRQWKO H[SUHVVHVDSURSHUUHDWRQKLSEHWZHQYDULDEOHVLQDSK\VLFDOSURFHVV
LWZ LOOEHGLP HQVLRQDOO KRP RJHQHRXV WKDWLV HDFK RI LWV DGGLWLYH WHUP V ZLOOKDYH WKH VDP
GLP HQVLRQV

&RQVLGHU%HUQRXOOL¶VHTXDWRQ IRULQFRP SUHVVLEOHIORZ

$$\frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{const} \qquad (5.6)$$

(DFKWHUP LQFXGLQJ WKHFRQVDQWKDVGLP HQVLRQVRI YHORFLWVTXDUHG RU^/ 7í ` 7KHHTXDWRQL
GLP HQVLRQDOO KRP RJHQHRXVDQGJLYHVSURPHUUHVXOWVIRUDQ FRQVLVWHQWV\VWHP XQLW
6WXGHQWV FRXQWRQ GLP HQVLRQDOKRP RJHQHLW\ DQG XVH LWWR FKHFN WKHP VHOYHV Z KHQ WKH FDQQ
TXLWHUHP HP EHUDQHTXDWRQGXULQJ DQH[P) RUH[P SOH Z KLFK LVLW

$$S = \tfrac{1}{2} g t^2? \qquad \text{or} \qquad S = \tfrac{1}{2} g^2 t? \qquad (5.7)$$

%\ FKHFNLQJ WKHGLP HQVLRQV Z HUHPHFWKH VHFRQG IRUP DQG EDFN XS RXUIDXOWV P HP RU : HDU
H[SORLWLQJ WKHSULQFLSOHRI GLP HQVLRQDOKRP RJHQHLW\ DQGWKLVFKDSWHUPLP S0 H[SORLWLWXUKHU

9 DULDEOHVDQG&RQVWDQWV

(TXDWRQV DQG DOR LOOXVWDWH VRP H RWKHUIDFWRUV WKDWRWHQ HQHUJLQR D GLP HQVLRQDO
DQDOVLV

- ' LP HQVLRQDOYDULDEOHV DUH WKH TXDQWLWHV WKDWDFWXDOO YDU GXULQJ D JLYHQ FDVH DQ
Z RXOG EH SORWWHG DJDLQWWHDFK RWKHUWR VKRZ WKHGDWD ,Q(T WKH DUH6 DQGWLQ(T

VKH DUHS 9 DQG] $OKDYH GLPHQVLRQV DQG DOFDQEHQRQGLPHQVLRQDO]HG DV I
GLPHQVLRQDOOHVVWHFKQLTXH

- ' LPHQVLRQDOFRQVWDQWV PD\ YDU IURP FDVH WR FDVH EXW DUH KHOG FRQVWDQW GXULQJ DJLYH
UXQ ,Q(T VKH DUH6 9 DQGJ DQG LQ(T VKH DUH! J DQG& 7KH DOKDY
GLPHQVLRQV DQG FRQFHLYDEO\ FRXOG EH QRQGLPHQVLRQDO]HG EXW KH DUH QRW DOO XVHG W
KHSS QRQGLPHQVLRQDO]HWKH YDULDEOHV LQ WKH SUREOP
- 3XUH FRQVWDQWV KDYH QR GLPHQVLRQV DQG QHYHU GLG 7KH DULVH IURP PDWKHPDWF
PDQLSXODWLRQV ,Q ERWK(TV DQG VKH DUHò DQG WKH H[SRQHQW ERWK RI ZKLF
FDPH IURP DQLQWHJUDWLRQ $W WKH PDWKHPDWDOIXQFWLRQ VXFK DV OQ H[S FRV RU- L
GLPHQVLRQDWV
- $QJOHV DQG UHYROXWLRQV DUH GLPHQVLRQDWV 7KH SUHUHGXQWLRQDQDQJOHV WKHUDGLDQ
ZKLFK PDNHV LWFOHDU WKDW DQJOHV DUH ,QDNHPDQDU DUHYROXWLRQV CEUDGLDQV
- &RXQWLQJ QXPEHUV DUH GLPHQVLRQDWV) RU H[DPSOH LI ZH WDONHG HQHUJ\ (VR (WK
FRHILFLHQW LV GLPHQVLRQDWV

1 RWH WKDW LQWHJUDWLRQ DQG GLIIHUHQWLDWLRQ RI DQ HTXDWLRQ PD\ FKDQJH WKH GLPHQVLRQV EXW QRW WKH
KRPRJHQHLW\ RI WKH HTXDWLRQ) RU H[DPSOH LQWHJUDWLRQ RUGLIIHUHQWLDW(T

$$\int S\,dt = S_0 t + \tfrac{1}{2}V_0 t^2 + \tfrac{1}{6}gt^3 \qquad (5.8a)$$

$$\frac{dS}{dt} = V_0 + gt \qquad (5.8b)$$

,Q WKH LQWHJUDWHG IRUP D HY-U WHUP KDV GLPHQVLRQV RI ^/ 7` ZKLOH LQ WKH GHULYDWLYH IRUP
E HY-U WHUP LV DY-OFLW^/ 7í `

) LQDOO\ VRPH SK\VLFDO YDULDEOHV DUH QDWXUDOO\ GLPHQVLRQDWV E\ YLUWXH RI WKHLU GHILQLWLRQ D
UDWLRV RI GLPHQVLRQDO TXDQWLWLHV 6RPH H[DPSOHV DUH VWUDLQ FKDQJH LQ OHQJWK SHU XQLW OHQJWK
3RLVVRQ¶V UDWLR UDWLR RI WDQVYHUVH VWUDLQ WR ORQJLWXGLQDO VWUDLQ DQG VSHFLILF JUDYLW\ UDWLR RI GHQVL
WR VWDQGDUG ZDWHU GHQVLW\

7KHP RVY HEHKLQG GLPHQVLRQDOOHVVLVWKDWDQ GLPHQVLRQDOO KRPRJHQHRXV HTXDWLRQ FDQEH
ZULWWHQ LQ DQ HQWLUHO\ HTXLYDOHQW QRQGLPHQVLRQDO IRUP WKDWLV PRUH FRPSDFW 8VXDOO\ WKH UH DU
PXOWLSOH PHWKRGV RI SUHVHQWLQJ RQH¶V GLPHQVLRQDW GDWD RU WKRU / HWXV LOOXVWUDWH WKHVH FRQFHSW
PRUH WKRURXJKO\ E\ XVLQJ WKH IDQQQJ ERG\ UHDWLRQ DV DQH[DPSOH

$PELJXLWV 7KH&KRLFHRI 9DULDEOHVDQG6FDOLQJ 3DUDP HWHUV

(TXDWLRQ LVIDPLOLDUDQGVLPSOH\HWLOOXVWUDWHVPRVWRI WKHFRQFHSWRI GLPHQVLRQDOOHVV
,WFRQWDLQV ILYH VPEROV 6 6 9 WJ ZKLFK ZH PD\ GLYLGH LQRXUWKLQNLQJ LQWR YDULDEOHV DQ
SDUDP HWHUV 7KHYDULDEOHV DUH WKHWKUHH ZHZLVK WRSORWWKHEDVLF RXWSXWV RI WKH H[SHULP HQWRUWKHRU
LQWKLVFDVH 6 YHUVXV W7KHSDUDP HWHUV DUH WKRVHTXDQWLWLHV ZKRVHHIIHFWRQ WKHYDULDEOHV ZH VK
NQRZ LQWKLVFDVH6 9 DQGJ $OPRVWDQ\ HQJLQHHULQJ VWXG\ FDQEHVXEGLYLGHGLQWKLVPDQQHU

7RQRQGLPHQVLRQDOL]HRXUHVXOW ZHQHHGWRNQRZ KRZ PDQ GLPHQVLRQV DUHFRQVLGHG DPRQ

RXU YDULDEOHV DQG SDUDPHWHUV LQWKLV FDVH RQO VZR GHQVLW^/` DQG WHPH^7` &KHFN HDFK WHUP YHULI\ WKLV

$$\{S\} = \{S_0\} = \{L\} \qquad \{t\} = \{T\} \qquad \{V_0\} = \{LT^{-1}\} \qquad \{g\} = \{LT^{-2}\}$$

$ PRQJ RXU SDUDPHWHUV ZH WKHUHRUH VHOHFW VZR RI WKH WKUHH SDUDPHWHUV DV RFDOHG UHSHDWLQ YDULDEOHV XVHG WR GHILQH GLPHQVLRQOHVV YDULDEOHV : KDWHYHU PDLQ ZLOO EH WKH ³EDVLF SDUDPHWHUV ZKRVH HIIHFWV ZH ZLVK WR VHH LQ RXU SORWV 7KH VHFRQG FKRLFH ZLOO QRW DIIHFW WKH FRQFOXVLRQV RU GDWD RQ WKH IRUP RI WKH UHVXOWDQW &HDLO WKH UHVXOWV PEJXLW LQ WKH VHFRQG FKRLFH VRPHWKLQJ WKDW WHQG YH[H WKH EHJLQQLQJ H[SHULPHQWDU %XW WKH DPEJXLWLHV GHDEHUDWH , WSXUSRVHLV WR VKRZ DSDUWFXODUHIIHFWLQ DQG WKHFKRLFHLV\RXUV\RPDNH

) RU WKH IDQQLQJ ERG\ SUREOP ZH VHOHFWDQ VZR RI WKH WKUHH SDUDPHWHUV WR EH VFDOHG SDUDPHWHUV 7KXV ZHKDYHWKUHHRSWRQV / HVXGLVFXVDQGGLVSOD\ WKHP LQWXLQ

Option 1: 6FDOLQJ SDUDPHWHUV6 DQG9 WKHHIIHFWRI JUDYLWJ

) LUVWXVH WKH VFDOLQJ SDUDPHWHUV 6 9 WR GHILQH GLPHQVLRQOHVV GLVSODFHPHQWDQG WPH 7KHUHLVRQO RQHVXLWDEOHGHILQLWLRQIRUHDFK

$$S^* = \frac{S}{S_0} \qquad t^* = \frac{V_0 t}{S_0} \tag{5.9}$$

6XEVWLWXWLQJWKHVHYDULDEOHVLQWR(T DQG FOHDQHYHUWKLQJ XS XQLQWHUHVWLQJ 7KHUHVXOWLVRXUILUVWSORWRQ

$$S^* = 1 + t^* + \frac{1}{2}\alpha t^{*2} \qquad \alpha = \frac{gS_0}{V_0^2} \tag{5.10}$$

7KLV UHVXOWLV VKRZQ SORWWHG LQ) LJ D 7KHUHLV D VLQJOH GLPHQVLRQOHVV SDUDPHWU. ZKLFK VKRZV VKHUHWKHHIIHFWRI JUDYLW ,WFDQQRWXKRZ WKHGLUHFWHIIHFWRI 6 DQG9 VLQFHWKHVHZR DU KLGGHQLQWKHRUGLQDWHDQGDEVFLVVD : HVHHWKDWJUDYLWLQFUHDVHVWKHSDUDERODF UDWRI IDOORUW! EXWQRWWKHLQLWLDOVORSHDWW : HZRXOGOHDUQWKHVDPHIURP IDOOLQJ ERG\ GDWD DQGWKHSORWZLWKO H[SHULPHQWDODFFXUDF\ ZRXOGORRNOLNH) LJ D

Option 2: 6FDOLQJ SDUDPHWHUV9 DQGJ WKHHIIHFWRI LQLWLDOGLVSODFHPHQW6

1RZ XVHWKHQHZ VFDOLQJ SDUDPHWHUV 9 J WR GHILQH GLPHQVLRQOHVV GLVSODFHPHQWDQG WPH $JDLQWKHUHLVRQO RQHVXLWDEOHGHILQLWLRQ

$$S^{**} = \frac{Sg}{V_0^2} \qquad t^{**} = t\frac{g}{V_0} \tag{5.11}$$

6XEVWLWXWLQJWKHVHYDULDEOHVLQWR(T DQG FOHDQHYHUWKLQJ XS DJDLQ 7KHUHVXOWLV RXUVHFRQG RSWRQ

$$S^{**} = \alpha + t^{**} + \frac{1}{2}t^{**2} \qquad \alpha = \frac{gS_0}{V_0^2} \tag{5.12}$$

7KLV UHVXOWLVSORWWHG LQ) LJ E 7KHVDPHVLQJOHSDUDPHWU. DJDLQDSSHDUV DQG KHUHWKHZV WKH

HIHFWR LQLWLDOGLVSODFHPHQWZ KLFKPHUHO PRYHVWKHFXUYHVXSZ DUGZ LWKRXWFKDQJLQJ WKHLUVKDSH

Option 3: 6FDOLQJ SDUDPHWHUV 6 DQGJ WKHHIFWR LQLWLDOVSHHG9

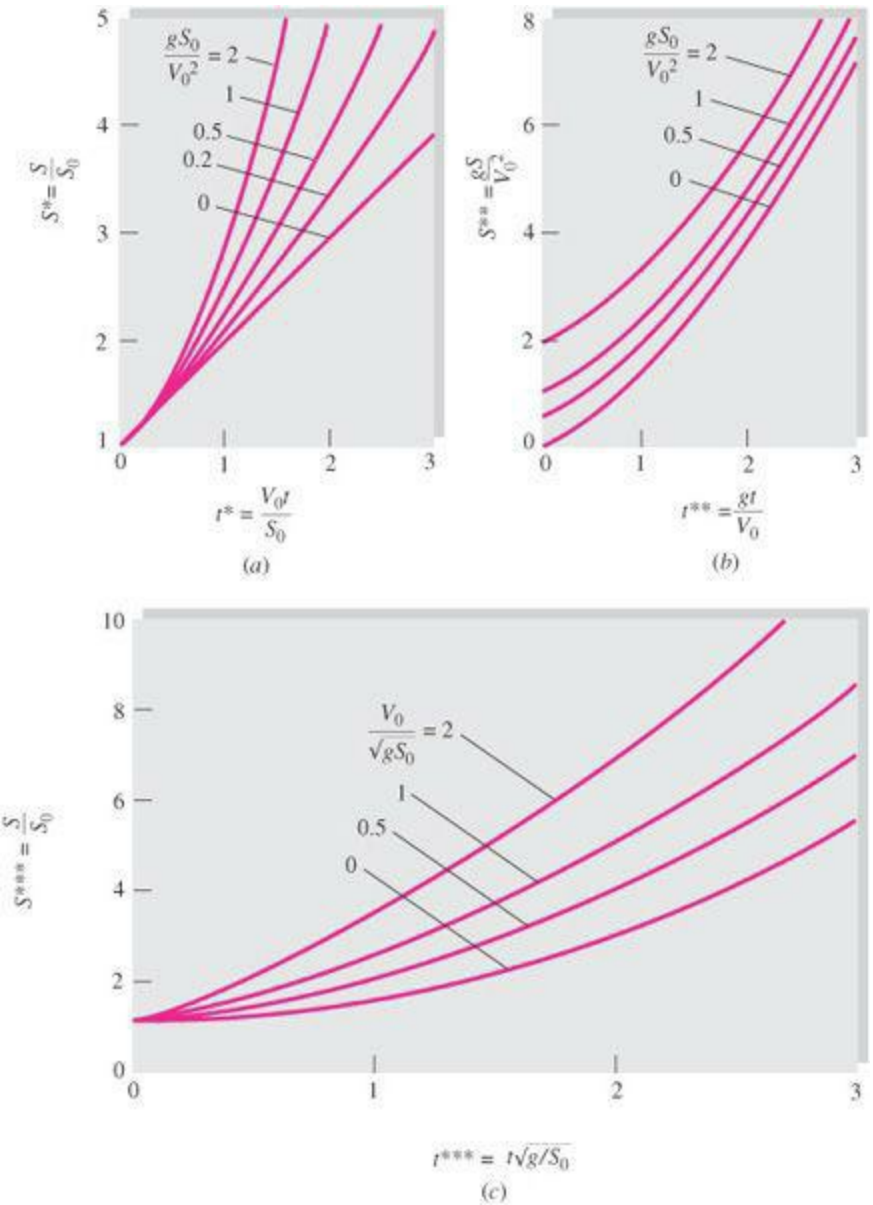
) LQDOO XVHWKHVFDOLQJ SDUDPHWHUV 6 J WR GHILQHGLPHQVLRQOHVV GLVSODFHPHQWDQG WPH\$JDLQWKHUHLVRQO RQHVXLWDEOHGHILQLWLRQ

$$S^{***} = \frac{S}{S_0} \qquad t^{***} = t\left(\frac{g}{S_0}\right)^{1/2} \tag{5.13}$$

6XEWLWXWLQJWKHVHYDULDEOHVLQWR(T DQG FOHDQHYHUWKLQJ XS DVXVXDO 7KHUHVXOWLV RXUWKLUG DQGILQDORSWLRQ

$$S^{***} = 1 + \beta t^{***} + \frac{1}{2}t^{***2} \qquad \beta = \frac{1}{\sqrt{\alpha}} = \frac{V_0}{\sqrt{gS_0}} \tag{5.14}$$

7KLVILQDOSUHVHQWDWLRQLVVKRZQLQ) LJ F 2QHDJDLQWKHSDUDPHWHU. DSSHDUV EXWZHKDYH UHGHILQHGLWXSVLGHGRZQβ = 1/√α VRWKDWRXUGLVSOD\ SDUDPHWHU9 LVLQWKHQXPHUDWRUDQGLVOQHDU 7KLV LV RXU IUHH FKRLFH DQG VLPSOLILHV WKH GLVSOD\ ) LJXUH F VKRZV VWDQGDUGLQWHJUDWLRQ LQFUHDVHVWKHIDQJOH GLVSODFHPHQW



) LJ 7KUHH HQWLWLHV HTXLYDOHQWGLPHQVLRQOHVV SUHVHQWDWLRQV RI WKH IDQQLQJ ERG\ SUREOHP (T WKH HIIHFWLRI *a* JUDYLW *b* LQWDOGLVSDFHPHQW DQG *c* LQWDOYHORFLW \$OOSDV FRQWDLQWKH VDPHLQIRUPDWLRQ

1RWH WKDWLQ DOO WKUHH RSWLRQV WKH VDPH SDUDPHWU. DSSHDUV EXWKDV D GLIIHUHQWPHDQLQ GLPHQVLRQOHVV JUDYLW LQWDOGLVSDFHPHQWDQG LQWDOYHORFLW 7KHJUDSKV ZKLFK FRQWLQH[DFWV WK VDPHLQRUPDWLRQ FKDQJHWKHLUDSSHDUDQFHWRUHOHFWWKHVHGLIIHUHQFHV

: KHUHDV WKH RUJLQDO SUREOHP (T LQYROYHG ILYH TXDQWLWV WKH GLPHQVLRQOHV SUHVHQWDWLRQVLQYROYHRQO WKUHH KDYLQJ WKHIRUP

$$S' = \mathrm{fcn}(t', \alpha) \qquad \alpha = \frac{gS_0}{V_0^2} \qquad (5.15)$$

7KH UHGXFWLRQ í VKRXOG HTXDOWKH QXPEHUR IXQGDPHQWDOGLPHQVLRQV LQYROYHG LQ WK SUREOHP ^/ 7` 7KLVLGHDOHGWRWKHSLWKHRUHP 6HF

6HOHFWLRQ RI 6FDOLQJ 5HSHDWLQJ 9DULABOHV

7KHVHOHFWLRQRI VFDOLQJ YDULABOHVLVOHIWWR WKHXVHU EXWKHUHDUHVRPHJXLGHOLQHV ,Q(T LLVQRZ FOHDUWKDWWKH VFDOLQJ YDULABOHVZHUH! 9 DQG/ VLQFHWKH\ DSSHDULQERWIRUFHFRHILFLHQ DQG5H\QROGVQXPEHU: HFRXOGWKHQLQWURGXFHIURP (T DVWKHYDULABOHV IRUFHYHUVXVGLPHQVLRQOHVV YLVFRVLW VLQFHHDFKDSSHDUVLQRQO RQHGLPHQVLRQOHVJURXS 6LPLODUO LQ(T WKHVFDOLQJ YDULABOHVZHUHVHOHFWHGIURP 6 9 J QRW6 W EHFDXVHZHZLVKHGWRSO 6 YHUXVVWLQWKHILQDOUHVXOW

7KHIROORZLQJ DUHVRPHJXLGHOLQHVIRUVHOHFWLQJ VFDOLQJ YDULABOHV

7KH PXWQRWRUP DGLPHQVLRQOHVJURXS DPRQJ WKHPVHOYHV EXWDGGLQJ RQHPRUHYDULABO ZLOOIRUP DGLPHQVLRQOHVTXDQWLW) RUH[DPSOH \WWWSRZHUVRI ! 9 DQG/

$$\rho^a V^b L^c = (ML^{-3})^a (L/T)^b (L)^c = M^0 L^0 T^0 \quad \text{only if} \quad a = 0,\ b = 0,\ c = 0$$

,Q WKLV FDVH ZHFDQ VHHZK\ WKLV LV VR 2QO! FRQWLQV WKH GLPHQVLRQ^0 ` DQG RQO 9 FRQWLQV WKH GLPHQVLRQ^7` VR QR FDQFHOODWLRQ LV SRVVLEOH ,I QRZ ZHDGG VRWKH VFDOLQ JURXS ZHZLOOREWLQ WKH 5H\QROGV QXPEHU ,I ZHDGG) VRWKH JURXS ZHIRUP WKHIUF FRHILFLHQW

' RQRWHOHFWRXSXWYDULABOHVIRU\RXUVFDOLQJ SDUDPHWUV ,Q(T FHUWLQO GR QR VHOHFW\ ZKLFK\RX ZLVK VRLVOOWIRU\RXSORW1 RUZDV VHOHFWHG IRUZHZLVKHG WRSO IRUFHYHUVXVYLVFRVLW

,I FRQYHQLHQWVHOHFWSRSXODU QRWREVFXUH VFDOLQJ YDULABOHVEHFDXVHWKH\ ZLOODSSHDULQD RI \RXUGLPHQVLRQOHVJURXSV 6HOHFWGHQVLW QRWXUDFHWWQLRQ 6HOHFWEERG GHQJWK QRWXUDF URXJKQHVV 6HOHFWXWXPHP YHORFLW QRWSHGRI VRXQG

7KHH[DPSOHVKDWWRQORZ ZLOOPDNHWKLVFHDU 3UREOHP DVJQPHQWP LJKWJLYHKLQW 6XSSRVHZHZLVK WRVWXG GUDJ IRUFHYHUVXVYHORFLW 7KHQZHRXGQRWXVH9 DVD VFDOLQ

SDUDPHWULF(T : HZ RXOG XVH ! / LQWRDG DQG WKH ILQDO GLPHQVLRQOHVV IXQFWLRQ ZRXOG EH FRPH

$$C_F' = \frac{\rho F}{\mu^2} = f(\mathrm{Re}) \qquad \mathrm{Re} = \frac{\rho V L}{\mu} \tag{5.16}$$

,QSORWWLQJ WKH VHGDW ZHZRXOG QRWEH DEOHWR GLVFHUQ WKH HIFWR ! RU VLQFH WKH DSSHDUL ERWK GLPHQVLRQOHVV JURXSV 7KH JURXSLQJ &) • DJDLQZRXOG PHDQ GLPHQVLRQOHVV IRUFH DQG 5HLVQRZ LQWUSUHWHG DV HLWKHU GLPHQVLRQOHVV YHORFLWV RUV]H 7KH SORWZRXOG EH TXLWH GLIHUHQWFRPSDUHG (T DQWRXJK LWFRQWLQV H[DFWO WKH VDPH LQRUPDWRQ 7KH GHYHORSPHQWRI SDUDPHWULV VXFK D &) • DQG5HIURP WKHLQLWLDOYDULDEOHVLVWKHVXEVHFWRQ WKHSLWKHRUHP 6HF

6RPH3HFXOLDU(QJLQHHULQJ(TXDWLRQV

7KHIRXQGDWRQRI WKHGLPHQVLRQDOQDOVLVPHWKRGUHVWVRQ WZR DVXPSWLRQV 7KHSURSRVH SK\VLFDO UHODWLRQ LV GLPHQVLRQDOO KRPRJHQHRXV DQG DOO WKH UHOHYDQWYDULDEOHV KDYH EHH LQFOXGHGLQWKHSURSRVHGUHODWLRQ

,I D UHOHYDQWYDULDEOH LV PLVVLQJ GLPHQVLRQDO DQDOVLV ZLOO IDLO JLYLQJ HLWKHU DQJHEUDL GLIILFXOWLHV RU ZRUVH \LHOGLQJ D GLPHQVLRQHVV IRUPXODWRQ WKDWGRHV QRWUHVROYH WKH SURFHVV $ WWSLFDOFDVHLV0 DQQLQJ¶VRSHQFKDQQHOIORZ GLVFXVVHGLQ([DPSOH

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{1}$$

6LQFH9 LVYHORFLW5 LVDUDGLXV DQGQDQG6 DUHGLPHQVLRQOHVV WKHIRUPXOLVQRWGLPHQVLRQDO KRPRJHQHRXV 7KLVVKRXOGEHDZDUQLQJ WKDW WKHIRUPXOFKDQJHVLI WKHXQLWV RI 9 DQG5 FKDQJI DQG LI YDOG LWUHSUHVHQW D YHU VSHFLDO FDVH (TXDWRQ LQ ([DPSOH SUHGDWV WK GLPHQVLRQDOQDOVLVWFKQLTXHDQG LV YDOG RQO IRUZDWUQURXJK FKDQQHOV DWPRGHUDWH YHORFLW DQGGUDJHUDGLLQ%* XQLW

6XFK GLPHQVLRQDO LQKRPRJHQHRXV IRUPXODV DERXQG LQ WKH K\GUDXOF \$QRWK H[DPSOHLVWKH+D]HQ: LOOLDPVIRUPXOD> @RUYROXPHIORZ RI ZDWUWKURXJK DWXDLJKWPRRWSLSH

$$Q = 61.9 D^{2.63} \left(\frac{dp}{dx}\right)^{0.54} \tag{5.17}$$

ZKHUH' LV GLDPHWUDQGGS G[LV WKH SUHVVXUH JUDGLHQW6RPH RI WKHVH IRUPXODV DULVH EHFDXV QXPEHUV KDYH EHHQ LQVHUWG IRU IOXLG SURSHUWV DQG RWKHU SK\VLFDO GDWD LQRU SHUFHQW GHJLWPDW KRPRJHQHRXVIRUPXODV: HVKDOOQRWJLYH WKHXQLWV RI (T VR DYRLG HQFRXUDJLQJ LWXVH

2Q WKH RWKHU KDQG VRPH IRUPXODV DUH ³FRQWUWF WKDWFDQQRWEH PDGH GLPHQVLRQOHVV KRPRJHQHRXV 7KH³ YDULDEOHV WKH UHDWFDQQRWEH DQDO]HGE\ WKHGLPHQVLRQDODQDOVLV WKHVHIRUPXODV DUHUDZ HPSLULFLVP VFRQVHQHQWRDVP DOOJURXS RI VSHFLDOXW +HUHDUH WKUH H[DPSHV

$$B = \frac{25{,}000}{100 - R} \tag{5.18}$$

$$S = \frac{140}{130 + \text{API}} \tag{5.19}$$

$$0.0147 D_E - \frac{3.74}{D_E} = 0.26 t_R - \frac{172}{t_R} \tag{5.20}$$

(TXDWLRQ UHODWHV WKH %ULQHOO KDUGQHVV %RI DP HDVXUHG WR LWV 5RFNZHOO KDUGQHVV 5 (TXDWLR
UHODWHV WKH VSHFLILF JUDYLW6 RI DQRLOWLWGHQVLWLQGHJUHHV $3, (TXDWLRQ UHODWHV WKH
YLVFRVLW RI DQ TXLG LQ ' (RUGHUHV (QJHU WR LWV YLVFRVLW VA LQ 6D\EROWHFRQGV 6XFK IRUPXOD
KDYH DFHUWDLQ XVHXOQHVV ZKHQ FRPPXQLFDWLQJ EHWZHHQ IHOORZ VSHFLDOLVWV EXW ZHFDQQRW KDQGOH WKH
KHUH 9DULDEOHV QN %ULQHOO KDUGQHVV DQG 6D\EROWYLVFRVLW DUH QRWXLWHG WR DQ0 / 7, GLPHQVLRQ
VVWHP

7KH3L7KHRUHP

7KHUHDUHVHYHUDOPHWKRGV RI UHGXFLQJ DQXPEHUR GLPHQVLRQDOYDULDEOHVLQWR DVPDOOHUQXPEH
RI GLPHQVLRQOHVV JURXSV 7KHILUVWWKHPHJLYHQKHUHZDVSURSRVHGLQ E\ %XFNLQJKDP > @DQ
LV QRZ FDOOHG WKH%XFNLQJKDP 3L7KHRUHP 7KH QDPHSL FRPHV IURP WKH PDWKHPDWLFDO QRWDWLRQ
PHDQLQJ D SURGXFWRI YDULDEOHV 7KH GLPHQVLRQOHVV JURXSV IRXQG IURP WKH WKHRUHP DUH SRZH
SURGXFWVGHQRWHGE\ HW 7KHPHWKRGDOORZV WKHSLJURXSVWREHIRXQGLQVHTXHQWLDORUGH
ZLWKRXWUHVRUWLQJ WRIUHH[SRQHQW

7KHILUVWSDUWRI WKHSL WKHRUHP H[SODLQVZKDWGXFWLRQLQYDULDEOHVWRH[SHFW

7DNHWKHVSHFLILFFDVHRI IRUFHRQDQLPPHUVHGERG\ (T FRQVLQVILYHYDULDEOHV) / 8 !
DQG GHVFULEHGE\ WKUHGLPHQVLRQV^0 / 7` 7KXVQ DQGM' 7KHUHRUHLWVDJRRGJXHVWWKD
ZHFDQUHGXFHWKHSUREOP WRNSLJURXSV ZLWKN QíM• í $QGWKLVLVH[DFW ZKDWZH
REWDLQHG WZR GLPHQVLRQOHVV YDULDEOHV3 &) DQG3 5H 2QUHRFFDVLRQVLWPD\ WDNHPRUHS
JURXSVWKDQWKLVPLQLPXP VHH([DPSOH

,I D SK\VLFDOSURFHVV VDWLVILHV WKH3' + DQG LQYROYHVQ GLPHQVLRQDOYDULDEOHV LWFDQ E
UHGXFHG WR D UHODWLRQ EHWZHHQRQ N GLPHQVLRQOHVV YDULDEOHV RU3V 7KH UHGXFWLRQM Q íN
HTXDOV WKHPD[LPXP QXPEHURI YDULDEOHV WKDWGR QRWRUP DSLDPRQJ WKHPVHOYHVDQG LVDQ\ D\
OHVV WKDQRUHTXDOWR WKHQXPEHURI GLPHQVLRQVGHVFULELQJ WKHYDULDEOHV

7KHVHFRQGSDUWRI WKHWKHRUHP VKRZVKRZ WRILQGWKHSLJURXSVRQHDWDWLPH
7RBEHVSHFLILF VXSSRVHWKHSURFHVVLQYROYHVILYHYDULDEOHV

$$v_1 = f(v_2, v_3, v_4, v_5)$$

6XSSRVHWKHUHDUHWKUHHGLPHQVLRQV^0 / 7` DQGZHVHDUFKDURXQGDQGILQGWKDWWQGHHGM 7KH
N íDQGZHH[SHFWIURP WKHWKHRUHP WZRDQGRQO WZRSLJURXSV 3LFNRXWWKUHHFRQYHQLHQ
YDULDEOHVWKDWGR QRWRUP DSL DQGVXSSRVHWKHVHWXQRXWBH# # DQG# 7KHQWKHWZRSLJURXS
DUHIRUPHGE\ SRZHUSURGXFWVRI WKHVHWKUHHSULPDU\DQGLWHUDWHG# RU#

$$\Pi_1 = (v_2)^a (v_3)^b (v_4)^c v_1 = M^0 L^0 T^0 \qquad \Pi_2 = (v_2)^a (v_3)^b (v_4)^c v_5 = M^0 L^0 T^0$$

+ HUHZ HKDYH DUEL WDU O FKRVHQ# DQG# WKH DGGHG YDULDEOHV WR KDYH XQLW[SRQHQW (TXDWL
H[SRQHQWV RI WKH YDULRXV GLPHQVLRQV LVJXDUDQWHHG E\ WKH WKHRUHP WR JLYH XQLTXH YDOXHV RI D E DQG I
IRUHDFK SL \$QG WKH DUELWUDU\ FRQVWDQW 3 FRQVLVWV# DQG RQO 3 FRQVLVWV# ,WV DYHU
QHDWX\WWP RQFH\RXJHWXG WR WKH SURFHGXUH : HVKDOOLOXWUDWLWZ LWKVHYHUDO H[DPSOHV

) LQG WKH UHGXFLRQ M WKH QVHOHFWWMFDQQ YDULDEOHV WKDWGR QRWRUP DSL DPRQJ WKHPVHOHV
(DFK GHVLUHG SL JURXS Z LOOEH D SRZ HUSURGXFWRI WKHVHMYDULDEOHVSOXVRQH DGGLWLRQDOYDULDEOH
Z KLFK LV DWLJQHG DQ FRQYHQLHQWQRQ HURH[SRQHQW(DFK SL JURXS WKXVIRXQG LVLQGHSHQGHQW

7\SLFDO VL[WWMSV DUHLQYROYHG

/ LWWDQG FRXQWWKH Q YDULDEOHV LQYROYHG LQ WKH SUREOHP ,I DQ LP SRUWDQWYDULDEOHV DU
PLWLQJ GLPHQVLRQDODQDO VLVZ LOOI DLO
/ LWWKH GLP HQVLRQVRI HDFK YDULDEOHDFFRUGLQJ WR{$MLT\Theta$} or {$FLT\Theta$}. \$ OWWLVJLYHQLQ7DEO

7DEOH ' LPHQVLRQVRI) OXLG 0 HFKDQLFV3URSHUWLHV

Quantity	Symbol	Dimensions	
		$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}

) LQGM ,QWDOO JXHVVMHTXDOWR WKH QXPEHUR GLIIHUHQWGLP HQVLRQV SUHVHQWDQG OHRN IRU
YDULDEOHV WKDWGR QRWRUP DSL SURGXFW,I QR OXFN UHGXFHMEV DQG OHRN DJDLQ : LWK SUDFWH
\RX Z LOOILQGMUDSLGO
6HOHFWWMFDQQ SDUDP HWHUV WKDWGR QRWRUP DSL SURGXFW0 DNH VXUH WKH SOHDVH \RX DQ
KDYH VRP HJHQHUDOLW SRVLEOH EH FDXVH WKH Z LOOWKHQ DSSHDULQ HYHU RQ RI \RXUSL JURXSV
3LFN GHQLWLRUYHOFLWLRUOHQJWK ' RQRWSLFN VXUFHWHQVLRQ IRUH[DPSOH RU \RX Z LOOI RUP VL
GLIIHUHQWQGHSHQGHQW HEHUQXPEHUSDUDP HWHUVDQG WKRURXJKO DQDO \RXUFRODEDJXHV
\$GG RQH DGGLWLRQDO YDULDEOH WR \RXUMUHSHDWQJ YDULDEOHV DQG IRUP D SRZ HU SURGXF
\$OJHEUDLFDOO ILQG WKH H[SRQHQWWKDWP DNH WKH SURGXFWGLP HQVLRQOHVV 7U WR DUDQJH IRU\RX
RXVBXWRUGHSHQGHQWYDULDEOHV IRUH SUHVXUH GURS WRUTXH SOZ HU WR DSSHDULQ WKH QXP HUDWRU
DQG\RXSORWZ LOOORRNEHWWU ' RWKLVVHTXHQWDO DGGLQJ RQHQHZ YDULDEOHHDFK WLP H DQG\R
Z LOOILQGDOOQí M NGHVLUHGSL SURGXFWV
: ULWH WKH ILQDOGLP HQVLRQOHVV IXQFWLRQ DQG FKHFN WKH WWMP V WR P DNH VXUH DOOSL JURXSV DU
GLP HQVLRQOHVV

5HSHDWWKHGHYHORSPHQWR (T IURP (T XVLQJWKHSLWKHRUHP

6RXWLRQ

6WHS
: ULWHWKHIXQFWLRQDOGHSHQGHQFHRIWKHYDULDEOHV

$$F = f(L, U, \rho, \mu) \quad \text{there are five variables } (n = 5)$$

6WHS
/LVWGLPHQVLRQVRI HDFKYDULDEOH) URP 7DEOH

F	L	U	ρ	μ
$\{MLT^{-2}\}$	$\{L\}$	$\{LT^{-1}\}$	$\{ML^{-3}\}$	$\{ML^{-1}T^{-1}\}$

6WHS
) LQGM 1R YDULDEOHFRQWLQV WKHGLPHQVLRQ, DQG VRMLVGHWVWKDQRUHTXDOWR 0/7 : H
LQVHFWWKHQXPEHUGVHWDW 8 DQG! FDQQRWRUP DSLJURXSEHFDXVHRQ! FRQWLQVPDWDQGRQ
8 FRQWLQVWPH 7KHUHRUHMGRHVHTXDO DQGQí M í N 7KHSLWKHRUHP JXDUDQWHVIR
WKLVPUEOP WKDWWKHUHZLOOEH[DFWO VZRLQGHSHQGHQWGLPHQVLRQOHVJURXSV

6WHS
6HOHFWUHSHDWQJ MYDULDEOHV 7KHJURXS/ 8 ! ZHIRXQGLQWS ZLOOGRILQH

6WHS
&RPELQH/ 8 ! ZLWKRQHDGGLWLRQDOYDULDEOHLQVHTXHQFH WRILQGWKHVZRSLSURGXFW
) LWWDGGIRUFHWRILQG <RXPD\ VHOHFWDQ H[SRQHQWRQWKLVDGGLWLRQDOWP DV\RXSOHDVH W
SDFHILWLQWKHQXPHUDRUORGHQRPLQDRUWR DQ SRZHU6LQFH) LVWKHRXSXWRUGHSHQGHQWYDULDEOH
ZHVHOHFWWRRDSSHDUWKHILWWSRZHULQWKHQXPHUDU

$$\Pi_1 = L^a U^b \rho^c F = (L)^a (LT^{-1})^b (ML^{-3})^c (MLT^{-2}) = M^0 L^0 T^0$$

Equate exponents:

Length: $a + b - 3c + 1 = 0$

Mass: $c + 1 = 0$

Time: $-b \quad -2 = 0$

ZHFDQVROYHH[SOLFLWIRU

D ± E ± F ±

7KHUHRUH

$$\Pi_1 = L^{-2}U^{-2}\rho^{-1}F = \frac{F}{\rho U^2 L^2} = C_F \qquad Ans.$$

7KLV LV H[DFWO WKH ULJKWSL JURXS DV LQ(T %\ YDU\LQJ WKH H[SRQHQWRQ) ZH FRXOG KDY IRXQG RWKHU HTXLYDOHQW JURXSV VXFK DV 8/!)

) LQDOO DGG YLVFRVLW\ WR/ 8 DQG! WR ILQG 3 6HOHFWDQ SRZHU\RX OLNH IRU YLVFRVLW\ %\ KLQGVLJKWDQG FXWWP ZHVHOHFWWKH SRZHUí WRSODFHILWLQWKHGHQRPLQDWRU

$$\Pi_2 = L^a U^b \rho^c \mu^{-1} = L^a (LT^{-1})^b (ML^{-3})^c (ML^{-1}T^{-1})^{-1} = M^0 L^0 T^0$$

Equate exponents:

Length:	$a + b - 3c + 1 = 0$
Mass:	$c - 1 = 0$
Time:	$-b \quad + 1 = 0$

from which we find

D E F

$$\Pi_2 = L^1 U^1 \rho^1 \mu^{-1} = \frac{\rho UL}{\mu} = \mathrm{Re} \qquad Ans.$$

6WHS

: HNQRZ ZH DUH ILQLVKHG WKLV LVWKH VHFRQG DQG ODVWSL JURXS 7KH WKHRUHP JXDUDQWHHV WKDWWKH IXQFWLRQDO UHODWLRQVKLS PXVWEHRI WKH HTXLYDOHQWRUP

$$\frac{F}{\rho U^2 L^2} = g\left(\frac{\rho UL}{\mu}\right) \qquad Ans.$$

ZKLFKLVH[DFWO(T

(; $0 3/ (

7KHSRZHULQSXW8 WR DFHQWULXJDOSXPS LV DIXQFWLRQ RI WKHYROXPHIORZ 4 LPSHOOHUGLDPHWHU' URWDWLRQDOVSHHG 9 DQGWKHGHQVLW\! DQGYLVFRVLW\ RI WKHIOXLG

3 I4 ' !

5HZULWHWKLVDVDGLPHQVLRQOHVVUHODWLRQVKLS +LQW8VH9 ! DQG' DVUHSHDWLQJ YDULDEOHV

6ROXWLRQ

6WHS
&RXQWKHYDULDEOHV 7KHUHDUHV[GRQ¡WRUJHWWKHRQHRQWKHOHIW3
6WHS
/LVWWKHGLPHQVLRQVRI HDFKYDULDEOHIURP 7DEOH 8VHWKH^)/7,` VVWHP

P	Q	D	Ω	ρ	μ
$\{FLT^{-1}\}$	$\{L^3T^{-1}\}$	$\{L\}$	$\{T^{-1}\}$	$\{FT^2L^{-4}\}$	$\{FTL^{-2}\}$

6WHS
)LQGM/ XFN\ XV ZHZ HUHVRCGVRXVH 9 ! ' DVUHSHDWQJ YDULDEOHV VR VXUHO M WKHQXPEHU RI GLPHQVLRQV)/7 " &KHFNWKDWWKHVHWKUHHGRQRWRUP DSLJURXS

$$\Omega^a\rho^bD^c = (T^{-1})^a(FT^2L^{-4})^b(L)^c = F^0L^0T^0 \quad \text{only if} \quad a = 0,\ b = 0,\ c = 0$$

<HV M 7KLVZDVQRWDVREYLRXVDVWKHVFDOQJ JURXS / 8 ! LQ([DPSOH EXWLWVWKH: HQRZ NQRZ IURP WKHWKRUHP WKDWDGGLQJ RQHPRUHYDULDEOHZ LOOLQGHGIRUP DSLJURXS
6WHS D
&RPELQH 9 ! ' ZLWKSRZHU3 WRILQGWKHILUVWSLJURXS

$$\Pi_1 = \Omega^a\rho^bD^cP = (T^{-1})^a(FT^2L^{-4})^b(L)^c(FLT^{-1}) = F^0L^0T^0$$

Equate exponents:

Force:	b	$+ 1 = 0$
Length:	$-4b + c$	$+ 1 = 0$
Time:	$-a + 2b$	$- 1 = 0$

6RYH DQJHEUDLFDOO WR REWDLQD í E í DQGF í 7KLV ILUVWSL JURXS WKH RXWSX GLPHQVLRQOHVYDULDEOH LVFDOOHGWKHSRZHUFRHILFLHQWRI DSXPS &3

$$\Pi_1 = \Omega^{-3}\rho^{-1}D^{-5}P = \frac{P}{\rho\Omega^3D^5} = C_P$$

6WHS E
&RPELQH ! ' ZLWKIORZ UDWH4 WRILQGWKHVHFRQGSLJURXS

$$^{D}!^{E}\,'^{F}4 \quad 7^{\pm D})7\ /^{\pm E}/^{F}/\ 7^{\pm}\ )\ /\ 7$$

$IWHUHTXDWQJ H[SRQHQW ZHQRZ ILQGD í E DQGF í 7KLVVHFRQGSLJURXSLVFDOOHWKHIORZ FRHILFLHQWRI DSXPS &4

$$\Pi_2 = \Omega^{-1}\rho^0 D^{-3} Q = \frac{Q}{\Omega D^3} = C_Q$$

6WHS F

&RPELQH ! ' ZLWKYLVFRVLW\ WRILQGWKHWKUHGDQGGDWDSLJURXS

D!E' F 7± D)7 /± E / F)7/±) / 7

7KLVWLPH D í E í DQGF í RU3 ! ' DVRUWR 5HQROGVQXPEHU

6WHS

7KHRULJLQDOUHDWRQEHWZHHQV[YDULDEOHVLVQRZ UHGXFHGWRWKUHGLPHQVLRQOHVJURXSV

$$\frac{P}{\rho\Omega^3 D^5} = f\left(\frac{Q}{\Omega D^3}, \frac{\mu}{\rho\Omega D^2}\right) \qquad Ans.$$

&RPPHQW7KHVHWKUHHUHDUHWKHFODVVLFDOFRHILFLHQWXVHGWRFRUUHODWHSXPS SRZHULQ&KDS

(; $0 3/ (

$WWDZ YHORFLWLHV OPLQDUIRZ WKHYROXPHIORZ 4 WKURXJKDVPDOOERUHWXEHLVDIXQFWLRQRQO
RI WKHWXEHUDGLXV5 WKHIOXLGYLVFRVLW\ DQGWKHSUHVVXUHGURSSHUXQLWWXEHOHQJWKGS G[8VLQJ WK
SLWKHRUHP ILQGDQDSSURSULDWHGLPHQVLRQOHVVUHODWLRQVKLS

6RXWLRQ

: ULWHWKHJLYHQUHODWLRQDQGFRXQWYDULDEOHV

$$Q = f\left(R, \mu, \frac{dp}{dx}\right) \quad \text{four variables } (n = 4)$$

0 DNHDQDWWR WKHGLPHQVLRQVRI WKHVHYDULDEOHVIURP 7DEOH XVLQJ WKH^0 / 7` V\VWHP

Q	R	μ	dp/dx
$\{L^3T^{-1}\}$	$\{L\}$	$\{ML^{-1}T^{-1}\}$	$\{ML^{-2}T^{-2}\}$

7KHUHDUHWKUHHSULPDU\ GLPHQVLRQV 0 / 7 KHQFHM' %\ WDOODQGHUURUZHGHWHUPLQHWKDW3
DQGGSG[FDQQRWEHFRPELQHGLQWRDSLJURXS 7KHQM DQGQí M í 7KHUHLVRQO RQ
SLJURXS ZKLFKZHILQGE\ FRPELQLQJ4 LQDSRZHUSURGXFWZLWKWKHRWKHUWKUHH

$$\Pi_1 = R^a \mu^b \left(\frac{dp}{dx}\right)^c Q^1 = (L)^a (ML^{-1}T^{-1})^b (ML^{-2}T^{-2})^c (L^3T^{-1})$$
$$= M^0 L^0 T^0$$

Equate exponents:

Mass: $b + c = 0$

Length: $a - b - 2c + 3 = 0$

Time: $-b - 2c - 1 = 0$

6RONLQJ VLPXOWDQHRXVO\ ZHREWDLQD í E DQGF í 7KHQ

$$\Pi_1 = R^{-4} \mu^1 \left(\frac{dp}{dx}\right)^{-1} Q$$

or $$\Pi_1 = \frac{Q\mu}{R^4(dp/dx)} = \text{const}$$ *Ans.*

6LQFH WKHUH LV RQO RQH SL JURXS LWP XUWHTXDO D GLPHQVLRQOHVV FRQVWDQW 7KLV LV DV IDU D GLPHQVLRQDO DQDOVLV FDQ WDNH XV 7KH ODP LQDU IORZ WKHRU RI 6HF VKRZV WKDWWKH YDOXH RI WKH FRQVWDQW $-\frac{\pi}{8}$

(; $0 3/ (

$ WXP HWKDWWKH WSG HOHFWRQ/ RI DFDQLGHYHUEHDP LV DIXQFWLRQ RI WKH WSG ORDG3 EHDP CHQJWK/ DUHD PRP HQWRI LQHUWLD, DQG PDWULDO PRGXXV RI HODVWLFLW(WKDWLV / I 3 / , (5HZULWWWKH IXQFWLRQLQGLPHQVLRQOHVIRUP DQG FRPPHQWLWFRPSO[LWDQGWKHSHFXODUYDOXHRI M

6RXWLRQ

/LWWKHYDULDEOHVDQGWKHLUGLPHQVLRQV

δ	P	L	I	E
$\{L\}$	$\{MLT^{-2}\}$	$\{L\}$	$\{L^4\}$	$\{ML^{-1}T^{-2}\}$

7KHUHDUHILYHYDULDEOHV Q DQGWKUHHSULPDU GLPHQVLRQV 0 / 7 KHQFHM' %XWWK D ZHPD ZHFDQQRWLQGDQ FRPELQDWLRQ RI WKHYDULDEOHVWKDWGRHVQRWFRP DSL JURXS 7KLV L EHFDXVH ^0 ` DQG ^7` RFFXUFRQ LQ3 DQG(DQG RQ LQ WKH VDPH IRUP ^0 7í ` 7KXV ZH KDY HQFRXQWHUD VSHFLDOFDVH RI M ZKLFK LVOHVV WKDQ WKH QXPEHU GLPHQVLRQV 0 / 7 7RJDL PRUH LQWLJKWLQR WKLV SHFXODULW \RX VKRXG UHZRUN WKH SUREOP XVLQJ WKH) / 7 V\VWP R

GLPHQVLRQV < RXZ LOOILQGWKDWRQO ^) ` DQG^/ ` RFFXULQWKHVHYDULDEOHV KHQFHM
: LWKM ZHVHOHFW/ DQG(DVWZR YDULDEOHVWKDWFDQRWRUP DSLJURXS DQGWKHQDGGRWKH
YDULDEOHVWRIRUP WKHWKUHHGLPLSLV

/D(E, / D 0/± 7± E / 0 / 7

IURP ZKLFK DWWUHTXLQJ H[SRQHQW ZHILQGWKDWD í E RU3 ,/ 7KHQ

/D(E3 / D 0/± 7± E 0/7± 0 / 7

IURP ZKLFKZHILQGD í E í RU3 3 (/ DQG

/D(E/ / D 0/± 7± E / 0 / 7

IURP ZKLFKD í E RU3 // 7KHSURPHQLRQDWVIXQFWLRQV3 I3 3 RU

$$\frac{\delta}{L} = f\left(\frac{P}{EL^2}, \frac{I}{L^4}\right) \qquad Ans.\ (1)$$

7KLVLVDFRPSOH[WKUHHYDULDEOHIXQFWLRQ EXWGLPHQVLRQDOODQDOVLVDORQHFDQWDNHXVQRIXUWKHU
&RPPHQW : H FDQ³LPSURYH (T E\ WDNLQJ DGYDQWDJH RI VRPH SK\VLFDOUHDVRQLQJ D
/DQJKDDUSROWVRXW S @) RUVPDOOHDWF GHOFWLRQV/ LVSURSRUWLRQDOWR3 DQGLQYHUV
SURSRUWLRQDOWRPRPHQWRI LQHUWLD, 6LQFH3 DQG, RFFXUVHSDUDWHO LQ(T WKLVPHDQVWKDW3 PX
EHSURSRUWLRQDOWR3 DQGLQYHUVHO SURSRUWLRQDOWR3 7KXV IRUWKHVHFRQGLWLRQV

$$\frac{\delta}{L} = (\text{const})\frac{P}{EL^2}\frac{L^4}{I}$$

or
$$\delta = (\text{const})\frac{PL^3}{EI} \qquad (2)$$

7KLVFRXOGQRWEHSUHGLFWHGE\ DSXUHGLPHQVLRQDOODQDOVLV 6WHQWKRI PDMRUDOVWKHRU SUHGLFW
WKDWWKHYDOXHRI WKHFRQVWDQW$

Q QDOVLVRQDWM6WHS E\ 6WHS 0 HWKRGE\ ,SVHQ

7KH SL WKHRUHP PHWKRG IXUWH[SODLQHG DQG LOOXVWUDWHG LV RIWQ FDOOHG WKHUHSHDWQJ YDULDEOH
PHWKRGRI GLPHQVLRQDOODQDOVLV 6HOHFWWKHUHSHDWQJ YDULDEOHV DGGRQHP RUH DQG\ RXJHWD SLJURXS
7KH ZULWUQHV LW7KLV PHWKRG LV VWUDLJKWRUZ DUG DQG V\VWHPDWLFDOO UHYHDOV DOOWKH GHVLUHG S
JURXSV + RZ HYHU WKHUH DUH GUZ EDFNV $OOSL JURXSV FRQWDLQWKH VDPH UHSHDWQJ YDULDEOHV DQ

PLJKWDFNYDULHWRUHIHUUHGWR DQG RQHPXVWVRPHWLPHVODERURXVO FKHFNWKDWWKHVHOHFW
UHSHDWQJ YDULDEOHVGRQRWRUP DSLJURXSDPRQJWKHPVHOYHV VHH3URE 3

,SVHQ> @JXJJHWWDQHQWHO GLIHUHQWSURFHGXUH DVWKS E\ VWHSPHWKRGWKDWREWLQVDOORI WKHS
JURXSVDWRQFH ZLWKRXWDQ FRQWQJ RUFKHFNLQJ 2QHVLPSO VXFFHVVLYH HOPLQDWLRQ HDFK GLPHQVLR
LQWKH GHVLUHG IXQFWRQE\ GLYLVRQ RUPXOWLSOLFDWRQ / HWXVLOOXVWUDWH ZLWKWKH VDPHFODVLFDOGUD
IXQFWRQSURSRVHGLQ(T 8QGHUQHDWKHYDULDEOHV ZULWRXWWKHGLPHQVLRQVRI HDFKTXDQWLW

$$\underset{\{MLT^{-2}\}}{F} = \text{fcn}(\underset{\{L\}}{L}, \underset{\{LT^{-1}\}}{V}, \underset{\{ML^{-3}\}}{\rho}, \underset{\{ML^{-1}T^{-1}\}}{\mu}) \qquad (5.1)$$

7KHUHDUHWKUHHGLPHQVLRQV ^0 / 7` (OPLQDWWKHP VXFFHVVLYHO E\ GLYLVRQRUPXOWSOLFDWRQE
DYDULDEOH 6WDUWZLWKPDV ^0 ` 3LFN DYDULDEOHWKDWFRQWDLQV PDV DQG GLYLGHLWLQWR DOOWKHRWK
YDULDEOHVZLWKPDVGLPHQVLRQV: HVHOHFW GLYLGH DQGUHZULWWKHIXQFWLRQ

$$\underset{\{L^4T^{-2}\}}{\frac{F}{\rho}} = \text{fcn}\left(\underset{\{L\}}{L}, \underset{\{LT^{-1}\}}{V}, \underset{\{ML^{-3}\}}{\cancel{\rho}}, \underset{\{L^2T^{-1}\}}{\frac{\mu}{\rho}}\right) \qquad (5.1a)$$

: HGLGQRWGLYLGHLQR/ RU9 ZKLFKGRQRWFRQWDLQ^0 ` (TXDWLRQ D DWLWWOHRNV WKDQJH
EXWWFRQWDLQVILYHGLPHQVLRQDOYDULDEOHVDQGWKHVDPHLQRUPDWRQDV(T
: HVHWKDW LVQR ORQJHULPSRUWDQWEHFDXVHQR RWKHUYDULDEOHFRQWDLQV^0 ` 7KXVGLVFDUG!
DQG QRZ WKHUH DUH RQO IRXU YDULDEOHV 1H[WHOPLQDWWWPH ^7` E\ GLYLGLQJ WKH WPH FRQWDLQLQ
YDULDEOHVE\ VXLWDEOHSRZHUVRI VD\ 9 7KHUHVXOWV

$$\underset{\{L^2\}}{\frac{F}{\rho V^2}} = \text{fcn}\left(\underset{\{L\}}{L}, \underset{\{LT^{-1}\}}{\cancel{V}}, \underset{\{L\}}{\frac{\mu}{\rho V}}\right) \qquad (5.1b)$$

1RZ ZHVHHWKDW9 LVQR ORQJHUUHOHYDQWWLQFHRQO 9 FRQWDLQV WPH ^7` ) LQDOO HOPLQDWW ^/`
WKURXJKGLYLVRQE\ VD\ DSSURSULDWHSRZHUVRI / LWHO

$$\underset{\{1\}}{\frac{F}{\rho V^2 L^2}} = \text{fcn}\left(\cancel{L}, \underset{\{L\}}{\frac{\mu}{\rho VL}}\right) \qquad (5.1c)$$

1RZ / E\ LWHOLVQR ORQJHUUHOHYDQWDQGVRGLVFDUGLWDOVR 7KHUHVXOWLVHTXLYDOHQWWR(T

$$\frac{F}{\rho V^2 L^2} = \text{fcn}\left(\frac{\mu}{\rho VL}\right) \qquad (5.2)$$

,Q,SVHQ]V VWHS E\ VWHSPHWKRG ZHILQGWKHIRUHHFRHILFLHQWVDIXQFWRQVRO0 RI WKH5H\QROG
QXPEHU : HGLG QR FRXQWQJ DQG GLG QRWMLQGM : HMXVWVXFFHVVLYHO HOPLQDWG HDFK SUPDU
GLPHQVLRQE\ GLYLVRQZLWKWKHDSSURSULDWHYDULDEOHV

5HFDOO([DPSOH ZKHUHZHGLVFRYHUHG DZ NZ DUG0 WKDWWKHQXPEHURI UHSHDWQJ YDULDEOH
ZDVOHVV WKDQ WKH QXPEHURI SUPDU GLPHQVLRQV ,SVHQ]V PHWKRG DYRLGV WKLV SUHOPLQDU FKHFN
5HFDOOWKHEHDP GHOHFWLRQSUREOP SURSRVHGLQ([DPSOH DQGWKHYDULRXVGLPHQVLRQV

$$\underset{\{L\}}{\delta} = f(\underset{\{MLT^{-2}\}}{P}, \underset{\{L\}}{L}, \underset{\{L^4\}}{I}, \underset{\{ML^{-1}T^{-2}\}}{E})$$

)RUWKHILUVWWKUHH GHVLJQ GHWXVHOHPLQDWH^0` E\ GLYLGLQJ E\ (: HRQH KDYH WR GLYLGHLQWR3

$$\underset{\{L\}}{\delta} = f\left(\underset{\{L^2\}}{\frac{P}{E}}, \underset{\{L\}}{L}, \underset{\{L^4\}}{I}, \underset{\{ML^{-1}T^{-2}\}}{\cancel{E}}\right)$$

: HVHH WKDW ZHPD\ GLVFDUG(DVQRORQJHUUHOHYDQWDQGGWKHGLPHQVLRQ^7` KDV YDQLVKHGDORQ ZLWK^0` : HQHGRQ HOHPLQDWH^/` E\ GLYLGLQJ E\ VD\ SRZHUVRI / LWHO

$$\frac{\delta}{L} = \underset{\{1\}}{\text{fcn}}\left(\underset{\{1\}}{\frac{P}{EL^2}}, \underset{\{L\}}{\cancel{L}}, \underset{\{1\}}{\frac{I}{L^4}}\right)$$

' LVFDUG/ LWHO DVQRZ LUUHOHYDQWDQGZHREWLQ$QZHU VR([DPSOH

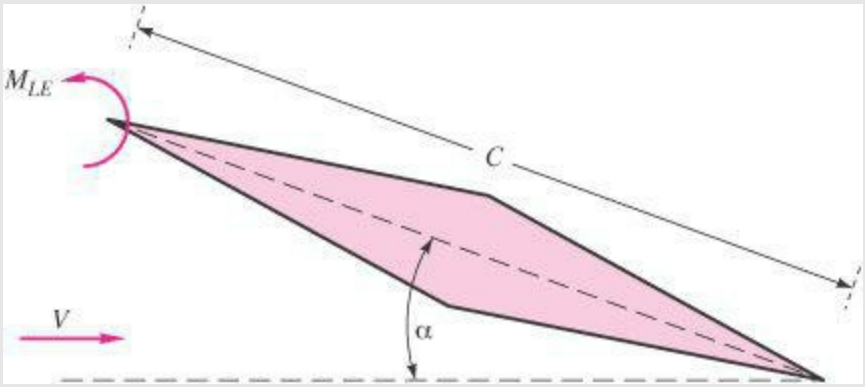
$$\frac{\delta}{L} = \underset{\{1\}}{\text{fcn}}\left(\underset{\{1\}}{\frac{P}{EL^2}}, \underset{\{1\}}{\frac{I}{L^4}}\right)$$

,SVHQJVDSSURDFKLVDJDLQVXFFHVVIXO7KHIDFWWKDW^0` DQG^7` YDQLVKHGLQWKHVDPHGLYLVRQL SURRI WKDWWKHUHDUHQR VZRUHSHDWLQJ YDULDEOHVWKLVWLPH QRWWKHWKUHHWKDWZRXOGEHLQHUHGE\ WK SUHVHQFHRI ^0` ^/` DQG^7`

(; $0 3/ (

7KH OHDGLQJ HGJH DHURG\QDPLF PRPHQW/(RQ D VXSHUVRQLF DLUIRLOLV D IXQFWLRQ RI LWV FKRU OHQJWK& DQJOHRI DWWDFN DQG VHFWLRQ GHVLJQ9 GHQVLW! VSHHGRI VRXQ D DQG VSHFLILF KHDWUDWRN)LJ (7KHUH LV D YHU ZHDN HIHFWRI DLU YLVFRVLW ZKFK L QHJOHFWGKHUH

(



8VH,SVHQ¶VPHWKRGVRUHZULWWKLVIXQFWLRQLQGLPHQVLRQDOIRUP

6RXWLRQ

: ULWHRXWKHJLYHQIXQFWLRQDQGGDWWKHYDULDEOHV¶GLPHQVLRQV^0 / 7` XQGHUQHDWK

$$\underset{\{ML^2/T^2\}}{M_{LE}} = \text{fcn}(\underset{\{L\}}{C}, \underset{\{1\}}{\alpha}, \underset{\{L/T\}}{V}, \underset{\{M/L^3\}}{\rho}, \underset{\{L/T\}}{a}, \underset{\{1\}}{k})$$

7ZRRI WKHP . DQGN DUHDOHDG\ GLPHQVLRQOHVV / HDYHWKHP DORQH WKH ZLOOEHSLJURXSVLQWR
ILQDOIXQFWLRQ<RXFDQHDPLQDWHDQ GLPHQVLRQ: HFKRRVHPDV^0 ` DQGGLYLGHE\ !

$$\underset{\{L^5/T^2\}}{\frac{M_{LE}}{\rho}} = \text{fcn}(\underset{\{L\}}{C}, \underset{\{1\}}{\alpha}, \underset{\{L/T\}}{V}, \cancel{\rho}, \underset{\{L/T\}}{a}, \underset{\{1\}}{k})$$

5HFDOO,SVHQ¶VUXOHV 2QO GLYLGHILQRYDULDEOHVFRQWDLQLQJ PDV LQWKLVFDVHRQO0 / (DQGWKH
GLVFDUGWKHGLYLVRU! 1RZ HDPLQDWHWP H^7` E\ GLYLGLQJ E\ DSSURSULDWHSRZHUVRI D

$$\underset{\{L^3\}}{\frac{M_{LE}}{\rho a^2}} = \text{fcn}\left(\underset{\{L\}}{C}, \underset{\{1\}}{\alpha}, \underset{\{1\}}{\frac{V}{a}}, \cancel{a}, \underset{\{1\}}{k}\right)$$

)LQDOO HDPLQDWHP^/` RQWKHOHWWLGHE\ GLYLGLQJ E\ &

$$\underset{\{1\}}{\frac{M_{LE}}{\rho a^2 C^3}} = \text{fcn}\left(\cancel{C}, \underset{\{1\}}{\alpha}, \underset{\{1\}}{\frac{V}{a}}, \underset{\{1\}}{k}\right)$$

: HHQGXS ZLWKIRXUSLJURXSVDQGUHRJQL]H9 DDVWKH0 DFKQXPEHU0 D ,QDHURG\QDPLFV WK
GLPHQVLRQOHVV PRPHQWLV RIWHQ FDOOHG WKHPRPHQWFRHILFLHQW&0 7KXV RXUILQDOUHVXOWFRXOG E
ZULWWHQLQWKHFRPSDFWRUP

$$C_M = \text{fcn}(\alpha, \text{Ma}, k) \qquad \textit{Ans.}$$

&RPPHQWV 2XUDQDOVLVLVILQH EXWH[SHULPHQWDQG WKHRU DQG SK\VLFDOUHDVRQLQJ DOOLQGLFDW
WKDW0 / (YDULHVPRUHVWURQJO ZLWK9 WKDQZLWKD 7KXVDHURG\QDPLFLWWFRPPRQO GHILQHWKHPRPHQ
FRHILFLHQWDV&0 0 / (!9 & RUVRPHWKLQJ VLPLODU: HZLOOVWXG\ WKH DQDOVLV RI VXSHUVRQLF
IORZHVDQGPRPHQWLQ&KDS

1RQGLPHQVLRQDOL]DWLRQ RI WKH%DVLF(TXDWLRQV

: HFRXOG XVH WKH SLWKHRUHP PHWRG RI WKH SUHYLRXV VHFWLRQ WR DQDO]H SUREOHP DIWU SUREOHP DIWU SUREOHP ILQGLQJ WKH GLPHQVLRQOHVV SDUDPHWHUV WKDWJRYHUQ LQ HDFK FDVH 7H[WERRNV RI GLPHQVLRQDODQDOVLV>RUH[DPSOH @GRWKLV $QDOWHUQDWLYH DQGYHU SRZHUXOWFKQLTXHLVWRDWWDF WKHEDVLF HTXDWLRQV RI IORZ IURP &KDS (YHQWKRXJK WKHVH HTXDWLRQV FDQQRWEHVROYHGLQJHQHUDO WKH ZLOOUHYHDOEDVLF GLPHQVLRQOHVV SDUDPHWHUV VXFK DV WKH 5H\QROGV QXPEHU LQWKHLUSRSHUIRUP DQG SURPSUSRVLWLQJ FOHVVWR KHQWKH DUHQHJOLJLEOH 7KHERXQGDU\ FRQGLWLRQV PXVWDOVR EH QRQGLPHQVLRQDO]HG

/ HWXV EUIHO DSSO WKLV WHFKQLTXH WR WKH LQFRPSUHVVLEOH IORZ FRQWLQXLW DQG PRPHQWXP HTXDWLRQV ZLWKFRQVWDQWYLVFRVLW

$$\text{Continuity:} \qquad \nabla \cdot \mathbf{V} = 0 \tag{5.21a}$$

$$\text{Navier-Stokes:} \qquad \rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{V} \tag{5.21b}$$

7\SLFDOERXQGDU\ FRQGLWLRQVIRUWKHVHWZR HTXDWLRQV DUH 6HFW

$$\begin{aligned} &\text{Fixed solid surface:} && \mathbf{V} = 0 \\ &\text{Inlet or outlet:} && \text{Known } \mathbf{V}, p \\ &\text{Free surface, } z = \eta\text{:} && w = \frac{d\eta}{dt} \qquad p = p_a - \Upsilon(R_x^{-1} + R_y^{-1}) \end{aligned} \tag{5.22}$$

: HRPLWWKH HQHUJ\ HTXDWLRQ DQG DVLJQLWV GLPHQVLRQOHVV IRUP LQ WKH SUHYLRXV 3URE 3

(TXDWLRQV DQG FRQVLQWKHWKUHHEDVLF GLPHQVLRQV0 / DQG7 $OOYDULDEOHVS 9 [\ ] DQGWFDQEHQRQGLPHQVLRQDO]HGE\ XVLQJ GHQVLW DQG VW R UHHUHQFH FRQVDQW WKDWPLJKWEH FKDUDFWHULVWLF RI WKHSDUWFXODUIORZ

5HHUHQFHYHORFLW 8 5HHUHQFHOHQJWK /

) RUH[DPSOH 8 PD\ EHWKHLQOHWRUXSVWUHDP YHORFLW DQG/ WKHGLDPHWUR DERG\ LPPHUVHG LQ WKHVWUHDP

1RZ GHILQHDOOUHOHYDQWGLPHQVLRQOHVV YDULDEOHV GHQRWLQJ WKP E\ DQDVWHULVN

$$\mathbf{V}^* = \frac{\mathbf{V}}{U} \qquad \nabla^* = L\nabla$$

$$x^* = \frac{x}{L} \quad y^* = \frac{y}{L} \quad z^* = \frac{z}{L} \qquad R^* = \frac{R}{L} \tag{5.23}$$

$$t^* = \frac{tU}{L} \quad p^* = \frac{p + \rho g z}{\rho U^2}$$

$OOWKHVH DUH IDLU REYLRXV H[FHSWIRUS ZKHUH Z HKDYH LQWRGXFHG WKH SH]RPHWF SUHVVXU DVXPLQJ WKDW\ LVXS 7KLVLVDKLQGVJJHVWHGE\ %HUQRXOOL VHTXDWLRQ

6LQFH! 8 DQG/ DUH DOO FRQVWDQW WKH GHULYDWLYHV LQ(TV FDQ DOOEH KDQGOHG L GLPHQVLRQOHVVIRUP ZLWKGLPHQVLRQOFRHILFLHQW) RUH[DPSOH

$$\frac{\partial u}{\partial x} = \frac{\partial (Uu^*)}{\partial (Lx^*)} = \frac{U}{L}\frac{\partial u^*}{\partial x^*}$$

6XEVWLWXWLQJ LQWR (TV DQG DQG GLYLGLQJ WKURXJKRXW E\ WKH FRPPRQ GLPHQVLRQDOFRHIILFLHQWLQWKH VPHZDV DVZHKDQGGHG (T +HUH DUH WKH UHVXOWLQJ GLPHQVLRQOHVVHTXDWLRQVRI PRWLRQ

Continuity:
$$\nabla^* \cdot \mathbf{V}^* = 0 \qquad (5.24a)$$

Momentum:
$$\frac{d\mathbf{V}^*}{dt^*} = -\nabla^* p^* + \frac{\mu}{\rho UL}\nabla^{*2}(\mathbf{V}^*) \qquad (5.24b)$$

7KHGLPHQVLRQOHVVERXQGDU\ FRQGLWLRQVDUH

Fixed solid surface:
$$\mathbf{V}^* = 0$$
Inlet or outlet:
$$\text{Known } \mathbf{V}^*, p^*$$

Free surface, $z^* = \eta^*$:
$$w^* = \frac{d\eta^*}{dt^*}$$
$$p^* = \frac{p_a}{\rho U^2} + \frac{gL}{U^2}z^* - \frac{\Upsilon}{\rho U^2 L}(R_x^{*-1} + R_y^{*-1}) \qquad (5.25)$$

7KHVH HTXDWLRQV UHYHDO D VRPHWKLQJ RI IRXU GLPHQVLRQOHVV SDUDPHWHUV RQH LQ WKH 1DYLHU6WRNHV HTXDWLRQ DQG WKUHH LQ WKH IUHH VXUIDFH SUHVVXUH ERXQGDU\ FRQGLWLRQ

' LPHQVLRQOHVV3DUDPHWHUV

,Q WKH FRQWLQXLW\ HTXDWLRQ WKHUH DUH QR SDUDPHWHUV 7KH 1DYLHU6WRNHV HTXDWLRQ FRQWDLQV RQ JHQHUDOO DFFHSWHGDVWKHPRVWVLPSOHQRQWULYLDOPHFKDQLFV

$$\text{Reynolds number Re} = \frac{\rho UL}{\mu}$$

,WLV QDPHG DIWHU 2VERUQH 5H\QROGV ± D %ULWLVK HQJLQHHU ZKR ILUVWSURSRVHG LWV 5H RI &KDS 7KH5H\QROGVQXPEHUL DQ DVVLPSRUWDQWZLWKRUZLWKRXWDIUHHVXUIDFH DQG FDQ EH QHJOHFWHG LQ IORZ UHJLRQV DZD\ IURP KLJK YHORFLW\ JUDGLHQWV IRUH[DPSOH DZD\ IURP VROLG VXUIDFHV MHWV RUZDNHV

7KHQR VOLS DQG LQOHWH[LWERXQGDU\ FRQGLWLRQVFRQWDLQQR SDUDPHWHUV 7KHIUHH VXUIDFH SUHVVXUH FRQGLWLRQFRQWDLQV WKUHH

$$\text{Euler number (pressure coefficient) } \mathrm{Eu} = \frac{p_a}{\rho U^2}$$

7KLV LV QDPHG DIWHU / HRQKDUG (XOHU ± DQG LV UDUHO LPSRUWDQWXQGHUVWRRG WKH SUHVXU
GURSVORZ HQRXJK WRFDXVHYDSRUIRUPDWRQ FDYLWDWRQ LQDQDTXLG 7KH(XOHUQXPEHULVRIWHQZULWW
LQWHUPVRI SUHVXUHGLIIHUHQFHV (X ûS !8 ,I ûS LQYROYHVYDSRUSUHVXUHS# LWLVFDOOHGWK
FDYLWDWRQQXPEHU&D S_D í S_# !8

7KHVHFRQGIUHHVXUDFHSDUDPHWHULVPXFKPRUHLPSRUWDQW

$$\text{Froude number Fr} = \frac{U^2}{gL}$$

,WLVQDPHGDIWHU: LOOLDP) URXGH ± D %ULWLVK QDYDODUFKLWHFWZKR ZLWK KLV VR
5REHUWGHYHORSHGWKHVKLSPRGHOWHVWLQJWHFKQLTXHVIRUIUHH VXUIDFH
IORZV VKLSUHVLVWDQFH VXUIDFHZDYHV RSHQFKDQQHOV 7KH) URXGHQXPEHULVWKHGRPLQDQWHIHFW
IUHH VXUIDFH IORZV DQG LV WRWDOO XQLPSRUWDQWL WKHUH LV QR IUHH VXUIDFH &KDSWHU LQYHVWLDWH
) URXGHQXPEHUHIHFWVLQGHWDLO

7KHILQDOIUHHVXUIDFHSDUDPHWHULV

$$\text{Weber number We} = \frac{\rho U^2 L}{\Upsilon}$$

,WLVQDPHGDIWHU0 RULV: HEHU ± RI WKH3RO\WHFKQLF,QVWLWXWHRI %HUOLQ ZKR GHYHORSHG
WKHODZ VRI VLPLODULWHQHULPRGHOLQIRUP ,WZDV: HEHUZKRQDPHG5HDQG) UDWH5H\QROGVDQ
) URXGH 7KH: HEHUQXPEHULVLPSRUWDQWQ LI LWLV RI RUGHUXQLW RUOHVV ZKLFK\WSLFDOO RFFXU
ZKHQWKHVXUIDFHFXUYDWXUHLVFRPSDUDEOHLQVL]HWR WKHOLTXLG GHSWK VXFK DVLQGURSOHWV FDSLOODU
IORZV UISSOHZDYHV DQG YHU VPDOOK\GUDXOLF PRGHOV ,I: HLVODUJH LWWHIHFWVPD\ EHQHJOHFWHG

,I WKHUHLV QR IUHH VXUIDFH) U (X DQG: H GURS RXWHQWHO H[FHSWIRU WKH SRVLEOHOW R
FDYLWDWRQ RI D OLTXLG DWYHU VPDOO(X 7KXV LQORZ VSHHG YLVFRXVIORZV ZLWK QR IUHH VXUIDFH WK
5H\QROGVQXPEHULVWKHRQO LPSRUWDQWGLPHQVLRQOHVVSDUDPHWU

&RP SUHVVLEOLOW 3DUDP HWHUV

,QKLJK VSHHGIORZ RI DJDVWKHUHDUHVLJQLILFDQWFKDQJHVLQSUHVXUH GHQVLW DQGWHPSHUDWXUHWKD
PXWWEHUHODWHGE\ DQHTXDWRQRI VWDWVXFK DVWKHSHUHFWJDVODZ (T 7KHVHWKUHRG\QDPL
FKDQJHVLQWRGXFHWZR DGGLWLRQDOGLPHQVLRQOHVVSDUDPHWUVPHQWLRQHGEUIH0 LQHDUHUFKDSWU

$$\text{Mach number Ma} = \frac{U}{a} \qquad \text{Specific-heat ratio } k = \frac{c_p}{c_v}$$

7KH0 DFK QXPEHULVQDPHGDIWU(UQVW0 DFK ± DQ$ XVWULDQSK\VLFLVW7KHHIHFWRI
LVRQO VOLJKWXR PRGHUDW EXW0 D H[HUW D VWURQJ HIHFWRQ FRPSUHVLEOH IORZ SURSHUWLHV LI LWL
JUHDWHUWKDQDERXW 7KHVHHIHFWVDUHVXGLHGLQ&KDS

2 VFLOODWLQJ) ORZ V

,I VKHIORZ SDWWHUQLVRVFLOODWLQJ DVHYHUDOPHWUHQWLVVKURXJKVKHLQOHWERXQGDU\ FRQGLWLRQ) RUH[DPSOH VXSSRVHVKDVKHLQOHWVWUHDP LVRI VKHIRUP

$$X \quad 8 \ FRV\&W$$

1RQGLPHQVLRQDOL]DWLRQ RI VKLVUHODWLRQ\LHOGV

$$\frac{u}{U} = u^* = \cos\left(\frac{\omega L}{U} t^*\right)$$

7KHDUJXPHQW RI VKHFRVLQHFRQWDLQV VKHQHZ SDUDPHWHU

$$\text{Strouhal number St} = \frac{\omega L}{U}$$

7KHGLPHQVLRQOHVVIRUPHV DQGPRPHQW IUFWLRQ DQGKHDWWDQVHU DQGVR RQRI VXFKDQRVFLOODWLQ IORZ ZRXOGEHDIXQFWLRQRI ERWK5H\QROGV DQG6WRXKDOXPEHUV 7KLVSDUDPHWULVQJLQJLQWKHZLGH 6RPHIORZVVKDWWRXPLJKWJXHVVWKDWDVHSHUHFW VWDGH DFWXDOO KDYHDQRVFLOODWRU SDWWHUQWKDWV GHSHQGHQWRQWKH5H\QROGVQXPEHU$QH[DPSOHLVWKHSURGLF YRUWH[VKHGGLQJ EHKLQGDEOXQWERG LPPHUVHGLQDVWDG\ VWUHDP RI YHORFLW8) LJXUH DVKRZVDQDUUD\ RI DSWLQGXFW YRUWH[VVKH IURP DFLUFXODUF\OLQGHULPPHUVHGLQDVWDG\ FURVVIORZ 7KLVUHJXODU SHULRGLF VKHGGLQJ LVFDOOHG . iUPiQYRUWH[VWUHHWDWU7 YRQ. iUPiQ ZKR H[SODLQHG LWWKHRUHWFDOO LQ 7KH VKHGGLQ RFFXUVLQWKHUDQJH 5H ZLWKDQDYHUDJH6WRXKDOXPEHU&G CB §) LJXUH E VKRZVPHDVXUHGVKHGGLQJIUHTXHQFLHV

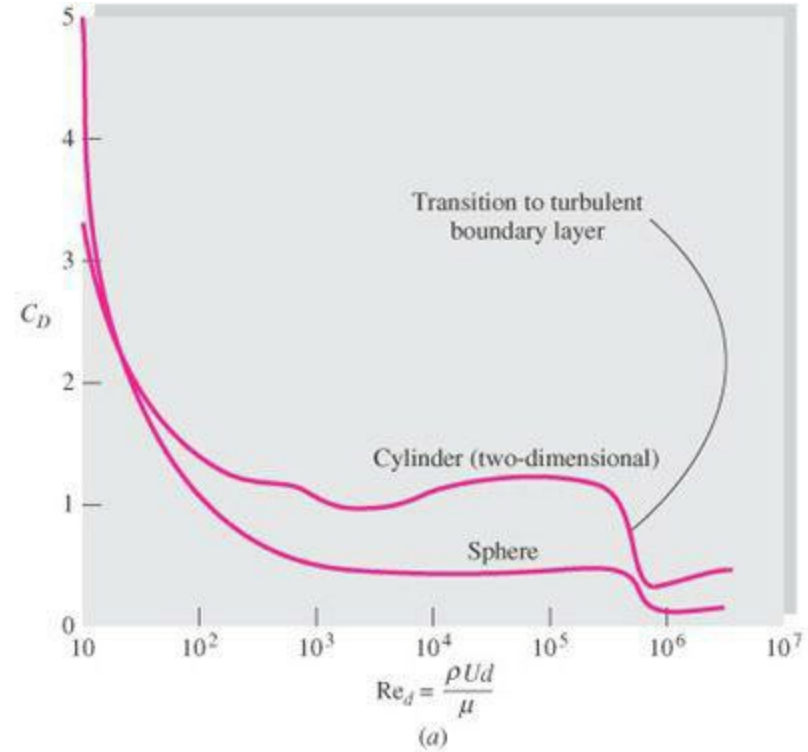
5HVRQDQFH FDQ RFFXUL D YRUWH[VKHGGLQJ IUHTXHQF\ LV QHDU D ERG\¶V VWUFWXUDO YLEUDWR IUHTXHQF\ (OHFWUF VDQVPLWWRQ ZLWV VLQJ LQ WKH ZLQG XQGHUHD PRRULQJ OQHV JDOORS DVFHUDO FXUHQWVSHHGV DQG VOHQGHUVWUXFWXUHV IOXWWUDWLWFDOZ LQG RUYHKLFOHVSHHGV $ VWULNLQJ H[DPSOHL VKHGLVDXWRXVIDLXUH RI VKH7DFRPD1DUURZV VXVSHQVLRQEULGJHLQ ZKHQZLQGH[FLWHG YRUWH VKHGGLQJ FDXVHG UHVRQDQFH ZLWK WKHQDWXUDO VRUVRQDOR VFLOODWRQV RI WKHEULGJH 7KHSUREOP ZD PDJQLILHGE\ WKHEULGJHGHFNQRQOQHDUWWQHVV ZKLFKRFFXUHGZKHQWKHKDQJHUZHQWVODFNGXULQ WKHRVFLOODWRQ

2 VKHU' LP HQVLRQOHVV3DUDP HWHUV

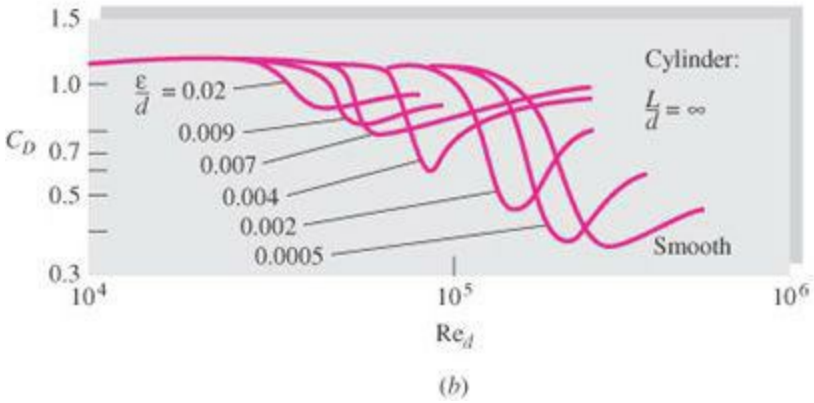
: HKDYHGLVFXVVHGVHYHQLPSRUWDQWSDUDPHWHUV LQIOXLG PHFKDQLFV DQG WKHUHDUHRWKHUV) RX DGGLWLRQDO SDUDPHWHUV DULVH IURP QRQGLPHQVLRQDOL]DWLRQ RI WKH HQHUJ\ HTXDWLRQ DQG LV ERXQGDU\ FRQGLWLRQV 7KHVH IRXU 3UDQGWQXPEHU (FNHUWQXPEHU * UDVKRI QXPEHU DQG ZDO WHPSHUDWXUHUDWLR DUHGLVFXVVHGLQ7DEOH IXUWKHUFDVH\RXIDLOWRVROYH3UREOP 3 \$QRWKHULPSRUWDQ SDUDPHWHUV VXUSULVLQJ SDUDPHWHUV WKHZDOOURXJKQHVVUDWLRë/ LQ7DEOH 6OJKWFKDQJHVLQ VXUIDFHURXJKQHVVKDYHDVWURQJ HIHFWQWKHWXUEXOHQWRZ RUKLJK5H\QROGV QXPEHUUDQJH DVZH

7DEOH ' LPHQVLRQOHVV* URXSVLQ) OXLG0 HFKDQLFV

Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$\mathrm{Re} = \frac{\rho U L}{\mu}$	$\frac{\text{Inertia}}{\text{Viscosity}}$	Almost always
Mach number	$\mathrm{Ma} = \frac{U}{a}$	$\frac{\text{Flow speed}}{\text{Sound speed}}$	Compressible flow
Froude number	$\mathrm{Fr} = \frac{U^2}{gL}$	$\frac{\text{Inertia}}{\text{Gravity}}$	Free-surface flow
Weber number	$\mathrm{We} = \frac{\rho U^2 L}{\Upsilon}$	$\frac{\text{Inertia}}{\text{Surface tension}}$	Free-surface flow
Rossby number	$\mathrm{Ro} = \frac{U}{\Omega_{\text{earth}} L}$	$\frac{\text{Flow velocity}}{\text{Coriolis effect}}$	Geophysical flows
Cavitation number (Euler number)	$\mathrm{Ca} = \frac{p - p_v}{\rho U^2}$	$\frac{\text{Pressure}}{\text{Inertia}}$	Cavitation
Prandtl number	$\mathrm{Pr} = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$\mathrm{Ec} = \frac{U^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific-heat ratio	$k = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow
Strouhal number	$\mathrm{St} = \frac{\omega L}{U}$	$\frac{\text{Oscillation}}{\text{Mean speed}}$	Oscillating flow
Roughness ratio	$\frac{\epsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent, rough walls
Grashof number	$\mathrm{Gr} = \frac{\beta \Delta T g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Rayleigh number	$\mathrm{Ra} = \frac{\beta \Delta T g L^3 \rho^2 c_p}{\mu k}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Temperature ratio	$\frac{T_\infty}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Friction factor	$f = \frac{h_f}{(V^2/2g)(L/d)}$	$\frac{\text{Friction head loss}}{\text{Velocity head}}$	Pipe flow
Skin friction coefficient	$c_f = \frac{\tau_{\text{wall}}}{\rho V^2/2}$	$\frac{\text{Wall shear stress}}{\text{Dynamic pressure}}$	Boundary layer flow



Cylinder length effect ($10^4 < \mathrm{Re} < 10^5$)	
L/d	C_D
∞	1.20
40	0.98
20	0.91
10	0.82
5	0.74
3	0.72
2	0.68
1	0.64



) LJ 7KH SURRI RI SUDFWLFDOLPHQWRQDOQDOVLV GUDJ FRHILFLHQWV RI D FLQGHUDQ VSKHUH *a* GUDJ FRHILFLHQWRI D VPRRWK FLQGHUDQG VSKHUH GDWD IURP PDQ VRXUFHV *b* LQFUHDVHGURXJKQHVVFDXVHVHDUOHUWUDQVLWLRQWRDVXEFULWLFDOUHJLPH

7KLV ERRN LV SULPDULO FRQFHUQHG ZLWK 5H QROGV 0DFK DQG)URXGH QXPEHUIHFWV ZKLF GRPLQDWH PRVW IOXLG SUREOHV 1RW VR GLVFRYHUHG VKHVH SDUDPHWUV H[FHSW/ VLPSO E\ QRQGLPHQVLRQDOL]LQJ WKHEDVLF HTXDWRQVZLWRXWDFWXDOO VROYLQJ WKHP

,I WKHUHDGHULVQRWIDPLDUZLWKWKH SDUDPHWUJLYHQLQ7DEOH 5H FRQWLQVDOLWR RYHU GLPHQVLRQOHVSDUDPHWUVLQXVHLQHQJLQHHULQJ

$ 6XFFHVVIXO$SSOLFDWLRQ

' LPHQVLRQDODQDOVLVIXQ EXWGRHVIWZRUN' <HV LI DOLP SRUDQWYDULDEOHVDUHLFFXGHGLQWK SURSRVHGIXQFWLRQ WKHGLPHQVLRQOHVIXQFWLRQIRXQG E\ GLPHQVLRQDODQDOVLV ZLOOFRQOSVHDOWKH GDWDRQRDVLQJOHFXUYHRUVHWRFXUYHV

$QH[DPSOHRI WKHVXFFHVVRI GLPHQVLRQDODQDOVLVJLYHQLQ) LJ IRUWKHPHDVXUHGGUDJ R VPRRWKFLQGHUVDQGVSKHUHV 7KHIBZ LVQRUPDOWRWKHDFLVRI WKHFLQGHUZKLFKLVH[WHPHO GRQJ / G""'' 7KHGDWDIURP PDQ VRXUFHV IRUERWK OTXLGV DQG JDVHV DQG LQFXGHERGLHV IURF

VHYHUDOPHWULVLQGLPHQVLRQOHVVGUDJEDOOVDQGEDOORRQV PP LQVL]H %RVWFXUYHVLQ) LJ
 D DUHHQWHUHG H[SHULPHQWDO VHDQDOVLVRI LPPHUVHGERG\ GUDJ LVRQHRI WKHZHDNHVWDUHDVR
PRGHUQICXLG PHFKDQLFVWKRU ([FHSWRUGLJLDOFRPSXWUFDOFXODWRQV WKHUH LVQWWH WKRU IR
FA\QQGHUDQGVSKHUHGUDJ H[FHSWFUHSLQJ IORZ 5H
7KH5H\QROGVQXPEHURI ERWKERGLHVLVEDVHGRQGLPHWUKHQHWKHQRWDWRQ5H %XWWKHGUD
FRHILFLHQWDUHGHILQHGLIHUHQWO

$$C_D = \begin{cases} \dfrac{\text{drag}}{\frac{1}{2}\rho U^2 Ld} & \text{cylinder} \\ \dfrac{\text{drag}}{\frac{1}{2}\rho U^2 \frac{1}{4}\pi d^2} & \text{sphere} \end{cases} \qquad (5.26)$$

7KH\ ERWKKDYH DIDFWRU EHFDXVH WKH WUP ! 8 RFFXUV LQ %HUQRXOOL¶V HTXDWLRQ DQG ERWKDU
EDVHGRQWKHSURMHFWHGDUHD WKDWV WKHDUHDRQHVHHVZKHQORRNLQJ WRZDUGWKHERG\ IURP XSWUHDP
7KHXVXDOGHILQLWLRQRI & LVWKXV

$$C_D = \frac{\text{drag}}{\frac{1}{2}\rho U^2(\text{projected area})} \qquad (5.27)$$

+RZHYHUQHVKRXOGFDUHXOO FKHFNWKHGHILQLWLRQVRI & 5H DQGWKHONHEHRUHXVLQJ GDWD
WKHOWUDXUH $LURLOV IRUH[DPSOH XVHWKHSODQRUP DUHD
)LJXUH DLVIRUORQJ VPRRWKFA\QQGHUV ,I ZDOURXJKQHVDQGFA\QQGHUHQWWKDUHLQFXGHGD
YDULDEOHV ZHREWLQIURP GLPHQVLRQDODQDOVLVDFRPSOH[WKUHSDUDPHWUIXQFWLRQ

$$C_D = f\left(\mathrm{Re}_d, \frac{\epsilon}{d}, \frac{L}{d}\right) \qquad (5.28)$$

7RGHVFUEHWKLVIXQFWLRQFRPSOHWO ZRXGUHTXLUH RUPRUH H[SHULPHQWV&)' UHVXO
7KHUHRUHLWVFXWRP DU WR H[SORUHWKHHQJWKDQGURXJKQHVHIHFWVHSDUDWO WRHVWDEOLKWHQGV
7KHVDEOHZLW) LJ DVKRZ VWKHHQJWKHIHFWZLWK]HUR ZDOURXJKQHV $VHQJWKGHFUHDVH
WKHGUDJ GHFUHDVHVE\ XS WR SHUFHQW3K\VLFDOO WKHSUHVXUHLV³UHOHYHG DWWKHHQGV DV WKHIORZ
LVDOORZHGWRVNLWDURXQGWKHVSVLQWDGR GHOHFWQJ RYHUDQGXQGHUWKHERG\
) LJXUH EVKRZ VWKHHIHFWRI ZDOURXJKQHVIRUDQLQLQWO ORQJ FA\QQGHU 7KHVKDS GURS LQ
GUDJ RFFXUVDWORZ HU5H DVURXJKQHVFDXVHVDQHDUOHUWUDQVLWLRQWRWXUEXOHQWZDNHURQWKH
VXUIDFHRI WKHERG\ 5RXJKQHVKDVWKHVPHHIHFWRQSHUHGUDJ DIDFWWKDWVH[SORLWHG
GHOEHUDWHGLPSOQJ RI JRO EDOOVWRJLYHWKP OHVGUDJ DWWKHLUIOJKWV§ 6HH) LJ '
) LJXUH LVDWWSLFDOH[SHULPHQWDOWXG RI DIXLG PHFKDQLFV SUREOP DIGHG E\ GLPHQVLRQ
DQDOVLV $VWPHDQG PRQH\ DQG GHPDQG DORZ WKHFRPSOHWWKUHSDUDPHWUHODWLRQ FRXO
EHILOHGRXWE\ IXUWKHU[SHULPHQWV

(; $0 3/ (

$ VPRRWFA\QQGHU FP LQGLPHWUDQG FP ORQJ LVWHVWHGLQDZLQGWXQQHOIRUDFURVWORZ R
P VRI DLUDW *f& DQG DW 7KHPHDVXUHGGUDJ LV " 1 D ' RHVWKLVGDWDSRLQWDJUH
ZLWKWKHGDWDLQ) LJ " E &DQWKLVGDWDSRLQWEHXVHGWRSUHGLFWWKHGUDJ RI DFKLPQH P L

GLDPHWUDQG P KLJK LQZ LQGVDW f& DQG DW" ,I VR ZKDWLV WKH UHFRPPHQGHG UDQJHRI ZLQ
YHORFLWLHV DQG GUDJ IRUFHV IRU WKLV GDWD SRLQW F : K\ DUH WKH DQZHUV WR SDUW E DQG D VWKH VDPH
UHJDUGLQJ WR WKH FKLPQH\ KHLJKW DV/ G'

6RXWLRQ

) RU DLU DW f& DQG DW VHD! NJ P DQG (í NJ P V 6LQFH WKH VW
FRQGLWLRQV G LWWKRXGE HFRPSDUHG ZLWK WKH DEXDQGYDOXH& § LQWKH DEC
WR WKH UJKW) LJ D) LUVW FDOFXODWH WKH 5H\QROGV QXPEHU RI WKH VWUWA\QGHU

$$\mathrm{Re}_d = \frac{\rho U d}{\mu} = \frac{(1.2\ \mathrm{kg/m^3})(45\ \mathrm{m/s})(0.01\ \mathrm{m})}{1.8\mathrm{E}{-5}\ \mathrm{kg/(m\ -\ s)}} = 30{,}000$$

<HV WKLV LV LQ WKH UDQJH 5H QXWG LQ WKH WDEOH 1RZ FDOFXODWH WKH WWWGUD
FRHILFLHQW

$$C_{D,\mathrm{test}} = \frac{F}{(1/2)\rho U^2 L d} = \frac{2.2\ \mathrm{N}}{(1/2)(1.2\ \mathrm{kg/m^3})(45\ \mathrm{m/s})^2(0.2\ \mathrm{m})(0.01\ \mathrm{m})} = 0.905$$

<HV WKLV LV FORVH DQG FHUWDLQO ZLWKLQ WKH UDQJHRI “ SHUFHQWXDWGE\ WKH WWWUHVXOW $QV
D
6LQFH WKH FKLPQH\ KDV/ G ZHFDQXVH WKH GDWD LI WKH5H\QROGV QXPEHUUDQJHLV FRUUHFW

$$10^4 < \frac{(1.2\ \mathrm{kg/m^3})U_{\mathrm{chimney}}(1\ \mathrm{m})}{1.8\ \mathrm{E}{-5}\ \mathrm{kg/(m\cdot s)}} < 10^5 \quad \text{if} \quad 0.15\,\frac{\mathrm{m}}{\mathrm{s}} < U_{\mathrm{chimney}} < 1.5\,\frac{\mathrm{m}}{\mathrm{s}}$$

7KHVH DUH HQJLQHHULQJ VR WKH WWWGDWSRLQWV RVYHU XVHXO $QV E
7KH GUDJ IRUFH LQ WKLV UDQJH DUH DOR QHJOLJLEO VP DOO

$$F_{\min} = C_D\frac{\rho}{2}U_{\min}^2 L d = (0.91)\left(\frac{1.2\ \mathrm{kg/m^3}}{2}\right)(0.15\ \mathrm{m/s})^2(20\ \mathrm{m})(1\ \mathrm{m}) = 0.25\ \mathrm{N}$$

$$F_{\max} = C_D\frac{\rho}{2}U_{\max}^2 L d = (0.91)\left(\frac{1.2\ \mathrm{kg/m^3}}{2}\right)(1.5\ \mathrm{m/s})^2(20\ \mathrm{m})(1\ \mathrm{m}) = 25\ \mathrm{N}$$

7U WKLV \RXUHO &KRRVH DQ VL]H IRUWKH FKLPQH\ HYHQVRPHWKLQJ VLOO OLNH PP
PP <RX ZLOOJHWWKH VDPH UHVXOWIRU8 DQG) DVLQSDUWE DERYH 7KLVLVEHFDXVHWKHSURGXF
8GRFFXUVLQ5H DQG LI / G WKHVDPHSURGXFWRFFXUVLQWKHGUDJIRUFH) RUH[DPSOHIRU5

$$Ud = 10^4\frac{\mu}{\rho} \quad \text{then } F = C_D\frac{\rho}{2}U^2 L d = C_D\frac{\rho}{2}U^2(20d)d = 20C_D\frac{\rho}{2}(Ud)^2 = 20C_D\frac{\rho}{2}\left(\frac{10^4\mu}{\rho}\right)^2$$

(; $0 3/ (

7HOHSKRQHZLUHVDUHVDLGWR³VLQJ´ LQWKHZLQG &RQVLGHUDZLUHRI GLDPHWHU PP $WZKDWVHD OHYHOZLQGYHORFLWLV LI DQ ZLOOWKHZLUHVLQJ DPLGGOH& QRWH'

6RXWLRQ

) RUVHDOHYHODLUWDNH § (í P V) RUQRQPXLFDUHDGHUV PLGGOH& LV +] 0 HDXUVKHGGLQJUDWHSDWWHUQLQ) LJ E 2YHUDZLGHUDQJH WKH6WURXKDOQXPEHUDSSURSULDWHO ZKLFKZHFDQWDNHDVDILUVWJXHVV 1RWHWKDW& Œ I WKHVKHGGLQJIUHTXHQF\ 7KXV

$$\mathrm{St} = \frac{fd}{U} = \frac{(262\ \mathrm{s}^{-1})(0.008\ \mathrm{m})}{U} \approx 0.2$$

$$U \approx 10.5\ \frac{\mathrm{m}}{\mathrm{s}}$$

1RZ FKHFNWKH5H\QROGVQXPEHUWRVHHLI ZHIDOOLQWRWKHDSSURSULDWHUDQJH

$$\mathrm{Re}_d = \frac{Ud}{\nu} = \frac{(10.5\ \mathrm{m/s})(0.008\ \mathrm{m})}{1.5\ \mathrm{E}{-5}\ \mathrm{m}^2/\mathrm{s}} \approx 5600$$

,Q) LJ E DW5H PDVEH6WLVDOWKHKJKHUDWDERXW 7KXVDVOJKWOLPSURYH HWWPDWLV

$$U_{\text{wind}} = (262)(0.008)/(0.21) \approx 10.0\ \text{m/s} \qquad \textit{Ans.}$$

0 RGHOLQJ DQG,W3LWIDOOV

6RIDUZHKDYHOHDUQHGDERXWGLPHQVLRQDOKRPRJHQHLW\DQGWKHSL WKHRUHP PHWKRG XVLQJ SRZH SURGXFW IRU FRQYHUWLQJ D KRPRJHQHRXV SK\VLFDO UHDWLRQ WR GLPHQVLRQOHVV IRUP 7KLV L VWUDLJKWIRUZDUGPDWKHPDWLFDO EXWHUWLQHQWLQHULQJ GLIILFXOWLHVQHGWREHGLVFXVVHG) LUVWZHKDYHPRUHRUOHVVWDNHQIRUJUDQWHGWKDWWKHYDULDEOHVWKDWIHFWWKHSURFHVVFDQE GDWHGDQGDQDO]HG$FWXDOO VHOHFWLRQRI WKHLPSRUWDQWYDULDEOHVUHTXLUHVFRQVLGHUDEOHPXGJPHQWDQ H[SHULHQFH 7KHHQJLQHUPXVWGHFLGH IRUH[DPSOH ZKHWKHUYLVFRVLW\FDQEHQHJOHFWHG $UHWKHU

VLJQLILFDQWSHUDWLRQ HIIHFW ,V VXUIDFH WHQVLRQ LPSRUWDQW: KDVDERXW ZDOURJKQHVV" (DFK S
JURXS WKDWV UHDOLQHG LQFUHDVHV WKH [SHQVHDQG HIRUWHTXLUHG -XGJPHQWLQVHFWLQJ YDULDEOHV ZLO
FRPHWKURXJK SUDFWLFHDQGPDWXULW\WKLVERRNVRXOGSURYLGHVRPHRI WKHQHFHVVDU\ [SHULHQFH
2QH WKH YDULDEOHV DUH VHOHFWHG DQG WKH GLPHQVLRQDODQDOVLV LV SHUIRUPHG WKH [SHULPHQW
VHNVWRDFKLHYHVLPLODULWHWZHHQWKHPRGHOWKHSURWRWSHWRE HGHVJQHG : LWKVXILFLHQ
WWXQQJ WKHPRGHOGDWZ LOOUHYHDOWKHGHVLUHGGLPHQVLRQOHVVIXQFWLRQEHWZHHQYDULDEOHV

$$\Pi_1 = f(\Pi_2, \Pi_3, \ldots \Pi_k) \qquad (5.29)$$

: LWK(T DYDLODEOHLQFKDUWJUDSKLFDO RUDQDOWLFDOIRUP ZHDUHLQD SRVLWRQWRHQXU
FRPSOHWVLPLODULWEHWZHQPRGHODQGSURWRWSH $ IRUPDOWDWPHQWZRXOGEHDVIROORZV
7KLVIROORZVPDWKHPDWLFDO IURP(T ,I$\Pi_{2m} = \Pi_{2p}$, $\Pi_{3m} = \Pi_{3p}$, DQG VR IRUWK(T
JXDUDQWHHVWKDWWKH GHVLUHG RXSXWΠ_{1m} will equal Π_{1p}. %XWWKLV LV HDVLHU VDLG WKDQ GRQH DV Z HQRZ
GLVFXV 7KHUHDUHSHFLDO[HGW[WRQPRGHOWWXQQJ > ± @

) ORZ FRQGLWLRQV IRU D PRGHO WWXVWDUH FRPSOHWHO VLPLODU LI DOU HHHYDQWGLPHQVLRQOHV
SDUDPHWUVKDYHWKHVDPHFRUUHVSRQGLQJ YDOXHVIRUWKHPRGHODQGWKHSURWRWSH

,QWKDG RI FRPSOHWVLPLODULW WKHHQJLQHHULQJ OWUDUXHVSHDNVRI SDUWFXODUWSHVRI VLPLODUW
WKHPRWWRPPRQEHLQJ JHRPHWUF NLQHPDWF G\QDPLF DQGWKHUPDO/ HVXVFRQVLGHUDFKVHSDUDWHO

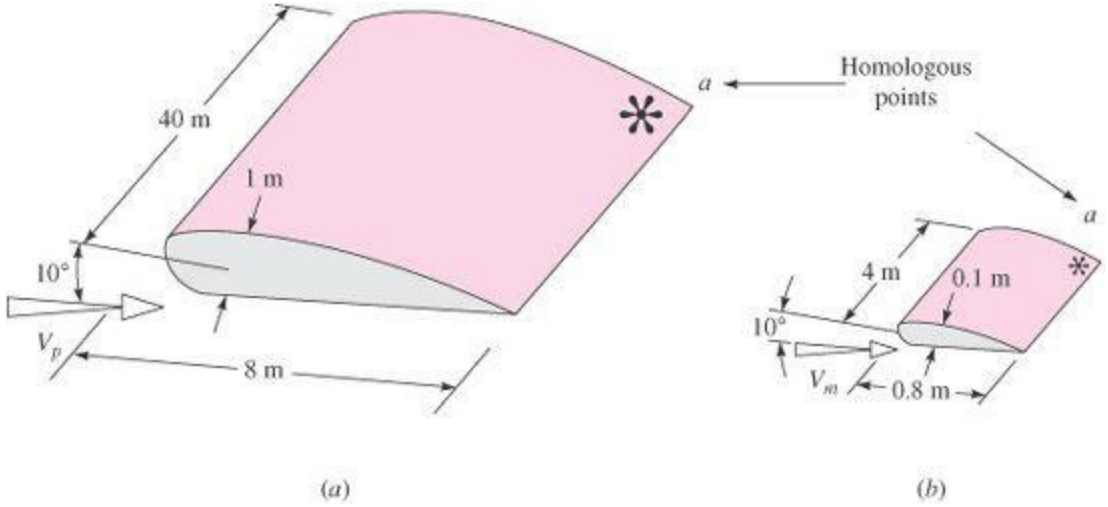
* HRP HWUF6LP LODULW

* HRP HWUF VLP LODULW FRQFHUQV WKHOHQJWKGLPHQVLRQ^/` DQGPXWEHHQXUEDQ VHQVLEO
PRGHOWWXQQJ FDQSURFHHG $ IRUPDOGHLQLWRQLVDVIROORZV
1RWWKDWVDOOOHQJWKVFDOHVPXWEHWKHVDPH ,WVDVLI \RXWRRNDSKRWRJUDSKRI WKHSURWRWSHDQ
UHGXFHGLWRUHQGHUHGLWXQLOLWLWWG WKHVL]HRI WKHPRGHO,I WKHPRGHOVWREHPDGHRQHWWQWKH
SURWRWSHVL]H LWVOHQJWK ZLGWK DQGKHJKWPXWWDFKEHRQHWWQWKDVOLJHQWUHVHDUFKHUVHSDUDWO
HQWUHVKDSHPXWEHRQHWWQWKDVOLJHDQGWWFKQLFDOO ZHVHDNRI KRPRORJRXVSRLQW ZKFK DU
SRLQWVWKDWKDYHWKHVDPHUHODWLYHORFDWRQ)RU[DPSOH WKHQRVHRI WKHSURWRWSHLVKRPRORJRXVL
WKHQRVHRI WKHPRGHO7KHOHIWZ LQJWSRI WKHSURWRWSHLVKRPRORJRXVWRWKHOHIWZ LQJWSRI WKHPRGHO
7KHQJHRPHWUF VLPLODULW UHTXLUHV WKDWDOOKRPRORJRXVSRLQWEHUHDWHG E\ WKHVDPHOQHDUVFDO
UDWR 7KLVDSSOHVWRWKHIOXLGJHRPHWW DVZHOODVWKHPRGHOJHRPHWW

$ PRGHODQG SURWRWSHDUHJHRPHWUFDOO VLPLODULW DQGRQO LI DOOERG GLPHQVLRQVLQDO
WKUHHRRUGLQDWWKDYHWKHVDPHOQHDUFDOHUDWR

$ OODQJOHV DUH SUHVHUYHG LQ JHRPHWUF VLPLODULW $ OOIGRZ GLUHFWRQV DUH SUHVHUYHG 7K
RUHODWWRQVRI PRGHODQG SURWRWSHZLWKUHVSHFWWRWKHVXUURXQGLQJVPXWEHLGHQWFDO

) LJXUH LOOXWUDWWVD SURWRWSHZLQJ DQG DRQHWWQWKVFDOHPRGHO7KPRGHOHQJWKV DUH D
RQHWWQWKDVOLJHWKDWWKH GHVLUHG DWDFNZLWKUHVSHFWWRWKHIUHVWWP LVWKHVDPHIRUERWKPRG
DQG SURWRWSH *QRW* $ OSK\VFDOGHWLOV RQWKHPRGHOPXWEHVFDOHG DQG VRPH DUHUDWKH
VXEWWDQG VRPHWPHVRYHUOBRNHG

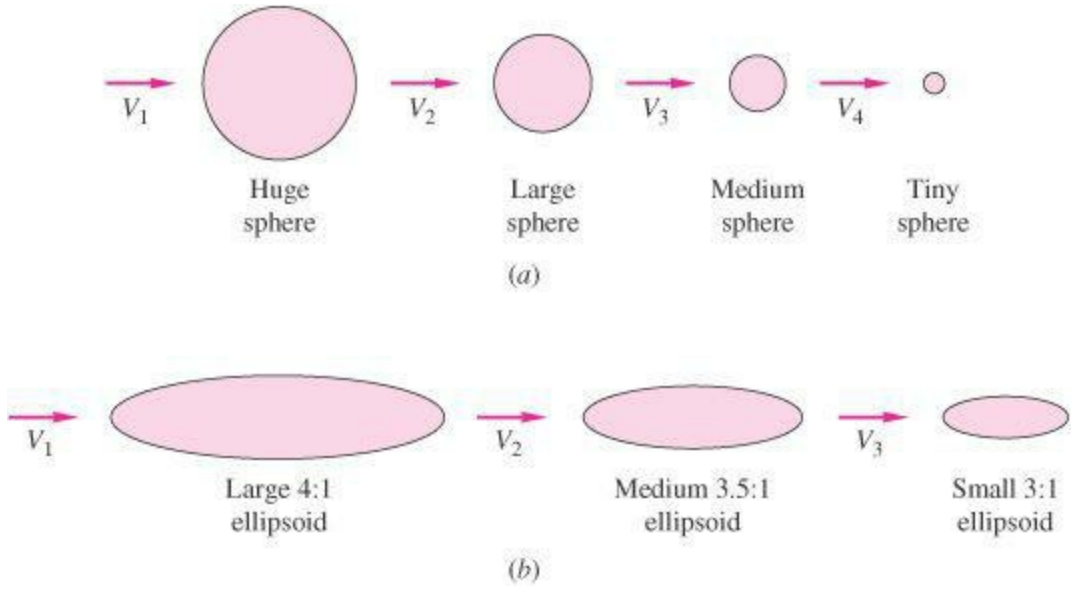


) LJ * HRP HWULF VLP LODULW LQ PRGHOLQJ *a* SURWRW\SH *b* RQH WHQWK VFDOH PRGHO

7KH PRGHO RQ HUDGLXV PXVW EH RQH WHQTK DV ODUJH
7KH PRGHO VXUIDFH URXJKQHVV PXVW EH RQH WHQTK DV ODUJH
,I WKH SURWRW\SH KDV D PP EORQGDU OD\HU WKLS ZLWH P IURP WKH OHDGLQJ HGJH WKH PRGHO VKRXOG KDYH D PP WLS ZLWH P IURP LWV OHDGLQJ HGJH
,I WKH SURWRW\SH LV FRQWUDFWHG ZLWK SURWXGLQJ IDXWHQHUV WKH PRGHO VKRXOG KDYH KRP RQH JRX SURWXGLQJ IDXWHQHUV RQ HWHQTK DV ODUJH

\$QG VR RQ \$Q GHSDUWXUH IURP WKH VHGHULOY LV D YLRODWRQ RI JHRPHWULF VLPLODULW DQG PXVW LG E\ H[SHULPHQWDO FRP SDUVRQ WR VKRZ WKDW WKH SURWRW\SH EHKDYLRU Z DV QRWWLJQLILFDQW DIIHFWHG E\ WKH GLVFUHSDQF\

0 RGHOV WKDW DSSHDU VLP LODU LQ VKDSH EXW WKDW FRQWDLQ YLRODWLJHRP HWULF VLP LODULW VKRXG QRWE FRP SDUHG H[FHSWDW RXUZ QULN) LJXUH LOOXVWDWHV WKLV SRLQW 7KH VSKHUHV LQ) LJ D DUH D JHRP HWULFDOO VLP LODU DQG FDQ EH WHVWHG RQ HLWKHU 5H\QROGV QXP EHU) URXGH QXP EHU RU WKH CNH LV P DWFKHG %XW WKH HDSVRLGV LQ) LJ E P HUHO ORRN VLP LODU 7KH DFWXDO KDYH GLIHUHQWO QH DU VFDOH UDWRV DQG WKHUHRUH FDQQRWEH FRP SDUHG LQ D UDWRQDO PDQQH HYHQ WKRXJK WKH PD\ KDYH LGHQWLFDO 5H\QROGV DQG) URXGH QXP EHUV DQG VR RQ 7KH GDWD ZLOOQRWE WKH VDP H IRU WKHVH HDSVRLGV DQG DQ DWWP SWRW3 FRP SDUH WKH P LV D PDWWUR RI URXJK HQJLQHHULQ MXGJP HQW



) LJ * HRP HWUF VLP LODULW DQG GLVWLP LODULW RI IGRZ V a VLP LODU b GLVWLP LODU

. LQHP DWF 6LP LODULW

. LQHP DWF VLP LODULW UHTXLUHV WKDW WKH PRGHO DQG SURWRW SH KDYH WKH VDP H GHQJWK VFDOH UDWR DQ WKH VDP H WHP H VFDOH UDWR 7KH UHVXOWLQJ WKDW WKH YHORFLW VFDOH UDWR ZLOO EH WKH VDP H IRU ERWK \$ / DQJKDU> @XDWLW/LW

/ HQJWK VFDOH HTXLYDOHQFH VLP SO LP SOHV JHRP HWUF VLP LODULW EXWWP H VFDOH HTXLYDOHQFH P D UHTXLUH DGGLWRQDO GQ QDP LF FRQVLGHUDWRQV\XFK DV HTXLYDOHQFH RI WKH5H\QROGV DQG0 DFK QXP EHUV

7KH PRWRQ RI WZR VLPLDU VKDPS DUH NLQHP DWFDOO VLP LODU LI KRP RGRJRXV SDUWFOHV OH D KRP RGRJRXVSRLQWDWKRP RGRJRXV\WP HV

2QH VSHFLDO FDVH LV LQFRP SUHVVLEOH IUFWRQDW IGRZ ZLWK QR IUHH VXUDFH DV\NHWKHG LQ) LJ D 7KHVH SHUFHFWOLG IGRZ V DUH NLQHP DWFDOO VLP LODU ZLWK LQGHSHQGHQWWKQJWK DQG WP H VFDOH DQG QR DGGLWRQDOSDUDP HWUV DUHQHFHVDU\ VHH&KDS IRUIXUWKHUGHWDLOV

) LJ ' \QDPLF VLPLODULW LQ VXEVRQLF JDVHV IORZ 0 RGHO DQG SURWRWSH \LHOG LGHQWLF
KRP RQRJRXV IRUFH SRO JRQV LL WKH 5H\QROGV DQG) URXGH QXPEHUV DUH WKH VDP H FRUUHVSRQGLQ
YDOXHV *a* SURWRWSH *b* PRGHO

' LVFUHSDQFLHV LQ : DWHU DQG \$LU 7HVWLQJ

7KH SHUFHQWDJH GLQDPLF VLPLODULW VKRZ QLQ) LJ LV PRUH RI DGUHDP WKDQ D UHDOLWH EHFDXVH WKH
HTXLYDOHQFH RI 5H\QROGV DQG) URXGH QXPEHUV FDQ EH DFKLHYHG RQO E\ GUDP DWF FKDQJHV LQ IOX
SURSHUWHV ZKHUHDV LQ IDFWP RWP RGHOWWLQJ LV VLP SO GRQH ZLWK ZDWUURUDU WKH FKDSHWWOXLG
DYDLODEOH

) LUWFRQVLGHU K\GUDXOF PRGHO WWXQQ ZLWK D IUHH VXUDFH ' \QDPLF VLPLODULW UHTXLU
HTXLYDOHQWY URXGHQXPEHUV (T DQG HTXLYDOHQW5H\QROGV QXPEHUV

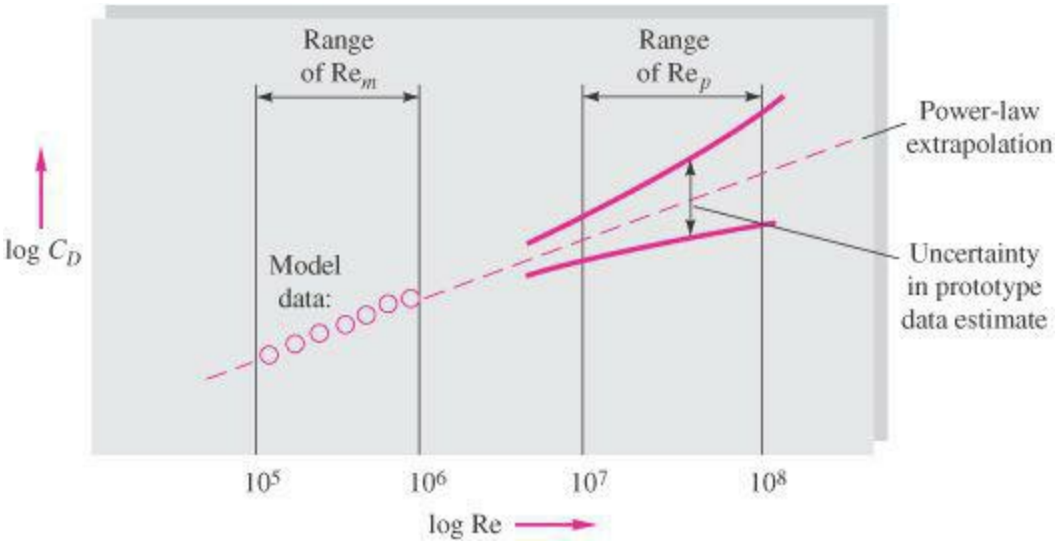
$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p} \qquad (5.34)$$

%XWEHWK YHORFLW DQG OHQJWK DUH FRQWUDLQHG E\ WKH) URXGH QXPEHU(TV DQG
7KHUHRUH IRUDJLYHQOHQJWKVFDOHUDWR. (T LVWKHRQO LI

$$\frac{\nu_m}{\nu_p} = \frac{L_m}{L_p}\frac{V_m}{V_p} = \alpha\sqrt{\alpha} = \alpha^{3/2} \qquad (5.35)$$

) RUH[DPSOH IRUD RQH WWQWK VFDOH PRGHO. DQG. 6LQFH $_S$ LV XQGRXEWGO
ZDWU ZHQHG DIXLG ZLWK RQO VLP HV WKH NLQHP DWF YLVFRVLW RI ZDWU WR DFKLHYH G\QDP LL
VLP LODULW 5HHUULQJ WR7DEOH ZHVHWKDWWKLVLVLP SRVVLEOH (YHQPHUFXU KDVRQO RQH QLQWK WK
NLQHP DWF YLVFRVLW RI ZDWU DQG DPHUFXU K\GUDXOF PRGHOZRXOG EH H[SHQLYH DQG EDG IRU \RX
KHDOWK ,Q SUDFWLFH ZDWU LV XVHG IRU ERWK WKH PRGHO DQG WKH SURWRWSH DQG WKH 5H\QROGV QXPEH
VLP LODULW LV XQDYRLGDEO YLRODWHG 7KH) URXGH QXPEHULV KHOG FRQVWDQWWLQFH LWW WKH GRP LQDQ
SDUDPHWU LQ IUHH VXUDFH IORZ V 7\SLFDOO WKH5H\QROGV QXPEHUR WKH PRGHOIORZ LV WR VP DOOE\ I
IDFWRUR VR \$ VVKRZ QLQ) LJ WKHORZ 5H\QROGV QXPEHUPRGHOGDWDUH XVHG WR HWWLP DL

E\ H[WDSRODWRQ WKH GHVLUHG KLJK 5H\QROGV QXPEHU SURWRWSHGDW $V WKH ILJXUH LQGLFDWHV WKHUH LV REYLRXV) FRQVLGHUDEOH XQFHUWDLQW LQ XVLQJ VXFK DQ H[WDSRODWRQ EXW WKH UHVXOWV RI WKH SUDFWLFDO DQDOLQDLYHLQK\GUDXOLF PRGHOWHVWLQJ



) LJ 5H\QROGV QXPEHU H[WDSRODWRQ RU VFDOLQJ RI K\GUDXOLF GDWD ZLWK HTXDO) URXG QXPEHUV

6HFRQG FRQVLGHUDWLRQ LQ DLU ZLWK QR IUHH VXUIDFH 7KH LPSRUWDQ SDUDPHWHUV/DUH WKH5H\QROGV QXPEHUDQG WKH0DFK QXPEHU(TXDWLRQ VKRXOG EH VDWLVILHG SOX WKH FRPSUHVVLELOLW\ FRQGLWLRQ

$$\frac{V_m}{a_m} = \frac{V_p}{a_p} \tag{5.36}$$

(OLPLQDWLRQ RI 9_P 9_S EHWZHHQ DQG JLYHV

$$\frac{\nu_m}{\nu_p} = \frac{L_m}{L_p}\frac{a_m}{a_p} \tag{5.37}$$

6LQFH WKH SURWRWSHLV QR GRXEWDQ DUURSHDWRQ ZHQHHG DZ LQG WXQQHOLQJ RI ORZ YLVFRVLW DQ KJK VSHHG RI VRXQG +\GURJHQ LV WKH RQO SUDFWDOH[DPSOH EXWFHDU LWLV VRR H[SHQVLYH DQ GDQJHURXV 7KHUHIRUH ZLQG WXQQHOV QRUPDO RSHUDW ZLWK DLU DV WKH ZRUNLQJ IOXLG &RROQJ DQ SUHVXUL]LQJ WKH DLUZ LOOEULQJ(T LQWR EHWWUDJUHPHQWEXWQRWHQRXJK WR VDWLV\ DOHQJWK VFDO UHGXFWLRQ RI VD RQH WHVWQK 7KHUHIRUH 5H\QROGV QXPEHU VFDOLQJ LV DOR FRPPRQO YLRODWHG L DHURG\QDPLF WHVWLQJ DQGDQH[WDSRODWRQQN HWKDWLQ) LJ LVUHTXLUHGKHUHDOR

7KHUHDUH VSHFLDOL]HGPRQJUDSKVGHYRWHGHQWLUH WR ZLQG WXQQHOWHVWLQJ ORZ VSHHG> @KLJ VSHHG > @DQG D GHWDLOHG JHQHUDO GLVFXVVLRQ > @7KH IROOWLQJ H[DPSOH LOOXVWUDWHV PRGHOLQ GLVFUHSDQFLHVLQDHURQDXWFDOWHVWLQJ

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$ SURWRWSHDLUSODQH ZLWKDFKRUGOHQJWKRI P LVWRIO DW D DW NP VDQGDUG DOWXGH $ RQHHLJKWKVFDOHPRGHOLVWREHWHVWHGLQDKHOXP ZLQG WXQQHODW f& DQG DW) LQGWKHKHOXP

VWWWHFWRQYHORFLWVWKDWZ LOOPDWFK D WKH0 DFKQXPEHUR E WKH5H\QROGVQXPEHUR WKHSURGXFWSH
,QHDFKFDVHFULWHULD]HWKHODPLQDU WUDQVLWLRQ G\QDPLF VLPLODULW F : KDWKLJK SUHVVXUHLQWKHKHDXP WKQHOZ LO
PDWFKERWK WKH 0 DFK DQG 5H\QROGV QXPEHUV" G : K\ GRHV SDUW F VWLOO QRWDFKLHYH G\QDPL
VLPLODULW"

6RQXWLRQ

) RUKHOXP IURP 7DEOH\$ 5 P V . N DQGHWWP DW $_{+H}$ § (í NJ P
V IURP WKHSRZHUODZ Q LQWKHWDEOH D &DOFXODWWKHKHOXP VSHHGRI VRXQGDQGYHORFLW

$$a_{He} = \sqrt{(kRT)_{He}} = \sqrt{(1.66)(2077\ \text{m}^2/\text{s}^2\text{K}) \times (373\ \text{K})} = 1134\ \text{m/s}$$

$$\text{Ma}_{air} = \text{Ma}_{He} = 2.0 = \frac{V_{He}}{a_{He}} = \frac{V_{He}}{1134\ \text{m/s}}$$

$$V_{He} = 2268 \frac{\text{m}}{\text{s}} \qquad \textit{Ans. (a)}$$

) RUG\QDPLF VLPLODULW WKH5H\QROGVQXPEHUVVKRXOGDOR EHHTXDO) URP 7DEOH\$ DWDQDOWWXG
R P UHDG! $_{DU}$ NJ P D_{DU} P V DQGHWWP DW $_{DU}$ § (í NJ P V IURP
WKHSRZHUODZ Q LQ7DEOH\$ 7KHDLUYHORFLWVLV9 $_{DU}$ 0 D D_{DU} P V 7K
PRGHOFKRUGOHQWKLV P P 7KHKHOXP GHQVLWVLV! $_{+H}$ S57 $_{+H}$ 3D >
P V . . @ NJ P 1RZ FDOFXODWWKHVZR5H\QROGVQXPEHUV

$$\text{Re}_{C,air} = \left.\frac{\rho VC}{\mu}\right|_{air} = \frac{(0.4125\ \text{kg/m}^3)(599\ \text{m/s})(1.6\ \text{m})}{1.48\ \text{E}-5\ \text{kg/(m} \cdot \text{s)}} = 26.6\ \text{E6}$$

$$\text{Re}_{C,He} = \left.\frac{\rho VC}{\mu}\right|_{He} = \frac{(0.131\ \text{kg/m}^3)(2268\ \text{m/s})(0.2\ \text{m})}{2.32\ \text{E}-5\ \text{kg/(m} \cdot \text{s)}} = 2.56\ \text{E6}$$

7KHPRGHO5H\QROGVQXPEHUV WPHVOHVWVWKDQWKHSURWRW\SH 7KLVLVWWSLFDOZ KHQXVLQJ VPDO
VFDOHPRGHOV 7KHWWWHVXOWVPXWEHH[WSRODWHGIRU5H\QROGVQXPEHUHIFW
E 1RZ LJQRUH0 DFKQXPEHUDQGGHWWKHPRGHO5H\QROGVQXPEHUPDWKWKHSURWRW\SH

$$\text{Re}_{He} = \text{Re}_{air} = 26.6\ \text{E6} = \frac{(0.131\ \text{kg/m}^3)V_{He}(0.2\ \text{m})}{2.32\ \text{E}-5\ \text{kg/(m} \cdot \text{s)}}$$

$$V_{He} = 23{,}600 \frac{\text{m}}{\text{s}} \qquad \textit{Ans. (b)}$$

7KLV LV ULGLFXORXV D K\SHUVRQLF 0 DFK QXPEHUR VXLWDEOH IRUHVFDSLQJ IURP WKH HDUWK
JUDYLW 2QHVKRXOGP DWFK WKH0 DFKQXPEHUVDQGFRUUHFWRUDOWHU5H\QROGVQXPEHU
F 0 DWFKERW5H\QROGVDQG0 DFKQXPEHUVE\ LQFUHDVLQJ WKHKHOXP GHQVLW

$$V_{He} = 2268 \frac{\text{m}}{\text{s}}$$

7KHQ

$$\mathrm{Re}_{\mathrm{He}} = 26.6\,\mathrm{E6} = \frac{\rho_{\mathrm{He}}(2268\ \mathrm{m/s})(0.2\ \mathrm{m})}{2.32\,\mathrm{E}-5\ \mathrm{kg/(m\cdot s)}}$$

6RONHIRU

$$\rho_{\mathrm{He}} = 1.36\,\frac{\mathrm{kg}}{\mathrm{m}^3} \quad p_{\mathrm{He}} = \rho RT|_{\mathrm{He}} = (1.36)(2077)(373) = 1.05\,\mathrm{E6\ Pa} \qquad \textit{Ans. (c)}$$

$ PDWKLVSRVWEHLZHLQFUHDVHWKHWXQQHOSUHVVXUHE\ DIDFWRURIWQDGDXQLQJWDN
G (YHQZLWK0DDQG5HPDWKHGZHDUHWWOOQRWGQDPLFDOO VLPLODUEHFDXVHWKHVZRJDV
KDYHGLIIHUHQWSHFLILFKHDWDWRV N+H DQGN'U 7KLVGLVFUHSDQF\ ZLOOFDXVHVXEVDQW
GLIIHUHQFHVLQSUHVXUHGHQVLW DQGWMPSHUDWXUHWKURXJKRXWSHUQFIORZ

) LJXUH VKRZVDK\GUDXOF PRGHORI WKH%OXHWRQH/DNH' DP LQ: HVW9LUJLQLD 7KHPRG
LWHOLVORFDWHGDWWKH8 6 $UP\ : DWHUZD\V([SHULPHQW6WDWLRQLQ9LFNVEXUJ 06 7KHKRUL]RQ
VFDOHV ZKLFKVXSHUHOHYDWHVYHUWLFDOVFDOHFDQDOR EH ZLWRXWLQFUHDVLQJVLJQLILFD
VXUIDFHWHQVLRQ : HEHUQXPEHU HIHFW 9HORFLWLHVDUHVFDOHGE\ WKH)URXGHQXPEHU+RZHYHUWK
SURWRWSH5HQROGVQXPEHUZKLFKLVRI RUGHU (FDQQRWEHPDWKHGKHUH 7KHHQJLQHHUVVHWWK
5HQROGVQXPEHUDWDERXW(KJKHQRXJKIRUDUHDVRQDEOHDSSUR[LPDWLRQ RI SURWRWSHWXEXOHQ
ICRZ YLVFRXVHIHFW 1RWWKHLQWHUHVWXEXOHQFHEHORZ WKHGDP 7KHGRZQWHDP EHG RUDSURQ RI I
GDP PXVWEHVWUHQJWKHQHGWXFXODO WRDYRLGEHGHURVLRQ

) LJ +\GUDXOF PRGHORI WKH%OXHWRQH/DNH' DP RQWKH1HZ 5LYHUQHDU+LQWRQ : H
9LUJLQLD 7KHPRGHOVFDOHLV EHWK YHUWFDO DQGKRUL]RQWDO DQG WKH5HQROGVQXPEHU

WKRXJK IDUEHORZ WKH SURWRWSH YDOXH LV VHWKLJK HQRXJK IRUWKH IORZ WR EH WXUEXOHQW*(Courtesy of the U.S. Army Corps of Engineers Waterways Experiment Station.)*

) RUK\GUDXOLF PRGHOV RI ODUJHUVFDOH VXFK DV KDUERUV HVWXDULHV DQG HPEDQPHQW JHRPHWUL VLPLODULW\PD\ EH YLRODWHGRUGRQHFHVVLW7KHYHUWLFDOVFDOHZLOOEHGLVWRUWHGWRGYRLG: HEHUQPEH HIIHFW) RUH[DPSOH WKHKRU]RQWDOVFDOHPD\ EH ZKLOHWKHYHUWLFDOVFDOHLVRQD 7KX WKHPRGHOFKDQQHOPD\ EHGHSHUHDWHGWRLWKRU]RQWDOGLPHQVLRQ6LQFHGHSHUSDWDJHVIORZ PRUHHILFLHQW WKHPRGHOFKDQQHOERWWRP PD\ EHGHEHUDWHGURXJKHQHGWRFUHDWWKHIULFWRQHYH H[SHFWHGLQWKHSURWRWSH

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7KHSUHVXUHGURSGXHWRIULFWRQIRUIORZ LQDORQJ VPRRWKSLSHLV DIXQFWLRQRI DYHUDJHIORZ YHORFLW GHQVLW YLVFRVLW DQGSLSHOHQJWKDQGGLDPHWU ûS IFQ9 ! / ' : HZLVKWRNQRZ KRZ ûSYDULHVZLWK9 D 8VHWKHSLWKHRUHP WRUHZULWWKLVIXQFWLRQLQGLPHQVLRQOHVVIRUP E 7KH SORWWKLVIXQFWLRQ XVLQJWKHIROORZLQJGDWDIRUWKUHHSLSHVDQGWKUHHIOXLGV

D, cm	*L*, m	*Q*, m³/h	Δ*p*, Pa	ρ, kg/m³	μ, kg/(m · s)	*V*, m/s*
1.0	5.0	0.3	4,680	680†	2.92 E-4†	1.06
1.0	7.0	0.6	22,300	680†	2.92 E-4†	2.12
1.0	9.0	1.0	70,800	680†	2.92 E-4†	3.54
2.0	4.0	1.0	2,080	998‡	0.0010‡	0.88
2.0	6.0	2.0	10,500	998‡	0.0010‡	1.77
2.0	8.0	3.1	30,400	998‡	0.0010‡	2.74
3.0	3.0	0.5	540	13,550§	1.56 E-3§	0.20
3.0	4.0	1.0	2,480	13,550§	1.56 E-3§	0.39
3.0	5.0	1.7	9,600	13,550§	1.56 E-3§	0.67

*$V = Q/A$, $A = \pi D^2/4$.
†Gasoline.
‡Water.
§Mercury.

F 6XSSRVHLWLVIXUWKHUNQRZQWKDWûS LV SURSRUWLRQDOWR/ ZKLFK LV TXLWHWUXHIRUORQJ SLSH ZLWK ZHOOURXQGHG HQWUDQFHV 8VH WKLV LQIRUPDWLRQ WR VLPSOLI\ DQG LPSURYH WKH SL WKHRUHP IRUPXODWLRQ 3ORWWKHGLPHQVLRQOHVVGDWDLQWKLVLPSURYHGPDQQHUDQGFRPPHQWRQWKHUHVXOWV

6RXWLRQ

7KHUHDUHVL[YDULDEOHVZLWKWKUHHSULPDU GLPHQVLRQVLQYROYHG^0 / 7` 7KHUHRUHZH H[SHFWWKD M í SLJURXSV: HDUHFRUHFWRUZHFDQILQGWKUHHYDULDEOHVWKDWGRQRWRUP DSLSURGXFW IRUH[DPSOH ! 9 / &DUHXOD VHOHFWWKUHH M UHSHDWQJ YDULDEOHV EXWQRWQFXGLQJ ûS RU9 ZKLFK ZH SODQ WR SORWYHUXV HDFK RWKHU: H VHOHFW ! ' DQG WKH SL WKHRUHP JXDUDQWHHV WKDWWKUH LQGHSHQGHQWSRZHUSURGXFWJURXSVZLOORFFXU

$$\Pi_1 = \rho^a \mu^b D^c \Delta p \qquad \Pi_2 = \rho^d \mu^e D^f V \qquad \Pi_3 = \rho^g \mu^h D^i L$$

or

$$\Pi_1 = \frac{\rho D^2 \Delta p}{\mu^2} \qquad \Pi_2 = \frac{\rho V D}{\mu} \qquad \Pi_3 = \frac{L}{D}$$

: HKDYHRPLWWGWKHDJHEUDNILQGLQJ D E F G H I J K L E\ VHWWQJ DOOH[SRQHQWVWR]HUR0
/ 7 7KHUHRUHZHZLVKWRSORWWKHGLPHQVLRQOHVVUHODWLRQ

$$\frac{\rho D^2\,\Delta p}{\mu^2} = \text{fcn}\left(\frac{\rho V D}{\mu}, \frac{L}{D}\right) \qquad \textit{Ans. (a)}$$

: HSORW YHUVXV ZLWK DVDSDUDPHWHU7KHUHZLOOEHQLQHGDWDSRLQWV)RUH[DPSOH WK
ILUVWURZ LQWKHGDWDKHUH\LHOGV

$$\frac{\rho D^2\,\Delta p}{\mu^2} = \frac{(680)(0.01)^2(4680)}{(2.92\text{ E-}4)^2} = 3.73\text{ E9}$$

$$\frac{\rho V D}{\mu} = \frac{(680)(1.06)(0.01)}{2.92\text{ E-}4} = 24{,}700 \qquad \frac{L}{D} = 500$$

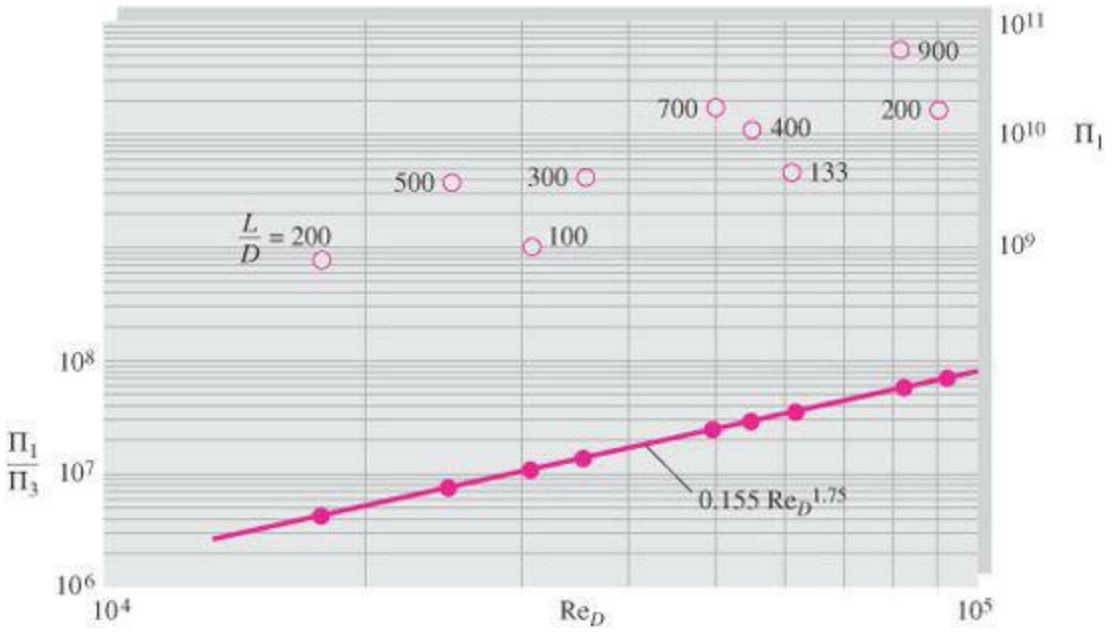
7KHQLQHGDWDSRLQWVDUHSORWWHGDVWKHRSHQFLUFOHVLQ) LJ 7KHYDOXHVRI/ ' DUHOLVWHGIR
HDFKSRLQWDQGZHVHHDVLJQLILFDQWQHQWKHIHFW,QIDFWLZHFRQFHQWKHRQ VWRSRLQWVWKDWKDY
WKHVDPH/ ' ZHFRXOGVHH DQGFURVSORWYHUL\ WKDWûSLQFUHDVHVQQHDU ZLWK/ D
VDWLVIGLQWKHODVWSDUWRI WKHSUREOHP 6LQFH/ RFFXUVRQO LQ3 / ' WKHIXQFWLRQ3 IFQ3 3
PXWWHGXFHWR3 / ' IFQ3 RUVLPSO DIXQFWLRQLQYROYLQJ RQO VWR SDUDPHWHUV

$$\frac{\rho D^3\,\Delta p}{L\mu^2} = \text{fcn}\left(\frac{\rho V D}{\mu}\right) \qquad \text{flow in a long pipe} \qquad \textit{Ans. (c)}$$

: HQRZ PRGLI\ HDFKGDWDSRLQWLQ) LJ E\ GLYLGLQJ LWE\ LW/ ' YDOXH)RUH[DPSOH IRUWK
ILUVWURZ RI GDWD!' ûS / ((: HUHSORWWKHVHQHZ GDWDSRLQWVDVVRO
FLUFOHVLQ) LJ 7KH\ FRUUHDWHDOPRVWSHUIHFW LQRDVWDLJKWOLQHSRZHUODZ IXQFWLRQ

$$\frac{\rho D^3\,\Delta p}{L\mu^2} \approx 0.155\left(\frac{\rho V D}{\mu}\right)^{1.75} \qquad \textit{Ans. (c)}$$

$ OOQHZWRQDQVPRRWKSLSHIORZ VVKRXOGFRUUHDWHLQWKLVPDQQHU7KLVH[DPSOHLVDYDULDWRQR
WKHILUVWFRPSOHWHG VXFFHVVIXOGLPHQVLRQOD DQDOVLV SLSHIORZ IURFWLRQ SHUIRUPHG E\ 3UDQGWO
VWXGHQW3DXO%ODVLXV ZKR SXEOLVKHG D UHDGDEOH SORWLQ) RUWKLV UDQJH RI WXUEXOHQWIORZ
5H\QROGVQXPEHUV WKHSUHVVXUHGURSLQFUHDVHVDSSUR[LPDWHO DV9



) LJ 7ZR GLIHUHQW FRUUHODWLRQV RI WKH GDWD LQ ([DP SOH 2 SHQ FLUFOHV ZKHQ SORWWHG $\rho D \hat{u} p \mu$ YHUVXV 5H $_D$ / D LV D SDUDPHWHU RQH LWLV NQRZQ WKDW $\hat{u} p$ LV SURSRUWLRQDO WR L, D UHSORWV VRGG FLUFOHV RI $\rho D \hat{u} p L \mu$ YHUVXV 5H $_D$ FRROODSVH LQWR D VLQJOH SRZHU FXUYH

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7KH VPRRWK VSKHUH GDWD SORWWG LQ) LJ D UHSUHVHQWG LQ GLPHQVLRQDWV GUDJ YHUVXV GLPHQVLRQDWV YLVFRVLW VLQFH ! 9 G ZHUH VHOHFWHG DV VFDOLQJ RU UHSHDWLQJ YDULDEOHV D 5HSORWWKHVH GDWD V GLVSOD\ WKH HIHFWRI GLPHQVLRQDWV YHORFLW RQ WKH GUDJ E 8VH \RXU QHZ ILJXUH WR SUHGLFW WKH WHUPLQDO]HUR DFFHOHUDWRQ YHORFLW RI D FP GLDPHWHU VWHHOEDOO 6* IDOOLQJ WKURXJK ZDWHU DW f&

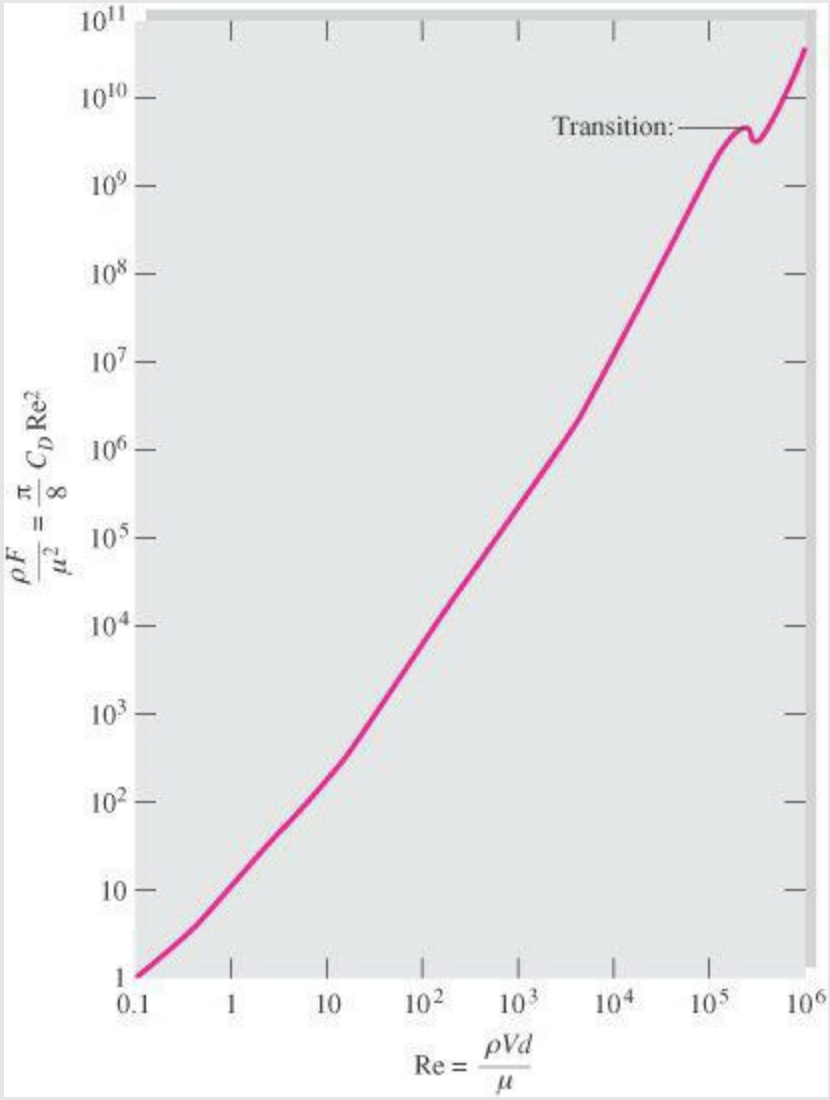
6RXWLRQ

- $VXPSWLRQV) LJ DLVYDOLG IRUDQ VPRRWK VSKHUHLQWKDW6HQRGVQXPEHUDQJH
- $SSURDFK D)RUP SLJURXSV IURP WKHIXQFWLRQ) IFQG 9 ! LQVXFK DZD WKDW\ L SORWWG YHUVXV9 7KHDQZHUZDVDOHDG JLYHQDV(T EXWGHWXVUHYLHZ WKHVWMSV 7K SURSHUVFDOLQJ YDULDEOHVDUH ! G ZKLFKGRQVWRUP DSL 7KHUHRUHM DQGZHH[SHFW í M í SLJURXSV 6NSSLQJ WKHDJHEUD WKH DULHDVIRGRZV

$$\Pi_1 = \rho^a \mu^b d^c F = \frac{\rho F}{\mu^2} \qquad \Pi_2 = \rho^a \mu^b d^c V = \frac{\rho V d}{\mu} \qquad \textit{Ans. (a)}$$

: HPD\ UHSORWWKHGDWDRI) LJ DLQWKLVQHZ IRUP QRWLQJ WKDW A CE &' 5H 7KH UHSORWV VKRZQDV) LJ 7KHGDWD LQFUHDVHVUDSLGO ZLWK YHORFLW XS WR WDQVLWLRQ ZKHU WKHUHLVDVQJKWGURS DWWUZKLFKLWQFUHDVHVPRUHWKDQHYHU,I IRUFHLVNQRZQ ZHPD\ SUHGLF

YHORFLWIURP WKHILJXUH DQGYLFHYHUVD



) LJ &URVV SORWRI VSKHUHGUDJ GDWDIURP) LJ *a* WRVKRZ GLPHQVLRQOHVV IRUFH YHUVXV GLPHQVLRQOHVV YHORFLW

- 3URSHUWLHV YDOXHV IRU SDUWE

$$\rho_{\text{water}} = 998 \text{ kg/m}^3 \qquad \mu_{\text{water}} = 0.001 \text{ kg/(m-s)}$$
$$\rho_{\text{steel}} = 7.86\rho_{\text{water}} = 7844 \text{ kg/m}^3.$$

- 6ROXWRQ WR SDUWE) RUWHUPLQDOYHORFLW VKHGUDJ IRUFHHTXDOVWKHQHWZHLJKWRI VKHVSKHU LQZDWHU

$$F = W_{\text{net}} = (\rho_s - \rho_w)g\frac{\pi}{6}d^3 = (7840 - 998)(9.81)\left(\frac{\pi}{6}\right)(0.01)^3 = 0.0351 \text{ N}$$

7KHUHRUHWKHRUGLQDWHRI) LJ LVNQRZQ

$$\text{Falling steel sphere:} \quad \frac{\rho F}{\mu^2} = \frac{(998 \text{ kg/m}^3)(0.0351 \text{ N})}{[0.001 \text{ kg/(m} \cdot \text{s})]^2} \approx 3.5 \text{ E7}$$

) URP) LJ D $\rho F/\mu^2 \approx 3.5$ E7 D P DJQL\LQJ JODVV UHYHDOV WKDW5H§ (7KHQD FUXG HXLP DWRI WKH WUPLQDO IDOOY-DFLWLV

$$\frac{\rho V d}{\mu} \approx 20{,}000 \quad \text{or} \quad V \approx \frac{20{,}000[0.001\ \text{kg/(m} \cdot \text{s)}]}{(998\ \text{kg/m}^3)(0.01\ \text{m})} \approx 2.0\ \frac{\text{m}}{\text{s}} \qquad \textit{Ans. (b)}$$

- &RPPHQWV %HWWHU DFFXUDF\ FRXOG EH REWDLQHG E\ H[SDQGLQJ WKH VFDOH RI) LJ LQ WK UHJLRQ RI WKH JLYHQ IRUFH FRHILFLHQW+RZHYHU WKHUH LV FRQVLGHUDEOH XQFHUWDLQW\ LQ SXEOLVKHG GUDJ GDWD IRU VSKHUHV VR WKH SUHGLFWHG IDOOY-DFLWLV SUREDEO\ XQFHUWDLQ E\ DW-HDVW SHUFHQW

1 RWH WKDW WKH IRXQG WKH DQWZHUGLUHFW IURP) LJ : HFRXOG XVH) LJ D DOR EXWZRXO KDYH WR LWHUDWH EHWZHHQ WKH RUGLQDWH DQG DEVFLVVD WR REWDLQ WKH ILQDO UHVXOWVLQFH9 LV FRQVLQHG L EHWZHHQ WKH SORWWHG YDULDEOHV

6XPP DU\

&KDSWHUV DQG SUHVHQWHG LQWURGXFHV GLIIHUHQWLDO PHWRGV RI PDWHPDWLFDO DQDOVLV RI IOXL IORZ 7KLV FKDSWHU LQWURGXFHV WKH WKLUG DQG ILQDO PHWRG H[SHULPHQWDWRQ DV VXSSOHPHQWG E\ WK WHFKQLTXH RI GLPHQVLRQDO DQDOVLV 7HWWDQG H[SHULPHQWDWLXXHGERWWRWWHQJWKRUH DQG WR SURYLGHXVH XCHQJLQHULQJ UHVXOWZKHQRU\ LVLQDGHTXDW

7KH FKDSWHUEHJLQV ZLWK D GLVFXVVLRQ RI VRPH IDPLDU SK\VLFDOUHDWRQV DQG KRZ WKH FDQE UHFDWWLQ GLPHQVLRQDOWV IRUP EHFXVH WKH\ VDWW\ WKH SULQFLSOH RI GLPHQVLRQDO KRPRJHQHLW\ $ JHQHUDOWFKQLTXH WKH SL WKHRUP LV WKHQ SUHVHQWG IRUV\WWP DWFDO ILQGLQJ DVHWRI GLPHQVLRQO SDUDPHWV E\ JURXSLQJ D OWRI YDULDEOHV WKDWJRYHUQ DQ SDUWFXODU SK\VLFDO SURFHW \$ VHFRQ WFKQLTXH ,SVHQJV PHWRG LV DOR GHVFULEHG \$OWLQDWO GLUHFWDSSOFDWRQ GLPHQVLRQDODQDOVL VR WKHEDVLF HTXDWRQV IOXLGPHFKDQFV\LHOGV WKHIXQGDPHQWDOSDUDPHWJRYHUQLQJ IORZ SDWWHUQ 5H\QROGVQXPEHU)URXGHQXPEHU3UDQGWQXPEHU0DFKQXPEHU DQGRWKHUV

,WLV VKRZQ WKDWP RGHOWWXQJ LQ DLU DQG ZDWU RIWQ OHDGV WR VFDOLQJ GLIILFXOWHV IRU ZKLF FRP SURPLVHV PXWEH PDGH 0DQ PRGHOWWXVGR QRWDFKLHYH WXHG GDQPLF VLPLODULW 7KH FKDSW HQGV E\ SRLQWLQJ RXWWKDWFDXLF GLPHQVLRQDW FKDUW DQG GDW FDQ EH PDQLSXODWG DQG UHFDWWR SURYLGH GLUHFWWRROWRQV WR SUREOPV WKDWZRXOG RWKHUZLVHEH TXLWH FXPEHUP H DQG OBERURXVO LWWUDWYH

3UREOPV

0 RWWR WKH SUREOPV KHUHQDUHIDLU VWDLJKWRUZ DUG 0 RUHGLIILFXOWRUSHQHGHDVLJQPHQ DUHOBEHOHG ZLWK DQ DWWULVN 3UREOPV OBEHOHG ZLWK DQ((6 LFRQ ZLOOBEHQHILWURP WKH XVH RI WK (QJLQHULQJ (TXDWRQ 6ROYHU ((6 ZKLH SUREOPV OBEHOHG ZLWK D FRPSXWHU PD\ UHTXLU WKH XVH RI D FRPSXWU 7KH WDQGDUG HQG RI FKDSWUSUREOPV 3 WR3 FDWHJRUL]HG LQ WK SUREOP OWWKHUH DUHIROBZHGE\ ZRUGSUREOPV: WR: IXQGDPHQWDOV RI HQJLQHULQJ H[DF SUREOPV) (WR) (FRPSUHHQLYH DSSOHGSUREOPV& WR& DQG GHVLJQ SURMFW' DQG'

3UREOHP ' LVWXUEDQFH

Quantity	Symbol	Dimensions $MLT\Theta$	Dimensions $FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

,QWURGXFWRQ G\QDPLF VLPLODULW\

- 3)RU D[LDO IORZ WKURXJK D FLUFXODU WXEH WKH 5H\QROGV QXPEHU IRU WDQVLWLRQ V WXUEXOHQFH LV DSSUR[LPDWHO\ >VHH(T @EDVHG RQ WKH GLDPHWHU UDQGDYHUJ YHORFLW\ ,I G FP DQG WKH IOXLG LV NHURVHQH DW f& ILQG WKH YRXPHIORZ UDWH L P KWDWFXVWVWDQWLRQ
- 3 $ SURWRWSHDXAP RELOHLVGHVLJQHG IRUFRQGZHDWHULQ' HQYHU&2 í f& N3D ,WV GLDJ IRUFHLV WR EH WWXAG RQ D RQH VHYHQWK VFDOH PRGHOLQ D ZLQG WXQQHODW PLK f& DQG DWP ,I WKH PRGHODQG SURWRWSHDUH WR VDWLV\ G\QDPLF VLPLODULW ZKDWSURWRWSHYHORFLW LQPLK QHGVWR EHPDWKHG' &RPPHQWRQ\RXUHVXOW
- 3 7KHWDQVHURI HQHJ\ E\ YLVFRXVGLVLSDWRQLV GHSHQGHQWXSRQ YLVFRVLW WKHUPD FRQGXFWLYLW\ WHP DQG WHP WSHUDWXUH7 * URXS WKHVHTXDQWLWLHV LI SRVLEOH LQWR WKHGLPHQVLRQOHVV%ULQP DQQXPEHU ZKLFK LV SURPLQHQWLQ WKLV SUREOHP
- 3 : KHQ WXUQLQ ZDWU DW f& IORZLQJ DW P V DQ FP GLDPHWHU VSKHUH KDV PHDVXUHG GUDJ RI 1 : KDWZLOOEH WKH YHORFLW DQG GUDJ IRUFH RQ D P GLDPHWHU ZHDWHU EDOORRQ PRRUHG LQ VHDOHYHO WDQGDUG DLU XQGHU G\QDPLFDOO VLPLODU FRQGLWLRQV"
- 3 $QDXAP RELOHKDV D FKDUDFWULVWF HQJWK DQG DUHD RI IWDQG IW UHVHFWLYHO : KHQWXUQLQVHDOHYHOWDQGDUG DLU LWKDV WKH IROORZLQJ PHDXUHG GUDJ IRUFHYHUX VSHHG

7KH VDPH FDU WDYHOV LQ &RORUDGR DW PLK DWDQ DOWLWXGH RI P 8VLQ

/ HIEYUH RQ ZDAUDW *f*& VWDUWQJ IURP UHWWZLWK J DFFHOHUDWRQLQD FP
GLDPHWUSLSH VKRZHG WDQVLWRQDWW V 8VH WKLV GDWD WR HVWLPDWH D WK
WDQVLWRQ WLPH DQG E WKH WDQVLWRQ 5HQRQV QXPEHU 5HI IRU ZDAUIORZ
DFFHOHUDWQJ DW P V LQD FP GLDPHWUSLSH

3 9RUWH[VKHGGLQJ FDQEH XVHG WR GHVLJQ DYRUW[IORZPHWU) LJ $ EXQWURO
VWUHWFKHG DFURVV WKH SLSH VKHGV YRUWLFHV ZKRVH IUHTXHQF\ LV UHDG E\ WKH VHQVR
GRZQVWUHDP 6XSSRVH WKH SLSH GLDPHWU LV FP DQG WKH URG LV D FYOLQGHU R
GLDPHWU PP ,I WKH VHQVRU UHDGV FRXQWV HVWLPDWH WKH YROXPH
IORZ UDWH R ZDWHULQP K +RZ PLJKW WKHPHWUUHDFWRRWKHUOTXLGV"

3 $ ILVKQHWLVPDGH R PP GLDPHWU VWULQJV NQRWWGLQR î FP VTXDUHV (VWUHWFK
WKH KRUVHSRZHUUHTXLUHG WR WRZ IWR WKLV QHWWQJ DW NQLQVHZ DWUDW *f*&
7KHQHWSDQHLVQRUPDOWR WKH IORZ GLUHFWRQ

3 7KHUDGLRDQWQQDRQD FDUEHJLQV WR YLEUDWH ZLOO DW +] ZKHQ WKH FDULVGULYHQ
PLK RYHUDUXWWG URDG WKDWDSSURPLPDWV DW VQLQH ZDYHR P SDUXGH FP DQ
ZDYHOHQJWK P 7KH DQWQQD GLDPHWU PP ,V WKH YLEUDWRQ GXH WR WK
URDGRUWR YRUWH[VKHGGLQJ"

3) ORZ SDWWD ORQJ F\OLQGHU R VTXDUH FURVVVHFWLRQ UHXOW LQ PRUH GUDJ WKDQ WK
FRPSDUEOH URXQG F\OLQGHU + HUH DUH GDWD WDN LQ D ZDWU WXQQHO IRU D VTXDU
F\OLQGHUR VLGHOHQJWK E FP

D 8VH WKH VHGDWR SUHGLFWWKH GUDJ IRUFH SHUXQLWG HSWK R ZLQG EORZLQJ DW P V L
DLUDW *f*& RYHUD WDOO VTXDUHFKLP QH R VLGHOHQJWK E FP E ,V WKHUHDQ XQFHUWDLQ
LQ\RXUHWXP DW"

3 7KH VLPSOL VXSSRUWG FDUERQ WXEH URG R) LJ 3 LV VXEMHFWG WR I
FURVVIORZ WUHDP R DLUDW *f*& DQG DW) RUZKDWWHDP YHORFLW8 ZLOOWKHUR
FHQWUGHOHFWRQEHDSSUR[LPDWO FP"

3

3) RUWKH WXEH URG R 3URE 3 DW ZKDWDLUWHDP YHORFLW8 ZLOOWKHURGEHJLO
WRYLEUDWDWDWUDOO LQUHVRQDQFH LQLWWILUWPRGH DKDO VLQHZDYH" +LW&RQX
DYLEUDWRQW[W> @XQGHU³ OWAUDOEHDP YLEUDWRQ´

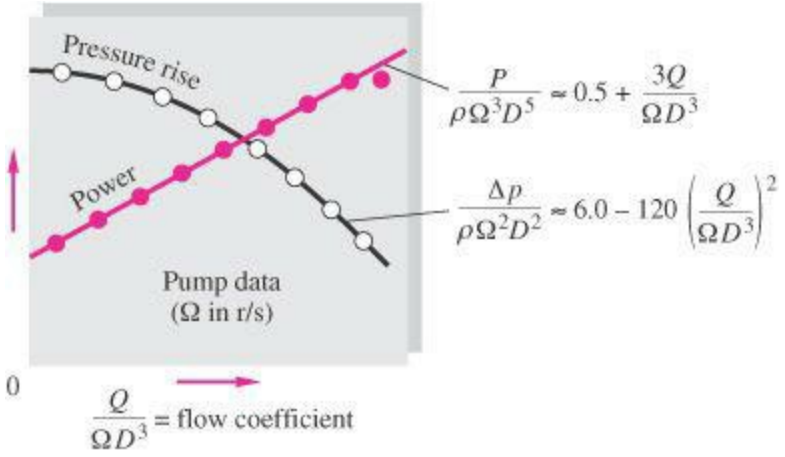
3 $ ORQJ VOHQGHU VPRRWK FP GLDPHWU IOXJSROHEHQGV DOOUPLQJ LQ PLK VHD
OHYHOZ LQGV FDXLQJ SDWWRWF FLW]HQ/WRJDS $QHQJLQHUFOLPVWKHSROHZ LO
EHQG OHW LI LW VXUFH LV GHOEHUDWWO URXJKHQG ,V VKH FRUHFW DWOHDV
TXDOWDWLYHO"

6FDOLQJ RI PRGHO GDWD

3 7KH WKUXVW RI DIUHH SURSHOOHU LV KHDYLO\ GHSHQGHQW RQ PDQLIROG GHVLJQ DQG WKH RYDOO RQUDWLQJ FRQGLWLRQ' DQG WKH IRUZDUG YHORFLW9 9LVFRXV HIHFW DUH VLJQLILFDQW RQ WKH 7HVW RI D FP GLDPHWUR RGHOLDUFDWSURSHOOHU LQ D VHD OHYHO ZLQG WXQQHO \LHOGV WKH IROORZLQJ WKUXVWGDWD YHORFLW RI P V D 8VH WKLV GDWD WR PDNH D FXUYH ILW YHUVXV 9 DQG GLPHQVLRQOHVV SORW E 8VH WK GLPHQVLRQOHVV GDWD WR SUHGLFW WKH WKUXVW LQ D VLPLODU P GLDPHWU SURRWSH SURSHOOHU ZKHQ URDWLQJ DW UPLQ DQG LQ)LJ DW PLK DW P VWDQGDUG DOWLWXGH

Rotation rate, r/min	4800	6000	8000
Measured thrust, N	6.1	19	47

3 EES ,I YLVFRVLW9 LV QHJOHFWHG \WSLFDOSXPS IORZ UHVXOWVIURP ([DPSOH DUHVKRZ LQ)LJ 3 IRUDPRGHOSXPS\WXGLQZDWU7KHSUHVVXUHULVHGFUHDVHVDQGWK SRZHU UHTXLUHG LQFUHDVHV ZLWK WKH GLPHQVLRQOHVV IORZ FRHILFLHQW &XUYH ILW H[SUHVVLRQV DUH JLYHQIRUWKHGDWD 6XSSRVHDVLPLODUSXPSRI FP GLDPHWU EXLOWWR PRYHJDVROLQHDW *& DQG DIORZ UDWHRI P K ,I WKHSXPSURWDWH VSHHGLV UV ILQG D WKHSUHVVXUHULVHDQG E WKHSRZHUHTXLUHG



3

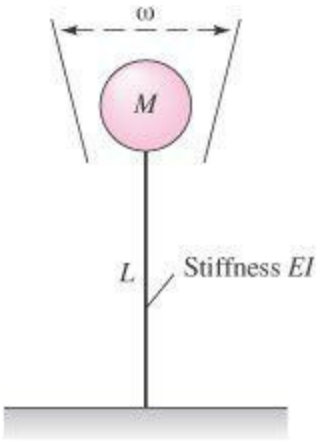
3 ([WHQG3UREOHP DVIRORZV / HWWKHPD[LPXP PDVVIORZ P DJDLQEHDIXQFWLRQR VDQN SUHVVXUHS DQG WHPSHUDWXUH7 JDV FRQVWDQWV DQG QR]]OH GLDPHWU' EX UHSODFHF$S E\ WKHVSHFLILF KHDWDWLR N)RUDQLUWDQNDW N3DDQG . ZLWK FP QR]]OH GLDPHWU H[SHULPHQWV VKRZ DPDVVIORZ RI NJ V D &DQ WKL GDWDEHXVHGWRFRUUHODWHQR[\JHQVDQ' E ,I VR HWWP DWWKHR[\JHQPDVVIORZ L WKHVDQNFRQGLWLRQ/DUH N3DDQG . ZLWKDQR]]OHGLDPHWURI FP

3 7KH SUHVVXUH GURS SHUXQLWOHQJWK ûS/ LQ VPRRWK SLSH IORZ LV NQRZQ WR EH I IXQFWLRQ RQO RI WKH DYHUDJH YHORFLW9 GLDPHWU' DQG IOXLG SURSHUWLHV! DQG 7KH IRORZLQJ GDWZ HUH REWDLQHG IRUIORZ RI ZDWUDW *& LQDQ FP GLDPHWU SLSH P ORQJ

9HUI\ WKDWWKHVH GDWD DUH VOJKWO RXWLGH WKH UDQJH RI)LJ : KDWV D VXLWDEOH SRZHU ODZ FXUYH ILWWRU WKH SUHVHQWGDW' 8VH WKHVH GDWZ HWWP DWWKH SUHVVXUH GURS IRUIORZ R NHURVHQHDW *& LQDVPRRWKSLSHRI GLDPHWU FP DQGOHQJWK P LI WKHIORZ UDWHLV P K

Q, m^3/s	0.005	0.01	0.015	0.020
Δp, Pa	5800	20,300	42,100	70,800

3 7KH QDWXUDO IUHTXHQF\ & RI YLEUDWRQ RI DPDV0 DWWDFKHG WR DURG DVLQ) LJ 3 GHSHQGV RQO RQ0



3 DQG WKH VWLIQHVV(, DQG GHQJWK/ RI WKH URG 7HVWV ZLWK D NJ PDV DWWDFKHG WR D FDUERQVWHOURG RI GLDPHWHU PP DQG GHQJWK FP UHYHDOHG QDWXUDO IUHTXHQF\ RI +] 8VHWKHVHGDWD WR SUHGLFW WKH QDWXUDO IUHTXHQF\ RI D NJ PDV DWWDFKHG WR D DOXPLQLXP DOR\ URG RI WKH VDPH VL]H

3 ,QVWUXEXHQWORZ QHDUDIODWZDOO WKHORFDOYHORFLWX YDULHV RQO ZLWK GLVWDQFH\ IURP WKH ZDOO ZDOWKHUWHV2 DQG IOXLGSURSHUWLHV! DQG 7KHIORZ LQJ GD ZHUH WDNHQ LQ WKH 8QLYHUVLWR 5KRGH,VODQG ZLQGWXQQHOIRUDLUIORZ ! VOXJ IW (VOXJ IW V DQG2 GEI IW D 3ORWWKHVHGDWDLQWK IRUP RI GLPHQVLRQDWVX YHUVXV GLPHQVLRQDWV\ DQG VXJJHVWD VXLWDEOHSRZHUODZ FXUYHILW E 6XSSRVHWKDWWKHWXQQHOVSHHGLVIUHDHGXQGX IWVDW LQ (VWLPDWHWKHQHZ ZDOOVKDUWXHVV LQGEI IW

y, in	0.021	0.035	0.055	0.080	0.12	0.16
u, ft/s	50.6	54.2	57.6	59.7	63.5	65.9

3 $ VRUSHGR P EHORZ WKH VXUIDFH LQ *f& VHZ DWUFDYLDWWV DWD VSHGRI P ZKQ DWRVSKHUF SUHVXUH LV NPD ,I 5H\QROGV QXPEHUDQG)URXGH QXPEHHIHFWDUHQHJOLJLEOH DWZKDWVSHGZLOOLWFDYLDWZKQUXQQLQJ DWDGHSWKRI P" $VZKDWGHSWKVKRXOGLWEHWRDYRLGFDYLWDWRQDW P V"

3 $ VXGHQWQHGVWR PHDXUHWKHGUDJRQDSURWSHRI FKDUDFWULWLF GLPHQVLRQG PRYLQJ DWYHORFLWV8S LQDLUDWXDQGDUGDWRSKHUF FRQGLWLRQV +HFRQWUFWV PRGHORI FKDUDFWULWLF GLPHQVLRQG VXFKWKDWWKHUDWRGS LVVRPHIDFWRUI +H WKHQPHDXUHVWKHGUDJRQWKHPRGHODWGQDPLFDOO VLPLODUFRQGLWLRQV DOR ZLWK DLUDWXDQGDUGDWRSKHUF FRQGLWLRQV 7KHVXGHQWFDLP VWKDWWKHGUDJ IRUFHR WKHSURWSHZLOOEHHLGHQWFDOWR WKDWPHDXUHGRQWKHPRGHO,VWKLVFDLP FRUUHFW ([SODLQ

3)RUWKHURWDWLQJ FDQGHUIXQFWLRQRI3URE 3 LI / ! ' WKH SUREOP FDQ EH UHGXFHGWRRQO WZRJURXSV) !8 /' YHUVXV 9' 8 +HUHDUH[SHULPHQWDOGDW IRUDFDQGHU FP LQGLDPHWUDQG P ORQJ URWDWQJ LQVLDOHYHOLUZLWK8 P V D 5HGXFHWKLVGDWWRWKHWZRGLPHQVLRQDWJURXSVDQGPDNHDSORWV E 8V

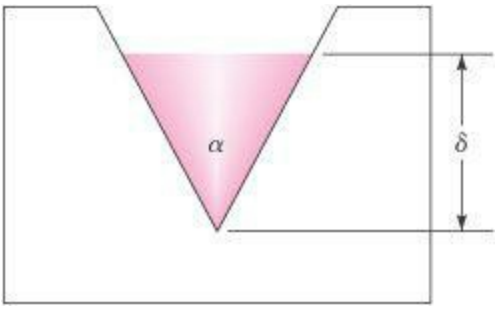
VKLV SORW WR SUHGLFW WKH OLIW RI DF\OLQGHU ZLWK' FP / FP UDGLXV UHY PLQLQ ZDWHU8 P V

Ω, rev/min	0	3000	6000	9000	12000	15000
F, N	0	850	2260	2900	3120	3300

3 $ VLPSOH IORZ PHDVXUHPHQW GHYLFH IRU WHDPV DQG FKDQQHOV LV D QRWFK RI DQJOH. FXWORXW WKH VLGH RI DGDP DV VKRZQ LQ) LJ 3 7KH YROXPHIORZ 4 GHSHQGV RQ. WKH DFFHOHUDWRQ RI JUDYLWJ DQG WKH KHLJKW/ RI WKH XSWUHDP ZDWHU VXUIDFH DERYH WKH QRWFK YHUWH[7HVWV RI DPRGHOQRWFK RI DQJOH. *f* \LHOG WKH IORORZ LQIORZ UDWHGDWD

δ, cm	10	20	30	40
Q, m^3/h	8	47	126	263

D) LQG DQDPHQVLRQOHVV FRUUHODWLRQ IRUP WKH GDWD E 8VH WKH PRGHOGDWD WR SUHGLFW WKH IORZ UDWH RI DSURWRW\SH QRWFK DOR RI DQJOH. *f* ZKHQ WKH XSWUHDP KHLJKW/ LV P



3

3 EES $ GLDPRQG VKDSHGERG RI FKDUDFWHULVWLF OHQJWK LQ KDV WKH IORORZ LQJ PHDVXUHG GUDJ IRUFHV ZKHQ SODFHG LQDZ LQG WXQQHODW WHDOHYHOV DQG DW DLIIHUHQW DQJOHV RUHQWDWLRQ 8VH WKHVH GDWD WR SUHGLFW WKH GUDJ IRUFH RI DVLPLODU LQGLPRQG SODFHG DW LPLODU RUHQWDWLRQ LQ *f*& ZDWIORZ LQJ DW P V

V, ft/s	30	38	48	56	61
F, 1bf	1.25	1.95	3.02	4.05	4.81

3 7KH SUHVXUHGURS LQDYHQXULPHWU) LJ 3 YDULHV RQO ZLWK WKH IOXLG GHQVLW SLSH DSSURDFK YHORFLW DQG GLPHWUDWR RI WKH PHWU $ PRGHOYHQXULPHWU VWXGLQZDWUDW *f*& VKRZ VD NBDGURS ZKHQ WKH DSSURDFK YHORFLW V $ JHRPHWULFDOO VLPLODU SURWRW\SH PHWUL V XVHG WR PHDVXUH JDVRQQH DW *f*& DQG IORZ UDWRI P PLQ ,I WKH SURWRW\SH SUHVXUH JDJH LV PRVWDFFXUDW DW NBD ZKDWWKRXOG WKH XSWUHDP SLSHGLPHWUE"

3 $ RQHVW HDWK VFDOHPRGHORI DOLJHFRPPHUFLDOODUFUDIW WV WWXQQHODW *f*& DQG DW 7KH PRGHOFKRUG OHQJWK LV FP DQG LWV ZLQJ DUHD LV P 7HVWV HXOV IRUWKH GUDJ RI WKH PRGHOUHDVIROORZV

,QWKHVSLULWRI) LJ XVH WKLVGDWD WR HVWLPDWH WKH GUDJ RI WKH IXOOVFDOH DLUFUDIW ZKHQ IO LQ DW PLK IRUWKHVDPH DQJOHRI DWWDFN DW IWXDQGDUDOWXGH

V, mi/h	50	75	100	125
Drag, N	15	32	53	80

3 7KH SRZHU3 JHQHUDWHG E\ DFHUWDLQ WLQGPLOOGHVJQGHSHQGV RQ LWV GLDPHWU' WK DLUGHQVLW! WKH ZLQG YHORFLW9 WKH URWDWRQUDW9 DQG WKH QXPEHUR EODGHV Q D : UDW WKLV UHODWRQVKLS LQGLPHQVLRQOHVV IRUP \$ PRGHOZ LQGP LOO RI GLDPHWU FP GHYHORSV N DWWDFKHG YHOZKHQ9 P V DQG ZKHQURWDWQJ DW

UPLQ E : KDVSRZHUZLOOEHGHYHORSHGE\ DJHRPHWULFDOO DQG GQDPLFDOO
VLPLODUSURSRVHSHRGLDPHWU P LQZLQGVRI P VDW P VDQGDUGDOWXGH
F : KDWLVWKHDSSURSULDWURDWRQUDWLRI WKHSURSRVHG'

3 $RQHWHQWKVFDOHPRGHORI DVXSHUFRQF ZLQJ WHWXG DW P VLQDLUDW *& DQG
DW VKRZV D SLWFKLQJ PRPHQWRI N1 P ,I 5HQROGV QXPEHUHIHFWV DL
QHJOLJLEOH ZKDWZLOOWKHSLWFKLQJ PRPHQWRI WKHSURSRVHZLQJ EHLI LWLVIOLQJ D
WKHVDPH0 DFKQXPEHUDW NP VDQGDUGDOWXGH'

) URXGHDQG0 DFK QXPEHUVFDOLQJ

3 $FFRUGLQJ WR WKHZHE VLWH8 6* 6 ' DL0 : DWU' DWD IRUWKH1 DWRQWKHPHDQIORZ
UDWLQWKH1HZ 5LYHUQHDU+LORQ : 9 LV IWV ,I WKHK\GUDXOLF PRGHOL
)LJ LVWRPDWKWKLVFRQGLWRQZLWK)URXGHQXPEHUVFDOLQJ ZKDWLVWKHSURSH
PRGHOIORZ UDWH'

3 $ IWGRQJ PRGHORI DVKLS LVWHVWLQGIUHVKZDWUWRZ VDQN 7KHPHDXUHGGUD
PD\ EH VSOLWLQWR ³IULFWLRQ GUDJ 5HYQROGV VFDOLQJ DQG ³ZDYH GUDJ)URXG
VFDOLQJ 7KHPRGHOGDWDUHDVIROORZV

Tow speed, ft/s	0.8	1.6	2.4	3.2	4.0	4.8
Friction drag, lbf	0.016	0.057	0.122	0.208	0.315	0.441
Wave drag, lbf	0.002	0.021	0.083	0.253	0.509	0.697

7KHSURSRVHGVKLS LV IWGRQJ (VLPDWLWVRRDOGUDJ ZKHQFUKLVLQJ DW NQL
VHDZDWUDW *&

3 $ GDP VSLOOZD\ LVRXEHWWHGE\ XVLQJ)URXGHVFDOLQJ ZLWKDRQHWKLUWHWKVFDO
PRGHO 7KHPRGHOIORZ KDVDQDYHUDJHYHORFLWRI P VDQGDYROXPHIORZ R
P V: KDWZLOOWKHYHORFLWDQGIORZ RI WKHSURSRVHEH' ,I WKHPHDXUH
IRUFHRQDFHUWDLQSDUWRI WKHPRGHOV 1 ZKDWZLOOWKHFRUUHVSRQGLQJIRUFHR
WKHSURSRVHEH'

3 $ SURSRVHVSLOOZD\ KDV DFKDUDFWHULVWLF YHORFLW RI P VDQGDFKDUDFWHULVWL
OHQJWKRI P $ VPDOOPRGHOVFRQWUXFWHGE\ XVLQJ)URXGHVFDOLQJ : KDWLVWK
PLQLPXP VFDOHUDWLRRI WKHPRGHOWKDWZLOOHQVXUHWKDWWKHPLQLPXP : HEHUQXPEH
LV "%RWKIORZVXVHZDWUDW *&

3 $Q(DW&RDVWHUXDU KDVDWGDOSHUURGRI K WKHVHPLGLXQDOOXQDUWGH DQ
WGDOFXUHQWRI DSSUR[LPDWH FP V ,I DRQHILYHKXQGUHGWKVFDOHPRGHOV
FRQWUXFWHGZLWKWGHVGULYHQE\ DSXPS DQGVWRUDJHDSSDUDWX ZKDWVKRXOGWK
SHUURGRI WKHPRGHOWGHVEHDQGZKDWPRGHOFXUHQWSHGVDUHH[SHFWG'

3 $ SURSRVHVKLS LV PORQJ DQGGHVLJQHGWRFUXLVHDW P VDERXW NQ ,WGUD
LVWREHVLPXODWGE\ D PORQJ PRGHOSXOOHGLQDWRZ WDQN)RU)URXGHVFDOQ
ILQG D WKHWRZ VSHHG E WKHUDWR RI SURSRVHVWRPRGHOGUDJ DQG F WKHUDWR R
SURSRVHVWRPRGHOSRZHU

3 $QDLUSODQH RI RYHUDOOOHQJWK IWLVGHVLJQHGWRI() DW P VDW P VDQGDU
DOWXGH $ RQHWKLUWHWKVFDOHPRGHOVWREHWHVWHGLQDSUHVVXUL]HGKHOXP ZLQ
WXQQHODW *& : KDWLVWKHDSSURSULDWWXQQHOSUHVVXUHLQDWP" (YHQDWWKLV KJK
SUHVVXUH H[DFWGQDPLFVLPLODULWLVQRWDFKLHYHG : K\"

3 $ RQHILIWHWKVFDOHPRGHORI DPLOLWDU DLUSODQH LVWHVWHG DW P VLQD ZLQ

VXQOHDVWHDOHYHOFRQGLWLRQV 7KHPRGHOZLQJDUHDLV FP 7KHDQJOHRI DWWDFN
LV GHJUHHV ,I WKHPHDVXUHGPRGHOOLQHV 1 ZKDWLVWKHSURRWSHOLWXVLQ
0 DFKQXPEHUVFDQQ ZKHQLWWOHVDW P VDQGDUGDOWXGHXQGHUGLQDPLFDO
VLPLODUFRQGLWLRQV" 1RWH %HFDUHXOZLWKWKHDUHDVFDQQ
3 $ RQHIRUWKWKVFDOHPRGHORI DVKLS¶VSURSHOOHULVWHVWHGLQDWZ VDQNDW UPL
DQG H[LELWV DSRZHURXSXWRI IW GEI V $FFRUGLQJ WR)URXGH VFDQQ ODZ\
ZKDWKRXOG WKH UHYROXWLRQV SHUPLQXWH DQG KRUVHSRZHURXSXWRI WKH SURRWSL
SURSHOOHUEHXQGHUG\QDPLFDO VLPLODUFRQGLWLRQV"
3 $ SURRWSHRFHDQSODWRUP SLQQLQJ LVH[SHFWHG WRHQFRXQWHUFXUUHQWVRI FP VDQ
ZDYHVRI VSHULRG DQG P KHLJKW,I DRQHILWHQKVFDOHPRGHOLVWHVWHG LQ
ZDYH FKDQQHO ZKDWFXUUHQWVSHHG ZDYH SHULRG DQG ZDYH KHLJKWWKRXOG EH
HQFRXQWHUHG\ WKHPRGHO"

,QYHQWXVHUVFDQQ RI WKHGDWD

3 $V VKRZQ LQ([DPSOH SXPS SHUIRUPDQFH GDWD FDQ EH QRQGLPHQVLRQDOL]HG
3URREOHP 3 JDYH WWSLFDOGLPHQVLRQOHVV GDWD IRUFHQWULXJDOSXPS³KHDG´ +
ûS!J DVIROORZV

$$\frac{gH}{n^2D^2} \approx 6.0 - 120\left(\frac{Q}{nD^3}\right)^2$$

ZKHUH4 LVWKHYROXPHIORZ UDWH QWKHURWDWLQUV DQG' WKHLPSHOOHUGLDPHWHU
7KLVWSHRI FRUUHODWLRQDOORZVRQHWRFRPSXWH+ ZKHQ ! , ' DUHNQRZQ D 6KRZ KRZ
WRUHDUUDQJHWKHVHSLJURXSVVRWKDWRQHFDQVL]HWKHSXPS WKDWV FRPSXWH' GLUHFW ZKH
4 + Q DUHNQRZQ E 0DNHDFXUHEXWHIFWLYHSORWRI \RXUQHZ IXQFWLRQ F $SSO
SDUWE WRWKHIORZLQJ H[DPSOH) LQG' ZKHQ+ P 4 P V DQGQ UV
) LQGWKHSXPSGLDPHWHUIRUWKLVFRQGLWLRQ
3 6RYH3UREOHP 3 IRUJ0FHULQDW *& XVLQJ WKHPRGLILHGVSKHUHGDJSORWRI)LJ

3 ,Q3UREOHP 3 LWZRXOGEHGLIILFXOWWRVROYHIRU9 EHFDXVHLWDSSHDUVLQDOOWKUHH
WKHGLPHQVLRQOHVVSXPSFRHILFLHQW6XSSRVHWKDWLQ3UREOHP 9 LVXQNQRZQEX
' FP DQG4 P K7KHIOLGLVJDVROLQHDW *& 5HVFDOHWKHFRHILFLHQW
XVLQJ WKH GDWD RI3UREOHP 3 WR PDNH D SORWRI GLPHQVLRQOHVV SRZHU YHUVX
GLPHQVLRQOHVV URWDWLQJ (QWUWKLVSORWWR ILQG WKHPD[LPXP URWDWLQJ9
IRUZKLFKWKHSRZHUZLOOQRWH[FHHG :
3 0RGLI\3UREOHP 3 DVIROORZV / HW0 UVDQG4 P KIRUDJHRPHWULFDOO
VLPLODUSXPS : KDWLVWKHPD[LPXP GLDPHWHUL WKHSRZHULVORWWR H[FHHG :
6RYH WKLV SUREOHP E\ UHFDOQLQJ WKH GDWD RI)LJ 3 WR PDNH D SORWR
GLPHQVLRQOHVSRZHUYHUVXVGLPHQVLRQOHVGLDPHWU(QWUWKLVSORWGLUHFWO WR ILQG
WKHGHVLUHGGLDPHWU
3 : DOIUHGWRQ2Z IRUWKLEXHQMORZ DWYHORFLW8 LQ D SLSH RI GLDPHWU' ZD
FRUUHODWHG LQ ZLWK D GLPHQVLRQOHVFRUUHODWRQ E\ /XGZLJ 3UDQGWO VWXGHQ

+ %ODVLXV

$$\frac{\tau_w}{\rho U^2} \approx \frac{0.632}{(\rho UD/\mu)^{1/4}}$$

6XSSRVH WKDW! 8 2Z ZHUH DOONQRZQ DQG LWZ DV GHVLUHG WR ILQG WKH XQNQRZQ
YHORFLW8 5HDUUDQJHDQG UHZULWH WKH IRUPXOD VR WKDW8 FDQ EH ILPPHGLDWHO FDOFXODWG
3 . QRZ LQ WKDWûS LV SURSRUWLRQDO WR/ UHVFDOH WKH GDWD RI ([DPSOH WR SO
GLPHQVLRQOHVûS YHUVXV GLPHQVLRQOHVV YLVFRVLW 8VH WKLV SORW WR ILQG WKH YLVFRVLW
UHTXLUHG LQ WKH ILUVW RI GDWD LQ([DPSOH LI WKH SUHVXUH GURS LV LQFUHDVHG W
N3D IRU WKH VDPH ILOW UDWH WKHQ WKH DQG GHQVLW
3 7KH WDGLWRQDO0RRG\ \SH SLSH IULFWLRQ FRUHODWLRQ LQ&KDS LV RI WKH IRUP

$$f = \frac{2\Delta pD}{\rho V^2 L} = \text{fcn}\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right)$$

ZKHUH' LV WKH SLSH GLDPHWU/ WKH SLSH OHQJWK DQG0 WKH ZDOO URXJKQHVV 1RWH WKL
SLSH DYHUDJH YHORFLW9 LV XVHG RQ ERWK VLGHV 7KLV IRUP LV PHDQWWR ILQG ûS ZKHQ9 L
NQRZQ D 6XSSRVH WKDWûS LV NQRZQ DQG ZHZLVK WR ILQG9 5HDUUDQJH WKH DERYH IXQFWR
VR WKDW9 LV LVRODWHG RQ WKH OHIW KDQG VLGH 8VH WKH IRUOORZLQJ GDWD IRU0' WR PDN
DSORWWR \RXUQHZ IXQFWRQ ZLWK\RXUYHORFLWSDUDPHWUDV WKH RUGLQDWH WKH SORW
E 8VH\RXUSORWWR GHWHUPLQH9 LQP V IRU WKH IROORZLQJ SLSH IORZ ' FP 0
FP / P IRUZDWHU DW *& DQG DW 7KHSUHVXUHGURSûS LV N3D

f	0.0356	0.0316	0.0308	0.0305	0.0304
$\rho VD/\mu$	15,000	75,000	250,000	900,000	3,330,000

: RUG3 UREOHPV

- : ,Q SHUFHQWR GDWDQDOVLVFDVHV WKH³UHGXFLQJ IDFWRU MZKLFK RZHV WKHQXPEH
QRI GLPHQVLRQOHVV YDULDEOHV WRQí MGLPHQVLRQOHVV JURXSV H[DFWO HTXDO WKHQXPEH
RI UHOHYDQWGLPHQVLRQV 0 / 7 4 ,QRQH FDVH ([DPSOH WKLV ZDV QRWWR
([SODLQLQZRUGVZK\ WKLVVLWXDWLRQKDSSHQV
- : &RQVLGHUWKH IROORZLQJ HTXDWLRQ GRODUELOOS LQ ,V WKLV UHDVRQDEOH
LQFRQVLVWHQW' RHVLWDLW\ WKH3' +": K\"
- : ,QP DNLQJ DGLPHQVLRQOHVV ZKDWXOHVGR\RXIRUHKRRVLQJ\RXUVFDOH
YDULDEOHV"
- : ,QDQHUHGLWRQ WKHZULWUDWNHGWKHIROORZLQJ TXHVWLRQDERXW)LJ 3: KLF
RI WKH WKUHH JUDSKV LV D PRUH HIHFWLYH SUHVHQWDWLRQ"´ : K\ ZDV WKLV D GXPI
TXHVWLRQ"
- : 7KLVFKDSWHUGLVFXVVHVWKHGLIILFXOWVRI VFDOQJ 0DFK DQG5H\QROGV QXPEHUV WRJHWKH
DQDLVSDQH DQG) URXGHDQG5H\QROGV QXPEHUV WRJHWKHU DVKLS * LYHDQH[DPSO
RI DIGRZ WKDWZRXOG FRPELQH0DFK DQG) URXGH QXPEHUV : RXOGWKHUHEH VFDOQ
SUREOHPV IRUFRPPRQIOXLGV"
- : : KDWLVGLIIHUHQWDERXWDYHUVPDOOPRGHORI DZHLURUGDP)LJ 3 WKDWZRXO

PDNHWKHWAWWUHXOWGLILFXOWRUHDWRWKHSURRWSH"

- : : KDWHOHDUH\RX WXGLQJ WKLV WUP" * LYH DQH[DPSOH RI DSRSXODUHTXDWRQF IRUPXODIURP DQRWKHUFRXUVH WKHUPRG QDPLFV WUHQJWKRI PDWULDOV RUWKHONH WK GRHVQRWWDWW\ WKHSULQFLSOHRI GLPHQVLRQDOKRPRJHQHLW ([SODLQZKDWLVZURQ DQGZKHWKHULWFDQEHPRGLILHGWRHKRPRJHQHRXV

- : 6RPHFRQGLJHV VXFKDV&RRUDGR6WDW8QLYHUVLW KDYHHQYLURQPHQWDOZLQGWXQQH WKDWFDQEH XVHG WR WXG SKHQRPHQD ONH ZLQG IORZ RYHUFLW EXLOGLQJV : KD GHWDLOVRI VFDOQJ PLJKWEHLPSRUWDQWVXFKWXGLHV"

- : ,I WKHPRGHOVFDOHUDWR LV. $/_P/_S$ DV LQ(T DQG WKH: HEHUQXPEHULV LPSRUWDQWKRZ PXWWKHPRGHODQG SURRWSH VXUFHWHQVLRQEHUHDWG WR. IR GQDPLF VLPLODULW"

- :)RUDWSLFDOLQFRPSUHVLEOHYHORFLW SRWQWDOODQDOVLVLQ&KDS ZHVRYH Ë VXEVHFWWR NQRZQ YDOXHV RI %Ë%Q RQ WKH ERXQGDULHV : KDWGLPHQVLRQOHV SDUDPHWHUVJRYHUQWKLV\WSHRI PRWRQ"

) XQGDPHQWDOVRI (QJLQHHULQJ ([DP 3UREOHPV

-)(* LYHQWKHSDUDPHWHU8 / J ! WKDWDIHFWDFHUWDLQGLIBRZ SUREOHP WK UDWR9 / J LVXVXDOO NQRZQDVWKH
  YHORFLWWKHDG
  %HUQRXOOLKHDG
) URXGHQXPEHU
  NLQHWF HQHJ\
  LPSDFWHQHJ\

-)($ VKLS P ORQJ GHVLJQHGWRFUXLVHDW NQ LVWREHWHWWHHQGLQDWRZ WDQNZLWK PRGHO P ORQJ 7KHSSURSULDWHWRZ YHORFLWLV
  P V
  P V
  P V G P V
  P V

-)($ VKLS P ORQJ GHVLJQHGWRFUXLVHDW NQ LVWREHWHWWHHQGLQDWRZ WDQNZLWK PRGHO P ORQJ ,I WKHPRGHOZDYHGUDJ LV 1 WKHHWWP DWGIXOOL]HVKLSZDY GUDJ LV
  1
  1
  1
  1
  1

-)($ WWDOHWWXDU LVGRPLQDWGE\ WKHV+PLGXQDOOXQDUWGH ZLWKDSHURGRI K ,I D PRGHORI WKHHWXDU LVWWWG ZKDWKRXGEHWKHPRGHOWGDSHURG'
  V
  PLQ
  PLQ
  PLQ

PLQ

-)($ IRRWEDOOPHDQVWREHWKURZQDW PLKLQVHDOHYHODLU! NJP
  (1 VP LVWREHWWXGXLQJDRQHTXDUWHUFDOHPRGHOLQDZDWHUVWQQHO!
  NJP 1 VP)RUGQDPLFVLPLODULW ZKDWLVWKHSURSHUPRG-
  ZDWHUYHORFLW"
  PLK
  PLK
  PLK
  PLK
  PLK

-)($ IRRWEDOOPHDQVWREHWKURZQDW PLKLQVHDOHYHODLU! NJP
  (1 P LVWREHWWXGXLQJDRQHTXDUWHUFDOHPRGHOLQDZDWHUVWQQHO!
  NJP 1 VP)RUGQDPLF VLPLODULW ZKDWLVWKHUDWR R
  SURWRWSHIRUFHWRPRGHOIRUFH"

-)(&RQVLGHUDTXLGIORZ RI GHQVLW! YLVFRVLW DQG YHORFLW8 RYHUDYHU VPDO
  PRGHOVSLQQLQJ RI HQJWKVFDOH/ VXFKWKDWWKHGLPHQVLRQOHVVFRHILFLHQW<
  LVLPSRUWDQW7KHTXDQWW! 8 / < LQWKLVFDVHLVLPSRUWDQWDQGLVFDOHGWKH
  FDSLOODU ULVH
)URXGHQXPEHU
  3UDQGWQXPEHU
  : HEHUQXPEHU
  %RQGQXPEHU

-)(,I DVWUHDP IORZLQJ DWYHORFLW8 SDVWDERG RI HQJWK/ FDXVHVDIRUFH) RQWKHERG
  WKDWGHSHQGVRQD RQ8 / DQGIXLGYLVFRVLW WKHQ) PXWWEHSURSRUWRQDOWR
  !8/
  !8 /
  8 /
  8/
  8/

-)(,Q VXSHUVRQF ZLQG WXQQHOWWXQJ LI GLIIHUHQWJDVHV DUH XVHG G\QDPLF VLPLODUW
  UHTXLUHVWKDWWKHPRGHODQGSURWRWSHKDYHWKHVDPH0 DFKQXPEHUDQGWKHVDPH
  (XOHUQXPEHU
  \SHHGRI VRXQG
  \XDQJQDWRQHQXPEHU
)URXGHQXPEHU
  \SHFLILF KHDWDWR
-)(7KH 5HQROGV QXPEHUIRU D IWGLDPHWU VSKHUH PRYLQJ DW PLK WKURXJ
  VHDZDWU \SHFLILF JUDYLW YLVFRVLW (1 VP LVDSSUR[LPDWO

-) (7KH (NPDQ QXPEHU LPSRUWDQWLQ SK\VLFDO RFHDQRJUDSK\ LV D GLPHQVLRQOHV
FRPELQDWLRQ RI / ! DQG WKH HDUWK¶V URWDWLRQ UDWH 9 ,I WKH (NPDQ QXPEHUL
SURSRUWLRQDO WR 9 LWVKRXOGVDNHWKHIRUP
! /
/ !
! /
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! /
-) ($ YDULG EXWSUREDEO XVHIWV GLPHQVLRQOHVV JURXS LV JLYHQ E\ 7RJ /.
ZKHUHHYHUVWKLQJ KDVLWVXVXDOPHDQLQJ H[FHSW : KDVDUHWKHGLPHQVLRQVRI ."
, /± 7±
, /± 7±
, 0 /±
, ± /7±
, /7±

&RP SUHKHQVLYH3 URE OHP V

- & (VWPDWQJ SLSHZDOOIUFWLRQLVRQHRI WKHPRVWFRPPRQWDVNVLQIOXLGVHQJLQHHULQ
)RUORQJ FLUFXODUURXJK SLSHVLQWXEXOHQWORZ ZDOOKHDU2 LVDIXQFWLRQRI GHQVLW
! YLVFRVLW DYHUDJHYHORFLW9 SLSHGLPHWUG DQGZDOOURXJKQHVVHLJKW7KX\
IXQFWLRQDO ZHFDQZULW2 IFQ! 9 Gë D 8VLQJ GLPHQVLRQDODQDOVL\
UHZULWWKLVIXQFWLRQLQGLPHQVLRQOHVIRUP E\ FHUWDLQSLSHKDVG FP DQGë
PP)RUIORZ RI ZDWHUDW *& PHDVXUHPHQWVKRZ WKHIRORZLQJ YDOXHVRI ZDW
VKHDUVWUHVV

3ORWWKHVHGDWD XVLQJ WKH GLPHQVLRQOHVIRUP REWDLQHG LQ SDUWD DQG VXJJHVWD FXUYH IL
IRUPXOD ' RHV\RXUSORWVHYHDOWKHQWXHIXQFWLRQDOUHODWLRQREWDLQHGLQSDUWD"

Q, gal/min	1.5	3.0	6.0	9.0	12.0	14.0
τ_w, Pa	0.05	0.18	0.37	0.64	0.86	1.25

- & : KHQ WKH IOXLG H[LWQJ D QR]]OH DV LQ) LJ 3 LV D JDV LQWRDG RI ZDWL
FRPSUHVVLEOHWPD\ EHLPSRUWDQWHVSHFLDOO LI XSWUHDP SUHVVXUHS LVODUJHDQG H[L
GLPHQVLRQ LVVPDOO,QWKLVFDVH WKHGLIIHUHQFHS í S LVQRORQJHUFRQVWDQWDQJ DQ
WKHJDVPDVVIORZ P" UHDFKHVDPD[LPXP YDOXHWKDWGHSHQGVRQS DQGG DQGDOR R
WKHDEVROXWXSWUHDP WHPSHUDWXUH7 DQGWKHJDVFRQVWDQW5 7KXV IXQFWLRQDO P"
IFQS G 7 5 D 8VLQJ GLPHQVLRQDO DQDOVLV UHZULWWKLV IXQFWLRQ L
GLPHQVLRQOHVIRUP E\ FHUWDLQSLSHKDVG FP)RUIORZ RI DLU PHDVXUHPHQ
VKRZ WKHIRORZLQJ YDOXHVRI PDVVIORZ WKURXJKWKHQR]]OH

T_1, K	300	300	300	500	800
p_1, kPa	200	250	300	300	300
$\dot{m}$, kg/s	0.037	0.046	0.055	0.043	0.034

3ORWWKHVHGDWDLQWKHGLPHQVLRQOHVVIRUP REWDLQHGLQSDUWD ' RHV\RXUSORWWHYHDOWK HQWUHIXQFWLRQDOUHDWLRQREWDLQHGLQSDUWD"

- & 5HFRQVLGHUWKHIXOO GHYHORSHGGUDLQLQJ YHUWLFDORLOLP SUREOP VHH) LJ 3 D DQH[HUFLVH LQ GLP HQVLRQDO DQDOVLV / HWXWKH YHUWLFDOYHORFLW EH D IXQFWLRQ RQO F GLVWDQFH IURP WKH SDWH IOXLG SURSHUWLV JUDYLW DQG ILOP WKLFNQHVV 7KDWV Z IFQ[! J / D 8VH WKH SL WKHRUHP WR UHZULWWKLV IXQFWLRQ LQ WHUP VRI GLP HQVLRQOHV SDUDP HWUV E 9HUI\ WKDWWKH H[DFWVROXWLRQ IURP 3URE 3 LVFRQVLVWHQW WZ LWK\RX UHVXOWLQ SDUW D

- & 7KH7DFR,QF PRGHO FHQWUIXJDOSXPS KDVDQLPSHOOHUR GLDPHWU LQ : KHQSXPSLQJ *& ZDWUDW9 UPLQ WKHPHDVXUHGIORZ UDW4 DQG SUHVVXU UUHûSDUHJLYHQE\ WKHPDQXDFWXUHUDVIROORZ V

D $VXPLQJ WKDWûS IFQ! 4 ' 9 XVH WKH SL WKHRUHP WR UHZULWWKLV IXQFWLRQ L WHUP VRI GLP HQVLRQOHVSDUDP HWUV/DQG WKHQSORWWKH JLYHQGDWDIQGLP HQVLRQOHVIRUP E , LV GHWHUPLQHG WR XVH WKH VDP H SXPS UXQQLQJ DW UPLQ WR SXPS *& JDVROLQH DW JDOPLQ $FFRUGLQJ WR\RXUGLP HQVLRQOHVFRUUHODWLRQ ZKDWSUHVVXUHULVûS LVH[SHFWHG L GE LQ"

Q, gal/min	200	300	400	500	600	700
Δp, lb/in^2	36	35	34	32	29	23

- & ' RHV DQ DXWRP REOHP UDLR DQWQQD YLEUDW LQ UHVRQDQFH GXH WR YRUW[VKHGGLQJ &RQVLGHUDQDQWQQD RI CHQJWK/ DQG GLDP HWU' $FFRUGLQJ WR EHDP YLEUDWRQ WKHRU >VHH> @S @WKHILUWPRGHQDWXUDOIUHTXHQF\ RI D VROG FLUFXODUFDQWLOHYH EHDP LV&Q >(, !$/ @ ZKHUH(LVWKHPRGXOXV RI HODVWLFLW, LVWKHDUH PRPHQWRI LQHUWD ! LVWKHEHDP PDWHULDOGHQVLW DQG$ LVWKHEHDP FURVV-VHFWLR DUHD D 6KRZ WKDW&Q LV SURSRUWLRQDO WR WKHDQWQQD UDGLXV5 E ,I WKHDQWQQD L VWHO ZLWK/ FP DQG' PP HVWLPDWWKHQDWXUDOYLEUDWRQIUHTXHQF\ LQ+] F &RPSDUHZLWKWKHVKHGGLQJIUHTXHQF\ LI WKHFDUPRYHVDW PLK

' HVLJQ3URMHFWV

- ' : HDUHJLYHQDERUDWRU GDWD WDNHQE\ 3URI 5REHUW LFKKRI DQG KLV VXGHQWVDWK 8QLYHUVLWVRI 0 DVVDFKXVHWWV IRUWKHVSLQUDWRI D FXS DQHPRPHWU 7KHDQHPRPHW ZDVPDGHRI SLQJ SRQJ EDOOV G LQ VSDWLQKDO IDFLQJ LQRSSRVLWGLUHFWRQ DQG JOXHG WRWKLQ URGVSHJHG WR DFHQWUDO[OH 6HH) LJ 3 IRUDVNHWK 7KHU ZHUHIRXU URGV RI OHQJWKV O DQG IW7KHH[SHULPHQW GDWD IRUZLQGVSHHGOYHORFLW8 DQGURWDWRQUDW9 DUHDVIROORZ V

$VXPHWKDWWKHDQJXODUYHORFLW9 RI WKHGHYLFHLVDIXQFWLRQRI ZLQGVSHHG8 DUGHQVLW! DQG YLVFRVLW URGOHQJWKO DQG FXS GLDPHWUG)RUDOOGDWD DVXPHDULVDW DW DQG *& ' HILQHDSSURSULDWH JURXSVIRUWKLVSUREOP DQG SORWWKHGDWD LQWKLVGLPHQVLRQOHVP DQQHU &RPPHQWRQWKHSRVVLEOHXQFHUWDLQWVRI WKHUHVXOWV

$I = 0.212$		$I = 0.322$		$I = 0.458$		$I = 0.574$	
U, ft/s	Ω, r/min	U, ft/s	Ω, r/min	U, ft/s	Ω, r/min	U, ft/s	Ω, r/min
18.95	435	18.95	225	20.10	140	23.21	115
22.20	545	23.19	290	26.77	215	27.60	145
25.90	650	29.15	370	31.37	260	32.07	175
29.94	760	32.79	425	36.05	295	36.05	195
38.45	970	38.45	495	39.03	327	39.60	215

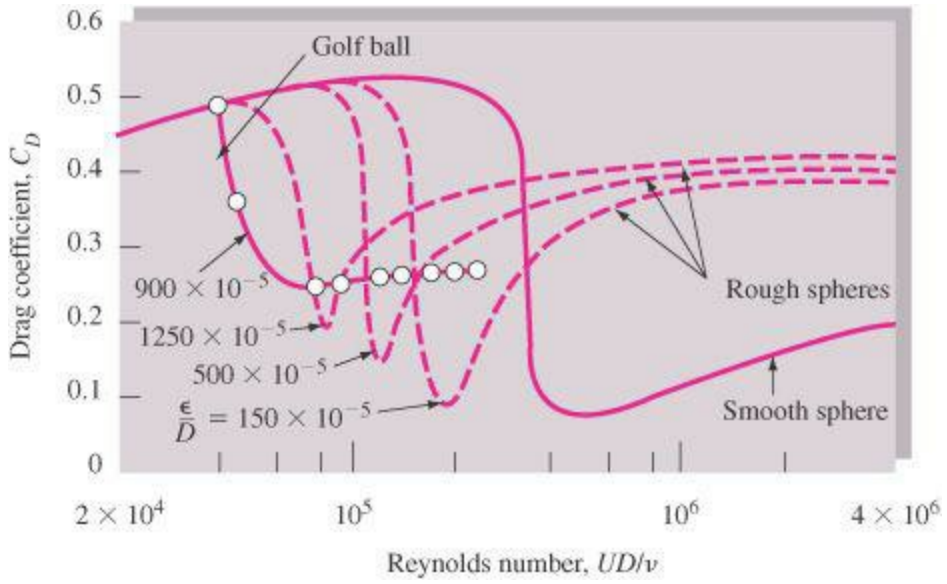
$ VDGHVLJQDSSOLFDWLRQ VXSSRVHZ HDUHWRXVHWKLVDQHPRPHWUJHRPHWU IRUDGDJH VFDG G FP DLUSRUWZ LQG DQHPRPHWU ,I ZLQG VSHHGV YDU XS WR P V DQG ZHGHVLUH D DYHUDJHUURUDWLRQUDW9 UPLQ ZKDWKRXGEHWKHSURSHUURGOHQJWK' : KDWDUHSRVLEO OPLWDWLRQV RI \RXUGHVLJQ' 3UHGLFWWKHH[SHFWHG9 LQUPLQ RI \RXUGHVLJQDVDIIHFWHGE\ ZLQ VSHHGVIURP WR P V

- ' %\ DQDORJ\ ZLWK WKH FDQGHUGUDJ GDWD LQ) LJ E VSKHUHV DOR VKRZ D WUQ URXJKQHVVHIHFWRQGUDJ DWDDWLQWKH5H\QROGV QXPEHUUDQJH (5H (ZKLFKDFFRXQWVIRUWKHGLPSOQJ RI JROEDOOVWRLQFUHDVHWKHLUGUDQFHWDYHOHG 6RPH H[SHULPHQWDOGDWDIRUURXJKHQHG VSKHUHV> @DUHJLYHQLQ) LJ ' 7KHILJXUHDOV VKRZ VWSLFDOJROEDOOGDW: HVHHWKDWWRPHURXJKHQHG VSKHUHVDUHEHWWUWKDQJRO EDOOV LQ VRPH UHJLRQV)RUWKH SUHVHQWWXG\ OHWXV QHJOHFWWKHEDOOV VSLQ ZKLFK FDXVHVWKHYHU LPSRUWDQWLIRUFHRU0 DJQXVHIHFWVHH) LJ DQGDVXPHWKDW WKHEDOOVKLWZLWKRXWVSLQDQGIROORZ VWKHHTXDWLRQV RI PRWLRQIRUSDQHPRWLRQ [ ]

$$m\ddot{x} = -F\cos\theta \qquad m\ddot{z} = -F\sin\theta - W$$

$$\text{where} \qquad F = C_D\frac{\rho}{2}\frac{\pi}{4}D^2(\dot{x}^2 + \dot{z}^2) \qquad \theta = \tan^{-1}\frac{\dot{z}}{\dot{x}}$$

7KHEDOOKDV DSDUWFXODU& 5H FXUYH IURP) LJ ' DQG LV VWUDLJKW ZLWK DQ LQLWL YHORFLW9 DQGDQJOH 7DNHWKHEDOOVDYHUDJHPDVVWREH J DQGLWVGLDPHWUREH FP $VVXPLQJ VHD OHYHOLUDQGDPRGHUDWHZLQGVSHHGV YDU XS WR P V DQG ZHGHVLUH D HTXDWLRQ RI PRWLRQ WR FRPSDUH WKH WUDMHFWRU RI ³URXJKHQHG VSKHUH´ WR DFXUYHGOJROEDO FDOFXODWLRQ &DQ WKH URXJK VSKHUHRXGLYH D QRUPDOJROEDOIRUDQ FRQGLWLRQ" : KDW URXJKQHVVHIHFWGLIIHUHQFHVRFFXUEHWZHHQDOWR LPSDFWGXIHUDQG VD\ 7LJHU: RRGV"



'

5 HIHUHQFHV

(%XFNLQJKDP ³2Q 3K\VLFDO 6LPLODU6\VWHPV ,QIXVLRQV RI WKH 8VH RI ' LPHQVLRQ
(TXDWLRQV´ 3K\V 5HY YRO QR SS ±
- ' $QGHUVRQ &RPSXWDWLRQDO) OXLG' \QDPLFV 7KH %DVLFV ZLWK $SSOLFDWLRQV 0 F* UDZ
+LOO 1HZ <RUN
3 : %UDGJPDQ ' LPHQVLRQDO$QDO\VLV <DOH8QLYHUVLW\3UHVV 1HZ +DYHQ &7 UH
HG
+ / / DQJKDUU' LPHQVLRQDO$QDO\VLVDQG WKH7KHRU RI 0RGHOV : LOH\ 1HZ <RUN
(& ,SVHQ8QLW ' LPHQVLRQV DQG' LPHQVLRQOHVV 1XPEHUV 0 F* UDZ +LOO 1HZ <RU

+ * +RUQXQJ ' LPHQVLRQDO$QDO\VLV ([DPSOHV RI WKH8VH RI 6\PPHWU ' RYHU 1HZ
<RUN
(6 7D\ORU' LPHQVLRQDO$QDO\VLVIRU(QJLQHHUV &DPEULGJH3UHVV 2[IRUG (QJODQG
* , %DUHQEODWW LPHQVLRQDO$QDO\VLV * RUGRQDQG%UHDFK 1HZ <RUN
$ & 3DPHU' LPHQVLRQDO$QDO\VLV DQG ,QIOXHQWLDO([SHULPHQWDWLRQ : RUOG 6FLHQWLI
3XEOLVKLQJ +DFNHQVDFN 1-
7 6]LWWV DQG 3 5R]D $SSOLHG' LPHQVLRQDO$QDO\VLV DQG 0RGHOLQJ 0 F* UDZ +LOO 1H
<RUN
5 (VQDXOW3HOWULH' LPHQVLRQDO $QDO\VLV DQG 0HWRGRORJ\ ) 5RXJH / DXVDQQ
6ZLW]HUODQG
5 . XUWK' LPHQVLRQDO$QDO\VLV DQG* URXS 7KHRU LQ$VWURSK\VLFV 3HUJDPRQ 1HZ <RUN

) - GH-RQJ ' LPHQVLRQDO$QDO\VLVIRU(FRQRPLVWV 1RUWK+ROODQG $PVWHUGDP
5 1DNRQ &KHPLFDO3URFHVV 6RIWZDUH ' LPHQVLRQDO$QDO\VLV 3UHQWLFH+DOO 8SSH
6DGGOH5LYHU 1-
' 5 0DGPHQW HG +\GURORJLF DQG +\GUDXOLF 0RGHOLQJ 6XSSRUW: LWK * HRJUDSKL
,QIRUPDWLRQ6\VWHPV (QYLURQPHQWDO6\VWHPV5HVHDUFK,QVWLWXWH 5HGODQGV &$
$ 0 &XUHQDQG/ ' 0 XQGD\ ' LPHQVLRQDO$QDO\VLVIRU0 HGV GHG ' HPDU7KRPVR
/ HDUQLQJ $GEDQ 1<
* 3 &UDJ &OQLFDO &DOFXODWLRQV 0 DGH (DV 6RIWZDUH 3URGXFWV 8VLQJ ' LPHQVLRQ
$QDO\VLV WKHG / LSSLQFRWW LQWHUQDWLRQDO: LOLQV %DOWLPRUH 0 '
0 =DNDULQ' LPHQVLRQDO$QDO\VLV DQG 6FDOH8S LQ &KHPLFDO(QJLQHHULQJ 6SULQJHU
9HUODJ 1HZ <RUN
: * -DFRE\ ' DW7KHRU DQG' LPHQVLRQDO$QDO\VLV 6DJH 1HZ EXU 3DUN &$
% 6FKHSDW]' LPHQVLRQDO$QDO\VLV LQ WKH %LRPHGLFDO6FLHQFHV 7KRPDV 6SULQJILHOG ,/

$ - 6PLWK' RVDJHDQG6ROXWRQ&DPEULGJH7KH' LPHQVLRQDO$QDO\VLV: D\ 0 RVE\ 6\
/ RXLV 0 2
- % %DXLQJZDLJKWHWDO) UDFWDO3K\VLRORJ\ 2[IRUG8QLY 3UHVV 1HZ <RUN

. - 1LNDV3ODQV$QDPHWK 7KH6FDQQJ RI) RUP DQG3URFHVV8QLY RI &KLFDJR3UHV
&KLFDJR

³) ORZ RI) OXLGV VKURXJK 9DOYHV) LWWQJV DQG3LSHV´ &UDQH9DOYH* URXS / RQJ %HDF
&$ QRZ XSGDWHGDVD&' 520 VHH KWWS ZZZ FUDQHYDOYHVFRP!

$ 5RVKNR ³2 Q WKH ' HYHORSPHQWRI 7XUEXOHQW. DNHV IURP 9RUWH[6WUHWV´ 1$&$ 5H

* : -RQHV -U ³8QVWDG / LW) RUFHV * HQHUDWHG E\ 9RUWH[6KHGGLQJ DERXWD / DJJ
6WDWRQDU 2VFLOODWQJ &\OLQGHU DW+LJK 5H\QROGV 1XPEHUV´ $60 (6\PS 8QWDG) OR

2 0 * UIILQ DQG 6 (5DPEHUJ ³7KH 9RUWH[6WUHWV. DNHV RI 9LEUDWQJ &\OLQGHU´-
) OXLG0HFK YRO SW SS ±

(QF\FORSHGLD RI 6FLHQFH DQG7HFKQRORJ\ WKHG 0F* UDZ +LOO1HZ <RUN

+ $ %HFNHU' LPHQVLRQDW3DUDPHWULV : LOH\ 1HZ <RUN

9 3 6LQJK HWDO HGV +\GUDXOLF 0RGHOQJ : DWHU 5HVRXUFHV 3XEOLFDWLRQV // &
+LJKODQGV5DQFK &2

- - 6KDUS +\GUDXOLF 0RGHOQJ %XWWHUZRUWK / RQGRQ

5 (WWP D +\GUDXOLF 0RGHOQJ &RQFHSWV DQG 3UDFWLFH $PHULFDQ 6RFLHWW RI &LYL
(QJLQHHUV 5HVWRQ 9$

5 ' %OHYLQV $SSOLHG) OXLG ' \QDPLFV + DQGERRN YDQ 1RVWUDQG 5HLQKROG 1HZ <RU

: - 3DOP ,,, 0HFKDQLFDO9LEUDWLRQ : LOH\ 1HZ <RUN

6 6 5R 0HFKDQLFDO9LEUDWLRQV WKHG 3UHQWLFH+DOO8SSHU6DGGOH5LYHU1-

* , %UHQEDWW6FDQQJ &DP EULGJH8QLYHUVLW3UHVV &DP EULGJH8.

/ -) LQJHUK ³8QVWDG $HURG\QDPLFV ([SHULPHQW-RXUQDO RI 6RDU (QHUJ
(QJLQHHULQJ YRO 1RY S

- % %DUBZ : + 5H DQG$ 3RSH/RZ 6SHHG: LQG7XQQHO7HVWQJ : LOH\ 1HZ <RU

%+ * RHWKHUW7UDQVRQLF: LQG7XQQHO7HVWQJ ' RYHU1HZ <RUN

$PHULFDQ,QVWLWXWH RI $HURQDXWLFV DQG$VWURQDXWLFV 5HFRPPHQGHG3UDFWLFH : LQG7XQQ
7HVWQJ YROV 5HVWRQ 9$

3 1 ' HVDL - 7 6FKRILHOG DQG0 (/LDQR ³) OJKWV HFRQWUXFWLRQ RI WKH0 DLV3DWWLQG
' LVN* DS %DQG3DUDFKXWH' UDJ &RHILFLHQWV´ - 6SDFHFUDIWDQG5RFNHWV YRO QR -XO
$XJXVW SS ±

– , DP LQGHEWHG WR3URI -DFTXHV/ HZDOOHR 6\UDFXVH8QLYHUVLW IRUVXJJHVWQJ RXWQLQJ DQ
FODUL\LQJ WKLVHQWUHGLVFXVLRQ

– 0 DNHWKH P SURSRUWRQDOWR6 DQGW' RQRWGHIQH GLPHQVLRQDWV WUPV XSVLGHGRZQ 6 R
6 9W 7KH SORWZ LOOOBRNIXQQ XVHUV RI \RXUGDWZ LOOEHFRQXHG DQG\RXUVSHUYLVRUZ LOOEH
DQJU ,WW/QRWDJRRGLGHD

– : HZ HUHOXFN\ WR DFKLHYH DVL]H HIHFWEHFDXVH LQWKLV FDVH/ DVFDQQJ SDUDPHWU GLG QR
DSSHDULQWKHGUDJ FRHILFLHQW

– 0 DNH D FOHYHU FKRLFH KHUH EHFDXVH DOO SL JURXSV ZLOO FRQWLQXH DVVHMYDULDEOHV LQ YDULRXV JURXSLQJV

– 7KHVH VZR PHWKRGV WKH SL WKHRUHP YHUVXV ,SVHQ DUH TXLWH GLIIHUHQW %RWK DUH XVHIXO LQ LQWHUHVWLQJ

– 5RXJKQHVV LV HDVLO\ RYHUORRNHG EHFDXVH LW WDNHV DOJKWJHRPHWUF HIHFWKDWGRHVQRWDSSHDULQWK HTXDWRQV RI PRWRQ ,WV DERXQGDU\ FRQGLWRQWKDWRQHPLJKWJHW

&KDSWHU 9LVFRXV) ORZ LQ' XFWV

7KLV FKDSWHU LV PRVWO DERXW SLSH IORZ 7KH SKRVR VKRZV D GUDP DWF SLSH HQJLQHHULQ
DFKLHYHP HQW WKH 7UDQV$ODVND 3LSHOLQH 6\VWHP 7$36 7KH SLSHOLQH UXQV DOP RVW PLG
NP IURP 3UXGKRH %D\ LQ WKH QRUWK WR WKH SRUWV RI 9DOGH] LQ WKH VRXWK ,WZ DV EXL
EHWZHHQ DQG DWD FRVWV RI HJKWELOOLRQ GROODUV 7KHUH DUH SXP SLQJ VWDWLRQV HDF
ZLWK IRXU SXP SV 7KH VLQJOH SLSH LV LQ P LQGLP HWHU DQG LV FDSDEOH RI FDUU LQJ XS WR
VWRP LOOLRQ EDUUHOV P RI RLO SHU GD\ RQ DWWS WKDWWDNHV DOP RVW[GD\ 6LQFH WKH RL
WHP SHUDWXUH LV WSLFDOO DERXW f& P DQ\ SRUWLRQV DUH FRROHG E\ DQ DP PRQLD V\VWHP WR DYRL
GDP DJH WR WKH SHUP DIURVW: K\ GRHV WKH SLSHOLQH]LJ]DJ" 6KRGG\ FRQWUXFWLRQ' 1R WKH]LJ]D
DOORZV IRU IOH[LEOH SLSHOLQH P RYHP HQWL GLVWXUEHG E\ HDUWKTXDNHV DYDODQFKHV RU VXGGHO
WHP SHUDWXUH FKDQJHV ***Photo © Corbis RF***

0 RWLYDWLRQ

7KLV FKDSWHU LV FRP SOHWHO GHYRWHG WR DQLP SRUWDQW SUDFWLFDO OXLG V HQJLQHHULQ SUREOP IORZ L
GXFWZ LWK YDUL RXV YHORFLWLHV YDUL RXV IOXLG V DQG YDUL RXV GXFWWKDSHV 3LSLQ V WHP V DUH HQFRXQWH
LQ DOP RWWYHU HQJLQHHULQ GHVLJQ DQG WKXV KDYH EHHQ WXGLHG[WHQVLYHO 7KHUH LV DVP DOOP RXQWR

WKHRU SOXVDOOJHDPRXQWRI H[SHULPHQWDWRQ

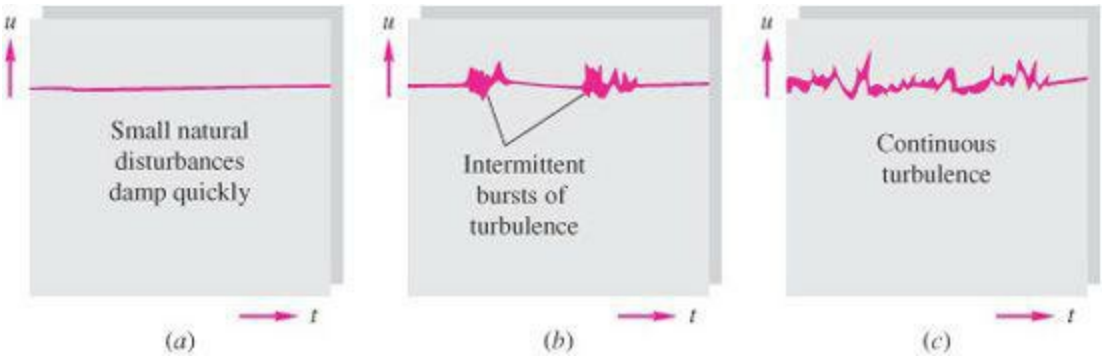
7KHEDVLF SLSLQJ SUREOHP LV WKLV * LYHQWKHSLSHJHRPHWU DQGLWVDGGHG FRPSRQHQW VXFK D ILWWQJV YDOYHV EHQGV DQGGLIIXVHUV SOXVWKHGHVLUHGIORZ UDWHDQGIOLG SURSHUWLHV ZKDWSUHVVU GURSLVQHHGHGWRGULYHWKHIORZ" 2I FRXUVH LWPD\ EHVDIHGLQDOWQDWLYHIRUP * LYHQWKHSUHVVU GURSDYDLODEOHIURP DSXPS ZKDWIORZ UDWHZLOOHQVXH" 7KHFRUHDWRQVGLVFXVVHGLQWKLVFKDSWD DUHDGHTXDWHWRVROYHPRVWXFK SLSLQJ SUREOHPV

5HYQROGV1XPEHU5HJLPHV

1RZ WKDWZHKDYHGHULYHGDQGVXGLHGWKHEDVLF IORZ HTXDWRQVLQ&KDS \RX ZRXOGWKLQNWKDW ZH FRXOGMXVWZKLS RII P\UDG EHDXWXOVROXWRQVLQOXWDQJ WKH IXOOUDQJHRI IOLG EHKDYLRU R FRXUVHH[SHULQJ DOOWKHVHGXDWRQDOUHVXOWLQGLPHQVLRQDWIORP XVLQJRXUQHZ WRROIURP &KDS GLPHQVLRQDOQDOVLV

7KHIDFWRI WKHPDWWULVWKDWQR JHQHUDOQDOVLVRI IOLG PRWRQ\HWH[LWV 7KHUHDUHVHYHUD GR]HQNQRZQSDUWFXODUVROXWRQV WKHUHDUHPDQ DSSUR[LPDWHGLJLWDOFRPSXWHUVROXWRQV DQGWKHU DUHDJUHDWPDQ H[SHULPHQWDOGDWD 7KHUHIRUH WKHRU DYDLODEOHLI ZHQHJOHFWXFK LPSRUDQ HIHFWVDVYLVFRVLW DQGFRPSUHVVLELOLW &KDS EXWWKHUHVQR JHQHUDOWKHRU DQGWKHUHP DQ H[SHULPHQWDOGDWD 7KHUHIRUHRQH RI WKHPDLQ JRDOVRI IOLG PRWRQ\HWH[LWV 7KHUHDUHDJUHDWPDQ GHSHQGHQWYDULDEOHV 7KHIORZ FKDUDFWHULVWLFVRI DQ LQFRPSUHVVLEOHIOLG EHKDYLRUDFFXUDWHPRGHUDWH5H QXPEHUV 7KHIORZ FKDUDFWHULVWLFVFKDQJH GUDPDWLFDOO\ DQG EHFRPHVLFKDOOHQJLQJ DQG GLIILFLQ VWXG\ 7KHFKDQJHRYHULVFDOOHGWUDQVLWLRQ RU VXEFULWLFDO5H QXPEHU 7KHIORZ FKDUDFWHULVWLFVDUH VWXGLHGDQGGRFXPHQWHG ,Q) LJ DZHVZ WKDWWUDQVLWLRQRI WKHF\OLQGHUDQG VSKHUHFRFXUHGDWDERXW5H î ZKHUHWKHVKDSS GURSLQWKHGUDJ FRHILFLHQ DSSHDUHG 7UDQVLWLRQGHSHQGV RQP DQ HIHFW VXFK DV ZDOOURXJKQHVV) LJ E RUIOXFWXDWLRQV LQ WKHIUHHVWUHDP EXWWKHSUHP DU SDUDPHWHUVLV WKH5H QROGV QXPEHU 7KHIORZ FKDUDFWHULVWLFVDUH GDWDWR VWXGLHGLQ DQG DJUHDWPDQ GDWDR WDQVLWLRQEXWRQO DVP DOOLP RXQWR WKHRU > WR @

7XUEXOHQFHFDQEHGHWHFWHGIURP DPHDVXUHPHQWV DVP DOO VHQVLWLYHIOXQWXPHQWXFK DV DKRW ZLUHDQHPRPHWU) LJ H RUDSLWRFOFWF SUHVVXUHWDQGXFHU 7KHIORZ ZLOODSSHDUWXEXOHQ RU DYHUDJHEXWZLOOIUHTXHQWO\ UDQGRP IOXFWXDWLRQVLI WKUEXOHQFHLVSUHVHQWDV VNHWKHGLQ) LJ , WKHIORZ LVOMP LQDU WKHUHP DV EHRFFDVLRQDOOQDWXUDOGLWXUEDQFHV WKDWGDPS RXWXLFNO) LJ D , WDQWLWRQLVRFFXUULQJ WKHUHZLOOEHVKDSS EXWWRI IOXHQWWXFWXDWLRQ) LJ E DV WK LQFUHDVLQJ 5H QROGV QXPEHUFDXVHV D EUHDNGRZQ RULQWDELOLW RI OMP LQDU PRWRQ $WXILFLHQW GJUH 5H WKH IORZ ZLOO IOXFWXDWH FRQWLQXDOO) LJ F DQG LV WHUPHGIXOO WXUEXOHQW 7K IOXFWXDWLRQ\ \WSLFDOO UDQJLQJ IURP WR SHUFHQWRI WKH DYHUDJH YHORFLW DUH QRWXWLFW SHUFRGE EXWDUH UDQGRP DQG HQFRSDVV D FRQWLQXRXV UDQJH RUVSHFWUP RI IUHTXHQFLHV ,Q D WSLFDOZ LQ WXQQHOIORZ DWKJK5H WKHWXUEXOHQWUHTXHQF\ UDQJHVIURP WR +] DQGWKHZDYHOHQJWKUDQJH IURP DERXW WR FP



)LJ 7KH WKUHH UHJLPHV RI YLVFRXV IORZ *a* ODPLQDU IORZ DW ORZ 5H *b* WUDQVLWLRQ D LQWHUPHGLDWH 5H *c* WXUEXOHQW ORZ DW KLJK 5H

(; $0 3/ (

7KH DFFHSWDQFH 5H QRQOLQHDU IRU IORZ LQ D FLUFXODU SLSH LV 5H $SSUR[ ) RU IORZ WKURXJK D FP GLDPHWHU SLSH DW ZKDWYHORFLW\ ZLOO WKLV RFFXU *f*& IRU D DLUIORZ DQG E ZDWHU IORZ"

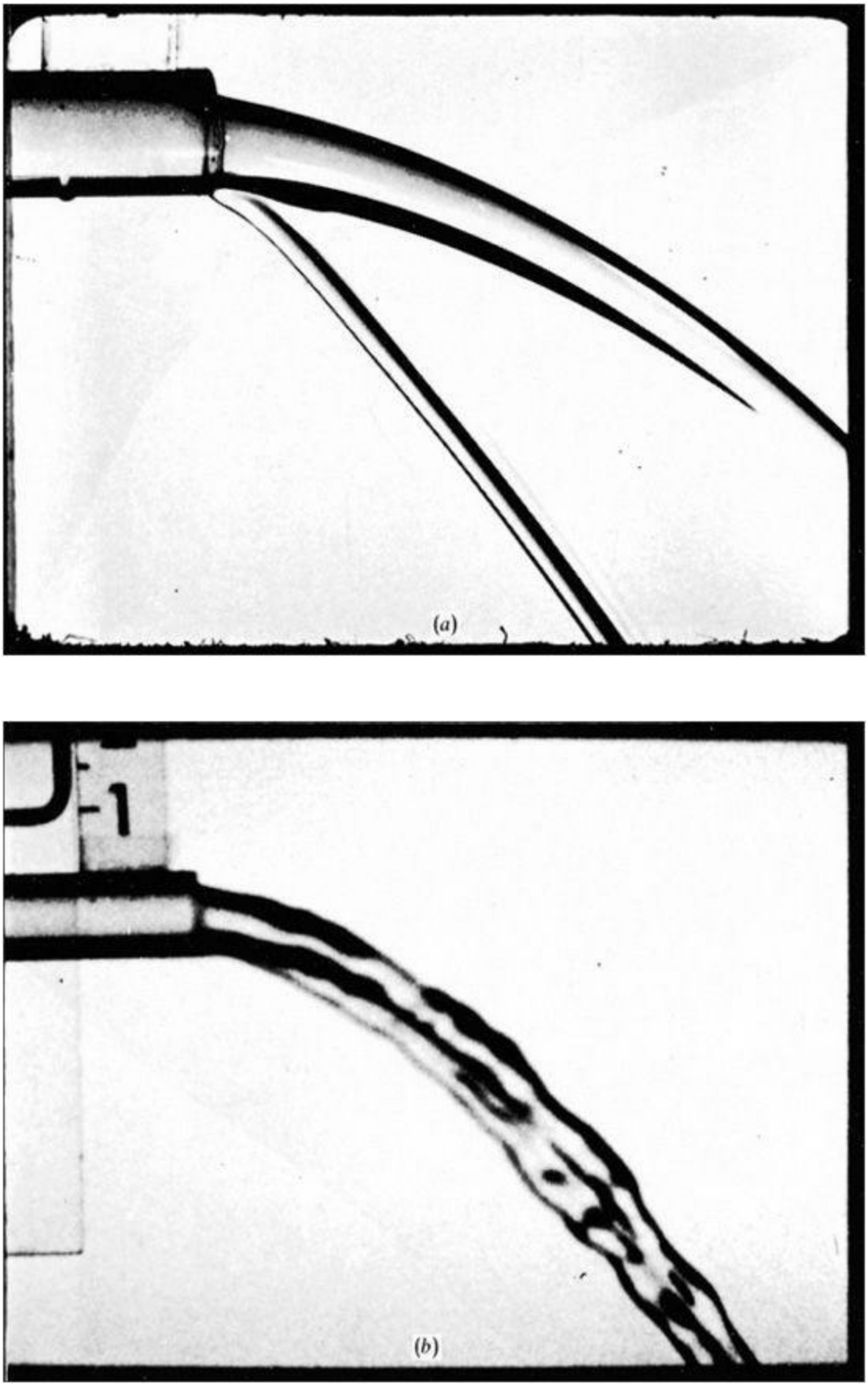
6RXWLRQ

$ OPRVWDOOSLSHIORZ IRUPXODV DUH EDVHG RQ WKH DYHUDJH YHORFLW9 4 $ QRWH QWUDQVLWLRQ RUDQ RWKHU SRLQWYHORFLW 7KXV WUDQVLWLRQ LV VSHFLILHG DW 9G § : LWK GNQRZ Q ZHLQWRGXFH WK DSSURSULDWH IOXLG SURSHUWLHV DW *f*& IURP 7DEOH $ DQG $

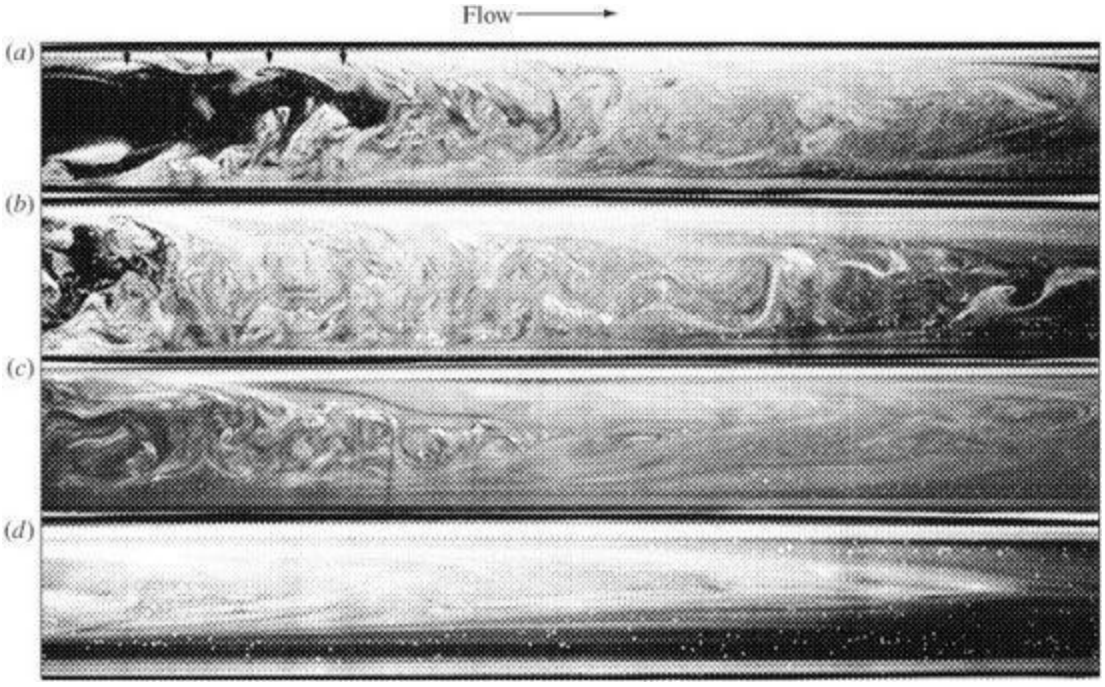
- $$(a)\ \text{Air:}\quad \frac{\rho V d}{\mu} = \frac{(1.205\ \text{kg/m}^3)V(0.05\ \text{m})}{1.80\ \text{E-5 kg/(m}\cdot\text{s)}} = 2300 \quad \text{or} \quad V \approx 0.7\,\frac{\text{m}}{\text{s}}$$
- $$(b)\ \text{Water:}\quad \frac{\rho V d}{\mu} = \frac{(998\ \text{kg/m}^3)V(0.05\ \text{m})}{0.001\ \text{kg/(m}\cdot\text{s)}} = 2300 \quad \text{or} \quad V = 0.046\,\frac{\text{m}}{\text{s}}$$

7KHVH DUH YHU ORZ YHORFLWLHV VR PRVW HQJLQHHULQJ DLUDQG ZDWUSLSH IORZ V DUH WXUEXOHQW QF ODPLQDU : HPLJKW[SHFWODPLQDUGXFWORZ ZLWK PRUH YLVFRXV IOXLGV VXFK DV OXEULFDWLQJ RLOV R JO\FHULQ

,QIUHVXUDFHIORZ V WXUEXOHQFHFDQEHREVHUYHGGLUHFWO) LJXUH VKRZV OQTXLG IORZ LWXLQ IURP WKHSHQHQGRI DWXEH 7KHORZ 5H QRQOLQHUWW) LJ D LVVPRRWKDQGODPLQDU ZLWKWK IDWFHQWUPRWRQ DQG VORZ HUZDOIORZ IRUPLQJ GLIHUHQWXDWFWLRQVRQG E\ DQTXLG VKHW7K KJKHU5H QRQOLQHUWXUEXOHQWORZ) LJ E LVXQWDG DQGLUUHJXODUEXWZKHQDYHUDJHGRYH WP H LVVWDG DQGSUHGLFWDEOH



) LJ) OBZ LVXLQJ DWFRQWDQWWSHHGIURP DSLSH *a* KLJK YLVFRVLW OBZ 5H\QROGV QXP EHU DP LQDUI OBZ *b* OBZ YLVFRVLW KLJK 5H\QROGV QXP EHU WXUEXOHQW OBZ *(National Committee for Fluid Mechanics Films, Education Development Center, Inc., © 1972.)*



) LJ) RUP DWRQ RI D VXUIDFWDQW LQ SLSH IORZ *a* DQG *b* QHDU WKH HQWUDQFH *c* VRPHZKDWGRZQVWUHDP *d* IDU GRZQVWUHDP ***Courtesy of Cambridge University Press–P. R. Bandyopadhyay, "*** $VSHFWV RI WKH (TXLOLEUXP 3XII LQ 7UDQVLWLRQDO3LSH) ORZ ´***Journal of Fluid Mechanics, vol. 163, 1986, pp. 439–458.)***

+RZ GLG VXUIDFWDQW IRUP LQWLGHWKHSLSH' 7KHOLQHUSDUDERORF IORZ SURILOH ZKLFK LV VLP LQ VR (T EHFDPH XQVWDEOH DQG DV5H§ EHJDQ WR IRUP ³VOXJV´ RU³SXIIV´ RI LQWHQV VXUIDFWDQW $ SXII KDV D IDVWPRYLQJ IURQWDQG D VORZ PRYLQJ UHDU DQG PD\ EH YLVXDOL]HG E\ H[SHULPHQWLQJ ZLWK JODVV WXEH IORZV WKURXJKHG E\ %DQGRSDGK\D > @1 HDUWKHQGDQFH) LJ DDQGE WKHUHLVDQLUHJXODUOPLQDU±WXUEXOHQWWUDQVLWLRQ DQGYRUWH[URODXS LV YLVLEOH) XUWKHUGRZQVWUHDP) LJ F WKH SXII EHFRPHVIXOO WXUEXOHQWDQGYHU DFWLYH ZLWKHDFKPRWRQVYLVLEOH)DUGRZQVWUHDP) LJ G WKHSXII LVFRQH VKDSHGDQGOHVDFWLYH ZLWK DIX]]\ LOOGHILQHGLQWHUIDFH VRPHWLPHVFDOOHGWKH³UHDPLQDUZRQ UHJLRQ

$ FRPSOHWH GHVFULSWLRQ RI WKH VWDWXVDOSHFWRI WXUEXOHQFHLV JLYHQ LQ5H ZKLOHWKRU DQGGDWDRQWDQWLRQHIIHFWVDUH JLYHQLQ5HV DQG $VKLVLQWRGXFWRU\ OHYHOZHPHUHO SRLQWRXWKDWWKHSUHP DU SDUDPHWULFIHFWQJ WDQWLRQLV WKH5H QROGVQXPEHU,I5H 8/ ZKHUH8 LV WKH DYHUDJHVWUHDP YHORFLW DQG/ LVWKH³ZLGWK´ RUWDQYHUWHWKLFNQHVV RI WKHVKHDUOD\HU WKHIORZ LQ DSSUR[LPDWHOHVRFFXU

- 5H KLJKO YLVFRXVOPLQDU³FUHHSLQJ´ PRWRQ
  5H OPLQDU WXUQJ 5H\QROGVQXPEHUGHSHQGHQFH
  5H OPLQDU ERXQGDU\ OD\HUWKRU XVHXO
  5H WDQVLWLRQWRWXUEXOHQFH
  5H WXUEXOHQWPRGHUDW5H\QROGVQXPEHUGHSHQGHQFH
  5H ' WXUEXOHQWWOQJKW5H\QROGVQXPEHUGHSHQGHQFH

7KHVHUHSUHVHQWDWLYH UDQJHVYDU VRPHZKDWZLWKIORZ JHRPHWU VXUIDFHURXJKQHVV DQGWKHOHY-RI IXFWXDWLQJVLQWKHIORZVWUHDP 7KHJUHDWPDMRULW RI RXUDQDO\VHVDUHFRQFHUQHGZLWKOPLQDUIORZ RUZLWKWXEXOHQWORZ DQGRQHVKRXOGQRWDOO GHVLJQDIORZ RSHUDWQLQWKHWUDQVLWLRQUHJLRQ

+ LVWRULFDO2 XWOLQH

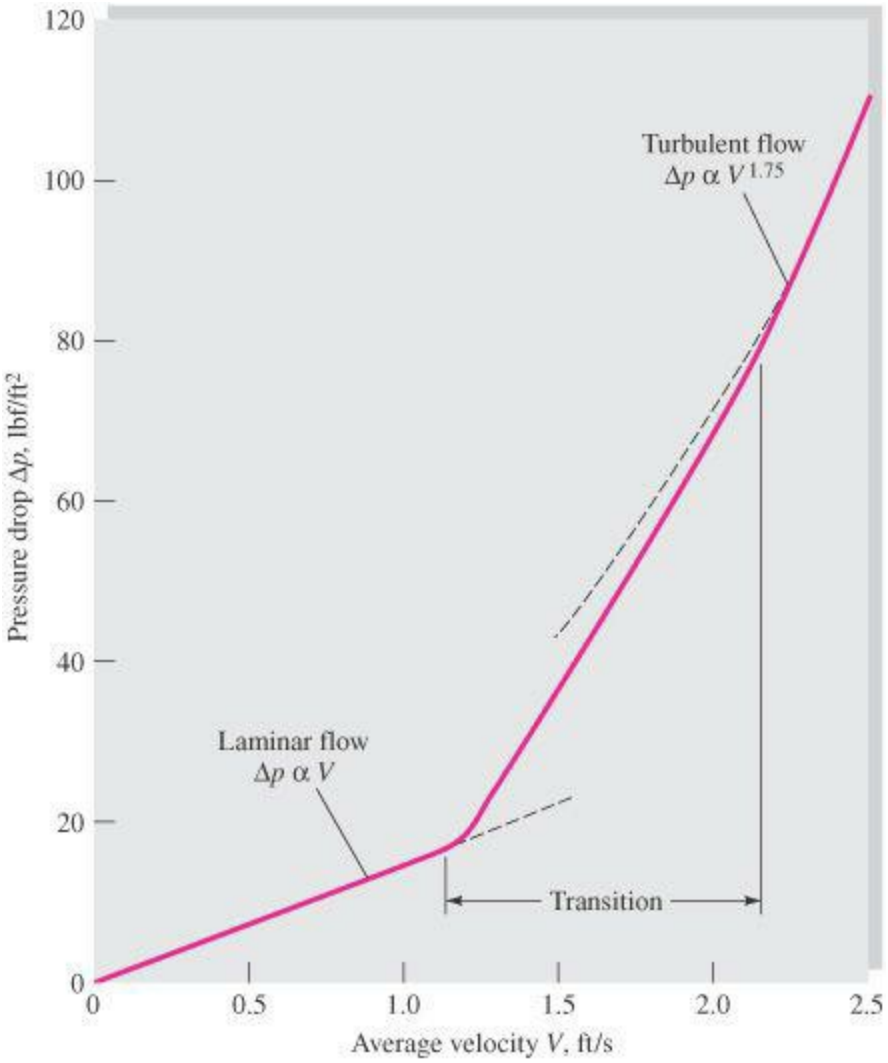
6LQFH VXUEXOHQW IORZ LV PRUH SUHYDOHQW WKDQ ODPLQDU IORZ H[SHULPHQWV KDYH REVHUYH VXUEXOHQFH IRUPHQXODV ZLWKRXW EHLQJ DZDUH RI WKH GHWDLOV %HRRE IORZ LQWURGXFHQWHV VRPH LQWHUHVWLQJ UHFRUG UDSLG IOXFWXDWLRQV DQG ZRUN-HV/ VLPSOH UHSRUWHG PHDQ YDOXHV RI YHORFLW\ SUHVVXUH IRUFH DQG VR RQ %XWXOHQFHFDQFKDQJHWKHPHDQYDOXHVGUDPDWLFDOO DVZLWKWHVKDS GURSLQGUDJ FRHIILFLHQW) LJ $ * HUPDQHQJLQHHUQDPHG* + / + DJHQILLWWUHSRUWHGLQ WKDWWKHUHPLJKWEHWZR UHJLPHVRI YLVFRXV IORZ +HPHDXUHG ZDWUIORZ LQORQJ EUDVSLSHV DQG GHGXFHGDSUHVVXUHGURSODZ

$$\Delta p = (\text{const})\frac{LQ}{R^4} + \text{entrance effect} \qquad (6.1)$$

7KLVLVH[DFWO RXUODPLQDUIORZ VFDOQJ ODZ IURP ([DPSOH EXW+ DJHQGLGQRWUHDOL]HWKDWWK FRQWDQWZ DVSURSRUWLRQDOWRWKHIOXLGYLVFRVLW\

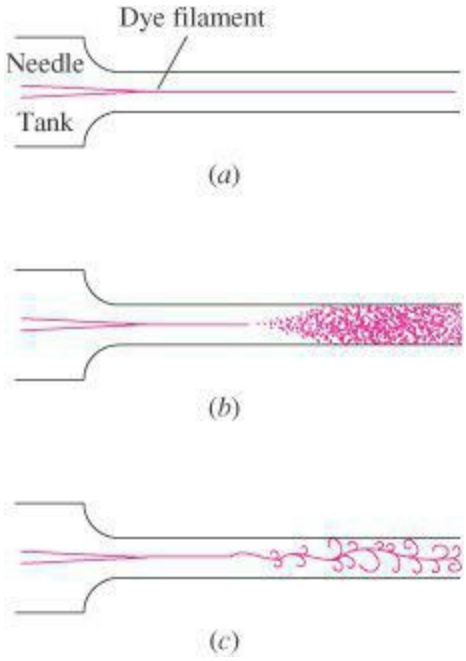
7KHIRUPXODEURNHGRZQ DV+ DJHQLQFUHDVHG4 EH\RQGDFHUWDLQOLPLW WKDWLV SDUWWKHFULWFD 5H\QROGVQXPEHU DQGKHVDWGLQKLVSDSHUWKDWWKHUHPXWEHDVHFRQGPRGHRI IORZ FKDUDFWHUL]H E\ ³WXUQ PRYHPHQWV RI ZDWUIRUZKLFK ûS YDULHV DV WKH VHFRQG SRZHURI WKH GLVFKDUJH« ´ +I DGPLWWHGWKDWWKHFRXOGQRWFODULI\ WKHUHDVRQVIRUWKHFKDQJH

$ \WSLFDOH[DPSOHRI + DJHQ¶V GDWDLV VKRZ QLQ) LJ 7KHSUHVVXUHGURS YDULHVOQHDU ZLV 9 4 $ XS WR DERXW IWV ZKHUHWKHUHLV D VKDUS FKDQJH $ERYHDERXW9 IWV WKHSUHVVXU GURSLVQHDU TXDGUDWF ZLWK9 7KHDFWXDOSRZHUûS 9 VHPVLPSRWLEOHRQGLPHQVLRQDOJURXQG EXWLVHDVLO H[SODLQHGZKHQWKHGLPHQVLRQOHVVSLSHIORZ GDWD) LJ DUHGLVSOD\HG



) LJ ([SHULPHQWDOHYLGHQFHRI WKDQVLWLRQIRUZDWHU LQD $ LQVPRRWKSLSH IWORQJ

,Q 2VERUQH5H\QROGV D%ULWLVK HQJLQHHULQJ SURIHVVRU VKRZHGWKDWWKHFKDQJHGHSHQGHGR
WKHSDUDPHWHU! 9G QRZ QDPHGLQKLVKRQRU %\ LQWURGXFLQJ DG\HWWHDNLQRXDSLSHIORZ 5HYQROG
FRXOGREVHUYHWDQVLWLRQDQGGWKUEXOHQFH +LVVNHWFKHV> @RI WKHIORZ EHKDYLRUDUHVKRZQLQ) LJ



) LJ 5H\QROGV¶VNHWFKHV RI SLSH IORZ WUDQVLWLRQ *a* ORZ VSHHG ODPLQDU IORZ *b* KLJK VSHHG WXUEXOHQW ORZ *c* VSDUN SKRWRJUDSK RI FRQGLWLRQ *b* ***(From Ref. 4.)***

,I ZH H[DPLQH +DJHQ¶V GDWD DQG FRPSXWH WKH 5H\QROGV QXPEHU DW WKH REVHUYHG 5H$ 7KH IORZ EHFDPH IXOO\ WXUEXOHQW DW 5H$ 7KH DFFHSWHG GHVLJQ YDOXH IRU SLSH IORZ WUDQVLWLRQ LV QRZ WDNHQ DV

$$Re_{d,crit} \approx 2300 \qquad (6.2)$$

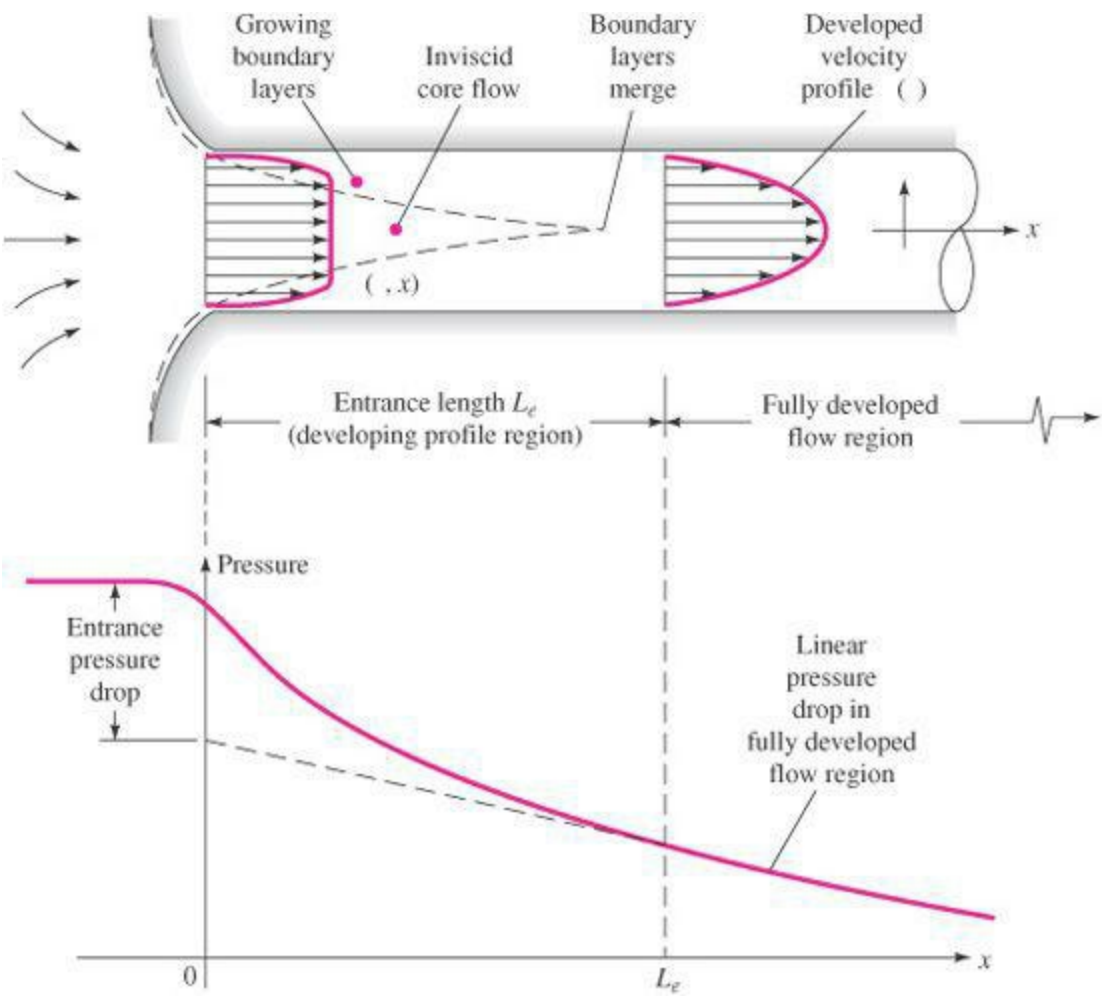
7KLV LV DFFXUDWH IRU FRPPHUFLDO SLSHV) LJ DOWKRXJK ZLWK VPHFLDO FDUH LQ SURYLGLQJ I URXQGHG HQWUDQFH VPRRWK ZDOOV DQG DVVXUH\ LQGHSHQGHQW 5H$ FULW FDQ EH GHD\HG XQWLO PXFK KLJKH YDOXHV 7KH VWXG\ RI WUDQVLWLRQ LQ SLSH IORZ ERWK H[SHULPHQWDO DQG WKHRHWLFDO FRQWULEXWLRQV WR EH IDVFLQDWLQJ VXEMHFW IRU UHVHDUFKHUV DV GLVFXVVHG LQ DUHFHQW UHYLHZ DUWLFOH > @1RWH 7KH YDOXH R LVIRUWDQWLQSLSHV 2WKHUJHRPHWULHV VXFK DV SODWHV DLUIRLOV FXUYHG SDWKV KDY FRPSOHWHO GLIIHUHQWWDQWLQH5H\QROGVQXPEHUV 7UDQVLWLRQ DOR RFFXUV LQ H[WUHPHOR V DURXQG ERGLHV VXFK DV WKH VSKHUH DQG F\OLQGHU) LJ / XGZLJ 3UDQGWO D* HUPDQHQJLQHHULQJ SURHVVRU VKRZHG LQ WKDWWKH WKLQ ERXQGDU OD\H VXUURXQGLQJ WKH ERG\ DV XQGHUJRLQJ WDQVLWLRQ IURP ODPLQDU WR WXUEXOHQW ORZ 7KHUHDWUWKH IRUF FRHILFLHQW DERG\ DV DFNQRZOHGJHG WR EH DIXQFWLRQ RI WKH5H\QROGV QXPEHU(T @ 7KHUHDUHQR H[WHQVLYH WHRUHWLFDO H[SUHVVLRQV RI ODPLQDUIORZ LQVWDELOLW\ WKDW H[SODLQ WKH IORZ FKDQJHV WR WXUEXOHQFH 5HHUHQFH LV DQ DGYDQFHG WH[WERRN RQ WKLV VXEMHFW

/ DPLQDU IORZ WKHRU LV QRZ ZHOO GHYHORSHG DQG PDQ VROXWLRQV DUH NQRZQ > @EXW DQDO VHV FDQ VLPXODWH WKH ILQH VFDOH UDQGRP IOXFWXDWLRQV RI WXUEXOHQW IORZ WKHRU LV VHPLHPSLULFDO EDVHG RQ GLPHQVLRQDO DQDO VLV DQG SK\VLFDO UHDVRQLQ LWV FRQFHQWUDWLRQ RQ WKH PHDQ IORZ SURSHUWLHV RQ DQG WKH PHDQ RI WKH IOXFWXDWLRQV QRWWKHUUDSLG YDULDWLRQV 7KH WXUEXOHQW IORZ ³WKHRU´ SUHVHQWG KHUH LQ&KDSV DQG LV XQEHOLHYDEO FUXGH \HW XSULJKWO HIIHFWLYH : HVKDOODWWP SWDUDWRQDOSSURDFKWKDWSODFHV WXUEXOHQW DQDO VLVRQ DILUP SK\VLFD EDVLV

,QWHUQDOYHUXV([WHUQDO9LVFRXV) ORZV

%RWK PDWHULDO DQG WXUEXOHQW ORZ PD\ EH HLWKHU LQWHUQDO WKDWV ³ERXQGHG´ E\ ZDOOV RU H[WHUQ DQG XQERXQGHG 7KLV FKDSWUWHDWV LQWHUQDO ORZV DQG&KDS WXGLHV H[WHUQDO ORZV

$Q LQWHUQDO ORZ LV FRQWDLQHG E\ WKH ERXQGLQJ ZDOOV DQG WKH YLVFRXV HIHFWV ZLOO JURZ DQ PHWDQG SHUPHDW WKH HQWLUH) LJXUH VKRZV DQ LQWHUQDO ORZ LQ D ORQJ GXFW7KHUH LV D HQWDQFH UHJLRQ ZKHUH D QHDU LQYLVFLG XSWUHDP ORZ FRQYHUJHV DQG HQWHUV WKH WXEH 9LVFRX ERXQGDU OD\HUV JURZ GRZQWUHDP UHGXFLQJ WKH D[LDO ORZ U [DWWKH ZDOO DQG WKHUHV DFFHOHUDWLQJ WKHFHQWUFRUHIORZ WKURXJKRXW WKH OHQJWK RI SHUWLEHFRQWLQXHTXLHPHQW



) LJ ' HYHORSLQJ YHORFLW SURILOHV DQG SUHVVXUH FKDQJHV LQ WKH HQWDQFH RI DGXFWORZ

$$Q = \int u\,dA = \text{const} \qquad (6.3)$$

$ VD ILQDO VWHS IURP WKH HQWDQFH WKH ERXQGDU OD\HUV PHUJH DQG WKH LQYLVFLG FRU GLVSSHDU 7KHVH EHIORZ LV WKH QHQWUDO YLVFRXV DQG WKH D[LDOYHORFLW DGXVWV OUKWV IXUWKHUQ DW /H LWQR ORQJHUFKDQJHV ZLWK[DQG LV VDLG WR EH IXOO GHYHORSHG X§ X U RQ ' RZ QWHP R[/ H WKH YHORFLW SURILOHV FRQWDQWWKH ZDOWKHUL FRQWDQWDQG WKH SUHVVXUHGURSV OQHDU Z LWK[IRUHWKHUPLQDUURUXEXOHQWORZ $OWKHVHGHDLVYDUH VKRZQ) LJ ' LPHQVLRQDODQDO VLV VKRZV WKDWWKH 5H\QROGV QXPEHUV WKH RQ SDUDPHWUL DIIHFWQJ HQWDQF OHQJWK ,I

$$L_e = f(d, V, \rho, \mu) \qquad V = \frac{Q}{A}$$

then

$$\frac{L_e}{d} = g\left(\frac{\rho V d}{\mu}\right) = g(\mathrm{Re}_d) \qquad (6.4)$$

) RUODP LQDU IORZ > @WKH DFFHSWHG FRUUHODWLRQ LV

$$\frac{L_e}{d} \approx 0.06\,\mathrm{Re}_d \qquad \text{laminar} \qquad (6.5)$$

7KH PD[LPXP ODPLQDUHQWDQFH OHQJWK DV5H$FUW LV/$ G ZKLFK LV WKH ORQJHVW GHYHORSPHQWOHQJWK SRVVLEOH

,QWXLEOHQWVORZ WKHERXQGDU\ OD\HUV JURZ IDVWHU DQG/$ LV UHODWLYHO\ VKRUWHU)RUGHFDGHV WKH ZULWHUKDV IDYRUHG DV[WK SRZHUODZ HXWP DWL$ EXWUHFHQW&)' UHVXOWV FRPPXQLFDWH E\ )DELHQ$QHPHWDQG VSDUDWH E\ 6XNDQG' DK LQGLFDWH WKDWD EHWWHU WXLEXOHQWQDQFH OHQJWK FRUUHODWLRQLV

$$\frac{L_e}{d} \approx 1.6\,\mathrm{Re}_d^{1/4} \quad \text{for} \quad \mathrm{Re}_d \le 10^7 \tag{6.6}$$

6RPHFRPSXWHGWXLEXOHQWQDQFH OHQJWKHWXP DWVDUHWKXV

Re_d	4000	10^4	10^5	10^6	10^7
L_e/d	13	16	28	51	90

1RZ GLDPHWHUV PD\ VHHP ³ORQJ´ EXWWSLFDOSLSHIORZ DSSOLFDWLRQV LQYROYH DQ/ GYDOXHR RUPRUH LQZKLFK FDVH WKHHQDQFHHIHFWPD\ EHQHJOHFWHG DQG D VLPSOHDQDO\VLV PDGHIR IXOO GHYHORSHG IORZ 7KLV LV SRVVLEOH IRUERWK ODPLQDUDQG WXLEXOHQWORZ V LQFOXGLQJ URXJK ZDOO DQG QRQFLUFXODUFURVVVHFWLRQV

(; $0 3/ (

$ LQGLDPHWHUZDWUSLSHLV IWORQJ DQG GHOLYHUVZDWUDW JDOPLQDW *HWKDFWRUV WKLVSLSHLVWDNHQXS E\ WKHHQDQFHUHJLRQ

6RXWLRQ

&RQVHUW

$$Q = (5\ \text{gal/min})\frac{0.00223\ \text{ft}^3/\text{s}}{1\ \text{gal/min}} = 0.0111\ \text{ft}^3/\text{s}$$

7KHDYHUDJHYHORFLWLV

$$V = \frac{Q}{A} = \frac{0.0111\ \text{ft}^3/\text{s}}{(\pi/4)[(\frac{1}{2}/12)\ \text{ft}]^2} = 8.17\ \text{ft/s}$$

) URP 7DEOH UHDG IRUZDWU î í P V î í IWV 7KHQWKHSLSH5H\QROGQXPEHULV

$$\mathrm{Re}_d = \frac{Vd}{\nu} = \frac{(8.17\ \text{ft/s})[(\frac{1}{2}/12)\ \text{ft}]}{1.09 \times 10^{-5}\ \text{ft}^2/\text{s}} = 31{,}300$$

7KLV LV JUHDWHU WKDQ KHQFH WKH IORZ LV IXOO WXUEXOHQW DQG (T DSSOLHV IRU HQWUDQFH OHQJWK

$$\frac{L_e}{d} \approx 1.6\,\mathrm{Re}_d^{1/4} = (1.6)(31{,}300)^{1/4} = 21$$

7KH DFWXDO SLSH KDV/ G $L/d = (60\text{ ft})/[(\tfrac{1}{2}/12)\text{ft}] = 1440$ + HQFH WKH HQWUDQFH UHJLRQ WDNHV XS WKH IUDFWLRQ

$$\frac{L_e}{L} = \frac{21}{1440} = 0.015 = 1.5\% \qquad \textit{Ans.}$$

7KLV LV D YHU\ VPDOO SHUFHQWDJH VR WKDW WKH HIIHFW RI HQWUDQFH ORVV RQ WKH WRWDO SUHVVXUH GURS LV YHU\ VPDOO

6KRXOG WKH VFDQEHD YLUWHLQGXFHG IORZ LI RQH ZLVKHV WR PDLQWDLQ WKH IORY LVFLGFRUH) RU [DPSOH D³ORQJ´ ZLGH WXQQHO ZRXOG EH UHFLFXODWHG VLQFH WKH YHVFRXV FRUH ZRXOG LQYDGH WKH SXUSRVH R VLPXODWLQJ IUHH IOJKW FRQGLWLRQV $ WXSLFDO DERUDWRU\ IORZ VSHHG ZLQG WXQQHO WHVWV LV P LG GLDPHWHU DQG P ORQJ ZLWK 9 P V ,I ZH WDNH DU î í P VIURP 7DEOH WKHQ 5H G î DQG IURP (T / H G§ 7KH WXUWXOHQFH KDV/ G ZKLFK LV PXFK VKRUWHU WKDQ WKH GHYHORSPHQWDO WKH $ WKH HQG RI WKH VHFWLRQ WKH ZDOO ER XQGDU\ OD\HUV DUH RQO FP WKLFN OHDYLQ FP RI LQYLVFLG FRUH VXLWDEOH IRU PRGHOLQJ

$Q H[WUDQDOIORZ KDV QR UHVWULFWLQJ ZDOOV DQG LV IUHH WR H[SDQG QR PDWWHU KRZ VKLFN WKH YHVFRX OD\HUV RQ WKH LPPHUVHG ERG\ PD\ EH FRPH 7KXV IDU IURP WKH ERG\ WKH IORZ LV QHDUO LQYLVFLG DQ RXU DQDO WXFDO WHFKQLTXH WHDWG LQ&KDS LV WR SDAK DQ LQYLVFLG IORZ VROXWLRQ RQR D YLVFRX ERXQGDU\ OD\HU VROXWLRQ FRP SXWG IRU WKH ZDOO UHJLRQ 7KHUH LV QR H[WUDQDO HTXLYDOHQW IXOO GHYHORSHG LQWHUQDOIORZ

+ HDG/ RVV±7KH) UFWLRQ) DFWRU

: KHQDSSO\LQJ SLSH IORZ IRUPXODV WR SUDFWLFDOSUREOHPV LWLV FXVWRPDU WR XVH D FRQVRYRXP DQDO VLV &RQVLGHULQFRP SUHVVLEOHIORZ EHWZHHQV DQG R WKHLQFOLQHGFRQVWDQWDU SLSHLQ) LJ 7KHRQH GLP HQVLRQDOFRQVHUYDWLRQ (T UHGXFHV WR

4 4 FRQVWDU9 9 9

VLQFHWKHSLSHLVR FRQVWDQWDUHD 7KHVWDG\ IORZ HQHUJ\ HTXDWLRQ EHFRPHV

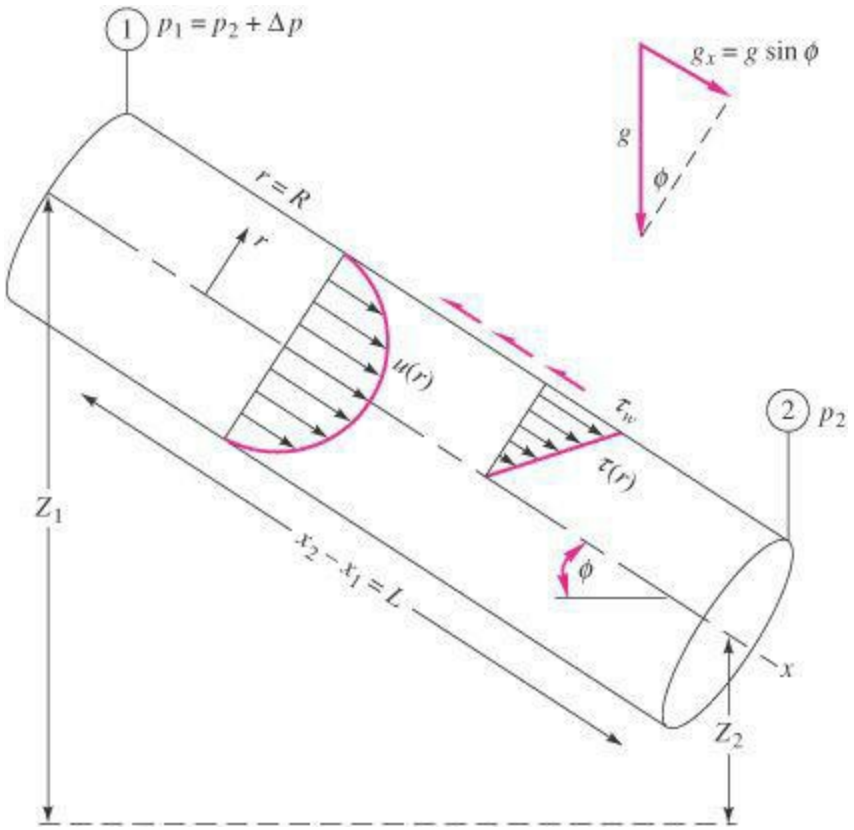
$$\left(\frac{p}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_2 + h_f \qquad (6.7)$$

VLQFHWKHUHLVQR SXPS RUWXUELQHEHWZHHQ DQG) RUIXOO GHYHORSHG IORZ WKHYHORFLWSURILO

VKDSHLVWKHVDPHDVWHFWRQV DQG 7KXV. . DQG VLQFH9 9 (T UHGXFHVWRKHDGORV
YHUXVSUHVVXUHGURSDQGHOHYDWLRQFKDQJH

$$h_f = (z_1 - z_2) + \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) = \Delta z + \frac{\Delta p}{\rho g} \qquad (6.8)$$

7KHSLSHKHDGORVVHTXDOVWKHFKDQJHLQWKHVXP RI SUHVVXUHDQGJUDYLWKHDG WKDWV WKHFKDQJ
LQKHJKWRI WKHK\GUDXOLFJUDGHOLQH +* /



) LJ &RQWROYROXPHRI VWHDG\ IXOO GHYHORSHGIORZ EHWZHHQVZRVHFWRQVLQDQLQFOLQH
SLSH

)LQDOO DSSO\ WKHPRPHQWXP UHDWRQ VR WKHFRQWROYROXPHLQ) LJ DFFRXQWLQJ IR
DSSOLHG[GLUHFWLRQIRUFHVGXHWRSUHVVXUH JUDYLW DQGVKHDU

$$\sum F_x = \Delta p(\pi R^2) + \rho g(\pi R^2)L \sin\phi - \tau_w(2\pi R)L = \dot{m}(V_2 - V_1) = 0 \quad (6.9a)$$

5HDUUDQJHWKLVDQGZHILQGWKDWWKHKHDGORVVLVDOVRUHDGLO\GRZ DOOVKHDUVWUHVV

$$\Delta z + \frac{\Delta p}{\rho g} = h_f = \frac{2\tau_w}{\rho g}\frac{L}{R} = \frac{4\tau_w}{\rho g}\frac{L}{d} \qquad (6.9b)$$

ZKHUHZ HKDYHVXEVWLWXWHG û] / VLQÉ IURP WKHJHRPHWU RI) LJ 1 RWHWKDWWUDGLWLRQDOWHV F
ZKHWKHUWKHSLSHLVKRUL]RQWDORUWLOWHG WKHKHDGORVVLVSURSRUWLRQDOWR WKHZDOOVKHDUVWUHVV
+RZ VKRXOG ZH FRUUHODWH WKH KHDG ORVV IRU SLSH IORZ SUREOHPV" 7KH DQVZHU ZDV JLYHQ
FHQWXU DQG D KDOI DJR E\ -XOXV: HLVEDFK D* HUPDQ SURIHVVRU ZKR LQ SXEOLVKHG WKH ILUV
PRGHUQ WH[WERRN ON K\GURG\QDPLFV (TXDWLRQ E VKRZV WKDW K, LV SURSRUWLRQDO / G DQG GDU

VXFK DV + DJHQW VLQ) LJ VKRZ WKDW IRU WXUEXOHQW IORZ K, LV DSSUR[LPDWHO SURSRUWLRQDO WR 9 7K SURSRVHG FRUUHODWLRQ WXUQV RXW WR EH IHDVLEOH DQG DFFXUDWH DV LQ LV

$$h_f = f\frac{L}{d}\frac{V^2}{2g} \quad \text{where} \quad f = \text{fcn}\left(\text{Re}_d, \frac{\varepsilon}{d}, \text{duct shape}\right) \tag{6.10}$$

7KH GLPHQVLRQOHVV SDUDPHWHU I LV FDOOHG WKH ' DUF\ IULFWLRQ IDFWRU DWH + HQU ' DUF\ : D) UHQFK HQJLQHHU Z KRVH SLSH IORZ H[SHULPHQW LQ ILUVW HVWDEOLVKHG WKH HIIHFWV URXJKQHVV RQ SLSH UHVLVWDQFH 7KH TXDQWLW 0 LV WKH Z DOO URXJKQHVV KHLJKW Z KLFK LV LP SRUWDQW LQ WXUEXOHQW ÀRZ SLSH IORZ : HDGGHG WKH³GXFWVKDSH´ HIIHFWV (T VR U HP LQG XV WKH VTXDUH DQG WULDQJXODU GXFWV KDYH D VRPHZ KDW GLIIHUHQW UHFWDQJXODU GXFWV SLSH \$FWXDOO DQG RUGLQDU DQG RWKHU RQ FLUFXODU GXFWV Z LOO EH GLVFXVVHG LQ WKH VHFWLRQ V WKDW IROORZ %\ HTXDWLQJ (TV DQG Z HILQG DQ DOWHUQDWLYH IRUP IRU IULFWLRQ IDFWRU

$$f = \frac{8\tau_w}{\rho V^2} \tag{6.11}$$

) RUQRFLUFXODU GXFW Z HP XVW LQWURGXFH D VRPHZ KDW DUELWUDU\ VWDWHPHQW VLQFH WKH GHILQLWLRQ RI WKH ' DUF\ IULFWLRQ IDFWRU LV SUHVHQWHG DV DQ LQGHSHQGHQW GHILQLWLRQ RI WKH VHFWLRQ V WKDW IROORZ

/ DPLQDU) XOO ' HYHORSHG 3LSH) ORZ

\$QDO WFDO VROXWLRQV FDQ EH REWDLQ GHULYHG IRU ODPLQDU IORZ V HLWKHU FLUFXODU RU QRQFLUFXODU &RQVLGHU IXOO GHYHORSHG3RLVHXLOOH IORZ LQ D URXQG SLSH RI GLDPHWUG UDGLXV5 &RPSOH DQDO WFDOUHVXOWZ HUHJLYHQLQ6HFW / HWXVUHYLHZ WKRVHIRUPXODVKHUH

$$u = u_{\max}\left(1 - \frac{r^2}{R^2}\right) \quad \text{where} \quad u_{\max} = \left(-\frac{dp}{dx}\right)\frac{R^2}{4\mu} \quad \text{and} \quad \left(-\frac{dp}{dx}\right) = \left(\frac{\Delta p + \rho g \Delta z}{L}\right)$$

$$V = \frac{Q}{A} = \frac{u_{\max}}{2} = \left(\frac{\Delta p + \rho g \Delta z}{L}\right)\frac{R^2}{8\mu}$$

$$Q = \int u dA = \pi R^2 V = \frac{\pi R^4}{8\mu}\left(\frac{\Delta p + \rho g \Delta z}{L}\right) \tag{6.12}$$

$$\tau_w = |\mu \frac{du}{dr}|_{r=R} = \frac{4\mu V}{R} = \frac{8\mu V}{d} = \frac{R}{2}\left(\frac{\Delta p + \rho g \Delta z}{L}\right)$$

$$h_f = \frac{32\mu LV}{\rho g d^2} = \frac{128\mu LQ}{\pi \rho g d^4}$$

7KH SDUDERORLG YHORFLW SURILOH KDV DQ DYHUDJH YHORFLW9 Z KLFK LV RQH KDOI RI WKH PD[LPXP YHORFLW 7KH TXDQWLWû S LV WKH SUHVVXUHGURS LQD SLSH RI OHQJWK/ WKDWV GS G[LVQHJDWLYH 7KHV IRUPXODV DUH YDOLG Z KHQHYHU WKH SLSH 5H\QROGV QXPEHU 5H G ! 9G LV OHVV WKDQ DERXW 1RW WKDWZ LV SURSRUWLRQDO9 VHH) LJ DQG LVLQGHSHQGHQW RI GHQVLW DQG YLVFRVLW 7KHVH IXQGDPHQWDO LV]HUR 1HWKHU RI WKH HIIHFWV WKH HLQWXEXOHQW

: LWK Z DOO VKHDU WUHVV NQRZ Q WKH 3RLVHXLOOH IORZ IULFWLRQ IDFWRU LV HDVLO GHWHUPLQHG

$$f_{\text{lam}} = \frac{8\tau_{w,\text{lam}}}{\rho V^2} = \frac{8(8\mu V/d)}{\rho V^2} = \frac{64}{\rho V d/\mu} = \frac{64}{\text{Re}_d} \qquad (6.13)$$

,QODPLQDUIORZ VKHSLSHIULFWRQIDFWRUGHFUHDVHVLQYHUVHO ZLWK5H\QROGVQXPEHU7KLVIDPRX IRUPXODLVHIHFWLYH EXWRWQWKHDJHEUDFUHDWRQV/R (TV DUHPRUHGLUHFWRUSUREOPV

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$QRLOZLWK! NJ P DQG P VIORZVXSZDUGWKURXJKDQLQFOLQHGSLSHDVVKRZQ LQ) LJ (7KHSUHVVXUHDQGHOHYDWLRQDUHNQRZQDWVHFWLRQV DQG P DSDUW$VVXPLQJWKDW ODPLQDUIORZ D YHUL\ WKDWWKHIORZ LVXS E FRPSXWHK EHWZHHQ DQG DQGFRPSXWH F 4 G 9 DQG H 5H ,VWKHIORZ UHDOO ODPLQDU'

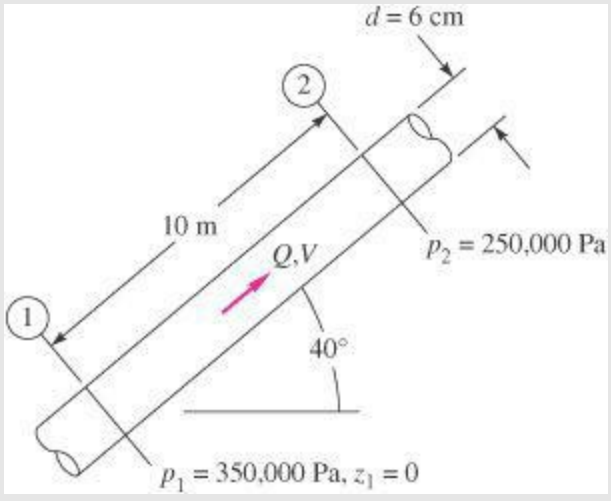
6RROXWLRQ

3DUWD

) RUODPLQDUXVH FDOFXODWH

! NJ P P V NJ P V
] û/ VLQ f P P

(



7KHIORZ JRHVLQWKHGLUHFWLRQRI IDOOQJ + * / WKHUHRUHFRPSXWWKHK\GUDXOLF JUDGHOQHKHLJK DWHDFKVHFWLRQ

$$\mathrm{HGL}_1 = z_1 + \frac{p_1}{\rho g} = 0 + \frac{350{,}000}{900(9.807)} = 39.65\ \mathrm{m}$$

$$\mathrm{HGL}_2 = z_2 + \frac{p_2}{\rho g} = 6.43 + \frac{250{,}000}{900(9.807)} = 34.75\ \mathrm{m}$$

7KH+* / LVGRZ HUDWH FWRQ KHQFHWKHIGRZ LVXSIURP WR DVDVXPHG \$QV D

3DUWE

7KHKHDGORVVLVWKHFKDQJHLQ+* /

$$h_f = \mathrm{HGL}_1 - \mathrm{HGL}_2 = 39.65\ \mathrm{m} - 34.75\ \mathrm{m} = 4.9\ \mathrm{m} \qquad \textit{Ans. (b)}$$

+DDWKHOHQJWKRI WKHSLSHLVTXLWHODUJHKHDGORVV

3DUWF

: HFDQFRPSXWH4 IURP WKHYDULRXVGLPLQDUIGRZ IRUPXODV QRWDE(T

$$Q = \frac{\pi\rho g d^4 h_f}{128\mu L} = \frac{\pi(900)(9.807)(0.06)^4(4.9)}{128(0.18)(10)} = 0.0076\ \mathrm{m^3/s} \qquad \textit{Ans. (c)}$$

3DUWG

' LYLGH4 E\ WKHSLSHDUHDWRJHWKHDYHUDJHYHORFLW

$$V = \frac{Q}{\pi R^2} = \frac{0.0076}{\pi(0.03)^2} = 2.7\ \mathrm{m/s} \qquad \textit{Ans. (d)}$$

3DUWH

: LWK9 NQRZ Q WKH5H\QROGVQXPEHUL

$$\mathrm{Re}_d = \frac{Vd}{\nu} = \frac{2.7(0.06)}{0.0002} = 810 \qquad \textit{Ans. (e)}$$

7KLVLVZHOOEHORZ WKHWUDQVLWLRQYDOXH5H\ VRZHDUHIDLU FHUWDLQWKHIGRZ LVODPLQDU
1RWHWKDWE\ XVLQJ HQWUDO VROXWLRQV, XQLW PHWHU VHFRQGV NLORJUDPV QHZWRQV IR
DOOYDULDEOHVZHDYRLGWKHQHGIRUDQ FRQYHUVLRQIDFWRULQWKHFDOFXODWLRQV

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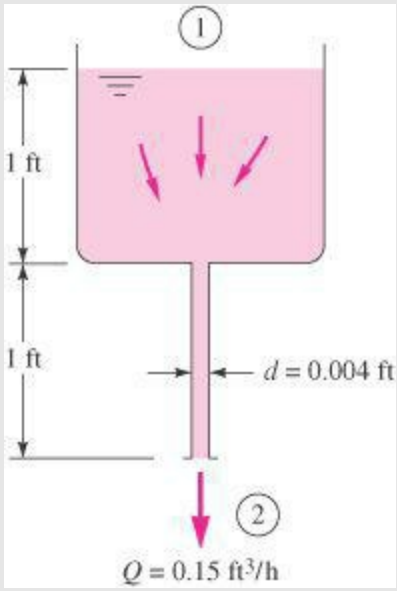
(



$ QDTXLGRI VSHFLILF ZHJKWJ GE IWIORZ VE\ JUDYLWVKURXJKD IWWDQNDQGD IWFDSLODUV VXEHDWD UDWH RI IWK DV VKRZQ LQ) LJ (6HFWLRQV DQG DUH DWDWRVSKHUF SUHVVXUH 1HJOHFWLQJ HQWUDQFH HIIHFWV FRPSXWH WKH YLVFRXV WR WKH QDQG

6RXWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZV/ IWG IWDQG4 IWK
- $VVXPSWLRQV / DPLQDU IXOO GHYHORSHG LQFRPSUHVLEOH 3RLVHXLOOH SLSH IORZ $VVRVSKHUF SUHVXUHDWWHFWLRQV DQG 1HJOLJLEOHYHORFLWDWXUDFH 9 §
- $SSURDFK 8VHFRQVQXLWDQGHQHUJ\ WRILQGWKHKHDGORVVDQGWKHQFHWKHYLVFRVLW
- 3URSHUWLYDOXHV * LYHQ! J GEI IW ILJXUHRXW VOXJ IWLI QHGHG
- 6RXWLRQVWHS) URP FRQVQXLWDQGWKHNQRZQIORZ UDWH GHWHUPLQH9

$$V_2 = \frac{Q}{A_2} = \frac{Q}{(\pi/4)d^2} = \frac{(0.15/3600)\text{ft}^3/\text{s}}{(\pi/4)(0.004\text{ ft})^2} = 3.32\text{ ft/s}$$

: ULWWKHHQHUJ\ HTXDWLRQEHWZHHQ DQG FDQFHODQJWKUP V DQGILQGWKHKHDGORVV

$$\frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f$$

or
$$h_f = z_1 - z_2 - \frac{\alpha_2 V_2^2}{2g} = 2.0\text{ ft} - 0\text{ ft} - \frac{(2.0)(3.32\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 1.66\text{ ft}$$

- &RPPHQW: HLQWRGXFHG. IRUODPLQDUSLSHIORZ IURP (T ,I ZHIRUJRW ZHZRXOGKDYHFDOFXODWHG
- 6ROXWLRQVWHS : LWKKHDGORVVNQRZQ WKHYLVFRVLWIRORZVIURP WKHODPLQDUIRUPXOD L (TV

$$h_f = 1.66\ \text{ft} = \frac{32\,\mu LV}{(\rho g)d^2} = \frac{32\mu(1.0\ \text{ft})(3.32\ \text{ft/s})}{(58\ \text{lbf/ft}^3)(0.004\ \text{ft})^2} \quad \text{solve for } \mu = 1.45\text{E-}5\ \frac{\text{slug}}{\text{ft-s}} \quad \textit{Ans.}$$

- &RPPHQW: HGLGQ¶WQHHGWKHYDOXHRI ! ² WKHIRUPXOD FRQWDLQV! J EXWZKR NQHZ" 1RV DQRWKDW/ LQWKLVIRUPXODLVWKHSLSHHQJWKRI IWQRWWKHWRWDOHOHYDWLRQFKDQJH
-) LQDOFKHFN &DOFXODWWKH5H\QROGVQXPEHUWRVHHLI LWLVOHVVWKDQ IRUODPLQDUIORZ

$$\text{Re}_d = \frac{\rho V d}{\mu} = \frac{(1.80\ \text{slug/ft}^3)(3.32\ \text{ft/s})(0.004\ \text{ft})}{(1.45\text{E-}5\ \text{slug/ft-s})} \approx 1650 \qquad \text{Yes, laminar.}$$

- &RPPHQW 6RZHGLGQ¶HG! DVWUDLJKWIRFDOXODW5H8Q[SHFWHGFRPPHQW) RUWKLVKHD GRV WKHUHLVDVHFRQG VWUEXHQWVRXWRQ DVZHVKDOOVHHLQ([DPSOH

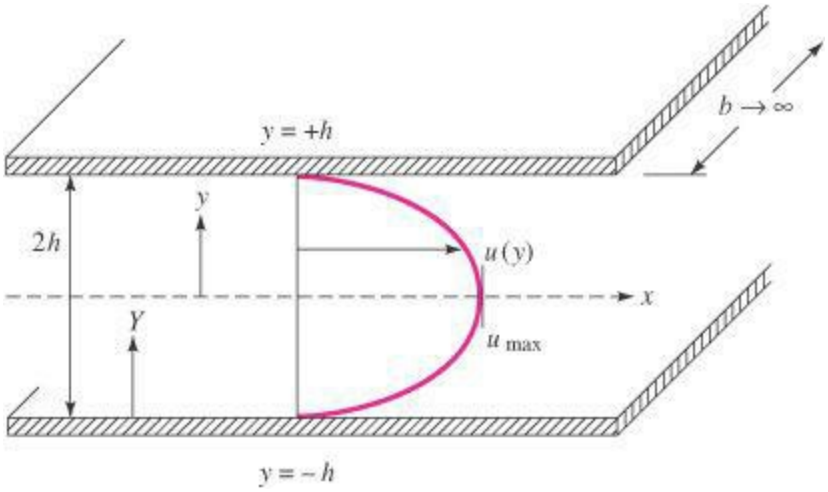
7XUEXOHQFH0RGHOLQJ

7KURXJKRXWWKLVFKDSWUZHDVXPHFRQWDQWGHQVLW DQG YLVFRVLW DQG QR WKHUPDOLQWUDFWLRQ V WKDWWKRQO WKHFRQWLQXLW DQGPRPHQWXP HTXDWLRQVDUHWREHVROYHGIRYHORFLW DQGSUHVXUH

$$\text{Continuity:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\text{Momentum:} \quad \rho \frac{d\mathbf{V}}{dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{V} \qquad (6.14)$$

VXEVHFWWR QR VOLS DWWKH ZDOOV DQG NQRZQ LQOHWDQG H[LWFRQGLWLRQV : H VKDOOVDYH RXUIUHH VXUIDFHVROXWLRQVIRU&KDS



) LJ) XOO GHYHORSHG IORZ EHVWHQSDUDOOHOSODWHV

: HZ LOOQRWZ RUNZ LWK WKH GLIIHUHQWLDO HQHUJ\ UHODWLRQ (T LQWKLV FKDSWHU EXWLWV YHU
LPSRUWDQWEHKDYLRUKDVWDQVIHUFDOFXODWLRQV DQG IRUJHQHUDOXQGHUVWDQGLQJ RI GXFWORZ SURFHVVH
7KHUH LV ZRUN EHLQJ GRQH E\ SUHVVXUH IRUFHV WR GULYH WKH IOXLG WKURXJK WKH GXFW: KHUH GRHV WKL
HQHUJ\ JR' 7KHUH LVQR ZRUN GRQH E\ WKH ZDOOVKDUVWUHVVHV EHFDXVHWKH YHORFLW\ DWWKH ZDOOV]HU
7KH DQVZHU LV WKDW SUHVXUH ZRUN LV EDODQFHG E\ YLVFRXV GLVLSDWLRQ LQ WKH ILQDO UHJLRQ RI WKH IORZ 7K
LQWUDOR RI WKH GLVLSDWLRQ IXQFWLRQ - IURP (T RYHU WKH IORZ ILHOG ZLOO HTXDOWKH SUHVXU
ZRUN $QH[DPSOHRI WKLV IXQGDPHQWDOYLVFRXV IORZ HQHUJ\ EDODQFHLVJLYHQLQ3URE&KP &

%RWK ODPLQDU DQG WXUEXOHQW IORZ V VDWLV\ (TV) RU ODPLQDU IORZ ZKHUH WKH DUH QR
UDQGRP IOXFWXDWLRQV ZH JR ULJKW WR WKH DXWDFN DQG VROYH WKH P IRU D YDULHW\ RI JHRPHWULHV > (
HDYLQJ PDQ PRUH RI FRXUH IRUWKH SUREOPV

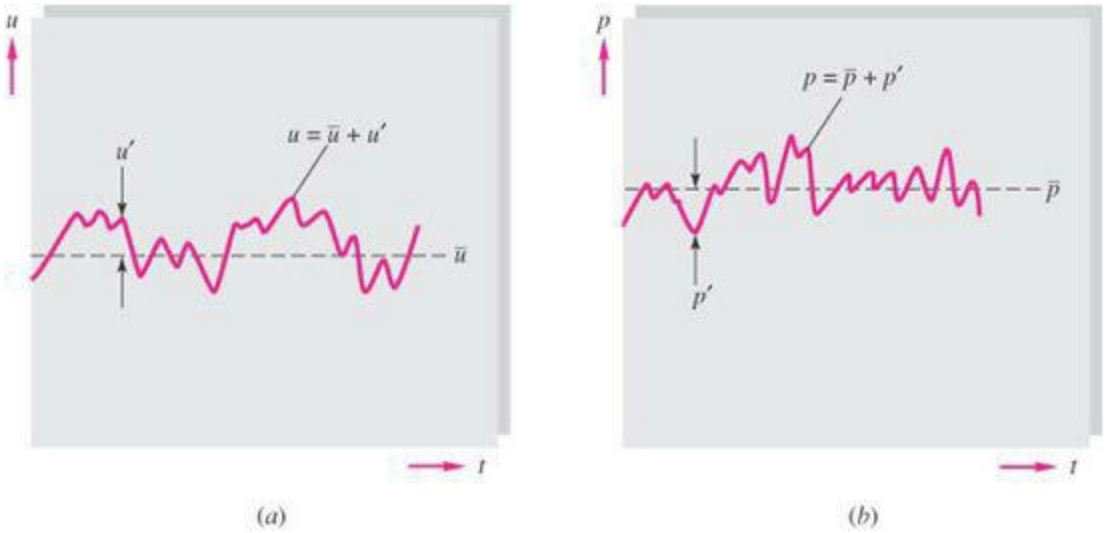
5HYQROGV¶7LPH$YHUDJLQJ &RQFHSW

) RUWXUEXOHQW IORZ EHFDXVHRI WKH IXFWXDWLRQV HYHU YHORFLW\ DQG SUHVXUH WHUP LQ(TV L
DUDSLG YDU\LQJ UDQGRP IXQFWLRQ RI WLPH DQG VSDFH $WSUHVHQWRXU PDWK PDWFVFDQQRWKDQGOHVXF
LQWDQDQHRXVIXFWXDWLQJ YDULDEOHV 1 RVLQJHSDUR UDQGRP IXQFWLRQVE9 E[\ ] WDQGS [\ ] V
LVNQRZ QWR EHD VRXWLRQ VR(TV 0 RUHRYHU RXUDWWHQWLRQ DVHQJLQHHUVLVWRZDUGWKH DYHUDJH
RUPHDQ YDOXHVRI YHORFLW SUHVXUH VKHDUVWUHVV DQGWKH0NHLQDKJK 5HYQROGV QXPEHU WXUEXOHQ
IORZ 7KLVDSSURDFKOHG2VERUQH5HYQROGVLQ WR UHZULW(TV LQWHUPVRI PHDQRUWPH
DYHUDJHGWXUEXOHQWYDULDEOHV

7KHWPHPHDQÌ RI DWXUEXOHQWXQFWLRQX [\ ] WLVGHILQHGE\

$$\bar{u} = \frac{1}{T}\int_0^T u\,dt \qquad (6.15)$$

ZKHUH7 LV DQ DYHUDJLQJ SHULRG WDNHQ WR EH ORQJHU WKDQ DQ\ VJQLILFDQWSHULRG RI WKH IXFWXDWLRQ
WKHPVHOYHV 7KH PHDQ YDOXHV RI WXUEXOHQWYHORFLW\ DQG SUHVXUH DUH LOOXVWUDWHG LQ) LJ) R
WXUEXOHQWJDVDQGZDWUIORZV DQDYHUDJLQJ SHULRG7 § VLVXXDOO TXLWHGHTXDW



) LJ ' HILQLWLRQ RI PHDQ DQG IXFWXDWLQJ WXUEXOHQWYDULDEOHV *a* YHORFLW *b* SUHVXUH

7KH IOXFWXDWLQJ• LVGHILQHGDVWKHGHYLDWLRQRI XIURP LWVDYHUDJHYDOXH

$$u' = u - \overline{u} \tag{6.16}$$

DOVR VKRZQLQ) LJ , WIROORZV E\ GHILQLWLRQ WKDWWKHIOXFWXDWLQJKDV]HURPHDQYDOXH

$$\overline{u'} = \frac{1}{T}\int_0^T (u - \overline{u})\,dt = \overline{u} - \overline{u} = 0 \tag{6.17}$$

+ RZ HYHU WKH PHDQ VTXDUH RI D IOXFWXDWLQJ LV QRW]HUR DQG LV D PHDVXUH RI WKHLQWHQVLW RI WK WXUEXOHQFH

$$\overline{u'^2} = \frac{1}{T}\int_0^T u'^2\,dt \neq 0 \tag{6.18}$$

1 RU LQ JHQHUDO DUH WKH PHDQ IOXFWXDWLQ SURGXFWV VXFK DV $\overline{u'v'}$ DQG $\overline{u'p'}$]HUR LQ D WSLFD WXUEXOHQWIORZ

5H\QROGV¶ LGHDZ DVWR VSOLWWFK SURSHUWLQRPHQSOX IOXFWXDWLQJ YDULDEOHV

$$u = \overline{u} + u' \quad v = \overline{v} + v' \quad w = \overline{w} + w' \quad p = \overline{p} + p' \tag{6.19}$$

6XEVWLWXWWKHVHLQR(TV DQG WDNHWKHWLPHPHDQRI HDFKHTXDWLQ 7KHFRQWLQXLWUHDWLR UHGXFHVWR

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0 \tag{6.20}$$

ZKLFKLVQRGLIIHUHQWIURP DOOPLQDUFRQWLQXLWUHDWLQ

+ RZ HYHU HDFK FRPSRQHQWRI WKHPRPHQXP HTXDWLQ E DWWUWLPHDYHUDJLQJ ZLOOFRQWDL PHDQ YDOXHV SOXV WKUHHPHDQ SURGXFWV RUFRUUHDWLQV RI IOXFWXDWLQJ YHORFLWLHV 7KHPRVWLPSRUWDQ RI WKHVHLV WKHPRPHQXP UHDWLQJLQWKHPDLQZLQGZ GLUHFWLQ WKHIRUP

$$\rho\frac{d\overline{u}}{dt} = -\frac{\partial \overline{p}}{\partial x} + \rho g_x + \frac{\partial}{\partial x}\left(\mu\frac{\partial \overline{u}}{\partial x} - \rho\overline{u'^2}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial \overline{u}}{\partial y} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial \overline{u}}{\partial z} - \rho\overline{u'w'}\right) \tag{6.21}$$

7KH WKUHH FRUUHDWLQ WUP $-\rho\overline{u'^2}$, $-\rho\overline{u'v'}$, and $-\rho\overline{u'w'}$ DUH FDOOHG WXEXOHQWWUHVVHV EHFDXVH WKH KDYH WKH VDPH GLPHQVLRQV DQG RFFXUULJKWDORQJVLGH WKH QHZWRQLDQ ODPLQDU VWUHVV /Ì /[DQG VR RQ $FWXDOO WKH\ DUH FRQYHFWLYH DFFHOHUDWLQWUP ZKLFK ZK\ WKHGHQVLW DSSHDUV QR VWUHVVHV EXWWKH\ KDYH WKHP DWKHP DWFDOOHIIHFWR WUHVV DQG DUH VR WUP HG DOPRWXQLYHUVDOO LQWKH QDWXUH

7KH WXUEXOHQWWUHVVHV DUH XQNQRZ QD SUROLDQGPXWEHUHDWHG E\ H[SHULPHQWWR JHRPHW DQ IORZ FRQGLWLRQV DVGHVFULEHG LQ5HV VR) RUWXQDWL LQGXFWDQG ERXQGDU\ OD\HUIORZ WKHVWUHV $\rho\overline{u'v'}$ DVRFLDWHG ZLWK GLUHFWLQ\ QORUPDO WR WKH ZDOO VGRPLQDQW DQG ZH FDQ DSSUR[LPDW ZLW

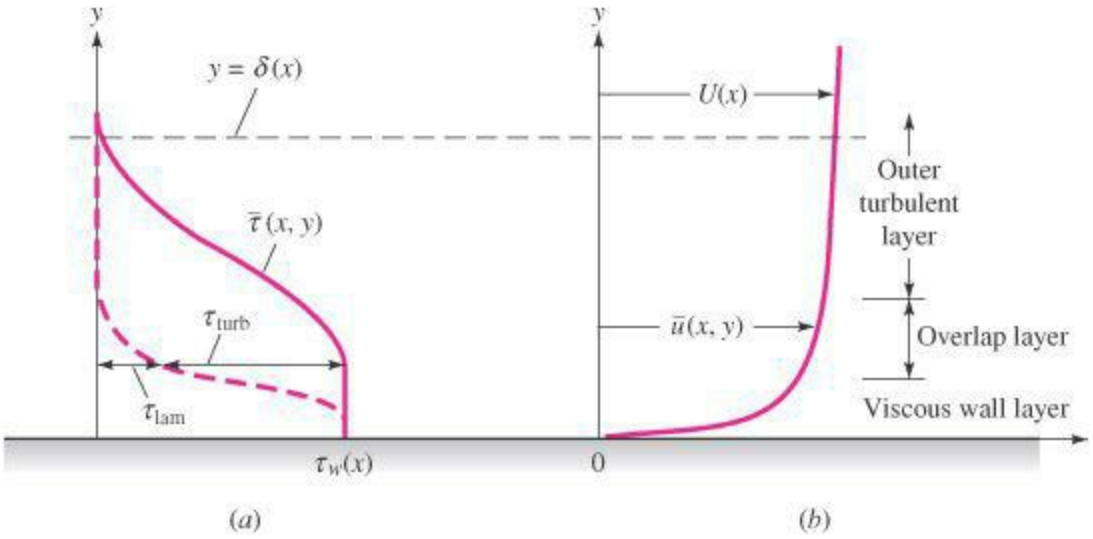
H[FHOOHQWDFFXUDF\ DVLPSOLILFDWLRQ

$$\rho \frac{d\bar{u}}{dt} \approx -\frac{\partial \bar{p}}{\partial x} + \rho g_x + \frac{\partial \tau}{\partial y} \qquad (6.22)$$

Z KHUH

$$\tau = \mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'} = \tau_{\text{lam}} + \tau_{\text{turb}} \qquad (6.23)$$

) LJXUH VKRZV VWKH GLVWULEXWLRQ RI 2OP DQG 2VXE IURP WSLFDOP HDVXUHPHQWV DFURVV D VXE EOD
VKHDUOD\HUQHDUDZDOO/ DPLQDUVKHDUVFRPLQDWHZDOWKHZDOOD\HU DQGVXEOD\HUWKHD
GRPLQDWHVLQWKHRXWHUOD\HU 7KHUHLVDQLQWHUPHGLDWHUHJLRQ FDOOHGWKHRYHUODS OD\HU ZKHUHERW
OPLQDUDQGVXEOD\HUHOPSRUWDQW7KHVHWKUHHUHJLRQVDUHOEHOGLQ) LJ



) LJ 7\SLFDOYHORFLW DQG VKHDUGLVWULEXWLRQVLQVXEOD\HUDQGRXWHUOD\HU D ZDOO *a* VKHDU *b* YHORFLW

,QWKHRXWHUOD\HU2VXE LVWZR RUWKUHHRUGHUVRI PDJQLWXGHJUHDWHUWKDQ2OP DQGYLFHYHUVDLQWK
ZDOOD\HU7KHVHH[SHULPHQWDOGDWDFRQILUPHQDEOHXVWRXVHDFXUDWHEXWVLPSOHODZWRGHVFULEHWKHYHORFLW
GLVWULEXWLRQÌ \ DFURVVDVXEOD\HUDOOD\HU

7KH/ RJDULWKPLF2 YHUODS/ DZ

: HKDYHVHHQLQ) LJ WKDWWKHUHDUHWKUHHUHJLRQVLQVXEOD\HUDOOD\HU

: DOOD\HU 9LVFRXVWKHDUGRPLQDWHV
2 XWHUOD\HU 7XUEXOHQWWKHDUGRPLQDWHV
2 YHUODS OD\HU %RWKWUSHVRI WKHDUHLPSRUWDQW

) URP QRZ RQGHWXVDJUHHWRGURSWKHRYHUEDUIURP YHORFLWÌ / HW2 EHWKHZDOOWKHDUWUHVV DQG
GHW DQG8 UHSUHVHQWWKHWKLFNQHVVDQGYHORFLWDWWKHHGJHRI WKHRXWHUOD\HU\ /
) RUWKH ZDOOOD\HU 3UDQGWOGHGXFHG LQ WKDWX PXWWEH LQGHSHQGHQWRI WKH VKHDUOD\H

VKLFNQHVV

$$u = f(\mu, \tau_w, \rho, y) \qquad (6.24)$$

%\ GLPHQVLRQDODQDOVLV VKLVLVHTXLYDOHQWRR

$$u^+ = \frac{u}{u^*} = F\left(\frac{yu^*}{\nu}\right) \qquad u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2} \qquad (6.25)$$

(TXDWLRQ LVFDOOHGWKHODZ RI WKHZDOO DQGWKHTXDQWLWX LVWKUHGWKHIULFWLRQYHORFLW
EHFDXVHILWKDVGLPHQVLRQV^/ 7í ` DQGRXJKLWVQRWDFWXDOO DIORZ YHORFLW
6XEVHTXHQW . iUPiQ LQ GHGXFHGWKDWX LQWKHRXWHUOD\HULVLQGHSHQGHQWRI PROHFXOD
YLVFRVLW EXWGHSHQGVRQIURP WKHWXUEP YHORFLW8 PXWGHSHQGRQWKHOD\HUWKLFNQHVV/ DQGWK
RWKHUSURSHUWLHV

$$(U - u)_{\text{outer}} = g(\delta, \tau_w, \rho, y) \qquad (6.26)$$

$ JDLQ E\ GLPHQVLRQDODQDOVLVZHUHZULWWKLVDV

$$\frac{U - u}{u^*} = G\left(\frac{y}{\delta}\right) \qquad (6.27)$$

ZKHUHX KDVWKHVDPHPHDQLQJDVLQ(T (TXDWLRQ LVFDOOHGWKHYHORFLW GHIHFWODZ
IRUWKHRXWHUOD\HU

%RWKWKHZDOOODZ DQGWKHGHIHFWODZ DUHIRXQGWRHDFFXUDWHIRUDZLGHYDULHWR
H[SHULPHQWDOWXEXOHQWGXFWDQGERXQGDU OD\HUIORZV> WR @7KH\ DUHGLIIHUHQWIRUP \HWWKH
PXWRYHUODS VPRRWKO LQWKHLQQHUGDWOD\HU ,Q & % 0 LOOLNDQVKRWHGWKDWWKLVFDQEHWUX
RQO LI WKHRYHUODS OD\HUYHORFLW YDULHVORJDULWKPLFDOO ZLWK\

$$\frac{u}{u^*} = \frac{1}{\kappa} \ln \frac{yu^*}{\nu} + B \qquad \text{overlap layer} \qquad (6.28)$$

2 YHUWKHIXOOUDQJHRI WKUEXOHQWPRRWKZDOOIORZV WKHGLPHQVLRQOHVVFRQVWDQWV DQG%DUHIRXQ
WR KDYH WKH DSSUR[LPDWH YDOXHV § DQG% § (TXDWLRQ LV FDOOHG WKHORJDULWKPLF
RYHUODS OD\HU

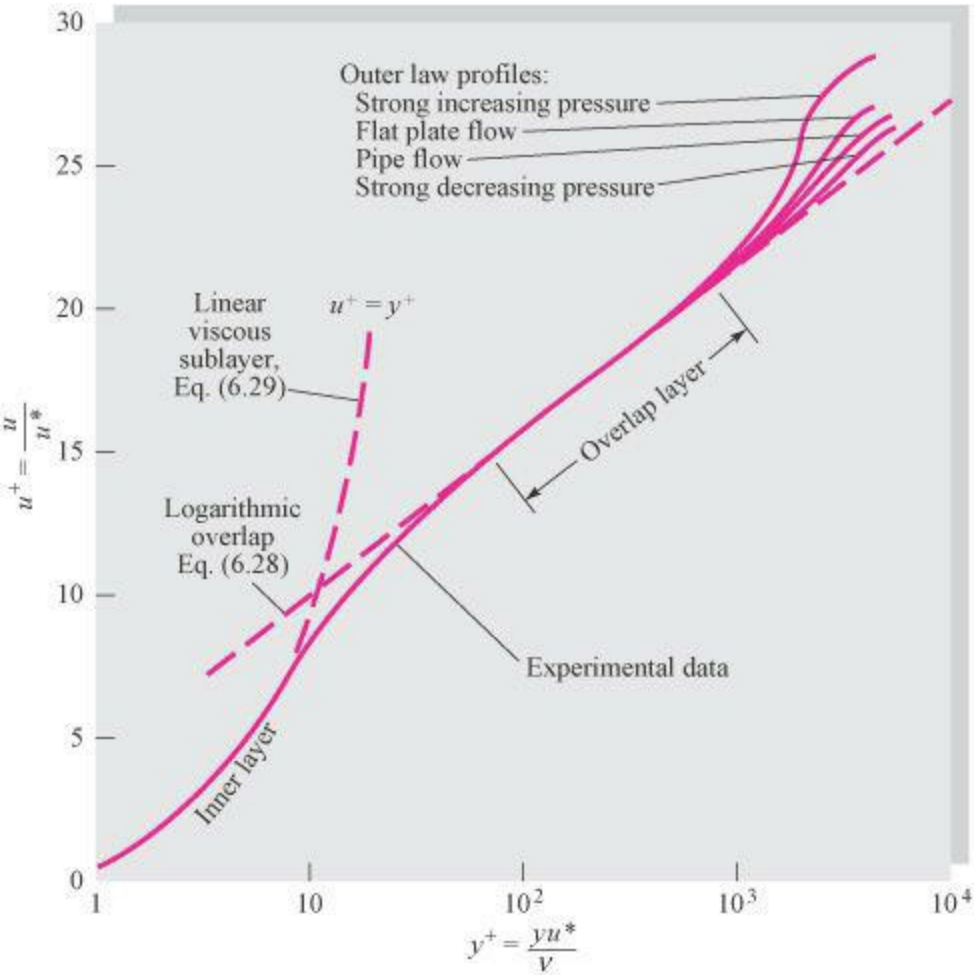
7KXV E\ GLPHQVLRQDOUHDVRQLQJ DQG SK\VLFDOLQVLJKWZH LQHUWKDWD SORWRI X YHUVXV OQ\ LQ I
WKUEXOHQWWKHDUOD\HU ZLOOVKRZ D FXUYHG ZDOUHJLRQ D FXUYHG RXWHUUHJLRQ DQG D WUDQVLWLRQDO
ORJDULWKPLF RYHUODS) LJXUH VKRZV WKDWWKLV LV H[DFWO WKH FDVH 7KHIRXURXWHUOD\ SURILOH
VKRZ QDOOPHUJH VPRRWKO ZLWKWKHORJDULWKPLF RYHUODS OD\ EXWKDYH GLIIHUHQWP DJQLWXGHV EHFDXV
WKH YDU LQH[WHQVLRQSUHVWXUHJUDGLHQW7KHZDOOODZ LVXQLTXHDQGIRUORZ VWKHOQH DUYLVFRXVUHDVRQ

$$u^+ = \frac{u}{u^*} = \frac{yu^*}{\nu} = y^+ \qquad (6.29)$$

IURP WKHZDOOWR DERXW WKHUHJWUFXUYLQJ RYHUWR PHJHZLWKWKHORJDULWKPLF OD\ DWDERX
\

%HDHYHLWRUORW) LJ ZKLFKLVQRWKLQJ PRUHWKDQDVKUHZGFRUUHODWLRQRI YHORFLWSURILOH

LV WKH EDVLV IRU PRVW [LXLQJ ³WKH RU´ RI WXUEXOHQW WKH DUIORZ V 1RWH WKDW HKDYH QRW WROYHG DQ
HTXDWRQV DVDOO EXWKDYH PHUHO H[SUHVVHG WKH WHUPZ LVH YHORFLWLQDQHDWRUP



) LJ ([SHULP HQWDO YHUL LFDWRQ RI WKH LQQHU RXWHU DQG RYHUODS OD\ HU OD ZV UHDVRQ
YHORFLW SURILOHV LQ WXUEXOHQW ZDOO IORZ

7KHUH LV VHUHQGLSLW LQ) LJ 7KH ORJDULWKP LF OD Z LQWHGR IMXWWEHLQJ D VKRU
RYHUODSSLQJ OQN DFWXDOO DSSUR[LPDWH Q HDU WKH HQWH YHORFLW SURILOH H[FHSWRU WKH RXWHU OD Z
ZKHQ WKH SHUHXUE LV LQFUHDVLQJ WXRQJO GRZQWHP DV LQ D GLIIXVHU 7KH LQQHU ZDOO OD Z WSLFDO
H[WHQGV RYHUO WV WKDQ SHUF HQWR WKH SURILHDG FDQEHQHJOHFWHG 7KXV ZHFDQXVH(T DV D
H[FHOHQWDSSUR[LPDWRQ WR VROYH QHDU HYHU WXUEXOHQWORZ SUREOP SUHVHQWHG LQ WKLV DQG WKH QH[
FKDSWU 0DQ DGGLWRQDODSSOFDWRQV/DUHJLYHQLQ5HV DQG

$GYDQFHG0 RGHOLQJ &RQFHSWV

7XUEXOHQFH PRGHOLQJ LV D YHU DFWLYH ILHOG 6FRUHV RI SDSHUV KDYH EHHQ SXEOLVKHG WR PRU
DFFXUDWH VLP XODWWKH WXUEXOHQWXWXHVLQ(T DQG WKLU\ DQG] FRP SRQHQW 7KLV UHVHDUF
QRZ DYDLODEOH LQ DGYDQFHG WH[W V> @JRHV ZHOOEH\RQG WKH SUHVHQWERRN ZKLFK LV FRQFHQWUD
WKH XVH RI WKH ORJDULWKP LF OD Z IRU SLSH DQG ERXQGDU OD\ HU SUREOPV) RU[DP SOH / 3UDQG
ZKR LQYHQWHG ERXQGDU OD\ HU WKHRU LQ OD\ HU SURSRVHG DQHGG YLVFRVLW PRGHO RI WKH5 H QROG
WXUEV\VWHP LQ(T

$$-\rho\,\overline{u'v'} = \tau_{\text{turb}} \approx \mu_t \frac{du}{dy} \quad \text{where} \quad \mu_t \approx \rho l^2 \left|\frac{du}{dy}\right| \tag{6.30}$$

7KH WHUP W Z KLFK LV D SURSHUW RI WKH IOXLG LV FDOOHG WKH HGG YLVFRVLW DQG FDQ EH PRGHOHG LQ YDULRXV Z D\V 7KH PRVW SRSXODU IRUP LV(T Z KHUH O LV FDOOHG WKH PL[LQJ OHQJWK R WKH VKLEXOHQW HGGLHV DQG RJRXV W PHDQ IUHH SDW LQ PROHFXODU WKHRU 1HDU D VROG Z DOO OL DSSUR[LPDWHO SURSRUWLRQDO W GLVWDQFH IURP WKH Z DOO DQG. iUPiQVXJJHVWHG

$$l \approx \kappa y \quad \text{where} \quad \kappa = \text{Kármán's constant} \approx 0.41 \tag{6.31}$$

$ V D KRPHZ RUN DVLJQPHQW3URE 3 \RX PD\ VKRZ WKDW(TV DQG HDG WR WK RJDULWKPLF OD Z QHDU D Z DOO

0 RGHUQ WXUEXOHQFH PRGHOV DSSUR[LPDW WKUHH GLPHQVLRQDO WXUEXOHQW IORZ V DQG HPSOR DGGLWLRQDO SDUWLDO GLIIHUHQWLDO HTXDWLRQV IRU VXFK TXDQWLWLHV DV WKH WXUEXOHQFH NLQHWF HQHJ\ WK WXUEXOHQW GLVLSDWLRQ DQG WKH VL[5H\QROGV VWUHVVHV) RUGHUDLV VH5HV DQG

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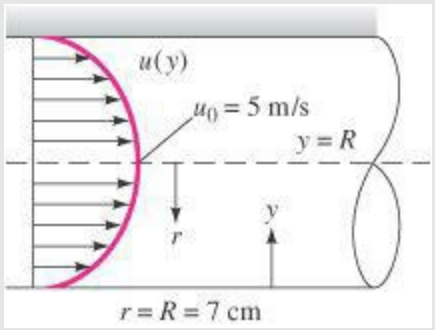


$ LU DW f& IORZ V WKURXJK D FP GLDPHWU WXEH XQGHU IXOO GHYHORSHG FRQGLWLRQV 7K FHQWHUOLQH YHORFLW LVX P V (VWLPDWH IURP) LJ D WKH IULFWLRQ YHORFLW X DQG E WKH Z DO VKHDU VWUHVV2Z

6RXWLRQ

- 6\VWHP VNHWFK) LJXUH(VKRZ V WXEXOHQW SLSHIORZ Z LWK X P V DQG5 FP
- $VVXPSWLRQV) LJXUH VKRZ V WKDW WKH ORJDULWKPLF OD Z (T LV DFFXUDWH DOO WKH Z D WR WKH FHQWHUOLQH RI WKH WXEH
- $SSURDFK 8VH(T WR HVWLPDWH WKH XQNQRZ QIULFWLRQ YHORFLW X
- 3URSHUWLHV) RUDLUDW f& ! NJ P DQG (P V
- 6ROXWLRQ 6WHS ,QWHUSRODWH JLYHQ GDWD LQR(T DW 5 WKH FHQWHUOLQH 7KH RQ XQNQRZ QLV X

$$\frac{u_0}{u^*} = \frac{1}{\kappa}\ln\left(\frac{Ru^*}{\nu}\right) + B \quad \text{or} \quad \frac{5.0\ \text{m/s}}{u^*} = \frac{1}{0.41}\ln\left[\frac{(0.07\ \text{m})u^*}{1.51\text{E-}5\ \text{m}^2/\text{s}}\right] + 5$$

$ QWRXJKWKHORJDULWKP PDNHVLWDZ NZ DUG RQHFDQLWHUDWHWKLVE\ KDQGWRILQGX 2 URQHFDQRSH
((6 DQGWSHRXWDVLQJOHVWDWHPHQWRI (T

XWDU QQ XWDU (

$Q QRPLQDOJXHVV HJ X ZLOOGR ((6 LPPHGLDWHO UHXLQVWKHFRUUHFWRXWRQ

$$u^* \approx 0.228\ \text{m/s} \qquad \textit{Ans. (a)}$$

$$\tau_w = \rho u^{*2} = (1.205)(0.228)^2 \approx 0.062\ \text{Pa} \qquad \textit{Ans. (b)}$$

- &RPPHQWV 7KHORJDULWKPLF ODZ VROYHG HYHUWKLQJ 7KLV LV D SRZHUXOWHFKQLTXH XVLQJ D H[SHULPHQWDOYHORFLWFRUUHDWRQ VR DSSUR[LPDWJHQHUDOVXEHQWORZ V <RX PD\ FKHFN WKH5H\QROGVQXPEHU5H LVDERXW GHILQLWHO VXEHQWORZ

7XUEXOHQW3LSH)ORZ

)RUWXUEXOHQWSLSHIORZ ZHQHHG QRWWROYH DGLIIHUHQWLDOHTXDWRQ EXWLQVWHDG SURFHHG ZLWKWK ORJDULWKPLF ODZ DVLQ([DPSOH $VXPHWKDW/T FRUUHODWHVWKHORFDOPHDQYHORFLWX U DQ WKHZD\ DFURVVWKHSLSH

$$\frac{u(r)}{u^*} \approx \frac{1}{\kappa}\ln\frac{(R-r)u^*}{\nu} + B \qquad (6.32)$$

ZKHUHZHKDYHUHSODFHG\ E\ 5 í U &RPSXWWKHDYHUDJHYHORFLWIURP WKLVSURILOH

$$V = \frac{Q}{A} = \frac{1}{\pi R^2}\int_0^R u^*\left[\frac{1}{\kappa}\ln\frac{(R-r)u^*}{\nu} + B\right]2\pi r\,dr$$

$$= \frac{1}{2}u^*\left(\frac{2}{\kappa}\ln\frac{Ru^*}{\nu} + 2B - \frac{3}{\kappa}\right) \qquad (6.33)$$

,QWRGXFLQJ DQG% ZHREWDLQ QXPHULFDOO

$$\frac{V}{u^*} \approx 2.44\ln\frac{Ru^*}{\nu} + 1.34 \qquad (6.34)$$

7KLVORRNVRQO PDJLFDOO LQWHUHVWLQJ XQWLOZHUHDOL]HWKDW X LVGLUHFWO UHODWHGWRWKH' DUF

IULFWLRQ IDFWRU

$$\frac{V}{u^*} = \left(\frac{\rho V^2}{\tau_w}\right)^{1/2} = \left(\frac{8}{f}\right)^{1/2} \tag{6.35}$$

0 RUHRYHU WKH DJUHHPHQW RI WKH ORJDULWKP LQ LV HTXLYDOHQW WR

$$\frac{Ru^*}{\nu} = \frac{\frac{1}{2}Vd}{\nu}\frac{u^*}{V} = \frac{1}{2}\mathrm{Re}_d\left(\frac{f}{8}\right)^{1/2} \tag{6.36}$$

,QWURGXFLQJ DQG LQWR(T FKDQJLQJ WR D EDVH ORJDULWKP DQG UHDUUDQJLQJ ZH REWDLQ

$$\frac{1}{f^{1/2}} \approx 1.99 \log (\mathrm{Re}_d f^{1/2}) - 1.02 \tag{6.37}$$

,QRWKHU ZRUGV E\ VLPSOI FRPSXWLQJ WKH PHDQ YHORFLW IURP WKH ORJDULWKPLF ODZ FRUUHODWRQ ZI REWDLQ D UHDWRQ EHWZHHQ WKH IUFWRQ IDFWRU DQG 5H\QROGV QXPEHU IRUW[WEHQWSLSH IORZ 3UDQG GHULYHG(T LQ DQG WKHQ DGXVWHG WKH FRQVWDQWV WR ILW[WUFWRQ GDWD EHWWU

$$\frac{1}{f^{1/2}} = 2.0 \log (\mathrm{Re}_d f^{1/2}) - 0.8 \tag{6.38}$$

7KLV LV WKH DFFHSWHG IRUP XOD IRU D VPRRWK ZDOOHG SLSH 6RPH QXPHULFDO YDOXHV PD\ EH OLVWHG IRQORZV

Re_d	4000	10^4	10^5	10^6	10^7	10^8
f	0.0399	0.0309	0.0180	0.0116	0.0081	0.0059

7KXVI GURSVE\ RQO DIDFWRUR RYHUD IRQGLQFUHDVHLQ5H\QROGV QXPEHU (TXDWR LV FXPEHUVRPH WR VROYH LI 5H LV NQRZQ DQGI LV ZDQWHG 7KHUH DUH PDQ DOWLQDWLYH DSSUR[LPDWRQVLQWKHOWHUDWXUHIURP ZKLFKI FDQEHFRPSXWHG[SOLFLW IURP 5H

$$f = \begin{cases} 0.316\,\mathrm{Re}_d^{-1/4} & 4000 < \mathrm{Re}_d < 10^5 \quad \text{H. Blasius (1911)} \\ \left(1.8 \log \dfrac{\mathrm{Re}_d}{6.9}\right)^{-2} & \text{Ref. 9} \end{cases} \tag{6.39}$$

%DVLXV D WXGHQWRI 3UDQGWO SUHVHQWHG KLV IRUPXOD LQ WKH ILUWFRUUHODWRQ HYHUPDGH RI SLS IUFWRQ YHUXV 5H\QROGV QXPEHU $OWKRXJK KLV IRUPXOD KDV D OLPLWHG UDQJH LWLOOXWUDWV ZKDW ZH KDSSHQLQJ LQ) LJ WR+DJHQV SUHVXUH GURS GDWD)RUDKRU]RQWDOSLSH IURP (T

$$h_f = \frac{\Delta p}{\rho g} = f\frac{L}{d}\frac{V^2}{2g} \approx 0.316\left(\frac{\mu}{\rho V d}\right)^{1/4}\frac{L}{d}\frac{V^2}{2g}$$

or

$$\Delta p \approx 0.158\, L\rho^{3/4}\mu^{1/4}d^{-5/4}V^{7/4} \tag{6.40}$$

DWDOO WXUEXOHQW5H\QROGV QXPEHUV 7KLV H[SODLQV ZK\ +DJHQV GDWD IRU SUHVXUH GURS EHJLQV LQFUHDVH DV WKH SRZHU RI WKH YHORFLW LQ) LJ 1RWH WKDWWS YDULHV RQO VOLJKWO ZLWK YLVFRVLW ZKLFK LV FKDUDFWHULVWF RI WXUEXOHQWOWRZ ,QWURGXFLQJ $Q = \frac{1}{4}\pi d^2 V$ LQWR(T ZH REWDL

WKHDOWLQDWLYHIRUP

$$\Delta p \approx 0.241 L \rho^{3/4} \mu^{1/4} d^{-4.75} Q^{1.75} \qquad (6.41)$$

)RUDJLYHQIORZ UDWH4 WKHWXUEXOHQWSUHVVXUHGURS GHFUHDVHVZLWKGLDPHWHUYHQRUHVKDSSO WKDQWKHODPLQDUIRUPXOD 7KXVWKHTXLFNHVWZD\ WRUHGXFHUHTXLUHGSXPSLQJ SRZHUVWKHSLSHVL]H DOWKRXJK RI FRXUVH WKHODUJHUSLSHLVPRUHH[SHQVLYH ' RXEOLQJ WKHSLSHVL]H GHFUHDVHVûS E\ DIDFWRURI DERXW IRUDJLYHQ4 &RPSDUH(T ZLWK([DPSOH DQG) LJ

7KHPD[LPXP YHORFLWLQWXUEXOHQWSLSHIORZ LVJLYHQE\ (T HYDOXDWGDWU

$$\frac{u_{\max}}{u^*} \approx \frac{1}{\kappa} \ln \frac{Ru^*}{\nu} + B \qquad (6.42)$$

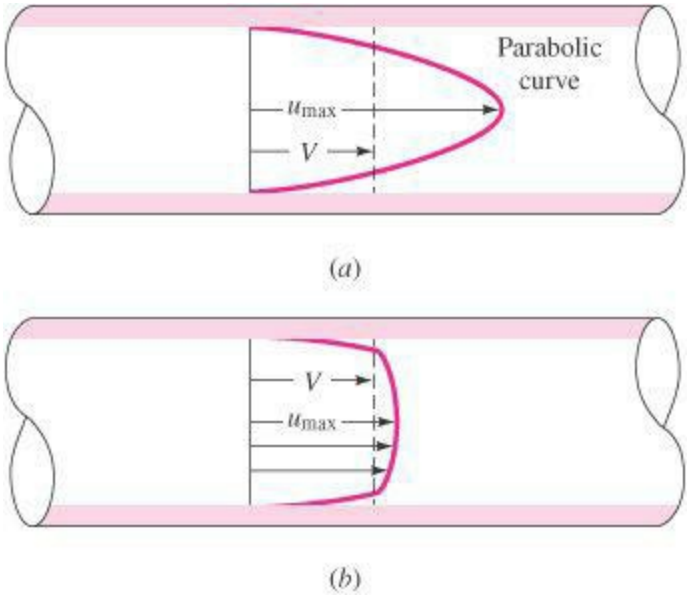
&RPELQLQJ WKLV ZLWK(T ZH REWDLQ WKH IRUPXOD UHODWLQJ PHDQ YHORFLW WR PD[LPXP YHORFLW

$$\frac{V}{u_{\max}} \approx (1 + 1.3\sqrt{f})^{-1} \qquad (6.43)$$

6RPHQXPHULFDOYDOXHVDUH

Re_d	4000	10^4	10^5	10^6	10^7	10^8
$V/u_{\max}$	0.794	0.814	0.852	0.877	0.895	0.909

7KHUDWLR YDULHVZLWKWKH5H\QROGVQXPEHUDQGLVPXFKODUJHUWKDQWKHYDOXHIRU SUHGLFWHGIR DOOODPLQDUSLSHIORZ LQ(T 7KXVDVXEVWDQWLDOSURILOH DVVKRZQLQ) LJ E LVYHU IODWLQWKHFHQWUDODQGGURSVRII VKDUSO WR]HURDWWKHZDOO



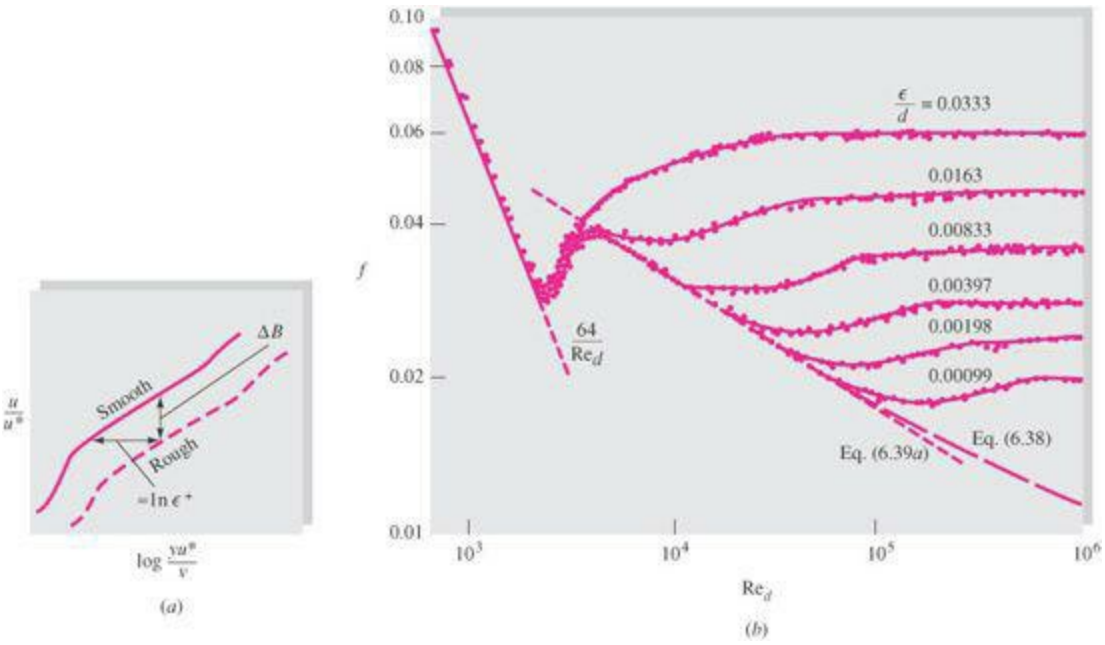
) LJ &RPSDULVRQ RI ODPLQDUDQG WXUEXOHQWSLSHIORZ YHORFLW SURILOHV IRU WKH VDPH YROXPHIORZ *a* ODPLQDUIORZ *b* WXUEXOHQWIORZ

(IIHFWRI 5RXJK : DOOV

,WZ DV QRWHG QXPHULFDOO\ E\ &RROHEURRN > @DQG WXUEXOHQFH URXJKQHVV KDV DQ HIIH
RQ IULFWLRQ UHVLVWDQFH ,W WXUQV RXW WKDW WKH HIIHFWLYH JHRPHWULF SDUDPHWHU GRZ DQG DOO WK
GPLQDQW IRUP XODV GHULYHG LQ WKLV VHFWLRQ DUH YDOLG IRU URXJK ZDOOV DOR %XW WXUEXOHQW ORZ LV VWURQJO
DIIHFWHG E\ URXJKQHVV ,Q) LJ WKH OQH DUYLVFRXV VXEOD\HU [WQGV RXWQDO WR\ \X 7KX\
FRPSDUHG ZLWK WKH GLPHQVLRQ WKH VXEOD\HU WKLFNQHVV\ LV RQO

$$\frac{y_s}{d} = \frac{5\nu/u^*}{d} = \frac{14.1}{\mathrm{Re}_d f^{1/2}} \qquad (6.44)$$

) RUH[DPSOH DW5H G DQG\ G DZ DOOURXJKQHVV RI DERXW GZ LQ
EUHDNXS WKH VXEOD\HUDQG SURIRXQGO FKDQJHV WKH ZDOODZ LQ) LJ
0 HDVXUHPHQWV RI X\ LQ WKH EXIIHU UHJLRQ ZHUH E\ 3UDQGWO VWXGHQW1LNXUDGVH> @WKRZ D
LQ) LJ D WKDW URXJKQHVV KHLJKW ZLOO IRFH WKH RJDULWKP ODZ SURILOH RXWZDUGV RQ WKH DEVFLVV
E\ DQDPRXQW SSUR[LPDWHO HTXDO WR ZKHUH ∈ X 7KH VORSH RI WKH RJDULWKP ODZ UHPDLQ
WKH VDPH EXW WKH VKLIWRXWZDUGV FDXVHV WKH FRQVWDQW %WR EH OHVV E\ DQDPRXQW%§ QQ



) LJ (IIHFWRI ZDOOURXJKQHVV RQ VXUEXOHQWSLSH IORZ *a* 7KH ORJDULWKPLF RYHUDO
YHORFLW SURILOH VKLIW GRZQ DQG WR WKH ULJKW *b* H[SHULPHQWV ZLWK VDQG JUDLQ URXJKQHVV E\
1LNXUDGVH > @WKRZ D V\VWHPDWLF LQFUHDVH RI WKH VXUEXOHQW UHVLVWDQFH ZLWK WKH URXJKQHV
UDWLR

1LNXUDGVH > @WLP XODWHG URXJKQHVV E\ JOXLQJ XQLIRUP VDQG JUDLQV RQWR WKH LQQHU ZDOOV RI WK
SLSHV +H WKHQ PHDVXUHG WKH SUHVVXUH GURSV DQG IORZ UDWHV DQG FRUUHODWHG IULFWLRQ IDFWRU YHUVX
5H\QROGV QXPEHULQ) LJ E : HVHH WKDW WKH LQLWLDO IULFWLRQ UHVXOWV EHORZ EXW WXUEXOHQW IULFWLRQ DW
DQRQVHWSRLQW LQFUHDVHV PRQRWRQLFDOO ZLWK WKH URXJKQHVV UDWLR G)RUDQ JLYHQ G WKH IULFWLRQ
IDFWRU EHFRPHV FRQVWDQW XOO URXJK DWKLJK 5H\QROGV QXPEHUV 7KHVH SRLQWV RI FKDQJH DUH FHUWDL
YDOXHV RI∈ ∈ X

$\frac{\epsilon u^*}{\nu} < 5$: *hydraulically smooth* walls, no effect of roughness on friction

$5 \le \frac{\epsilon u^*}{\nu} \le 70$: *transitional* roughness, moderate Reynolds number effect

$\frac{\epsilon u^*}{\nu} > 70$: *fully rough* flow, sublayer totally broken up and friction independent of Reynolds number

) RUIXOO URXJKIORZ ∈ ! WKHORJODZ GRZQWKLWWi%LQ) LJ DLV

$$\Delta B \approx \frac{1}{\kappa} \ln \epsilon^+ - 3.5 \qquad (6.45)$$

DQGWKHORJDULWKP ODZ PRGLILHGIRUURXJKQHVVEHFRPHV

$$u^+ = \frac{1}{\kappa} \ln y^+ + B - \Delta B = \frac{1}{\kappa} \ln \frac{y}{\epsilon} + 8.5 \qquad (6.46)$$

7KHYLVFRVLWYDQLVKHV DQGKHQFHIXOO URXJKIORZ LVLQGHSHQGHQWRI WKH5H\QROGVQXPEHU,I Z LQWUDW(T VRREWDLQWKHDYHUDJHYHORFLWLQWKHSLSH ZHREWDLQ

$$\frac{V}{u^*} = 2.44 \ln \frac{d}{\epsilon} + 3.2$$

or $$\frac{1}{f^{1/2}} = -2.0 \log \frac{\epsilon/d}{3.7} \qquad \text{fully rough flow} \qquad (6.47)$$

7KHUHLVQR5H\QROGVQXPEHUHIIHFWKHQFHWKHKHDGORVV YDULHV H[DFWO DV WKHVTXDUH RI WK YHORFLWLQWKLVFDVH 6RPHQXPHULFDOYDOXHVRI IULFWRQIDFWRUPD\ EHOLVWHG

ϵ/d	0.00001	0.0001	0.001	0.01	0.05
f	0.00806	0.0120	0.0196	0.0379	0.0716

7KHIULFWRQIDFWRULQFUHDVHVE\ WPHVDVWKHURXJKQHVVLQFUHDVHVE\ DIDFWRURI ,QWK WDQVLWLRQDOURXJKQHVV UHJLRQ VDQG JUDLQV EHKDYH VRPHZKDWGLIIHUHQWO IURP FRPPHUFLDO URXJI SLSHV VR) LJ EKDVQRZ EHHQUHSODFHGE\ WKH0RRG FKDUW

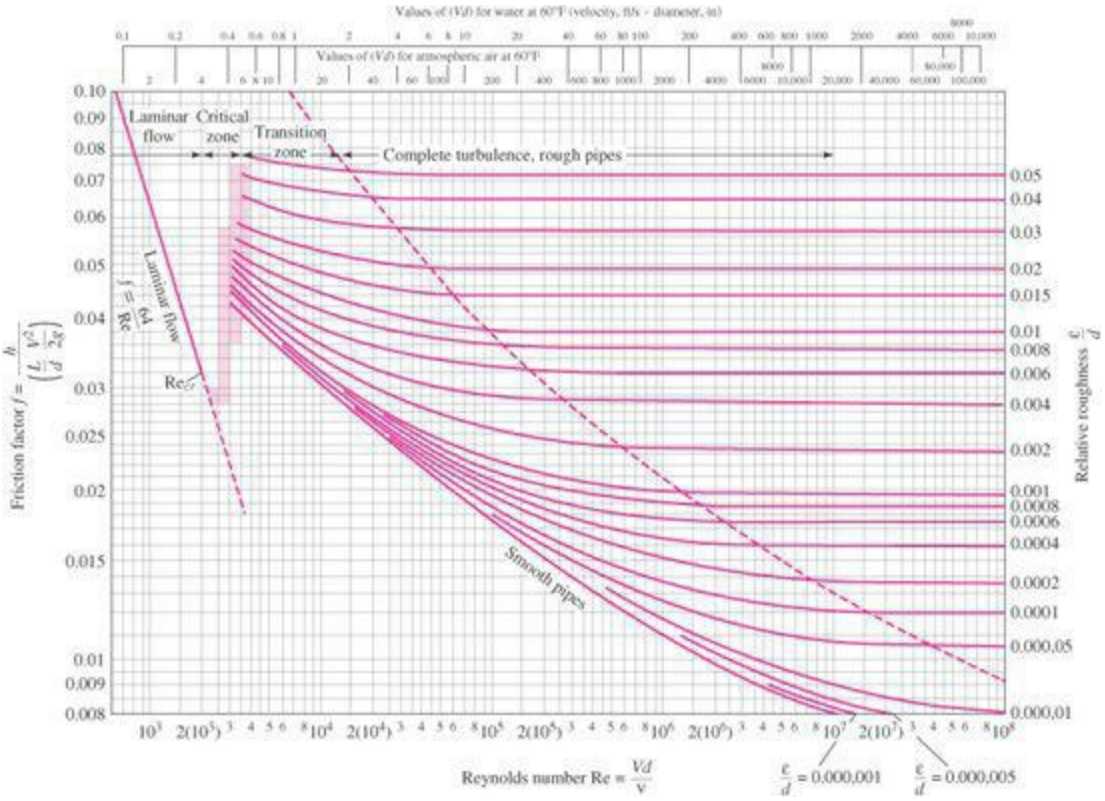
7KH0RRG &KDUW

,Q WKFRYHUWKHWDQVLWLRQDO URXJKUDQJH &RROEURRN> @FRPELQHG WKH VPRRWKZDOO>(T @DQGIXOO URXJK>(T @UHDUUDQJHGLQWRDFRPYHULQLHQWIRUPXOD

$$\frac{1}{f^{1/2}} = -2.0 \log\left(\frac{\epsilon/d}{3.7} + \frac{2.51}{\mathrm{Re}_d f^{1/2}}\right) \qquad (6.48)$$

7KLVLVWKHDFFHSWHGGHVLJQIRUPXODIRUWXUEXOHQWXUEXOHQW ,WZDVSURSRVHGLQ E\ 0RRG> LQRWZKDWLVQRZ FDOOHGWKH0RRG FKDUWRUSLSHIULFWLRQ) LJ 7KH0RRG FKDUWLVSUREDEO WKH PRVWXVHGGUDZLQJ DQG XVHXOILJXUH LQ IOXLG PHFKDQLFV ,WLV DFFXUDWH WR " SHUFHQWIRU GHVLJ

FDOFXODWRQVRYHUWKHIXOOUDQJHVKRZQLQ) LJ ,WFDQEHXVHGIRUFLUFXODUDQGQRQFLUFXODU 6HF
SLSHIORZVDQGIRUURSHQFKDQQHOIORZV &KDS 7KHGDWDFDQHYHQEHDGDSWHGDVD
DSSUR[LPDWLRQVRERXQGDU\OD\HUIORZV &KDS



) LJ 7KH 0 RRG\ FKDUWIRU SLSH IULFWRQ ZLWK VP RRWK DQG URXJK ZDOOV 7KLV FKDUWL
LGHQWLFDOWR(T IRUWXUEXOHQWIORZ *(From Ref. 8, by permission of the ASME.)*

(TXDWLRQ LVFXPEHUVRPHWRHYDOXDWHIRUI LI 5H LVNQRZQ DOWKRXJKLWHDVLO\ \LHOGVWRWK
((6 (TXDWLRQ6RLVYHU\$QDOWHUQDWLYHH[SOLFLWIRUPXODJLYHQE\ +DDDDG> @DV

$$\frac{1}{f^{1/2}} \approx -1.8 \log\left[\frac{6.9}{\mathrm{Re}_d} + \left(\frac{\epsilon/d}{3.7}\right)^{1.11}\right] \qquad (6.49)$$

YDULHVOHVVWKDQ SHUFHQWURP (T
7KH VKDGHG DUHD LQ WKH 0 RRG\ FKDUWLQGLFDWHV WKH UDQJH ZKHUH WDQJLWLRQ IORZ LQDU V
VXUEXOHQWIORZ RFFXUV 7KHUHDUHQR UHOLDEOHIULFWLRQIDFWRUVLQWKLVUDQJH 5H 1RWH
WKDWWKHURXJKQHVVFXUYHVDUHQHDUO\ KRUL]RQWDOLQWKHIXOO URXJKUHJLPHVRWKHGDVKHGOLQH
)URP WWWZLWKFRPPHUFLDOSLSHV UHFRPPHQGHGYDOXHVIRUDYHUDJHSLSHURXJKQHVDUHOLVWHG
7DEOH

7DEOH 5HFRPPHQGHG5RXJKQHVV9DOXHVIRU&RPPHUFLDO XFW

Material	Condition	ε ft	ε mm	Uncertainty, %
Steel	Sheet metal, new	0.00016	0.05	±60
	Stainless, new	0.000007	0.002	±50
	Commercial, new	0.00015	0.046	±30
	Riveted	0.01	3.0	±70
	Rusted	0.007	2.0	±50
Iron	Cast, new	0.00085	0.26	±50
	Wrought, new	0.00015	0.046	±20
	Galvanized, new	0.0005	0.15	±40
	Asphalted cast	0.0004	0.12	±50
Brass	Drawn, new	0.000007	0.002	±50
Plastic	Drawn tubing	0.000005	0.0015	±60
Glass	—	Smooth	Smooth	
Concrete	Smoothed	0.00013	0.04	±60
	Rough	0.007	2.0	±50
Rubber	Smoothed	0.000033	0.01	±60
Wood	Stave	0.0016	0.5	±40

([DP SOH

&RPSXWH WKH RWV RI KHDG DQG SUHVVXUH GURS LQ IWRI KRU]RQWDO LQGLDP HWHU DVSKDOWHG FDVW LURQ SLSH FDUU\LQJ Z DWHU Z LWK DP HDQ YHORFLW\ RI IWV

6ROXWLRQ

- 6\VWHP VNHWFK 6HH) LJ IRU DKRU]RQWDO SLSH Z LWK û] DQG K¡ SURSHUWLHV DQG ûS
- $VVXPSWLRQV 7XUEXOHQW IORZ DVSKDOWHG KRU]RQWDO FDVW LURQ SLSH G IW/ IW
- $SSURDFK) LQG 5H G DQG ε G HQWHU 0 RRG\ FKDUW) LJ ILQG I WKHQ K¡ DQG ûS
- 3URSHUWLHV) URP 7DEOH$ IRU Z DWHU FRQYHUWQJ WR %* XQLWV!
  VOXJ IW (VOXJ IWV
- 6ROXWLRQ VWHS &DOFXODWH 5H G DQG WKH URXJKQHVV UDWLR $ VDFUKWK 0 RRG\ SURYLGHG Z DW DQG DLUYDOXHV RI ³9G DWWKH WRS RI) LJ WR ILQG 5H G 1R OHWV FDOFXODWH LW WR OHDUQ WKH SURFHGXUH

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{(1.94\ \text{slug/ft}^3)(6\ \text{ft/s})(0.5\ \text{ft})}{2.09\text{E-}5\ \text{slug/(ft} \cdot \text{s)}} \approx 279{,}000 \qquad \text{(turbulent)}$$

) URP 7DEOH IRU DVSKDOWHG FDVW LURQ ε IW7KHQ FDOFXODWH

ε G IW IW

- 6ROXWLRQ VWHS) LQG WKH IULFWLRQ IDFWRU IURP WKH 0 RRG\ FKDUW RU IURP (T ,I \RX XVH WKH 0 RRG\ FKDUW) LJ \RX QHHG SUDFWLFH) LQG WKH DQHRQ WKH ULJKW VLGH IRU ε G

DQG IRO RZ LWEDFN WR WKH OHWXQ LQ LWKLWWKH YHUWLFDO OLQH IRU 5H § (5HDG DSSURSULDWH

I § >RUFRPSXWH IURP (T SHUKDSV XVLQJ ((6@

- 6RO WR QWS &DOFXODWH IURP (T DQG IURP (T IRU DKRUL]RQWDO SLSH

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.02)\left(\frac{200\text{ ft}}{0.5\text{ ft}}\right)\frac{(6\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} \approx 4.5\text{ ft} \qquad \textit{Ans.}$$

$$\Delta p = \rho g h_f = (1.94\text{ slug/ft}^3)(32.2\text{ ft/s}^2)(4.5\text{ ft}) \approx 280\text{ lbf/ft}^2 \qquad \textit{Ans.}$$

- &RPPHQWV ,QJLYLQJ WKLV H[DPSOH 0 RRG > @XDWHG WKDWWKLV HWWP DWH HY-QIRUFHDQQHZ SLSH FDQEHFRQVLGHUHGDFFXUDWHRQO WR DERXW SHUFHQW

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2 LO Z LWK! NJ P DQG P V IORZ V DW P V WKURXJK P R PP GLDP HWHU F DWLRQ SLSH ' HWHUP LQH D WKH KHDG ORVV DQG E WKH SUHVVXUH GURS LI WKH SLSH VORSH GRZ QDW f LQ WKH IORZ GLUHFWLRQ

6RO XWLRQ

) LUVW FRP SXWH WKH YHORFLW IURP WKH NQRZ QIORZ UDWH

$$V = \frac{Q}{\pi R^2} = \frac{0.2\text{ m}^3/\text{s}}{\pi(0.1\text{ m})^2} = 6.4\text{ m/s}$$

7KHQ WKH 5H QROGV QXPEHUV

$$\text{Re}_d = \frac{Vd}{\nu} = \frac{(6.4\text{ m/s})(0.2\text{ m})}{0.00001\text{ m}^2/\text{s}} = 128{,}000$$

) URP 7DEOH ∈ PP IRU FDVWLURQ SLSH 7KHQ

$$\frac{\epsilon}{d} = \frac{0.26\text{ mm}}{200\text{ mm}} = 0.0013$$

(QWHU WKH 0 RRG FKDUWRQ WKH ULJKW DW G \RX Z LOO KDY H WR LQWHUSRODWH DQG PRYH WKH OHWWR LQWHUVHFW Z LWK 5H 5HDG I § >IURP (T IRU WKHVH YDOXHV Z H FRXOG FRP SXWH I @7KHQ WKH KHDG ORVV IS

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.0225)\frac{500\text{ m}}{0.2\text{ m}}\frac{(6.4\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 117\text{ m} \qquad \textit{Ans. (a)}$$

) URP (T IRUWKHLQFOLQHGSLSH

$$h_f = \frac{\Delta p}{\rho g} + z_1 - z_2 = \frac{\Delta p}{\rho g} + L\sin 10°$$

$$\text{or} \quad \Delta p = \rho g[h_f - (500\text{ m})\sin 10°] = \rho g(117\text{ m} - 87\text{ m})$$

$$= (900\text{ kg/m}^3)(9.81\text{ m/s}^2)(30\text{ m}) = 265{,}000\text{ kg/(m}\cdot\text{s}^2) = 265{,}000\text{ Pa} \qquad \textit{Ans. (b)}$$

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5 HSHDW ([DPSOH VRVHHWKHWUHLVDQ SRVLEOHVXEVWDQWLDO VRXUFHRI IRUDVPRRWK ZDOOHGSLSH

6RXUVRQ

,Q ([DPSOH ZHHVWLPDWHGDKHDGORVVK § IWDVXPLQJ ODPLQDUH[LWORZ . §) R VKLVFRQGLWLRQWKHIULFWLRQIDFWRULV

$$f = h_f\frac{d}{L}\frac{2g}{V^2} = (1.66\text{ ft})\frac{(0.004\text{ ft})(2)(32.2\text{ ft/s}^2)}{(1.0\text{ ft})(3.32\text{ ft/s})^2} \approx 0.0388$$

) RUODPLQDUIORZ 5H G I § DVZHVKRZHGLQ ([DPSOH +RZHYHUIURP WKH0RRG FKDUW) LJ ZHVHHWKDW DOORFFXUHVSRQGVVR DVXEVWDQWLPRRWKZDO FRQGLWLRQ DW5H G § ,I WKHIORZ DFWXDOO ZHUHVXEVWDQWLDOWZHVKRXOGFKDQJHRXUQLQHWHHQWU IDFWRUVR. § >(T @ZKHQHWHVWHFRUUHFWHGK § IWDQGI § : LWKI NQRZQ ZI FDQHVWLPDWWKH5H QROGVQXPEHUIURP RXUIRUPXODV

$$\text{Re}_d \approx 3250 \quad [\text{Eq. (6.38)}] \qquad \text{or} \qquad \text{Re}_d \approx 3400 \quad [\text{Eq. (6.39}b\text{)}]$$

6RWKHIORZ PLJKWKDYHEHHQWXUEXOHQWLQZKLFKFDVHWKHYLVFRVLWRI WKHIOXLGZRXOGKDYHEHHQ

$$\mu = \frac{\rho V d}{\text{Re}_d} = \frac{1.80(3.32)(0.004)}{3300} = 7.2\times 10^{-6}\text{ slug/(ft}\cdot\text{s)} \qquad \textit{Ans.}$$

7KLVLVDERXW SHUFHQWGWVWKDQRXUODPLQDUHVWLPDWLQ ([DPSOH 7KHPRUDOLVWKDW FDSLOODU IORZ 5H QROGVQXPEHUEHORZ DERXW WKDYRLGVXFKGXSOLFDWLVRQWUROV

: HKDYHFRQYHUJHGVRWKHVLJQLILFDQWLJXUHV 7KXVRXUDQVZHURQLV

$$V = 4.84 \text{ m/s}$$

$$Q = V\left(\frac{\pi}{4}\right)d^2 = (4.84)\left(\frac{\pi}{4}\right)(0.3)^2 \approx 0.342 \text{ m}^3/\text{s} \qquad \textit{Ans.}$$

7KHLWHUDWLYHDSSURDFKLVVWDLJKWRUZDUGDQGQRWRRRQHURXV VRLWVURXWLQHO XVHGE\ HQJLQHHU
2EYLRXVO\WKLVUHSHWLWLYHSURFHGXUHLVLGHDOIRUDSHUVRQDOFRPSXWHU

(QJLQHHULQJ (TXDWLRQ6RONHU ((6 6RONXWLRQ

EES ,Q((6 RQHVLPSOHQHWV WKHGDWDDQG WKHDSSURSULDWHTXDWLRQ/ HWWLQJ WKHVRIWZDUHGR WK
UHWW&RUHFWXQLWP XWR FRXUHEHXVHG)RUWKHSUHVHQWSURMHFW WKHGDWDFRXOGEHHQWHGDV6,
rho=950 nu=2E-5 d=0.3 L=100 epsod=0.0002 hf=8.0 g=9.81
7KHDSSURSULDWHTXDWLRQ/DUHWKH0 RRG\ IRUPXOD SOXVWKHGHILQLWLRQ/RI 5H\QROGVQXPEH
YROXPHIORZ UDWHDVGHWHUPLQHGIURP YHORFLW DQGWKH' DUF\ KHDGORVVIRUPXOD

```
Re=V*d/nu
Q=V*pi*d^2/4
f=(-2.0*log10(epsod/3.7+2.51/Re/f^0.5))^(-2)
hf=f*L/d*V^2/2/g
```

((6 XQGHUVWDQGVWKDW7SL´ UHSUHVHQWV 7KHQKLW762/9(´ IURP WKHPHQX ,I HURUVKDV
EHHQ HQWHUHG ((6 ZLOOFRPSODLQ WKDWWKH VXVWHP FDQQRWEH VROYHG DQG DWWHPSWRV H[SODLQ ZK
2WKHUZLVH WKHVRIWZDUHZLOOLWHUDWH DQGLQWKLVFDVH((6 SULQWVWKHFRUUHFWVROXWLRQ

```
Q=0.342 V=4.84 f=0.0201 Re=72585
```

7KHXQLWVDUHVHOHGRXWQD VHSDUDWHOLWWOHV>P NJ V 1@7KLVHOHJDQWSURFKVRHQJLQHULQ
SUREOP VROLQJ KDVRQHGUZEDFN? QPHO WKDWWKHXVHUIDLOVWRFKHFNWKHVROXWLRQIRUHQJLQHULQ
YLDELOLW\ )RU[PSOH DUHWKHGDW\WSHGFRUUHFW" ,VWKH5H\QROGVQXPEHUXEXHQW

7\SH 3UREOP) LQG WKH3LSH' LDP HWU

7KH0 RRG\ FKDUWLV HSHFLDOO DZNZDUGIRUILQGLQJ WKHSLSH V]H VLQFHGRFFXUVLQDOOWKUH
SDUDPHWUV I 5H DQG G)XUWKHUPRUHGSHQGVRQZKHWKHUZHNQRZ WKHYHORFLWRUWKHIORZ UDWH: H
FDQQRWNQRZ ERWK RUHOHZHFRXOGLPPHGLDWHO FRPSXWH $d = \sqrt{4Q/(\pi V)}$
/ HWXVDVXPHWKDWZHNQRZ WKHIORZ UDWH4 1RWWKDWWKLV UHTXLUHV XVRUHGHILQHWKH5H\QROG
QXPEHULQWUVRI 4

$$\mathrm{Re}_d = \frac{Vd}{\nu} = \frac{4Q}{\pi d\nu} \qquad (6.52)$$

7KHQ LI ZH FKRRVH 4 ! DV VFDOQJ SDUDPHWHUV VR HOLPLQDWH ZH REWDLQ WKH IXQFWLRQDO UHODWLRQVKLS

$$\mathrm{Re}_d = \frac{4Q}{\pi d\nu} = \mathrm{fcn}\left(\frac{gh_f Q^3}{L\nu^5}, \frac{\epsilon\nu}{Q}\right)$$

DQG FDQ WKXV VROYHG ZKHQ WKH UJKWKDQG VLGH LV NQRZQ 8QIRUWXQDWHO WKH ZULWHU NQRZV RI QI IRUPXODV IRU WKLV UHODWLRQ +HUH LWV WHPV UHDVRQDEOH WR IRUP D SRWHQWLDOO\ IXQFWLRQDO IRUP XOD DQG VLPSOI VHWXS WKH SUREOHP DV DQLWLDWLRQ QLQWHUP VRI WKH0RRG FKDUWYDULDEOHV ,QWKLV FDVH ZH DOV KDYH WR VHWXS WKH IUFWLRQ IDFWRU QI WHUP VRI WKH IORZ UDWH

$$f = h_f \frac{d}{L}\frac{2g}{V^2} = \frac{\pi^2}{8}\frac{gh_f d^5}{LQ^2} \qquad (6.53)$$

7KHIRQRZLQJ VWHS H[DPSOHV LOOXVWUDWH WKH LWHUDWLRQ

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: RUN([DPSOH EDFNZDUG DVXPLQJ WKDW4 P VDQG= PP DUHNQRZ QEXWWKDW FP LVXQNQRZ Q 5HFDOO P ! NJ P (P V DQGK P

,WHUDWLYH6RXWLRQ

) LUVWZ UWKHGLP HWULQWP VRI WKHIUFWRQIDFWRU

$$f = \frac{\pi^2}{8}\frac{(9.81\ \mathrm{m/s^2})(8\ \mathrm{m})d^5}{(100\ \mathrm{m})(0.342\ \mathrm{m^3/s})^2} = 8.28d^5 \quad \text{or} \quad d \approx 0.655f^{1/5} \qquad (1)$$

LQ6, XQLW $OR ZUWKH5H\QROGVQXPEHUDQGURXJKQHVUDWLQWP VRI WKHGLP HWU

$$\mathrm{Re}_d = \frac{4(0.342\ \mathrm{m^3/s})}{\pi(2\ \mathrm{E\text{-}5\ m^2/s})d} = \frac{21{,}800}{d} \qquad (2)$$

$$\frac{\epsilon}{d} = \frac{6\ \mathrm{E\text{-}5\ m}}{d} \qquad (3)$$

* XHVVI FRP SXWHGIURP WKHQFRP SXWH5HGIURP DQG GIURP DQGFRP SXWHDEHWWU IURP WKH0RRG FKDUWRU(T 5HSHDWXQWO IDLO UDSLG FRQYHUJHQFH +DYLQJ QR LQWZ HWXP DWIRUI WKHZULWHUJXHVVHVI § DERXWQWKHPLGGOHRI WKHWUEXOHQWSRUWRQRI WKH0RRG FKDUW 7KHIRQRZLQJ FDOFXODWRQVUHVXOW

7\SH 3UREOHP) LQG WKH 3LSH/ HQJWK

,QGHWJQLQJ SLSLQJ V\VWHPV LWV GHVLJQHUV EHVW HWLPDWH WKH SSURSULDWH SLSH GLDPHWHU SXPS SRZHU DQG IORZ UDWH 7KHSXPS KHDG ZLOO DIIHFW WKH SLSLQJ KHDG ORVV ,I PLQRU ORVVHV DUH QHJOHFWHG WKH KRUL]RQWDO SLSHOHQJWK IROORZV IURP ' DUF\¶V IRUPXOD

$ SXPS GHOLYHUV KS WR ZDWHU DW f) IORZLQJ LQ D LQGLPHWUDSKDOHG FDVWLURQ KRUL]RQWDO SLSH D/ IW/ : KDWV WKH SURSHU SLSHOHQJWK WR PDWK WKHVHFRQGLWRQV"

6RXWLRQ

- $SSURDFK) LQGK IURP WKHNQRZQSRZHUDQG ILQGI IURP 5H DQG G 7KHQILQG/
- : DWHU SURSHUWLHV) RUZ DWHU DW f) 7DEOH$ FRQYHUWQJ WR %* XQLWV! VOXJ IV DQG (í VOXJ IW/ V
- 3LSHURXJKQHVV) URP 7DEOH IRUDSKDOHG FDVWLURQ IW
- 6RXWLRQVWHS) LQG WKHSXPS KHDG IURP WKHIORZ UDWHDQG WKHSXPS SRZHU

$$Q = AV = \frac{\pi}{4}(0.5\text{ ft})^2\left(6\frac{\text{ft}}{\text{s}}\right) = 1.18\frac{\text{ft}^3}{\text{s}}$$

$$h_{\text{pump}} = \frac{\text{Power}}{\rho g Q} = \frac{(0.6\text{ hp})[550(\text{ft}\cdot\text{lbf})/(\text{s}\cdot\text{hp})]}{(1.94\text{ slug/ft}^3)(32.2\text{ ft/s}^2)(1.18\text{ ft}^3/\text{s})} = 4.48\text{ ft}$$

- 6RONWRQVWHS &RPSXWHWKHIULFWLRQIDFWRUIURP WKH&RROEURRNIRUPXOD (T

$$\text{Re}_d = \frac{\rho V d}{\mu} = \frac{(1.94)(6)(0.5)}{2.09\text{ E}-5} = 278{,}500 \qquad \frac{\varepsilon}{d} = \frac{0.0004\text{ ft}}{0.5\text{ ft}} = 0.0008$$

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{\text{Re}_d\sqrt{f}}\right) \quad \text{yields} \quad f = 0.0198$$

- 6RONWRQVWHS) LQG WKHSLSHOHQJWKIURP WKH' DUF\ IRUPXOD

$$h_p = h_f = 4.48\text{ ft} = f\frac{L}{d}\frac{V^2}{2g} = (0.0198)\left(\frac{L}{0.5\text{ ft}}\right)\frac{(6\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}$$

Solve for $L \approx 203\text{ ft}$ *Ans.*

- &RPPHQWV 7KLV LV 0 RRG¶V SUREOHP ([DPSOH WKLQHG DURXQG VR WKDWWKH OHQJWK LV XQNQRZQ

) ORZ LQ 1 RQFLUFXODU ' XFWV

,I WKH GXFWLQJ RQFLUFXODU WKHDQDOVLVRI IXOO GHYHORSHGIORZ IRORZ VWKDWWR WKHFLUFXODUSLSHEX LVP RUHFRP SOLFDWHGDQGDJHEUDLFDO) RUOP LQDUIORZ RQHFDQVROYHWKH H[DFWHTXDWRQVRI FRQWQXW DQG PRP HQWXP) RUWXUEXOHQWIORZ WKHORJDULWKP ODZ YHORFLW SURILOHFDQ EH XVHG RU EHWWU DQ VLP SOHU WKHK\GUDXOF GLP HWULDQH[FHOHQWDSSUR[LP DWRQ

7KH+ \GUDXOF ' LDP HWU

) RUDQRQFLUFXODUGXFWWKHFRQWROYROXP HFRQFHSWRI) LJ LVXWLOLYDOG EXWWKHFURVV VHFWLRQD DUHD\$ GRHV QRWHTXDOC5 DQG WKH FURVV VHFWLRQDOSHULP HWUZ HWWG E\ WKH VKDUWWHW GRHV QR HTXDO C5 7KHP RP HQWXP HTXDWRQ D WKXVEHFRP HV

$$\Delta p\,A + \rho g A\,\Delta L\sin\phi - \bar{\tau}_w \mathcal{P}\,\Delta L = 0$$

or

$$h_f = \frac{\Delta p}{\rho g} + \Delta z = \frac{\bar{\tau}_w}{\rho g}\frac{\Delta L}{A/\mathcal{P}} \qquad (6.55)$$

7KLV LV LGHQWLFDOWR(T E H[FHSWWKDW WKH VKDUWWHW LV DQ DYHUDJH YDOXH LQWUDW DURXQG WKH SHULP HWUDQG WKH OHQJWK VFDOH\$ WDNHV WKH SODFH RI WKH SLSHUDGLXV5) RUWKLV UHDVR

DQRQFLUFXODUGXFWLVVDLGWRKDYHDK\GUDXOLFUDGLXV5$ GHILQHGE\

$$R_h = \frac{A}{\mathcal{P}} = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} \tag{6.56}$$

7KLV FRQFHSWUHFHLYHV FRQVLGHUDEOH DWWHQWLRQ LQ RSHQFKDQQHO IORZ &KDS ZKHUH WKH FKDQQHO FURVV VHFWLRQ LV DOPRVW QHYHU FLUFXODU ,I E\ FRPSDULVRQ WR(T IRU SLSH IORZ ZH GHILQH WKH IULFWLRQ IDFWRU LQ WHUPV RI DYHUDJH VKHDU

$$f_{\mathrm{NCD}} = \frac{8\bar{\tau}_w}{\rho V^2} \tag{6.57}$$

ZKHUH1&' VDQGVIRUQRQFLUFXODUGXFWDQG9 4 $ DVXVXDO(T EHFRPHV

$$h_f = f\frac{L}{4R_h}\frac{V^2}{2g} = f\frac{L}{D_h}\frac{V^2}{2g} \tag{6.58}$$

7KLV LV HTXLYDOHQWWR(T IRU SLSH IORZ H[FHSWWKDWG LV UHSODFHG E\ 5$ 7KHUHRUH ZH FXWRP DULO GHILQHWKH K\GUDXOLF GLDPHWUDV

$$D_h = \frac{4A}{\mathcal{P}} = \frac{4 \times \text{area}}{\text{wetted perimeter}} = 4R_h \tag{6.59}$$

: HVKRXOGVWUHVVWKDWWKHZHWWHG SHULPHWULQFOXGHV DOOVXUIDFHV DFWG XSRQE\ WKH VKHDUVWUHVV) RU[DPSOH LQDFLUFXODUDQQXOXV ERWKWKHRXWHUDQGWKHLQQHUSHULPHWUVKRXOGEHDGGHG 7KHIDFWKDWW$ HTXDOV 5$ LVMXVWRQHRI WKRVHWKLQJV &KDOLWXS WRDQHQJLQHHUV KXPRU 1RWWKDW IRUWKHGHUHQHUDWLRQDOFDVHRI DFLUFXODUSLSH '$ G5 G5 5 DVH[SHFWHG

: HZRXOGWKHUHRUHH[SHFWE\ GLPHQVLRQDODQDOVLVWKDWWKLVIULFWLRQIDFWRUI EDVHGRQK\GUDXOL GLDPHWUDV LQ(T ZRXOGFRUUHODWHZLWKWKH5H\QROGVQXPEHUDQGURXJKQHVVUDWLREDVHGRQ WKHK\GUDXOLF GLDPHWU

$$f = F\left(\frac{VD_h}{\nu}, \frac{\epsilon}{D_h}\right) \tag{6.60}$$

DQGWKLVLVWKHZD\ WKHGDWDUHFRUHODWHG %XWZHVKRXOGQRWQHHGDULO H[SHFWWKH0 RRG\ FKDU) LJ WR KROG H[DFWO LQWHUPV RI WKLV QHZ GHQVLW\ FDOH $QG LWGRHVQRWEXWLWV VXSULQJIC DFFXUDWH

$$f \approx \begin{cases} \dfrac{64}{\mathrm{Re}_{D_h}} & \pm 40\% \quad \text{laminar flow} \\ f_{\mathrm{Moody}}\left(\mathrm{Re}_{D_h}, \dfrac{\epsilon}{D_h}\right) & \pm 15\% \quad \text{turbulent flow} \end{cases} \tag{6.61}$$

1RZ GHWXVGRRNDWWRPHSDUWFDVHV

) ORZ EHWZHHQ 3DUDOOHO 3ODWHV

3URFHHG WKH VLPSOHVW QRQFLUFXODU GXFW ORZ LV IXOO GHYHORSHG IORZ EHWZHHQ SDUDOOHO SODWHV I GLVWDQFH K DSDUWDV LQ) LJ $VVXPH GLQWKH ILJXUH WKH ZLGWK E≫ K VR WKH IORZ LV HVVHQWLDOO VWR GLPHQVLRQDO WKDW X X\ RQO 7KHK\GUDXOLF GLDPHWHU LV

$$D_h = \frac{4A}{\mathcal{P}} = \lim_{b\to\infty} \frac{4(2bh)}{2b + 4h} = 4h \qquad (6.62)$$

WKDW VZLFH WKH GLVWDQFH EHWZHHQ WKH SODWHV 7KH SUHVVXUH JUDGLHQW VFRQWDQW íGS G[ûS/ ZKHUH/ LV WKH OHQJWK R WKH FKDQQHO DORQJ WKH[D[LV

/ DP LQDU) ORZ 6RXWLRQ

7KH ODP LQDU VROXWLRQ ZDV JLYHQ LQ6HFW LQ FRQQHFWLRQ ZLWK) LJ E / HWXV UHYLHZ WKRV UHVXOWV KHUH

$$u = u_{\max}\left(1 - \frac{y^2}{h^2}\right) \quad \text{where} \quad u_{\max} = \frac{h^2}{2\mu}\frac{\Delta p}{L}$$

$$Q = \frac{2bh^3}{3\mu}\frac{\Delta p}{L}$$

$$V = \frac{Q}{A} = \frac{h^2}{3\mu}\frac{\Delta p}{L} = \frac{2}{3}u_{\max} \qquad (6.63)$$

$$\tau_w = \mu\left|\frac{du}{dy}\right|_{y=h} = h\frac{\Delta p}{L} = \frac{3\mu V}{h}$$

$$h_f = \frac{\Delta p}{\rho g} = \frac{3\mu LV}{\rho g h^2}$$

1RZ XVH WKH KHDG ORVV WR HYDOXDWH WKH ODP LQDU IULFWLRQ IDFWRU

$$f_{\text{lam}} = \frac{h_f}{(L/D_h)(V^2/2g)} = \frac{96\mu}{\rho V(4h)} = \frac{96}{\text{Re}_{D_h}} \qquad (6.64)$$

7KXV LI ZH FRXOG QRW ZRUN RXW WKH ODP LQDU WKHRU DQG FKRVH WR XVH WKH DSSUR[LPDWH) LJ DV(T VWDWHV

-XVW DV LQ FLUFXODU SLSH ORZ WKH ODP LQDU VROXWLRQ DERYH EHFRPHV XQXVDEOH DW DERXW5H WKDQVLWLRQ RFFXUV DQG WXUEXOHQW ORZ UHVXOW

7XUEXOHQW) ORZ 6RXWLRQ

) RU WXUEXOHQW ORZ EHWZHHQ SDUDOOHO SODWHV ZH FDQ DJDLQ XVH WKH ORJDULWKP ODZ (T DV DSSUR[LPDWLRQ DFURVV WKH HQWLUH FKDQQHO XVLQJ QRW EXW DZDOOFRRUGLQDWH< DV VKRZQ LQ) LJ

$$\frac{u(Y)}{u^*} \approx \frac{1}{\kappa}\ln\frac{Yu^*}{\nu} + B \qquad 0 < Y < h \tag{6.65}$$

7KLV GLVWULEXWLRQ RRNV YHU PXFK OLNH WKH IDPLWLDU EXON SURILOH IRU SLSH IORZ LQ) LJ E DQ WKH PHDQ YHORFLWLV

$$V = \frac{1}{h}\int_0^h u\,dY = u^*\left(\frac{1}{\kappa}\ln\frac{hu^*}{\nu} + B - \frac{1}{\kappa}\right) \tag{6.66}$$

5HFDOOLQJ WKDW9 X I Z HVHHWKDW(T LVHTXLYDOHQWWR DSDUDPHWULFIUDFWLRQDO 5HDUUDQJLQJ DQGFDOLQJXSWKHFRQVWDQWWUPV Z HREWLQ

$$\frac{1}{f^{1/2}} \approx 2.0\log\left(\mathrm{Re}_{D_h} f^{1/2}\right) - 1.19 \tag{6.67}$$

Z KHUH Z H KDYH LQWRGXFHG WKH K\GUDXOLF GLDPHWHU' K K 7KLV LV UHPDUNDEOH FORVH WR WK VPRRWK Z DOO SLSH IUFWRQ ODZ (T 7KHUHRUH Z H FRQFXGH WKDWWKH XVH RI WKH K\GUDXOL GLDPHWHU LQ WKLV VXEVWLWXWLRQ LV TXLWH VXFFHVVIXO 7KDWXLQV RXWR EH WKH IRUPRQFLFXOD VXEVWLWXWRZ VDOR

(TXDWLRQ FDQEHEURXJKWLQWR H[DFWDJUHHPHQWZLWKWKHSLSHODZ E\ UHZULWLQJ LWLQWKHIRUP

$$\frac{1}{f^{1/2}} = 2.0\log\left(0.64\,\mathrm{Re}_{D_h} f^{1/2}\right) - 0.8 \tag{6.68}$$

7KXV WKH WXEXOHQWUFWLRQ LV SUHGLFWHG PRVWDFFXUDWO Z KHQ Z H XVH DQ HIIHFWLYH GLDPHWHU' H HTXDOWR WPHV WKH K\GUDXOF GLDPHWHU 7KH HIIHFWRQ I LWH0 LV PXFK OHVV DERXW SHUFHQWO PRWW: HFDQFRPSDUHZ LWK(T IRUODPLQDUIORZ Z KLFK SUHGLFWHG

$$\text{Parallel plates:} \qquad D_{\text{eff}} = \frac{64}{96}D_h = \frac{2}{3}D_h \tag{6.69}$$

7KLVFORVHUHPEODQFH ' K YHUXV ' K RFFXUVRRIWQLQRQFLUFXODUGXFWORZ WKDWZI VDNHLWWRE HDJHQHUDOXOHIRUFRPSXWQJ WXEXOHQWUFWLRQLQGXFW

$$D_{\text{eff}} = D_h = \frac{4A}{\mathcal{P}} \quad \text{reasonable accuracy}$$

$$D_{\text{eff}} = D_h\frac{64}{(f\,\mathrm{Re}_{D_h})_{\text{laminar theory}}} \quad \text{better accuracy} \tag{6.70}$$

-RQHV> @KRZ VWDWWKHIIFWLYH DPLQDUGLDPHWULGHDFRODSVHVDOOGDWIRUUHFWDQJXODUGXFW RI DUEWUDU KHJKWRZ LGWKUDWR RQWKH0 RRG\ FKDUWRUSLSHIORZ : HUHFRPPHQGWKLVIGHDIRUDQ QRQFLUFXODUGXFW

(; $0 3/ (

) OXLGIORZ VDWQDYHUDJHYHORFLWRI IWVEHWZHHQRU]RQWDOSDUDOOHDSODWVDGLVWDQFHRI L DSDUW) LQGWKHKHDGORVVDQGSUHVVXUHGURSIRUHDFK IWRI OHQJWKIRU VOXJVIWDQG D

IWVDQG E IWV $VXPHVPRRWKZDOOV

6ROXWLRQ

3DUWD

7KHYLVFRVLW\ ! î í VOXJ IWV 7KHVSDFLQJ LV K LQ IWDQG'K K IW7KH5HQROGVQXPEHULV

$$\mathrm{Re}_{D_h} = \frac{VD_h}{\nu} = \frac{(6.0\ \text{ft/s})(0.4\ \text{ft})}{0.00002\ \text{ft}^2/\text{s}} = 120{,}000$$

7KHIRZ LVWKHUHRUHWXUEXOHQW)RUUHDVRQDEOHDFFXUDF\ VLPSOH RRN RQWKH0RRG\ FKDUW) LJ IRUVPRRWKZDOOV

$$f \approx 0.0173 \quad h_f \approx f\frac{L}{D_h}\frac{V^2}{2g} = 0.0173\,\frac{100}{0.4}\,\frac{(6.0)^2}{2(32.2)} \approx 2.42\ \text{ft} \qquad \textit{Ans. (a)}$$

6LQFHWKHUHLVQR FKDQJHLQHOHYDWLRQ

$$\Delta p = \rho g h_f = 1.9(32.2)(2.42) = 148\ \text{lbf/ft}^2 \qquad \textit{Ans. (a)}$$

7KLVLVWKHKHDGORVVDQGSUHVVXUHGURSSHU IWR FKDQQHO)RUPRUHDFFXUDF\ VWN-$D_{\text{eff}} = \frac{64}{96}D_h$ IURP ODPLQDUWKHRU VKHQ

$$\mathrm{Re}_{\text{eff}} = \tfrac{64}{96}(120{,}000) = 80{,}000$$

DQGIURP VKH0RRG\ FKDUWHDGI § IRUVPRRWKZDOOV 7KXVDEHWWHUHVWLPDWLVV

$$h_f = 0.0189\,\frac{100}{0.4}\,\frac{(6.0)^2}{2(32.2)} = 2.64\ \text{ft}$$

and $$\Delta p = 1.9(32.2)(2.64) = 161\ \text{lbf/ft}^2 \qquad \textit{Better ans. (a)}$$

7KHPRUHDFFXUDWHIRUPXODSUHGLFWVIULFWLRQDERXW SHUFHQWKLJKHU

3DUWE

&RPSXWH ! VOXJ IWV 7KH5HQROGVQXPEHUV VKHUHRU VKHIORZ LVODPLQDU VLQFH5HLVOHVVWKDQ

<RXFRXOGXVHWKHODPLQDUIORZ IULFWLRQIDFWRU(T

$$f_{\text{lam}} = \frac{96}{\text{Re}_{D_s}} = \frac{96}{1200} = 0.08$$

from which $$h_f = 0.08 \frac{100}{0.4} \frac{(6.0)^2}{2(32.2)} = 11.2 \text{ ft}$$

and $$\Delta p = 1.9(32.2)(11.2) = 684 \text{ lbf/ft}^2 \qquad \textit{Ans. (b)}$$

$ OWLQDWH \RXFDQILQHWXHWKH5HQROGVQXPEHUDQGJRGLUHFWVWRWKHDSSURSULDWHOLQHUHVL
IRUPXODH T

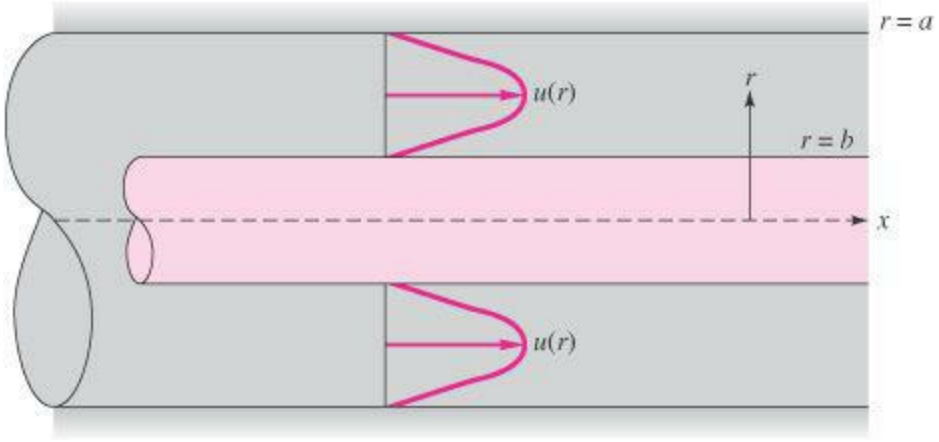
$$V = \frac{h^2}{3\mu} \frac{\Delta p}{L}$$

or $$\Delta p = \frac{3(6.0 \text{ ft/s})[0.0038 \text{ slug/(ft} \cdot \text{s)}](100 \text{ ft})}{(0.1 \text{ ft})^2} = 684 \text{ slugs/(ft} \cdot \text{s}^2) = 684 \text{ lbf/ft}^2$$

and $$h_f = \frac{\Delta p}{\rho g} = \frac{684}{1.9(32.2)} = 11.2 \text{ ft}$$

7KLVLVRQHRIWKRVH SHUKDSVXQH[SHFWHG SUREOPVZKHUHWKHOPLQDUIUFWRQLVJUHDWUWKDQWK
WXUEXOHQWUFWRQ

) ORZ WKURXJK D &RQFHQWULF $QQXOXV



) LJ) XOO GHYHORSHG IORZ WKURXJK D FRQFHQWULF DQQXOXV

&RQVLGHUWWDGG D[LDOOOPLQDUIORZ LQWKHDQQXODUVSDFHEHWZHHQWZR FRQFHQWULF F\OLQGHUV DV L
) LJ 7KHUH LV QR VOS DWWKH LQQHU U E DQG RXWUDGLXV U D) RUX X U RQO WK
JRYHUQLQJ UHDWLRQV(T '

$$\frac{d}{dr}\left(r\mu \frac{du}{dr}\right) = Kr \qquad K = \frac{d}{dx}(p + \rho gz) \qquad (6.71)$$

,QWUDWWKLVWZLFH

$$u = \frac{1}{4}r^2\frac{K}{\mu} + C_1 \ln r + C_2$$

7KHFRQVWDQWVDUHIRXQGIURP WKHVWR QR VOLS FRQGLWRQV

$$u(r = a) = 0 = \frac{1}{4}a^2\frac{K}{\mu} + C_1 \ln a + C_2$$

$$u(r = b) = 0 = \frac{1}{4}b^2\frac{K}{\mu} + C_1 \ln b + C_2$$

7KHILQDOVROXWRQIRUWKHYHORFLWSURILOHLV

$$u = \frac{1}{4\mu}\left[-\frac{d}{dx}(p + \rho gz)\right]\left[a^2 - r^2 + \frac{a^2 - b^2}{\ln (b/a)}\ln\frac{a}{r}\right] \qquad (6.72)$$

7KHYROXPHIORZ LVJLYHQE\

$$Q = \int_b^a u\, 2\pi r\, dr = \frac{\pi}{8\mu}\left[-\frac{d}{dx}(p + \rho gz)\right]\left[a^4 - b^4 - \frac{(a^2 - b^2)^2}{\ln (a/b)}\right] \qquad (6.73)$$

7KH YHORFLW SURILOHX U UHVHPEOHV D SDUDEROD ZUDSSHG DURXQG LQ D FLUFOH WR IRUP D VSO
GRXJKQXWDVLQ) LJ

,WLVFRQXVLQJ WRHDVHWKHIULFWLRQIDFWRUURQWKHZDOOVKHUHDUHWZR VKHDUWHVWVHV
WKHILQDOWHVWVEHLQJ JUHDWUWKDQWKHRXWU,WLVEHWWUWR GHILQHI ZLWKUHVSHFWWR WKHKHDGORVV DVL
(T

$$f = h_f\frac{D_h}{L}\frac{2g}{V^2} \qquad \text{where } V = \frac{Q}{\pi(a^2 - b^2)} \qquad (6.74)$$

7KHK\GUDXOLF GLDPHWUIRUDQDQQXOXVLV

$$D_h = \frac{4\pi(a^2 - b^2)}{2\pi(a + b)} = 2(a - b) \qquad (6.75)$$

,WLVVWLFHWKHFRUDQFH UDWKHUQNHWKHSDUDOOHOSODWUHVXOWRI VWLFHWKHGLVWDQFHEHWHQSODW
>(T @

6XEVWLWXWLQJ K_f ' _K DQG9 LQR(T ZHILQGWKDWWKHIULFWRQIDFWRUIRUDPLQDUIORZ LQI
FRQFHQWULF DQQXOXVLVRI WKHIRUP

$$f = \frac{64\zeta}{\mathrm{Re}_{D_h}} \qquad \zeta = \frac{(a - b)^2(a^2 - b^2)}{a^4 - b^4 - (a^2 - b^2)^2/\ln (a/b)} \qquad (6.76)$$

7KHGLPHQVLRQOHVV WHUP LV D VRUWRI FRUUHFWRQ IDFWRUIRUWKH K\GUDXOLF GLDPHWU: H FRXO
UHZ ULWH(T DV

Concentric annulus: $f = \frac{64}{\mathrm{Re}_{\mathrm{eff}}}$ $\mathrm{Re}_{\mathrm{eff}} = \frac{1}{\zeta}\mathrm{Re}_{D_h}$ (6.77)

6RPHQXPHULFDOYDOXHVRI I 5H$_K$ DQG'$_{HI}$ ' $_K$ DUHJLYHQLQ7DEOH $JDLQ OPLQDUDQXOL IORZ EHFRPHVXQWDEOHDV6H$_K$ §

7DEOH / PLQDU) UILFWRQ) DFWRUVIRUD&RQFHQWULF \$QQXOXV

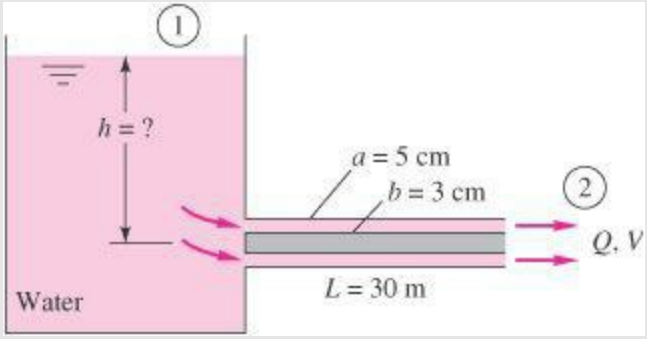
b/a	$f\,\mathrm{Re}_{D_h}$	$D_{\mathrm{eff}}/D_h = 1/\zeta$
0.0	64.0	1.000
0.00001	70.09	0.913
0.0001	71.78	0.892
0.001	74.68	0.857
0.01	80.11	0.799
0.05	86.27	0.742
0.1	89.37	0.716
0.2	92.35	0.693
0.4	94.71	0.676
0.6	95.59	0.670
0.8	95.92	0.667
1.0	96.0	0.667

) RUWXEXODUIORZ WKURXJKDFRQFHQWULF DQQXOXV WKHDQDO\VLVPLJKWSURFHHGE\ SDWKLQJ WRJHWKHU WZR ORJDULWKPLF OD\ SURILOHV RQHJRLQJ RXWURP WKHLQQHUZDOOWRPHHWKHRXWHUFRPLQJ IURP WKH RXWHUZDOO: HRPLWWXFKDVFKHPHKHUHDQGSURFHHGGLUHFWO WR WKHIULFWRQIDFWRU\$FFRUGLQJ WR WKH JHQHUDO UKHS SURSRVHG LQ(T WXEXODUIORZ LV SUHGLFWHG ZLWK H[FHOOHQWDFFXUDF\ E\ UHSODFLQJ G LQ WKH 0 RRG\ FKDUWE\ ' $_{HI}$ Dí E ZLWK YDOXHV OLVWHG LQ7DEOH 7KLV LGH LQFXGHVURXJKQHVVDOR UHSODFH GLQWKHFKDUWE\ ' $_{HI}$) RUDTXLFNGHVLJQQXPEHUZLWKDERXW SHUFHQWDFFXUDF\ RQHFDQVLPS) XVHWKHK\GUDXOLFGLDPHWHU' $_K$ Dí E

(; \$0 3/ (

: KDWKRXOGWKHUHVHUYRLUOHYHOKEHWRPDLQWDLQDIORZ RI P VWKURXJKWKHFRPPHUFLDOXWDQXV P ORQJ VKRZQLQ) LJ (" 1HJOHFWQDQFHHIHFWVDQGWDNH! NJ P DQG Î Í P VIRUZDWU

(



6RXWLRQ

- $VVXPSWLRQV) XOO GHYHORSHG DQQXODU IORZ PLQRU ORVVHV QHJOHFWHG
- $SSURDFK ' HWHUPLQH WKH 5H\QROGV QXPEHU WKHQ ILQG I DQG K, DQG WKHQ H K
- 3URSHUWLHV * LYHQ! NJ P DQG (P V
- 6ROXWLRQ VWHS &DOFXODWH WKH YHORFLW\ K\GUDXOLF GLDPHWHU DQG 5H\QROGV QXPEHU

$$V = \frac{Q}{A} = \frac{0.01\ \text{m}^3/\text{s}}{\pi[(0.05\ \text{m})^2 - (0.03\ \text{m})^2]} = 1.99\frac{\text{m}}{\text{s}}$$

$$D_h = 2(a - b) = 2(0.05\ \text{m} - 0.03\ \text{m}) = 0.04\ \text{m}$$

$$\text{Re}_{D_h} = \frac{VD_h}{\nu} = \frac{(1.99\ \text{m/s})(0.04\ \text{m})}{1.02\text{E-}6\ \text{m}^2/\text{s}} = 78{,}000 \quad \text{(turbulent flow)}$$

- 6ROXWLRQ VWHS $SSO WKH WDG IORZ HQHUJ\ HTXDWLRQ EHWZHHQ VHFWLRQV DQG

$$\frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f$$

or

$$h = \frac{\alpha_2 V_2^2}{2g} + h_f = \frac{V_2^2}{2g}\left(\alpha_2 + f\frac{L}{D_h}\right) \qquad (1)$$

1RWH WKDW K) RUV XUE XOHQW IORZ IURP (T F ZH HYDOXDWH DW. §

- 6ROXWLRQ VWHS ' HWHUPLQH WKH UHJLPHV DQG WKH IULFWLRQ IDFWRU) URP 7DEOH IR QHZ FRPPHUFLDO VWHHOSLSH PP 7KHQ

$$\frac{\epsilon}{D_h} = \frac{0.046\ \text{mm}}{40\ \text{mm}} = 0.00115$$

) RU D UHDVRQDEOH HVWLPDWH XVH 5H WR HVWLPDWH WKH IULFWLRQ IDFWRU IURP (T

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{0.00115}{3.7} + \frac{2.51}{78{,}000\sqrt{f}}\right) \quad \text{solve for } f \approx 0.0232$$

) RU VOJKWOY EHWWU DFFXUDF\ ZH FRXOG XVH' HI ' K) URP 7DEOH IRU E D 7KHQ' HI PP PP ZKHQ H 5H K ∈ ' HI DQG I HI § 8VLQJ WKH DWWU HVWLPDWH ZH ILQG WKH UHTXLUHG UHVHUYRLU HY HO IURP (T

$$h = \frac{V_2^2}{2g}\left(\alpha_2 + f_{eff}\frac{L}{D_h}\right) = \frac{(1.99\ \text{m/s})^2}{2(9.81\ \text{m/s})^2}\left[1.03 + 0.0257\frac{30\ \text{m}}{0.04\ \text{m}}\right] \approx 4.1\ \text{m} \qquad \textit{Ans.}$$

- &RPPHQWV 1RWH WKDW ZH GR QRW XVH SDFH' K E\ ' HI LQ WKH KHDG ORVV WHUP I/ ' K ZKLFK FRPH

IURP D PRPHQWXP EDODQFH DQG UHTXLUHV K\GURJHQ GLPHWHU ,I ZH XVHG WKH VLPSOH UIUDFWR
HWXP DW I § ZHZRXOGREWDLQK § P RUDERXW SHUFHQWBOZHU

2 WKHU1 RQFLUFXODU&URVV6HFWLRQV

,Q SUDFWLSH DQ GXFWURVV VHFWLRQ FDQ EH VRQYHG DQDOWFDOO IRUWKH ODPLQDUIORZ YHORFLW
GLVWULEXWLRQ YROXPHIORZ DQG IULFWLRQIDFWRU 7KLVLVEHFDXVHDQ FURVVHFWLRQFDQEHPDSSHGRQR
FLUFOHE\ WKH PHWKRGV RI FRPSOH[YDULDEOHV DQG RWKHU SRZHUXO DQDOWFDO WHFKQLTXHV DUH DOV
DYDLODEOH 0DQ H[DPSOHVDUHJLYHQE\ : KLWH> SS ± @%HUNHU> @DQG2 OVRQDQG: ULJK
> SS ± @5HHUHQFH LVGHYRWHGWRWKH VXEMHFWDUGXFWORZ

,Q JHQHUDO KRZHYHU PRVWXQXVXDOGXFWVHFWLRQV KDYH VWUDLJKW DFDGHPLF DQG QRWFRP PHUFL
YDOXH : HOLVWKHUHQR WKHUHFDQJXODUDQG LVRVFHOHV WUDQJXODUVHFWLRQV LQ7DEOH OHDYLQJ RWKH
FURVVVHFWLRQVIRU\RXVWXGHQWVHUHVHDUFK

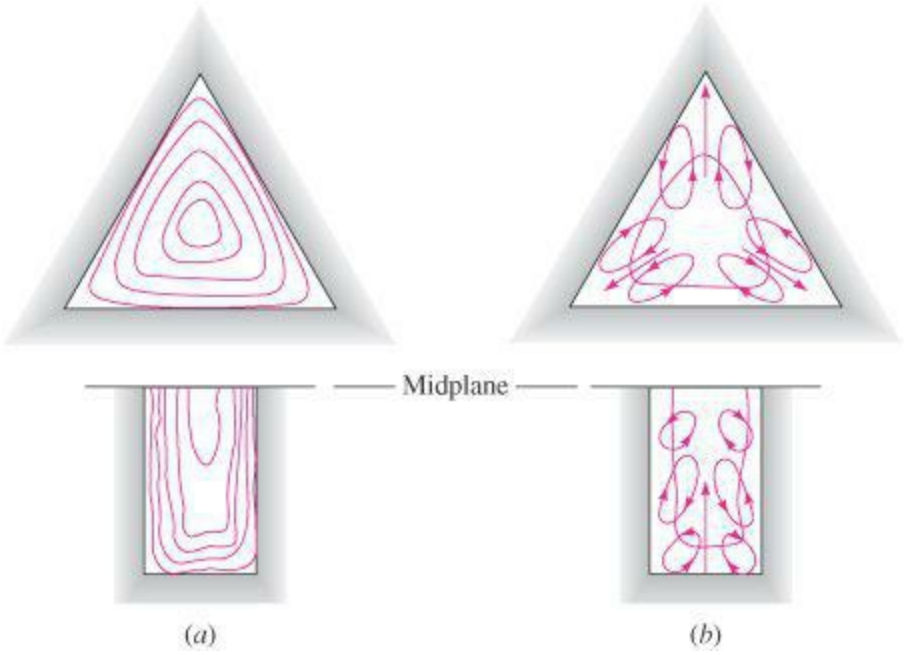
7DEOH / DPLQDU) UIFWLRQ&RQWDQW I5HIRU5HFWDQJXODUDQG7UDQJXODU' XFWV

Rectangular		Isosceles triangle	
b/a	$f\mathrm{Re}_{D_h}$	θ, deg	$f\mathrm{Re}_{D_h}$
0.0	96.00	0	48.0
0.05	89.91	10	51.6
0.1	84.68	20	52.9
0.125	82.34	30	53.3
0.167	78.81	40	52.9
0.25	72.93	50	52.0
0.4	65.47	60	51.1
0.5	62.19	70	49.5
0.75	57.89	80	48.3
1.0	56.91	90	48.0

) RUWXUEXOHQWIORZ LQDGXFWRI XQXVXDOFURVVVHFWLRQ RQHVKRXOGUHSODFHGE\ ' $_K$ RQWKH0 RRG
FKDUWI QR OP LQDUWKHRU LVDYDLODEOH ,I OP LQDUUHVXOWVDUHNQRZ Q VXFK DV7DEOH UHSODFHGE\
' $_{HI}$ > I5H@$_K$ IRUWKHSDUWFXODUJHRPHWU RI WKHGXFW

) RUOP LQDUIORZ LQUHFWDQJOHV DQG WUDQJOHV WKHZDOOIULFWLRQYDULHVJUHDW EHLQ ODUJHVWQHD
WKHPLGSRLQWV RI WKHVLGHV DQG]HUR LQ WKH FRUQHUV ,QWXEXOHQWIORZ WKURXJK WKHVDPH VHFWLRQV WK
VKHDUL Q HDO FRQWDQWDORQJ WKHVLGHV GURSSLQJ RI VKDUS WR]HUR LQ WKH FRUQHUV 7KLVLVEHFDXVHF
WKHSKHQRPHQRQ RI WXUEXOHQWVHFRQGDU IORZ LQZKLFK WKHUH DUH QHUR PHDQ YHORFLWWV Y DQG Z L
WKHSDQH RI WKHFURVVVHFWLRQ 6RPHPHDVXUHPHQWV RI D[LDOYHORFLW DQG VHFRQGDU IORZ SDWWHUQV DU
VKRZ Q LQ) LJ DV VNHWFKHG E\ 1 LNXUDGVHLQKLV GLVWLQJXLVKHG 7KHVHFRQGDU IORZ ³FHOO
GULYHQ WKHPHDQIORZ WRZDUG WKHFRUQHUV VR WKDWWKH D[LDOYHORFLW FRQWRXUV DUH VLPLODU WR WKHFURV
VHFWLRQ DQG WKH ZDOO VKDUL Q FRQWDQW7KLV LV ZK\ WKH K\GUDXOF GLPHWHU FRQFHSW VL
VXFFHVVIXOIRUWXEXOHQWIORZ / DPLQDUIORZ LQD VTXDJKWQRQFLUFXODUGXFWKDV QR VHFRQGDU IORZ $ C
DFFXUDWWKHRUHWFDOSUHGLFWLRQRI WXEXOHQWVHFRQGDU IORZ KDV\HWWREHDFKLHYHG DOWKRXJK QXPHUL
PRGHOVDUHRWQVXFFHVVIXO> @

) LJ



,OOXVWUDWLRQ RI VHFRQGDU\ VXUIDFH FHOOXODU GXFWV *a* D[LDO PHDQ YHORFLW\ FRQWRXUV *b* VHFRQGDU\ IORZ LQ SDQH FHOOXODU PRGHOV*(After J. Nikuradse, dissertation, Göttingen, 1926.)*

(; $0 3/ (

$ LU Z LWK! VXJ IWDQG IWV LV IRUFHG WKURXJK DKRUL]RQWDOTXDUH E\ LQGXFW IWORQJ DW IWV) LQG WKHSUHVVXUHGURS L∈ IW

6RXWLRQ

&RPSXWH WKHPHDQYHORFLW\DQGK\GUDXOLFGLDPHWHU

$$V = \frac{25\ \text{ft}^3/\text{s}}{(0.75\ \text{ft})^2} = 44.4\ \text{ft/s}$$

$$D_h = \frac{4A}{\mathcal{P}} = \frac{4(81\ \text{in}^2)}{36\ \text{in}} = 9\ \text{in} = 0.75\ \text{ft}$$

) URP 7DEOH IRUE D WKHHIHFWLYHGLDPHWHULV

$$D_{\text{eff}} = \frac{64}{56.91} D_h = 0.843 \text{ ft}$$

whence
$$\text{Re}_{\text{eff}} = \frac{VD_{\text{eff}}}{\nu} = \frac{44.4(0.843)}{0.000157} = 239{,}000$$

$$\frac{\epsilon}{D_{\text{eff}}} = \frac{0.0003}{0.843} = 0.000356$$

) URP VKH0 RRG\ FKDUWUHDGI 7KHQVKHSUHVVXUHGURS LV

$$\Delta p = \rho g h_f = \rho g \left(f \frac{L}{D_h} \frac{V^2}{2g} \right) = 0.00237(32.2) \left[0.0177 \frac{100}{0.75} \frac{44.4^2}{2(32.2)} \right]$$

or
$$\Delta p = 5.5 \text{ lbf/ft}^2 \qquad \textit{Ans.}$$

3UHVVXUHGURS LQDLUGXFWVLVXVXDOO VP DOOEHFDXVHRI VKHORZ GHQVLW

0 LQRUR/ RFDO/ RVVHVLQ3LSH6\ VWHPV

) RUDQ SLSHV\ VWHP LQDGGLWLRQ WR VKH0 RRG\ \WSH IULFWLRQ ORVV FRP SXWHG IRU WKH HQWLUH SLSH-
VKHUHDUHDGGLWLRQDO VR FDOOHG P LQRU ORVVHV RU ORFDO ORVVHV GXH WR

3LSHHQWUDQFHRUH[LW
6XGGHQH[SDQVLRQRUFRQWUDFWLRQ
%HQGV HOERZ V VWHV DQG RWKHUILWWQJV
9DOYHV RSHQRUSDUWLDOO FORVHG
* UDGXDOH[SDQVLRQV/RUFRQWUDFWLRQV

7KH ORVVHV PD\ QRWEH VR P LQRU IRUH[DPSOH D SDUWLDOO FORVHG YDOYH FDQ FDXVH D JUHDW
SUHVXUHGURS WKDQ DORQJ SLSH
6LQFH WKH IORZ SDWWHUQ LQ ILWWQJV DQG YDOYH LV TXLWH FRP SOH[WKH WKHRU LV YHU ZHDN 7K
ORVVHV DUH FRP P RQO P HDVXUHG H[SHULP HQWDOO DQG FRUUHODWHG Z LWK WKH SLSH IORZ SDUDP HWHUV 7K
GDWD HSHFLDOO IRU YDOYH DUH VRP HZ KDWG HSHQG HQW RQ WKH SDUWF XODU P DQX DFWXUHU¶V GHVLJQ VR WKD
WKHYDOXHVOLVWHGKHUHP XVWEHWDNHQDV/ DYHUDJHGHVLJQHWWP DWV> @

7KHP HDVXUHG P LQRU ORVV LV XVXDOO JLYHQ DV D UDWLR RI WKH KHDG ORVVK, ûS !J WKURXJK WK
GHYLFHWR WKH YHORFLW\KHDG9 J RI WKH DVVRFLDWHG SLSLQJ\ VWHP

$ OWKRXJK. LVGLP HQVLRQOHVV LWRI WHQ LV QRW FRUUHODWHG LQ WKH OLWHUDWXUH Z LWK WKH 5H\QROGV QXP EH-
DQG URXJKQHVV UDWLR EXW DWKHUP S0 Z LWK WKH UDZ VL]HRI WKH SLSHIQ VD\ LQFKHV \$ P RWDOGDWD DU
UHSRUWHG IRU VXUEXHQWORZ FRQGLWLRQV

\$ VLQJOH SLSHV\ VWHP P D\ KDYH P DQ P LQRU ORVVHV 6LQFH DOOLUH FRUUHODWHG Z LWK9 J WKH

FDQEHVXPPHGLQRDVLQJOHVRDOXWLRQ RWLI WKHSLSHKDVFRQVDQWGLDPHWU

$$\Delta h_{tot} = h_f + \Sigma h_m = \frac{V^2}{2g}\left(\frac{fL}{d} + \Sigma K\right) \qquad (6.79)$$

1 RWH KRZ HYHU WKDW HPXWXP WKHRULHV VHSDUDWH LI WKHSLSHV] HFKDQJHVVR WKDW9 FKDQJHV 7KHHQJWK/ LQ(T LV WKHVRDOOHQJWKRI WKHSLSHD[LV

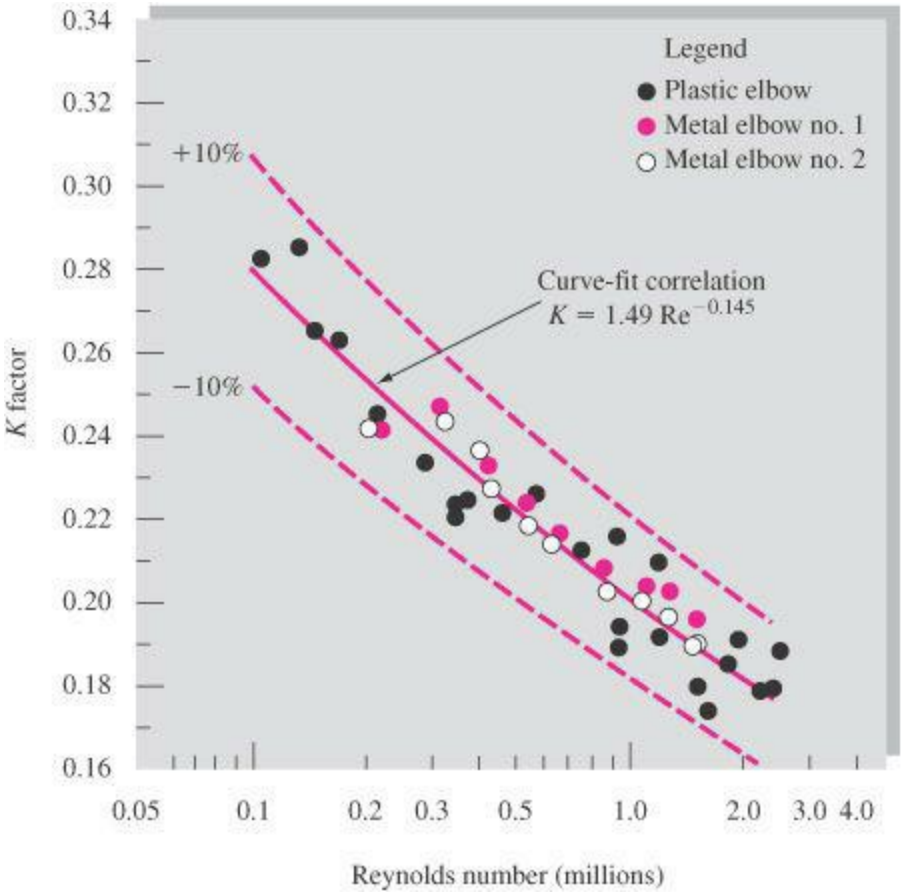
7KHUH DUH PDQ GLIIHUHQWYDOYH GHVLJQV LQ FRPPHUFLDO XVH) LJXUH VKRZ V ILYH WSLFD GHVLJQV D WKHJDWH ZKLFK VOLGHV GRZQDFURVWKH VHFWLRQ E WKHJOREH ZKLFK FORVHV DKROHLQ VSHFLDOLQHW F WKHDQJOH VLP LODUWR DJORBEHEXWZLWKD f WKLQ G WKHVZLQJ FKHFNYDOYH ZKLFK DOORZV RQO RQHZD\ IORZ DQG H WKHGLVN ZKLFK FORVHVWKHVHFWRQZLWKDFLUFXODUJDW 7KHJORBE ZLWKLWVWRUWXRXVIORZ SDWK KDVWKHKLJKHVWORVVZKHQIXOO RSHQ 0 DQ H[FHOOHQWGHDOLVYDERXWWKHV DQGRWKHUYDOYHVDUHJLYHQLQWKHKDQGERRNVE\ 6NRXVHQ> @DQG6PLWKDQG=DSSH> @

7DEOH OLVWVORVFRHILFLHQWV. IRUIRXUWSHVRI YDOYH WKUHDQGHVRI HOERZ ILWWQJ DQGVZF VWHFRQFHQWRQV) LWWQJVPD\ EHFRQFHQWRGE\ HLWKHULQWHUQDOFUHZVRUIODQJHV KHQFHWKHVZRQWQJV : H VHH WKDW JHQHUDOO GHFUHDVHV ZLWK SLSH VL]H ZKFK LV FRQVLVWHQWZ LWK WKH KJKHU 5H QROGQXPEHUDQG GHFUHDVHG URXJKQHVV UDWR RI ODUJH SLSHV : H WUHVWKDW7DEOH UHSUHVHQW GRVH DYHUDJHGP RQJ YDUURXVP DQXIDFWXUHUV VR WKHUHLVDXQFHUWDLQWDVKJKDV“ SHUFHQW

7DEOH 5HVLVWDQFH&RHILFLHQW. K_p >9 J @RU2 SHQ9DOYHV (OERZ V DQG7HHV

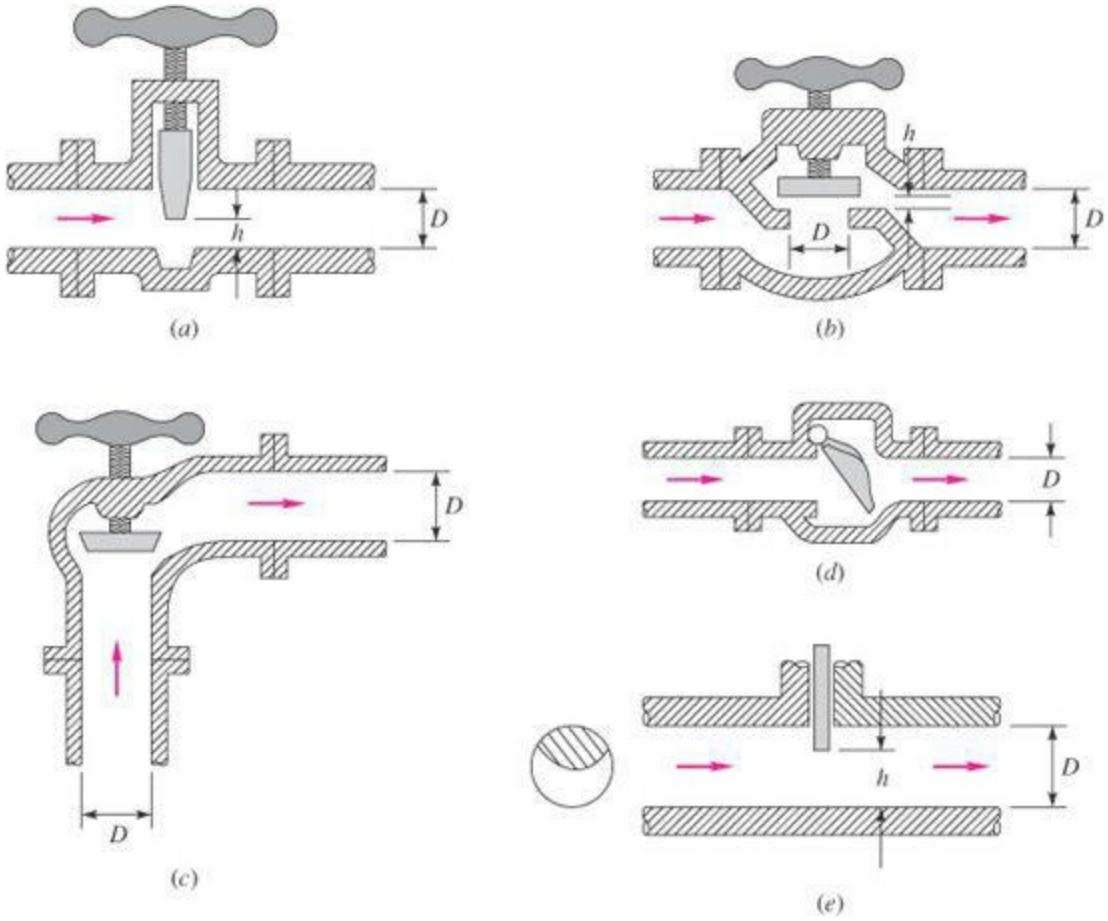
	Nominal diameter, in								
	Screwed				Flanged				
	½	1	2	4	1	2	4	8	20
Valves (fully open):									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows:									
45° regular	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° regular	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees:									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

,QDGGLWLRQ PRVWRI WKH GDWD LQ7DEOH DUH UHDLYHO RGG> @DQG WKHUHRUH EDVHG RI ILWWQJVPDQXIDFWXHGLQWKH V 0 RGHUQIRUJHG DQG PRGHG ILWWQJVPD\ \LHOG VRPHZKDWGLIHUHQ ORVWIDFWRUV RIWQOHVWKDQQLWHG7DEOH $QH[DPSOH VKRZ QLQ) LJ D JLYHV UHFWGD > @IRUIDLO VKRWWEHQG UDGLXVOBERZ GLDPHWU IQDQJHG f HOBERZV 7KHOBERZ GLDPHWU ZDV LQ 1RWH ILWWWKDW LV SORWWG YHUXV 5H QROGV QXPEHU UDWKHU WKDQ YHUXV WKH UDZ GLPHQVLRQDO SLSHGLDPHWUV LQ7DEOH DQG WKHUHRUH) LJ DKDVPRUHJHQHUDOW 7KHQRWH WKDWWKH. YDOXHV RI “ DUH VLJQLILFDQW OHVWKDQ WKH YDOXHV IRU f HOBERZV LQ7DEOH LQGLFDWQJ VPRRWKHU ZDOV DQG RU EHWWU GHVLJQ 2QH PD\ FRQFOXGH WKDW 7DEOH GDWD DU SUREDEO FRQVHUYDWLYH DQG OBERWIDFWRUV DUH HKJKO GHSHQGHQWRQ DFWDOGHVLJQ DQG PDQXIDFWXULQ IDFWRUV ZLWK7DEOH VHUYLQJ RQO DV DURXJK JXLGH

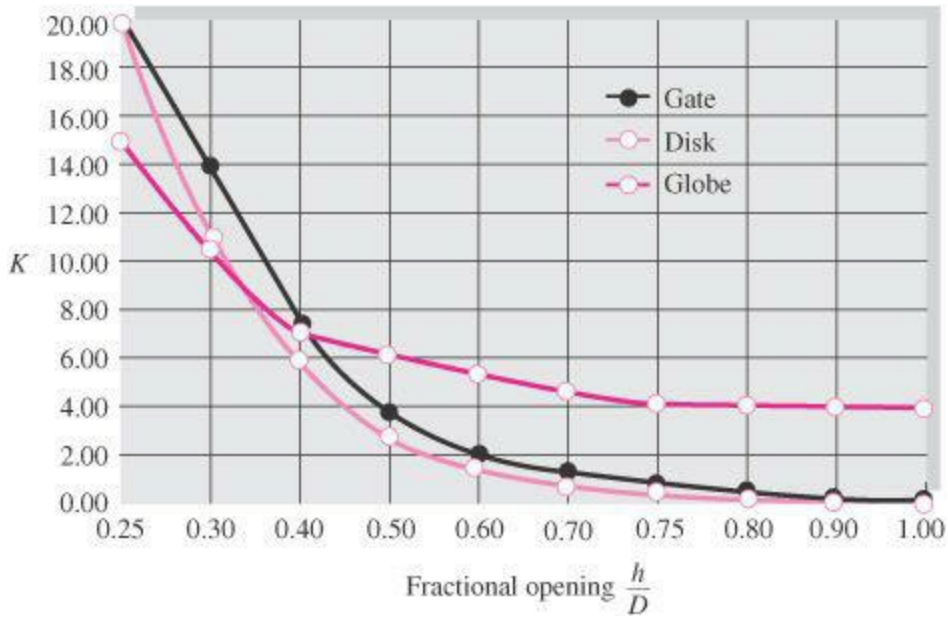


) LJ D 5HFHQWPHDVXUHG GDWD FRHIILFLHQWV IRU f HGERZV 7KHVH YDOXHV DUH GHVLJQ VWDQGDUGV VKRVHUHSRUWHGLQ7DEOH *(From Ref. 48, courtesy of R. D. Coffield.)*

7KHYDOYHGRWHVLQ7DEOH DUHIRUWKHIXOO RSHQFRQGLWLRQ / RWHVFDQEHPXFKKLJKHUIRU SDUWDOO RSHQYDOYH) LJXUH EJLYHVDYHUDJHGRWHVIRUWKUHHYDOYHVDVDIXQFWLRQRI ³SHUFHQWDJH RSHQ´ DVGHILQHGE\ WKHRSHQLQJGLVWDQFHUDWRK' VHH) LJ IRUWKHJHRPHWULHV $JDLQZI VKRXOGZDUQRI DSRWLEOHXQFHUWDLQWRI “ SHUFHQW2I DOOPLQRUGRWHV YDOYHV EHFDXVHRI WKHL FRPSOH[JHRPHWU DUH PRWWHQLWYH WR PDQXIDFWXUHV¶ GHVLJQGHWDLOV) RUPRUH DFFXUDF\ WK SDUWFXODUGHVLJQDQGPDQXIDFWXUVKRXOGEHFRQVXOWG> @



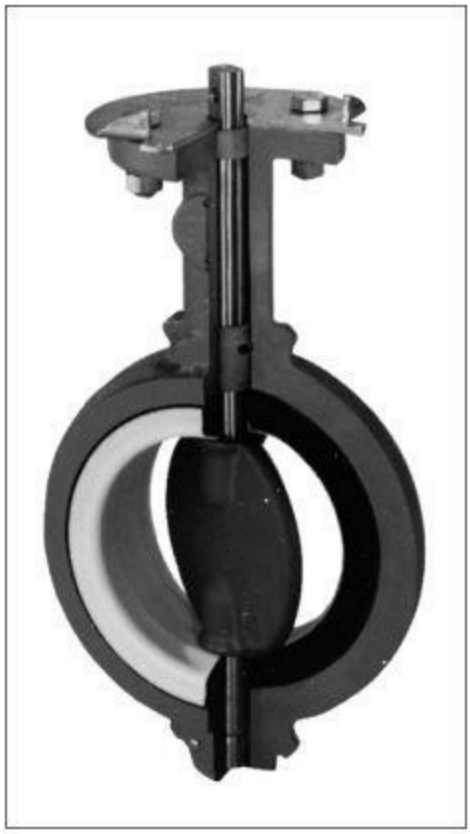
) LJ 7\SLFDOFRPPHUFLDOYDOYH JHRPHWULHV *a* JDWH YDOYH *b* JORE YDOYH *c* DQJOH YDOYH *d* VZLQJ FKHFN YDOYH *e* GLVN VWRSH JDWH YDOYH



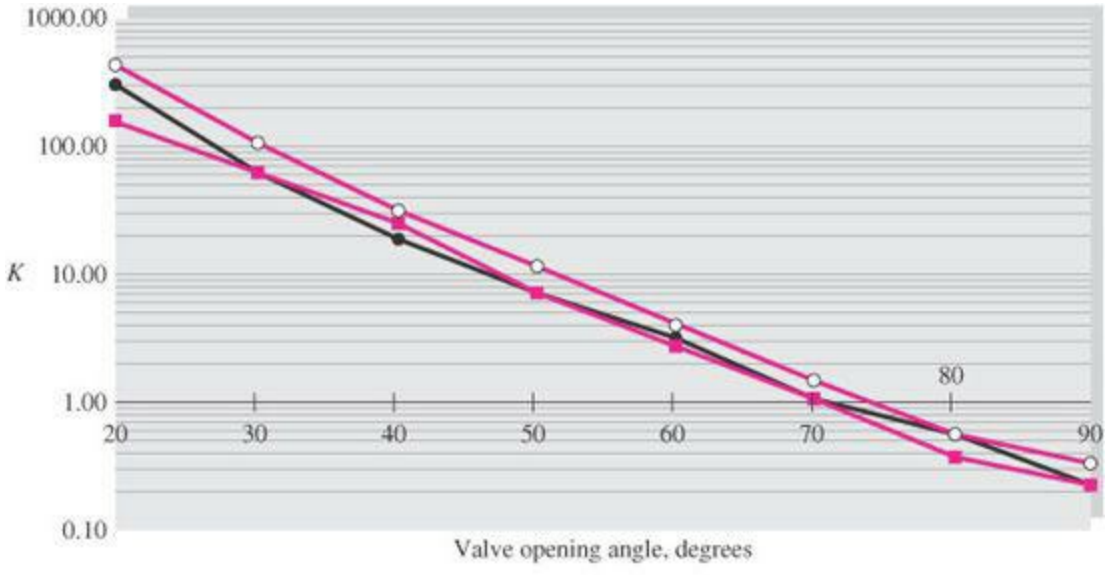
) LJ E $YHUDJH ORVV FRHIILFLHQWV IRU SDUWLDOO RSHQ YDOYHV VHHVNHWFKHVLQ) LJ

7KH EXWWHUIO YDOYH IR) LJ DLV D VWHP PRXQWHG GLVN WKDW ZKHQ FORVHG VHDWV DJDLQVW D 2 ULQJ RU FRP SOLDQW VHDOLQJ HDUWKHSLSH VXUIDFH \$ VLQJOH *f* WXUQ RSHQV WKH YDOYH FRPSOHWHO KHQFH WKH GHVLJQ LV LGHDO IRU FRQWROOEOH TXLFN RSHQLQJ DQG TXLFN FORVLQJ VLWXDWLRQV VXFK DV RFFXUULQ ILUH SURWHFWLRQ DQG WKH HOHFWULF SRZHU FRQGXVW + RZ HYHU FRQVLGHUDEOH GLQPLF WRUTXHLVQHGHGWR FORV

WKHVHYDOYHV DQGORVWHVDUHKLJKZKHQWKHYDOYHVDUHQHDUO FORVHG



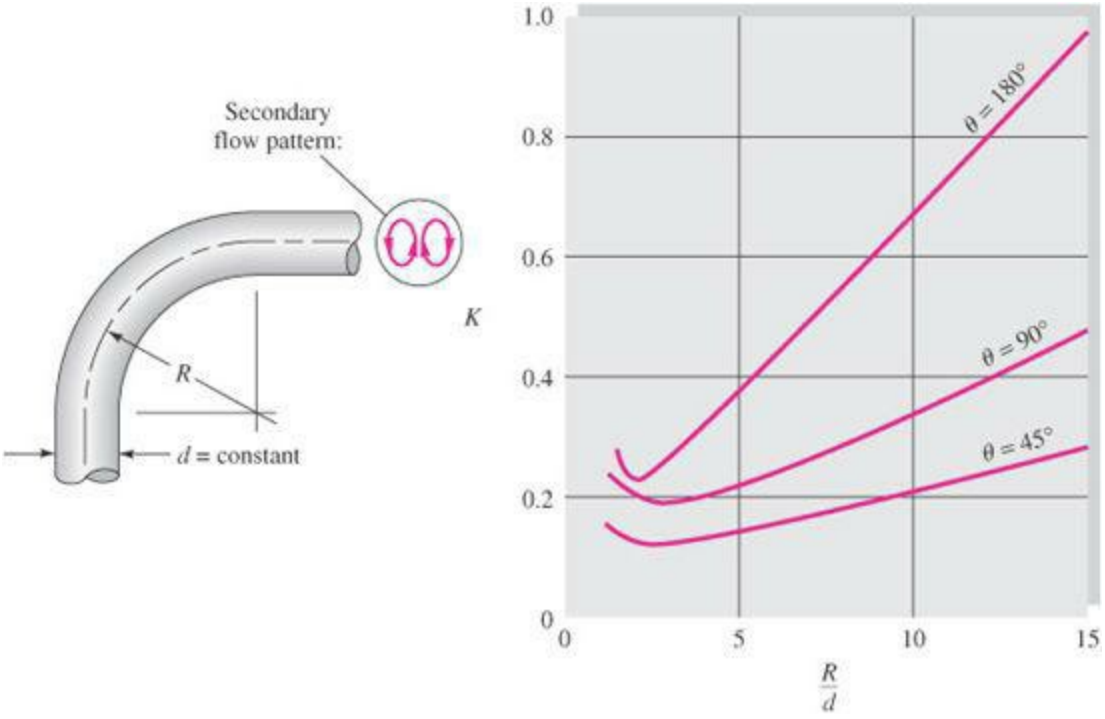
(*a*)



(*b*)

) LJ 3HUFHQWDJH RI EXWWHUIO YDOYHV *a* WSLFDO JHRPHWUL ***Courtesy of Tyco Engineered Products and Services*** *b* ORVV FRHIILFLHQWV IRUWKUHHGLIIHUHQWPDQXIDFWXUHUV

) LJXUH E VKRZV EXWWHUIO YDOYHORVV FRHIILFLHQWV DV D IXQFWLRQ RI WKH RSHQLQJ DQJOH IR WKUEXHQWVORZ FRQGLWLRQV LV FORVHG 7KHORVVHV DUH KXJH ZKHQ WKH RSHQLQJ LV VPDOO DQG. GURSSRI QHDUO H[SRQHQWLDOO ZLWKWKHRSHQLQJ DQJOH 7KHUHLV DIDFWRUR VSUHDGPRQJ WKHYDULRX PDQXIDFWXUHUV 1RWWKDW LQ) LJ E LV DXXDOO EDVHGRQWKHDYHUDJHSLSHYHORFLW9 4 \$ QR RQWKHLQFUHDVHGYHORFLWRI WKHIORZ DVLVSDWHVWKURXJKWKHQDUURZ YDOYHSDWDJH



) LJ 5HVLVWDQFH FRHILFLHQWV IRU VPRRWK ZDOOHG f f DQG f EHQGV DW5e_d DIWMU,WR> @

$ EHQG RU FXUYH LQ D SLSH DV LQ) LJ DQ DVVLJQHG DJHUWKDQ WKH VPSHVWDJKV SLSH0 RRG\ IULFWLRQ RRV GXHVR IORZ VHSDUDWLRQ RQ WKH FXUYHG ZDOV DQG DVZLOOQJ VHFRQGDU IORZ DULVLQJ IURP WKH FHQWULIXJDO FHOHUDWLRQ 7KHVPRRWK ZDOORV FRHILFLHQWV. LQ) LJ IURP WK GDWD RI ,WR> @DUH IRUWKDOORV LQFXGLQJ 0 RRG\ IULFWLRQ HIHFW 7KH VHSDUDWLRQ DQG VHFRQGDU IORZ ORVVHV GHFUHDVH ZLWK5 G ZKLHWKH0 RRG\ ORVVHV LQFUHDVHEHFDXVHWKHEHQG OHQJWK LQFUHDVH 7KHFXUYHV LQ) LJ WKXVVKRZ DPLQLPXP ZKHUHWKHVZR HIHFWFURVV ,WR> @JLYHVDFXUYHIL IRUPXODIRUWKH f EHQGLQWXEXOHQWORZ

$$90^\circ \text{ bend: } K \approx 0.388\alpha\left(\frac{R}{d}\right)^{0.84} \mathrm{Re}_D^{-0.17} \text{ where } \alpha = 0.95 + 4.42\left(\frac{R}{d}\right)^{-1.96} \geq 1 \quad (6.80a)$$

7KHIRUPXOD DFFRXQWVIRU5H\QROGV QXPEHU ZKLFK HTXDOV LQ) LJ &RPSUHKHQVLY UHYLHZ VR FXUYHG SLSH IORZ IRUERWKODPLQDUDQG WXUEXOHQWORZ DUH JLYHQE\ %HQHGHWWDO> @DQG IRU f EHQGVE\ 6SHGGLQJHWDO> @

$ V VKRZ Q LQ) LJ HQJDJHH ORVVHV DUH KLJKO GHSHQGHQWRQ HQJDJHH JHRPHWU EXW[L ORVVHV DUH QRW6KDSHGJHV RUSURWKVLRQV LQ WKHHQJDJHH FXUHODJH]RQHVRI IORZ VHSDUDWLRQ DQ OLJHOOHURXQGLQJ JHVDORQJ ZD\ DQGDZHOOURXQGHGHQJDJHH U G KDV DQHQHJOLJLEOH ORVV. $WD VXEP HJHGH[LWRQ WKH RWKHUHDQG WKH IORZ VHPS SDWHV RXWRI WK SLSH IORQR WKH OLJHGZ QWUHP UHVHUYRLUDQG ORVHV DOOLW YHOFLW KDGGXHVR YLVFRXVGLWLSDWQ 7KHUHRUH. IRUDOOVXEP HJHGH[LW QR P DWWKUKRZ ZHOOURXQGHG

) LJ) ORZ OR VVHV LQ D JUDGXDO FRQHFDOH[SDQVLRQ UHJLRQ DV FDOFXODWHG IURP * LEVRQ
VXJJHVWLRQ> @(T IRUD VPRRWK ZDOO

$ VIORZ HQWHUV D JUDGXDO H[SDQVLRQ RU LGLIIXVHU VXFK DV WKH FRQHFDOJHRPHWU RI) LJ WK
YHORFLW GURSV DQG WKH SUHVVXUH UHV + HDG ORVV FDQEH GHWHUPLQHG XVLQJ ORVV FRHIILFLHQWV L
WKH FRQHDQJOHLV WRR JUHDW$ WKLQ-HQWDQHERXQGDU OD\HUDVLQ) LJ FDXVHVDVHJKWW VPDOO
ORVV WKDQDIXOO GHYHORSHG LQOHWORZ 7KHIORZ ORVVLVDFRPELQDWLRQRI QRQLGHDOSUHVXUHUHFRYHU
SUXVZDOOIULFWLRQ 6RPHFRUUHODWLQ FXUYHVDUHVKRZQLQ) LJ 7KHORVVFRHIILFLHQW LVEDVHGR
WKHYHORFLWKHDGLQWKHLQOHWVPDOO SLSHDQGGHSHQGVXSRQFRQHDQJOH DQGWKHGLIXVHUGLPHQV
UDWRGG 7KHUHLV VFDWWHULQ WKH UHSRUWHG GDWD> @7KHFXUYHVLQ) LJ DUHEDVHGRQI
FRUUHODWLRQE\ $ + * LEVRQ> @FLWHGLQ5H

$$K_{\text{diffuser}} = \frac{h_m}{V_1^2/(2\,g)} \approx 2.61 \sin\theta \left(1 - \frac{d^2}{D^2}\right)^2 + f_{\text{avg}} \frac{L}{d_{\text{avg}}} \qquad \text{for} \quad 2\theta \le 45^\circ \qquad (6.82)$$

) RU ODUJH DQJOHV ! *f* GURS WKH FRHILFLHQW VLQ ZKLFK OHDYHV XV ZLWK D ORV
HTXLYDOHQWWR WKH VXGGHQ H[SDQVLRQ RI (T $ VVHHQ WKH IORPXOD LV LQ UHDVRQDEOH DJUHHPHQ
ZLWK WKH GDWD IURP 5H 7KHPLQLPXP ORVV OLHV LQ WKH UHJLRQ *f* ZKLFK LV WKH EHVW
JHRPHWU IRUDQ HILFLHQWGLIXVHU) RUDQJOHV OHVV WKDQ *f* WKH GLIXVHULV WRR ORQJ DQG KDV WRR PXFK
IULFWLRQ $QJOHV JUHDWHU WKDQ *f* FDXVH IORZ VHSDUDWLRQ UHVXOWLQJ LQ SRRU SUHVVXUH UHFRYHU
3URHVWRU* RUGRQ + RORZD\ SURYLGHG WKH ZULWWHQ UHFRPPH ZKHUH DQ LP SURYHG GLIXVHU
GHVLJQUHGXFHG WKH SRZHUUHTXLUHPHQWR DZLQG WKQQHOE\ SHUFHQW KS GHFUHDVH : HVKDO
ORRNDJDLQDWGLIXVHUVLQ6HF XVLQJ WKHGDWDR 5H

) RU D JUDGXDOFRQWDFWLRQ WKH ORVV LV YHU VPDOO DV VHHQ IURP WKH IORORZLQJ H[SHULPHQW

YDOXHV> @

Contraction cone angle 2θ, deg	30	45	60
K for gradual contraction	0.02	0.04	0.07

5HHUHQFHV DQG FRQWLQDGGLWRQDOGDWDRQPLQRUORVVHV

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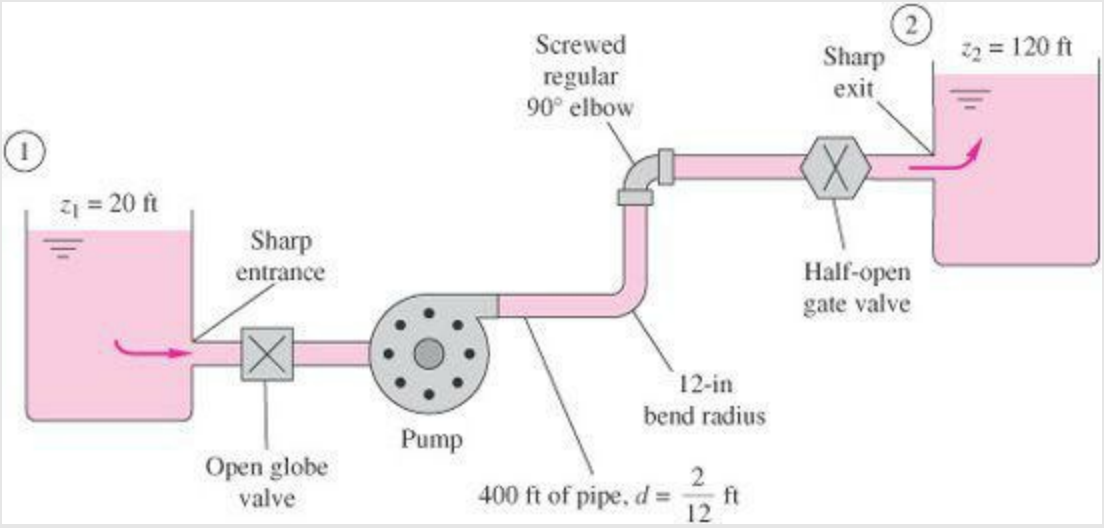
: DMU ! VOXJVIWDQG IWV LSXPSHGEHWHQWZRUHVHUYRLUVDW IW
WKURXJK IWA LQGLDPHWUSLSHDQGVHYHUDPLQRUORVVHV DVVKRZQLQ) LJ (7KHURXJKQHV
UDWLR G &RPSXWHWKHSXPSKRUVHSRZHUUHTXLUHG

(



6RXWLRQ

: UDWWKHVWADG\ IGRZ HQHUJ\ HTXDWRQEHWHQVHFWRQV DQG WKHWRUHVHUYRLUXUFHV

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + h_f + \sum h_m - h_p$$

ZKHUHK$_S$ LVWKHKHDGLQFUHDVHDFURVVWKHSXPS %XWWLQHS S DQG9 9 § VRQYHIRUWK
SXPSKHDG

$$h_p = z_2 - z_1 + h_f + \sum h_m = 120\text{ ft} - 20\text{ ft} + \frac{V^2}{2g}\left(\frac{fL}{d} + \sum K\right) \qquad (1)$$

1RZ ZLWKWKHIGRZ UDWHNQRZQ FDOFXODWH

$$V = \frac{Q}{A} = \frac{0.2\text{ ft}^3/\text{s}}{\frac{1}{4}\pi(\frac{2}{12}\text{ ft})^2} = 9.17\text{ ft/s}$$

1 RZ QDWDQG VXP WKHP LQRUGRW/FRKHILFLHQW
&DOFXODWH WKH5H\QROGV QXPEHUDQG SLSHIULFWLRQIDFWRU

$$\mathrm{Re}_d = \frac{Vd}{\nu} = \frac{9.17(\frac{2}{12})}{0.000011} = 139{,}000$$

) RU G IURP WKH0 RRG\ FKDUWDHDGI 6XEVWLWXWLQJ(T

$$h_p = 100\text{ ft} + \frac{(9.17\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}\left[\frac{0.0216(400)}{\frac{2}{12}} + 12.3\right]$$
$$= 100\text{ ft} + 84\text{ ft} = 184\text{ ft} \quad \text{pump head}$$

7KHSXPS PXVWSURYLGHDSRZHUWRWKHZDWHUI

3 SJ4K$_S$ > GEI IW@ IWV IW IWÂGEI V

7KHFRQYHUVLRQIDFWRULV KS IW GEI V7KHUHRUH

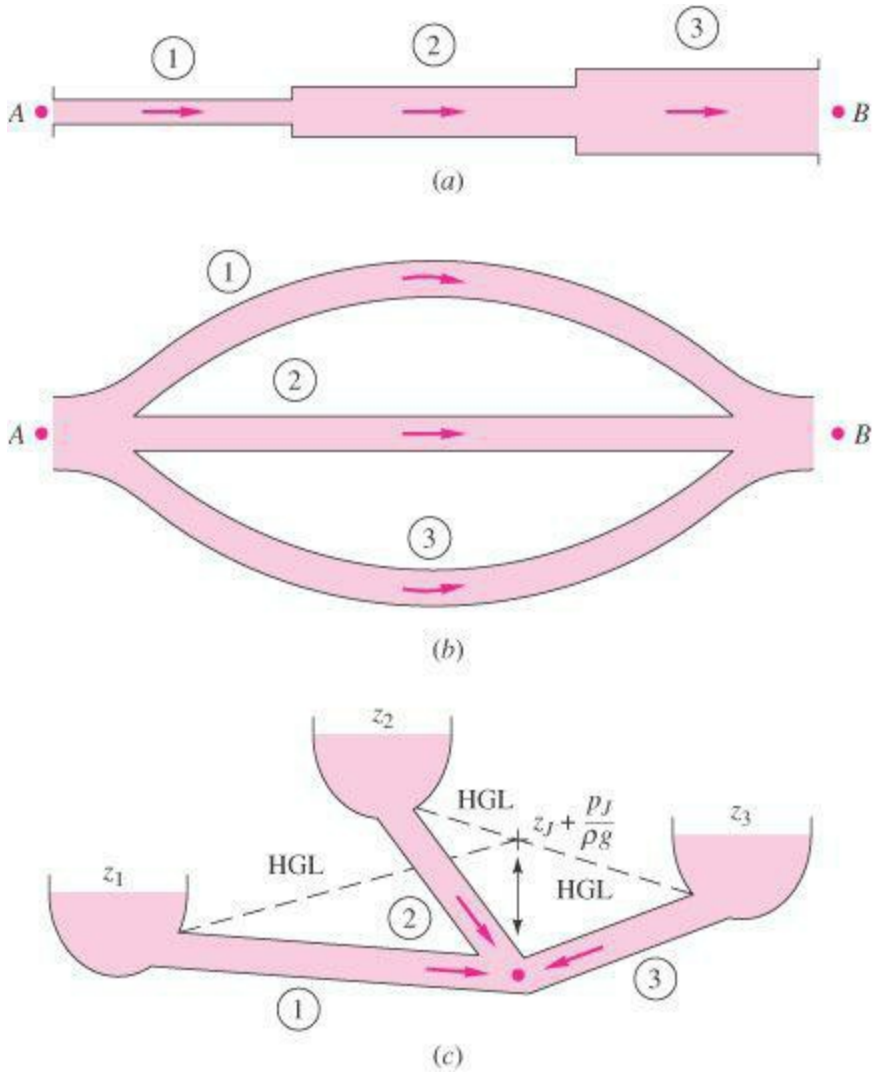
$$P = \frac{2300}{550} = 4.2\text{ hp} \qquad \textit{Ans.}$$

$\GORZ LQJ IRUDQHILFLHQF\ RI VR SHUFHQWDSXPS LVQHGHGZLWKDQLQSXWRI DERXW KS

Loss	K
Sharp entrance (Fig. 6.21)	0.5
Open globe valve (2 in, Table 6.5)	6.9
12-in bend (Fig. 6.20)	0.25
Regular 90° elbow (Table 6.5)	0.95
Half-closed gate valve (from Fig. 6.18*b*)	2.7
Sharp exit (Fig. 6.21)	1.0
	$\Sigma K = 12.3$

0 XOWLSOH3LSH6\VWHPV

,I \RX FDQ VROYH WKH HTXDWRQ/IRUURQH SLSH V\VWHP V \RX FDQ VROYH WKHP DOO EXW ZKHQ V\VWHP
FRQGLWLQV/ZR RUP RUH SLSHV FHULDQEDLF UXOHVPDNH WKH FDOFXODWRQV YHU VPRRW $Q UHVHPEODQFH
EHWZHHQ WKHVHUXOHVDQG WKHUXOHVIRUKDQGOLQJ HDFWLF FLUFXLWVRWRFLGHQWDO
) LJXUH VKRZ VWKUHH[DPSOHVRI PXOWLSOH SLSHV\VWHPV



) LJ ([DP SOH V R I P X OWL SOH SLSH V V WHP V *a* SLSH V LQ V HULH V *b* SLSH V LQ SDUDOOHO *c* WKH WKUHH UHVHUYRLUV MXQFWLRQ SUREOHP

3LSHVLQ6HULHV

7KHILUVWLVDVHWRI WKUHH RUPRUH SLSHVLQVHULHV 5XOH LVWKDWWKHIORZ UDWHLVWKHVDPHLQDO SLSHV

$$Q_1 = Q_2 = Q_3 = \text{const} \tag{6.83}$$

RU

$$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2 \tag{6.84}$$

5XOH LVWKDWWKHWRWDOKHDGORVVWKURXJKWKHV\VWHP HTXDOVWKHVXP RI WKHKHDGORVVLQHDFKSLSH

$$\Delta h_{A\to B} = \Delta h_1 + \Delta h_2 + \Delta h_3 \tag{6.85}$$

,QWHUPVRI WKHIULFWLRQDQGPLQRUORVVHVLQHDFKSLSH ZHFRXOGUHZULWHWKLVDV

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}\left(\frac{f_1 L_1}{d_1} + \sum K_1\right) + \frac{V_2^2}{2g}\left(\frac{f_2 L_2}{d_2} + \sum K_2\right) + \frac{V_3^2}{2g}\left(\frac{f_3 L_3}{d_3} + \sum K_3\right) \tag{6.86}$$

DQGVRRQIRUDQ\ QXPEHURI SLSHVLQWKHVHULHV 6LQFH9 DQG9 DUH SURSRUWLRQDOWR9 IURP (T (T LVRI WKHIRUP

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}(\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3) \tag{6.87}$$

ZKHUHWKH.L DUHGLPHQVLRQOHVVFRQVWDQWV ,I WKHIORZ UDWHLVJLYHQ ZHFDQHYDOXDWHWKHUHODWLYHKHDGORVVHVDQGKHQFHWKHVROXWLRQ ,I WKHKHDGORVVLVJLYHQ DQDOWHUQDWLYHSURFHGXUH LVUHTXLUHG I DQG I DOOGHSHQGRQ9 WKURXJK WKH5H\QROGV QXPEHU %HJLQE\ FDOFXODWLQJ I I DQGI DVVXPLQJ IXOO URXJK IORZ DQGWKH VROXWLRQ IRU9 ZLOOFRQYHUJHZLWKRQHORRSWKURXJKWKH SXUSRVH

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* LYHQLVDWKUHH SLSHVHULHVV\VWHP DVLQ) LJ D 7KHVROXWLRQSURFHGXUHSVLS$ í S% 3D DQGWKHHOHYDWLRQGURSV]$ í]% P 7KHSLSHGDWDDUH

Pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

7KHIOXLGLVZDWHU! NJ P DQG î í P V &DOFXODWHWKHIORZ UDWH4 LQP I

6RXWLRQ

7KH VWDUWLQJ SRLQW LV

$$\Delta h_{A\to B} = \frac{p_A - p_B}{\rho g} + z_A - z_B = \frac{150{,}000}{1000(9.81)} + 5\text{ m} = 20.3\text{ m}$$

) URP WKH FRQWLQXLWV UHODWLRQ WKH YHORFLWLHV DUH

$$V_2 = \frac{d_1^2}{d_2^2}V_1 = \frac{16}{9}V_1 \qquad V_3 = \frac{d_1^2}{d_3^2}V_1 = 4V_1$$

and

$$\mathrm{Re}_2 = \frac{V_2 d_2}{V_1 d_1}\mathrm{Re}_1 = \frac{4}{3}\mathrm{Re}_1 \qquad \mathrm{Re}_3 = 2\mathrm{Re}_1$$

1 HJOHFWLQJ P LQRU ORVVHV DQG VXEVWLWXWLQJ LQWR (T Z HREWDLQ

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}\left[1250f_1 + 2500\left(\frac{16}{9}\right)^2 f_2 + 2000(4)^2 f_3\right]$$

or

$$20.3\text{ m} = \frac{V_1^2}{2g}(1250f_1 + 7900f_2 + 32{,}000f_3) \qquad (1)$$

7KLV LV WKH IRUP WKDW Z DV KLQWHG DW LQ (T ,W WHPSWV WR EH GRPLQDWHG E\ WKH WKLUG SLSH ORV I %HJLQE\ HVWLPDWLQJ I I DQG I IURP WKH0 RRG\ FKDUWXOD URXJK UHJLPH

I I I

6XEVWLWXWLQ (T WR ILQG $V_1^2 \approx 2g(20.3)/(33 + 185 + 973)$ 7KH ILUVW HVWLPDWH WKXV LV9 P V IURP Z KLFK

5H § 5H 5H

+ HQFH IURP WKH0 RRG\ FKDUW

I I I

6XEVWLWXWLQ (T JLYHV WKH EHWWHU HVWLPDWH

$$V_1 = 0.565 \text{ m/s} \qquad Q = \tfrac{1}{4}\pi d_1^2 V_1 = 2.84 \times 10^{-3} \text{ m}^3/\text{s}$$

or $$Q = 10.2 \text{ m}^3/\text{h} \qquad \textit{Ans.}$$

$ VHFRQG LWHUDWLRQ JLYHV 4 P K DQHJOLJLEOHFKDQJH

3 LSHV LQ 3 DUDOOHO

7 KH VHFRQG P XOWLSOH SLSH V\ VWHP LV WKH SDUDOOHO IORZ FDVH VKRZ Q LQ)LJ E + HUH WKH SUHVVXUH GURS LV WKH VDP H LQ HDFK SLSH DQG WKH WRWDO IORZ LV WKH VXP RI WKH LQGLYLGXDO IORZ V

$$\Delta h_{A\to B} = \Delta h_1 = \Delta h_2 = \Delta h_3 \qquad (6.88a)$$

$$Q = Q_1 + Q_2 + Q_3 \qquad (6.88b)$$

,I WKH WRWDO KHDG ORVV LV NQRZ Q LW LV VWUDLJKWIRUZ DUG WR VROYH IRU 4L LQ HDFK SLSH DQG VXP WKHP D Z LOO EH VHHQ LQ ([DP SOH 7 KH UHYHUVH SUREOHP RI GHWHUP LQLQJ 4L Z KHQ Kf LV NQRZ Q UHTXLUHV LWHUDWLRQ (DFK SLSH LV UHODWHG WR Kf E\ WKH 0 RRG\ UHODWLRQ Kf I / G 9 J I4 & Z KHUH & CEJG / 7 KXV HDFK SLSH KDV QHDUO\ TXDGUDWLF QRQOLQHDU SDUDOOHO UHVLVWDQFH DQG KHDG ORVV LV UHODWHG WR WRWDO IORZ UDWH E\

$$h_f = \frac{Q^2}{(\sum\sqrt{C_i/f_i})^2} \qquad \text{where } C_i = \frac{\pi^2 g d_i^5}{8L_i} \qquad (6.89)$$

6LQFH WKH IL YDU\ Z LWK 5 H\ QROGV QXP EHU DQG URXJKQHVV UDWLR RQH EHJLQV (T E\ JXHVVLQJ YDOXHV RI IL IXOO\ URXJK YDOXHV DUH UHFRP P HQGHG DQG FDOFXODWLQJ D ILUVW HVWLP DWH RI Kf 7 KHQ HDFK SLSH \ LHOGV D IORZ UDWH HVWLP DWH 4L § &L Kf IL DQG KHQFH D QHZ 5 H\ QROGV QXP EHU DQG D EHWWHU HVWLP DWH RI IL 7 KHQ UHSHDW (T WR FRQYHUJHQFH

,W VKRXOG EH QRWHG WKDW ERWK RI WKHVH SDUDOOHO SLSH FDVHV² ILQGLQJ HLWKHU 4 RU Kf ² DUH HDVLO\ VROYHG E\ ((6 LI UHDVRQDEOH LQLWLDO JXHVVHV DUH JLYHQ

(; $0 3/ (

$ VVXP H WKDW WKH VDP H WKUHH SLSHV LQ ([DP SOH DUH QRZ LQ SDUDOOHO Z LWK WKH VDP H WRWDO KHDG ORVV RI P &RP SXWH WKH WRWDO IORZ UDWH 4 QHJOHFWLQJ P LQRU ORVVHV

6 ROXWLRQ

)URP (T D Z H FDQ VROYH IRU HDFK 9 VHSDUDWHO\

$$Q_i = (\pi/4)d_i^2 V_i, \quad \text{Re}_i = \rho V_i d_i/\mu$$

* XHVVIXOO URXJKIORZ LQSLSH I 9 P V KHQH5H 9 G) UR
VWH0 RRG\ FKDUWHDGI UHFRPSXWH9 P V 4 P K >7KLVSUREOHP FDQDOV
EHVROYHGIURP (T @
1H[WXHVVIRUSLSH I § 9 § P V VWHQ5H DQGKHQFHI 9
P V 4 P K
) LQDOO JXHVVIRUSLSH I § 9 § P V VWHQ5H DQGKHQFHI
9 P V 4 P K
7KLVLVVDWLVIDFWRU\ FRQYHUJHQFH 7KHVROXWLRQ UDWLV

4 4 4 4 P K

7KHVHVWUHSLSHVFDUV WLPHVPRUHIORZ LQSDUDOOHODQWKH GRLQVHUHV
EES 7KLVH[DPSOHLVGHDOIRU((2QHHQWU\VWKHSLSHGDWD/ L G L ∈ L WKHIOXLGSURSHUWLHV !
WKH GHILQLWLRQ $Q = (π/4)d_i^2 V_i, Re_i = ρV_i d_i/μ DQG h_f = f_i (L_i/d_i) (V_i^2/2g) SOXVWKH &RROEURRNIRUPXOD IR
HDFKIULFWLRQIDFWRUI L 7KHUHLVQR QHGWRXVHUHLWHUDWLRQDQGHGDVVXFK DV(T 6SHFLI\ WKDWL !
DQG5H L ! 7KHQ LI RQHQWU\V4 4 L P V ((6 TXLFNO VROYHVIRUK P
&RQVHUVH LI RQHQWU\VK P ((6 VROYHVIRU4 P K

7KUHH5HVHUYRLU-XQFWLRQ

&RQVLGHUWKHWKUHH[DPSOHRI DWKUHHUHVHUYRLUSLSHMXQFWLRQ DVLQ) LJ F ,I DOOIORZ VDU
FRQVLGHUHGSRVLWLYHWRZDUGWKHMXQFWLRQ WKHQ

$$Q_1 + Q_2 + Q_3 = 0 \qquad (6.90)$$

ZKLFK REYLRXVO LPSOLHV WKDWRQH RUWZR RI WKHIORZ V PXVWEH DZ D\ IURP WKH MXQFWLRQ 7K
SUHVVXUH PXVWFKDQJH WKURXJK HDFK SLSH VR DV WR JLYH WKH VDPH WUXH SUHVVXUH DWWKHMXQFWLRQ ,
RWKHUZRUGV OHWWKH+* / DWWKHMXQFWLRQKDYHWKHHOHYDWLRQ

$$h_J = z_J + \frac{p_J}{\rho g}$$

ZKHUHS LVLQJDJHSUHVVXUHIRUVLPSOLFLW 7KHQWKHKHDGORVVWKURXJKHDFK DWXP LQJ S S
S JDJH DWHDFKUHVHUYRLUXUDFH PXWWEHVXFKWKDW

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J \qquad (6.91)$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

: HJXHVWKHSRVLWRQK DQGVRQH(TV IRU9 9 DQG9 DQGKHQFH4 4 DQG4 LWHUDWLQJ XQWLOWKHICRZ UDWHVEDODQFHDWWKHMXQFWLRQDFFRUGLQJWR(T ,I ZHJXHVVK WRKLJK WKHVXP 4 4 4 ZLOOEHQHJDWLYHDQGWKHUHPHG\ LVWRUHGXFHK DQGYLFHYHUVD

(; $0 3/ (

7DNHWKHVDPHWKUHHSLSHVDVLQ([DPSOH DQGDVVXPHWKDWWKH FRQQHFWWKUHHUHVHUYRLUVD WKHVHVXUIDFHHOHYDWLRQV

] P] P] P

)LQGWKHUHVXOWLQJICRZ UDWHVLQHDFKSLSHQHJOHFWLQJPLQRUORVVHV

6RXWLRQ

$VDILUVWJXHVV WDNHK HTXDOWR WKHPLGGOHUHVHUYRLUKHLJKW] K P 7KLVVDYHVRQ FDOFXODWLRQ 4 DQGHQDEOHVXVWRJHWWKHOD RI WKHODQG

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h	L_i/d_i
1	40	−20	0.0267	−3.43	−62.1	1250
2	40	60	0.0241	4.42	45.0	2500
3	40	0		0	0	2000
					$\Sigma Q = -17.1$	

6LQFHWKHVXP RI WKHICRZ UDWHVWRZ DUG WKHMXQFWLRQLVQHJDWLYH ZHJXHVVHGK WRKLJK 5HGXFHK WR P DQGUHSHDW

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h
1	30	−10	0.0269	−2.42	−43.7
2	30	70	0.0241	4.78	48.6
3	30	10	0.0317	1.76	8.0
					$\Sigma Q = 12.9$

7KLVLVSRVLWLYH 4 DQGVR ZHFDQQQHDUO LQWHUSRODWHWR JHWDQ DFFXUDWH JXHVV K § P 0 DNHRQHILQDOOLWW

Reservoir	h_j, m	$z_i - h_j$, m	f_i	V_i, m/s	Q_i, m^3/h
1	34.3	−14.3	0.0268	−2.90	−52.4
2	34.3	65.7	0.0241	4.63	47.1
3	34.3	5.7	0.0321	1.32	6.0
					$\Sigma Q = 0.7$

7KLV LV FORVH HQRXJK KHQFH Z H FDQ FXODWH WKDW WKH IORZ UDWH LV P K WKURXJK UHVHUYRLU EDODQFHG E\ P K DW D IURP UHVHUYRLU DQG P K DW D IURP UHVHUYRLU

2QH IXUWKHU LWHUDWLRQ Z LWK WKLV SURFHGXUH Z RXOG JLYH K P UHVXOWLQJ LQ4 í 4 DQG4 P K WR WKDW4 WR WKUHH SODFHV DFFXUDF\ 3HGDJRJLFDO VSHDNLQJ Z H Z RXO WKHQ EH[KDXVWHG

3LSH1HWZRUNV

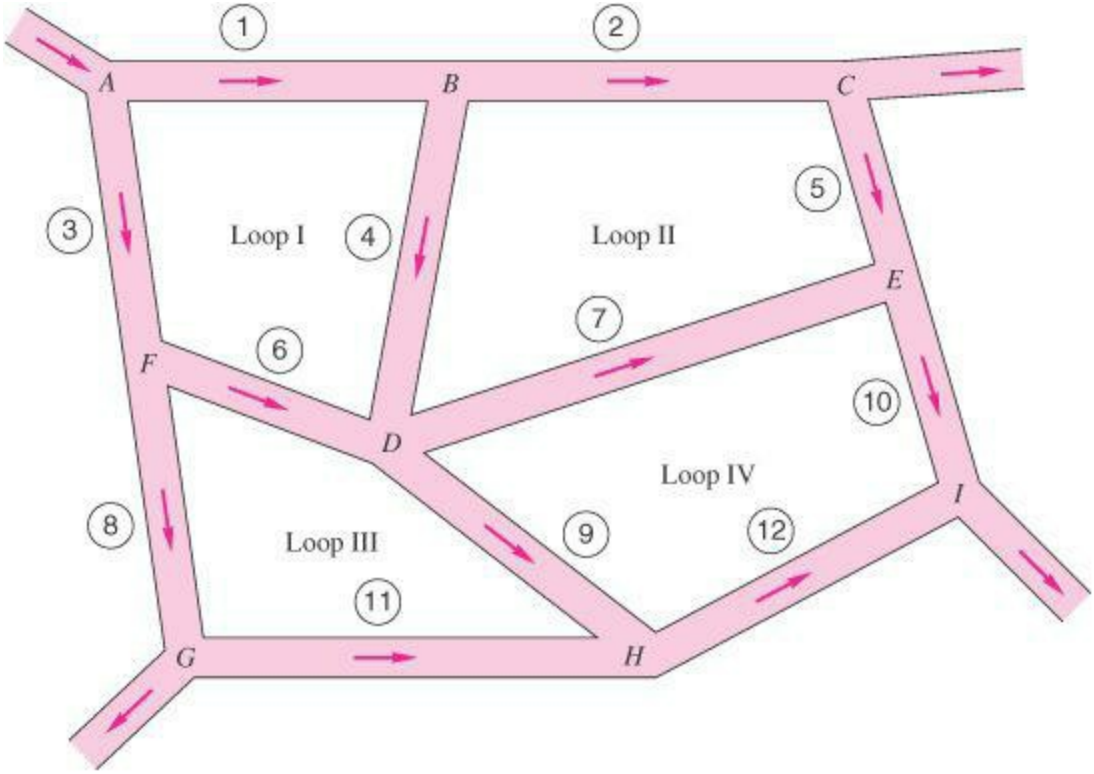
7KH XOWLPDWH DLP FDVH RI DPXOWLSLSH V\VWHP LV WKH SLSLQJ QHWZRUN LOOXVWUDWHG LQ) LJ 7KLV PLJK UHSUHVHQW D ZDWHU GLVWULEXWLRQ V\VWHP IRU DQ DSDUWPHQW RUVXEGLYLVLRQ RU HYHQ D FLW 7KLV QHWZRUN LV TXL FRPSO[DQJHUDLFDOO EXWRGRZ V WKH VDPH EDVLF UXOHV

7KH QHWZRUN LQR DQ NXQFWRQP XWEH]HUR

7KH QHWSUHVXUH FKDQJH DURXQG DQ FORVHG ORRS PXWEH]HUR ,QRWKHUZRUGV WKH+ / D HDFK MXQFWRQP XWKDYH RQH DQG RQH HOHYDWRQ

\$OOSUHVXUHFKDQJHVP XWVDWLV\ WKH0 RRG\ DQG PLQRUORVVIUFWRQFRUHDWRQV

%\ VXSSOLQJ WKHVH UXOHV WR HDFK MXQFWRQ DQG LQG-SHQG-QWORRS LQ WKH QHWZRUN RQH REWDLQV D V-RI VLPXOWDQHRXV HTXDWRQV IRU WKH IORZ UDWHV LQ HDFK SLSH OHJ DQG WKH+ / RU SUHVXUH DWHDF MXQFWRQ 6RXWRQ P D\ WKHQ EH REWDLQHG E\ QXP HULFDO LWUDWRQ DV ILLUWGHYHORSHG LQ D KDQ FDOFXODWRQ WHFKQLTXH E\ 3UR + DUG\ &URVV LQ > @ &RP SXWHU VROXWRQ RI SLSH QHWZRU SUREOP VLVQRZ TXLWH FRP P RQDQG FRYHUHG LQDVWDQGDUWRQHSHFLDO]HGWH[W> @1 HWZRUNDQDO VLV TXLWH XVHXOIRUUHDOZ DWUGLVWULEXWRQ V\VWHP VLI ZHOFDOEUDWG Z LWK WKH DFWXDO\VWHP KHDG GRV GDWD



)LJ 6FKHPDWLF RI DSSLSQJ QHWZRUN

([SHULPHQWDO XFWJ ORZV ' LIIXVHU3HUPDQFH

7KH0RRG\ FKDUWLV VXFK DJUHDWFRUHDWRQIRUWEHVRI DQ FURVVHFWRQZLWKDQ URXJKQHVVF IORZ UDWHWKDWZHPD\ EHGHXGHGLQRWKLQJWKDWKHZRUGRI LQWUQDOIORZ SUHGLFWRQLVDWRXIHH 1RWR 7KHWKHRU LVUHDEOHRQ IRUGXFWVRI FRQXDQWURVVHFWRQ \$VVRQDVWKHVHFWRQYDUHV ZHPXWUO SULQFLSDO RQH[SHULPHQWR GHWUPLQHWKHIORZ SURSHUWV \$VPHQWRQHGPDQ WPH EHRUH H[SHULPHQWDWRQLVDYLWDSDWR IOXLGPHFKDQFV

/LWHUDO VKRXDQGVRI SDSHUVLQWKHOWHUDWXUHSRWH[SHULPHQWDOGDWIRUVSHFLILF LQWUQDOOD H[WUQDOYLVFRXVIORZV :HKDYHDOHDG VHQVHYHUDOH[DPSOHV

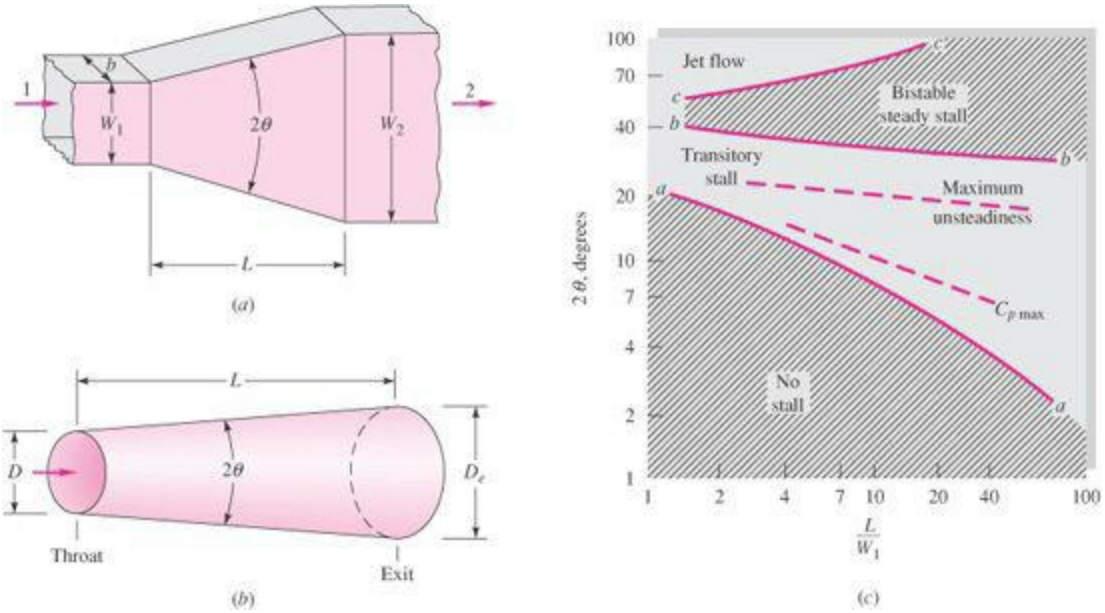
9RUWH[VKHGGLQJ IURP DF\OQGHU)LJ
' UDJ RI DSKHUHDQGDF\OQGHU)LJ
+\GUDXOF PRGHORI DGDP VSLOZD\ )LJ
5RXJK ZDOOSLSHIORZV)LJ
6HFRQGDU IORZ LQGXFWV)LJ
0 LQRUGXFWVRVFRHILFLHQW 6HF

&KDSWU ZLOOWHDWDJUHDWP DQ PRUHH[WQDOIORZ H[SHULPHQW HVSHFLDOO LQ6HF +HWZ VKDOOVKRZ GDWIRUQHWSHRI LQWUQDOIORZ WKHGLIIXVHU

' LIIXVHU3HUPDQFH

\$ GLIIXVHU VKRZ Q LQ)LJ D DQGE LV DQ H[SDQVRQ RU DUHD LQFUHDVH LQWQDOHG WR UHGXF

YHORFLWLQRUGHUWRUHFRYHUWKHSUHVVXUHKHDGRI WKHIORZ 5RXVHDQG,QH> @UHDVRQHGWKDWWKHDLU EHKLQGWKHGE\ FXWRPHUVRI WKHHDUO 5RPDQ DERXW 6&\$ ' 6& ZDVUXSSO\LQJWKH ZDWUIORZHG FRQGXLWRXV DQG ZDV EORFNHG DFFRUGLQJ WR SLSH VL]H 7KH LQJHQLRXV FXWRPHU GLVFRYHUHGWKDWWKH FRXOGLQFUHDVHWKHIORZ UDWHDFWXDOO\ IODULQJ WKHRXWOHWRI WKH SLSH



) LJ ' LIIXVHUJHRPHWUL DQGWSLFDOIORZ UHJLPHV *a* JHRPHWUL RI DIODWZDOOHGGLIIXVHU *b* JHRPHWUL RI DFRQFDOGLIIXVHU *c* IODWGLIIXVHUVWDELOLW\PDS*(From Ref. 14, by permission of Creare, Inc.)*

(QJLQHHUV KDYH DOZD\V GHVLJQHG GLIIXVHUV WR LQFUHDVH SUHVVXUH DQG UHGXFH NLQHWLF HQHUJ\ R GXFWVIORZ V EXWKQWKDERXW GLIIXVHUGHVLJQZDVDFRPELQDWLRQ RI DUWOXFN DQG YDUDP RXQ RI HPSLULFLVP 6PDOO FKDQJHV LQ GHVLJQ SDUDPHWHUV FDXVHG ODUJH FKDQJHV LQ SHUIRUPDQFH 7K %HUQRXOOLHTXDWLRQVHPHGKJKO VXSHFWDVDXHXOWRO

1HJOHFWLQJ ORVVHVDQG JUDYLW\HIHFW WKHLQFRPSUHVVLEOH%HUQRXOOLHTXDWLRQSUHGLFWVWKDW

$$p + \tfrac{1}{2}\rho V^2 = p_0 = \text{const} \qquad (6.92)$$

ZKHUHS LVWKHVWDWLQDWLRQSUHVVXHWKHIOXLGZRXOGDFKLHYHLI WKHIOXLGZHUHVORZHGWRUHVW9 ZLWKRXWORVVHV

7KHEDVLFRXSXWR DGLIIXVHULVWKHSUHVVXHUHFRYHU FRHILFLHQW&S GHILQHGDV

$$C_p = \frac{p_e - p_t}{p_{0t} - p_t} \qquad (6.93)$$

ZKHUH VXEVFULSWH DQGWPHDQWKHH[LWDQG WKHWKURDWRULQOHW UHVSHFWLYHO +LJKHU&S PHDQ EHWWUSHUIRUPDQFH

&RQVLGHUWKHIODWZDOOHGGLIIXVHULQ) LJ D ZKHUHVHFWLRQ LVWKHLQOHWDQGVHFWLRQ WKH H[\$SSOLFDWLRQRI %HUQRXOOL¶VHTXDWLRQ WRWKLVGLIIXVHUSUHGLFWVWKDW

$$p_{01} = p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2 = p_{02}$$

or
$$C_{p,\text{frictionless}} = 1 - \left(\frac{V_2}{V_1}\right)^2 \tag{6.94}$$

0 HDQZ KLOH VWDG RQH GLP HQVLRQDO FRQVLGHUV RXG UHTXLUHVWDW

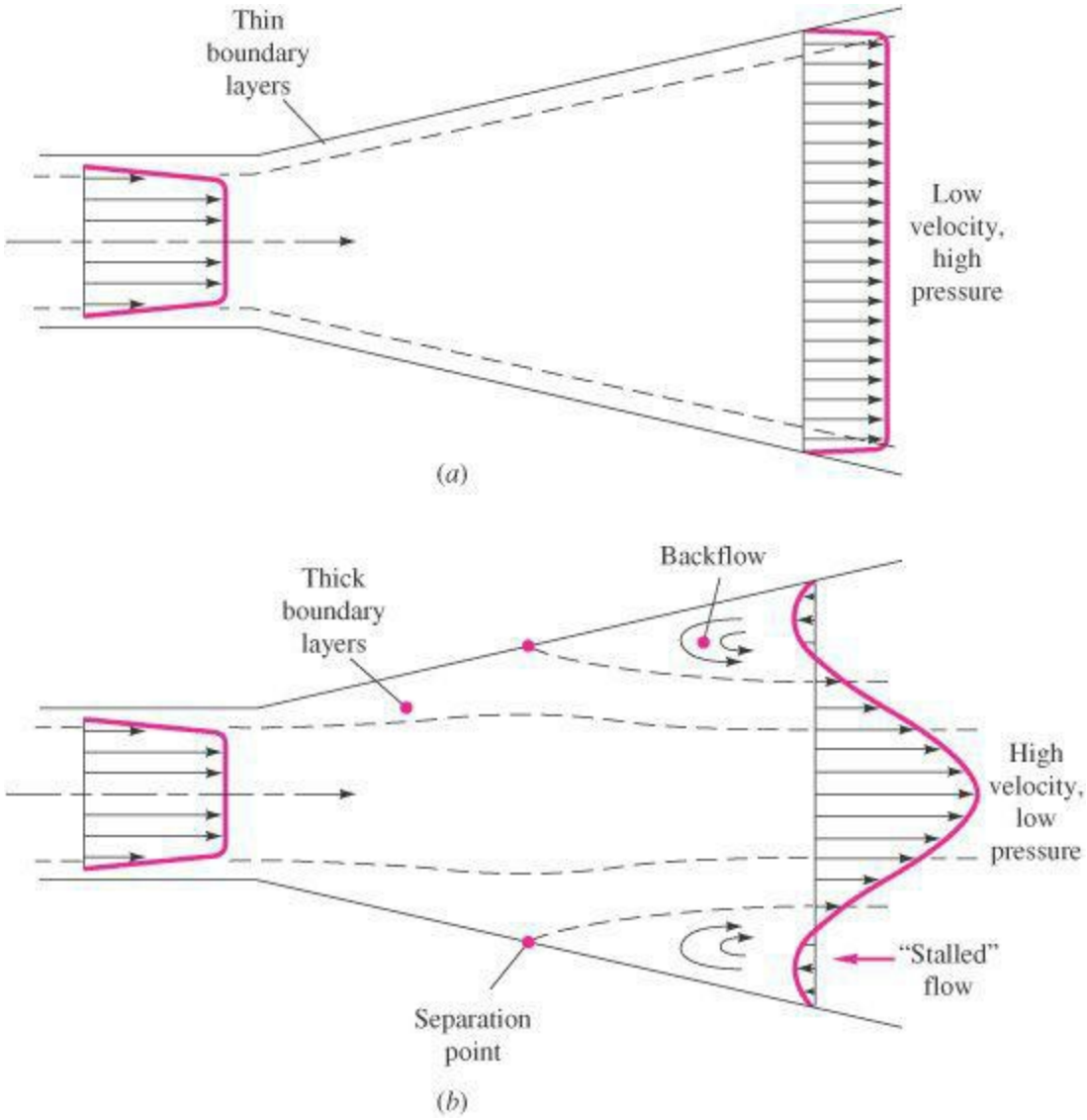
$$Q = V_1 A_1 = V_2 A_2 \tag{6.95}$$

&RPELQLQJ DQG ZH FDQ ZULWH WKH SHUFRUPDQFH LQ WHUPV RI WKHDUHD UDWLR $5 $ $ ZKLFK LVDEDVLF SDUDPHWHULQGLIIXVHUGHVLJQ

$$C_{p,\text{frictionless}} = 1 - (\text{AR})^{-2} \tag{6.96}$$

$ WSLFDOGHVLJQZRXG KDYH $5 IRUZKLFK(T SUHGLFWV&S RUQHDUO IXO UHFRYHU %XWLQIDFWP HDXUHG YDOXHVRI &S IRUWKLV DUHDUDWLR > @DUHRQO DVKJK DV DQG FDI EHDVGRZ DV

7KHEDVLF UHDVRQIRUWKHGLVFUHSDQF\ LVIORZ VHSDUDWLRQ DVVXPHWKHGLQ) LJ 7KHLQFUHDVLQ SUHVVXUH LQ WKH GLIIXVHULV DQ XQDYRUDEOHJUDGLHQW6HF ZKLFK FDXVHV WKH YLVFRXV ERXQGDU\ OD\HUV WR EUHDN DZ D IURP WKH ZDOOV DQG JURZ WKH SRUP WKH &RPSXWDWLRQDO IOXLG G\QDPLFV &)' FDQQRW SUHGLFWWKLVEHKDYLRU



) LJ ' LIIXVHU SHUIRUPDQFH *a* LGHDO SDWWHUQ ZLWK JRRG SHUIRUPDQFH *b* DFWXDO PHDVXUHG SDWWHUQ ZLWK ERXQGDU\ OD\HUV HSDUDWLRQ DQG UHVXOWLQJ SRRU SHUIRUPDQFH

$ VDQDGGHG FRPSOLFDWLRQ WR ERXQGDU\ OD\HUV HSDUDWLRQ WKH IORZ SDWWHUQ VLQDGLIIXVHUDUHKJKO YDULDEOH DQG ZHUH FRQVLGHUHG P\ WHUIRXVDQG HUDWF XQLO ZKHQ. OQHUHY HDGWKH WUXFWXUH WKH HSDWWHUQ ZLWK IORZ YLVXDOL]DWLRQ VLQDQ[LQVDPSOHZDWHUFKDQQHO

$ FRPSOHWH VWXEELQJ RI GLIIXVHU IORZ SDWWHUQ ZDV SXEOLVKHG LQ E\ )R[DQG . OQ > @DV VKRZQ LQ) LJ F 7KHUH DUH IRXU EDVLF UHJLRQV %HORZ OQHDU WKH UHV WUDG YLVFRX IORZ QR VHSDUDWLRQ DQG PRGHUDWH JRRG SHUIRUPDQFH 1RWH WKDW HYHQ D YHU VKRUW GLIIXVHU ZLO VHSDUDW RUXDOO LI WKDO DQJOHV JUHDWHUWKDQ *f*

%HZHHQ OQHVDD DQGEE LV D WDQVLWLRQ VWDOO SDWWHUQ ZLWK VWURQJ XQVWDG\ IORZ %HV SHUIRUPDQFH KLJKHVW&S RFFXUV LQ WKLV UHJLRQ 7KH WKLG SDWWHUQ EHZHHQEE DQG FF LV VWDG ELXDEOH WDOO IURP RQH ZDOO RQ 7KH WDOO SDWWHUQ PD\ IOLS IRS IURP RQH ZDOO WR WKH RWKHU DQ SHUIRUPDQFH LV SRRU

7KH IRXUWK SDWWHUQ DERYH OQHFF LV WKH WDOO VHSDUDWLRQ LV VR JURVV DQ SHUYDVLYH WKDW WKH P DLQXWDP LJQRUHV WKH ZDOO DQG VLPSO SDWHV RQ WKURXJK DWQ HDU FRQWDQWD 3HUIRUPDQFH LV H[WHPHO SRRU ULQWKLV UHJLRQ

' LPHQVLRQDO DQDO VLV RI DIODWZ DOHG RUFRQFDOGLIIXVHUV KRZ V WKDW&S VKRXOG GHSHQG RQ WK IRORZ LQJ SDUDPHWHUV

$Q VZRRI WKHIROORZLQJHRPHWFSDUDPHWUV

$UHDUDWR$5 $ $ RU ' H'

' LYHUJHQFHDQJOH

6OHQGHUQHVV/ : RU/ '

,QLWLDOVHLVRGVQXPEHU5H 9: RU5H 9'

,QLWLDOMDFKQXPEHU0D 9 D

,QLWLDEERXQGDU\ OD\HUEORFNDJHIDFWRU% $ $ ZKHUH$ LVWKHZDOODUHDEORFNHG R

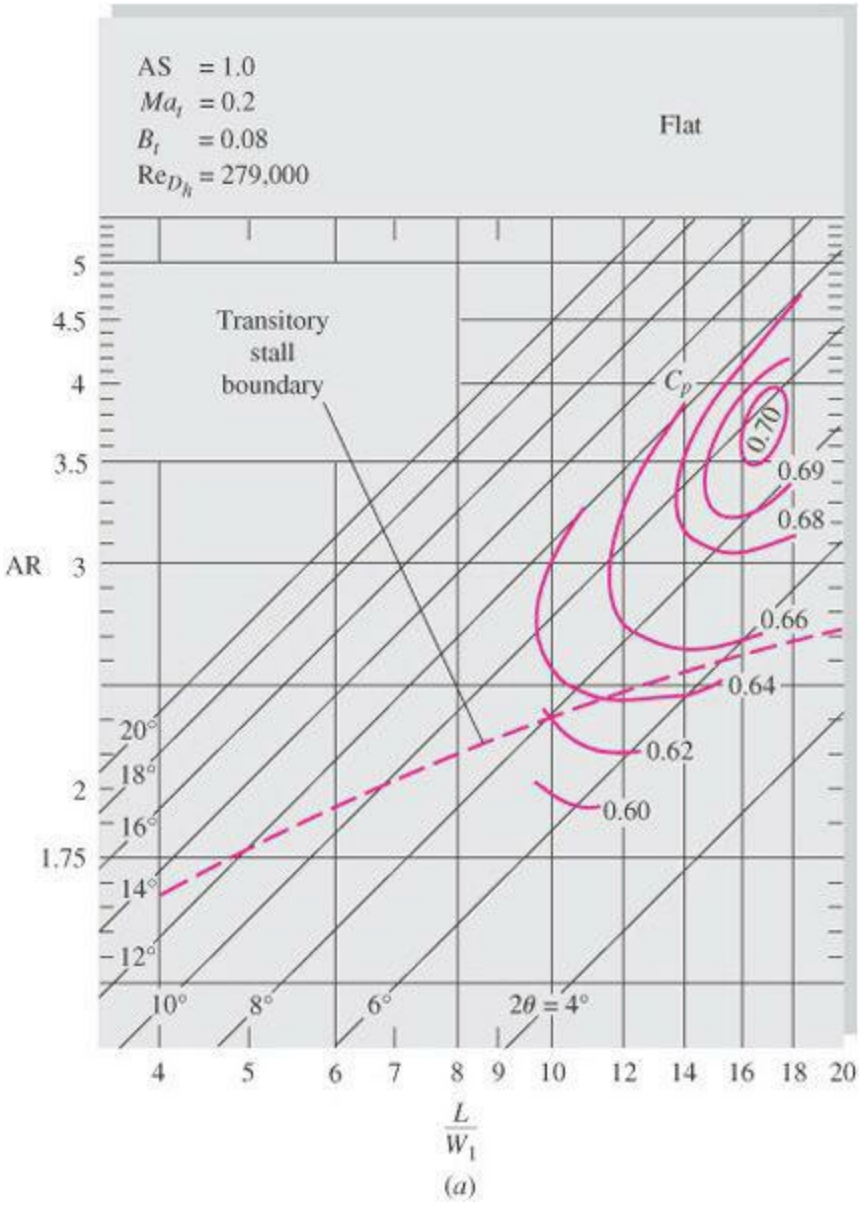
GLVSODFHG E\ WKHUHDUGHGERXQGDU\ OD\HUIORZ LQWKHLQLWLDWLQJ%YDULHVIURP WR

$ IODWZDOOHGGLIIXVHUZRXOGUHTXLUHQDGGLWLRQDOVKDSHSDUDPHWUL]DWLRQVVHFWLRQ

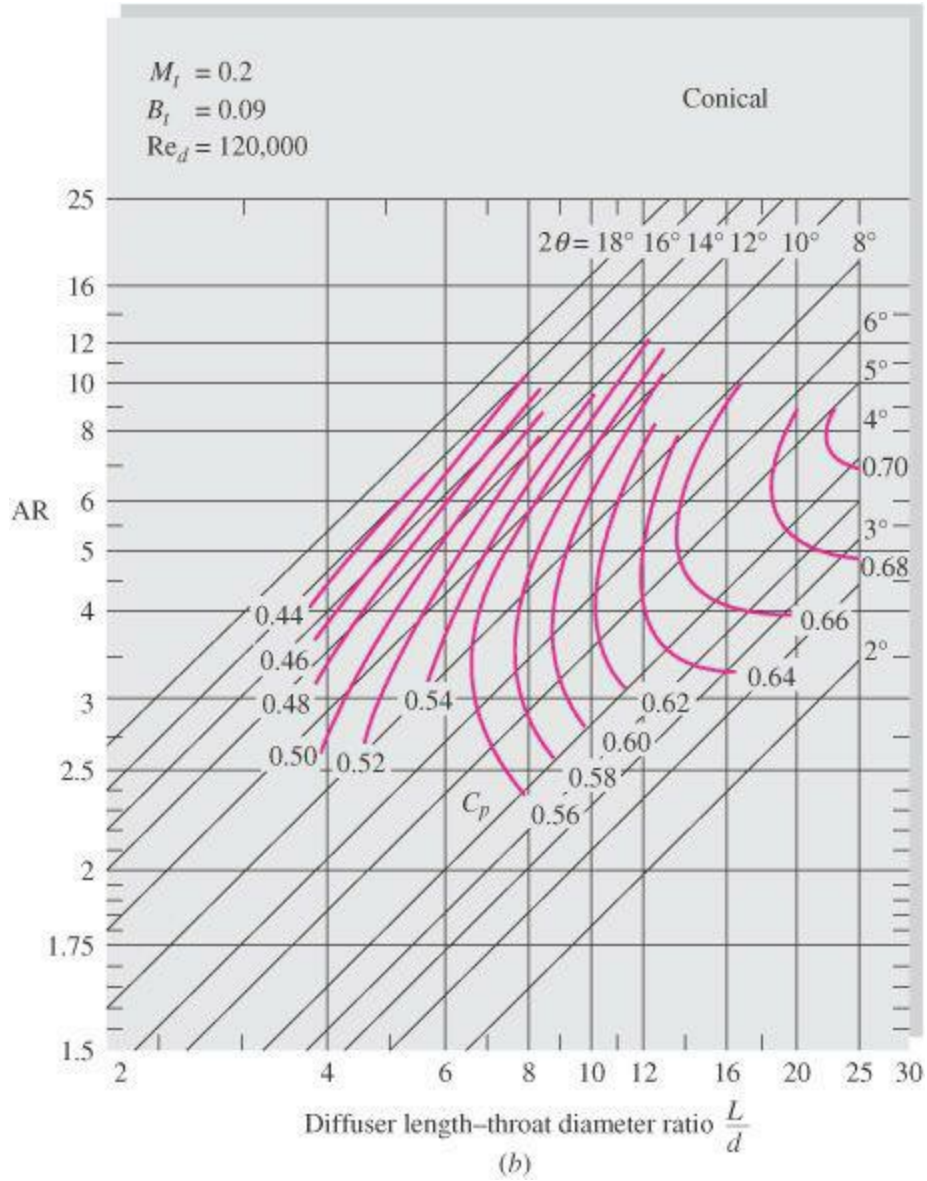
- $VSHFWDWR$6 E:

(YHQZLWKVLVIRUPLGDEOHOWZHKDYHPLWWGILYHSRVLEOHPSRUDQWHIHFW LQDWWXEXOHF-
LQDWWZLOOLQDWSURLOHYRUWLFLW VXSHUPSRVHGSXOVDWRQV DQGGRZQWHP REVWUXFWLRQ DOOR ZKH
RFFXULQSUDFWLFDOPDFKLQHU DSSOLFDWLRQV

7KHWKUHHPRWWPSRUDQWSDUDPHWUV DUH$5 DQG% 7\SLFDOSHURUPDQFHPDSVIRUGLIIXVHU
DUH VKRZQ LQ) LJ) RUWKLV FDVHRI VR SHUFHQWEORFNDJH ERWKWKH IODWZDOOHGDQG FRQFD
WWSHV JLYHDERXWWKHVDPHPD[LPXP SHURUPDQFH &S EXWDWGLIHUHQWGLYHUJHQFHDQJOHV
IODWWYHUWV ƒ FRQFDO %RWKWWSHVIDOOIDUVKRUWRI WKH%HUORXOLHWMDWWRI &S IODWDQ
FRQFDO SUIPDULO EHFDXVHRI WKEORFNDJHHIHFW



) LJ D 7\ SLFDOSHUIRUPDQFHPDSVIRUIODWZDOODQGFRQFDYLGHUVDWWLPLODUURSHUDWLQ
FRQGLWLRQV IODWZDOO*(From Ref. 14, by permission of Creare, Inc.)*



) LJ E 7\ SLFDOSHUFRUP DQFH P DSV IRU IODWZ DOODQG FRQLFDOGLIIXVHUV DWWP LODUSHUDWQ FRQGLWRQV FRQLFDOZ DOO *(From Ref. 14, by permission of Creare, Inc)*

) URP WKH GDWD RI 5H Z H FDQ GHWUPLQH WKDW LQ JHQHUDO SHUFRUP DQFH GHFUHDVHV Z LV EORFNDJH DQG LV DSSUR[LP DWHO WKH VDP HIFRUEORFNDJH DOHG DQG FRQLFDOGLIIXVHUV DV VKRZ QLQ7 DEC , QDOOFDVHV WKHEHWWFRQLFDOGLIIXVHULV WR SHUFHQWDQJHUWKDQ WKHEHWWDZ DOHG GHVLJ 7KHUHRUH LI GHQJWK LV OP LWG LQ WKH GHVLJQ WKH IDZ DOHG GHVLJQ Z LOOJLYH WKHEHWWUSHUFRUP DQFH GHSHQGLQJ RQGXFWURVVVHFWRQ

7DEOH 0 D[LPXP ' LIIXVHU3HUFRUP DQFH' DWD> @

Inlet blockage B_t	Flat-walled		Conical	
	C_p,max	L/W_1	C_p,max	L/d
0.02	0.86	18	0.83	20
0.04	0.80	18	0.78	22
0.06	0.75	19	0.74	24
0.08	0.70	20	0.71	26
0.10	0.66	18	0.68	28
0.12	0.63	16	0.65	30

7KHH[SHULP HQWDO GHVLJQRI DGLIIXVHULVDQH[FHOOHQW[DPSOHRI DVXFFHVVIXOLWHUDWXUHSWWRP LQPL]I

VKHXQGHVLUDEOHIHIHFWRI DGYHUWHSUHVXUHJUDGLHQWDQGIORZ VHSDUDWRQ

) OXLG0 HWHUV

$ OPRVWDOOSUDFWLFDOIOXLGHQJLQHHULQJ SUREOPVDUHDVRFLDWHGZLWKWHQHGIRUDQDFFXUDWHIORZ PHDVXUHPHQW7KHUHLVDQHGVRPHDVXUHORFDOSURSHUWLV YHORFLW SUHVXUH VWSHUDXH GHQVLW YLVFRVLW VKUEXOHQWLQWQVLW LQWUDUDG SURSHUWLV PDV IORZ DQG YROXPH IORZ DQGJORED SURSHUWLV YLVXDOLDWRQRI WKHQWHIORZ ILHOG : HVKDOOFRQHQWDWLQWKLVVHFWRRQRQYHORFLW DQ YROXPHIORZ PHDVXUHPHQW

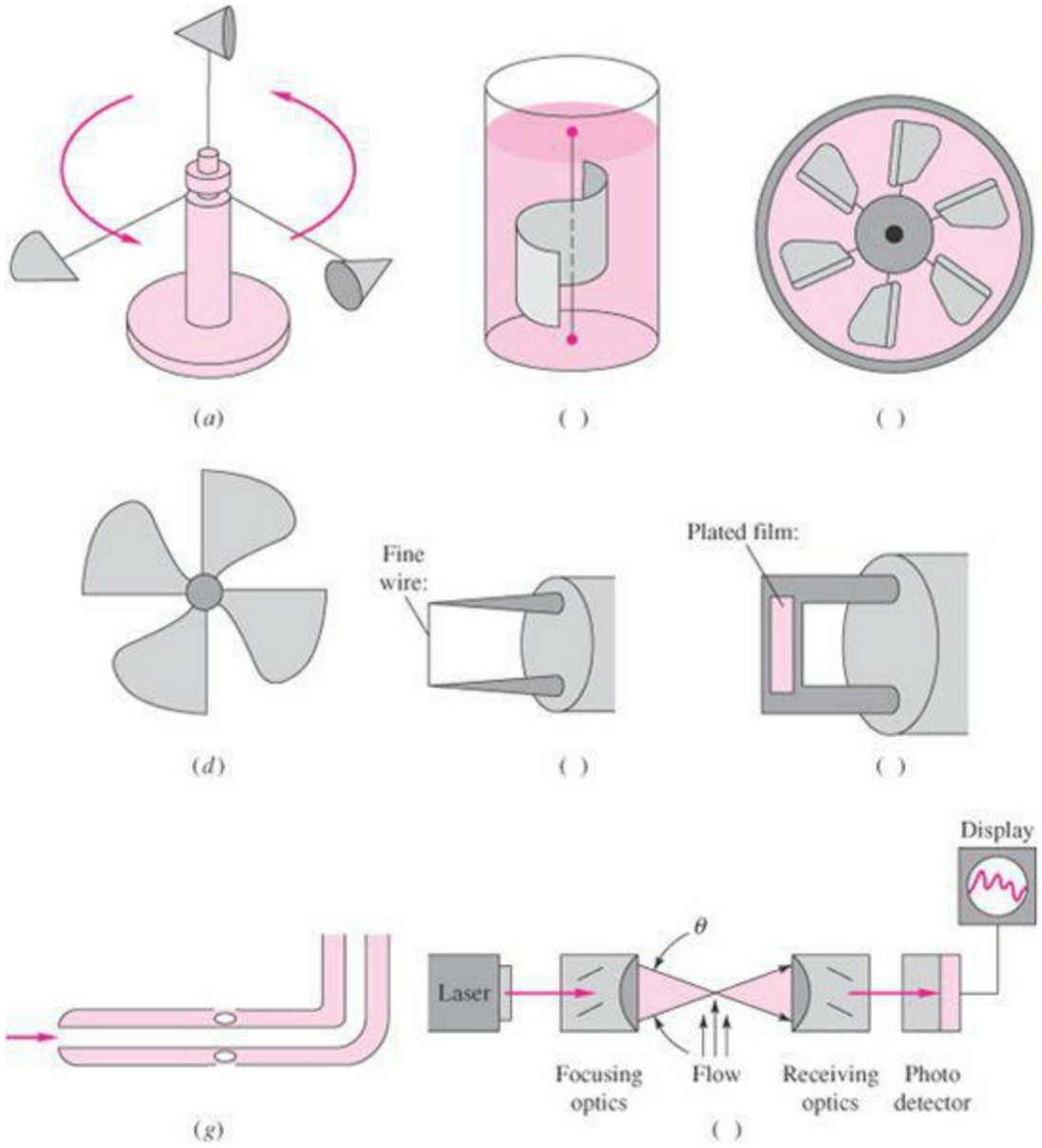
: HKDYHGLVFXVHG SUHVXUHPHDVXUHPHQWLQ6HF 0 HDVXUHPHQWRI RWKHUWKHUPRG\QDPL SURSHUWLV VXFKDVGHQVLW VWSHUDXH DQG YLVFRVLW LVEHIRQGWKHVFRSHRI WKLVWH[WDQGLVWHDG LQVSHFLDOLHGERRNVXFKDV5HV DQG * OREDOYLXDOL]DWRQWFKQLTXHVZHUHGLVFXVHGLQ6HF IRUORZ VSHHGIORZV DQGWKHVSHFLDOSURSWLFDOWFKQLTXHVXVHGLQKJK VSHHGIORZV DUHWHDWHGLQ 5H RI &KDS) ORZ PHDVXUHPHQWWFKHPHV VXLWDEOHIRUSHQFKDQQHODQG RWKHUIUHHVXUDFH IORZ VDUHWHDWHGLQ&KDS

/ RFDO9 HORFLW0 HDVXUHPHQW

9HORFLW DYHUDJHG RYHUD VPDOOUHJLRQ RUSRLQWFDQEHPHDVXHG E\ VHYHUDOGLIHUHQWSK\VLFD SULQFLSOHV OXVHGLQRUGHUIRI LQFUHDVLQJ FRPSOH[LW DQG VRSKLWWFDWRQ

7UDQVIHUU RI IORDWVRUQHXWDO EXR\ DQVSDUWFOHV
5RWDWQJ PHFKDQLFDOGHYLFHV
&XS DQHPRPHWU
6DYRQLXVURWRU
3URSHOOHUPHWU
7XUELQHPHWU
3LWRWWXEHWVWDWLF3UHVVXUHWDSV) LJ
(OHFWURPDJQHWF FXUUHQWPHWU
+RWZLUHVDQGKRWILOPV
/DVHUGRSSOHUDQHPRPHWU/ ' $

6RPHRI WKHVHPHWKRGVDUHVXLWDEOHIRGLQ) LJ



)LJ (LJKWVFRPPRQYHORFLWPHWHUV *a* VKUHHFXSDQHPRPHWHU *b* 6DYRQLXVURWRU *c* VXUELQHPRXQWHG LQ DGXFW *d* IUHHSURSHOOHU PHWHU *e* KRWZLUH DQHPRPHWHU *f* KRWILOP DQHPRPHWHU *g* SLWRWVWDWLFWXEH *h* DVHUGRSSOHUDQHPRPHWHU

) ORDWVRU%XR\DQW3DUWLFOHV

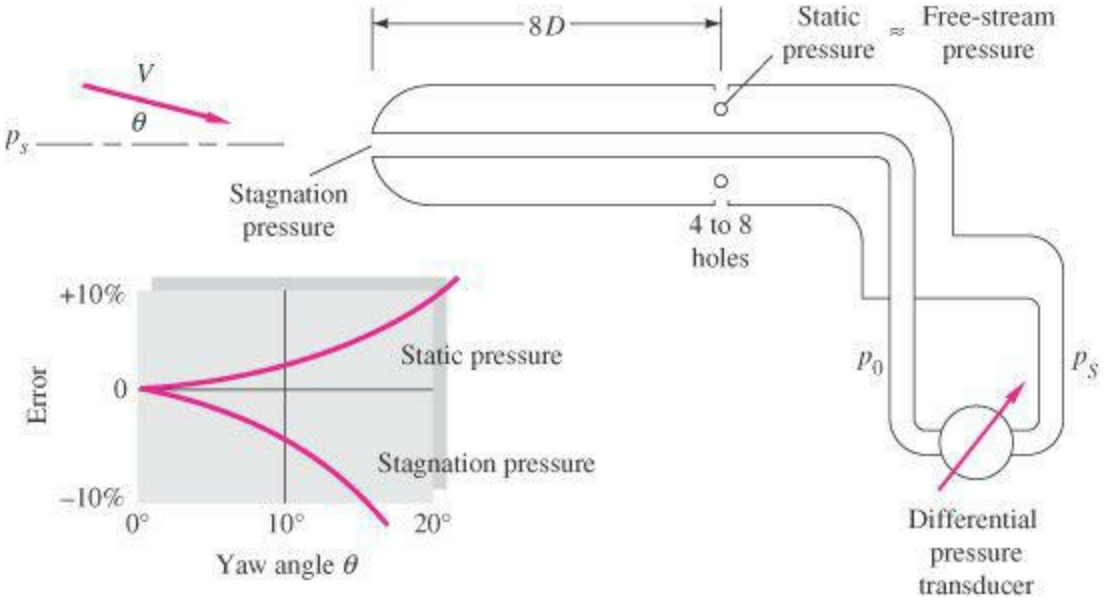
$ VLPSOHEXWHIIHFWLYHWHFKQLTXHIRU ORFDWLQJ YHORFLWYFDQEHIRXQGIURP YLVLEOHSDUWFOHVHQGDQHGL VKHIORZ ([DPSOHVLQFXGHIODNHVRQWKHVXUIDFHRI DFKDQQHOIORZ VPDOOQHXWUDOO EXR\DQWSKHUH PL[HGZLWKDQTXLG RUKYGURJHQEXEEOHV 6RPHWPHVJDVIORZ VFDQEHHVWPDWHGIURP WKHPRWRQR HQGDQHGGXWSDUWFOHV 2QHPXWHVWEDK ZKHWKHUWKHSDUWFOHPRWRQWKO VLPXODWHVWKHIOXL PRWRQ)ORDWDUHFRPPRQO XVHGWRWDFNWKHPRYHPHQWRI RFHDQZDWHUVDQGFDQEHGHVLJQHGV PRYHDWWKHVXUIDFH DORQJ WKHERWWRP RUDWDQ JLYHQGHSWK> @0 DQ RIILFLDOWGDOFXUHQWFKDU > @ZHUHREWDLQHGE\ UHDVLQJ DQG WPLQJ DIORDWQJ VPDUWFKHGWR DHQWKRI WWLQJ 2QHFD UHDVHZKROHJURXSRI VSDUVWRGHWHUPLQHDIORZ SDWWHUQ

5RWDWLQJ6HQVRUV

7KH UROOLQJ GHYLFHV RI)LJ D VRG FDQ EH XVHG LQ HLWKHU JDVHV RU OLTXLGV DQG WKHLU URODWR UDWH LV DSSUR[LPDWHO SURSRUWLRQDO WR WKH IORZ YHORFLW 7KH FXS DQHPRPHWHU)LJ D DQ 6DYRQLXV URWRU)LJ E DQG D VURODWH WKH VDPH ZD\ UHJDUGOHVV RI IORZ GLUHFWLRQ 7KH DU SRSXODU LQ DWPRVSKHULF DQG RFHDQRJUDSKLF DSSOLFDWLRQV DQG FDQ EH IILWWHG ZLWK D GLUHFWLRQ YDQH WR DOOJQ WKHP VHOYHV ZLWK WKH IORZ 7KH GXFWHG SURPHOOHU)LJ F DQG IUHH SURPHOOHU)LJ G PHWHUV PXWHEHDUJQHG ZLWK WKH IORZ SDUDOOHOLVP RI URWDWLQ 7KH FDQVHQVHUYHU EHFDXVH WKH ZLOOLQJQHVV RI IORZ FRQVOXVLRQV $OWKHVHURWDWLQ VHQVRUV FDQEH DXWFKHG WR RSSRVLWH GLUHFWLRQ $OWHUQDWLQ VHQVRUV FDQ EH HPSOR\HG E\ HDFK RP DJQHW FRUSS UIQ GHYLFHV IRUHLWKHUDFRQLQXRXVRUDGLJLWDOUHDGLQ RI IORZ YHORFLW $OKDYH WKH GLVDGYDQWDJH RI EHLQJ UHODWLYHO CDJH DQG WKXV QRWUHSUHVHQWLQ D [3SRLQW

3LWRW6WDWLF7XEH

$ VOHQGHUWXEH DQJHG ZLWK WKH IORZ)LJV J DQG FDQPHDVXUHORFDOYHORFLWE\ PHDQ RI DSUHVVXUHGLIIHUHQFH ,WKDVVLGHZDOKROHVWR PHDVXUHWKHVWDWF SUHVXUHS, LQWKHPRYLQJ VWUHDP DQGDKROHLQWKHIURQWWR PHDVXUHWKHVWDJQDWQ SUHVXUHS ZKHUHWKHVWUHDP LVGHFHOHUDWHGWR]HU YHORFLW ,QVWDG RI PHDVXULQJ S RUS, VHSDUDWH LWLV FXVWRPDU WR PHDVXUHWKHUGLIIHUHQFH ZLWK VD\ DWWDQGXFHU DVLQ) LJ



) LJ 3LWRWVWDWF VXEH IRUFRP ELQHG PHDVXUHPHQWV VWDWF DQG VWDJQDWQ SUHVXUH LQ PRYLQJ VWUHDP

,I 5H ! ZKHUH' LV WKH SURBHGLDPHWU WKH IORZ DUXQG WKH SURBHLVQHDU IUHHVWUHDQG%HUQRXOOL[V UHODWLRQ(T DSSOLHVZLWKJRRG DFFXUDF\ )RULQFRP SUHVVLEOHIORZ

$$p_s + \tfrac{1}{2}\rho V^2 + \rho g z_s \approx p_0 + \tfrac{1}{2}\rho(0)^2 + \rho g z_0$$

$ VXP LQJ WKDWWKHHOHYDWRQSUHVXUHGLIIHUHQFH! J]V í] LVQHJOLJLEOH WKLVUHGXFHVWR

$$V \approx \left[2\frac{(p_0 - p_s)}{\rho}\right]^{1/2} \qquad (6.97)$$

7KLV LV WKH3LWRWVRUPXOD QDPHG DIWHU WKH) UHQFK HQJLQHHU + HQUL GH 3LWRWZKR GHVLJQHG WK
GHYLFHLQ

7KH SUIP DU GLVDGYDQWDJHV RI WKH SLWRWWXEH LV WKDWLWP XVWEH DDJQHG ZLWK WKH IORZ GLUHFWRQ
ZKLFK PD\ EH XQNQRZQ) RU \DZ DQJOHV JUHDWHUWKDQ WKH UHDGHVXEVWDQWLDOHUURUVLQEHWKWKHS DQ
S\ PHDVXUHPHQWV DVVKRZQLQ) LJ 7KHSLWRWWXEHLVXVHXOLQGXVWULDOHQYLURQPHQWV IRUJDVHV
DFRPSUHVVLEOHDVFRUUHFWRQVLQHFWDU LI WKHVWHDP 0 DFKQXPEHULVKLJK &KDS %HFDXVHRI WK
VORZ UHVSRQVHRI WKHIOLGILOOHGWXEHVGHDGLQJ WR WKHSUHVXUHVHQVRUV LWLVQRWXVHXOIRUXQVWHDG
IORZ PHDVXUHPHQWV ,WGRHVUHPEDLSRLQWDQGFDQEHPDGHVPDOOHQRXJK WR PHDVXUH IRU[DPSOH
EORRGIORZ LQDUWHULHVDQGYHLQV ,WLVQRWXVLDEOHIRUORZ YHORFLWPHDVXUHPHQWQJDVHVEHFDXVHF
WKHVPDOOSUHVXUHGLIIHUHQFHVGHYHORSHG)RU[DPSOH LI 9 IWVLQWDQGDU DLU IURP (T
ZH FRPSXWHS í S HTXDOWR RQO GEI IW 3D 7KLV LV EHORZ WKH UHVROXWRQ RI PRV
SUHVXUHJDJHV

(OHFWURP DJQHWLF0 HWHU

,I DPDJQHWLF ILHOG LV DSSOLHG DFURVV DFRQGXFWQJ IOXLG WKH IOXLG PRWRQ ZLOOLQGXFH D YROWDJ
DFURVV WZR HOHFWURGHVSODFHGLQRUQHDUWKHIORZ 7KHHOHFWURGHVFDQEHWXWHPQHGRUEXWWQRWK
ZDOO DQGWKH\ FDXVHDLWHRUQR IORZ UHVLVWDQFH 7KHRXSXWV YHU VWURQJ IRUKLJKO FRQGXFWQJ IOXLG
VXFK DV OTXLG PHWDOV 6HDZ DWHUDOR JLYHV JRRG RXSXWDQG HOHFWURP DJQHWF FXUHQWPHWHU DUH L
FRPPRQXVHLQRFHDQRJUDSK\ (YHQORZ FRQGXFWLYLWIUHKZDWUFDQEHPHDVXUHG E\ DPSOLI\LQJ WK
RXSXWDQGLQXODWQJ WKHHOHFWURGHV &RPPHUFLDOLQXWPHQWDU HYDLODEOHIRUPRVWOTXLG ORZ VEX
DUHUHDWLYHO FRXQW (OHFWURP DJQHWF ORZ PHWHUV DUH WHDGLQ5H

+ RW/ LUH$ QHP RP HWHU

$ YHU ILQHZLUH G PP RUOHVV KHDWHGEHZHHQWZR VPDOOSUREHV DV LQ) LJ H LV
LGHDO VXLWDWR PHDVXUHUDSLGO IOXFWXDWQJ IORZ V VXFK DV WKH WXUEXOHQWHUXQGDU CDYHU 7KH LGH
GDWDVEDFNVWR ZRUNE\ / 9 . LQJ LQ RQKDWRVIURP ORQJ WKLQF\OQGHUV ,I HOHFWF SRZHUL
VXSSOHG WR KDWKH F\OQGHU WKH ORV YDUHV ZLWK IORZ YHORFLW DFURV WKH F\OQGHUDFFRUGLQJ W
. LQJ¶VODZ

$$q = I^2R \approx a + b(\rho V)^n \qquad (6.98)$$

ZKHU $n \approx \frac{1}{3}$ DVYHU ORZ 5H\QROGVQXPEHUVDQG HTXDO $\frac{1}{2}$ DWKJK 5H\QROGVQXPEHUV 7KHKRWZLU
QRUPDOO RSHUDWV LQ WKH KLJK 5H\QROGVQXPEHUUDQJH EXWWKRXGG EH FDOEUDWG LQ HDFK VLWXDWQ W
ILQG WKH EHWWLVD E DQGQ 7KH ZLUHFDQEHRSHUDWG HLWKHUDWFRQWDQWF XUUHQW VR WKDWWHVLWDQFH5
LVDPHDVXUHRI 9 RUDVFRQWDQWWHVLWDQFH5 FRQWDQWWHPSHUDWXUH ZLWK, DPHDVXUHRI YHORFLW ,
HLWKHUFDVH WKHRXSXWV DQRQOLQDUIXQFWQRQ 9 DQG WKHHTXLSPHQWKRXG FRQWLQDOOHUUHU
SURGXFHFRQYHQLHQWYHORFLWGDW 0DQ YDUHWVRI FRPPHUFLDOKRWZLUHHTXLSPHQWDYDLODEOH D
DUHGR LWIRXUHOGHVJQ/> @([FHOHQWGHWLQGGLVFXVLRQV RI WKHKRWZLUHDUHJLYHQLQ5H
%HFDXVHRI LWIUDJLW WKHKRWZLUHLVQRWXVLWGWRDQDOTXLGIORZ V ZKRVHKLJKGHQVLWDQGHQWDLQH

VHGLPHQWZ LOONQRFN WKHZ LUHULJKWRI $ PRUHVXEWOH\HWTXLWVHQVLWLYHDSSOLFDWLRQ
PHDVXUHPHQWVWKHKRWWLP DQHPRPHWU) LJ I $ WKLPHDQDF ILP XVXDOO SDWQP LVSDWW
RQRXDUHDLYHO WKLFNVXSSRUWZ KLFK FDQEHDZ HGJH DFRQH RUDF\ QQGHU 7KHRSHUDWRQLVVLPLO
WRWKHKRWZ LUH 7KHFRQHJLYHVEHWWHVSRQVHEXWLVODEOHWRHURUZ KHQWKHIORZ LV\ DZ HGWRLWVD[LV
+RWZ LUHV FDQHDLO EH DUUDQJHG LQ JURXSV WR PHDVXUH VZR DQG WKUHH GLPHQVLRQDO YHORFLW
FRPSRQHQW

/ DVHU ' RSSOHU $ QHP RP HWHU

,QWKH/ ' $ DODVHUEHDP SURYLGHVKJKO IRFXVHG FRKHUHQW PRQRFKURPDWF OJKWWKDWLV SDWH
WKURXJKWKHIORZ : KHQWKLVOJKWLV VFDWWHUHG IURP DPRYLQJ SDUWFOHLQWKHIORZ DVDQRQLV REVHUYH
FDQ GHWHFW D FKDQJH RUGRSSOHU VKLIW LQ WKH IUHTXHQF\ RI WKH VFDWWHUHG OJKW 7KH VKLIW LV
SURSRUWLRQDOWR WKHYHORFLW RI WKH SDUWFOH 7KHUHLVHVHQWLDOO]HUR GLVWXUEDQFH RI WKHIORZ E\ WK
ODVHU

) LJXUH K VKRZ V WKHSRSXODUGXDOEHDP PRGHRI WKH/ ' $ $ IRFXVLQJ GHYLFHVSODWWKHODV
LQRWZR EHDPV ZKLFKFURVV WKHIORZ DWDQDQJOH 7KHUHLVQWUHFWLRQ ZKLFKLVWKHPHDVXULQJ YROXP
RUUHVROXWLRQRI WKHPHDVXUHPHQWUHVPEOHVDQHOSRLGDERXW PP ZLGHDQG PP LQGLDPHWU
3DUWFOHVSDVVLQJ WKURXJK WKLV PHDVXULQJ YROXPH VFDWWUWKHEHDPV WKH WKQSDV WKURXJK UHFHLYLQ
RSWFV WR D SKRWRGHWHFWRU ZKLFK FRQYHUWV WKH OJKWWR DQ HOHFWULF VJQDO $ VJQDOSURFHVVRU WKH
FRQYHUWVHOHFWULF IUHTXHQF\ WR D YROWDJH WKDWFDQEHHLWHUGLVSOD\HGRUVWRUHG ,I LV WKHZDYHOHQJV
RI WKHODVHUOJKWWKHPHDVXUHGYHORFLWLVJLYHQE\

$$V = \frac{\lambda\,\Delta f}{2\sin(\theta/2)} \qquad (6.99)$$

0 XOWLSOHFRPSRQHQWVRI YHORFLWFDQEHGHWHFWHG E\ XVLQJ PRUHWKDQRQHSKRWRGHWHFWRUDQGRWK
RSHUDWQJ PRGHV (LWHUDWLYHUJRULWKPVFDQEHPHDVXUHGDVORQJ DV\RQJ DVFDWWULQJ SDUWFOHVUHPHQ
,QQTXLGV QRUPDOLPSULWHVVHUYHDVFDWWHUV EXWJDVHVPD\ KDYHVREVHGG 7KHSDUWFOHVPD
EHDVPDOODVWKHZDYHOHQJWKRI WKHOJKW$QRXJK WKHPHDVXULQJ YROXPHLVQRWDVPDOOZLWKDKF
ZLWH WKH/ ' $ LVFDSDEOHRI PHDVXULQJ VKUEXOHQWOXFWXDWQV

7KHDGYDQWDJHVRI WKH/ ' $ DUHDVIROORZV

1 RGLVWXUEDQFHRI WKHIORZ
+LJK VSDWDOUHVROXWQRI WKHIORZ ILHOG
9HORFLWGDWDWKDWUHLQGHSHQGHQWRI WKHIXLGWKHUPRG\QDPLF SURSHUWLHV
$QRXSXVYROWDJHWKDWLVOLQHDUZLWKYHORFLW
1 RQHGIRUFDOEUDWQ

7KHGLVDGYDQWDJHVDUHWKDWERWKWKHDSSDUDWXVDQGWKHIXLGPXWEHWUDQVSDUHQWROJKWDQGWKDWWK
FRWWLVKJK DEDVLF VWHP VKRZ QLQ) LJ KEHJLQVDWDERXW
2QHILQDOGHG DQ/ ' $ FDQPDS WKHHQWLUHIORZ ILHGLQPLQXWHVDO 7RVWXO DSSUHFLDWWK
SRZ HURI WKH/ ' $ RQHVKRXGH[PLQH IRULQXQFH WKHPD]LQJO GHWDLOHG WKUHHGLPHQVLRQDOIORZ
SURILHV PHDVXUHG E\ (FNDUGW> @LQ D KJK VSHG FHQWUXJDO FRPSUHVVRU LPSHOOHU ([WQVLY
GLVFXVLRQVRI ODVHUYHORFLPHW DUHJLYHQLQ5HV DQG

3DUWFOH, P DJH9 HORFLP HWU

7KLVSRSXODUQHZ LGHD FDOOHG3,9 IRUVKRUWP HDVXUHVQRWWXWDVLQJHSRLQWEXWLQXWDGP DSVWK
HQWUHILHOG RI IORZ \$QLQWXLWDWRQZDV VKRZQ LQ) LJ E 7KH IORZ LV VHHGHG ZLWK QHXWUDO
EXR\DQVPDUFOHV \$ SDQDUDHUQJKWKHWDFURVWKH IORZ LVSXOVHGWZLFHDQGSKRWRJUDSKHGWZLFH
,I ûULVWKHSDUFOHGLVSODFHPHQWYHFWRUYHUVXVKRUWP HûWDQHWUP DWRI LWVYHORFLWLV9 § ûUû\
\$ GHGLFDWHGFRP SXWUDSSOHVWKLVIRUPXODWRDZKROHFORXGRI SDUFOHVDQGWKXVP DSVWKHIORZ ILHOG
2QHFDQDOR XVHWKHGDWDWRFDOFXODWHYHORFLWJUDGLHQWDQGYRUWLFLWILHOGV 6LQFHWKHSDUFOHV D
ORRN DOLNH RWKHUFP HUDV P D\ EHQHHGHG WR LGHQW\ WKHP 7KUHHGLP HQVLRQDO YHORFLWLHV FDQ E
PHDVXUHG E\ WZR FDP HUDV LQ D VWUHRVFRSLF DUUDQJHP HQW7KH 3,9 PHWKRG LV QRWOLP LWHG WR WKRS
DFWLRQ 1HZ KJK VSHHG FDP HUDV XS WR IUDP HV SHU VHFRQG FDQ UHFRUG P RYLHV RI XQWDG
IORZ ILHOGV)RUIXUWKHUGHWDLOV VHHWKHP RQRJUDSK E\ 0 5DIHD> @

(; \$0 3/ (

7KHSLWRWWXEHFWE HR) LJ XVHVP HUFXU DVDP DQRP HWHUIOXLG : KHQLWV SDFHGLQDZDWHU
IORZ WKHP DQRP HWHUUHLJKWUHDGLQJ LVK LQ 1HJOHFWQJ \DZ DQGRVKHUHURV ZKDWV WKHIORZ
YHORFLW9 LQIWV"

6ROXWLRQ

)URP WKHVZR IOXLGP DQRP HWHUUHDGLQ ZLWK]\$] WKHSUHVXUHGLIIHUHQFHLVUHDGGWRK
E\

$$p_0 - p_s = \rho g h$$

7DNLQJ WKHVSHFLILF ZHLJKWRI P HUFXU DQGZDWHUIURP 7DEOH ZHKDYH

$$p_0 - p_s = (846 - 62.4\ \text{lbf/ft}^3)\frac{8.4}{12}\text{ft} = 549\ \text{lbf/ft}^2$$

7KHGHQVLW RI ZDWHULV VOXJVIW ,QVRGXFLQJ WKHVH YDOXHV LQWR WKHSLWRWWXEH
IRUP XOD ZHREWLQ

$$V = \left[\frac{2(549\ \text{lbf/ft}^2)}{1.94\ \text{slugs/ft}^3}\right]^{1/2} = 23.8\ \text{ft/s} \qquad \textit{Ans.}$$

6LQFHWKLVLVDORZ VSHHGIORZ QR FRP SUHVVLELOLW\FRUUHFWLRQLVQHGHG

9 ROXP H) ORZ 0 HDVXUHP HQW

,WLV RIWHQ GHVLUDEOH WR PHDVXUH WKH LQIOXHQFH RI YROXPH ICRZ SDWWLQ WKURXJK D GXF
$FFXUDWH PHDVXUHPHQWV ICRZ LVYLWDOLQEOODQJ FXWWHPHUV IRUDJLYHQDP RXQWV RI GTXLGRUJDVSDWLQ
WKURXJK DGXFW7KH GLIIHUHQWGHYLFHV DYDLODEOH WR PDNH WKHVH PHDVXUHPHQWV DUH GLVFXVVHG LQ JUH
GHWDLOLQ WKH $60(WH[WRQ IXLG PHWHUV> @7KHVH GHYLFHV VSDQLQWR VZR FDWHJ PHFKDQF
LQWXKPHQWVDQGKDGRVLQWXKPHQWV

7KHPHFKDQFDOLQWXKPHQWPHDVXUHDFWXDOOPDVRUYROXPHRI IXLGE\ WDSSLQJ LWDQGFRXQWLQ
LW7KHYDULRXVWSHVRI PHDVXUHPHQWDUH

0 DVPHDVXUHPHQW
: HJKLQJ WDQNV
7LQWQJ WDSV
9ROXPHPHDVXUHPHQW
9ROXPHWDQNV
5HFLSURFDWQJ SLVWRQV
5RWDWQJ VORWWHG GULQJV
1XWDWQJ GLVF
6QDLQJ YDQHV
* HDURUGRE GLP SHOHUV
5HFLSURFDWQJ EHOORZ V
6HDOHG GUXP FRP SDUWPHQW

7KHODWWHUHR WKHVH DUH VXLWDEOH IRUJDVICRZ PHDVXUHPHQW
7KHKHDGORVVGHYLFHVREWXFWWKHICRZ DQGFDXVHDSUHVVXUHGURS ZKLFKLVDPHDVXUHR ICX[

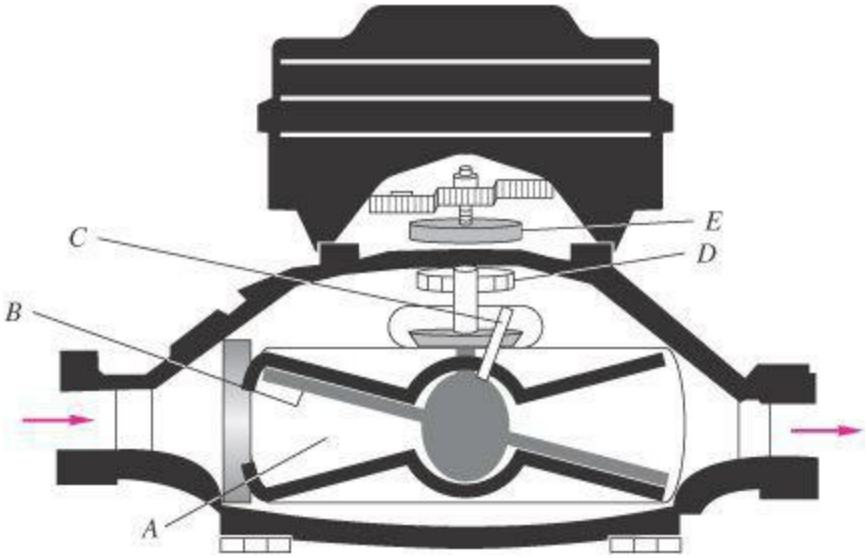
%HUQRXOOLWSHGHYLFHV
7KLQSODWHRUILFH
) ORZ QR]]OH
9HQWXULWXEH
) UHFWDQJORVGHYLFHV
&DSLOODU WXEH
3RURXVSOXJ

7KHIULFWLRQORVVPHWKVFDXVHDOOJHQRQUHFRYHUDEOHKHDGORVVDQGREWXFWWKHICRZ WRR PXFKV
EHJHQHUDOO XVHXO
6L[RWKHUZLGHO XVHGPHWKVRSHUDWQGLIIHUHQWSK\VFDOSULQFLSOHV

7XUELQHPHWU
9RUWH[PHWU
8OWUDVRQLF ICRZ PHWU
5RWDPHWU
&RULROLVPDVICRZ PHWU
/ DPLQDUICRZ HOHPHQW

1XWDWQJ ' LVF0 HWU

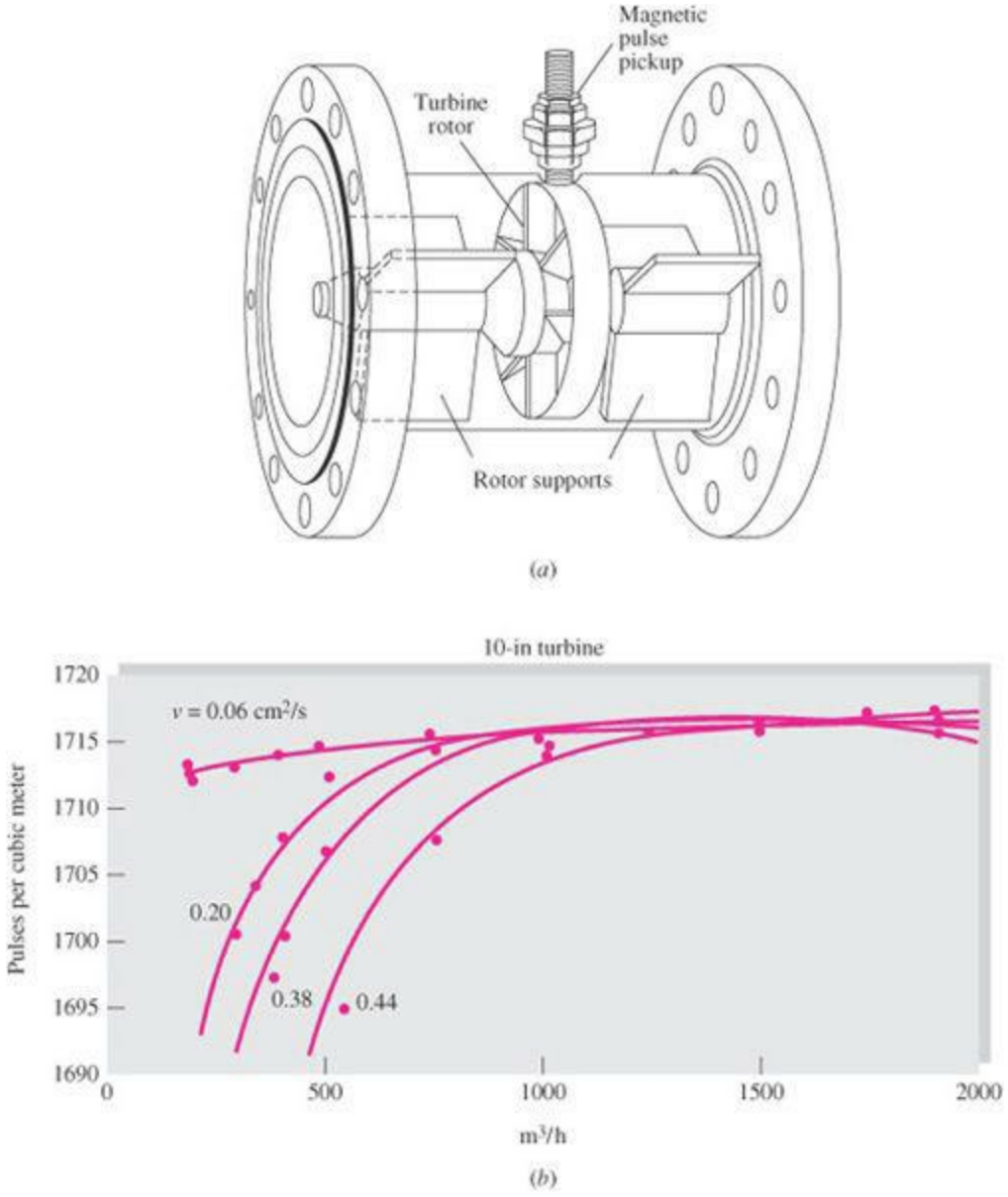
) RU PHDVXULQJ GLVF DQG WKH WXUELQH PHWHU) LJXUH VKRZV D FXWDZD\ VNHWFK RI D QXWDWLQJ GLVF PHWHU ZLGHO XVHG LQ ERWK ZDWHU DQG JDVROLQH GHOLYHU\ V\VWHPV 7KH PHFKDQLVP LV FOHYHU DQG SHUKDS EH\RQG WKH ZULWHUV\V FDSDELOLWLHV 7KH PHWHULQJ FKDPEHUV DVVHPEOHG DVSKHUH DQG FRQQHFW URWDWLQJ GLVF VHWDWDQ DQJOH WR WKH LQFRPLQJ IORZ 7KH IOXLG FDXVHV WKH GLVF WR QXWDWH VSLQ HFFHQWULFDOO DQG RQH UHYROXWLRQ FRUUHVSRQGV WR D FHUWDLQ IOXLG YROXPH SDVVLQJ WKURXJK 7RWD YROXPHV REWDLQHG E\ FRXQWLQJ WKH QXPEHU RI UHYROXWLRQV



) LJ &XWDZD\ VNHWFK RI D QXWDWLQJ GLVF IOXLG PHWHU*A* PHWHUHG YROXPH FKDPEHU*B* QXWDWLQJ GLVF*C* URWDWLQJ VSLQGOH*D* GULYH PDJQHW*E* PDJQHWF FRXQWHU VHQVRU*(Courtesy of Badger Meter, Inc., Milwaukee, Wisconsin.)*

7XUELQH0 HWHU

7KH WXUELQH PHWHU VRPHWLPHV FDOOHG DSURSHOOHU PHWHU LV DIUHH URWDWLQJ SURPHOOHU WKDW FDQ EH LQVHUWHG LQDSLSHOLQH $ \WSLFDO FURVV VHFWLRQ QLQ) LJ D 7KH DUHIORZ VWUDLJKWHQHUV XSVWUHDP RI WKH URWRU DQG WKH URWDWRQLV PHDVXUHG E\ HOHFWURPDJQHWF SLFNXS RI SXOVHV FDXVHG E\ SDVVDJH RI D SROQWRQ WKH URWRU 7KH URWRU URWDWRQLV DSSUR[LPDWH SURSRUWRQDO WR WKH YROXPH IORZ LQ WKH SLSH



) LJ 7KH VXUELQH PHWHU ZLGHO XVHG LQ WKH RLO JDV DQG ZDWHU VXSSO LQGXVWULHV *a* EDVL GHVLJQ *b* WSLFDO FDOEUDWRQ FXUYH IRU DUDQJHR FUXGH RLO*(Daniel Industries, Houston, TX.)*

/ LNHWKH QXDWQJ GLVF DP DWRUDGYDQDJHR WKH WXUELQH PHWULV WKDW DFK SXOH FRUHVSRQG VR ILQDWLQFUHPHQDO YROXP HR IOLG DQG WKH SXOHV DUH GLJLWDODQG FDQEH VXPPHGDLO / LTXLG IORZ WXUELQH PHWHUV KDYH DV IHZ DV WZR EODGHV DQG SURGXFH D FRQWDQWQXPEHU SXOHV SHU XQLWYROXP HRYHUD IORZ UDWH UDQJH ZLWK“ SHUFHQWDFFXUDF\ * DV PHWHUV QHGP DQ EODGHV VR SURGXFH VXILFLHQWRUTXHDQG DUHDFFXUDWWR“ SHUFHQW

6LQFH WXUELQH PHWHUV DUH YHU LQGLYLGXDOWF IORZ FDOEUDWRQ DQ DEVROXWQHFHVLW \$ WSLFD QTXLG PHWUFDOEUDWRQ FXUYH LV VKRZ QLQ) LJ E 5HVDUFKHUV DWWP SWQJ WR HVDEOK XQLYHUVD FDOEUDWRQ FXUYHV KDYHP HWZ LWKDWWHSUDFWFDOVXFFHVV DV DUHVXOWR P DQ DFWXLQJ YDULDELQWHV 7XUELQH PHWHUV FDQDOR EHXVHGLQXQRQLQGIORZ VLWXDWRQV VXFK DV ZLQGVRURHDQFUHQW 7KH FDQEHFRPSDFWHYQPLFURV]HZLWKZZR RUWKUHFRPSRQHWGLFDWRQV) LJXUH LOOXWDW DKDQGKHOGZLQGYHORFLWPHWHUWKDWXVHVDVHYQEODGHGWXUELQHZLWKDFDOEUDWGGLJLWDORXSXW7K DFFXUDF\ RI WKLVGHYLFHLVTXRWGDV“ SHUFHQW

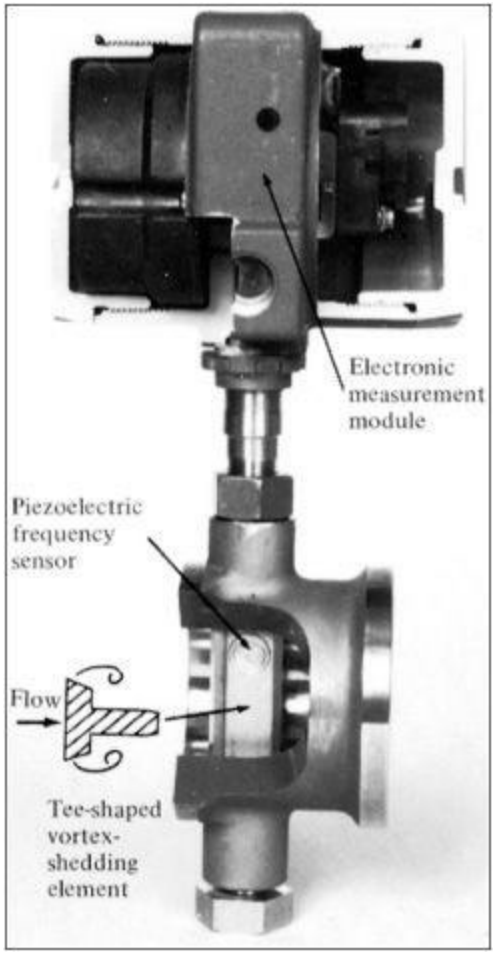
) LJ $ &RPPHUFLDO KDQGKHOG ZLQG YHORFLW VXUYH PHWHU *Courtesy of Nielsen-Kellerman Company.)*

9RUWH[ ) ORZ PHWHUV

5HFDOO IURP) LJ VKDWDEOH ERG SODFHG LQ DXQLIRUP FURVVIORZ VKHGV DOWHUQDWLQJ YRUWLFHV D D QHDU XQLIRUP 6WURXKDO QXPEHU 6W I/ 8 ZKHUH8 LV WKH DSSURDFK YHORFLW DQG/ LV I FKDUDFWHULVWLF ERG\ ZLGWK 6LQFH/ DQG 6WDUH FRQVWDQWV WKLV PHDQV WKDWWKH VKHGGLQJ IUHTXHQF\ L SURSRUWLRQDOWR YHORFLW

$$f = (\text{const})(U) \qquad (6.100)$$

7KH YRUWH[PHWHU LQWURGXFHV D VKHGGLQJ HOHPHQWDFURVV D SLSH IORZ DQG SLFNV XS WKH VKHGGLQ IUHTXHQF\ GRZQVWHDP ZLWK D SUHVVXUH XGXDVRQF RUKHDWWDQVIHUWSHR VHQVRU $ WSLFDOGHVLJQLV VKRZ QLQ) LJ



) LJ $ YRUWH[IORZPHWHU *Courtesy of Invensys p/c.)*

7KH DGYDQWDJHV RI DYRUWH[PHWHUV DUH DV IROORZV

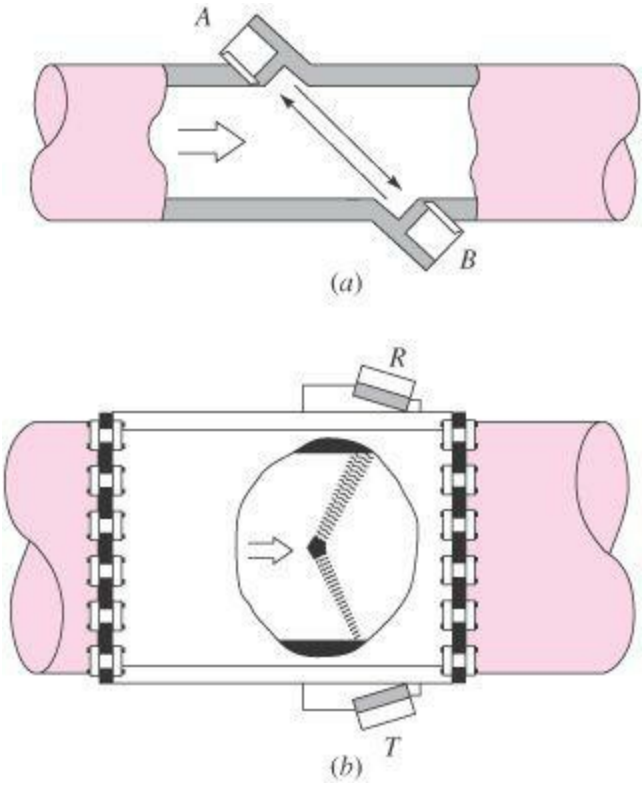
$ EURDG UDQJH RI PRYLQJ SDUWV
$ FFXUDF\ VR“ SHUFHQW RYHU DZ LGH IORZ UDWH XS WR
$ ELOLW\ WR KDQGOH YHU KRWRUY HU FROGIOXLGV
5 HTXLUHPHQW RQO D VKRUW SLSH OHQJWK
&DOLEUDWRQ LQYHQWRULHV RIOXLG GHQVLWLHV RUYLVFRVLW\

) RUIXUWKHUGHWDLOVVHH5H

8OWUDVRQLF) ORZPHWHUV

7KH VRXQG ZDYH DQDORJ RI WKH ODVHUYHORFLPHWURI) LJ K LV WKH XOWUDVRQLF IORZPHWHU 7Z F H[DPSOHV DUH VKRZQ LQ) LJ 7KH SXOVHWSH IORZPHWHU LV VKRZQ LQ) LJ D 8 SVWUHDP SLH]RHOHFWULF WUDQVGXFHU$ LV H[FLWHG ZLWK D VKRUWSXOVH SXOVH WKDWSURSDJDWHV DFURVV WKH IORZ V GRZQVWUHDP WUDQVGXFHU% 7KH DUULYDO WLPH WUJJHUV DQRWKHU SXOVH WR EH FUHDWHG DV$ UHVXOWLQJ LQI UHJXODU SXOVH IUHTXHQF\ I$ 7KH VDPH SURFHVV LV GXSOLFDWHG LQ WKH UHYHUVH GLUHFWLRQ IURP % WR$ FUHDWLQJ IUHTXHQF\ I% 7KH GLIIHUHQFH I$ í I% LV SURSRUWLRQDO WR WKH IORZ UDWH) LJXUH E VKRZV I GRSSOHUWSH DUUDQJHPHQWZKHUH VRXQG ZDYHV IURP WUDQVPLWWU7 DUH VFDWWHUHG E\ SDUWFOHV R FRQWDPLQDQWV LQ WKH IORZ VR UHFHLYHU5 &RPSDULVRQ RI WKH WZR VLJQDOV UHYHDOV D GRSSOHUIUHTXHQF VKLIWWKDWLV SURSRUWLRQDO WR WKH IORZ UDWH 8OWUDVRQLF PHWHUV DUH QRQLQWUXVLYH DQG FDQ EH GLUHFW

DWWDFKHGWRSLSHIORZVLQWKHILHOG) LJ F 7KHUTXRWGXQHUDLQWR “ WR SHUFHQWDQUHV
“ SHUFHQWRUPRUHGXHWRLUHJXODULWLHVLQYHORFLWSURILOHIOXLGWSHDWXH RU5HQROGVQXPEH
) RUIXUWKHUGHWDLOVHH5H



) LJ 8OWUDVRQLF IORZ PHWHUV *a* SXOVH WUSH *b* GRSSOHU VKLWWSH*(from Ref. 41)* *c* [
SRUWDEOH QRQLQYDVLYH LQWDOODWRQ*(Courtesy of Thermo Polysonics, Houston, TX.)*

5 RWDP HWHU

) LJ $ FRPPHUFLDO URWDPHWHU 7KH IORDWUDSHV LQ WKH WDSHUHG WXEH WR DQ HTXLOLEULXP SRVLWLRQ ZKLFK LV D PHDVXUH RI WKH IOXLG IORZ UDWH*(Courtesy of Blue White Industries, Huntington Beach, CA.)*

7KH YDULDEOH DUHD WDQSDUHQW URWDPHWHU RI) LJ KDV D IORDW WKDW XQGHUWKH DFWLRQ RI IORZ UHVLQ WKH YHUWLFDO WDSHUHG WXEH DQG VDFHUDWHG WR HTXLOLEULXP SRVLWLRQ IRU DQ JLYHQ IORZ UDWH $ VXGHQ Y[HUFLVH IRU WKH IRUFHV RQ WKH IORDW ZRXOG \LHOG WKH DSSURSULDWH RQ

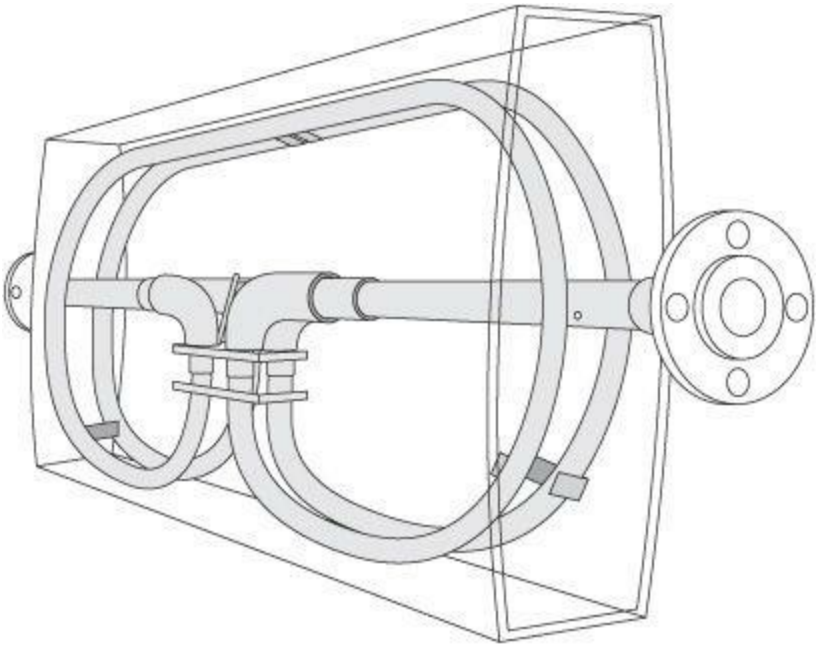
$$Q = C_d A_a \left(\frac{2W_{\text{net}}}{A_{\text{float}} \rho_{\text{fluid}}} \right)^{1/2} \qquad (6.101)$$

ZKHUH: $_{QHW}$ LV WKH IORDWV QHW ZHLJKW LQ WKH IOXLG D $_{WXEH}$ í $_{IORDW}$ LV WKH DQQXODU DUHD EHWZHHQ WKH IORDW DQG WKH WXEH DQG &$_G$ LV D GLPHQVLRQOHVV GLVFKDUJH FRHIILFLHQW IRU WKH DQQXODU FRQVWULFWLRQ IRUPHG IORZ) RUVOJKW VDSHUHG WXEHV $D YDULHV QHDUO\ OLQHDUO\ ZLWK WKH IORDWSRVLWLRQ DQG WKH WXEH PDY EH FDOLEUDWHG DQG PDUNHG ZLWK D IORZ UDWH VFDOH DV LQ) LJ 7KH UHVP HWU WKX SURYLGHV DUHDGLO YLVEOHPHDVXUH RI WKH IORZ UDWH &DSDFLWLVP DY EHFKDQJHG E\ XVLQJ GLIIHUHQWL[H IORDWV 2 EYLRXVO WKH WXEHP XWEH YHUWFDO DQG WKH GHYLFH GRHV QRWLYH DFFXUDWHGLQJV IRU IOXLG FRQGLQJ KJK FRQFHQWUDWLRQV RI EXEEOHV RU SDUWFOHV

&RULROLV0 DVV) ORZ P HWHU

0 RVWFRP PHUFLDOP HWHUV PHDVXUHYROXP HIORZ ZLWKPDVVIORZ WKHQFRP SXWHGE\ PXOWLSOLQJ E\ WKHQRP LQDOIOXLG GHQVLW\ $QDWWDFKHPHQWIRUGHQDOQDWHLV DPDVVIORZ PHWHU ZKLFKRSHUDWHV RQ WKH SULQFLSOHRI WKH&RULROLV DFFHOHUDWRQDVRFLDWHG ZLWKQRQLQHUWLDOFRRUGLQDWH>UHFDOO) LJ DQG WKH&RULROLV WHUP 9 î 9 LQ(T @7KHRXSXWRI WKHPHWHULGLUHFWO SURSRUWRQDOWR PDVV IORZ

) LJXUH LV DVFKHPDWF RI D&RULROLV GHYLFH WR EH LQVHUWHG LQRX SLSLQJ V\VWHP 7KH IORZ HQWHUV D GRXEOHORRS GRXEOHWXEH DUUDQJHPHQWZKLFK LV HDFWRPDQXIDFWXUHG YLEUDWHG DVD KLJI QDWXUDOIUHTXHQF\ DPSOLWXGH PP DQG IUHTXHQF\ ! +] 7KH XS IORZ LQGXFHV LQZ DUG ORR PRWRQ ZKLOHWKHGRZQIORZ FUHDWHVRXWZDUGORRPRWRQ ERWKGXHWRWKH&RULROLVHIHFW6HQVRUV D ERWK HQGV UHJLWHUD SKDVH GLIIHUHQFH WKDWLV SURSRUWRQDO WR PDVV IORZ 4 XRWHG DFFXUDF\ L DSSUR[LPDWH “ SHUFHQWRI IXOOVFDOH



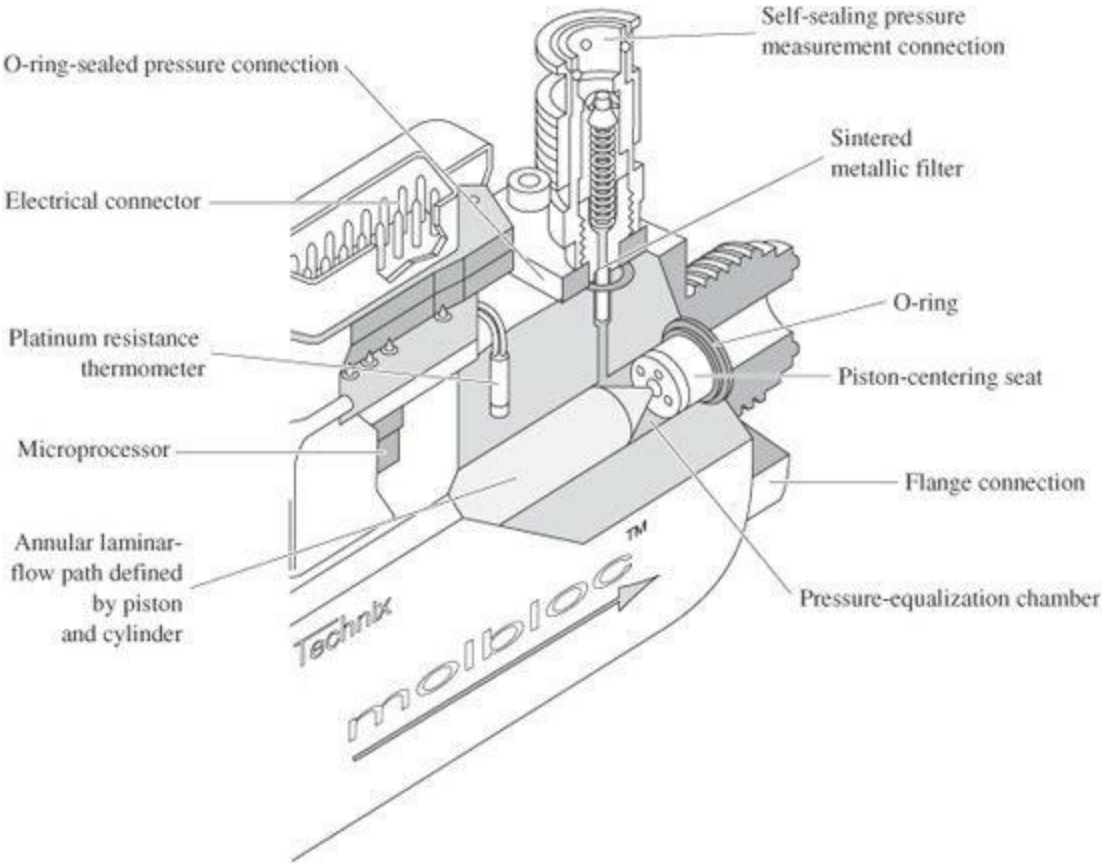
) LJ $ &RULROLVP DVVIORZ P HWHU *(Courtesy of ABB Instrumentation, Inc.)*

/ DP LQDU) ORZ (OHP HQWV

,QP DQ SHUKDSVP RVWFRP PHUFLDOIORZ PHWHUV WKHIORZ WKURXJKWKHP HWHULVWXEHQWDQG WK YDULDWRQRI IORZ UDWHZLWKSUHVXUHGURS LVQRQOLQHDU,QDP LQDUGXFWORZ KRZHYHU 4 LVQHDU SURSRUWRQDOWRûS DVLQ(T 4 >Œ5 / @ûS 7KXVDOOP LQDUIORZ VHQVLQJ HOHP HQWV DWWDFKHVLQFHILWFDOEUDWRQZLOOEHQHDU7RHQVXUHOP LQDUIORZ IRUZKDWRWKHUZLVHZRQG EHD WXEXOHQWFRQGLWLRQ DOORUSDUWRI WKHIOXLG LVGLVFDUGHG LQRV PDOSDWDJHV HDFK RI ZKLFK KDV DORZ OP LQDU 5HQROGVQXPEHU$ KRQH FRP E LVDSSXODUGHVLJQ

) LJXUH XVHV D[LDOIORZ WKURXJK DQDUURZ DQQXOXV WR HIFWOP LQDUIORZ 7KH WKHRU DJDL SUHGLFW4 ûS DVLQ(T +RZHYHU WKHIORZ LV YHU VHQVLWLYH WR SDWDJHV] H IRUH[DPSO KDOYLQJ WKH DQQXOXV FOHDUDQFH LQFUHDVHV ûS PRUH WKDQ HJKWWLPHV &DUHXO FDOEUDWRQ LV WK QHFHVDU ,Q) LJ WKH OP LQDUIORZ FRQF-SWKDV EH-HQ VXQK-V]HG LQRX DFRP SOHWH PDW IORZ

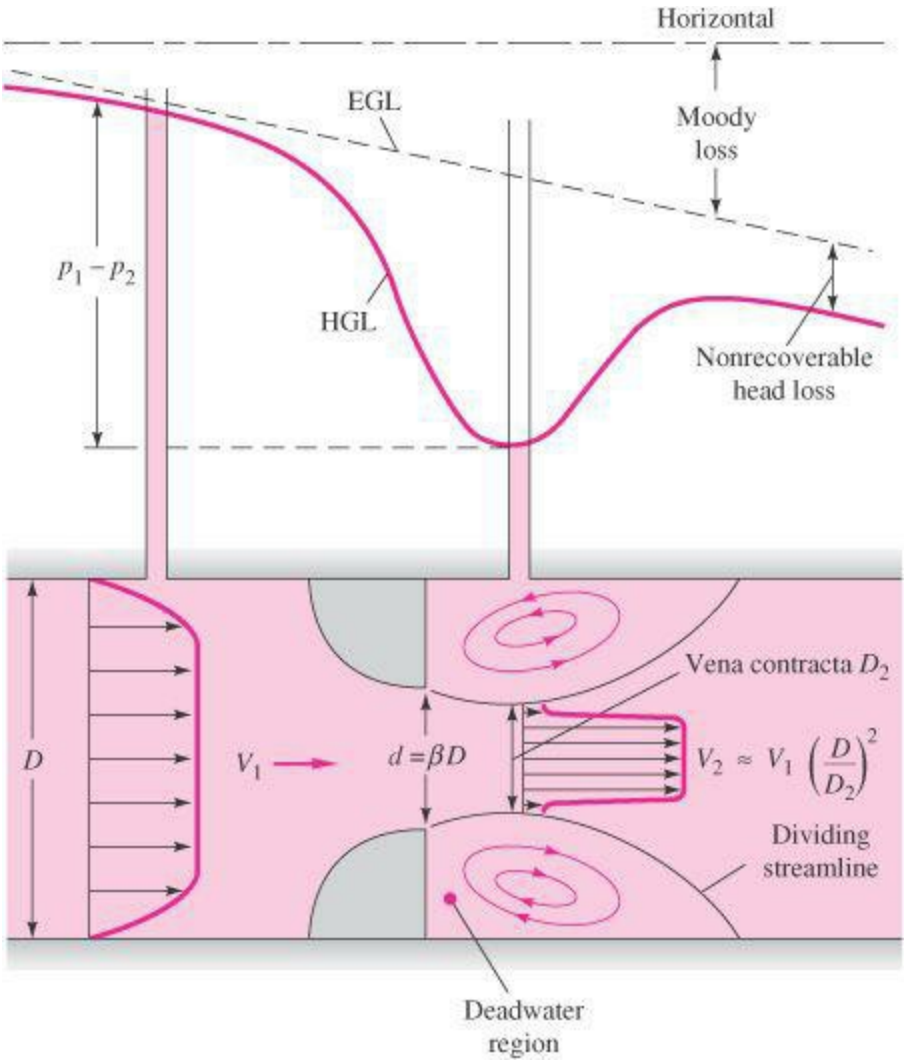
V\ VWHP Z LWK VWHS HUDWXUH FRQWURO GLIIHUHQWLDO SUHVVXUH PHDVXUHPHQWDQG DPLFURSURFHVVRU EDVHG FRQGLWLRQHG 7KH DFFXUDF\ RI WKLV GHYLFH LV UDWHG DW SHUFHQW



) LJ $ FRPSOHWH SUHVVXUH PHDVXUHPHQW XQLQJ D ODP LQDUIORZ HOHPHQW LQ WKLV FDVH D QDUURZ DQQXOXV 7KH IORZ UDWH LV OLQHDU SURSRUWLRQDO WR WKH SUHVVXUH GURS ***(Courtesy of Martin Girard, DH Instruments, Inc.)***

%HUQRXOOL2 EVWUXFWLRQ 7KHRU\

) LJ



9HORFLWLHV DQG SUHVVXUH FKDQJHV WKURXJK DJHQHUDO]HG %HUQRXOOL REWXFWLRQ P HWHU

&RQVLGHU WKH JHQHUDO]HG IORZ REWXFWLRQ VKRZQ LQ) LJ 7KH IORZ LQ WKH EDVLF GXFWLO GLP HWHU' LV IRUFHG WKURXJK DQ REWXFWLRQ RI GLP HWHUG WKH UDWLR RI WKH GHYLFH LV D NH SDUDP HWHU

$$\beta = \frac{d}{D} \tag{6.102}$$

$ IWHU GHYLQJ WKH REWXFWLRQ WKH IORZ PD\ QHFNGRZQ HYHQ PRUH WKURXJK D YHQD FRQWUDFWD GLP HWHU' G DV VKRZQ $SSO WKH %HUQRXOOL DQG FRQWLQXLW HTXDWLRQV IRU LQFRP SUHVVLEOH VWDG IULFWLRQOHVV IORZ WR HVWP DWH WKH SUHVXUHFKDQJH

Continuity: $$Q = \frac{\pi}{4}D^2V_1 = \frac{\pi}{4}D_2^2V_2$$

Bernoulli: $$p_0 = p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

(OP LQDWLQJ 9 Z HVROYH WKH HVHIRU9 RU4 LQWHUP VRI WKH SUHVXUHFKDQJHS í S

$$\frac{Q}{A_2} = V_2 \approx \left[\frac{2(p_1 - p_2)}{\rho(1 - D_2^4/D^4)}\right]^{1/2} \tag{6.103}$$

%XW WKLV LV VXUHO LQDFFXUDWH EHFDXVH Z HKDYH QHJOHFWHG IULFWLRQ LQ D GXFWORZ Z KHUH Z HNQRZ IULFWLRQ Z LOOEH YHU LPSRUDQW1 RUGR Z HZ DQWR JHWQRR WKH EXLQHVV RI PHDVXULQJ YHQD FRQWUDFW

UDWRV' GIRUXVHLQ 7KHUHZHDVXPHWKDW ' § DQGWKHQFDOLEUDWWKHGHYLFHWRIL
WKHUHDWRQ

$$Q = A_t V_t = C_d A_t \left[\frac{2(p_1 - p_2)/\rho}{1 - \beta^4} \right]^{1/2} \qquad (6.104)$$

ZKHUH VXEVFULSWVGHQRWHVWKHWKURDWRI WKHREVWUXFWLRQ 7KHGLPHQVLRQOHVVGLVFKDUJHFRHIILFLHQ
&G DFFRXQWVIRUWKHGLVFUHSDQFLHVLQWKHDSSUR[LPDWHDQDO VLV %\ GLPHQVLRQOHVDQDO VLVIRUDJLYH
GHVLJQZHH[SHFW

$$C_d = f(\beta, \mathrm{Re}_D) \quad \text{where} \quad \mathrm{Re}_D = \frac{V_1 D}{\nu} \qquad (6.105)$$

7KHJHRPHWULFIDFWRULQYROYLQJ LQ LVFDOOHGWKHYHORFLW RI DSSURDFK IDFWRU

$$E = (1 - \beta^4)^{-1/2} \qquad (6.106)$$

2QHFDQDOVR JURXS &G DQG(LQ(T VRIRUP WKHGLPHQVLRQOHVVIORZ FRHILFLHQW

$$\alpha = C_d E = \frac{C_d}{(1 - \beta^4)^{1/2}} \qquad (6.107)$$

7KXV(T FDQEHZ ULWWQLQWKHHTXLYDOHQWRUP

$$Q = \alpha A_t \left[\frac{2(p_1 - p_2)}{\rho} \right]^{1/2} \qquad (6.108)$$

2EYLRXVO WKHIORZ FRHILFLHQWV FRUUHODWHGLQWKHVDPHPDQQHU

$$\alpha = f(\beta, \mathrm{Re}_D) \qquad (6.109)$$

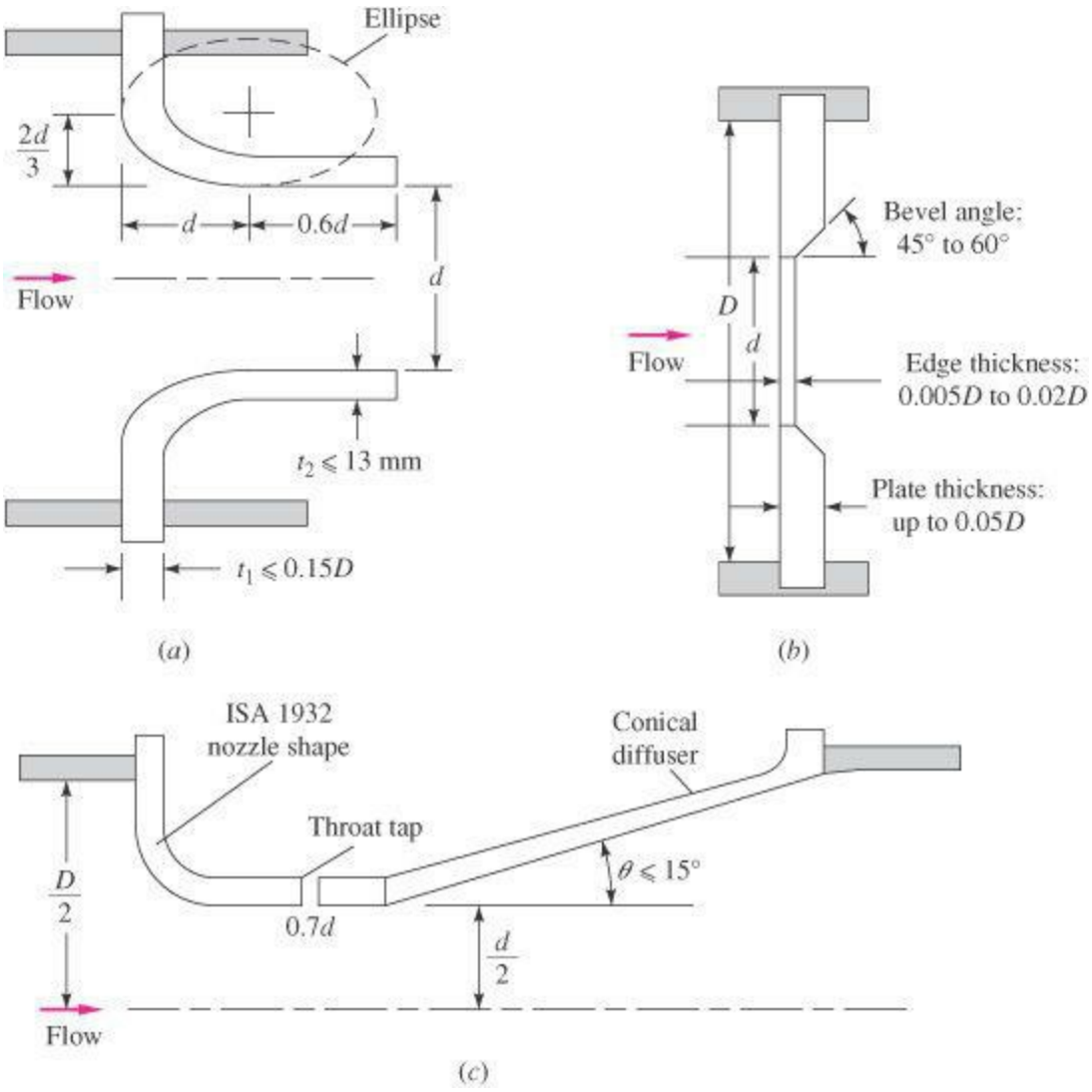
2FFDVLRQDOO RQHXVHVWKHWKURDW5H\QROGVQXPEHULQXQLWVRI WKHDSSURDFK5H\QROGVQXPEHU

$$\mathrm{Re}_d = \frac{V_t d}{\nu} = \frac{\mathrm{Re}_D}{\beta} \qquad (6.110)$$

6LQFHWKHGHVLJQSDUDPHWHUV DUHDVXPHGNQRZQ WKHFRUUHODWLRQRI . IURP(T RUR &(
IURP(T LVWKHGHVLUHGVROXWLRQWRWKHIOXLGPHFKDQLQJ SUREOP
7KHPDVVIORZ LVUHODWHGWR4 E\

$$\dot{m} = \rho Q \qquad (6.111)$$

DQGLVWKXVFRUUHODWHGE\ H[DFWO WKHVDPHIRUPXODV
) LJXUH VKRZV WKH WKUHH EDVLF GHVLFQV UHFRPPHQGHG IRU XVH E\ WKH ,QWHUQDWLRQDO
2UJDQL]DWLRQIRU6WDQGDUGL]DWLRQ ,62 > @WKHRULILFH QR]]OH DQGYHQXULWXEH



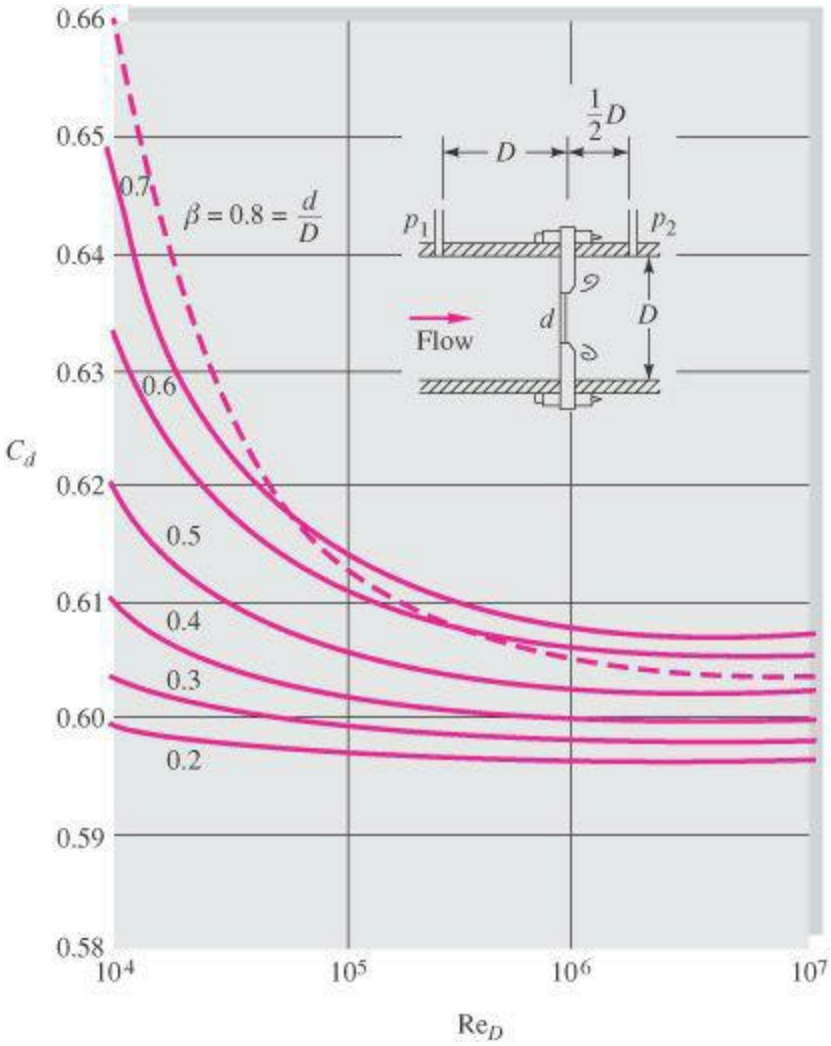
) LJ , QWHUQDWLRQDO VWDQGDUG VKDSHV IRU WKH WKUHH SULPDU\ %HUQRXOOL REVWUXFWLRQ WS PHWHUV *a* ORQJ UDGLXV QR]]OH *b* VKLQ SODWH RULILFH *c* YHQWXUL QR]]OH*(From Ref. 31 by permission of the International Organization for Standardization.)*

7KLQ 3ODWH2 UILFH

7KH WKLQ SODWH RULILFH) LJ E FDQ EH PDGH ZLWK LQWHUDQJHR VR H[FHSWWKDWWKH KROH GLDPHWHUG VKRXOG QRWEH OHVV WKDQ PP 7R PHDXUH S DQGS WKUHH WDSSLQJV DU FRPPRQO XVHG

&RQQHUDSVZKHUHWKHSODWHPHHWWKHSLSHZDOO
' ½ ' VDSV SLSHZDOOVDSVDW XSWUHDP DQG½ ' GRZQWUHDP
)ODQJHVDSV LQ PP XSWUHDP DQG LQ PP GRZQWUHDP RI WKHSODWH UHJDUGOHVV RI WKHVL]H'

7\SHV DQG DSSUR[LPDWH FRPPHUFLDO EXWWLQFH WKH IODQJHVDSV GR QRWWKH PXWHFRUUHODWHGVHSDUDWLRQ IRUHYHU VLQJOHVL]HRI SLSHLQZKLFK DIODQJHVDS SODWHLVXVHG>@



) LJ ' LVFKDUJH FRHIILFLHQWV RU D VKLQS SODWH RUILFH ZLWK D ½D VWDSV SORWWHG IURP (TV DQG b

) LJXUH VKRZV VWHDG\VFKDUJH FRHIILFLHQWV DQ RUILFH ZLWK' ½ ' RU WSH VDSV LQ WK 5H\QROGV QXPEHUUDQJH 5H VR RI QRUP DOXVH $QWKRXJK GHVLJQ GDWD FKDUW VXFK DV) LJ DUH DYDLODEOH IRUGHVLJQHUV> @WKH$60 (UHFRPPHQGV XVH RI WKH FXUYH ILWWRUP XDVGHYHORSHG E WKH,62 > @7KHEDVLF IRUP RI WKH FXUYH ILWV> @

$$C_d = f(\beta) + 91.71\beta^{2.5}\mathrm{Re}_D^{-0.75} + \frac{0.09\beta^4}{1-\beta^4}F_1 - 0.0337\beta^3 F_2 \qquad (6.112)$$

where $f(\beta) = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8$

7KHFRUHODWLRQIDFWRUV) DQG) YDU Z LWKWDS SRVLWLRQ

Corner taps: $F_1 = 0 \quad F_2 = 0$ (6.113*a*)

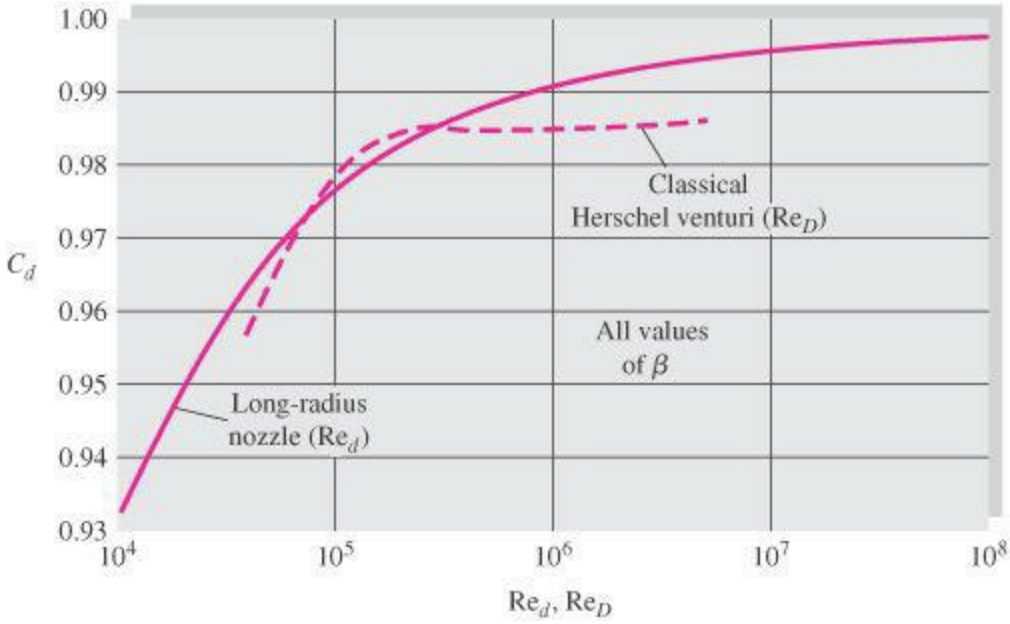
D; $\frac{1}{2}D$ taps: $F_1 = 0.4333 \quad F_2 = 0.47$ (6.113*b*)

$$\text{Flange taps:} \qquad F_2 = \frac{1}{D\ (\text{in})} \qquad F_1 = \begin{cases} \dfrac{1}{D\ (\text{in})} & D > 2.3\ \text{in} \\ 0.4333 & 2.0 \le D \le 2.3\ \text{in} \end{cases} \tag{6.113c}$$

1 RWH WKDW WKH IODQJH WDSV F QRW EH OLQJ JHRPHWULFDOO\ VLPLODU XVHUDZ GLDPHWULQLQFKHV LQ WKH IRUPXOD 7KH FRQWUDFWDQW Z LOO FKDQJH LI RWKHU GLDPHWUXQLWV DUH XVHG : HFDXWLRQHG DJDLQVW XVH GLDPHWULQLQFKHV LQ IRUPXODV LQ([DPSOH DQG(T DQG JLYH(T F RQO EHFDXVH IODQJH WDSV DUH Z LGHO XVHG LQ WKH 8QLWHG 6WDWHV

) ORZ 1 R]]OH

7KH IORZ QR]]OH FRPHV LQ WZR WSHV D ORQJ UDGLXV WSH VKRZ Q LQ) LJ D DQG D VKRUW UDGLXV WSH QRWKRZ Q FDOOHG WKH ,6$ QR]]OH > @7KH IORZ QR]]OH Z LWK LWV VPRRWK URXQGHG HQWUDQFH FRQYHUJHQFH SURILOH DOORZV WKH YHQD FRQWUDFWD DQG JLYHV GLVFKDUJH FRHILFLHQWV QHDU XQLW 7KH QR]]OH RYHUDOO EHORZ LV VWLOO D JRRG EHFDXVH WKHUH LV QR GLIIXVHU SURYLGHG IRU JUDGXDO H[SDQVLRQ



) LJ ' LVFKDUJH FRHILFLHQW IRU RQH UDGLXV QR]]OH DQG FODVVLFDO+ HUVFKHOWSH YHQWXUL

7KH,62 UHFRPPHQGHG FRUUHODWLRQ IRU RQH UDGLXV QR]]OH GLVFKDUJH FRHILFLHQWV

$$C_d \approx 0.9965 - 0.00653\beta^{1/2}\left(\frac{10^6}{\text{Re}_D}\right)^{1/2} = 0.9965 - 0.00653\left(\frac{10^6}{\text{Re}_d}\right)^{1/2} \tag{6.114}$$

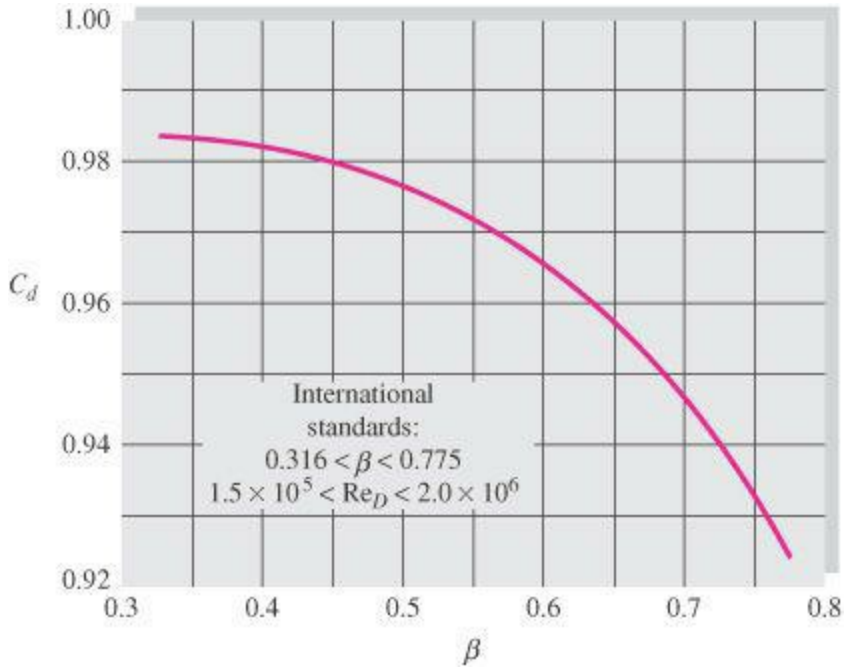
7KH VHFRQG IRUP LV LQGHSHQGHQW RI WKH UDWLR DQG LV SODWWHG LQ) LJ $ VLPLODU ,62 FRUUHODWLRQ LV UHFRPPHQGHG IRU WKH VKRUW UDGLXV,6$ IORZ QR]]OH

$$C_d \approx 0.9900 - 0.2262\beta^{4.1} + (0.000215 - 0.001125\beta + 0.00249\beta^{4.7})\left(\frac{10^6}{\text{Re}_D}\right)^{1.15} \tag{6.115}$$

) ORZ QR]]OHV PD\ KDYH YDOXHV EHWZHHQ DQG

9HQXUL0 HWHU

7KH VWDQGDUG ILQDO WSH RI REVWUXFWLRQ PHWHU LV WKH YHQXUL QDPHG LQKRQRU RI * LRYDQQL 9HQXU
± DQ,WDOLDQ SK\VLFLVW WKR ILUVW VWXGLHG FRQLFDO [SDQVLRQV DQG FRQWUDFWLRQV 7KH RULJLQD
RUFDVWLFDO YHQXUL ZDV LQYHQWHG E\ D8 6 HQJLQHHU &HPHQV + HUVFKHO LQ ,WFRQVLVWV RI
I FRQLFDO FRQWUDFWLRQ DVWDLJKW WKURDW GLDPHWHU GDQGGHQJWK WKHQD VR I FRQLFDO [SDQVLRQ
7KH GLVFKDUJH FRHILFLHQWV DQG WKH RQUHFRYHUDEOH RVVLVYHU VPDOO + HUVFKHOYHQXUL
DUHVHOGRP XVHGQRZ



) LJ ' LVFKDUJH FRHILFLHQWV RU DYHQXUL QR]]OH

7KH PRGHUQ YHQXUL QR]]OH) LJ F FRQVLVWV RI DQ ,6$ QR]]OH HQWDQFH DQG D FRQLFDO H[SDQVLRQ RI KDOI DQJOH R JUHDWHU WKDQ I ,WV LQWHQGHG WR EH RSHUDWHG LQ D QDUURZ 5H QRGH QXPEHUUDQJH RI î WR î ,WV GLVFKDUJH FRHILFLHQWV KRZ QLQ) LJ LV JLYHQ E\ WK ,62 FRUUHODWLRQ IRUP XOD

$$C_d \approx 0.9858 - 0.196\beta^{4.5} \qquad (6.116)$$

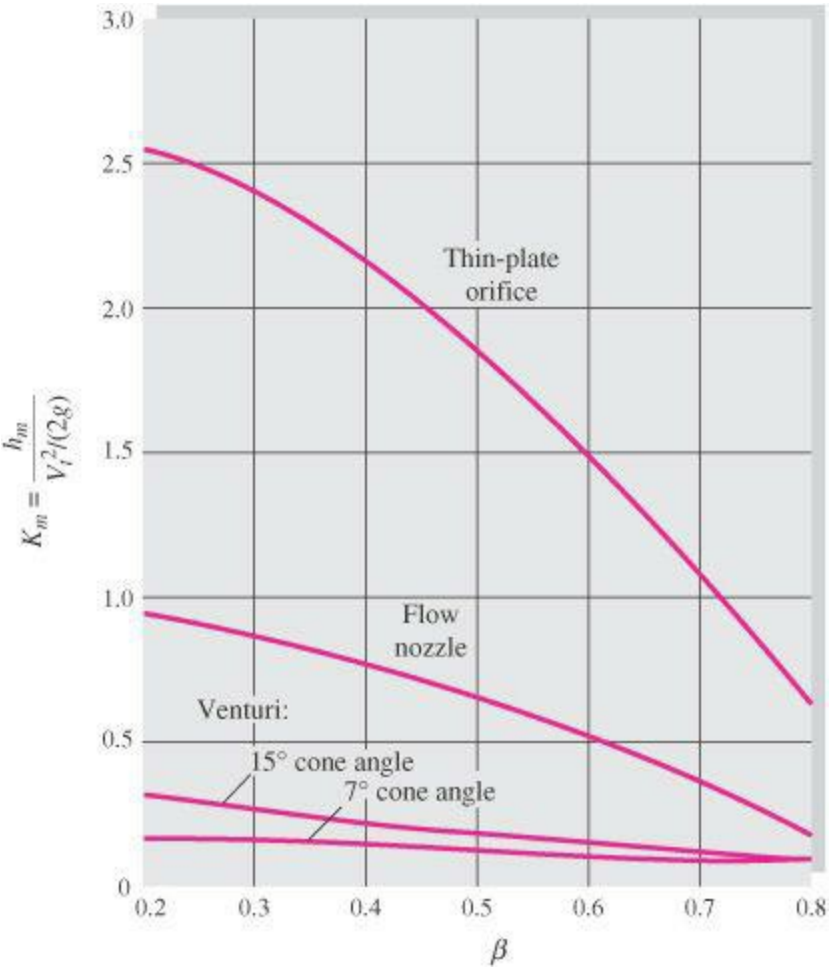
,WV LQGHSHQGHQW RI 5H ZLWKLQ WKH JLYHQ UDQJH 7KH + HUVFKHOYHQXUL GLVFKDUJH YDULHV ZLWK 5H EXW QRW ZLWK DV VKRZ QLQ) LJ %RWK KDYH YHU\ ORZ QHWORVVHV

7KH FKRLFH RI PHWHU GHSHQGV RQ WKH ORVV DQG WKH FRVW DQG FDQ EH LOOXVWUDWHG E\ WKH IROORZLQ WDEOH

Type of meter	Net head loss	Cost
Orifice	Large	Small
Nozzle	Medium	Medium
Venturi	Small	Large

$ VVR RI WQ KDSSHQV WKH SURGXFW RI LQHILFLHQF\ DQG LQLWLDO FRVW LV DSSUR[LPDWH FRQVDQW
7KH DYHUDJH QRQUHFRYHUDEOH KHDG ORVVHV IRU WKH WKUHH W\SHV RI PHWHUV H[SUHVVHG DV DIUDFWLRQ RI
WKH WKURDW YHORFLW\ KHDG $$\frac{V_t^2}{(2g)}$$ DUH VKRZ QLQ) LJ 7KH RULLFH KDV WKH JUHDWHVW ORVV DQG WK

YHQXUL WKH GHDWVDV GLVFXVVHG 7KH RUILFH DQG QR]]OH VLPXODWH SDUWLDOO FORVHG YDOYHV DV LQ) LJ E ZKLOH WKH YHQXUL LV D YHU PLQRU ORVV : KHQ WKH ORVV LV JLYHQ DV D IUDFWLRQ RI WKH PHDVXUH SUHVVXUHGURS WKH RUILFH DQG QR]]OH KDYH QHJOLJLEOH HTXDOORVWHV DV([DPSOH ZLOOLOXVWUDWH



) LJ 1 RQUHFRYHUDEOH KHDG ORVV LQ %HUQRXOOL EHWZXHHQ PHWHUV *Adapted from Ref. 30.*

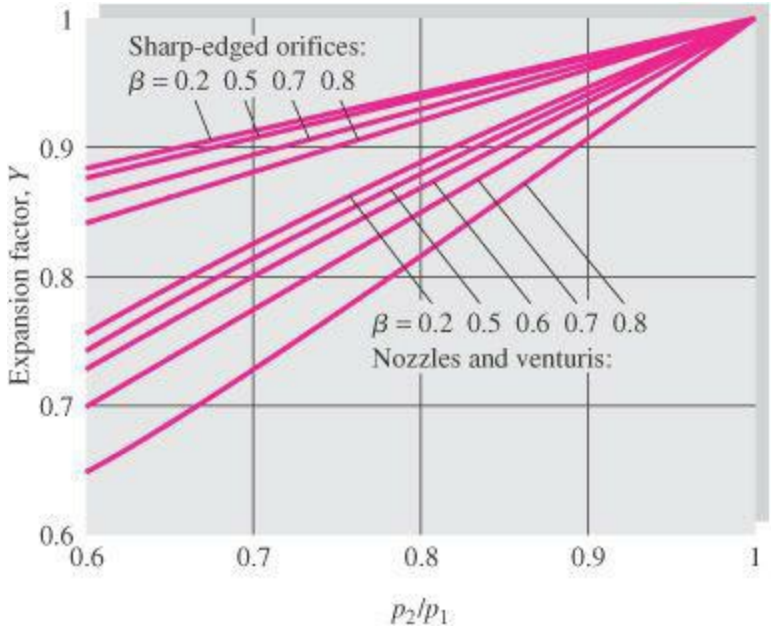
7KH RWKHU WSH RI LQWUXVLYH PHQW GLVFXVVHG HDUOLHU LQ WKLV VHFWLRQ FDQ DOVR VHUYH DV ORZ PHWHUV SURSHUO FRQWUXFWHG) RU[DPSOH D KRWZLUH PRXQWHG LQ D WXEH FDQ EH FDOLEUDWHG WR UHDG YROXPH ORZ UDWH UDWHUWKDQ SRLQWYHORFLW 6XFK KRWZLUH PHWHUV DUH FRP PHUFLDOO DYDLODEOH DV DUH RWKHU PHWHUV PRGLILHG WR XVH YHORFLW LQWUXVLYHV) RU IXUWKHU HGHDLOV VHH5H

&RP SUHVVLEOH* DV) ORZ &RUUHFWLRQ) DFWRU

7KH RUILFH QR]]OH YHQXUL IRUPXODV LQ WKLV VHFWLRQ DVXPH LQFRP SUHVVLEOH ORZ ,I WKH IOXLG LV JDV DQG WKH SUHVVXUH UDWLR S S LV QRW QHDU XQLW DFRP SUHVVLELOLW FRUUHFWLRQ LV QHHGHG (TXDWLR LVUHZ ULWWQ LQ WHUP VRI P DVV ORZ DQG WKH XSVWUHDP GHQVLW!

$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1(p_1 - p_2)}{1 - \beta^4}} \quad \text{where} \quad \beta = \frac{d}{D} \qquad (6.117)$$

7KH GLPHQVLRQOHVV[SDQVLRQ IDFWRU< LV D IXQFWLRQ RI SUHVVXUH UDWLR DQG WKH WSH RI PHWHU 6RPH YDOXHV DUH SORWWHG LQ) LJ 7KH RUILFH ZLWK WKLQ[ZDOO HGJHV LV WKH OHDVW VHQVLWLYH IURP WKH YHQXUL RU WKH ORZ QR]]OH ZKLFK DUH GHVLJQHG WR HOLPLQDWH FRQWUDFWLRQ



) LJ &RP SUHVVLEOH IORZ H[SDQVLRQ IDFWRU Y IRU IORZ PHWHUV

(; $0 3/ (

: HZ DQWWR PHWHUWKH YROXPHWULF RI ZDWHU ! NJ P î í P V PRYLQ VKURXJK D PP GLDPHWHUSLSH DWDQ DYHUDJH YHORFLWY RI P V ,I WKH GLIIHUHQWLDOSUHVXUH JDJH VHOHFWHGVDFFXUDWHO DWS í S 3D ZKDWZ]HPHWHUVKRXOGEHVHOHFWHGIRULQWDOOQJ DQRUILFHZLWK' ½ ' VDSV E DORQJ UDGLXVIORZ QR]]OHRU F DYHQWXULQR]]OH" : KDWZRXOGEHWK QRQUHFRYHUDEOHKHDGORVVIRUHDFKGHVJQ'

6RXWLRQ

+HUHWKHXQNQRZQLV WKH UDWH RI WKHPHWU 6LQFHWKHGLVFKDUJHFRHILFLHQWLV DFRPSOFDWH IXQFWLRQRI LWWUDWRQR((6ZLOOEHQHFHVVDU : HDUHJLYHQ' P DQG9 P V 7KHSLSH-DSSURDFK5H\QROGVQXPEHULVWKXV

$$\mathrm{Re}_D = \frac{V_1 D}{\nu} = \frac{(2.0)(0.2)}{1.02 \times 10^{-6}} = 392{,}000$$

) RUDOWKUHHFDVHV> D VR F @WKHJHQHUDO] HGIRUPXOD KROGV

$$V_t = \frac{V_1}{\beta^2} = \alpha\left[\frac{2(p_1 - p_2)}{\rho}\right]^{1/2} \qquad \alpha = \frac{C_d}{(1 - \beta^4)^{1/2}} \qquad (1)$$

ZKHUH WKH JLYHQ GDWD DUH9 P V! NJ P DQG ûS 3D ,QVHUWLQJ WKHV NQRZQ YDOXHVLQWR(T JLYHVDUHDWRQEHWZHHQ DQG.

$$\frac{2.0}{\beta^2} = \alpha\left[\frac{2(50{,}000)}{1000}\right]^{1/2} \quad \text{or} \quad \beta^2 = \frac{0.2}{\alpha} \qquad (2)$$

7KH XQNQRZQ V DUH RU. DQG &G 3DUW D VR F GHSHQG RQ WKH SDUWFXODU FKDUWRU IRUP XQ QHGHG IRU &G IFQ5H : HFDQP DNHDQLQWDOJXHV § DQGLVWUDWVRFRQYHUJHQFH

3DUWD

) RUWKHRUILFHZLWK' ½ ' WDSV XVH(T RU) LJ 7KHLWUDWLYHVHTXHQFHLV

&G . &G .

: HKDYHFRQYHUJHGWRWKUHHILJXUHV 7KHSURSHURUILFHGLDPHWULV

$$d = \beta D = 112\ \text{mm} \qquad \textit{Ans. (a)}$$

3DUWE

) RUWKHRQJ UDGLXVIORZ QR]]OH XVH(T RU) LJ 7KHLWUDWLYHVHTXHQFHLV

&G . &G .

: HKDYHFRQYHUJHGWRWKUHHILJXUHV 7KHSURSHUQR]]OHGLDPHWULV

$$d = \beta D = 89\ \text{mm} \qquad \textit{Ans. (b)}$$

3DUWF

) RUWKHYHQWXULQR]]OH XVH(T RU) LJ 7KHLWUDWLYHVHTXHQFHLV

&G . &G .

: HKDYHFRQYHUJHGWRWKUHHILJXUHV 7KHSURSHUYHQWXULGLDPHWULV

$$d = \beta D = 89\ \text{mm} \qquad \textit{Ans. (c)}$$

&RPPHQWV 7KHVHPHWULV DUH RI VLPLODUVL]H EXWWKHLUKHDG ORVVHV DUH QRWWKHVDPH) URP) LJ IRUWKHVKDUHGILHUHQWKDSHVZHPD\ UHDGWKHWKUHH. IDFWRUVDQGFRPSXWH

K RUILFH P K QR]OH P K YHQXUL P

7KHYHQXULRVLVRQD DERXW SHUFHQWR WKHRUHWLFDOQR]]OHRVHV

6RXWLRQ

7KHLWHUDWLRQHQFRXQWHUHGLQWKLVH[DPSOHLVLGHDOIRUWKH((6 ,QSXWKHGDWDLQ6, XQLW

```
Rho = 1000 Nu = 1.02E-6 D = 0.2 V = 2.0 DeltaP = 50000
```

7KHQZULWHRXWKHEDVLFIRUPXODVIRU5H\QROGVQXPEHUWKURDWWHORFLWDQGIORZ FRHILFLHQW

```
   Re=V*D/Nu
  Vt=V/Beta^2
Alpha=Cd/(1-Beta^4)^0.5
  Vt=Alpha*SQRT(2*DeltaP/Rho)
```

)LQDOO LQSXWKHSURSHUIRUPXODIRUWKHGLVFKDUJHFRHILFLHQW)RU[PSOH IRUWKHIORZ QR]]OH

```
Cd=0.9965-0.00653*Beta^0.5*(1E6/Re)^0.5
```

: KHQDVNHG WR 6RONH WKH HTXDWLRQ ((6 DWLLWVFRPSDLQV RI GLYLGLQJ E\ ]HUR 2QH PXWWKH VWKHQXS WKH 9DULDEOH,QRUPDWRQE\ QRWDOORZLQJ . RU&G WR EHQHJDWLYH DQG LQSDUWFXODU E\ FRQLQLQJ WR LWVSUDFWFDOUDQJH ((6 WKHQUHGLO DQQRXQFHVFRUHFWDQZHUVIRUWK IORZ QR]]OH

$QSKD &G %HWD

(; $0 3/ (

$ QRQJUDGXDQR]]OHRI GLPHWU FP LV XVHG WR PHWUDLUIORZ LQD FP GLPHWUSLSH-8SWUHDP FRQGLWLRQVDUHS NBDDQG7 *f& ,I WKHSUHVVXUHGURSWKURXJKWKHQR]]OHLV NBD HVWLPDWWKHIORZ UDWLQP V

6RXWLRQ

- $VXPSWLRQV 7KHSUHVVXUHGURSV SHUFHQWRZHQHGWKHFRPSUHVVLELOLWIDFWRU< DQ (T LVDSSOLFDEOHWRWKLVSUREOP
- $SSURDFK)LQG! DQG&G DQGDSSO(T ZLWK
- 3URSHUWYDOXHV * LYHQS DQG7 ! S 57 > @ NJ P 7KHGRZQWUHDP SUHVWXUHLVS í NBD KHQFHS S $W *f& IURP 7DEO

$ WKHYLVFRVLWRI DLUDV (NJ P V

- 6RODWRQVWDWV ,QWURGXFWLRQ DSSO(T E\ JXHVWLQJ IURP)LJ WKDW&G §)URP)LJ IRUDQR]]OHZLWKS S DQG UHDG< § 7KHQ

$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1(p_1 - p_2)}{1 - \beta^4}} \approx (0.98)(0.80)\frac{\pi}{4}(0.06\text{ m})^2\sqrt{\frac{2(1.87\text{ kg/m}^3)(60{,}000\text{ Pa})}{1 - (0.6)}}$$

$$\approx 1.13\frac{\text{kg}}{\text{s}}$$

1RZ HVWLPDWH5H6 SXWWLQJ LWLQWKHFRQYHQLHQWPDVIORZ IRUP

$$\text{Re}_d = \frac{\rho V d}{\mu} = \frac{4\dot{m}}{\pi\mu d} = \frac{4(1.13\text{ kg/s})}{\pi(2.17\text{ E-5 kg/m} - \text{s})(0.06\text{ m})} \approx 1.11\text{E6}$$

5HVXOLQJ WR)LJ ZHFRXOGUHDGDVOJKWO EHWWU&G § 7KXVRXUILQDOHVWLPDWLV

$$\dot{m} \approx 1.14\text{ kg/s} \qquad Ans.$$

- &RPPHQWV)LJXUH LVQRWWXWD³FKDUWIRUHQJLQHHUVWRXVHFDVXDOO ,WVEDVHGRQWK FRPSUHVVLEOHIORZ WKHRU RI&KDS 7KHUH ZHPD UHDWLJQWKLVH[DPSOHDVDWKHRU

6XPPDU\

7KLVFKDSWUKDVEHQFRQFHQWUDWHGZLWKLQWURDOSLSHDQGGXFWORZV ZKLFK DUHSUREDEO WKHPRV FRPPRQSUREOPVHQFRXQWHUHGLQHQJLQHHULQJ IOXLGPHFKDQFV 6XFKORZV DUHYHU VHQVLWLYH WR 5H1RROGV QXPEHUDQGFKDQJHIURP ODPLQDUWXUEXOHQWORZ DV WKH5H1RROGV QXPEHUV LQFUHDVHV

7KHYDULRXV5H1RROGV QXPEHUUHJLPHVZHUHRXWOLQHG DQGDVHPLHPSLULFDOSSURDFKWRWXEXOHO IORZ PRGHOQJ ZDVSUHVHQWHG 7KHFKDSWUWKHQPDGHDGHWDLOHG DQDOVLVRI IORZ WKURXJK DVWUDLJK FLUFXODUSLSH OHDGLQJ WRWKHIDPRXV0RRG\ FKDUW) LJ IRUWKHIULFWLRQIDFWRU3RZOEHXVHVR WKH0RRG\ FKDUWZHUHGLVFXVVHGIORU UDWHDQGVL]LQJ SUREOPV DVZHOODVWKHDSSOLFDWLRQRI WK 0RRG\ FKDUWWRQRQFLUFXODUGXFWXXLQJ DQHTXLYDOHQWGXFW7GLDPHWHU 7KHDGGLWLRQ PLQRUORVVH GXHWRYDOYHV HOERZV ILWWQJV DQGRWKHUGHYLFHVZDVSUHVHQWHGLQWKHIRUP RI ORVVFRHILFLHQWVRE LQFRUSRUDWHGDORQJ ZLWK0RRG\VSHFIULFWLRQORVVHV 0 XOWLSOHSLSHVVWHPVZHUHGLVFXVVHGEULHO DQGZHUHVHQWRE HTXLWFRPSOH[DJHEUDLFDOO DQGDSSURSULDWHFRPSXWHUURXWLQ

' LIIXVHUV/DUHDGGHGWRGXFWWRRLQFUHDVHSUHVVXUHUHFRYHU DWKHH[LWRI DV\VWHP 7KLUEHKDYLR ZDVSUHVHQWHGDVH[SHULPHQWDOGDWDVLQHWKHWKRU RI UHDOLIIXVHUVLVVWXGLQRWZHOOGHYHORSHG 7K FKDSWUHQGHG ZLWK D GLVFXVVLRQ RI IORZ PHWHUV HVSHFLDOO WKH SLWRWVWDWF WXEH DQG WKH %HUQRXO REWXFWLRQWSHRI PHWHU)ORZ PHWHUVDOR UHTXLUHFDUHXO[SHULPHQWDOFDOEUDWLRQ

3UREOHPV

0 RVW RI WKH SUREOHPV KHUH LQ DUHDV OLNH GHVLJQ DQDO VLV DQG 0 RUH GLIILFXOW RUHV HQFRXQWHUHG LQ DFWXDO SUDFWLFH WKDQ DUH GHVFULEHG LQ WKH WH[W 3UREOHPV ODEHOHG ZLWK DQ DVWHULVN 3UREOHPV ODEHOHG ZLWK DQ ((6 LFRQ ZLOO EH IRXQG LQ WKH XSSHU ULJKW (QJLQHHULQJ (TXDWLRQ 6ROYHU ((6 ZKLFK SURJUDPV ODEHOHG ZLWK D FRPSXWHU PD\ UHTXLUH WKH XVH RI D FRPSXWHU 7KH VWDQGDUG HQG RI FKDSWHU SUREOHPV 3 VR3 FDWHJRULHG LQ WKH SUREOHP OLVWV HUH DUH RQH HG E\ ZRUGSUREOHPV: VR: IXQGDPHQWDOV RI HQJLQHHULQJ H[DF SUREOHPV) (VR) (FRPSUHKHQVLYH SUREOHPV & VR& DQG GHVLJQ SURMHFWV' DQ '

3UREOHP ' LVWULEXWLRQ

Section	Topic	Problems
6.1	Reynolds number regimes	P6.1–P6.5
6.2	Internal and external flow	P6.6–P6.8
6.3	Head loss—friction factor	P6.9–P6.11
6.4	Laminar pipe flow	P6.12–P6.33
6.5	Turbulence modeling	P6.34–P6.40
6.6	Turbulent pipe flow	P6.41–P6.62
6.7	Flow rate and sizing problems	P6.63–P6.85
6.8	Noncircular ducts	P6.86–P6.98
6.9	Minor or local losses	P6.99–P6.110
6.10	Series and parallel pipe systems	P6.111–P6.120
6.10	Three-reservoir and pipe network systems	P6.121–P6.130
6.11	Diffuser performance	P6.131–P6.134
6.12	The pitot-static tube	P6.135–P6.139
6.12	Flowmeters: the orifice plate	P6.140–P6.148
6.12	Flowmeters: the flow nozzle	P6.149–P6.153
6.12	Flowmeters: the venturi meter	P6.154–P6.159
6.12	Flowmeters: other designs	P6.160–P6.161
6.12	Flowmeters: compressibility correction	P6.162–P6.163

5 H\ QRROGV QXP EHU UHJLP HV

- 3 $ Q HQJLQHHULQJ FDOLPV WKDW WKH IORZ RI 6$ (: RLO DW *f*& WKURXJK D FP GLDPHWHU VPRRWK SLSH DW PLOORQ 1 K LV OLPLQDU ' R\ RX DJUHH' $ PLOORQ QHZWRQV LV I ORWWR WKLV VRXQGV OLNH DQ ZIXOO KJK IORZ UDWH
- 3 7KH SUHVVXUH UDWH RI 1 RUWK 6RSH FUXGH RLO WKURXJK WKH $ODVND 3LSHOLQ VHH WKH FKDSWHU RSHQHU SKRWR LV DERXW EDUUHOV SHU GD\ EDUUHO 8 6 JDOORQV : KDW ZRXOG EH WKH PDLPXP UDWH LI WKH IORZ ZHUH FRQWDLQHG WR EH ODPLQDU $VXPH WKDW $ODVND QRLOLW) LJ $ RI WKH $SSHQG[DW *f&
- 3)RORZLQJ XS3URE 3 VXSSRVH WKH $ODVND 3LSHOLQH ZHUH FDUU\LQJ PLOORQ 8 6 JDOORQV SHU GD\ RI 6$ (: RLO DW *f&" (VWLPDWH WKH 5H\QROGV QXP EHU ,V WKH IORZ ODPLQDU'
- 3)RU IORZ RI 6$ (RLO WKURXJK D FP GLDPHWHU SLSH IURP) LJ $ IRUZKLF IORZ UDWH LQ P KZRXOG ZHH[SHFW WKDW QLWRQ WR WXUEXOHQFH DW D *f& DQG E *f&"
- 3 ,Q IORZ SDWWHUQ ERG\ RU ZDOO HDU WKDW QLWRQ WR WXUEXOHQFH FDQ EH LQGXFHG E\ SODFLQJ DWUS ZLUHV RQ WKH ZDOO DFURVV WKH IORZ DV LQ) LJ 3 ,I WKH WUS ZLUHV) LJ 3 LV SODFHG ZKHUH WKH ORFDOYHORFLWLV 8 LWZ LOO WULJJHU WXUEXOHQFH LI 8G ZKHUH G LV WKH ZLUH GLDPHWU> S @,I WKH VSKHUH GLDPHWULV FP DQG WDQQLWRQ LV REVHUYHG D

5H ZKDWLVWKHGLDPHWURIWKHWULSZLUHLQPP"

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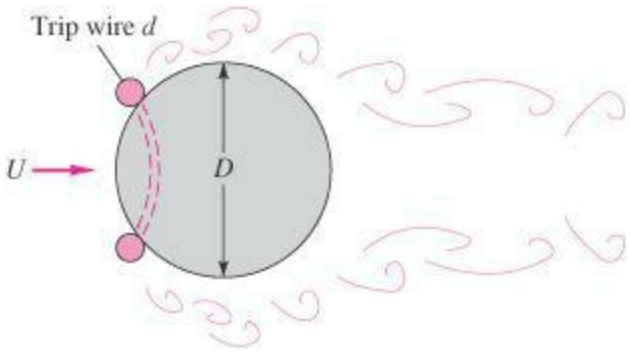

,QWHUQDOQG H[WHUQDOIORZ

- 3)RUIORZ RI DXQIRUP VWHDP SDUDGHOWRDVKDSHGVSODWH VWDQGLQJRQWKHEXQGDU OD\HURQWKHSODWHPD\ RFFXUDW5H ! 8[§ (ZKHUH8 LVWKHDSSURDFH YHORFLW DQG[LVGLVWDQFHDORQJ WKHSODWH ,I8 P V GHWHUPLQHWKHGLVWDQFH[IRUWK IRORZLQJ IXLGV DW *f& DQG DW D K\GURJHQ E DU F JDVROLQH G ZDWHU H PHUFXU DQG I JOFHULQ
- 3 &RQG DSSUR[LPDWHGDVSXUHZDWHUDW *f& LVWRILOODQ R\ FRQGLQHU 8 6 JI IORZ WKURXJKD PP GLDPHWUWXEH (VWDPDWWKHPLQPXP ILODQJ WPHLI WKHWXE IORZ LVWRUHPDLQODPLQDU)RUZKDWFRQ ZDWU WPSHUDWXUHZRXOGWKLVPLQPXP WPHE PLQ"
- 3 : KHQZDWUDW *f& LVLQWDG\WXEHQWORZ WKURXJKDQ FP GLDPHWUSLSH WK ZDOOVKHDUVWUHVV 3D : KDWLVWKHD[LDOSUHVVXUHJUDGLHQW%S%p LI WKHSLSH LV D KRUL]RQWDOQG E YHUWFDOZLWKWKHIORZ XS"

+ HDG ORVVHV IULFWLRQ IDFWRU

- 3 $ OJKWWQLG ! § NJ P IORZ V DWDQ DYHUDJH YHORFLW RI P V WKURXJK I KRUL]RQWDO VP RRWK WXEH RI GLDPHWU FP 7KH IOXLG SUHVVXUH LV PHDVXUHG DW F LQWHUYDOV DORQJ WKHSLSH DVIRORZ V

x, m	0	1	2	3	4	5	6
p, kPa	304	273	255	240	226	213	200

(VWLPDWH D WKH VRXOO KHDG ORVV LQ PHWHUV E WKH ZDOOVKHDU VWUHVV LQ WKH IXOO GHYHORSHGVHFWLRQRI WKHSLSH DQG F WKHRYHUDOOIULFWLRQIDFWRU
- 3 : DWHUDW *f& IORZ VWKURXJK DQLQFOLQHG FP GLDPHWUSLSH $VWHFWLRQV$ DQ %WKHIORZLQJ GDWDDUHWDNHQ S$ N3D 9$ P V]$ P DQGS% N3D 9% P V]% P : KLFKZD\ LVWKHIORZ JRLQJ": KDWLVWKHKHDGORVVLQPHWHUV"
- 3 : DWHUDW *f& IORZ VXSZDUGDW P VLQD FP GLDPHWUSLSH 7KHSLSHQHV EHWZHHQSRLQWV DQG LV P DQGSRLQW LV P KLJKHU$ PHUFXU PDQRPHWHU FRQQHFWHG EHWZHHQ DQG KDVDUHDGLQJ K PP ZLWS KLJKHU D : KDWLVWKHSUHVVXUHFKDQJH

S í S " E : KDWLV WKH KHDG ORVW LQ PHWUV" F ,V WKH PDQRPHWUH DGLQJ SURSRUWLRQDO WR WKH DGORVW" ([SODLQ G : KDWLV WKH IULFWLRQ IDFWRU IRU WKH IORZ"

,Q3UREV WR QHJOHFW PLQRU ORVVHV

/ DP LQDUSLSHIORZ ±±QR P LQRU ORVVHV

- 3 $ PP GLDP HWUHFDSLOODU\ WXEH LV XVHG DV DYLVFRP HWUIRURLOV : KHQ WKH IORZ UDWH LV P K WKH PHDVXUHG SUHVVXUH GURS SHU XQLW OHQJWK LV N3DP (VWDWH DW WK YLVFRVLWLHV WKH IOXLG ,V WKH IORZ ODPLQDU" &DQ \RX DOVR HVWLPDWH WKH GHQVLWLHV IOXLG'
- 3 $ VRGD VWUDZ LV FP ORQJ DQG PP LQ GLDP HWU ,WV GHOLYHU\ FRQG FRQ DSSUR[LPDWHG DV ZDWHU DW f& DQG UDWHV FP V D : KDWLV WKH KHDG ORVV WKURXJK WK VWUDZ" : KDWLV WKH DIIHFWLYH JUDGLHQW %S % LI WKH IORZ LV E YHUWFDO XS RU F KRUL]RQWDO &DQ WKH KXP DQ QXQ GHOLYHU WKLV PXFK IORZ"
- 3 : DWHU DW f& LV WR EH VLSKRQHG WKURXJK DWEH P ORQJ DQG PP LQ GLDP HWU DW LQ) LJ 3 ,V WKH UHDO KHJKW IRU ZKLFK WKH IORZ PLJKW QRW EH ODPLQDU" : KDWLV WK IORZ UDWH L + FP" 1HJOHFW WKH WXEH FXUYDWXUH

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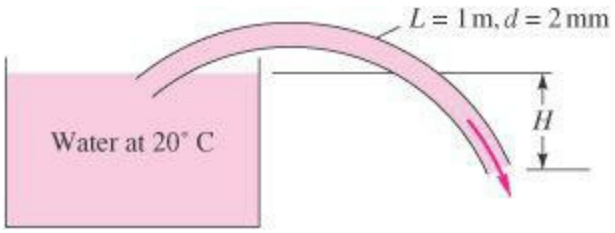
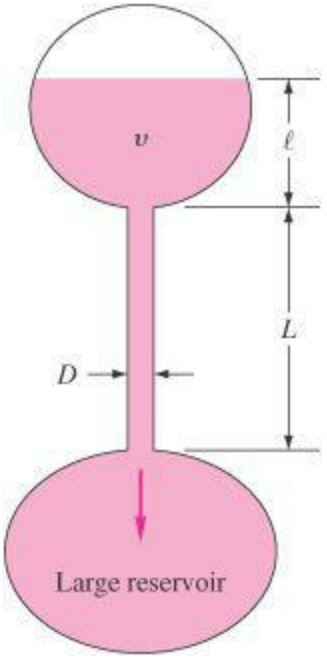


- 3 3URHVRU* RUGRQ+ RQRZ D\ DQG KLV WXGHQWV DWWKH 8QLYHUVLWL RI 1HZ %UXQZ LFN ZHQWWR D IDWWRRG HP SRUXP DQG WUHG WR GULQN FKRFRODWH VKDNHV ! § NJ P § NJ P V WKURXJK IDWWHG V PP LQGLDP HWU DQG FP ORQJ D 9HUL\ WKDWWKH LUKXP D OXQJV ZKLFK FDQ GHYHORS DSSUR[LPDWH 3DR YDFXXP SUHVVXUH ZRXOG EH XQDEOH GULQN WKH PLOVKDNHV WKURXJK WKH YHUWFDO WUHZ E $ WXGHQWFXW FP IURP KLV WUHZ DQ SURFHHGHG WR GULQN KDSSLO : KDW DWWR P LOVKDNHV ZDV SURGXFHG E\ WKLV WUDW\"
- 3) OLG ICRZ V WKURXJK DYROXPHUDW4 WKURXJK DOAJHSLSHDQG WKHQGLYLGHV LQ WZR VPDOOSLSHV WKH OAJHUR ZKLFK KDV DQ LQWLGHGLDP HWU RI PP DQG FDUULHV WKUH WLPHV WKH IORZ RI WKH VPDOOHUSLSH %RWK VPDOOSLSHV KDYH WKH VDPH OHQJWK DQG SUHVXU GURS ,I DOOIORZ VDUHODP LQDU HWUP DW WKH GLDP HWU RI WKH VPDOOHUSLSH
- 3 $ FDSLOODU\ YLVFRP HWU P HDVXUHV WKH WP HUHTXLUHG IRU D VPHFLILHG YROXPH%R OTXLG WR IORZ WKURXJK D VP DOOERUH JODVV WXEH DV LQ) LJ 3 7KLV WDQLWXP HLV WKH FRUUHDWG ZLWK IOXLG YLVFRVLW) RU WKH VWXP VKRZQ D GHULYHDQ DSSUR[LP DW IRUP XO IRU WKH WP HUHTXLUHG DVXP LQJ ODP LQDU IORZ ZLWK QR HQWDQFH DQG H[WRXVH E ,I/ FP O FP % FP DQG WKH IOXLG LV ZDWHU DW f& ZKDW FDSLOODU\ GLDP HWU ZLO UHVXOWLQ DWDQLWP HWR VHFRQGV"



- 3 7RGHWHUPLQHWKHYLVFRVLWRI DOLTXLGRI VSHFLILF JUDYLW \RXILOOVWRDGHSW RI FP DODUJHFRQHLQHUWKDWGULDQVWKURXJK D FP ORQJ YHUWLFDOWXEHDWDFKHGWRWK ERWWRP 7KHWXEHGLDPHWHULV PP DQGWKHUDWHRI GULQLQJ LVIRXQGWREH FP V: KD LV\RXUHWLPHDWRI WKHILOGYLVFRVLW\" ,VWKHWXEHIORZ ODPLQDU"
- 3 $QRLO6* LVXVHGIURP WKHSLSHLQ)LJ 3 DW4 IWK: KDWLVWK NLQHPDWLF YLVFRVLWRI WKHRLOLQIWV" ,VWKHIORZ ODPLQDU"

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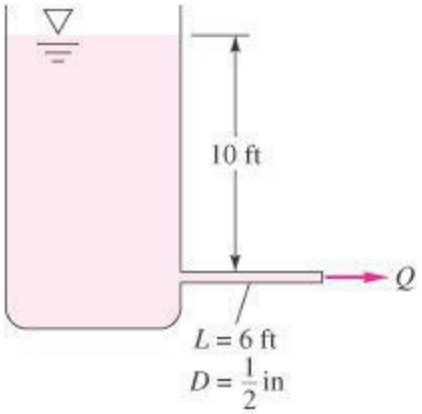
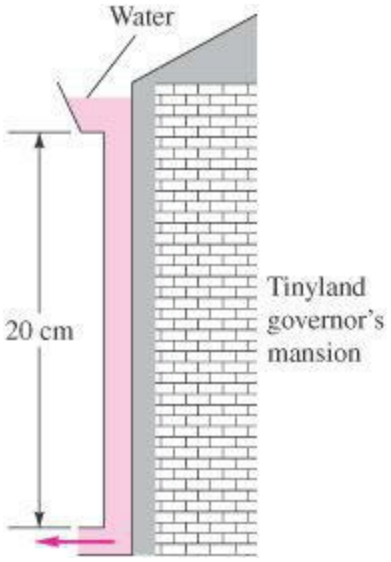


- 3 7KH RLOWDQN LQ 7LQ ODQG DUH RQO FP KLJK DQG WKH GLVFKDUJH WR WK 7LQ ODQG RLOWKFN\WKURXJK DVPRRWKWXEH PP LQGLDPHWHUDQG FP ORQJ 7KHWXEH[LWL RSHQWRWKHDWPRVSKHUHDQG EHORZ WKHWDQNVXUIDFH 7KHIOXLGLVPHGLXP IXHORLO! NJ P DQG NJ P V (VWLPDWHWKHRLOIORZ UDWHLQFP K
- 3 ,Q 7LQ ODQG KRXVHV DUH OHVV WKDQ D IRRWKJK 7KH UDLQDOO LV ODPLQDU 7K GULQSLSH LQ)LJ 3 LVRQO PP LQGLDPHWHU D : KHQWKHJXWWULVIXOO ZKDWLV WK UDWHRI GULQLQJ" E 7KHJXWWULVGHVLJQHGIRUDVXGGHQUDLQWRUP RI XS WR PP SHUKRX) RUWKLV FRQGLWLRQ ZKDWLV WKH PD[LPXP UDWH DWWDFQ EH GULQHG VXFFHVVIXOO" F : KDWLV5H"



- 3 $ VWDG SXVK RQ WKH SLVWRQ LQ) LJ 3 FDXVHV D IORZ UDWH 4 FP V WKURXJK WKH QHHGOH 7KH IOXLG KDV! NJ P DQG NJ P V : KDW RUHH) LV UHTXLUHG WR PDLQWDLQ WKH IORZ"

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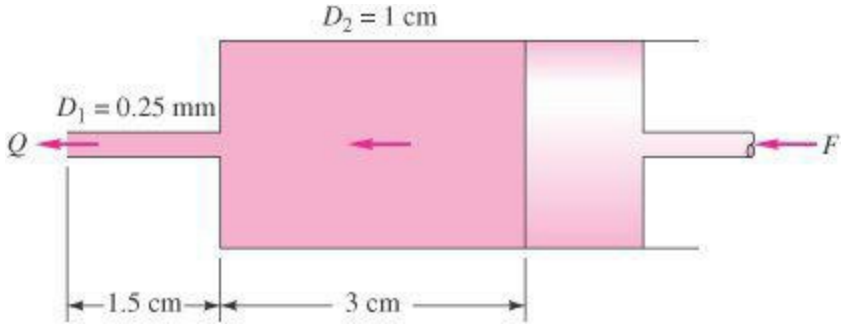

- 3 6$(RLO DW *& IORZ VLQ DYHUWLFDO SLSH RI GLDPHWHU FP ,WV IRXQG WK WKH SUHVVXUH LV FRQVWDQW WKURXJKRXW WKH IOXLG : KDW V WKH RLO IORZ UDWH LQ K" ,V WKH IORZ XS RU GRZ Q'
- 3 7Z R WDQNV RI Z DWHU DW *& DUH FRQQHFWHG E\ D FDSLOODU WXEH PP LQ GLDPHWHU DQG P ORQJ 7KH VXUIDFH RI WDQN LV FP KLJKHU WKDQ WKH VXUIDFH RI WDQN D (WXP DW WKH IORZ UDWH LQ K ,V WKH IORZ ODPLQDU' E) RU Z KDW WXEH GLDPHWHU Z LOO 5H EH "
- 3) RU WKH FRQLJXUDWRQ VKRZ QLQ) LJ 3 WKH IOXLG LV HW KDOD RKRO DW *& DQ WKH WDQN VDUH YHU Z LGH) LQG WKH IORZ UDWH Z KLFK RFFXUV LQ K ,V WKH IORZ ODPLQDU'

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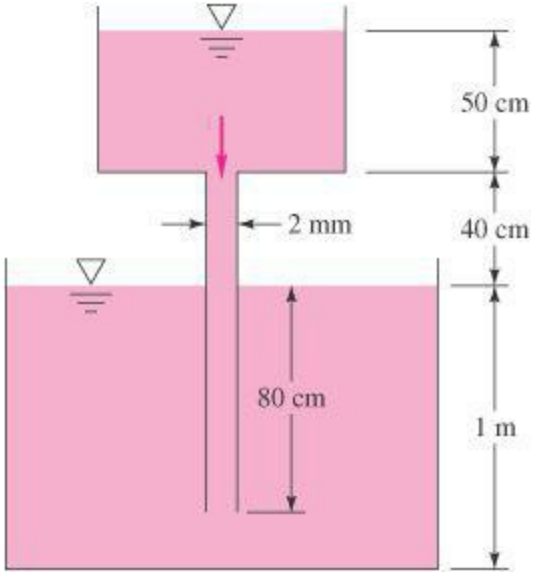

- 3 7Z RRLOWDQVDUHFRQQHFWHGE\ WZR PORQJSLSHV DVLQ)LJ 3 3LSH LV FP LQGLDPHWUDQGLV PKLJKHUWKDQSLSH ,WLVIRXQGWKDWWKHIORZ UDWHLQSLSH LVWZLF DVODUJHDVWKHIORZ LQSLSH D : KDWLVWKHGLDPHWURI SLSH " E \$UHERWKSLSHIORZ V ODPLQDU" F : KDWLVWKHIORZ UDWHLQSLSH P V" 1HJOHFWPLQRUORVVHV
- 3 /HWXVDVVXPH3URE 3 LQV\PERODFIDVKLRQ XVLQJ)LJ 3 \$OOSDUDPHWHU DUHFRQVWDQWH[FHSWWKHXSSHUDQGORZHU= W)LQGDQH[SUHVVLRQIRUWKHIORZ UDW4 WDVI IXQFWLRQRI= W 6HWXSDGLIIHUHQWLDOHTXDWLRQ DQGVROYHIRUWKHWLPHWWRGUDLQWKHXSSH WDQNFRPSOHWHO \$VXPHTXDVLVWDWLF\ODPLQDUIORZ

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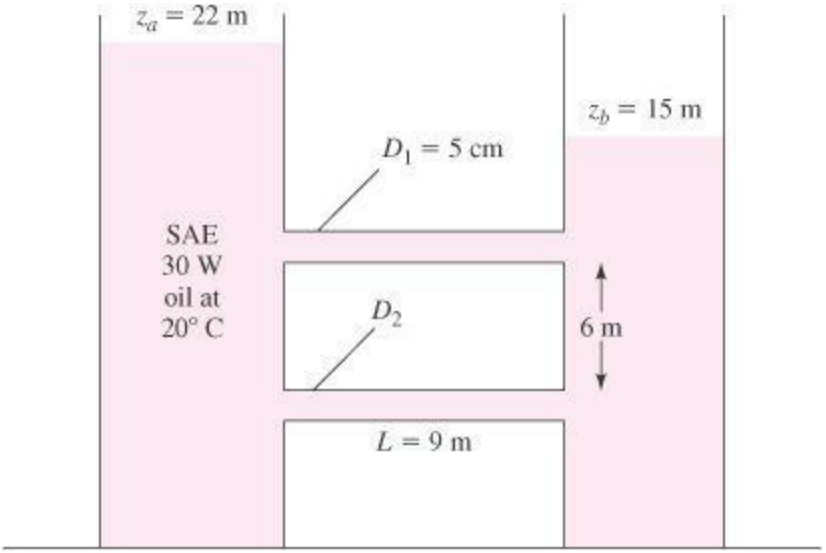

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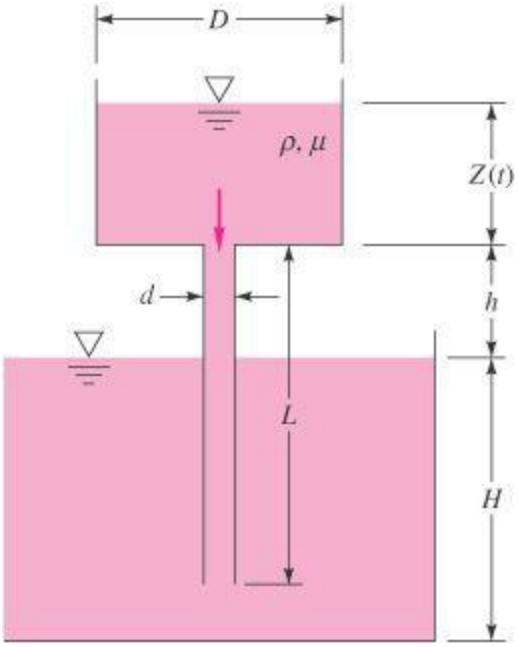

- 3) RUVWDJKAMQLQJ DQG VPRRWKLQJ DQDLUFRZ LQD FP GLDPHWUGXFWWKHGXFW SDFNHGZLWKD³KRQH\FRPE´ RI WKLQWDZVRI CHQJWK FP DQG GLDPHWU PP DVLQ) LJ 3 7KHLQDHWCRZ LVDLUDW N3DDQG *f& PRYLQJ DWDQDYHUDJHYHORFLWRI P\ (WUPDMWKHSUHVXUHGURSDFURVVWKHKRQH\FRPE
- 3 6$(: RLODW *f& ICRZ VWKURXJKDWXDJKWSLSH P CRQJ ZLWKGLDPHWU FP 7KHDYHUDJHYHORFLWV P V D ,VWKHICRZ ODPLQDU' &DFXODWH E WKHSUHVXUHGURS DQG F WKHSRZHUUHTXLUHG G ,I WKHSLSHGLDPHWULV GRXEOHG IRUWKHVDPHDYHUDJH YHORFLW E\ ZKDWSHUFHQWGRHVWKHUHTXLUHGSRZHULQFUHDVH'
- 3 6$(RLODW *f& ICRZ VWKURXJKWKH FP GLDPHWUYHUWFDOSLSHRI) LJ 3) RUWKHPHUFXU PDQRPHWUHDGLQJ K FP VKRZQ D FDOFXODWWKHYROXPHICRZ UDWH P KDQG E WDWWKHGLIHUHQRI ICRZ

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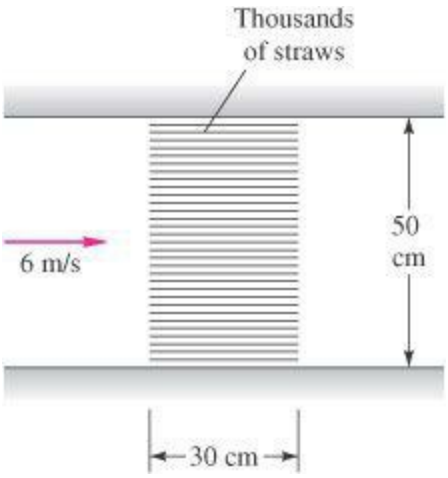

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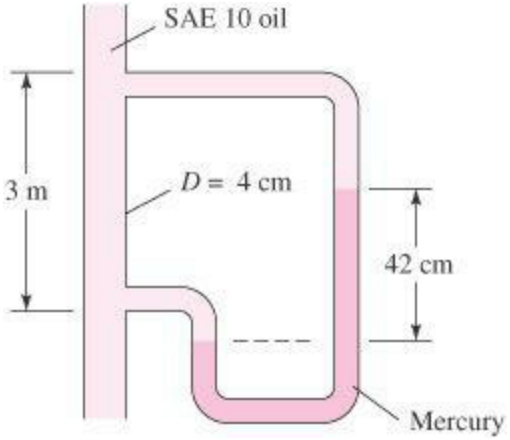
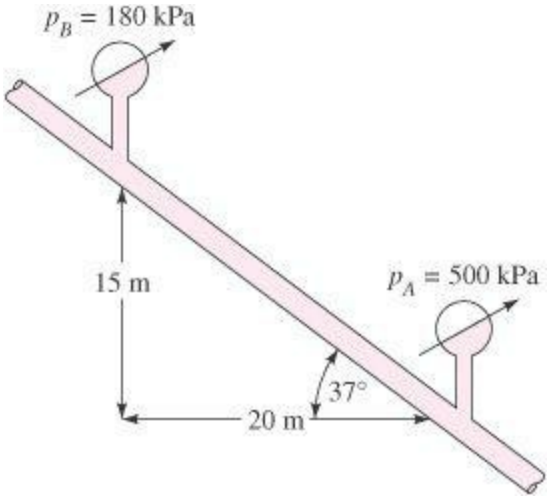


- 3 $ ODP LQDU IORZ HOHPHQW/) (0 HULDP ,QWUKPHQW&R PHDVXUHVORZ JDVIOR UDWHVZLWKDEXQGDQFHRI FDSLOODU\ WHEHVRUGXFWSDFNHGLQVLGHDODUJHRXWHUWXEH &RQVLG-R\JHQDW *& DQG DWP IORZLQJ DW IWPLQLQD LQGLDPHWUSLSH D ,VWKHIORZ WXUEXOHQWZKHQDSSURDFKLQJ WKHHOHPHQW E ,I WKHUHDUH FDSLOODU\ WEHV/ LQ VHOFWDWEHGLDPHWUWRNHS5H EHORZ DQGDOR WRNHSWKHWEHSUHWXUHGURSQ JUHDWUWKDQ GEH LQ F ' RWKHWEHVVHOFWGLQSDUWE ILWQFH0 ZLWLQWKHDSSURDFK SLSH'

- 3 6$(RLODW *& IORZ VLQWKH FP GLDPHWUSLSHLQ LJ 3 ZKLFKVORSH DW) RUWKHSUHWXUHPHDVXUHPHQWVKRZQ GHWHUPLQH D ZKHWKHUWKHIORZ LV XS R GRZQDQG E WKHIORZ UDWHLQP K

3



- 3 : DWUDW *& LVSXPSHGIURP DUHVHUYRLUWKURXJK DYHUWFDOWEH IWORQJ DQ WKLQLQGLDPHWU7KHSXPS SURYLGHVDSUHVXUHULVHR GEH LQ WKHIORZ 1HJOHF HQWDQFHORVVHV D &DOFXODWWKHH[LWYHORFLW E $SSUR[LPDWO KRZ KJK ZLOOWKHH[L ZDWUWWWULVH' F 9HUI\ WKDWWKHIORZ LVODPLQDU

7XUEXOHQFHPRGHOLQJ

- 3 ' HULYHWKHWPHDYHUDJHG[PRPHQWXP HTXDWRQ E\ GLUHFWXEVWXWRQR (TV LQRWKHPRPHQWXP HTXDWRQ ,WLVFRQYHQLHQWWR ZULWWKHFRQYHFWLY

DFFHOHUDWRQV

$$\frac{du}{dt} = \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw)$$

ZKLFKLVYDOLGEHFDXVHRI WKHFRQWLQXLW\UHODWLRQ (T

- 3 ,QWKHRYHUODSSLQJ)LJ D WXUEXOHQWWKHDULVOLJH ,I ZHQHJOHFWYLVFRVLW\ZI FDQUHSODFH(T E\ WKHDSSUR[LPDWHYHORFLW\JUDGLHQWXQFWLRQ

$$\frac{du}{dy} = fcn(y, \tau_w, \rho)$$

6KRZ E\ GLPHQVLRQDODQDOVLV WKDWWKLVHDGV\RWKHRUJDULWKPLF RYHUODSUHODWLRQ

- 3 7KHIROORZLQJ WXUEXOHQWYHORFLW\GDWX\ IRUDLUDW *f* DQG DW QHDU\ VPRRWKIODWZDOOZHUHWDNHQZLWKH8QLYHUVLWRI 5KRGH,VODQGZLQGWXQQHO

y, in	0.025	0.035	0.047	0.055	0.065
u, ft/s	51.2	54.2	56.8	57.6	59.1

(VWLPDWH D WKHZDOOVKHDUVWUHVVDQG E WKHYHORFLWX DW LQ

- 3 7ZRLQLQWHUSDUDGLXDQFHK DSDUWDUHSDUDWHGWKH[SODQHZLWKWHXSSHU SODWH PRYLQJ DWSHHG9 DV LQ)LJ 3 7KHUH LV DIXLG RI YLVFRVLW\ DQG FRQVWDQ SUHVVXUH EHWZHHQ WKH SODWHV 1HJOHFWLQJ JUDYLW\ DQG DVVXPLQJ LQFRPSUHVVLEOH WXUEXOHQ IORZ X\ EHWZHHQ WKH SODWHV XVHWKHRJDULWKPLF ODZ DQG DSSURSULDWHERXQGDU\ FRQGLWLRQ WR GHULYH D IRUPXOD IRU GLPHQVLRQOHVV ZDOO VKHDU VWUHVV YHUVXV GLPHQVLRQOHVV SODWH YHORFLW\ 6NHWFK DWSLFDOVKDSHRI WKHSURILOHX\

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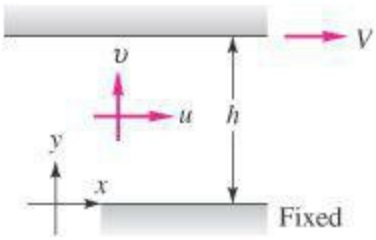

- 3 6XSSRVHLQ)LJ 3 WKDWWK FPP WKHIOXLGLQZDWUDW *f*& DQGWKHIORZ L WXUEXOHQWWRWKDWWKHRJDULWKPLF ODZ LVYDOLG ,I WKHVKHDUVWUHVVLQWKHIOXLGLV 3D ZKL LV9 LQP V"
- 3 %\ DQDORJ\ ZLWKODPLQDUVKHDU2 GXG\ 7 9 %RXVLQHVT LQ SRVWXODWH WKDWWXUEXOHQWVKHDUFRXOG DOVR EH UHDWHG WR WKHPHDQ YHORFLW\JUDGLHQW2 $XE ∈ GXG\ ZKHUHE LVFDOOHG WKHHGG\ YLVFRVLW\DQG LVPXFKODUJHUWKDQ ,I WKHORJDULWKPLF RYHUODZ ODZ (T LVYDOLGZLWK2 $XE § 2 VKRZ WKDW § ! X\
- 3 7KHRGRUH YRQ .iUPiQ LQ WKHRUL]HG WKDWWXUEXOHQWVKHDU FRXOG E UHSUHVHQWHG E\ 2 $XE ∈ GXG\ ZKHUHE ! \ XGXG\X LVFDOOHG WKHPL[LQJ OHQJWK HGG YLVFRVLW\DQG § LV.iUPiQ¶V GLPHQVLRQOHVVPL[LQJ OHQJWKFRQVWDQW> @\$ VXPLQ WKDW2 $XE § 2 QHDU WKH ZDOO VKRZ WKDWWKLV H[SUHVVLRQ FDQ EH LQWHJUDWHG WR \LHOG WK

GRJDULWKPLF RYHUDSDZ (T

7XUEXOHQWSLSHIORZ±±QRPLQRUORVVHV

- 3 7ZRUHVHUYRLUV ZKLFKGLIIHULQVXUIDFHHOHYDWLRQE\ P DUHFRQQHFWHGE\ P RI QHZ SLSHRI GLDPHWU FP ,I WKHGHVLUHGIORZ UDWHLVDWOHDVW 1 VRI ZDWUI *f& FDQWKHSLSHPDWHULDOEHPDGHRI D JDOYDQL]HGLURQ E FRPPHUFLDOWHDRU F FDVWLURQ" 1HJOHFWPLQRUORVVHV

- 3)OXLGIORZVWKDGLO DWYROXPHUDW4 WKURXJKDGUJHKRU]RQWDOSLSHDQGWKH GLYLGHVLQRWZRVPDOOSLSHV WKHGUJHURI ZKLFKKDVDQLQVLGHGLPHWURI PP DQ FDUULHVWKUHHWLPHVWKHIORZ RI WKHVPDOOSLSHV %RWKVPDOOSLSHVKDYHWKHVDPHGQJWKDQ SUHVVXUHGURS ,I DOOIORZV DUHWXUEXOHQWDW5H QHDU HVWLPDWWKHGLDPHWURI WK VPDOOSLSH

- 3 $ UHVHUYRLUVXSSOLHVZDWUWKURXJK P RI FP GLDPHWUFDWLURQ SLSHWR I VXUEQHWKDW[WDFW KSIURP WKHIORZ 7KHZDWUWKHQ[KDXVWVWR WKHDWPRVSKHU

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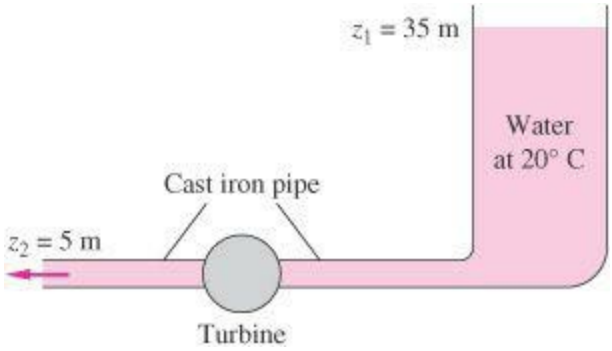


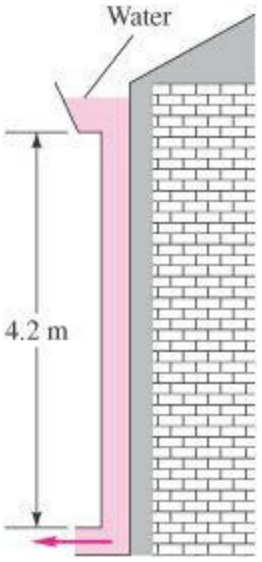
1HJOHFWPLQRUORVVHV D $VVXPLQJWKDW§ ILQGWKHIORZ UDWH ZKLFKUHVXOWLQ FXELF SRO QRPLDO ([SODLQ ZK\ WKHUH DUHWZR GHJLWP DWVROXWRQV E)RU[WD FUHGV VROYHIRUWKHIORZ UDWHVXVLQJWKHDFXDOIULFWRQIDFWRUV

- 3 0HUFXU DW *f& IORZVWKURXJK P RI PP GLDPHWUJODVVWXELQJ DWDQDYHUDJ YHORFLWRI P V (VWLP DWWKHKHDGORVVLQP DQGWKHSUHVVXUHGURSLQ3D

- 3 EES 2LO 6* DQG (P V IORZVDW JDOPLQWKURXJK D L DVSKDOWHG FDVWLURQ SLSH 7KHSLSHLV PLORQJ DQG VORSHV XSZDUG DW *f LQWKHIORZ GLUHFWRQ &RPSXWWKHKHDGORVVLQIWDQGWKHSUHVVXUHFKDQJH

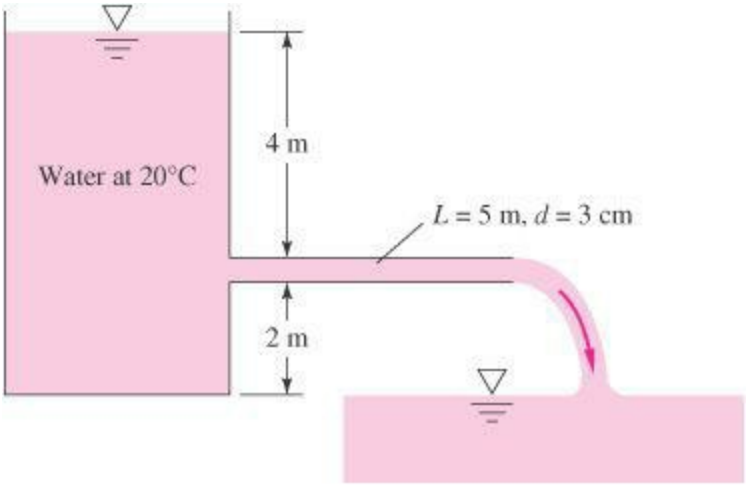
- 3 5HSHDW3UREOHP 3 IRUWKHVDPHGDW E\ XVLQJ WKHPRUH H[DFWXEHQWORZ IRUPXODVWRFRPSXWWKHYROXPHIORZ UDWHLQJDOORQVSHUPLQXW

- 3 7KHJXWWUDQGVPRRWKGULQSLSHLQ)LJ 3 UHPRYHUDLQZDWUIURP WKHURRI R DEXLOGLQJ 7KHVPRRWKGULQSLSHV FP LQGLDPHWU D : KHQWKHJXWWULVIXOOHWXPD WKHUDWHRI GUDLQLQJ E 7KHJXWWULVGHVLJQHGIRUDVXGGHQUDLQWRUP RI XS WR LQFKHVSH KRXU)RUWKLVFRQGLWLRQ ZKDWLVWKHPD[LPXP URRI DUHDWKDWPDQEHGUDLQHGVXFFHVVIXOO"



- 3 6KRZ WKDWL (T LV DFFXUDWH WKH SRVLWLRQLQ DWKUEXOHQWSLSHIORZ ZKHU ORFDOYHORFLWX HTXDOY DYHUDJHYHORFLW9 RFFXUVH[DFWO DWU 5 LQGHSHQGHQWWKI WK 5H\QROGV QXPEHU
- 3 7KHWDQN±SLSHVXWXP RI) LJ 3 LVWRGHOYHUDWHDW P KRI Z DWUDW *f& WRWKHUHVHUYRLU: KDWLVWKHPD[LPXP URXJKQHVVKHLJKW DOORZDEOHIRUWKHSLSH"

3



- 3 (VWDQRODW *f& IORZV DW 8 6 JDOPLQWKURXJK D KRUL]RQWDOFDVWLURQ SLS ZLWK/ P DQGG FP 1HJOHFWLQJ HQWUDQFHHIIHFWVHVWLPDWH D WKHSUHVVXUHJUDGLHQ GS G[E WKHZDOOKHDUWWHVV2 DQG F WKHSHUFHQWDJHUHGXFWLRQLQIULFWLRQIDFWRULI WK SLSHZDOOYDUHSODFHGKHGWRDVPRRWKVXUIDFH
- 3 7KH YLVFRXV VXEOD\HU) LJ LV QRUPDOO GHV WKDQ SHUFHQWRI WKH SLSI GLDPHWUDQG WKHUHRUHYHU GLIILFXOWSURPEHZLWKDILQLWH]HGLQWKPHQW,QDQHIRUWW JHQHUDWH WKLFNVXEOD\HUIRUSURELQJ 3HQQXYDQD 6WDW 8QLYHUVLW EXLOWD SLS ZLWKDIGRZ RI JOFHULQ $VXPHDVPRRWK LQGLPHWUSLSHZLWK9 IWV/DQGJOFHULQ DW *f& &RPSXWWKHVXEOD\HUWKLFNQHVVLQLQFKHVDQGWKHSXPSLQJKRUVHSRZHUHTXLUHGI SHUFHQWHILFLHQF\ LI / IW
- 3 7KHSLSHIORZ LQ) LJ 3 LVGULYHQE\ SUHVVXU]HGDLULQWKHWDQN: KDWJDI SUHVXUHS LVQHGHGWRSURYLGHD *f& ZDWUIORZ UDW4 P K"

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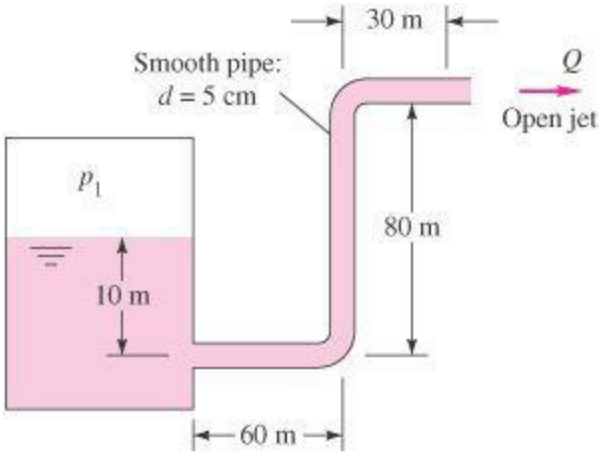
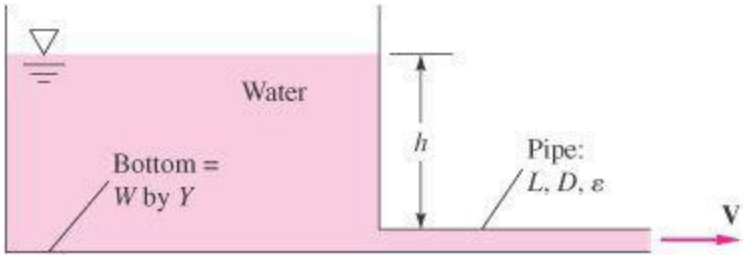


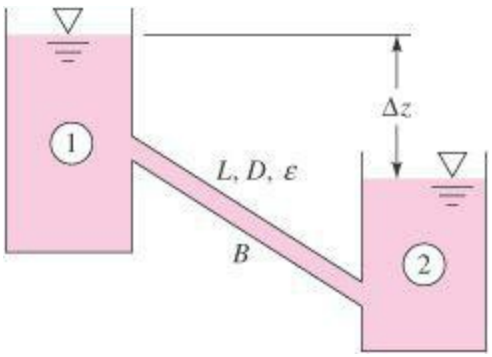
- 3 : DWUDW f& IORZ VE\ JUDYLW VKURXJK D VPRRWK SLSH IURP RQH UHVHUYRLU WR I
ORZ HURQH 7KH HOHYDWLRQ GLIIHUHQFH P 7KH SLSH LV P ORQJ ZLWK DGLDP HWUR
FP &DOFXODWH WKH H[SHFWHG IORZ UDWH LQP K 1HJOHFW PLQRU ORVVHV
- 3 $ VZLPPLQJ SRRO E\ <E\ KGHHSLV WR E HHPSWHG E\ JUDYLW WKURXJK WKH ORQ
SLSH VKRZQ LQ) LJ 3 $VVXPLQJ DQ DYHUDJH SLSH IULFWLRQ IDFWRU I DQG QHJOHFWLQJ PLQR
ORVVHV GHULYH DQ H[SUHVVLRQ IRU WKH WLPH WR HPSW\ WKH WDQN IURP DQ LQLWLDO YHORFLW

3



- 3 EES 7KH UHVHUYRLV LQ) LJ 3 FRQQHFWHG WR D WDQN RI VPRRWK ZDO
/ P DQG G FP ZKHUH WKH ORZ UDWH LQP K EHIRUH û] P"

3



- 3 7KH $OD 3LSH DQG 1HWZRUN SURSHUUSKRVRKDVDGHVLJQ ORZ UDWH R (
JDOORQV SHU GD\ RI FUXGH RLO DW f& VHH) LJ $ D $VXPLQJ DJDLQVW DQ HHWWLP DWWKH VRDOSUHVXUHGURS UHTXLUHG IRUWKH PLGHWS E ,I WKHUH DUH QLQHHTXDO
VSDFHG SXPSV HWWLP DWWKH KRUVHSRZHU HDFK SXPS PXVW GHOLYHU

- 3 $SSO\ VKHDQDO\VLVRI 3URE 3 \ VRVKHIRORZLQJGDWD / HW P < P K
P / P ' FP DQG D %\ GHWHUPLQK P DQG P DVUHSUHVHQWDWLY
GHSWKV HWWP DWWKHDYHUDJHIUFWRQIDFWRU 7KHQ E HWWP DWWKHWP HVRGULQWKHSRRO
- 3)RUWKHVXWWP LQ3URE DSXPSLVXVHGDWQJKVWRGULYHZDWUEDFNVRVK
XSSHUUHVHUYRLU,I WKHSXPSGHOYHUV : VRVKHZDWU HWWP DWWKHIORZ UDW
- 3 7KHIORORZLQJGDWZHUHEWLQHGIRUIORZ RI *& ZDWUDW P KVKURXJKI
EDGO FRURGHG FP GLDPHWUSLSHWKDWVORSHVGRZQZDUGDWDQJOHRI S N3D
] P S N3D] P (WWP DW D WKHURXJKQHVVUDWR RI WKHSLSHDQG E WK
SHUFHQWDFFHFKDQJHLQKHGGORVVLWKHSLSHZHUHVPRRWKDQGWKHIORZ UDWWKHVDPH
- 3 ,QWKHVSLUDWR +DDDQG¶V H[SORFLWSLSHIUFWRQIDFWRUDSSUR[LPDWRQ(T
-HSSVRQ> @SURSRVHGWKHIORORZLQJ H[SOFLWRUPXOD

$$\frac{1}{\sqrt{f}} \approx -2.0 \log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{5.74}{\mathrm{Re}_d^{0.9}}\right)$$

D ,VWKLVLGHQWFDOWR+DDDQG¶VIRUPXODZLWKMXVWDVLPSOHUHDUDQJHPHQW([SODLQ E
&RPSDUH-HSSVRQ¶VIRUPXODWR+DDDQG¶VIRUDIHZ UHSUHVHQWDWLY HYDOXHVRI WKUEXOHQW5H
DQG GDQGWKHUHUURUVFRPSDUHGWRWKH&RROEURRNIRUPXOD ' LVFXVEULHO
- 3 : KDWOHYHOK PXWWEH PDLQWDLQHGLQ) LJ 3 VR GHOYHUDIORZ UDWRI
IWVVWKURXJKWK LQFRPPHUFLDOWXEHSLSH'

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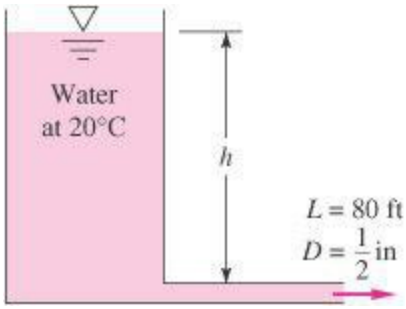

- 3 : DWUDW *& LVWREHSXPSHGWKURXJK IWRI SLSHIURP UHVHUYRLU WR DW
UDWRI IWV DVVKRZQLQ) LJ 3 ,I WKHSLSHLVFDWWLURQRI GLPHWU LQDQGWKHSXPS
LV SHUFHQWHIILFLHQWZKDWKRUVHSRZHUSXPSLVQHHGHG'

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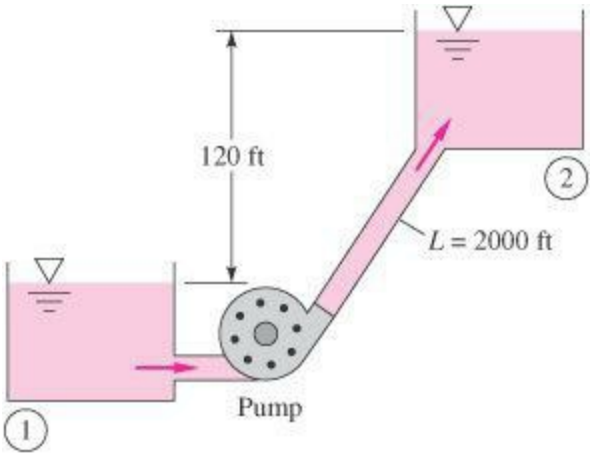

) OHRZ UDWHDQG VL]LQJ SUREOHPV

- 3 $ VDQNFRQGLQV P RI ZDWUDW *& DQG KDV DGUDZQFDSLODU RXOHWXEHD WKHERWWRP DVLQ)LJ 3) LQG WKHRXOHWYROXPHIOX[4 LQP KDWKLVLQXDQW

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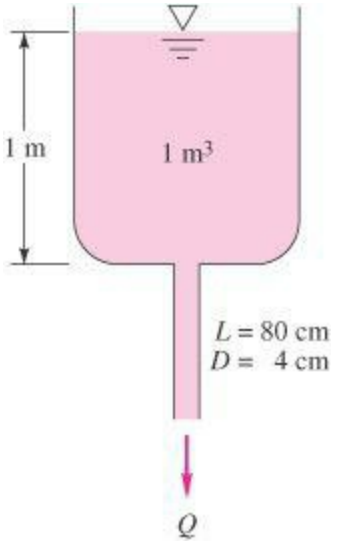

- 3) RUWKHVXVWHP LQ)LJ 3 VRQVHIRUWKHIOHRZ UDWHLQP KLWKHIOXLGLV6$(RLODW *& ,VWKHIOHRZ ODPLQDURUWXUEXOHQW
- 3 ,Q3UREOHP 3 WKHLQWDOIOHRZ LVWXUEXOHQW$VWKHZDWUGUDLQVRXWRI WKHVDQ ZLOOWKHIOHRZ UHYHUWWRODPLQDUPRQDVWKHVDQNEHFRPHVQHDUO HPSW' ,I VR DWZKDWDQ GHSWK' (VWLPDWWKHWLPH LQK VRGUDLQWKHVDQNFRPSOHWHO
- 3 (VWKOODORKRODW *& IORZ VWKURXJK D FP KRUL]RQWDGUDZQWEH F RQJ 7KHIXOO GHYHORSHGZDOOWKHUWXHVLV 3D (VWLPDWH D WKHSUHVXUHGURS E WK YROXPHIORZ UDWH DQG F WKHYHORFLWX DWU FP
- 3 $ VWUDLJKW FP FRPPHUFLDOWHOSLSHLV NP ORQJ DQG LV ODLG RQ D FRQWDQ VORSHRI *& : DWUDW *& IORZ VGRZQZDUG GXHWRJUDYLWRQ (VWLPDWWKHIORZ UDWH P K : KDWKDSSHQVLWKHSLSHOHQJWKLV NP"
- 3 7KH0 RRG\ FKDUW)LJ LVEHWWRUILQGLQJ KHDGORVV RUûS ZKHQ4 9 G DQG/ DUHNQRZQ ,WLVZ NZDUGIRUWKHVHFRQGWSHRI SUREOHP ILQGLQJ4 ZKHQK RUûS L

NQRZ Q VH([DPSOH 3UHSDUH D PRGLILHG 0 RRG\ FKDUW KRVH DEVFLVVDLVLQG HSHQGHQ
RI 4 DQG 9 XVLQJ GDVD SDUDPHWHUV IURP ZKLFK RQH FDQ LPPHGLDWHO UHDG WKH RUGLQDWH
ILQG GLPHQVLRQDWV 4 RU 9 8VH \RXU FKDUWV RVOYH([DPSOH

- 3) RU3URE 3 VXSSRVH WKHRQ SXPS DYDLODEOH FDQGLGDWH KS WR WKH IOXG
: KDWV WKH SURSHUSLSHVL]HLQLQFKHV WR PDLQWDLQ WKH IWVIGRZ UDWH

- 3 (WKGHQH JOFRODW *& IGRZ VWKURXJK P RI FDWDORQSLSHRI GLDPHWU FP
7KH PHDXUHG SUHVXUH GURS LV N3D 1HJOHFWPLQRU ORVVHV 8VLQJ D QRQWDUDWY
IRUPXODWRQ HVWLPDWWKH IGRZ UDWLQP K

- 3 ,WLV GHVLUHG WR VOYH3URE IRUWKH PRWWHRQRP LFDOSXPS DQG FDWDORU
SLSHV\VWHP ,I WKH SXPS FRVW SHUKRUVHSRZHUGHOYHUHG WR WKH IOXLG DQG WKH SLS
FRVW SHULQFKRI GLDPHWU ZKDWDUH WKH PLQLPXP FRVWDQG WKH SLSHDQG SXPS VL]HV
PDLQWDLQWKH IWVIGRZ UDWH" 0 DNHVRPHVLPSOLI\LQJ DVVXPSWLRQV

- 3 0RGLI\3URE 3 E\ GHWHUQJ WKH GLDPHWUEHXQNQRZQ) LQG WKH SURSHUSLS
GLDPHWUIRUZKLFK WKH SRROZLOOGUDLQLQDERXWZRKRXUVIODW

- 3 7KH0RRG\ FKDUW)LJ LVEHWWRUILQGLQJ KHDGORV RUûS ZKHQ4 9 G
DQG/ DUHNQRZQ,WLVDZNZDUGIRUWKHWKLG\WSHRI SUREOP ILQGLQJ GZKHQK RUûS DQ
4 DUH NQRZQ VH([DPSOH 3UHSDUH D PRGLILHG 0 RRG\ FKDUW ZKRVH DEVFLVVD L
LQGHSHQGHQWVR G XVLQJ DVDSDUDPHWU QRQGLPHQVLRQDO ZLWKRXW GIURP ZKLFKRQHFD
LPPHGLDWHO UHDG WKH GLPHQVLRQDWV RUGLQDWH WR ILQGG 8VH \RXUFKDUWWR VOYH([DPSO

- 3 7ZR UHVHUYRLUV ZKLFK GLIIHULQ VXUIDFH HOHYDWRQ E\ P DUH FRQQHFWG E\ I
QHZ FRPPHUFLDOWHSLSHRI GLDPHWU FP ,I WKH GHVLUHG IGRZ UDWLV 1 VRI ZDWUL
*& ZKDWLV WKH SURSHUOHQJWKRI WKH SLSH"

- 3 <RX ZLVK WR ZDWU \RXU JDUGHQ ZLWK IWRI LQGLDPHWU KRVH ZKRV
URXJKQHVVLV LQ: KDWZLOOEH WKH HGHOYHU LQIWV LI WKH JDJHSUHVXUHDWWKHIDXFHWV
GEI LQ" ,I WKHUHLV QR QR]]OH IXWWDQRSHQKRVHH[W ZKDWLV WKHPD[LPXP KRUL]RQW
GLVWDQFHWKH H[WWWZLOOFDUU"

- 3 7KHVPDOOWXEHQHLQ) LJ 3 H[WDFW : RI SRZHUIURP WKHZDWUIGRZ
%RWK SLSHV DUH ZURXJKWLURQ &RPSXWH WKH IGRZ UDW4 LQ P K : K\ DUH WKHUH VWI
VROXWRQV" : KLFK LVEHWWU"

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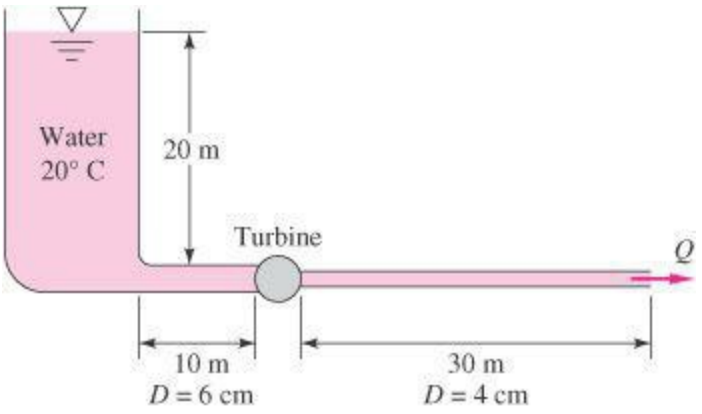

- 3 0RGLI\3URE 3 LQRX DQ HFRQRPLF DQDOVLV DV IROORZV / HWWKH P R

ZURXJKWURQSLSHKDYHDXQIRUP GLDPHWUG / HWWKHVWDG ZDWUIORZ DYDLODEOHEH4
P K 7KHFRVWRI WKHVXEELQHLV SHUZDWWGHYHORSHG DQGWKHFRVWRI WKHSLSLQJ LV
SHUFHQWPHWURI GLDPHWU 7KHSRZHUJHQHUDWGPD\ EHVROGIRU SHUNLORZDWWKRXU
) LQG WKH SURSHUSLSH GLDPHWUIRUPLQLPXP SD\EDFNWPH? WKDWLV WKHPLQLPXP WPHIR
ZKLFKWKHSRZHUVDOHVZLOOHTXDOWKHLQLWDOFRVWRI WKHVXVWP

- 3 ,Q) LJ 3 WKHFRQQHFWQJSLSHLVFRPPHUFLDOWHO FP LQGLDPHWU (VWP DW WKHIORZ UDWH LQP K LI WKHIOLGLVZDWUDW f& : KLFKZD\ LVWKHIORZ"

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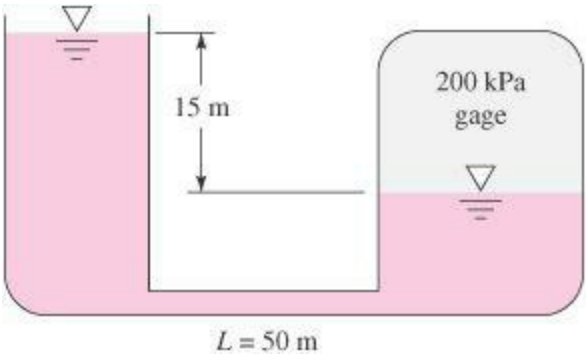

- 3 $ JDUGHQKRVHLVWRE HXVHGDVWKHUHVXUQOQHLQDZDWUDOOGLVSOD\ DWDPDOO,(
RUGHUWRVHOHFWWKHSURSHUSXPS \RXQHGWRNQRZ WKHURXJKQHVVKLGKWLQLGHWKHJDUGH
KRVH 8QRUWXQDWHO URXJKQHVWLQRUPDWRQLVQRWXSSOLHGE\ WKHKRVHPDQXIDFWXUHU6R\RX
GHYLVHDVLPSOHH[SHULPHQWWRPHDVXUHWKHURXJKQHVW 7KHKRVHLVDWWDFKHGWRWKHGULQR
DQDERYHJURXQGVZLPPLQJSRROWKHVXUIDFHRI ZKLFKLV P DERYHWKHKRVHRXWOHW<RX
HVWLPDWWKHPLQRUORVVFRHILFLHQWRI WKHHQWUDQFHUHJLRQDV DQGWKHGULQYDOYHKDVI
PLQRUORVV HTXLYDOHQWOHQJWK RI GLDPHWUV ZKHQ IXOO RSHQ 8VLQJ D EXFNHWDQ
VWRSZDWK \RXRSHQWKHYDOYHDQGPHDVXUHWKHIORZ UDWWRE î í P VIRUDKRU
WKDWLV P ORQJ DQGKDVDQLQVLGHGLDPHWURI FP (VWPDWWKHURXJKQHVWKHJKWV
PP LQVLGHWKHKRVH

- 3 7KHKDGYHUVXVIORZ UDWFKDUDFWHULVWLFVRI DFHQWULXJDOSXPS DUHVKRZQL
) LJ 3 ,I WKLVSXPS GULYHVZDWUDW f& WKURXJK P RI FP GLDPHWUFDVWLURQ
SLSH ZKDWZLOOEHWKHUHVXOWQJIORZ UDWLQP V"

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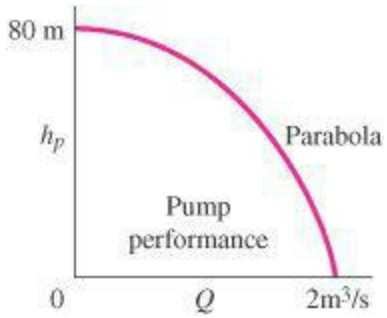

- 3 7KHSXPSLQ) LJ 3 LVXVHGWRGHOLYHUJDVROLQHDW f& WKURXJK P RI
FP GLDPHWUJDOYDQL]HGLURQSLSH (VWP DWWKHUHVXOWQJIORZ UDWLQP V 1RWWKDWWKH
SXPSKHDGLVQRZ LQPHWHUV RI JDVROLQH

- 3) OXLGDW *f*& IORZ VWKURXJK DKRU]RQDOJDOYDQ]HGLURQSLSH P ORQJ DQG FP LQGLDPHWHU 7KHZDOOVKHDUVWUHVV 3D &DOFXODWWKHIORZ UDWHLQP KLWKHIXLGL D JO\FHULQDQG E ZDWHU
- 3)RUWKHVXVWHP RI)LJ 3 OHW] P DQG/ P RI FDWLRQSLSH : KD LVWKHSLSHGLDPHWHUIRUZKLFKWKHIORZ UDWHZLOOEH P K"
- 3 ,WLVGHVLUHGWRGHOLYHU P KRI ZDWHUDW *f*& WKURXJKDKRU]RQWDOSKDOWH FDWLRQSLSH (VWXPDWWKHSLSHGLDPHWUHTXLUHGURSVREHH[DFWO NBDSHU P RI SLSHOHQJWK
- 3)RUWKHVXVWHP LQ3URE 3 DSXPS ZKLFKGHOLYHUV : RWKHZDWHU LV XVHGDWQJKWRUHLOOWKHXSSHUUHVHUYRLU 7KHSLSHGLDPHWULVLQFUHDVHGIURP FP WR SURYLGHPRUHIORZ ,IWKHUHXQGQWORZ UDWLV P K HVWLPDWWKHQHZ SLSHVL]H

1 RQFLUFXODUGXFWV

- 3 6$(RLODW *f*& IORZ VDWDQDYHUDJHYHORFLWR P VEHWZHHQWZR VPRRV SDUDOOHOKRU]RQWDOSODWHV FP DSDUW(VWXPDWD WKHFHQWUDOQHYHORFLW E WKHKHDGORV SHUPHWHU DQG F WKHSUHVVXUHGURSSHUPHWHU
- 3 $ FRPPHUFLDOVWHODQQXOV IWORQJ ZLWK LQ FRQQHFWVZR UHVHUYRLUV WKD GLIHULQVXUFH HKHLJKWE\ IW&RPSXWWKHIORZ UDWLQIWV WKURXJK WKHDQQXOLI WK IXLGLVZDWUDW *f*&
- 3 $QRLOFRROHUFRQVLVWV R PXOWSOH SDUDOOHOSODW SDVVDJHV DV VKRZQ LQ)LJ 3 7KHDYLODEOHSUHVVXUHGURSLV N3D DQG WKHIXLGLV6$(: RLODW *f*& ,I WK GHVLUHGWRWDOIORZ UDWLV P K HVWLPDWWKHDSSURSULDWQXPEHURI SDVVDJHV 7KHSODW ZDOOVDUHK\GUDXOLFDOO VPRRWK

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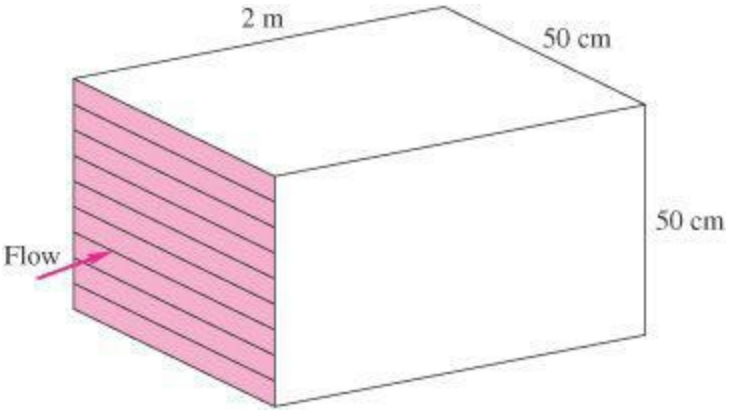

- 3 $QDQQXOVRI QDUURZ FOHDUDQFH FDXVHVDYHU ODJH SUHVVXUHGURSDQGLVXVHX DVDQDFFXUDWPHDVXUHPHQWRI YLVFRVLW ,I DVPRRWKDQQXOV P ORQJ ZLWKD PP DQG E PP FDUULHVDQRLOIORZ DW P V ZKDWLVWKHRLOYLVFRVLWVLI WKHSUHVVXUHGURSL N3D'
- 3 $ UHFDQJXODUWHHWLDOGXFWLV IWORQJ DQGKDVDIL[HGKHLJKW LQ 7K ZLGWK% KRZHYHU PD\ YDU IURP WR LQ $ EORZHUSURYLGHVDSUHVVXUHGURSRI 3D RI DLUDW *f*& DQG DW : KDWLVWKHRSWPXP ZLGWK%WKDWZLOOSURYLGHWKHPRVWDLUORZ L

IWV"

- 3 +HDWH[FKDQJHUVRIWQFRQVLWRI PDQ WLDQJXODUSDWDJHV 7\SLFDOLV) LJ 3 ZLWK/ FP DQGDQLVRVFHOHVWLDQJHFURVVVHFWLRQRI VLGHOHQJWKD FP DQGLQFOXGH DQJOH *f ,I WKH DYHUDJH YHORFLWV 9 P VDQG WKH IOXLG 6$(RLODW *f& HWUP DWWKH SUHVXUHGURS

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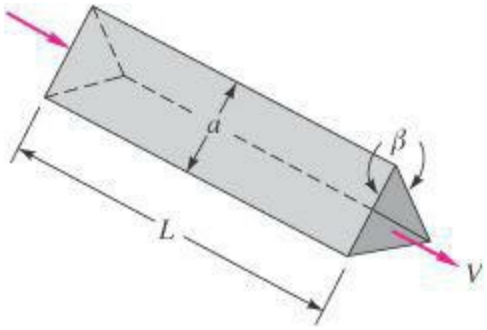

- 3 $ CDUJHURRP XVHVDIDQWRGUDZ LQDWRSSHUF DLUDW *f& WKURXJKD FP E FP FRPPHUFLDOVWHDGXFW P ORQJ DVLQ) LJ 3 (VWLPDWH D WKHDLUIORZ UDWHLQ P K LI WKHURRP SUHVXUHV 3DYDFXXP DQG E WKHURRP SUHVXUHLI WKHIORZ UDWHL P K 1HJOHFWPLQRUORVVHV

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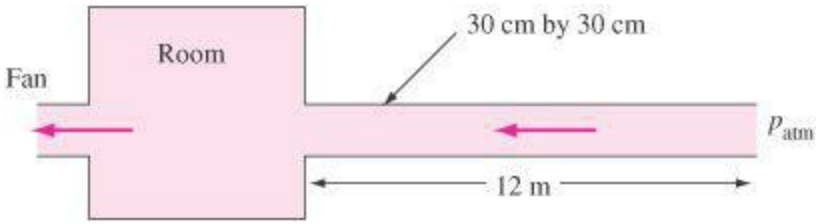

- 3 ,Q 0 RRG\¶V([DPSOH WKH LQFK GLDPHWU IWORQJ DSSHDOWG FDWWLURS SLSH KDV D SUHVXUH GURS RI DERXW CEI IW ZKHQ WKH DYHUDJH ZDWUYHORFLWV IW &RPSDUH WKLV WR DQDQQXODU FDWWLURQ SLSH ZLWK DQ LQQHU GLDPHWUR LQ DQG WKH VDPH DQQXODUDYHUDJHYHORFLWV IW D : KDWRXWUGLDPHWUZRXOG FDXVH WKHIORZ WRKDYH WKH VDPH SUHVXUHGURS RI CEI IW' E +RZ GR WKHFURVVVHFWLRQDUHDVFRPSDUH DQGZK\' 8VHWKHK\GUDXOF GLDPHWUDSSURPLPDWRQ

- 3 $LUDW *f& IORZV WKURXJK D VPRRWK GXFWRI GLDPHWU FP DWDQ DYHUDJH YHORFLWV P V ,WWKHQIORZV LQWR D VPRRWKVTXDUH GXFWRI VLGHOHQJWKD) LQG WKH VTXDUH GXFWV]HD IRUZKLFK WKH SUHVXUHGURS SHUP HWUZLOOEH H[DFWO WKH VDPH DV WKH FLUFXODU GXFW

- 3 $OWRXJK DQDOWFDOVROXWRQV DUH DYLODEOHIRUODP LQDUIORZ LQP DQ GXFWWKDSH> @ZKDWGR ZH GR DERXWGXFWV RI DUELWDU VKDSH' %DKUDPL HWDO> @SURSRVHWKDW EHWWUDSSURDFK WR WKHSLSHUHVXOM5H LVDFKLHYHGE\ UHSODFLQJ WKHK\GUDXOF GLDPHWU ' K E\ $ ZKHUH$ LV WKH DUHD RI WKH FURVVVHFWLRQ 7HVWWKLV LGHD IRU WKH LVRVFHOHV WULDQJOHV RI 7DEOH ,I WP HLV VKRUWDWDWDVWV f f DQG f : KDWGR\RX FRQFOXGH DERXWKLVLGHD'

- 3 $ IXHOFHOO> @FRQVLVWVRI DLU RUR[\JHQ DQGK\GURJHQPLFURGXFWVHSDUDWHG E\ DPHPEUDQHWKDWSURPRWHVSURWRQH[FKDQJHIRUDQHOHFWULF FXUUHQWDV LQ) LJ 3 6XSSRVHWKDWWKHDLUVLGH DW *& DQGDSSUR[LPDWH DW KDVILYH PP E\ PP GXFW HDFK P ORQJ 7KHWRWDOIORZ UDWHLV (NJ V D ' HWHUPLQHLWKHIORZ LVODPLQDUR WXUEXOHQW E (VWLPDWHWKHSUHVVXUHGURS 3UREOPP FRXUWHV\ RI ' U 3H]KPDQ6KLUYDQLDQ

- 3 $ KHDWH[FKDQJHUFRQVLVWVRI PXOWLSOHSDUDOOHOSODWHSDVVDJHV DVVKRZQLQ) LJ 3 7KHDYDLODEOHSUHVVXUHGURS LV NPD DQGWKHIOXLGLVZDWHUDW *& ,I WKHGHVLUH VWRODPLQDUR UDWHLV P K HWLPDWHWKHSSUHVSUHVVXUHQPEHURI SDVVDJHV 7KHSODWHZDOO DUHK\GUDXOLFDOO VPRRWK

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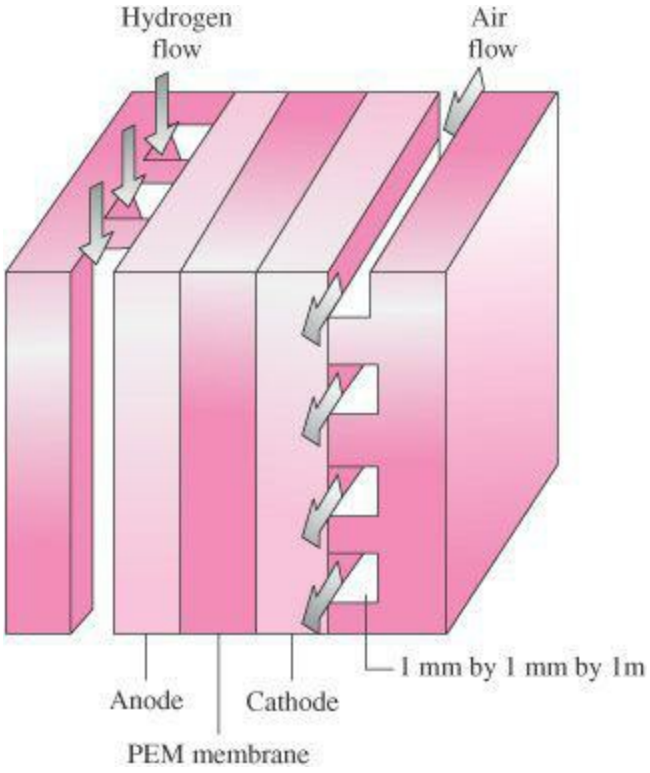

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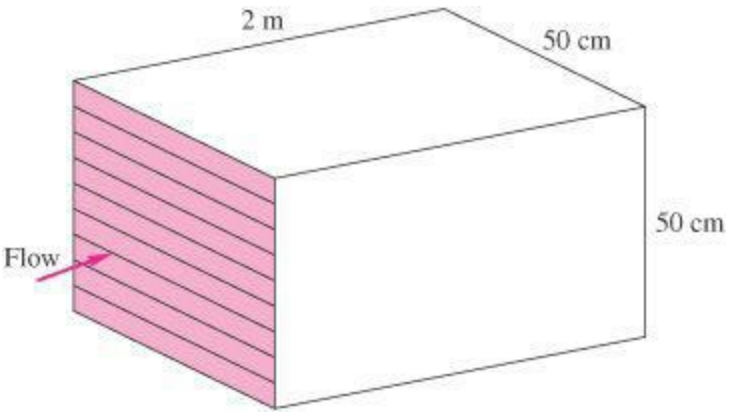

- 3 $ UHFWDQJXODUKHDWH[FKDQJHULV WR EH GLYLGHG LQWR VPDOOHU VHFWLRQV XVLQ VKHHWVRI FRPPHUFLDOO\ DYDLODEOH PP WKLFN DVVXPHWKHGLQ) LJ 3 7KHIORZ UDWHLV NJ V RI ZDWHUDW *& %DVLF GLPHQVLRQV DUH/ P : FP DQG+ FP : KDWLV WK

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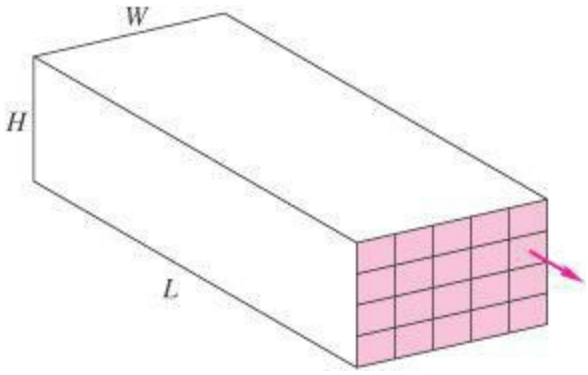

0 LQRUURUGRFDOOBXVHV

- 3 ,Q6HF LWZDV PHQWLRQHG WKDW5RPDQ DTXHGXFWFXWRPHUV REWDLQHG H[W ZDWUE\ DWWDFKLQJ DGLIIXVHUWR WKHLUSLSHH[LW)LJ 3 VKRZV DVLPXODWRQ DVPRRW LQDHWSLSH ZLWKRUZLWKRXW D f FRQFDOGLIXVHUH[SDQGLQJ WR D FP GLDPHWUH[LW7K SLSHHQWDQFHLVVKDUSHGJHG &DOFXODWWKHIORZ UDWH D ZLWKRXWDQG E ZLWKWKHGLIXVHU

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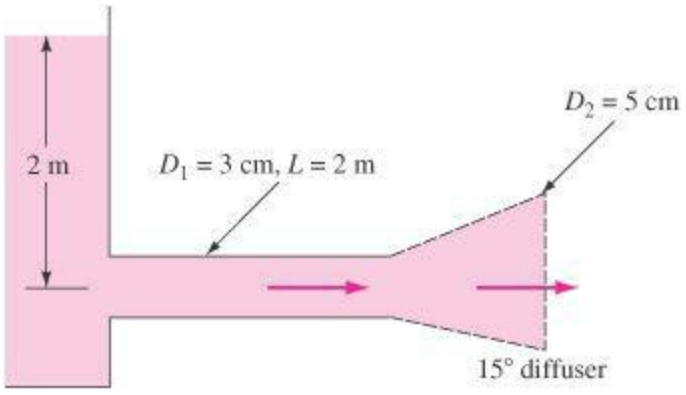

- 3 : DWUDW f) IORZ VDW JDOPLQWKURXJK D LQGLDPHWUFRPPHUFLDOWH SLSHWKDWLV IWORQJ $FFRUGLQJWR7DEOH KRZ PDQ f UHJXODUFUHZ HGHORZVFL EHDGGHGWRWKHV\VWHP EHRUHWKHYHUDOOSUHVXUHGURSH[FHHGV OEI LQ"
- 3 ,Q)LJ 3 DWKLFNILOWULVEHLQJ WHVWHG IRUORVV 7KHIORZ UDWHLQWKHSLSH LV P PLQ DQGWKHXSWUHDP SUHVVXUHLV N3D 7KHIOXLGLVDLUDW f& 8VLQJ WKHZDWHU PDQRPHWUHGLQJ HVWLPDWWKHORVVFRHILFLHQW RI WKHILOWU

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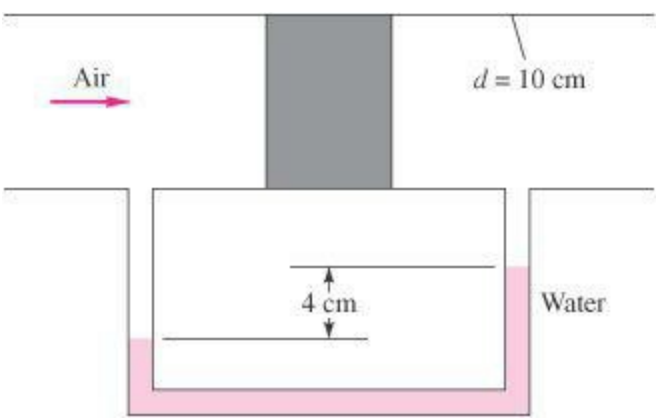

- 3 $ SHUFHQWHILFLHQWSXPS GHOLYHUV ZDWHU DW f& IURP RQH UHVHUYRLU DQRWKHU IWKLJKHU DV LQ) LJ 3 7KH SLSLQJ V\VWHP FRQWDLQV IWR JDOYDQL]H LURQ LQSLSH DUHQDQWHQDQFH VZRVFUHZHG f ORQJ UDGLXVHERZV DVFUHZHG RSH JDWH YDOYH DQG D VKDUS H[LW: KDWLV WKH LQSXWSRZHU UHTXLUHG LQ KRUVHSRZHU ZLWK DQ ZLWKRXW D f ZHOGHGVJQHGFRQFDOH[SDQVLRQGGHGWRWKH H[LW7KHIORZ UDWH LV IWV

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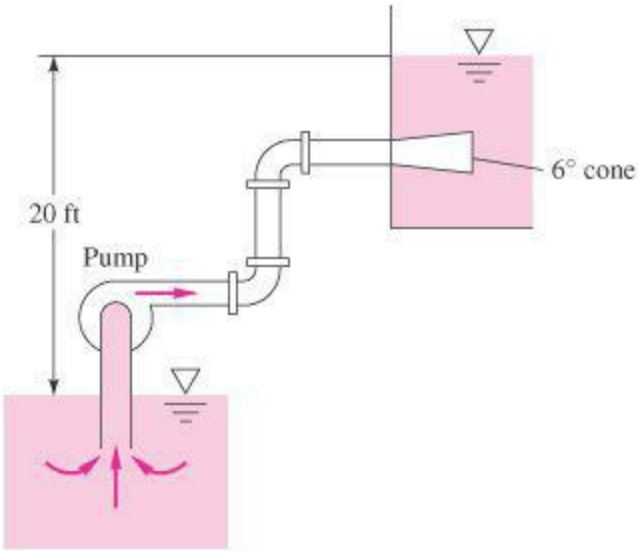

- 3 7KHUHVHUYRLVLQ) LJ 3 DUHFRQQHFWHG E\ FDVWLURQSLSHVNRLQGDEUXSWO ZLWKVKDUS HGJHGHQDQFH DQG H[LW,QFOXGLQJ PLQRUORVVHV HVWLPDWWKH IORZ RI WDWUD f& LI WKHVXUIDFHRI UHVHUYRLU LV IWKLJKHUWKDQWKDWRI UHVHUYRLU

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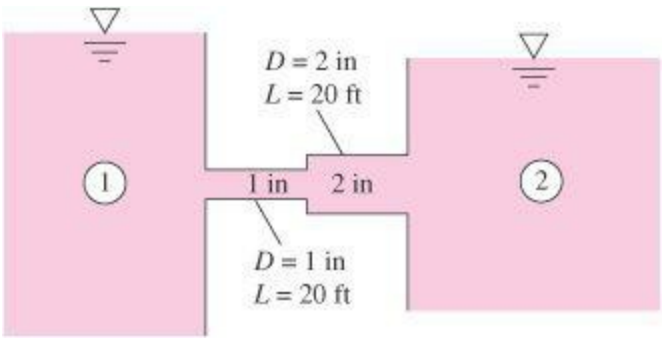

- 3 : DWUDW f& IORZV WKURXJK D VPRRWK KRUL]RQWDOSLSH DW P K 7KH SLS

GLDPHWHUV FP DQG LWV OHQJWK LV P D &RPSXWH WKH SUHVVXUH GURS E ,I DJDLQ YDOY
LV DGGHG ZKDW DUH RSHQLQJ K' ZLOO UHGXFH WKH IORZ UDWH E\ SHUFHQW IRU WKH VDPH
SUHVVXUH GURS'

- 3 7KH VWHP LQ)LJ 3 FRQVLVWV RI P RI FP FDVW LURQ SLSH WR
DQG IRXU ƒ IODQJHG HOERZV DIXOO RSHQ JORE HYDOYH DQG DVKOLS[L
LQDU DUHVHUYRLU ,I WKH HOHYDWLRQ DW SRLQW LV P ZKDW DJH SUHVXUH LV UHTXLUHG DW SRLQ
WR GHOLYHU P VRI ZDWHU DW ƒ& LQ WKH UHVHUYRLU'
- 3 7KH ZDWHU SLSH LQ)LJ 3 VORSHV XSZDUG DW ƒ 7KH SLSH KDV D L
GLDPHWHU DQG LV VPRRWK 7KH IODQJHG JORE HYDOYH LV IXOO RSHQ ,I WKH PHUFXU PDQRPHWH
VKRZV D LQGHIOHFWLRQ ZKDW LV WKH IORZ UDWH LQ IWV"
- 3 ,Q)LJ 3 WKH SLSH LV JDOYDQL]HG LURQ (VWDUW DW WKH SHUFHQWDJH LQFUHDVH L
WKH IORZ UDWH D LI WKH SLSH HQGDQFH LV FXW RII IODVK ZLWK WKH ZDOO DQG E LI WKH EXWWHUIO
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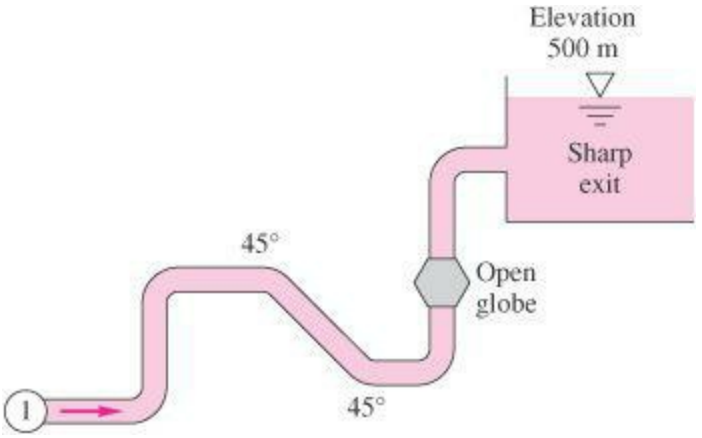

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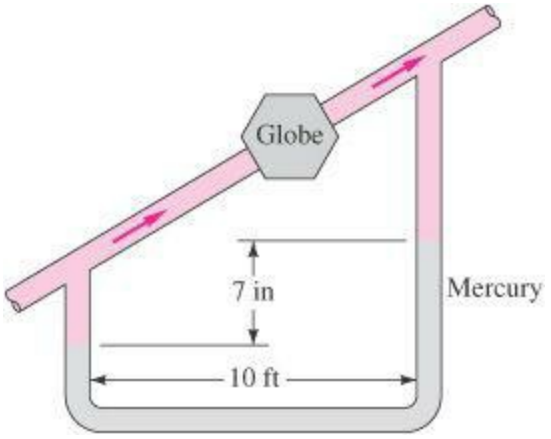

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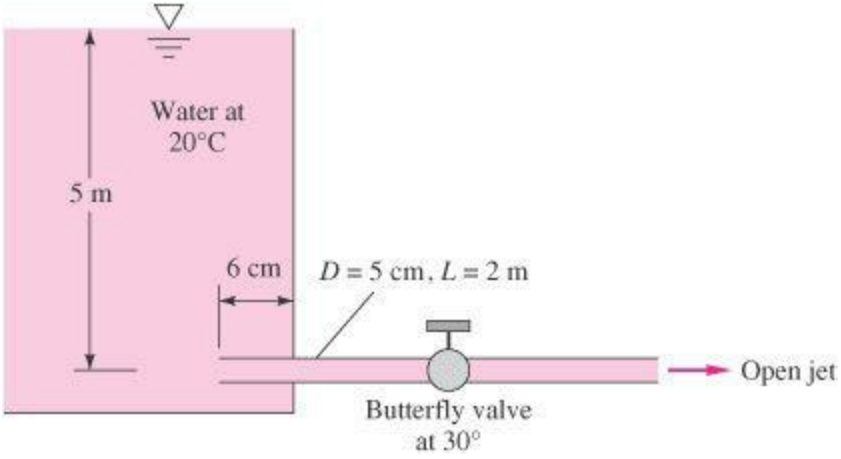

- 3 7KH ZDWHU SXPS LQ) LJ 3 PDLQWDLQV D SUHVVXUH RI SVLJ DWSRLQW 7KHUH LV DILOWHU DKDO RSHQGLVNYDOYH DQG VWR UHJXODUVFUHZHGHERZV 7KHUH DUH I RI LQGLPHWUFRPPHUFLDOWHDSLSH D ,I WKH IORZ UDWH LV IWV ZKDWLV WKH ORV FRHILFLHQWRI WKH ILOWHU E ,I WKH GLVNYDOYH LV ZLGH RSHQ DQG. ILOWU ZKDWLV WK UHVXOWLQJ IORZ UDWH"

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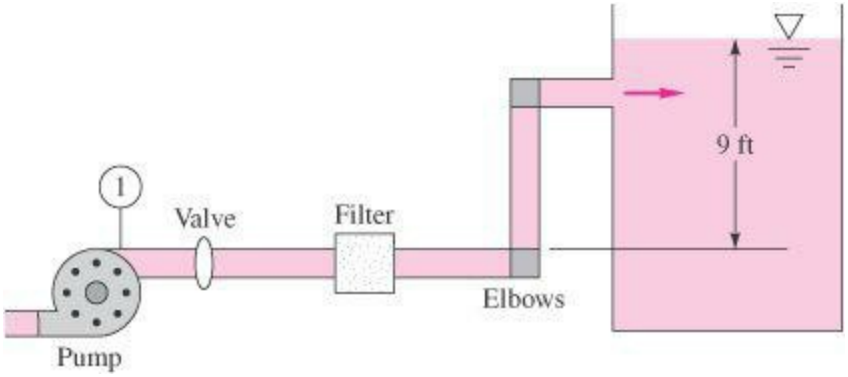

- 3 ,Q) LJ 3 WKHUH DUH IWRI LQSLSH IWRI LQSLSH DQG IWRI L SLSH DOOFDWWLURQ 7KHUH DUH WKUHH *f* HERZV DQG DQRSHQJORERHYDOYH DOOIODQJHG ,I WK H[LWKDYDLQRQLV]HUR ZKDWKRUVHSRZHULVH[WUDFWHG E\ WKH WXUELQH ZKHQWKH IORZ UDWH LV IWVRI ZDWUDW *f*&"

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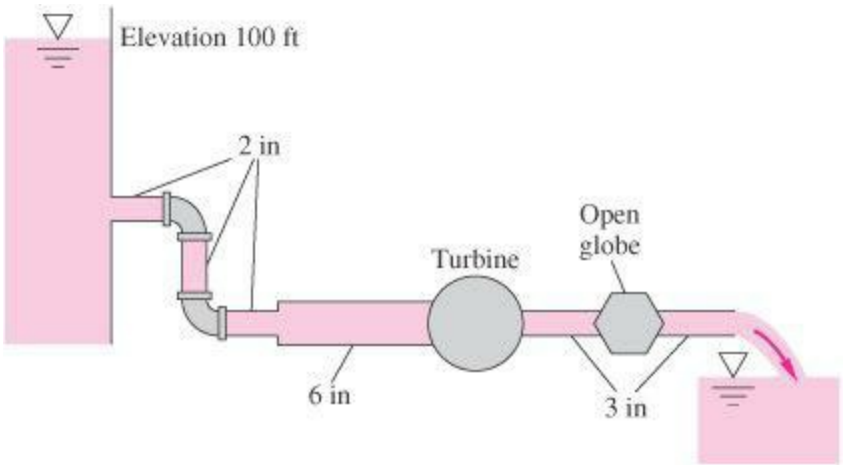

- 3 ,Q) LJ 3 WKH SLSH HQWUDQFH LV VKDUS HGJHG ,I WKH IORZ UDWH LV P V ZKDW SRZHU LQ: LVH[WUDFWHG E\ WKH WXUELQH"

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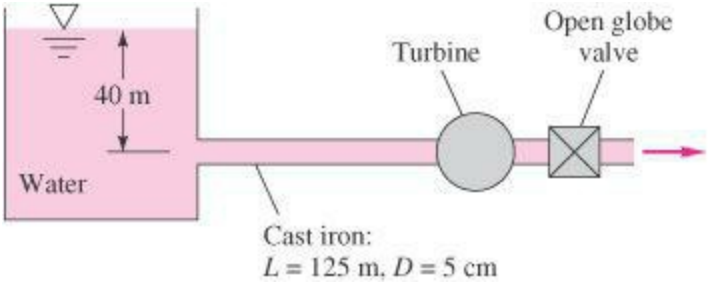

6HULHV DQG SDUDOOHO SLSH V\VWHPV

- 3) RUWKH SDUDOOHO SLSH V\VWHP RI) LJ 3 HDFK SLSH LV FDWLRQ DQG WK SUHVVXUHGURSS í S GE LQ &RPSXWH WKH WRWDO IORZ UDWH EHWZHHQ DQG LI WKH IULGL 6$(RLODW f&

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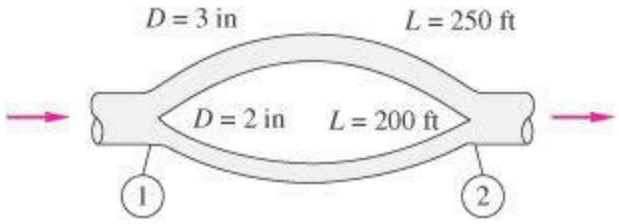

- 3 ,I WKH WZR SLSHV LQ) LJ 3 DUH LQWDG OLG LQ VHULHV ZLWK WKH VDPH WRWD SUHVVXUHGURSS RI GE LQ ZKDW ZLOO WKH IORZ UDWH EH' 7KH IULGL6$(RLODW f&
- 3 7KH SDUDOOHO JDOYDQL]HG LURQ SLSH V\VWHP RI) LJ 3 GHOLYHUV ZDWHU DW f& ZLWK D WRWDO IORZ UDWH RI P V ,I WKH SXPS SLSH ZLGHQHG QDQG QRWXQQLQJ ZLWK DOO FRQGLWLRQV GHWHUPLQH D WKH IORZ UDWH LQ HDFK SLSH DQG E WKH RYHUDOO SUHVVXUH GURS

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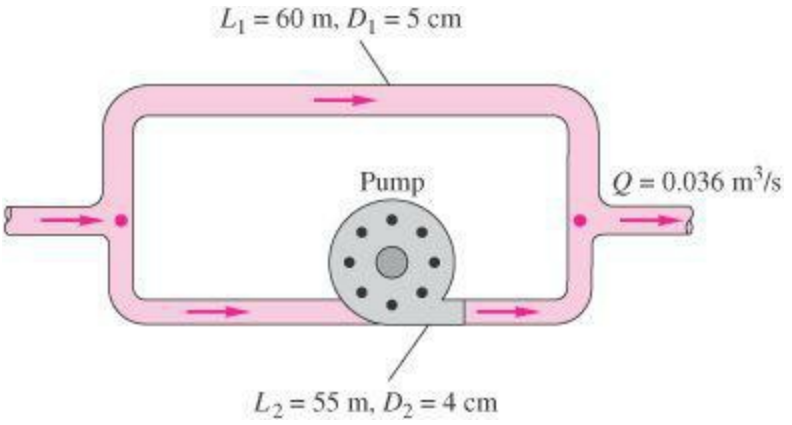
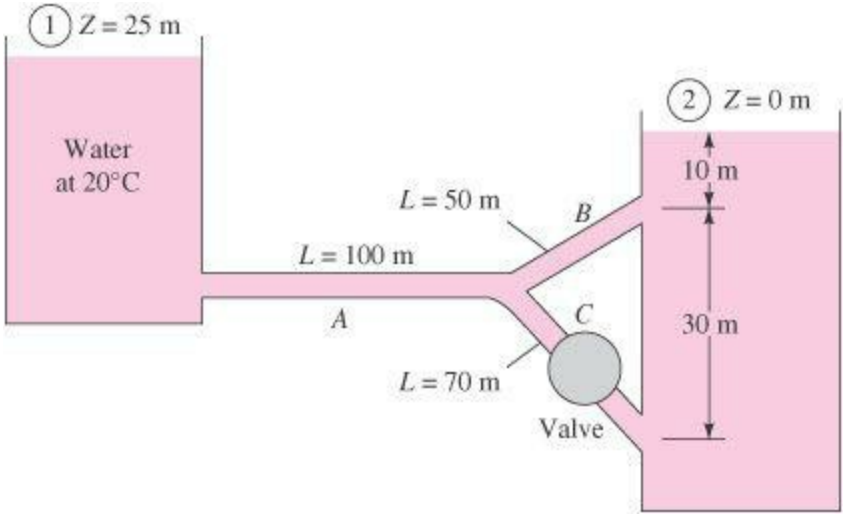
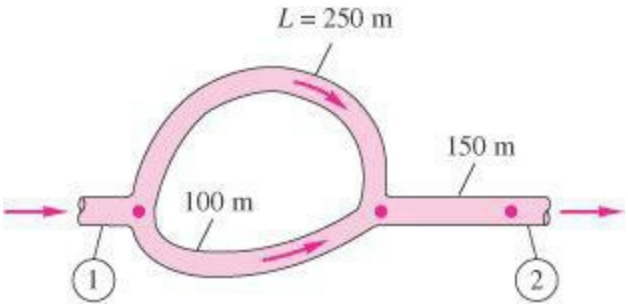


- 3 $ EOWZ HUVSSOLHVVWDQGDUG DLUWR DSHQP WKDWHHGVWR KROG[QDOVTXDU VKHHWP HDOGXFWZLWKVKDSHGHGHQGQHV 2QHGXFWLV IWORQJ ZLWKDFURVVHFWLR LQE\ LQ 7KHVHFRQGGXFWLV IWORQJ (DFKGXFWW[KDXWWWRWKHDWRSSHUH : KHQWK SHQP SUHVXUHLV GEI IWJDJH WKHYROXPHIORZ LQWKHORQJHUGXFWLVWKUHHWLP HVWK IORZ LQWKHVKRUWGXFW(VWPDWERWKYROXPHIORZVDQGWKHFURVVVHFWLRQ VL]HRI WK ORQJHUGXFW
- 3 ,Q)LJ 3 DOOSLSHVDUH FP GLDPHWUFDWWLRQ ' HWHUPLQHWKHIORZ UDWHIURP UHVHUYRLU LI YDOYH& LV D FORVHGDQG E RSHQ.
- 3)RUWKH VHULHVSDUDOOHOSLSHV DOOSLSHVDUH FP GLDPHWU DSSKDOWHGFDWLRQ ,IWKHWRWDOSUHVXUHGURSS í S N3D ILQGWKHUHXQLQJIORZ UDW 4 P KIRUZDWUDW *& 1HJOHFWPLQRUORVVHV
- 3 $ EOWZ HUGHOYHUVDLUDW P KWKHGXFWFLUFXLW)LJ 3 (DFKGXF LVFRPPHUFLDOWHHOEHQGRI VTXDUHFURVVVHFWLRQ ZLWKVLGHOHQJWKV D FP DQGD D FP $VVXPLQJ VHDOHYHODLUFRQGLWLRQV HVWPDWWKHSRZ HUUHTXLUHGLI WKHEOWZ H KDVDQHILFLHQF\ RI SHUFHQW1HJOHFWPLQRUORVVHV





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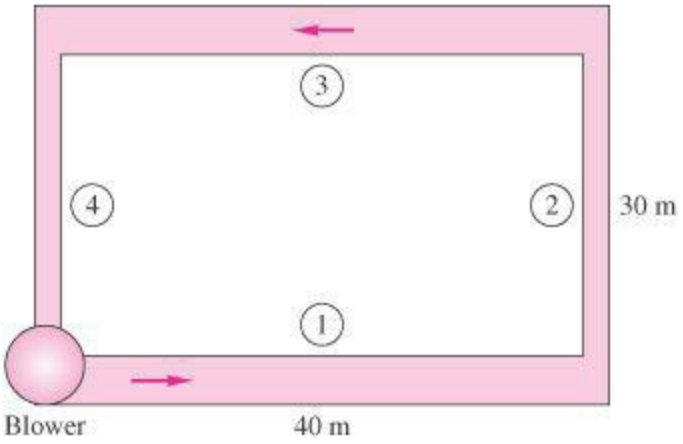

- 3) RU WKH SLSLQJ V\VWHP RI) LJ 3 DOO SLSHV DUH FRQFUHWH ZLWK I URXJKQHVV RI LQ 1HJOHFWLQJ PLQRU ORVVHV FRPSXWH WKH RYHUDOO SUHVVXUH GURS S í S L QE LQ LI 4 IWV 7KH IOXLG LV ZDWHU DW *f*&
- 3) RU WKH V\VWHP RI 3UREOHP 3 DJDLQ GHWHUPLQH WKH IOXLG EH6$(: RLO DW *f*& ZLWK ERWK SLSHV FDVWLURQ ,I WKH IORZ UDWH LQ WKH LQSLSH E LV IWPLQ HVWLPDWH WKH IORZ UDWH LQ WKH LQSLSH D LQIWPLQ

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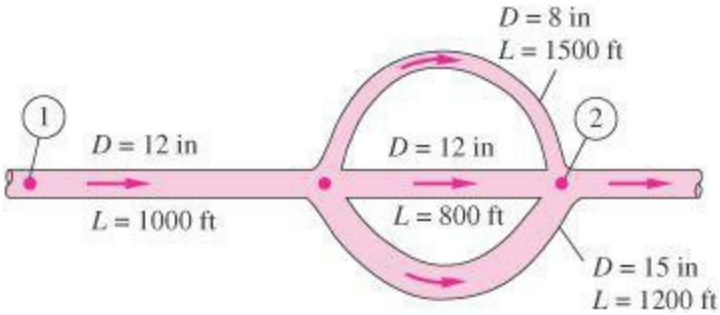


- 3 7KUHH FDVWLURQ SLSHV DUH DLG LQ SDUDOOHO ZLWK WKH VHGLPHQWV

Pipe	Length, m	Diameter, cm
1	800	12
2	600	8
3	900	10

7KH WRWDO IORZ UDWH LV P KR I ZDWUDW *f*& ' HWHUPLQH D WKH IORZ UDWH LQ HDFK SLSH DQG E WKH SUHVVXUH GURS DFURVV WKH V\VWHP

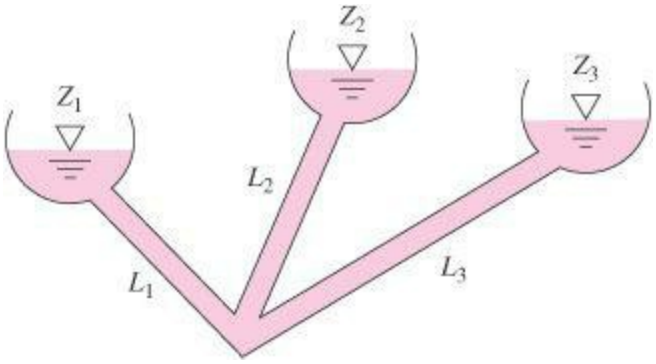
7KUHH UHVHUYRLUDQG SLSHQHWZRUN V\VWHPV

- 3 &RQVLGHU WKH WKUHH UHVHUYRLU V\VWHP RI) LJ 3 ZLWK WKH IROORZLQJ GDWD

$$L_1 = 95\ \text{m} \quad L_2 = 125\ \text{m} \quad L_3 = 160\ \text{m}$$
$$z_1 = 25\ \text{m} \quad z_2 = 115\ \text{m} \quad z_3 = 85\ \text{m}$$

$ OO SLSHV DUH FP GLDP HWHU XQLQVXODWHG FRQFUHWH PP &RPSXWH WKH VWDQG IORZ UDWH LQ DOO SLSHV IRU ZDWHU DW *f&*

3

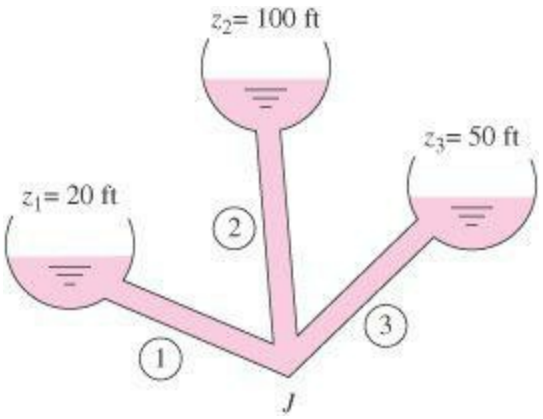


- 3 ((0 RGLI\ 3URE 3 DV IROORZV 5HGXFH WKH GLDP HWHU WR FP ZLWH PP DQG FRPSXWH WKH IORZ UDWH IRU ZDWHU DW *f&* 7KHQ GHWHUPLQH WKH V\VWHP PDQRPHWHU UHDGLQJ LQ3URE 3 EXW DVVXPH WKDW WKH IORZ UDWH &DQ \RX H[SODLQ WKLV GLIIHUHQFH'
- 3 0RGLI\ 3URE 3 DV IROORZV $OORZ WKH V\VWHP H[FHSW WKDW LV XQQRZ Q)LQG WKH YDOXH RI] IRU ZKLFK WKH IORZ UDWH LQ SLSH LV P V WR WZR DQG WKH MXQFWLRQ 7KLV SUREOHP UHTXLUHV LWHUDWLRQ DQG LVE\WXLWG WR DFRPSXWHU
- 3 7KH WKUHH UHVHUYRLU V\VWHP LQ) LJ 3 GHOLYHUV ZDWHU DW *f&* 7KH V\VWHP GDWD DUH DV IROORZV

$$D_1 = 8\ \text{in} \quad D_2 = 6\ \text{in} \quad D_3 = 9\ \text{in}$$
$$L_1 = 1800\ \text{ft} \quad L_2 = 1200\ \text{ft} \quad L_3 = 1600\ \text{ft}$$

$ OO SLSHV DUH JDOYDQL]HG LURQ &RPSXWH WKH IORZ UDWH LQ DOO SLSHV

3

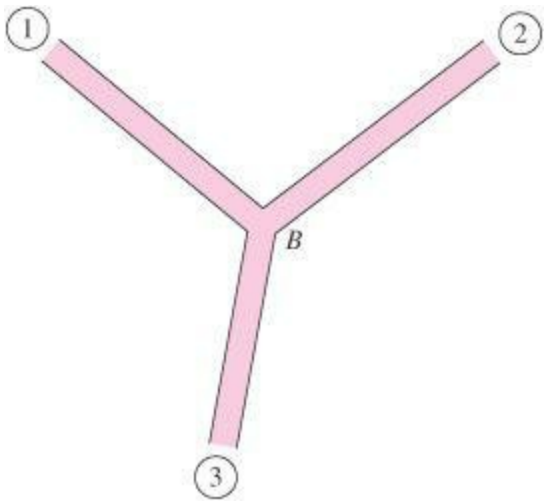


- 3 6XSSRVH WKDW WKH WKUHH FDVWLURQ SLSHV LQ 3UREOHP 3 DUH LQWHUFRQQHFWHG WR PHHW VPRRWKO DW SRLQW % DV VKRZ QLQ) LJ 3 7KH LQGLYLGXDO SUHVVXUHV LQ HDFK SLSH DUH

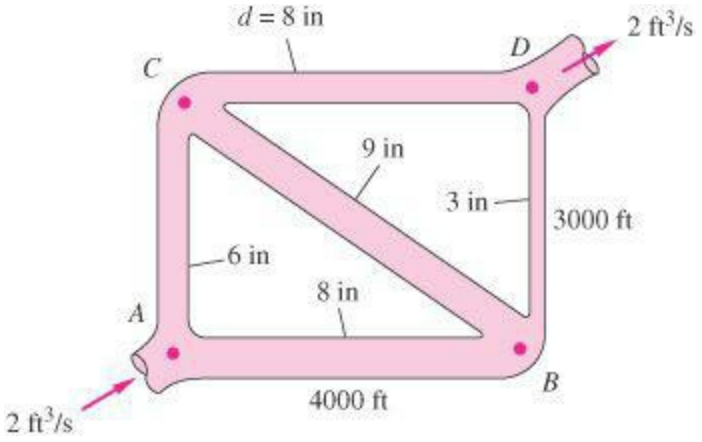
S N3DS N3DS N3D

7KH IOXLG LV ZDWHU DW *f*& 1HJOHFW PLQRU ORVVHV (VWLP DWH WKH IORZ UDWH LQ HDFK SLSH DQ ZKHWKHU LW LV ZRUWKZKLOH WR GRUDZ D IURP SRLQW %

3



- 3 0RGLI\ 3UREOHP 3 DV IROORZV / HW DOO GDWD EH WKH VDPH H[FHSW WKDW SLSH LV ILWWHG ZLWK D EXWWHUIO YDOYH) LJ E (VWLP DWH WKH SURSHUYDOYH RSHQLQJ DQJOH L GHJUHHV IRU WKH IORZ UDWH WKURXJK SLSH VR EH UHGXFHG WR IWV ZDG UHVHUYRLU 7KLV SUREOHP UHTXLUHV LWHUDWLQDQDOLVEHWWXLWHGWRDFRPSXWHU
- 3 ,Q WKH ILYH SLSH KRUL]RQWDO QHWZRUN RI) LJ 3 DVVXPH WKDW DOO SLSH KDYH DIULFWLRQ IDFWRU I) RU WKH JLYHQ LQIDWDQG H[LWGRZ UDWH IWV RI ZDWHU *f*& GHWHUPLQH WKH IORZ UDWH DQG GLUHFWLRQ LQ DOO SLSHV ,I $ GEI LQ JDJH GHWHUPLQH WKH SUHVVXUHV DW SRLQW% & DQG'



- 3 0 RGLI\ 3URE 3 DV IRGORZV / HWWKH LQDWWORZ UDWH DV$ DQG WKH H[L IORZ DW EHXQNQRZQ / HWS$ í S% GHI LQ &RPSXWWKH IORZ UDWH LQ DOOLYHSLSHV
- 3 ,Q) LJ 3 DOOIRXUKRUL]RQWDO FDWWLRQ SLSHV DUH P GHQJ DQG FP LG GLDPHWUDQGPHWDWWKQFWRQD GHOLYHULQJ ZDWUDW f& 7KHSUHVXUHVDUHNQRZ DWRXUSRLQWVDVVKRZQ

$$p_1 = 950\ \text{kPa} \quad p_2 = 350\ \text{kPa}$$
$$p_3 = 675\ \text{kPa} \quad p_4 = 100\ \text{kPa}$$

1HJOHFWLQJ PLQRUORVVHV GHWHUPLQHWKH IORZ UDWHLQHDFKSLSH

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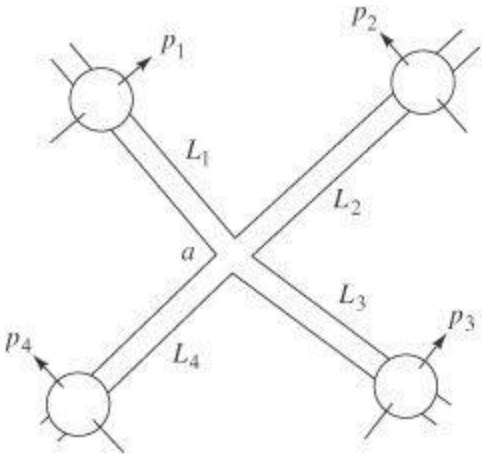
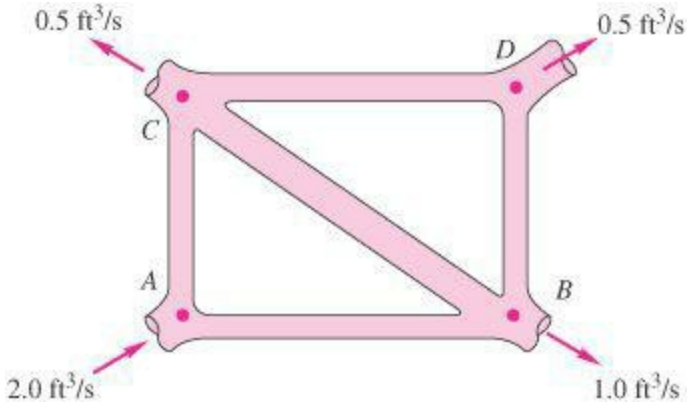


- 3 ,Q) LJ 3 GHQJWKV$%DQG% DUH DQG IWUHSHFWLYHO 7KI IUDFWRQIDFWRUV HYHUZKHUH DQGS$ GHI LQ JDJH $OOSLSHVKDYHDGLDPHWUR LQ)RUZDWUDW f& GHWHUPLQHWKH IORZ UDWHLQDOOSLSHVDQGWKHSUHVXUHVDWSRLQW% & DQG'

3

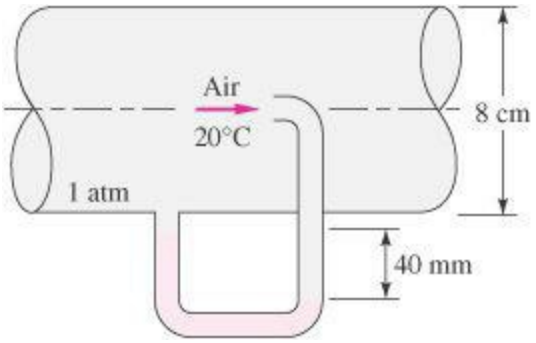


' LIIXVHUSHUFRUP DQFH

- 3 $ ZDWUXQQHOWWWHFWRQKDVD PGLDPHWUDQGIORZ SURSHUWLHV9 P V S NBD DQG7 *f*& 7KHERXQGDU\ OD\HUEORFNDJHDWWKHHQGRI WKHVHFWRQ LV SHUFHQW,I DFRQFDOLIXVHULVWREHDGGHGDWWKHHQGRI WKHVHFWRQWRDFKLHYHPD[LPXP SUHVVXUHUHFRYHU ZKDWKRXGLWDQJHHQJWK H[WGLDPHWU DQGH[WSUHVVXUHEH'
- 3)RU3URE 3 VXSSRVHZHDUHGPLWGE\ VSDFHWRDWRDOGLIXVHUHQJWKR P : KDWWKRXGWKHGLIXVHUDQJHH[WGLDPHWU DQGH[WSUHVVXUHEHIRUPD[LPXP UHFRYHU"
- 3 $ ZLQGWXQQHOWWWHFWRQLV IWWTXDUHZLWKIORZ SURSHUWLHV9 IWW S GEI LQ DEVROXWH DQG7 *f*) %RXQGDU\ OD\HUEORFNDJHDWWKHHQGRI WKHWWWHFWRQLV SHUFHQW) LQGWKHDQJHHQJWK H[WKHJKWDQGH[WSUHVVXUHRI DIODWZ DOOHGGLIXVHUDGGHRQWKHVHFWRQWRDFKLHYHPD[LPXP SUHVVXUHUHFRYHU
- 3)RU3URE 3 VXSSRVHZHDUHGPLWGE\ VSDFHWRDWRDOGLIXVHUHQJWKR IW: KDWWKRXGWKHGLIXVHUDQJHH[WKHJKWDQG H[WSUHVVXUHEHIRUPD[LPXP UHFRYHU"

7KHSLWRWWXEHWF WXEH

- 3 $QDLUSODQHXVHVDSLWRWWXEHDVDYHORFLPHWU 7KHPHDVXUHPHQW ZLWKWKHUXQHWUDQVGXFHULVWKHWRWDOWF WXEHSUHVVXUHR í " *f*& DVWDWF SUHVVXUHR " NBD DQGDSUHVVXUHGLIIHUHQFH S R í S V " 3D D (VWXPDWWKHDLUSODQH¶VYHORFLWDQG LWXQGHUVWDQG E ,VDFRPSUHVVLEOHORZFRUUHFWLRQHGHG'
- 3)RUWKHSLWRWWXEHWF SUHVVXUHDUDQJHPHQWR)LJ 3 WKHPDQRPHWUIOXLGV FRORUHG ZDWUDW *f*(VWXPDW D WKHFHQWHUOLQHYHORFLW E WKHSLSHYRXPHIORZ DQG F WKH VPRRWK ZDOOWKHUWXEHV
- 3)RUWKH *f*& ZDWUIORZ R)LJ 3 XVH WKHSLWRWWXEHWF DUDQJHPHQWR HVWXPDW D WKHFHQWHUOLQHYHORFLW DQG E WKHYRXPHIORZ LQWKH LQGLDPHWUVPRRWK SLSH F : KDWHUURULQIORZ UDWLVFDXVHGE\ QHJOHFWLQJ WKH IWHOHYDWLRQGLIIHUHQFH'



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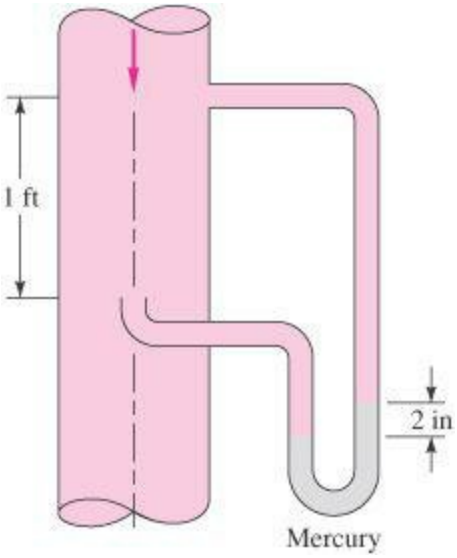
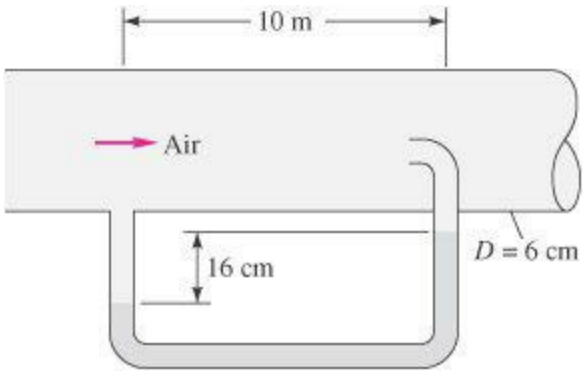


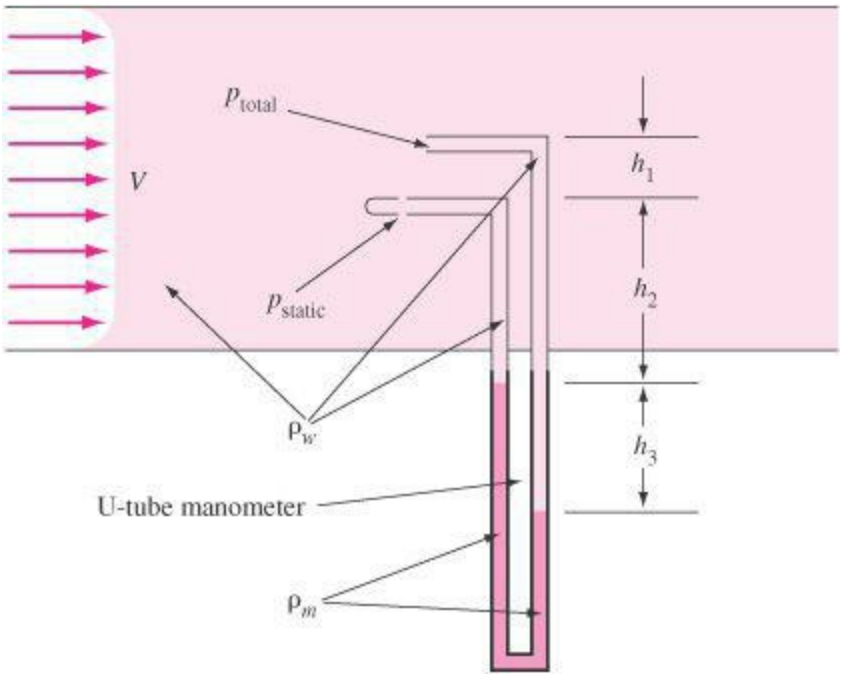
- 3 $QHQJLQHHUZKRWRRNFRQJHIXLGPHFKDQLFVRQDSDWK±IDLOEDLVKDVSDFH
WKH VWDWF SUHVXUH KRGH IDU XSVWHDP RI WKH VDJQDWRQ SUREH DV LQ) LJ 3 WKX
FRQVLGHULQJWKHSLWRWPHDVXUHPHQWVLGHXOO ZLWKSLSHIUFWRQORVVHV ,I WKHSLSHIORZ
LVDLUDW *& DQG DW DQGWKHPDQRPHWHUIOXLGLV0 HUFP UHGRLO 6* HWPD
WKHDLUFHQWHUOLQHYHORFLWIRUWKHJLYHQPDQRPHWHUUHDGLQJ RI FP $VXPHDVPRRWK
ZDOOHGWXEH

3



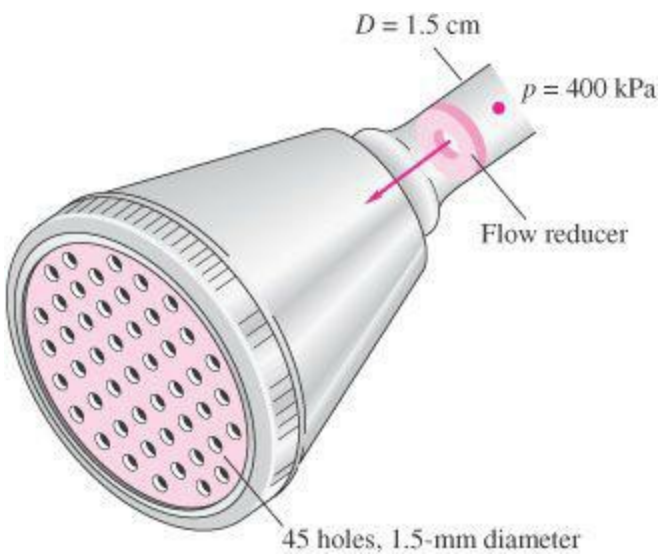
- 3 3URHVRU: DQWU7XQQHOQHGVWRPHDVXUHWKHIORZ YHORFLWLQDZDWUWXQQH
' XHWREXGJHWDU UHVWULFWLRQV KHFDQRWDIRUG DSLWRWWXEH SUREH EXWLQVWHDG LQVHUW
WRWDOKHDGSUREHDQGDVWDWF SUHVXUHSURBH DVVKRZQLQ) LJ 3 DGLVWDQFHK DSDU

IURP HDFK RWKHU %RWK SURELVDUHLQWKHPDLQIUHVWUHDP RI WKHZDWUXQQHOXQLIHFWG E\ WKHWKLQERXQGDU\ OD\HUVRQWKHVLGHZDOOV 7KHVWR SURELVDUHFRQQHFWHG DVVKRZQ D8 WXEHPDQRPHWU 7KHGHQVLW\DQG YHUWLFDOGLVWDQFHVDUHVKRZQ LQ) LJ 3 D : UL DQH[SUHVVLRQIRUYHORFLW9 LQWKHPVR WKHSDUDPHWUVLQWKHSURECOP E ,VLWFULWLFDOWKL GLVWDQFHK EHPHDVXUHGDFFXUDWH" F +RZ GRHVWKH[SUHVVLRQIRUYHORFLW9 GLIIHUIURP WKDWZKLFKZRXOGEHREWDLQHGLI DSLWRWWXEHSURECHDGEHQDYLGEOHDQGXVHGZLWKW VDPH8 WXEHPDQRPHWU



) ORZ PHWHUV WKHRULFHSODWH

- 3 . HURVHQHDW *& IORZ VDW P KLQD FP GLDPHWHUSLSH ,I D FP GLDPHWH WKLQSODWHRULILFHZLWKFRQHUWDSVLVLQWURGXFHG ZKDWZLOOWKHPHDVXUHGSUHVVXUHGURSEH L 3D'
- 3 EES * DVRQQHDW *& IORZ VDW P KLQD FP GLDPHWHUSLSH : HZLVKW PHWHUWKHIORZ ZLWKDWKLQSODWHRULILFHDQGDGLIIHUHQWDSSURXWHDQGXFHUWKDWWHGVEHW DWZERXW N3D : KDWLVWKHSURSHU UDWRIRUWKHRULILFH'
- 3 7KHVKRZHUKHDGLQ) LJ 3 GHOLYHUVZDWUDW *& \QRULILFHWSHIORZ UHGXFHULVWREHLQWURGXFHG 7KHXSWUHDP SUHVVXUHLVFRQWURQWDW N3D : KDWIORZ UDWH L JDOPLQ UHVXOWZLWKRXWWKHUHGXFHU' : KDWUHGXFHURULILFHGLDPHWUZRXOG GHFUHDVH WK IORZ E\ SHUFHQW



- 3 $ FP GLDPHWHUVPRRWKSLSHFRQWDLQVDQRULILFHSODWHZLWK VDSVDQG 7KHPHDVXUHGRULILFHSUHVVXUHGURSLV N3DIRUZDWHUDW *ZDWHUDW f& (VWLPDWHWK IORZUDWHLQP K: KDWLVWKHRQHFRYHUDEOHKHDGORVV"
- 3 :DWHUDW f& IORZVWKURXJKWKHRULILFHLQ) LJ 3 ZKLFKLVPRQLWRUHGE DPHUFXU PDQRPHWHU,IG FP D ZKDWLVKZKHQWKHIORZUDWHLV P KDQG E ZKDWLV4 LQP KZKHQK FP"

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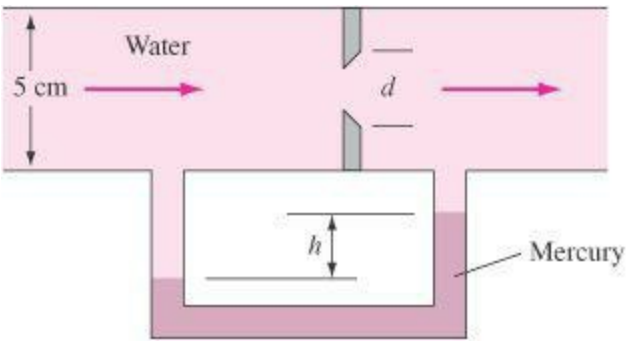
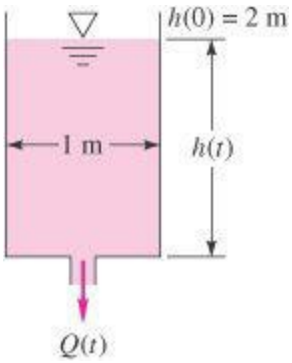


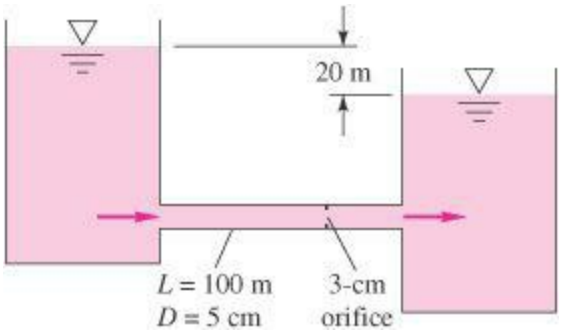
- 3 7KH PGLDPHWUDQNLQ) LJ 3 LVLQLWLDOO ILOOHGZLWKJDVROLQHDW f& 7KHUHLVD FP GLDPHWURULILFHLQWKHERWWRP ,I WKHRULILFHLVVXGGHQO RSHQHG HWLPDW WKHWLPHIRUWKHIXLGOHYHOWRGURSIURP WR P

3



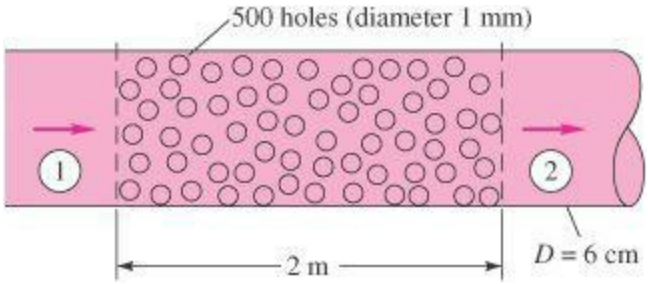
- 3 $ SLSHFRQQHFWLQJ WZR UHVHUYRLUV DV LQ) LJ 3 FRQWDLQV DWKLQSODV RUILFH) RUZDWHU IORZ DW *& HXVP DW D WHYROXPHIORZ WKURXJK WKHSLSHDQG E WK SUHVVXUHGURS DFURVV WKHRUILFHSODWH

3



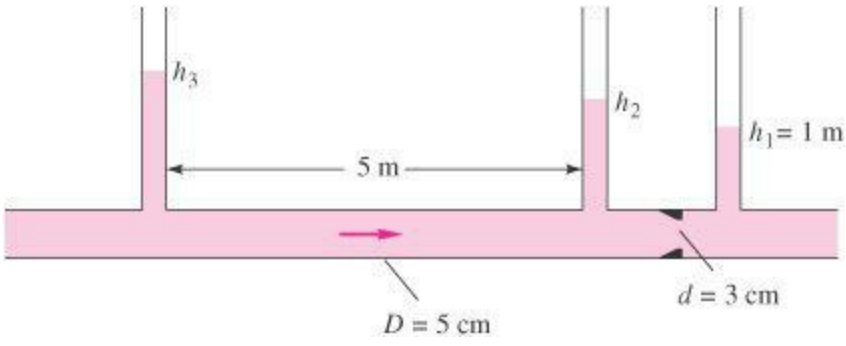
- 3 $LUIORZ VWKURXJK D FP GLDPHWUVPRRWKSLSHWKDWKDVD P ORQJ SHUPDW VHFWLRQFRQWDLQLQJ KROHV GLDPHWU PP DVLQ) LJ 3 3UHVVXUHRXWLGHWKHSLS LVVHDOHYHOWDQGDUDU,S N3DDQG4 P K HXVP DWS DQG4 DWXPLQ WKDWWKHKROHVDUHDSSUR[LPDWGE\ WKLQSODWRUILFHV +LQW$ PRPHQWXP FRQWROYROXPH PD\ EHYHU XVHXO

3



- 3 $ VPRRWKSLSHFRQWDLQLQJ HXDQRODW *& IORZVDW P KWKURXJKD%HUQRXO REVWUXFWLRQ DV LQ) LJ 3 7KUHH SLH]RPHWUWXEHV DUH LQVWDOOHG DV VKRZQ ,I WK REVWUXFWLRQLVDWKLQSODWRUILFH HXVP DWWKHSLH]RPHWUOHYHOV D K DQG E K

3



) IORZPHWUV WKHIORZ QR]]OH

- 3 ,QDODERUDWRU H[SHULPHQWDLUDW *& IORZ VIURP DODUJHVDQNURXJKD FP GLDPHWUVPRRWKSLSHLQRDVHDOHYHODWRSSKHUH DVLQ) LJ 3 7KHIORZ LVPHDVUH E\ DORQJ UDGLXVQR]]OHR FP GLDPHWUXVLQJ DPDQRPHWUZLWK0 HUHP UHGRLO 6* 7KHSLSHLV PORQJ 7KHPHDVXUHPHQWVRI VDQNSUHVVXUHDQGPDQRPHWUKHJKWDU DVIROORZV

8VHWKHVHGDWDWRFDOFXODWWKHIORZ UDWH4 DQG5H\QROGVQXPEHUV5H DQGPDNHDSOR RI PHDVXUHGIORZ UDWHYHUVXVVDQSUHVVXUH ,VWKHIORZ ODPLQDURUWXUEXOHQW&RPSDUHWK GDWD ZLWK WKHRUHWLFDO UHVXOWV REWDLQHG IURP WKH 0RRG\ FKDUWLQFOXGLQJ PLQRU ORVVHV ' LVFXVV

p_{tank}, Pa(gage):	60	320	1200	2050	2470	3500	4900
h_{mano}, mm:	6	38	160	295	380	575	820

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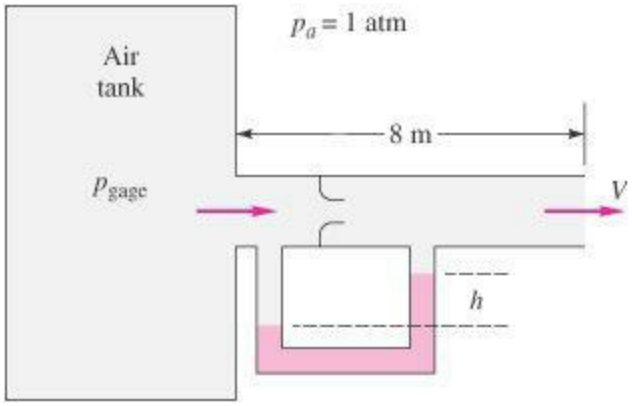
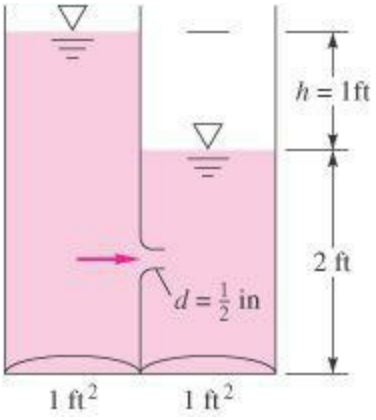


- 3 * DVROQHDW *& IORZ VDW P VWKURXJKD FP SLSHDQGLVPHWUHGE\ I FP ORQJ UDGLXVIORZ QR]]OH) LJ D : KDWLV WKH H[SHFWHG SUHVVXUHGURS DFURVV WK QR]]OH'
- 3 $QHQJLQHHUQHGVWRPRQLWRUDIORZ RI *& JDVROQHDWDERXW " JDOPL WKURXJK D LQGLDPHWUVPRRWKSLSH 6KHFDQXVHDQRULILFHSODWH DORQJ UDGLXV IORZ QR]]OH RUDYHQXUL QR]]OH DOOZLWK LQGLDPHWUWKURDW 7KHRQO GLIIHUHQWDOSUHVXU JDJHDYDLODEOHLV DFFXUDWHLQWKHUDQJH WR OEI LQ ' LVUHJDUGLQJ IORZ ORVVHV ZKLF GHYLFHLVEHVW
- 3 . HURVHQHDW *& IORZ VDW P KLQDQ FP GLDPHWUSLSH 7KHIORZ LVWRE PHWUHGE\ DQ,6$ IORZ QR]]OHVR WKDWWKH SUHVXUHGURS LV 3D : KDWLV WK SURSHUQR]]OHGLDPHWU'
- 3 7ZR ZDWHUVWUHDPV HDFK ZLWK EDVH DUHD RI IW DUH FRQQHFWHG E\ D LQ GLDPHWUORQJ UDGLXVQR]]OHDVLQ) LJ 3 ,I K IWDVVKRZQIRUW HVWLPDWHWK WPHIRUK WVWRGURSVR IW

) ORZ PHWHUV WKHYHQWXULPHWHU

- 3 * DVROQH DW *& IORZ V WKURXJK D FP GLDPHWUSLSH ,WLV PHWHUHG E\ PRGHUQYHQWXULQR]]OHZLWKG FP

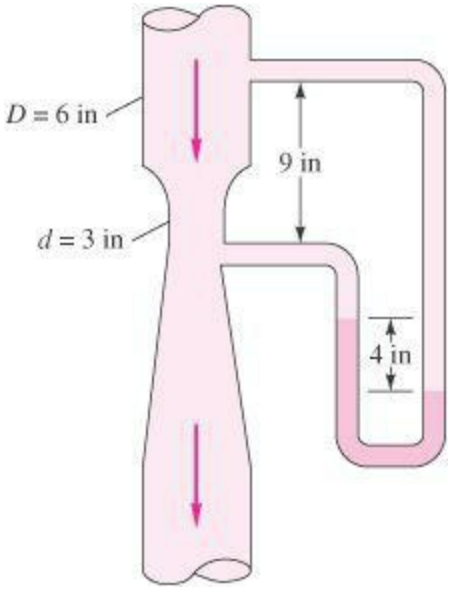


7KHPHDVXUHGSUHVVXUHGURSLV N3D (VWLPDWHWKHIORZ UDWHLQJDOORQVSHUPLQXWH

- 3 ,WLVGHVLUHG VR PHWHUPHKDQRODW *& IORZ LQJ WKURXJK D LQGLDPHWHUSLSH 7KH H[SHFWHG IORZ UDWH LV DERXW JDOPLQ 7Z R IORZ PHWHUV DUH DYDLODEOH D YHQXUL QR]]OHDQG DWKLQSODWHRULILFH HDFKZLWKG LQ 7KHGLIIHUHQWLDOSUHVVXUHJDJHRQKDQGL PRVWDFFXUDWHDWDWZERXW ± GEVLQ : KLFKPHWHULVEHWWHUIRUWKLVPRE'

- 3 (VWDQRODW *& IORZ V GRZ Q WKURXJK D PRGHUQ YHQXUL QR]]OH DV VKRZ Q HWWKH PHUFXU PDQRPHWHU UHDGLQJ LV DV VKRZ Q HWWKH IORZ UDWH L JDOPLQ

3



- 3 0 RGLI\ 3UREOHP 3 LI WKH IOXLG LV DLU DW *& HQWHULQJ WKH YHQXUL DW SUHVVXUH RI GEI LQ 6KRXOG DFFRPSUHVVLELOLW\ FRUUHFWLRQ EH XVHG'
- 3 : DWHU DW *& IORZV LQ D GRQJ KRUL]RQWDO FRPPHUFLDO FP GLDPHWHU SLSH WKDW WRQGLQV DFDXWLFDO+HUFKHOYHQXUL ZLWK D FP WKURDW7KH YHQXUL VFRQQHFWV WR D PHUFXU PDQRPHWHU ZKRVH UHDGLQJ LVK FP (VWLPDWH D WKH IORZ UDWH LQP K DQG E WKH WRWDO SUHVVXUHGLIIHUHQFHEHWZHHQSRLQWV FP XSVWUHDP DQG FP GRZQVWUHDP RI WKH YHQXUL
- 3 $ PRGHUQYHQXUL QR]]OHLV WR WR GLVFKDUJH IORZ ZLWK ZDWHU DW *& 7KHSLSH GLDPHWHU LV FP DQG WKH YHQXUL WKURDW GLDPHWHU LV FP 7KH IORZ UDWH L PHDVXUHG E\ DZHJK WDQN DQG WKH SUHVVXUHGURS E\ DZDWHU±PHUFXU PDQRPHWHU

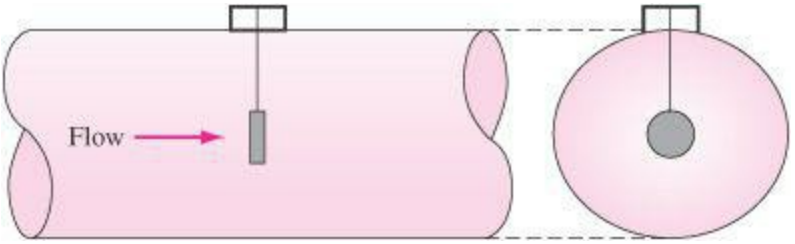
7KHPDVVIORZ UDWHDQGPDQRPHWUHDGLQVDUHDVIRORZV
8VH WKHVH GDWD WR SORWD FDOLEUDWRQ FXUYH RI YHQWXUL GLVFKDUJH FRHILFLHQWYHUVX
5HYRONVQXPEHU &RPSDUHZLWKWKHDFFHSWHGFRUUHODWRQ (T

$\dot{m}$, kg/s	0.95	1.98	2.99	5.06	8.15
h, mm	3.7	15.9	36.2	102.4	264.4

) ORZ PHWHUV RWKHUGHVLJQV

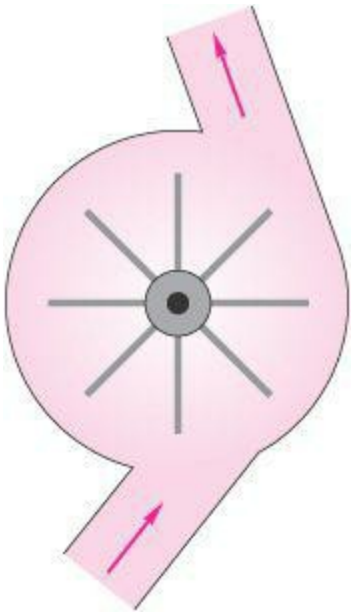
- 3 $QLQWUXPHQWSRSXODULQWKHEHYHUDJHLQGXVWU LVWKHVWUDLJHWORZPHWU LQ) LJ 3 $ VPDOOIODWGLVNLVPRXQWHGLQWKHFHQWHURI WKHSLSH VXSSRUWHGE\ DVWURQJ EXWWRQ URG D ([SODLQKRZ WKHIORZPHWUZRUNV E ,I WKHEHQGLQJ PRPHQW RI WKHURG LV PHDVXUHGDWWKHZDOOGHULYHDIRUPXODIRUWKHVWUDLQGYHORFLWRI WKHIORZ F /LVWDIHZ DGYDQWDJHVDQGGLVDGYDQWDJHVRI VXFKDQLQVWUXPHQW

3



- 3 $QLQWUXPHQWSRSXODULQWKHZDWUVXSSO LQGXVWU VNHWWKHGLQ) LJ 3 LV WKH VLQJOH MHWZDWU PHWU D +RZ GRHV LWZRUN' E : KDWGR \RX WKLQN D WSLFD FDOLEUDWRQFXUYHZRXOGORRNOLNH' F &DQ\RX FLWHIXUWKHUGHWDLOV IRUH[DPSOH UHODELOLW KHDGORVV FRVWV @

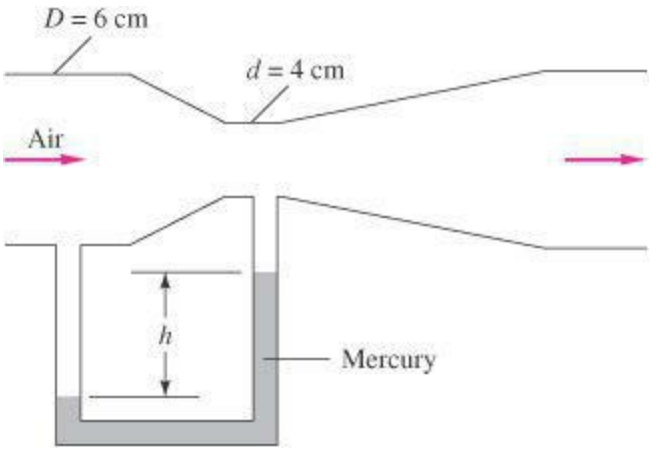
3



) ORZ PHWHUV FRP SUHVVLEOHOLW FRUUHFWLRQ

- 3 $ LU IORZ V DVKLJK VSHHG WKURXJK D + HUFKHDYHQXUL PRQVWHUHG E\ D PHUFXU PDQRPHWHU DV VKRZQ LQ) LJ 3 7KHXSWUHDP FRQGLWLRQV DUH N3D DQG *f& ,IK FP HWHUPLQDWH WKHPDVVIORZ LQNJ V +LQW7KHIORZ LVFRPSUHVVLEOH

3



- 3 0 RGLI\ 3UREOHP 3 DV IRORZV) LQG WKH PDQRPHWHU UHDGLQJ K IRUZKLFK WKH PDVVIORZ WKURXJK WKHYHQXUL VDSSUR[LPDWH NJ V +LQW7KHIORZ LVFRPSUHVVLEOH

: RUG3UREOHPV

- : ,QIXOO GHYHORSHG WXUEXOHQW IORZ WKHYHORFLW SURILOHV GR QRW FKDQJH ZK\" EXW WKH SUHVXUH GURS DORQJ WKH SLSH[V 7KXV WKHUHLV SUHVXUH ZRUN GRQH RQ WKH IOXLG , I VD\ WKH SLSHLVLQXODWHG IURP KHDWRV ZKHUHGRHV WKLVHQHUJ\ JR' 0DNHDWKHUPRG\QDPL DQDO\VLV RI WKHSLSHIORZ
- :) URP WKH0 RRG\ FKDUW) LJ URXJK VXUIDFHV VXFK DV VDQG JUDLQV RUUDJJH PDFKLQLQJ GR QRWDIIHFWODPLQDUIORZ &DQ\RX H[SODLQZK\" 7KH GRDIIHFWWKUEXOHQWORZ &DQ\RX GHYHORS RUVXJJHWWDQDQDO\WFDOPRGHORI WKUEXOHQWORZ Q+DUDURXJ VXUIDFHWKDWPLJKWEHXVHGWRSUHGLFWWKHQRZ QLQFUHDVHLQSUHVXUHGURS'
- : ' LIHUHQWDWRQ WKHOPLQDUSLSHIORZ VROXWRQ(T VKRZ VWKDWWKHIXL VKHDUWUHVV2U YDULHVOLQHDU IURP]HUR DWWKHD[LV WR2Z DWWKHZDOO,WVFOLPHGWKDWWKL LVDOR WXH DWODDWLQ WKHWPHPHQ IRUIXOO GHYHORSHGWXUEXOHQWORZ &DQ\RX YHUL\ WKLVFOLP DQDOWFDOO"
- : $ SRURXV PHGLXP FRQVLWV RI PDQ WLQ WRUWRXV SDWJHV DQG 5H\QROGV QXPEHU EDVHGRQSRUHV]HDUHXVDOO YHU\ RZ RUGHUXQLW,Q + ' DUA SURSRVH WKDWWKH SUHVXUH JUDGLHQWLQ D SRURXV PHGLXP ZDV GLUHFW SURSRUWRQDO WR WK YROXPHDYHUDJHGYHORFLW9 RI WKHIOXLG

$$\nabla p = -\frac{\mu}{K}\mathbf{v}$$

ZKHUH. LVWKH PHDVXUHG WKHSHUPHDELOLW RI WKHPHGLXP 7KLVLVQRZ FDOOHG' DUA¶VODZ R SRURXV IORZ &DQ\RX PDNHD 3RLVHXLOOH IORZ PRGHORI SRURXV PHGLD IORZ WKDWYHULI

' DUA\¶V ODZ" 0 HDQZKLOH DV WKH 5H\QROGV QXPEHUL FUHDVHV VR WKDW9. ! WK
SUHVVXUHGURS EHFRPHVQRQOLQHDU DVZDVVKRZQH[SHULPHQWDOO\ E\ 3 +) RUFKHLPHUD
HDUO DV 7KHIORZ LVVXDOOGHFLGHGO OPLQDU\HWKHSUHVVXUHJUDGLHQWLVTXDGUDWF

$$\nabla p = -\frac{\mu}{K}\mathbf{V} - C|V|\mathbf{V} \quad \text{Darcy-Forscheimer law}$$

Z KHUH& LV DQ HPSLULFDO FRQVWDQW &DQ \RX H[SODLQ WKH UHDVRQ IRU WKLV QRQOLQHD
EHKDYLRU'

SURECOPV TXHVWLRQV) XQGDPHQWDOV RI (QJLQHHULQJ ([DP 3URECOPV

-)(,QIORZ WKURXJK D VWUDLJKW VPRRWK SLSH WKH GLDPHWHU 5H\QROGV QXPEHU IF
WDQVLWLRQ WR WXUEXOHQFH LVJHQHUDOO\ WDNHQ WR EH

-)( )RUIORZ RI ZDWHUDW *& WKURXJK D VWUDLJKWVPRRWK SLSH DW P K WKH SLSI
GLDPHWHUIRUZKLFKWDQVLWLRQ WR WXUEXOHQFHRFFXUVLVDSSUR[LPDWO
FP
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-)( )RUIORZ RI RLO> NJ P V 6* @WKURXJK DORQJ VWUDLJKWVPRRWK
FP GLDPHWHUSLSHDW P K WKHSUHVVXUHGURS SHUPHWHULVDSSUR[LPDWO
3D
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-)( )RUIORZ RI ZDWHUDW5H\QROGV QXPEHUR (WKURXJK D FP GLDPHW
SLSHRI URXJKQHVVWKHJKW PP WKHDSSUR[LPDWO RRG IUFWLRQIDFWRULV

-)(0 LQRU ORVVHV WKURXJK YDOYHV ILWWLQJV EHQGV FRQWUDFWLRQV DQG WKH QH DL
FRPPRQO PRGHOHG DVSURSRUWLRQDOWR D WRWDOK HDG E VWUDLJK HDG F YHORFLW\K HDG G
SUHVVXUHGURS H YHORFLW\

-)($ VPRRWK FP GLDPHWUSLSH P ORQJ FRQQHFWVWRUHVUYRLV FRQWLQ ZDWUDW *& RQH RI ZKLFK KDV DVXUFH HOHYDWRQ P DQG WKH RWKHU DVXUFH HOHYDWRQ P ,I PLQRUORVVHVDUHQHJOHFWHG WKHHSHFWHGIORZ UDWWKURXJKWKHSLSHLV
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-)(,I LQ3URE)(WKHSLSHLVURXJKDQGWKHDFWXDOIORZ UDWLV P K WKHQWK H[SHFWHGDYHUDJHURXJKQHVVWKHLJKWR WKHSLSHLVDSSUR[LPDWHO
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-)(6XSSRVHIQ3URE)(WKHVWRUHVUYRLVDUHFRQQHFWHG QRWE\ DSLSH EXWE\ I VKDUS HGJHGRULILFHRI GLDPHWU FP 7KHQWKHH[SHFWHGIORZ UDWLVDSSUR[LPDWHO
  P K
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-)(2LO> NJ P V 6* @IORZ VWKURXJK D P ORQJ VPRRWK FP GLDPHWU SLSH 7KH PD[LPXP SUHVVXUH GURS IRU ZKLFK ODPLQDU IORZ LV H[SHFWHG L DSSUR[LPDWHO
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-)($LUDW *& DQG DSSUR[LPDWHO DW IORZ VWKURXJK D VPRRWK FP VTXDU GXFWDW IWPLQ 7KHH[SHFWHGSUHVVXUHGURS SHUPHWUR GXFWOHQJWKLV
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  3D
-)(: DWUDW *& IORZ VDW P KWKURXJK DVKDUS HGJHG FP GLDPHWURULILFHLQ FP GLDPHWUSLSH (VXP DWWKHH[SHFWHGSUHVVXUHGURS DFURVVWKHRULILFH
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-)(: DWUIORZ VWKURXJK DVWDLJKW FP GLDPHWUSLSH DVD GLDPHWU5HQRO QXPEHUR ,I WKH SLSH URXJKQHVV LV PP ZKDWLV WKH DSSUR[LPDW0RRG IUFWLRQIDFWRU'

-)(: KDWLV WKHK\GUDXOLF GLDPHWURI DUHFWDQJXODUDLUYHQWLODWLRQGXFWZKRVHFURV VHFWLRQLV P E\ FP"
  FP
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-)(: DWUDW *& IORZ VWKURXJK DSLSHDW JDOPLQZLWKDIULFWLRQKHDGORVVF IW. KDWLV WKHSRZHUHTXLUHGWRGULYHWKLVIORZ"
  N
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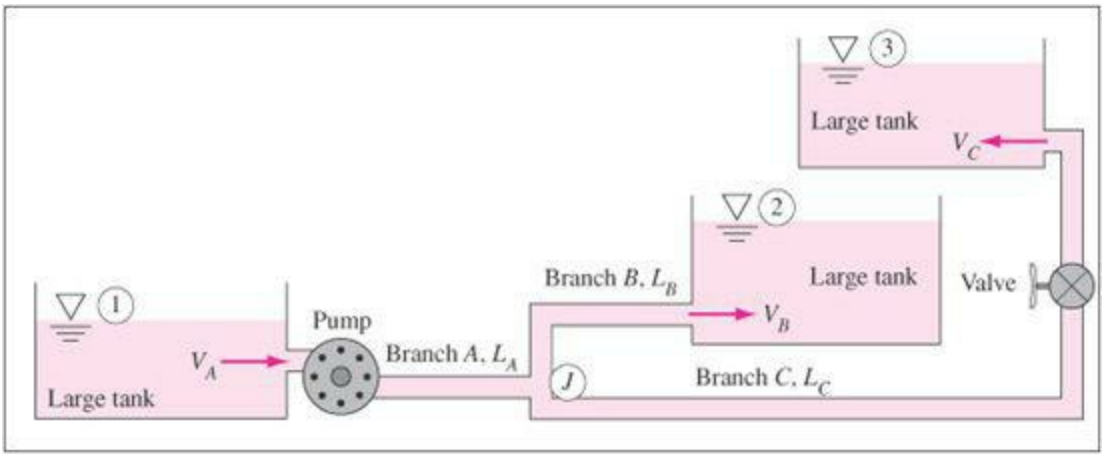
  N
-)(: DWUDW *& IORZ VDW JDOPLQWKURXJK DSLSH P ORQJ DQG FP L GLDPHWU,I WKHIULFWLRQKHDGORVWLV P ZKDWLV WKH0RRG\ IUFWLRQIDFWRU'

SUREOHPV TXHVWLRQV&RP SUHKHQVLYH3UREOHPV

- & $ SLWRWWXDWF SUREHZLOOEHXVHGWRPHDVXUHWKHYHORFLWGLVWULEXWLRQLQDZDW WKQQHODW *& 7KHVZR SUHVXUHQQHV IURP WKH SUREHZLOOEHFRQQHFWHGWRD8 WXE PDQRPHWUWKDWXVHVDQDTXLGRI VSHFLILF JUDYLW 7KHPDLPXP YHORFLWH[SHFWHGLQWK ZDWUWQQHOLV P V<RXUWHE LVWRVHOHFWDQDSSURSULDW8 WXEHIURP DPDQXIDFWXUH WKDWVXSSOLHV PDQRPHWUV RI KHLJKW DQG LQ 7KH FRWWLQFUHDV VJQLILFDQW ZLWKPDQRPHWUHLJKW. KLFKRI WKHVHVKRXOG\RXSXUFKDVH"
- & $ SXPS GHOLYHUV DVWDG\ IORZ RI ZDWU ! IURP DODUJH VDQN WR VWR RVWH KLJKHUHOHYDWRQ WDQN DVVKRZQLQ) LJ & 7KHVDPHSLSHRI GLDPHWUGDQGURXJKQHVLV XVHG WKURXJKRXW$OOPLQRU ORVVHV[FHSWWKURXJK WKH YDOYH DUH QHJOHFWHG DQG WKSDUWDOO FORVHGYDOYHKDVDORVVFRHILFLHQW $YDOYH 7XUEXOHQWORZ PD\ EHDVXPHGZLWKDO

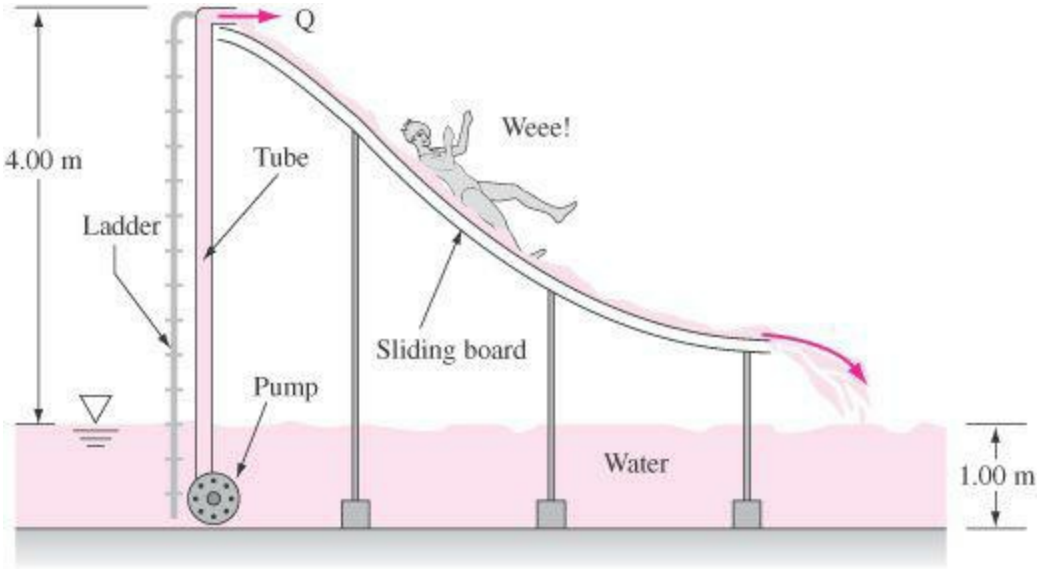
NLQHWF HQHUJ\ IOX[FRUUHFWLRQ FRHILFLHQW HTXDOV 7KH SXPS HDG+ LV DNQRZ
IXQFWLRQ RI 4$ DQG KHQFH DOVR RI 9$ 4$ $SLSH IRUH[DPSOH ZKHUH D DQG E DUH FRQVWDQ
6XEVFULSW UHIHUV WR WKH IXQFWLRQ SRLQWDWKH WHZKHUEUDQFK $VSDWLQR %DQG& 3LS
HQJWK/& LV PXFK ORQJHU WKDQ /% ,WLV GHVLUHG WR SURGLFWWKH SUHVXUH DW WKH WKUHH SLSH
YHORFLWLHV DQG IULFWLRQ IDFWRUV DQG WKH SXPS KHDG 7KXV WKHUH DUH HLJKWYDULDEOHV + 9$
9% 9& I$ I% I& S: ULWH GRZQ WKH HLJKWHTXDWLRQV QHHGHG WR UHVROYH WKLV SUREOHP EXWG
QRWWRYH VLQFH DQHDERUDWLWUDWLRQ SURFHGXUH RUDQHTXDWLRQVROYHUVXFK DV((6 ZRXO
EHUHTXLUHG

&



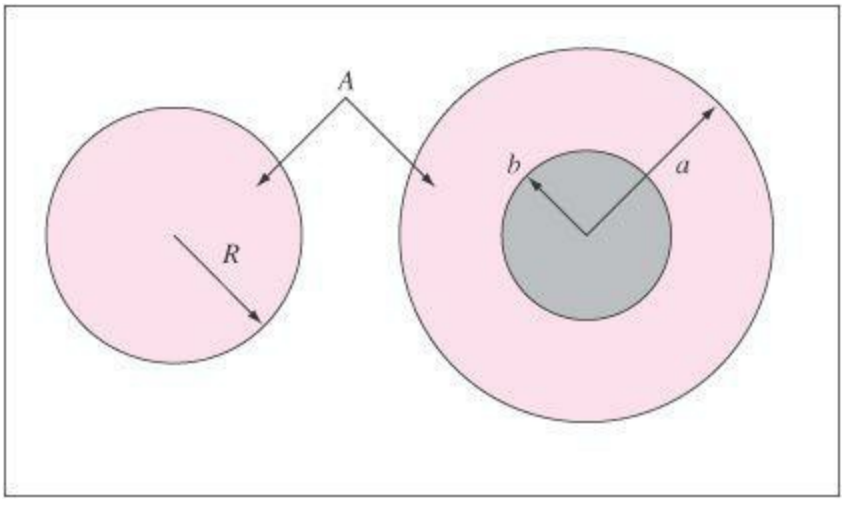
- & $ VP DOZ DWU VOGH LV WR EH LQVWDOOHG LQ GHDZ LPPLQJ SRRO 6HH) LJ & 7K
VOGH PDQXIDFWXUHU UHFRPPHQGV DFRQLQXRXV ZDWU IORZ UDW4 RI î í P V DERX
JDOPLQ GRZQ WKH VOGH WR HQVXUH WKDWWKH FXUUHQW GR QRWEXLOG WKH UERWWRP V $ SXPS
LV WR EH LQVWDOOHG XQGHU WKH VOGH ZLWK D P ORQJ FP GLDPHWU KRVH VXSSOLQ
VZ LPPLQJ SRRO ZDWU IRU WKH VOGH 7KH SXPS LV SHUFHQW HILFLHQW DQG ZLOO UHVWXOW
VXEPHUJHG P EHORZ WKH ZDWU VXUIDFH 7KH UHOJKQHVW LQVLGH WKH KRVH LV DERXW
FP 7KH KRVH GLVFKDUJHV WKH ZDWU DW WKH WRS RI WKH VOGH DV D IUHH MHWWRSHQ WR WK
DWPRVSKHUH 7KH KRVH RXOHWLV P DERYH WKH ZDWU VXUIDFH) RU IXOO GHYHORSH
WXUEXOHQWSLSH IORZ WKHNLQHWF HQHUJ\ IOX[FRUUHFWLRQ IDFWRUV DERXW ,JRUHDQ PLQR
ORVVHV KHUH $VVXPH WKDWV NJ P DQG% î í P VIRUWKLV ZDWU) LQG WK
EUDNHKRUVHSRZ HU WKDWV WKH DFWXDOKDWSRZ HULQZDWW UHTXLUHG WR GULYH WKH SXPS
- & 6XSSRVH \RX EXLOG D UXDOKRXVH ZKHUH \RX QHG WR UNQ D SLSH WR WKH QHDUH
ZDWU VXSSO ZKLFK LV IRUWKQDWH DWDQHOHYDWQRQ DERXW P DERYH WKDWWR \RXUKRXV
7KH SLSH ZLOOEH NP ORQJ WKH GLVDQFH WR WKH ZDWU VXSSO DQG WKH JDJH SUHVXUH DWWK
ZDWU VXSSO LV N3D <RX UHTXLUH DP LQP XP RI JDOPLQ RI ZDWU ZKHQ WKH HQGR
\RXU SLSH LV RSHQ WR WKH DWPRVSKHUH 7R PLQPL]H FRVW\RX ZDQW WR EX WKH VPDOOHVW
GLDPHWU SLSH SRVVLEOH 7KH SLSH\RX ZLOOXVH LVH[WUHPHO VPRRWK D) LQG WKH VPDOOHVW
ORVV IURP WKH SLSH LQOHWWR LWH[W1 HJOHFWDQ PLQRUORVVHV GXH WR YDOYH VHDERZ V HQWDQFH
OHQJWK DQG VR RQ VLQFH WKH OHQJWK LV VR ORQJ KHUH DQG PDWUORVVHV GRPLQDW $VXPH WK
RXOHWWR WKH SLSH LV RSHQ WR WKH DWPRVSKHUH E: KLFK LV PRUH LPSRUDQW WKDQ SURPS
WKH HDGORVV GXH WR HOYDWRQ GLIIHUHQFH RUWKH HDGORVV GXH WR SURWXUHGURS LQWKH SLSH
F) LQG WKH PLQLPXP UHTXLUHG SLSH GLDPHWU

&



- & : DWUDWURRP VWPSHUDWXUHIORZ VDWWKHVDPHYROXPHIORZ UDWH4 î í P V WKURXJKVWRGXFWRQHDURXQGSLSHDQGRQHDQDQXV 7KHFURVVVHFWLRQDODUHD$ R WKHVWRGXFWLVLGHQWLFDODQGDOZDOVDUHPDGHRI FRPPHUFLDOVWHHO%RWKGXFWDUHWK VDPHOHQJWK ,QWKHFURVVVHFWLRQVKRZQLQ) LJ & 5 PP DQGD PP D : KDWLVWKHUDGLXVEVXFKWKDWWKHFURVVVHFWLRQDODUHDVRI WKHVWRGXFWDUHLGHQWLFDO" E &RPSDUHWKHIULFWLRQDOKHDGORVVIRUWKHVWRGXFWRI SLSHIRUWKHVWRFDVHV DVXPLQJ IXOO GHYHORSHGIORZ) RUWKHDQQXOV GR ERWKDTXLFNHVWPDWXUQ WKHK\GUDXOLF GLDPHWHU DQGPRUHDFFXUDWHVWPDWXUQ WKHHIIHFWLYHGLDPHWUFRUUHFWRQ DQGFRPSDUH F , WKHORVVHV DUHGLIIHUHQWIRUWKHVWRFDVHV H[SODLQZK\ : KLFKGXFWLDQ LV PRUH ³HILFLHQW"

&



- & -RKQ/DXHU1$&$7HFK5HS JDYHYHORFLWGDWD *DLUORZ LQI VPRRWK FP GLDPHWUSLSHDW6H§ (7KHFHQWHUOLQH YHORFLW X&/ ZDV P V ' HWHUPLQH D WKHDYHUDJHYHORFLWE QXPHULFDOLQWHUDWRQ DQG E WKHZDOOVKHDUVWUHVV IURP WKHORJ ODZ DSSUR[LPDWH &RPSDUHZLWKWKH0 RRG\FKDUWDQGZLWK(T

u/u_{CL}	1.0	0.997	0.988	0.959	0.908	0.847	0.818	0.771	0.690
r/R	0.0	0.102	0.206	0.412	0.617	0.784	0.846	0.907	0.963

- & &RQVLGHU HQHUJ\ H[FKDQJH LQ IXOO GHYHORSHG ODPLQDU IORZ EHWZHHQ SDUDOOH SODWHV DVLQ(TV / HVWKH SUHVXUH GURS RYHUDOHQJWK/ EHûS &DOFXODWH WKH UDWH RI ZRUN GRQH E\ WKLV SUHVXUH GURS RQ WKH IXLG LQ WKH UHJLRQ [/ íK \ K DQG FRPSDUHZLWK WKH LQIRUPDWLRQ GLVSDWHG GXHWR WKH YLVFRXV IXQFWLRQ) IURP(T RYHUWKLV VDPH UHJLRQ 7KHWZR VHRXGEEHHTXDO([SODLQZK\ WKLVLVVR &DQ\RX UHDOWH WKYLVFRXVGUDJ IRUFHDQG WKHZDOOVKHDUWUHVVRWKLVHQHUJ\ UHVXOWV
- & 7KLV WH[WKDV SUHVHQWHG WKH WDGLWRQDO FRUUHODWRQV IRU WKH WXEXOHQW PRRWZ DI IUFWLRQ IDFWRU(T DQG WKHOZ RI WKHZDOO(T 5HFHQW JURXSV D 3ULQFHWRQ DQG 2UHJRQ> @KDYH PDGHQHZ IUFWLRQ PHDVXUHPHQW DQG VXJJHVWWK IROORZLQJ VPRRWKZDOOIUFWLRQODZ

$$\frac{1}{\sqrt{f}} = 1.930 \log_{10}(\mathrm{Re}_D\sqrt{f}) - 0.537$$

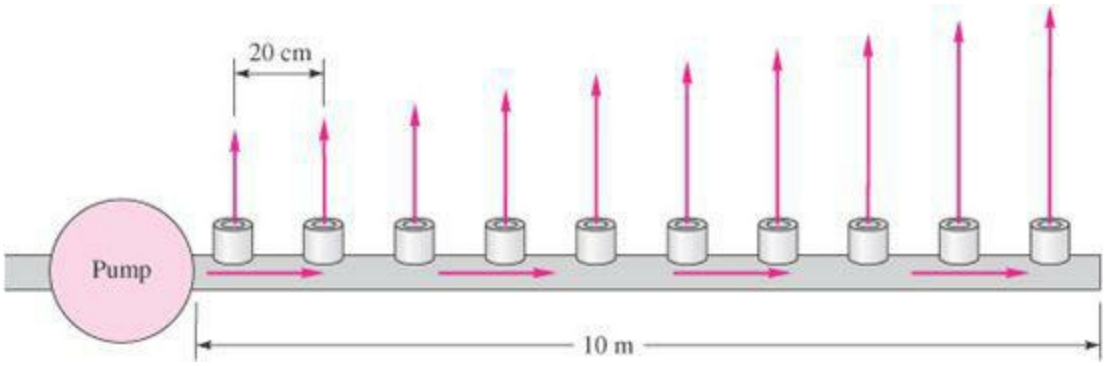
,QHDUOHUZRUN WKH DOR UHSRUWKDWEHWWUYDOXHVIRUWKHFRQVDQWV DQG%LQWKHORJ ODZ (T DUH § “ DQG%§ “ D &DOFXODWDIHZ YDOXHVRI LQWKH UDQJH (5H (DQGVHHKRZ WKHVZRIRUPXODVGLIHU E 5HDG5H DQGEULHO FKHF WKHILYHSDSHUVLQLWWELEORJUDSK\ 5HSRUWWR WKHFDVRQWKHJHQHUDOUHVXOWRI WKLVZRUN & \$ SLSHOLQH KDV EHHQ SURSRVHG WR FDUU QDWXUDOJDV PLOHV IURP \$ODVND1 RUWK6ORSH WR&DOJDU \$OEHUWD &DQDGD 7KH DVXPHG VPRRWK SLSHGLDPHWUZLOOEH LQ 7KH JDVZLOOEHDWKLJK SUHVXUH DYHUDJLQJ GEVLQ D : K\" 7KHSURSRVHGIORZ UDWH LV ELGGRQFXELF IHHWSHUGD\ DWWHDOHYHOFRQGLWLRQV E : KDWYROXPHIORZ UDWH D *f& ZRXOG FDUU WKHVDPHPDVVDWWKHKLJK SUHVXUH' F ,I QDWXUDOJDVLVDVVXPHGWREH PHWKDQH &+ ZKDWLV WKHVRXUSUHVXUHGURS" G ,I HDFK SXPSLQJ VWDWLRQFDQGHOLYHU KSWRWKHIORZ KRZ PDQ VWDWLRQVDUHQHHGHG'

SUREOHPV TXHVWLRQV' HVLJQ3URMHFWV

- ' \$ K\GURSRQLF JDUGHQXVHV WKH P ORJ SHUPDQHQWSLSHV\VWHP LQ)LJ ' WRGHOLYHUZDWHUDW *f& 7KHSLSHLV FP LQGLDPHWUDQGFRQVLVWVDFLUFXODUKROHHYHU FP \$ SXPS GHOLYHUVZDWHUDW N3D JDJH DWWKHHQWUDQFH ZKLOHWKHRWKHUHQGRI WKH SLSHLVFORVHG ,I \RX DWWHPSWHG IRUH[DPSOH 3UREOHP 3 \RXNQRZ WKDWWKHSUHVXUHQHDU WKHFORVHGHQGRI DSHUPDQHQW³PDQLRGG LVVXSSULVLQJO KLJK DQGWKHUHZLOOEHWRR PXFK IORZ WKURXJK WKH KROHVQHDUWKDWHQG 2QHUHPHG\ LV WR YDU WKH KROHVL]H DORQJ WKH SLSH DV RQH PRYHV DORQJ WKH SLSH D[LV 0 DNHDGHVLJQDQDOVLV SHUFDSVXLQJ DSHUWRQDOFRPSXWHU SURJUDP WKDWZLOOPDNH GLVWULEXWHWKHRSWLPXP KROH VL]H GLVWULEXWLRQ WKDWZLOOPDLQWDLQ D SHUWRQDOFRPSU WR SLFN WKHRSWLPXP KROH SDWWHUQ <RX DUH FRQVWUDLQHG WR SLFN KROHV VWDWLRQHUSRQG RQO WR FRPPHUFLDO QXPEHUHG PHWULF GULOOELWV/DYDLODEOHWRWKHVSLFDOP DFKLQHVKRS
- ' ,WLV GHVLUHG WR GHVLJQ D SXPS SLSLQJ V\VWHP WR NHHS D PLOORQJDOORQFDSDFLW ZDWHUWDQN ILOOHG 7KHSDQLV WR XVH D PRGLILHG LQVL]HDQG VSHHG YHUVLRQ RI WK

PRGHO FHQWUIXJDOSXPS PDQXDFWXUHG E\ 7DFR,QF &UDQWRQ 5KRGH,VODQ
7HVWGDWDKDYHEHHQSURYLGHGWRXVE\ 7DFR,QF IRUDVPDOOPRGHORI WKLVSXPS '
LQ 9 UPLQ WWWGZLWKZDWUDW f&

'



7KHVDQNLVWREHILOOHGGDLO ZLWKUDWHUFKLOO f& JURXQGZDWUIURP DQDTXLIHU ZKLF
LV PLIURP WKHVDQNDQG IWGRZHUWKDQWKHVDQN (VWPDWGGDLO ZDWUXVHLV PLOOR
JDOGD) LQDQ WPH VKRXOG QRWH[FHHG K SHUGD 7KH SLSLQJ V\VWP VKRXOG KDYH IRX
³EXWWUO´ YDOYHV ZLWK YDULDEOHRSHQLQJV VHH) LJ HDBRZ V RI YDUROV DQJOHV DQ
JDOYDQL]HGLURQSLSHRI DVL]HVWREHVHOHFWGLQWHGHVLJQ 7KHGHVLJQVKRXOGEHHFRQRPLFDO
ERWKLQFDSLWDOFRVWDQGRSHUDWQJ H[SHQVH 7DFR,QF KDVSURYLGHGWKHIROOWLQJ FRVWWWWP DW
IRUV\VWP FRPSRQHQWV

Q, gal/min	0	5	10	15	20	25	30	35	40	45	50	55	60
H, ft	28	28	29	29	28	28	27	26	25	23	21	18	15
Efficiency, %	0	13	25	35	44	48	51	53	54	55	53	50	45

<RXUGHVLJQWDVNLVWRVHOHFWDQHFRQRPLFDOSLSHVL]HDQGSXPSLPSHOHUVL]HDQGVSHGIR
WKLVWDVN XVLQJ WKHSXPS VWWWGDWDLQQRQGLPHQVLRQDOIRUP VHH3URE 3 DV GHVLJQGDW
: ULWWDEULH UHSRUWILYHWRVL[SDJHV VKRZLQJ \RXUFDOFXODWRQVDQGJUDSKV

Q, gal/min	0	5	10	15	20	25	30	35	40	45	50	55	60
H, ft	28	28	29	29	28	28	27	26	25	23	21	18	15
Efficiency, %	0	13	25	35	44	48	51	53	54	55	53	50	45

5 HIHUHQFHV

3 6 %HUQDUGDQG- 0 : DOOFH7XUEXOHQW)ORZ $QDOVLV0HDVXUHPHQWDQG3UHGLFWLRQ
: LOH\ 1HZ <RUN
+ 6FKODFKWQJHWDO %RXQGDU/D\HU7KHRU 6SULQJHU1HZ <RUN
) 0 : KLWH9LVFRXV) OXLG) ORZ GHG 0F* UDZ +LOO1HZ <RUN
2 5HIQROGV ³$Q ([SHULPHQWDO,QYHVWLJDWLRQ RI WKH &LUFXPVWDQFHV ZKLFK ' HWHUPLQ

: K+WK+U WK+ 0 RWRQ RI : DWU 6KDOO %H ' LUHFWRU 6LQRXV DQG RI WK+ / DZ RI 5HVLWDQFH I
3DUDGHD&KDQQHOV´ 3KLO 7UDQV 5 6RF YRO SS ±
3 * * ' UD]LQ ,QWURGXFWRQWR+\GURG\QDPLF6WDELOLW &DPEULGJH8QLYHUVLW3UHVV 1HZ
<RUN
+ 5RXVH DQG 6 ,QFH+LWRU RI +\GUDXOLFV ,RZD ,QVWLWXWH RI +\GUDXOLF 5HVHDUFK 6WD
8QLYHUVLW RI ,RZD ,RZD&LW
- 1LNXUDGVH ³6WUXFWXUHVHQHLQ5DXKHQ5RKUHQ´9' ,) RUVFKXQJVK (QJLQ
WDQV 1$&$ 7HFK 0HP
/) 0 RRG ³) UHFWRQ) DFWRUVIRU3LSH) ORZ´ $60 (7UDQV YRO SS ±
&) &RHEURRN ³7XUEXOHQW) ORZ LQ3LSHV ZLWK3DUWFXODU5HIHUHQFH WR WKH7UDQVLWL
EHWZHHQWKH6PRRWKDQG5RXJK3LSH/DZV´ - ,QVW&LY (QJ /RQG YRO ± SS
±
2 & -RQHV -U ³$Q,PSURYHPHQWLQWKH&DOFXODWRQ RI 7XUEXOHQW) UHFWRQ LQ5HFWDQJXO
' XFWV´ -) OXLGV(QJ -XQH SS ±
5 %HUNHU +DQGEXFKGHU3K\VLN YRO QR SS ± 6SULQJHU9HUODJ %HUOLQ
5 0 2 OVRQDQG6 - : ULJKW(VVHQWLDOVRI (QJLQHHULQJ) OXLG0 HFKDQLFV WKHG +DUSH
5RZ 1HZ <RUN
5 6FKLHWWHO0 RGHOLQJ DQG6LPXODWRQRI 7XUEXOHQW) ORZV : LOH 1HZ <RUN
3 : 5XQJHU-U HWDO ³' LIXVHU' DWD%RRN´ &UHDWH,QF 7HFK 1RVM +DQRYHU
1+
³) ORZ RI) OXLGV WKURXJK 9DOYHV) LWWLQJV DQG 3LSHV´ 7HFK 3DSHU &UDQH 9DOY
* URXS / RQJ %HDFK &$ QRZ XSGDWHGDVD&' 520 VHH KWWS ZZZ FUDQHYDOYHVFR
!
( ) %UDMU + : . LQJ - (/ LQGHOO DQG & < : HL + DQGERRN RI +\GUDXOLFV WKHG
0 F* UDZ +LOO1HZ <RUN
+ &URVV ³$QDO\VLVRI) ORZ LQ1HWZRUNVRI &RQGXLWVRU&RQGXFWRUV´8QLY ,OO %XOO
1RYHPEHU
3 . 6ZDPHH DQG $. 6KDUPD' HVLJQ RI : DWU 6XSSO 3LSH 1HWZRUNV : LOH
,QWHUVFLHQFH1HZ <RUN
* +HZLWWDQG& 9DVLOLDFRV 3UHGLFWRQ RI 7XUEXOHQW) ORZV &DPEULGJH8QLYHUVLW3UHV
1HZ <RUN
5 : -HSSVRQ $QDO\VLVRI) ORZ LQ3LSH1HWZRUNV %XWWHUZRUWK+HLQHPDQQ : REXUQ 0 $
5 :)R[DQG6 - . OQH ³) ORZ 5HJLPH' DWD DQG' HVLJQ0 HWKRGV IRU&XUYHG 6XEVRO
' LIXVHUV´ - %DVLF(QJ YRO SS ±
5 & %DNHU,QWURGXFWRU * XLGHWR) ORZ 0 HDVXUHPHQW GHG : LOH 1HZ <RUN
5 : 0 LOHU) ORZ 0 HDVXUHPHQW(QJLQHHULQJ +DQGERRN GHGLWRQ 0 F* UDZ +LOO1H
<RUN
% : DUUHQ DQG & : XQVFK HGV (YROXWRQ RI 3K\VLFDO2FHDQRJUDSK\ 0 , 7 3UHV
&DPEULGJH 0 $
8 6 ' HSDUWPHQWRI &RPPHUFH7LGDO &XUUHQW7DEOHV 1DWRQDO 2FHDQRJUDSKLF DQ
$WPRVSKHULF$GPLQLVWUDWLRQ : DVKLQJWRQ ' &
- $ 6KHUFOLII (OHFWURPDJQHWLF) ORZ 0 HDVXUHPHQW&DPEULGJH 8QLYHUVLW3UHV 1H
<RUN
- $ 0 LOOHU ³$ 6LPSOH / LQHDUL]HG +RW LUH $QHPRPHWHU´ -) OXLGV (QJ ' HFHPEH

SS ±

5 - * RQVWUXFWLQ HG) OXLG 0 HFKDQLFV 0 HDVXUHPHQWV G HG + HPLVSKHUH 1HZ <RUN

' (FNDUGW³' HVLJQHG) ORZ ,QVWUXPHQWDWLRQ ZLWKLQ D +LJK 6SHHG &HQWULIXJDO &RPSUHVV
,PSHOOHU -) OXLGV (QJ 6HSWHPEHU SS ±

+ 6 %HDQ HG) OXLG 0 HWHUV 7KHU7KHRU DQG $SSOLFDWLRQ VWHG \$PHULFDQ 6RFLHWRI
0 HFKDQLFDO (QJLQHHUV 1HZ <RUN

³0 HDVXUHPHQWV RI) OXLG) ORZ E\ 0 HDQV RI 2UILFH 3ODWHV 1R]]OHV DQG 9HQWXUL 7XE
,QVWUXPHQWDWLRQ &LUFXODU &RRV 6HFWLRQ &RQGXLW 5XQQLQJ) XOO ,QV 2UJDQ 6WDQG 5HS ' ,6
* HQHYD $SULO

% * HXUW (OHPHQWV RI ' LUHFWDQG / DUJH (GG\ 6LPXODWLRQ 5 7 (GZDUGV) ORXUHZ Q 3$

6 (+DDDQG ³6LPSOH DQG ([SOLFLW) RUPXODV IRU WKH) UFWLRQ) DFWRU LQ 7XUEXOHQW3LS
) ORZ ´ -) OXLGV (QJ 0 DUFK SS ±

5 . 6KDKDQG$ / / RQGRQ/ DPLQDU) ORZ) RUFHG &RQYHFWLRQ LQ' XFWV $FDGHPLF 1HZ
<RUN

3 / 6NRXVHQ 9DOYH+DQGERRN GHG 0 F* UDZ +LOO1HZ <RUN

: /L : ; &KHQ DQG 6 = ; LH ³1 XPHULFDO6LPXODWLRQ RI 6WHDG\,QGXFHG 6HFRQGDU
) ORZV ZLWK +\EULG) LQLWH $QDO\WF 0 HWKRG´ -RXUQDO RI +\GURG\QDPLFV YRO QR
' HFHPEHU SS ±

$6+5$(+DQGERRN²) XQGDPHQWDOV $6+5$($VVRFLDWH * $

) ' XUW $ 0 HDQJ DQG - + : KLWDZ 3UHFLSHV DQG 3UDFWLFH RI / DVHU' RSSOHU
$QHPRPHWHU GHG $FDGHPLF 1HZ <RUN

$ 3 / LWWQ HWDO / DVHU' RSSOHU DQG 3KDVH ' RSSOHU 0 HDVXUHPHQW7HFKQLTXH
6SULQJHU9HUDJ 1HZ <RUN

- ($PDGL (FKHQGX + =KX DQG (+ +JKDP ³$QDO\VLV RI 6LQJDOV IURP 9RUW
) ORZ PHWHUV´) ORZ 0 HDVXUHPHQWDQG,QVWUXPHQWDWLRQ YRO QR 2FW SS ±

* 9DV ³8QVWDWLRQDU) ORZ PHWHU%DVLFV´ 6HQVRUV YRO QR 2FW SS ±

$60 ( ) OXLG 0 HWHUV 5HVHDUFK &RPPLWWH ³7KH,62 $60 (2UILFH &RHILFLHQW(TXDWL
0 HFK (QJ -XO SS ±

5 ' %OHYLQV $SSOLHG) OXLG ' \QDPLFV + DQGERRN 9DQ 1RVWUDQG 5HLQKROG 1HZ <RUN

2 & -RQHV -U DQG - & 0 / HXQJ ³$Q ,PSURYHPHQWLQ WKH &DOLEUDWLRQ RI 7XUEXOH
) UFWLRQLQ6PRRWK&RQGXLW $QQXO´ -) OXLGV (QJ ' HFHPEHU SS ±

3 5 %DQGURSDGK\D ³$\SHFWVRI WKH(TXLOLEULXP 3XIILQ7UDQVLWLRQDO3LSH) ORZ ´ -) OXL
0 HFK YRO SS ±

, (,GHOFKLN + DQGERRN RI +\GUDXOF 5HVLVWDQFH G HG &5& 3UHVV %RFD 5DWRQ) /

6 . OHLQDQG : %HFNP DQ (QJLQHHULQJ (TXDWRQ 6RIWHU ((6 8QLYHUVLW\ RI : LVFRQVL
0 DGLVRQ : ,

5 ' &RIILHOG 3 7 0 F. HRZQ DQG 5 % + DPPRQG ³,UUHYHUVLEOH 3UHVVXUH / R
&RHILFLHQWV IRU 7Z R (OERZV LQ 6HULHV ZLWK 9DULRXV 2UHQWDWLRQ $QJOHV DQG 6HSDUDWR
' LVWDQFHV´ 5HSRUW $3' 7 %HWWV$ VRFLDWH 3RZHU/ DERUDWRU : HVW0 LIIOLQ 3$

+ ,VR ³3UHVVXUH / RVVHV LQ 6PRRWK 3LSH %HQGV´ - RXUQDO RI %DVLF (QJLQHHULQJ 0 DU
SS ±

$ + * LEVRQ ³2 QWH) OWZ RI : DWUKURXJK 3LSHVDQG 3DWDJHV´ 3UHF 5R\ 6RF / RQGRQ 6HU $ YRO SS ±
0 5DIHOHWDO 3DUWFOH ,PDJH 9HORFLPHWU $ 3UDFWLFDO* XLGH G HG 6SULQJHU 1HZ <RUN
3 6PLWK DQG 5 : =DSSH 9DOYH 6HOHFWRQ + DQGERRN WK HG * XO 3XEOLVKLQJ + RXVWRQ 7H[DV
6 $ %HUJHU / 7DOEHUWDQG / 6 <DR ³) OWZ LQ &XUYHG 3LSHV´ $QQXDO 5HYLHZ RI) OXL 0 HFKDQLFV YRO SS ±
3 / 6SHGGLQJ (%HQDUG DQG * 0 0 F1 DOO ³) OXLG) OWZ WKURXJK ƒ %HQGV ' HYHORSPHQWV LQ &KHPLFDO (QJLQHHULQJ DQG 0 LQHUDO 3URFHVVLQJ YRO QRV ± SS ±
5 5 . HUZHOO³5HFHQW 3URJUHVV LQ8QGHUVWDQGLQJ WKH 7UDQVLWLRQ 7XUEXOHQFH D3LSH-1RQOLQHDULW YRO SS 5 ±5
% - 0 F. HRQHWDO ³) UHFWRU) DFWRUV IRU 6PRRWK 3LSH) OWZ´-) OXLG 0 HFK YRO SS ±
0 %DKUDPL 0 0 <RYDQRYLFK DQG - 5 &XOKDP ³3UHVVXUH ' URS RI) XOO ' HYHORSH / DPLQDU) OWZ LQ0 LFURFKDQQHOV R\ $UEHWDU &URVV6HFWLRQ´-) OXLGV(QJLQHHULQJ YRO 6HSW SS ±
* 6 / DUDRQD $ 5LYDV DQG - & 5DPRV ³&RPSXWDWLRQDO0 RGHOLQJ DQG 6LPXODWLRQ R 6LQJOH-HW DWU0 HWU´ -) OXLGV(QJLQHHULQJ YRO 0 D\ SS ±
& 6SLHJHO' HVJQLQJ DQG%XLOGLQJ) XHO&HOOV 0 F* UDZ +LOO1HZ <RUN

– +RZ HYHU GLUHFWQXPHULFDOVLPXODWLRQ ' 1 6 RI OWZ 5H\QROGV QXPEHUXEXOHQFHLVQRZ TXLFRPPRQ> @

– 7KLVH[DPSOHZDVJLYHQE\ 0 RRG\ LQKLV SDSHU>@

– 7KHSDUDPHWU ZDVVXJJHVWHGE\ + 5RXVHLQ

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– -RQHV DQG / HXQJ > @VKRZ WKDWGDWD IRU DQQXDU IOWZ DOR VDWXU\ WKH HIHFWLYHOPLQDU GLPHWULGHD

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

&KDSWHU) OHZ 3DVW/P PHUVHG %RGLHV

7KLV SKRWRJUDSK LV QRW DOWHUHG 7KLV LV 5RFNHW0 DQ <YHV 5RXW IOLQJ ZLWK VHD GHVJQH ZLQJV VWDSSHG WR KLV EDFN SRZHUHG E\ IRXU* HUP DQ MHWP LQL VXUELQHV WKDWWRMO OBI R WKUXW2 QWKLV ZLGHD SXEOF]HG IOJKWLQ0 D\ 5RFNHW0 DQ MXP SHG IURP DQ DLUSODQH D IW FOP EHG WR IW WKHQ FUXLVHG DERXWWKH $GSV DW PLK IRU ILYH PLQXWV +I FHOEUDWG ZLWK IJXUH HJKW DQG D IXOO *URO,Q6HSWMP EHU 5RFNHW0 DQ FURVHG WK (QJOVK &KDQQHORQ D PLQXW IOJKW 5RFNHW0 DQ LV QRWXXSLG +H FDUULHV DQ HP HUJHQF SDUDFKXW EXWKDVXHGLWRQO IRUODQGLQJ *(Photo © AP Photo/Anja Niedringhaus)*

0 RVLYDWRQ

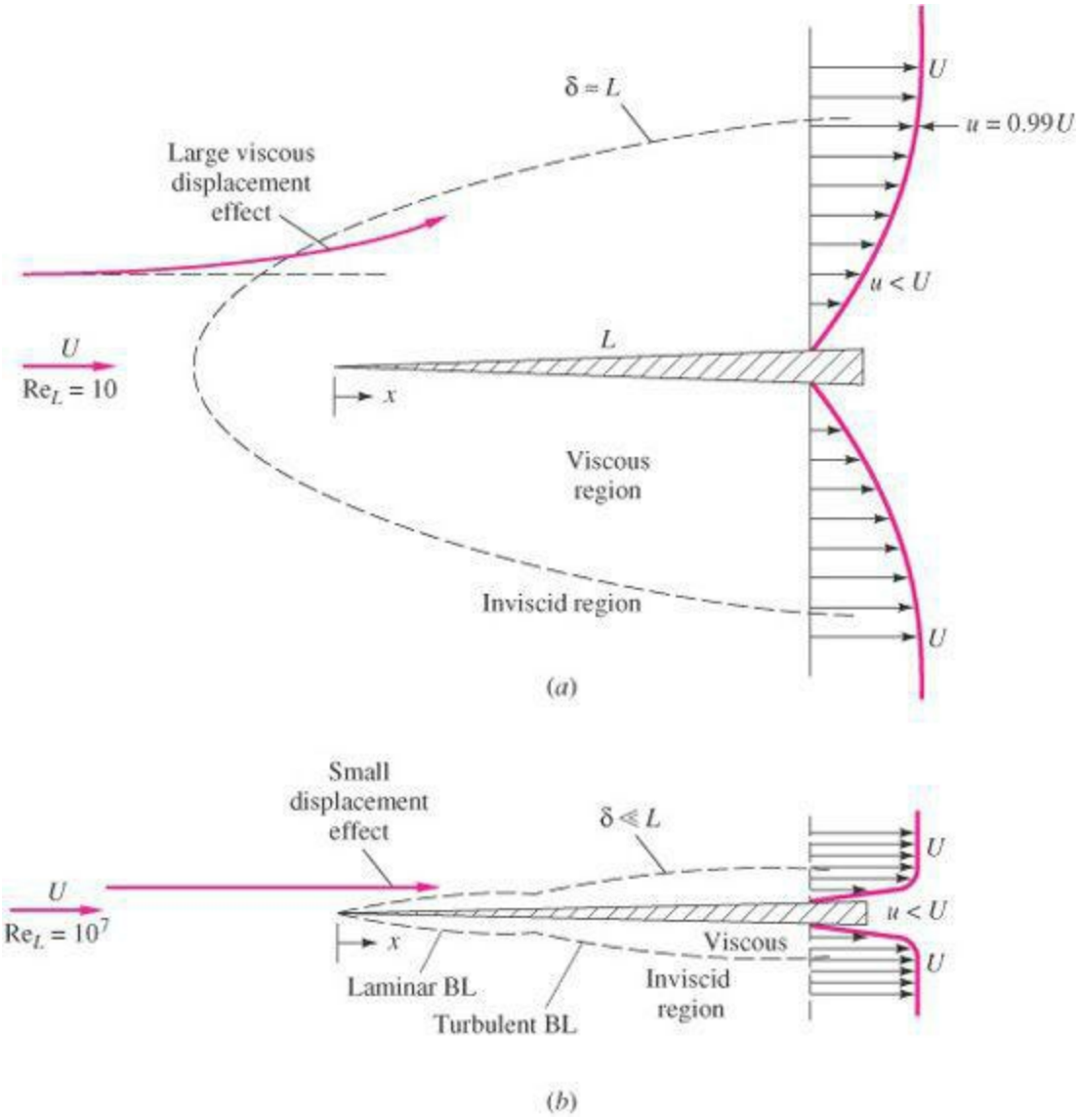
7KLV FKDSWHU LV GHYRWHG WR ³H[WUDÓ IOHZ V DURXQG ERGLHV LP PHUVHG LQ D IXLG VXUHDP 6XFK IOHZ ZLOKDYH YLVFRXV VKHDU DQG QR VOS HIHFWQHDUWKH ERG\ VXUIDFHV DQG LQLWZDNH EXWZL WWSLFDOO EHQHDO LQYLVFLG IDUIURP WKH ERG\ 7KHVH DUH XQFRQLQHG ERXQGDU OD\HU IOHZ V &KDSWHU FRQVLGHUHG ³LQWUQDÓ IOHZ V FRQLQHG E\ WKH ZDOOV R DGXFW,QWKDWFDVH WKH YLVFRX ERXQGDU OD\HU JURZ IURP WKH VLGHZDOOV P HWGRZQWUHDP DQG ILOOWKHQWHUGXFW9LVFRXV WKH GRPLQDQWHIHFW) RUH[DPSOH WKH0 RRG FKDUWR) LJ LV HVHQWLDOO D FRUUHDWLRQ R ZD WKH DUWXUHV IRUORQJ GXFWV R FRQWDQWURVVHFWLRQ

([WUDOIOHZ V DUH XQFRQLQHG IUHH WR H[SDQG QR PDWWU KRZ WKLFN WKH YLVFRXV OD\HU JURZ $OWKRXJK ERXQGDU OD\HUWKHRU 6HF DQG FRPSXWDWLRQDOIOLG G\QDPLFV &)' > @DUH KHSIXOL XQGHUVWDQGLQJ H[WUDOIOHZ V FRP SOH[ERG\ JHRPHWULHV XVXDOO UHTXLUH H[SHULP HQWDO GDWD RQ WK IRUFHV DQG PRP HQWV FDXVHG E\ WKH IOHZ 6XFK LPP HUVHG ERG\ IOHZ V DUH FRP PRQO HQFRXQWHG L HQJLQHHULQJ VWXGLHV DURXG QDP LFV DLUSODQHV URFNHWV SURMHFWLOHV K\GURG\QDP LFV VKLSV

VXEPDULQHV VRUSHGRV WDQVSRUWDWRQ DXWRPRELOHV WUDFNV FDUV ZLQGHQJLQHULQJ EXLOGLQJV EULGJHV ZDWHUZHUV ZLQGVKLHOGV DQGRFHDQHQJLQHULQJ EXR\V EUHDNZDWHUV SLOQJV FDEOHV PRRUHGLQWXPHQW 7KLVFKDSWHUSURYLGHVGDWDDQGDQDOYVLVWRDVVLVWGHV

5HQROGV1XPEHUDQG*HRPHWULF(IIHFWV

7KHWHFKQLTXHRI ERXQGDU\ OD\HU %/ DQDOVLVFDQEHXVHGWRFRPSXWHYLVFRXVHIIHFWVQHDUVROG ZDOOVDQGWR³SDWK WKHVHRQWKHRXWHUOLPLWLQJ PRWLRQ 7KLVSDWKLQJ LVPRUHVXFFHVVIXODVWK ERG\ 5HQROGVQXPEHUEHFRPHVODUJHUDVVKRZQLQ) LJ



) LJ &RPSDULVRQ RI IORZ SDWWDVKDUS IODWSODWH DWORZ DQG KLJK 5HQROGVQXPEHUV *a* ODPLQDU ORZ 5HIORZ *b* KLJK 5HIORZ

,Q) LJ DXQLRUP VWHDP 8 PRYHVSDUDOOHODWDVKDUS IODWSODWHRI OHQJWK/ ,I WKH5HQROGVQXPEHU8/ LVORZ) LJ D WKHYLVFRXVUHJLRQLVYHU EURDGDQG H[WHQGVIDUDKHDGDQGWKVLGHV RI WKH SODWH 7KH SODWH UHDUGV WKH RQFRPLQJ VWHDP JUDGXDOO DQG VPDOOFKDQJHV LQ IORZ SDUDPHWHUV FDXVH OLWWOH FKDQJHV LQ WKH SUHVVXUH GLVWULEXWRQ DORQJ WKH SODWH 7KXV DOKRXJK L SUHQFLSOHLWWKRXGEHSRVLEOHWRSDWK WKHYLVFRXVDQGLQYLVFLG ODYHUVLQDPDWKHPDWDOODQDOVLV WKHLULQWHUDFWRQVV WRQJ DQGQRQOHDU> WR @7KHUHLVQR H[WUQJ VLPSOHWKRU IRUH[WHQGOORZ

)LJ ,OOXVWUDWLRQ RI WKH VWUXFWXUH LQWHUFRQQHFWLRQ EHWZHHQ YLVFRXV DQG LQYLVFLG UHJLRQV LQ WKH UHD
RI EOXQWERG\ IORZ *a* LGHDOL]HG DQG GHILQLWH IDLOH SLFWXUH RI EOXQWERG\ IORZ *b* DFWXD
SLFWXUH RI EOXQWERG\ IORZ

7KH WKHRU\ RI VWUXFWXUH LQWHUFRQQHFWLRQ EHWZHHQ EOXQWERG\ YLVFRXV DQG LQYLVFLG OD\HUV LV QRWZH
GHYHORSHG)RZ V OLNH WKDWRI)LJ E DUH QRUPDOO VXGLHG H[SHULPHQWDOO RU ZLWK &)' > (
5HHUHQFH LVDQH[PSOHRI HIRUWVWRLPSURYHWKHWKHRU RI VHSDUDWHGIORZV 5HHUHQFH LVDQRWKH
WKHUERRNGHYRWHGWRVHSDUDWHGIORZ

(; $0 3/ (

$ ORQJ WKLQIODWSODWHLV SODFHG SDUDOOHOWR D IWV VWUHDP RI ZDWUDW *f*) $VZ KDWGLVDQFH[
IURP WKHOHDGLQJHGJHZ LOOWKHERXQGDU\ OD\HUWKLFNQHVVEH LQ'

6RXWLRQ

- $VXPSWLRQV)ODWSODWHIORZ ZLWK(TV DSSOLQJ LQWKHUDSSURSULDWHUDQJHV
- $SSURDFK * XHVV OPLQDUIORZ ILUVW,I FRQGLFWRU VW WXEXOHQWORZ
- 3URSHUWLYDOXHV)URP 7DEOH$ IRUZDWUDW *f* § (IWV
- 6RXWLRQVWHS : LWK/ LQ IWWW OPLQDUIORZ (T D

$$\frac{\delta}{x}\Big|_{\text{lam}} = \frac{5}{(Ux/\nu)^{1/2}} \quad \text{or} \quad \frac{1/12\ \text{ft}}{x} = \frac{5}{[(20\ \text{ft/s})x/(1.082\text{E-}5\ \text{ft}^2/\text{s})]^{1/2}}$$

Solve for $x \approx 513$ ft

3UHWW GRQ SODWH 7KLV GRHV QRW VRXQG ULJKW &KHFN WKH ORFDO 5H QXPEHU

$$\text{Re}_x = \frac{Ux}{\nu} = \frac{(20\ \text{ft/s})(513\ \text{ft})}{1.082\text{E-}5\ \text{ft}^2/\text{s}} = 9.5\text{E}8 \qquad (!)$$

7KLV LV LP SRVVLEOH VLQFH ODP LQDU ERXQGDU\ OD\HU IORZ RQO SHUVLVWV XS WR DERXW RU Z LW VSHFLDO FDUH WR DYRLG GLVWXUEDQFHV XS WR î

- 6RXWRQ WS 7U WXUEXOHQW (T E

$$\frac{\delta}{x} = \frac{0.16}{(Ux/\nu)^{1/7}} \quad \text{or} \quad \frac{1/12\ \text{ft}}{x} = \frac{0.16}{[(20\ \text{ft/s})x/(1.082\text{E-}5\ \text{ft}^2/\text{s})]^{1/7}}$$

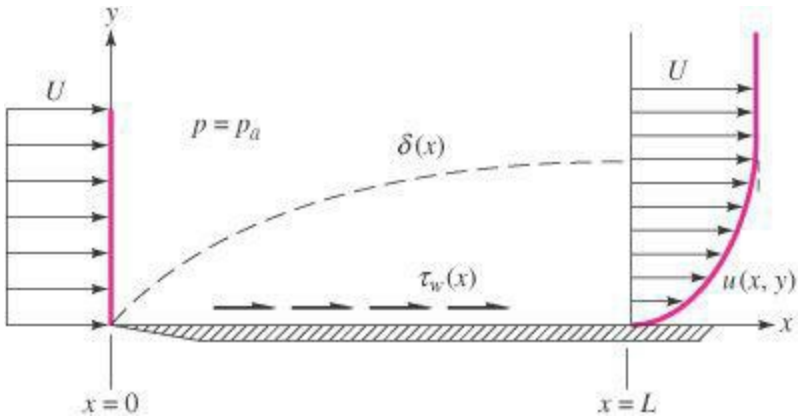
Solve for $x \approx 5.17$ ft *Ans.*

&KHFN 5H IWW IW (IWV (! 2. WXUEXOHQW

- &RPPHQWV 7KH IORZ LV WXUEXOHQW DQG WKH LQLWLDO OD\HU WKLFNHQV P EJXLW RI WKH WRU LV UHVROYHG

0 RP HQWXP ,QWHJUDO(VWLP DWHV

: KHQ Z H GHULYHG WKH PRP HQWXP LQWHJUDO UHDWLRQ (T DQG DSSOLHG LWWR D IODWSODW ERXQGDU\ OD\HULQ ([DPSOH Z H SURP LVHG WR FRQVLGHULWXUWKHULQ &KDS : HOO K-UH Z H DU- / HVXVUHYLHZ WKHSURE OP XVLQJ) LJ



) LJ * URZ WKRI DERXQGDU\ OD\HURQDIODWSODW

$ VKHDUOD\HURI XQLNRZQ WKLFNHVV JURZ VDORQJ WKH VKDUSIODWSODWLQ) LJ 7KHQR VOS Z D FRQGLWLRQ UHDGLO\ VWKHIORZ PDNLQJ LWLQWR D URXQGHG SURILHX [\ Z KLFK PHUJHV LQR WKH H[WHUQDO YHORFLWV8 FRQWLQXDWLD³ WKLFNHVV / [%\ XLOG]LQJ WKHFRQWROYROXPHR) LJ Z HIRXQ Z LWKRXWP DNLQJ DQ DVVXP SWLRQV DERXWDP LQDUYHUVXV WXUEXOHQW LQ([DPSOH WKDWWKHGUD

IRUFHRQVWHSODWHVJLYHQE\ VWHIROORZLQJ PRPHQXP LQWHJUDOFRVVWHH[SODQH

$$D(x) = \rho b \int_0^{\delta(x)} u(U - u)\, dy \qquad (7.2)$$

ZKHUHE LVWHSODWHZLGWKLQRRVWHSDSHUDQGVWHLQWHJUDWRQLVFDUULHGRXWDORQJ DYHUWFDOSODQH[FRQWDQW< RXVKRXOGUHYLHZ WHPRPHQXP LQWHJUDOUHODWRQ DQGLWXVHLQ([DPSOH

. iUPiQ¶V$QDOVLVRI WKH) ODW3ODWH

(TXDWRQ ZDVGHULYHGLQ E\ . iUPiQ> @ZKR ZURWHLWLQWKHFRQYHQLHQWRUP RI WK PRPHQXP WKLFNQHVV

$$D(x) = \rho b U^2 \theta \qquad \theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad (7.3)$$

0 RPHQXP WKLFNQHVV LV WKXV D PHDVXUH RI VRLDOSODWHGUDJ . iUPiQ WKHQ RQVLGHUHG DQ HTXDOLWKHLQWHJUDWHGZDOKDUWWHVDORQJ WKHSODWH

$$D(x) = b \int_0^x \tau_w(x)\, dx$$

or

$$\frac{dD}{dx} = b\tau_w \qquad (7.4)$$

0 HDQZKLOH WKHGHULYDWLYHRI(T ZLWK8 FRQVDQWLV

$$\frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

%\ FRPSDULQJ WKLV ZLWK(T . iUPiQDUULYHGDWZKDWLV QRZ FDOHG WKHPRPHQXP LQWHJUDUHODWRQIRUIODWSODWHERXQGDU\ OD\HUIORZ

$$\tau_w = \rho U^2 \frac{d\theta}{dx} \qquad (7.5)$$

,WLVYDOGIRUHLWKHUODPLQDURUWXUEXOHQWOD\SODWHIORZ

7R JHWD QXPHULFDOUHVXOWRUODPLQDUIORZ . iUPiQDVVXPHGWKDWWKH YHORFLW SURILOHVKDG D DSSUR[LPDWH SDUDEROLF VKDSH

$$u(x, y) \approx U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \qquad 0 \le y \le \delta(x) \qquad (7.6)$$

ZKLFKPDNHVLWSRVVLEOHWRHVWLPDWHERWKPRPHQXP WKLFNQHVVDQGZDOOVKHDU

$$\theta = \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy \approx \frac{2}{15}\delta$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \approx \frac{2\mu U}{\delta} \tag{7.7}$$

%\ VXEVWLWXWLQJ LQWR DQGUHDUUDQJLQJ ZHREWDLQ

$$\delta\, d\delta \approx 15\,\frac{\nu}{U}\,dx \tag{7.8}$$

ZKHUH ! : HFDQLQWHJUDWHIURP VR[DVXPLQJWKDW DW[WKHGHSWKLQFUHDVH

$$\frac{1}{2}\delta^2 = \frac{15\nu x}{U}$$

or

$$\frac{\delta}{x} \approx 5.5\left(\frac{\nu}{Ux}\right)^{1/2} = \frac{5.5}{\mathrm{Re}_x^{1/2}} \tag{7.9}$$

7KLV LV WKH GHVLUHG WKLFNQHVV HXLPDWH ,WLV DOD DSSUR[LPDWH RI FRXUVH SDUWRI . iUPiQ PRPHQWXP LQWHJUDOWKHRU\ > @EXWLWLV VWUDLJKWIRUZDUG DFFXUDWH EHLQJ RQO SHUFHQWKLJKHUWKDQ WK NQRZ QDFFHSWHG VROXWRQIRUODPLQDUIODWSODWHIORZ ZKLFK ZHJDYHDV/(T D

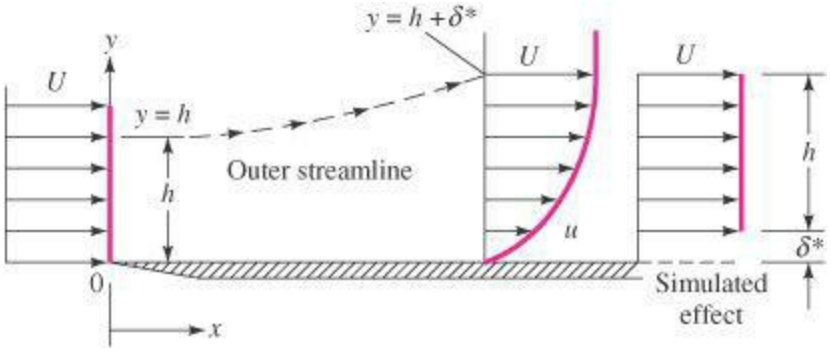
%\ FRPELQLQJ (TV DQG ZHDOR REWLQDVKHDUVWUHVVHXLPDWDORQJ WKHSODWH

$$c_f = \frac{2\tau_w}{\rho U^2} \approx \left(\frac{\frac{8}{15}}{\mathrm{Re}_x}\right)^{1/2} = \frac{0.73}{\mathrm{Re}_x^{1/2}} \tag{7.10}$$

$ JDLQ WKLV HXLPDWH LQ VSLWH RI WKH FUXGHQHVV RI WKH SURILOHDVXPSWRQ >(T @LV RQO SHUFHQWKLJKHUWKDQ WKH NQRZ QH[DFWODP LQDUSODWHIORZ VROXWRQ $\bar{c}_f = 0.664/\mathrm{Re}_x^{1/2}$, WHDVG LQ6HF 7KHGLPHQVLRQOHVVTXDQWLWF FDOOHG WKHVNLQIULFWLRQFRHIILFLHQWLVDQDORJRXVWRWKHIULFWLRQIDFWRU LQGXFW

$ ERXQGDU\OD\HUFDQEHMXGJHGDV³WKLQ LI VD\ WKHUDWR/ [LVOHVVWKDQDERXW 7KLVRFFXU D\$\delta/x = 0.1 = 5.0/\mathrm{Re}_x^{1/2}$ RUDW5H[ ) RU5H[OHVVWKDQ ZHFDQHXLPDWHWKDWERXQGDU\OD\H WKHRU IDLOVEHFDXVHWKHWKLFNQHVVHVWLPDWHGLVDVLJQLILFDQWIHFWRQWKHRXWULQYLVFLGIORZ 7KHXSSHUOLP RQ5H[IRUODPLQDUIORZ LVDERXW î ZKHUHP HDVXUHP HQWRQDVPRRWKIODWSODW> @WKRZ WKDWWKH IORZ XQGHUJRHVWUDQVLWLRQWRDWXUEXOHQWERXQGDU\OD\HU)URP î XSZDUGWKHWXUEXOHQW5H QRGH QXPEHUPD\ EHDUELWUDULO\ODUJH DQGDSUDFWLFDOOLPLWDSSUHFLDWLRQ LVREWDLQHG

' LVSODFHP HQW7KLFNQHVV



) LJ ' LVSODFHPHQWWKLFNHVV RI DERXQGDU\ OD\HU

$ QRWKHU LQWHUHVWLQJ HIIHFW RI DERXQGDU\ OD\HU LV LWV VPDOO EXW LQWHJUDO GLVSODFHPHQW WKLFNQHVV \$ VVKRZQ LQ) LJ RXWHU VWUHDPOLQHV PXVW GHIOHFW RXWZDUG DGLVWDQFH/ [WR YDLW\ FRQVHUYDWLRQ RI PDVV EHWZHHQ WKH LQIOHWDQG RXWOHW

$$\int_0^h \rho U b\, dy = \int_0^{\delta} \rho u b\, dy \qquad \delta = h + \delta^* \tag{7.11}$$

7KH TXDQWLW\/ LV FDOOHG WKH GLVSODFHPHQW WKLFNQHVV RI WKH ERXQGDU\ OD\HU 7R UHDOLVH WKLV \ FDQ HC DQG E IURP (T HYDOXDWH WKH OHIW LQWHJUDO DQG VOO DGG DQG VXEWUDFW8 IURP WKH ULJK LQWHJUDQG

$$Uh = \int_0^{\delta} (U + u - U)\, dy = U(h + \delta^*) + \int_0^{\delta} (u - U)\, dy$$

or

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy \tag{7.12}$$

7KXV WKH UDWLR RI / / YDULHV RQO Z LWK WKH GLPHQVLRQOHVV YHORFLW SURILOH VKDSH X8 ,QWURGXFLQJ RXU SURILOH DSSUR[LPDWLRQ LQWR Z H REWLQ E\ LQWHJUDWLQJ WKL DSSUR[LPDWHUHVXOW

$$\delta^* \approx \frac{1}{3}\delta \qquad \frac{\delta^*}{x} \approx \frac{1.83}{\mathrm{Re}_x^{1/2}} \tag{7.13}$$

7KHVHHVWLPDWHV DUH RQO SHUFHQW Z D\ IURP WKH H[DFW VROXWLRQ/ IRU ODPLQDU IODW SODWH JLYH LQ6HF 7.4: $\delta^* = 0.344\delta = 1.721x/\mathrm{Re}_x^{1/2}$ 6LQFH/ LV PXFK VPDOOHU WKDQ[IRU ODJH 5H DQG WKH RXWVWUHDPOLQHV DSSH9 8 LV SURSRUWLRQDO WR/ Z HFRQFOXGH WKDW WKH YHORFLW QRUPDO WR WKH Z DOO LV PXFK VPDOOHU WKDQ WKH YHORFLW SDUDOOHO WR WKH Z DOO 7KLV LV DNH\ DVVXPSWLRQ LQ ERXQGDU\ OD\HU WKHRU 6HF

: HDOR FRQFOXGH IURP WKH VXFFHVVIXO WKHVHLPSOHSDUDERODF HVWLPDWHV iUPiQ]V PRPHQWXP LQWHJUDO WKHRU LV HIIHFWLYH DQG XVHIXO0 DQ GHWDLOV RI WKLV WKHRU DUH JLYHQ LQ5HV VR

(; \$0 3/ (

$ UHOD WLYHO VPDOO VFDOH DLUFUDIW ZLQJ DW UHODWLYHO\ ORZ VSHHG VXFK DV D OLJKW &RQVLGHU IORZ DW 8 IW SHU VHFRQG DW VHD OHYHO IW&RPSXWH WKH ERXQGDU\ OD\HU WKLFNQHVV DW WKH WUDLOLQJ HGJH IRU D DLUFUDIW E ZLWK XU *LYHQ

6RXWLRQ

3DUW D

) URP 7DEOH $ $LU § (IWV 7KHUHIRUH WKH5H\QROGV QXPEHU WKXV

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{(1\ \text{ft/s})(1\ \text{ft})}{1.61\ \text{E-4 ft}^2/\text{s}} = 6200$$

6LQFH WKLV LV OHVV WKDQ WKH ICRZ LV SUHVXPHG ODPLQDU DQG VLQFH LWV JUHDWHU WKDQ WKH ERXQGDU\ OD\HUV UHDVRQDEO\ WKLQ) URP (T D WKH SUHGLFWHG ODPLQDU WKLFNQHVV

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{6200}} = 0.0634$$

or, at $x = 1$ ft, $\delta = 0.0634$ ft ≈ 0.76 in *Ans.* (*a*)

3DUW E

) URP 7DEOH $ $LU § (IWV 7KHUHIRUH5H\QROGV QXPEHUV

$$\mathrm{Re}_L = \frac{(1\ \text{ft/s})(1\ \text{ft})}{1.08\ \text{E-5 ft}^2/\text{s}} \approx 92{,}600$$

7KLV DJDLQ VDWLVILHV WKH ODPLQDU DQG WKLQQHVV FRQGLWLRQV 7KH ERXQGDU\ OD\HU WKLFNQHVV

$$\frac{\delta}{x} \approx \frac{5.0}{\sqrt{92{,}600}} = 0.0164$$

or, at $x = 1$ ft, $\delta = 0.0164$ ft ≈ 0.20 in *Ans.* (*b*)

7KXV HYHQ DW[XFK ICRZ YHORFLWLHV DQG VKRUWHQJWK ERWK DLUICRZ V DQG Z DWUICRZ V VDWLV\ WK ERXQGDU\ OD\HUDSSUR[LPDWLRQV

7KH%RXQGDU\ / D\HU(TXDWLRQV

,Q&KDSV DQG ZHOHDUQHGWKDWWKHUHDUHVHYHUDOGLIIHUHQWDSSURDFKHVWRVROYLQJ
> WR @1RQHDUHIRUH[WUDODIGRZ DURXQGLPPHUVHGERGLHV DOWKRXJK WKLVLVRQHRI WKHSULPDU
DSSOLFDWLRQVRI IOXLG PHFKDQLFV 1RH[DFWVROXWLRQVDUHNQRZQIRUWXUEXOHQWGRZ ZKRVHDQDOVL
WVSLFDO XVHVHPSLULFDOPRGHOLQJ GZVWRUHDWWLPHPHDQYDULDEOHV
7KHUHDUHSUHVHQW WKUHH WHFKQLTXHVXVHGWR VWXG\ H[WHUQDOIGRZV QXPHULFDO FRPSXWH
VROXWLRQV H[SHULPHQWDWLRQ DQG ERXQGDU\OD\HUWKHRU\
&RPSXWDWLRQDOIOXLGG\QDPLFVLVQRZ ZHOOGHYHORSHGDQGGHVFULEHGLQDGYDQFHGWH[WVXFK D
WKDWE\ 7X HWDO > @7KRXVDQGVRI FRPSXWHUVROXWLRQV DQG PRGHOVKDYH EHHQSXEOLVKHG H[HFXWR
WLPHV PHWK VL]HV DQG JUDSKLFDOSUHVHQWDWLRQVDUH LPSRVLQJ HDFK\HDU %RWKOLPLWDWLRQVDQG VWUHQ
IORZ VROXWLRQVKDYHEHHQSXEOLVKHG DQG VWUHQJWKHQHG DQG VXSSOHPHQWHG E\ H[SHULPHQWDO GDWD IR
IRUDEUH GLVFXVVLRQRI FRPSXWDUDQDOVLVLQ&KDS WKHVRSLF RI &)' LVEHIRQGRXUVFRSHKHUH
([SHULPHQWDWLRQLVWKHPRVWFRPPRQPHWKRG RI VXG\LQJ H[WHUQDOIGRZV &KDSWHU RXWOLQHG WK
WHFKQLTXH RI GLPHQVLRQDODQDOVLV DQG ZH VKDOOJLYH PDQ QRQGLPHQVLRQDOH[SHULPHQWDOGDWD IR
H[WHUQDOIGRZVLQ6HF
7KHWKLUG WRROLV ERXQGDU\ OD\HUWKHRU\ ILUVWVROXWLRQ E\ /XGZLJ 3UDQGWOQ : HVKDO
IROORZ 3UDQGWOVLGHDVKHDQG PDNHFHUWDLQRUGHUR PDJQLWXGHDVXPSWLRQV WK
1DYLHU6WRNHVHTXDWLRQV LQWR ERXQGDU\ OD\HUHTXDWLRQV WKDWDUH VROYHG UHDGLO\ HDVLO DQ
SDWKHGRQWR WKHRXWHULQYLVFLGIORZ ILHOG
2QH RI WKH JUHDWFKLHYHPHQWV RI ERXQGDU\ OD\HUWKHRU\ LV LWV DELOLW\ WR SUHGLFWWKH IORZ
VHSDUDWLRQ WKDWRFFXUV LQ DGYHUVH SUHVVXUH JUDGLHQW DV LOOXVWUDWHG LQ) LJ E %HUJ
ZKHQ3UDQGWOSXEOLVKHGKLVSLRQHHULQJ SDSHUQRRQHUHDOL]HGWKDWWXFK WKLQVKDUOD\HUVFRXO
FDXVH VXFK D JURVV HIHFWDV IORZ VHSDUDWLRQ (YHQ WKRGD\ KRZ HYHU ERXQGDU\ OD\HUWKHRU\ FDQQR
DFFXUDWHO SUHGLFWWKH EHKDYLRUR WKH VHSDUDWHG IORZ UHJLRQDQG LWV LQIXHQFH ZLWK WKH RXWHU IORZ
0RGHUQUHVHDUFK> @KDVIRFXVHGRQGHWDLOHG&)' VLPXODWLRQVRI VHSDUDWHG IORZ DQG WKHUHVXOWL
ZDNHV WRJDLQIXUWKHULQVLJKW

' HULYDWLRQIRU7ZR' LPHQVLRQDO) ORZ

: HFRQVLGHURQH VWHS GLPHQVLRQDOLQFRPSUHVVLEOHYLVFRXV IORZ ZLWK WKH[GLUHFWLRQDORQ
WKH ZDOODQG< QRUPDOWR WKH ZDOO DV LQ) LJ : HQHJOHFWJUDYLW ZKLFK LV LPSRUWDQWRQ LQ
ERXQGDU\ OD\HUV ZKHUH IOXLG EXR\DQF\ LV GRPLQDQW> 6HF @) URP &KDS WKH FRPSOHW
HTXDWLRQVRI PRWLRQFRQVLVWRI FRQWLQXLW DQG WKH[DQG< PRPHQWXP UHDWLRQV

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (7.14a)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (7.14b)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (7.14c)$$

7KHVH VKRXOG EH VROYHG IRUX % DQG S VXEMHFWWR WKSLFDO QR VOS LQDHW DQG H[LWERXQGDU
FRQGLWLRQV EXWLQIDFWWKH DUHWRR GLIILFXOWWR KDQGOHIRUPRVW[WKLQDO IORZ VH[FHSWZLWK&)'

,Q 3UDQGWOFRUHFWO GHGXFHG WKDWDVKHDUOD\HUPXWEHYHU WKLQL WKH5HQROGVQXPEHUL
ODUJH VR WKDWWKHIROORZLQJ DSSUR[LPDWLRQVDSSO

Velocities: $$v \ll u \tag{7.15a}$$

Rates of change: $$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y} \qquad \frac{\partial v}{\partial x} \ll \frac{\partial v}{\partial y} \tag{7.15b}$$

Reynolds number: $$\mathrm{Re}_x = \frac{Ux}{\nu} \gg 1 \tag{7.15c}$$

2 XUGLVFXVVLRQ RI GLVSODFHPHQWWKLFNQHVV LQ WKH SUHYLRXV VHFWLRQ Z DV LQIOXHQFHG WR MXVWLI\ WKHV
DVVXPSWLRQV

\$ SSO\LQJ WKHVH DSSUR[LPDWLRQVWR(T F UHVXOWLQ DSRZ HUXOLPSOLFDWLRQ

$$\underbrace{\rho\left(u\frac{\partial v}{\partial x}\right)}_{\text{small}} + \underbrace{\rho\left(v\frac{\partial v}{\partial y}\right)}_{\text{small}} = -\frac{\partial p}{\partial y} + \underbrace{\mu\left(\frac{\partial^2 v}{\partial x^2}\right)}_{\text{very small}} + \underbrace{\mu\left(\frac{\partial^2 v}{\partial y^2}\right)}_{\text{small}}$$

$$\frac{\partial p}{\partial y} \approx 0 \quad \text{or} \quad p \approx p(x) \quad \text{only} \tag{7.16}$$

,QRWKHUZRUGV WKH\ PRPHQWXP HTXDWLRQFDQEHQHJOHFWHGHQWLUHO DQG WKHSUHVVXUHYDULHVRQO
DORQJ WKHERXQGDU\ OD\HU QRWWKURXJK LW7KHSUHVVXUHJUDGLHQWWULP LQ(T E LVDVXPHGWR EH
NQRZ QLQDGYDQFHIURP %HUQRXOOL¶VHTXDWLRQDSSOLHGWRWKHIUHHVWUHDPIORZ

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = -\rho U \frac{dU}{dx} \tag{7.17}$$

3UHVXPDEO\ Z HKDYHDOUHDG\ PDGHWKHLQYLVFLGDQDOVLVDQG NQRZ WKHGLVWULEXWLRQRI 8 [DORQ
WKHZDOO &KDS

0 HDQZKLOH RQHWULP LQ(T E LVQHJOLJLEOHGXHWR(TV

$$\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2} \tag{7.18}$$

+RZ HYHU QHLWKHUWULP LQWKHFRQWLQXLW\UHODWLRQ D FDQEHQHJOHFWHG? DQRWKHUZDUQLQJ WKDW
FRQWLQXLW\LVDOZD\VDVDYLWDOSDUWRI DQ\ IOXLGIORZ DQDO\VLV

7KH QHWUHVXOWLV WKDWWKH WKUHH IXOOHTXDWLRQV RI PRWLRQ DUH UHGXFHG WR 3UDQGWO¶V ZHI
ERXQGDU\ OD\HUHTXDWLRQV IRUWZR GLPHQVLRQDOLQFRPSUHVVLEOHIORZ

Continuity:
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (7.19a)$$

Momentum along wall:
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \approx U\frac{dU}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \qquad (7.19b)$$

where
$$\tau = \begin{cases} \mu\dfrac{\partial u}{\partial y} & \text{laminar flow} \\ \mu\dfrac{\partial u}{\partial y} - \overline{\rho u' v'} & \text{turbulent flow} \end{cases}$$

7KHVHDUHVRPHWLPHVUHIHUUHGIRUX[\ DQG%[\ ZLWK8 [DVXPHGWREHDNQRZQIXQFWLRQIURP WK RXWHUIULFWLRQOLFGIORZ DQDOVLV 7KHUHDUHWZRERXQGDU\FRQGLWLRQVRQX DQGRQHRQ%

At $y = 0$ (wall): $u = v = 0$ (no slip) (7.20a)

As $y = \delta(x)$ (outer stream): $u = U(x)$ (patching) (7.20b)

8QDNHWKH1DYLHU6WRNHVHTXDWLRQV ZKLFKDUHPDWKHPDWLFDOO HOOLSWLF DQG PXVWEHVROYH VLPXOWDQHRXVO RYHUWKHHQWLUHIORZ ILHOG WKHERXQGDU\ DUHPDWKHPDWLFDO SDUDEROLF DQG DUHVROYHG E\ EHJLQQLQJ DWWKHOHDGLQJ HGJH DQG PDUFKLQJ GRZQWUHDP DV IDUDV\R QDNH VWRSSLQJ DWWKHVHSDUDWLRQSRLQWRUHDUHUL\RXSHU

7KHERXQGDU\ OD\HUHTXDWLRQVKDYHEHHQVROYHG IRUVFRUHVRI LQWHUHVWLQJ FDVHVRI LQWHUQDOO H[WHUQDOIORZ IRUERWKODPLQDUDQGWXUEXOHQWIORZ XWLOL]LQJ WKHLQYLVFLGGLVWULEXWLRQ8 [DSSURSULDWH VROHFKIORZ) XOOGHWDLOVRI ERXQGDU\ OD\HUWKHRU DQGUHVXOWDQGFRPSDULVRQZLWKH[SHULPHQWDU JLYHQLQ5HV VR +HUHZHVKDOOFRQLQHRXUHOYHVSULPDULO WRIODWSODWHVROXWLRQV 6HF

7KH) ODV\3ODVWM%RXQGDU\ / D\HU

7KHFODVVLF DQG PRVWWRXVHG VROXWLRQRI ERXQGDU\ OD\HUWKHRU LVIRUIODWSODWHIORZ DVLQ) LJ ZKLFKFDQUHSUHVHQWWKHKHUDOPLQDURUXXLEXOHQWORZ

/ DPLQDU) ORZ

)RUODPLQDUIORZ SDVWWKHSODWH WKHERXQGDU\ OD\HUHTXDWLRQV FDQEHVROYHG H[DFWO IRU DQG% DVXPLQJ WKDWWKHIUHHVWUHDP YHORFLW8 LVFRQVWDQWG8 G[7KHVROXWLRQ ZDVJLYHQE 3UDQGWOV VWXGHQW%ODVLXV LQ KLV GLVVHUWDWLRQ IURP * | WWQJHQ : LWK DQ LQJHQLRXV FRRUGLQDW WUDQVRUP DWLRQ %ODVLXV VKRZHG WKDWWKHGLPHQVLRQOHVV YHORFLW SURILOHX 8 LV D IXQFWLRQ RQO RI WK VLQJOHFRPSRVLWHGLPHQVLRQOHVV YDULDEOH \ >8 [@

$$\frac{u}{U} = f'(\eta) \quad \eta = y\left(\frac{U}{\nu x}\right)^{1/2} \qquad (7.21)$$

ZKHUHWKHSULPHGHQRWHVGLIIHUHQWLDWLRQZLWKUHVSHFWWR 6XEVWLWXWLRQRI LQWRWKHERXQGDU\ OD\HUHTXDWLRQV UHGXFHVWKHSUREOHP DWWKLVPXFK PDJHEUD WR D VLQJOH WKLUG RUGHU RQOQRQ RUGLQDU GLIIHUHQWLDOHTXDWLRQIRUI > ± @

$$f''' + \tfrac{1}{2} f f'' = 0 \qquad (7.22)$$

7KHERXQGDU\ FRQGLWLRQV EHFRPH

$$\text{At } y = 0: \qquad f(0) = f'(0) = 0 \qquad (7.23a)$$

$$\text{As } y \to \infty: \qquad f'(\infty) \to 1.0 \qquad (7.23b)$$

7KLV LV WKH %ODVLXV HTXDWLRQ IRU ZKLFK DFFXUDWH VROXWLRQV KDYH EHHQ REWDLQHG RQO E\ QXPHULFD LQWHJUDWLRQ 6RPH WDEXODWHG YDOXHV RI WKH YHORFLW SURILOH VKDSH• X 8 DUH JLYHQ LQ7DEOH 6LQFH X 8 DSSURDFKHV RQO DV\ \ ' LWLV FXVWRPDU WR VHOHFW WKH ERXQGDU OD\HU WKLFNQHVV DV WKDW SRLQW ZKHUH X 8) URP WKH WDEOH WKLV RFFXUV DW §

$$\delta_{99\%} \left(\frac{U}{\nu x} \right)^{1/2} \approx 5.0$$

or

$$\boxed{\frac{\delta}{x} \approx \frac{5.0}{\mathrm{Re}_x^{1/2}} \qquad \text{Blasius (1908)}} \qquad (7.24)$$

: LWK WKH SURILOH NQRZ Q %ODVLXV RI FRXUVH FRXOG DOVR FRP SXWH WKH Z DOO VKHDU DQG GLVSODFHPHQ WKLFNQHVV

$$\boxed{c_f = \frac{0.664}{\mathrm{Re}_x^{1/2}} \qquad \frac{\delta^*}{x} = \frac{1.721}{\mathrm{Re}_x^{1/2}}} \qquad (7.25)$$

1 RWH KRZ FORVH WKHVH DUH WR RXU ULQWUDOHVWLP DWV (TV DQG : KHQF L FRQYHUWHG WR GLP HQVLRQDO IRUP Z HKDYH

$$\tau_w(x) = \frac{0.332 \rho^{1/2} \mu^{1/2} U^{1.5}}{x^{1/2}}$$

7KH Z DOO VKHDU GURSV RI Z LWK[EHFDXVH RI ERXQGDU OD\HU JURZ WK DQG YDULHV DV Y HORFLW WR WK SRZ HU 7KLV LV LQFRQVLVWHQW ZLWK OP LQDUSLSHIORZ Z KHUH2 ⁄ 8 DQG LV LQGHSHQGHQW [

7DEOH 7KH %ODVLXV 9HORFLW 3URILOH> WR @

$y[U/(\nu x)]^{1/2}$	u/U	$y[U/(\nu x)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

,I 2Z [LVVXEVWLWXWHGLQR(T ZHFRPSXWHWKHWRWDOGUDJ IRUFH

$$D(x) = b\int_0^x \tau_w(x)\,dx = 0.664b\rho^{1/2}\mu^{1/2}U^{1.5}x^{1/2} \qquad (7.26)$$

7KH GUDJ LQFUHDVHV RQO DV WKH VTXDUH URRWRI WKH SODWH OHQJWK 7KH QRQGLPHQVLRQDOGUD FRHILFLHQWLVGHILQHGDV

$$C_D = \frac{2D(L)}{\rho U^2 bL} = \frac{1.328}{\mathrm{Re}_L^{1/2}} = 2c_f(L) \qquad (7.27)$$

7KXV IRUODPLQDUSODWH IORZ & HTXDOV WZLFH WKH YDOXHRI WKH VNLQIULFWLRQ FRHILFLHQWDWWK WUDLOLQJ HGJH 7KLVLVWKHGUDJRQRQHVLGHRI WKHSODWH

. iUPiQSRLQWVIURPWKHGUDJFRXOGDOVREHFRPSXWHGIURP WKHPRPHQWXP UHDWLRQ ,GLPHQVLRQOHVVIRUP (T EHFRPHV

$$C_D = \frac{2}{L}\int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad (7.28)$$

7KLVFDQEHUHZULWWQLQWHUPVRI WKHPRPHQXP WKLFNQHVVDWWKHWUDLOLQJ HGJH

$$C_D = \frac{2\theta(L)}{L} \qquad (7.29)$$

&RPSXWDWLRQRI IURP WKHSURILOHX 8 RUIURP & JLYHV

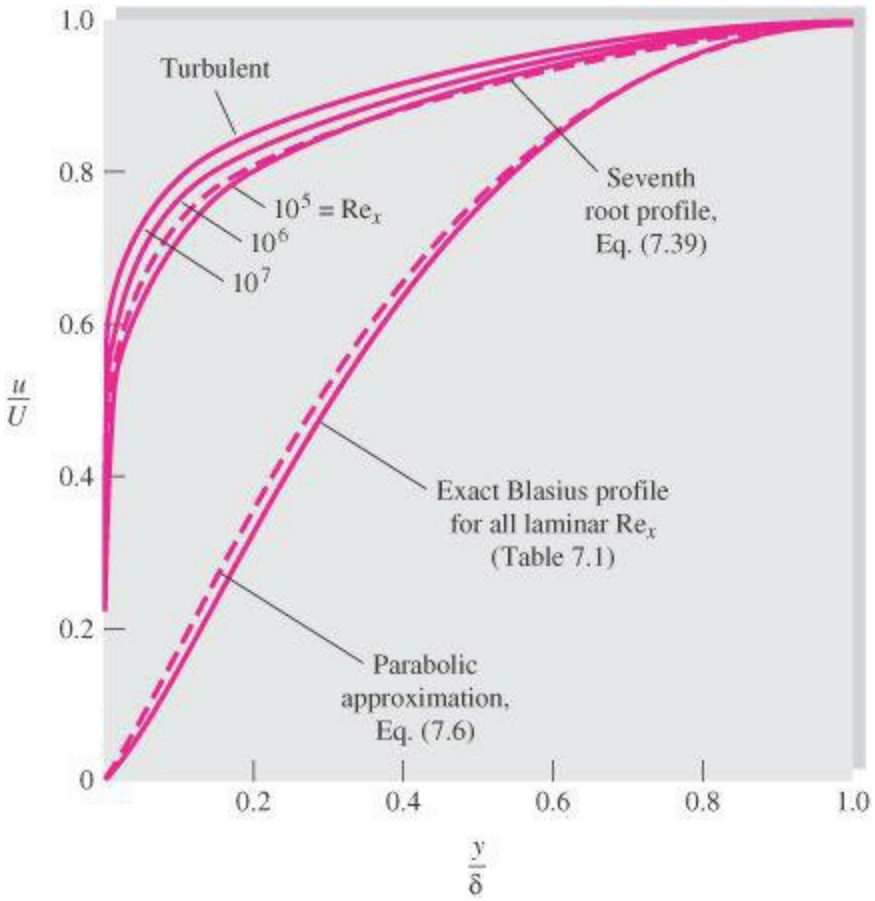
$$\frac{\theta}{x} = \frac{0.664}{\mathrm{Re}_x^{1/2}} \qquad \text{laminar flat plate} \qquad (7.30)$$

6LQFH/ LVVRLOOGHILQHG WKHPRPHQXP WKLFNQHVV EHLQJ GHILQHGWR FRUHODWHGDWGDWD VDNHQ IRUD YDULHWV RI ERXQGDU\ OD HUV XQGHU GLIHUHQ FRQGLWLRQV 7KH UDWR RI GLVSODFHPHQWWR PRPHQXP WKLFNQHVV FDOOHG WKH GLPHQVLRQOHVVSURILOHVKDSH IDFWRU LV DOVR XVHXO LQ LQWHUSUHWLQJ WKHRUHV) RUODPLQDUIODWSODWH IORZ

$$H = \frac{\delta^*}{\theta} = \frac{1.721}{0.664} = 2.59 \qquad (7.31)$$

$ ODUJH VKDSH IDFWRU WKHQLP SOLHV WKDW WERXQGDU\ OD\HUV HSDUDWLRQ LV DERXW RFFXU

,I ZH SORW WKH %ODVLXV YHORFLW SURILOH IURP 7DEOH LQWHIRUP RI X 8 YHUVXV \ / ZH FDQ VH ZK\ WKH VLP SOH LQWJUDO WKHRU JXHVV (T ZDV VXFK D JUHDW WXFFHVV 7KLV LV GRQH LQ) LJ 7KH VLP SOH SDUDERODF DSSUR[LP DWLRQ LV QRW VDWLVIDFWRU IURP WKH WXH %ODVLXV SURILOH KHQFH LW P RP HQWXP VKLFNQHVV ZLWKLQ SHUFHQW WKH WXH YDOXH $OR VKRZ QLQ) LJ DUH WKH WZ SLFDO XEOHQW OD SODW YHORFLW SURILOHV 1RWH KRZ VXUNLQJ GLIIHUHQWLQ VKDSH WKH\ DUH IURP WKH OP LQDU SURILOH ,QWHGG RI GHFUHDVLQJ SDUDERODFDOO WR]HUR WKH VXUEOHQW SURILOH DUH YHU IODWDQG WKQ GURS RI VKDUSO DWWKH ZDOO \V RX PLJKWJXHV WKH IRORZ WKH ORJDULWP LF OD VKDSH DQG WKXV FDQ EH DQDO]HG E\ PRP HQWXP LQWHJUDO WKHRU LI WKLV VKDSH LV SURPHU UHPUHVHQWG



) LJ &RP SDULVRQ RI GLP HQVLRQOHVV OP LQDU DQG WXUEXOHQW OD\SODW YHORFLW SURILOHV

7UDQVLWLRQ WR 7XUEXOHQFH

7KH OP LQDU IODW SODW ERXQGDU\ OD\HU HY HQWXDOO EHFRP HV WXUEXOHQW EXW WKH UHLV QR XQLTXH YDOX IRU WKLV FKDQJH WR RFFXU : LWK FDUH LQ SRO\KQJ WKH Z DOO DQG NHSLQJ WKH IUHH WXUEDP TXLHWRQH FD GHD\ WKH WUDQVLWLRQ 5H QROGV QXP EHUV 5H [W (> @+ RZ HYHU IRU WSLFDO FRP P HUFLDO VXUDFH DQG JXWW IUHH WXUEDP V DP RU UHDOLVWF YDOXHV

5H [W§ (

(T EHFRPHV

%XWWKHGHILQLWLRQRI WKHVNLQIULFWLRQFRHILFLHQW(T LVVXFKWKDWWKHIRGBZLQJ LGHQWLWK
KROGV

7KHUHRUH(T LVDVNLQIULFWLRQGDZ IRUWXUEXOHQWODWSDWIORZ

,WVDFRPSOLFDWHGGDZ EXWZHFDQDVVHDWWKHVWRYHIRUDIHZ YDOXHVDQGGUDWKHP
) RGBZLQJ DVXJJHVWLRQ3UDQGWZHFDQIRUJHWKHFRPSOH[ORJ IUFWLRQGDZ DQGVLPSO
ILWKHQXPEHUVLQWKHWDEOHWRDSRZHUGDZ DSSUR[LPDWLRQ

7KLVZHVKDOXVHDVWKHGHWKDQGVLGHRI(T) RUWKHUJKWKDQGVLGH ZHQHGDQHWWP DL
IRU [LQWHUPVRI / [,I ZHXVHWKHORJDULWKPLF GDZ SURFLOH ZHVKDOOEHXS WRRXUKSVL
ORJDULWKPLF LQWHJUDWLRQVIRUWKHPRPHQWXP WKLFNQHVV ,QWKDGZHIRGBZ DQRWKHUVXJJHWLRQ3UDQG
ZKR SRLQWHGRXWWKDWWKHWXUEXOHQWSUR ILOHVLQ) LJ FDQEHDSSUR[LPDWHG E\ DRQHVHYHQWKSRZH
GDZ

7KLV LV VKRZQ DV DGDWKHG GDQHLQ) LJ ,WLV DQ H[FHOOHQWLWWR WKHORZ 5H\QROGVQXPEH
WXUEXOHQWGDWD ZKLFK ZHUH DOO WKDWZHUH DYDLODEOH WR 3UDQGWO DWWKH WLPH : LWK WKLV VLPSO
DSSUR[LPDWLRQ WKHPRPHQWXP WKLFNQHVV FDQHDVLO EHHYDOXDWHG

: HDFFHSWWKLVUHVXOWDQGVXEVWLWXWH(TV DQG LQWR. iUPiQ¶VPRPHQWXP GDZ

$$c_f = 0.02\,\mathrm{Re}_\delta^{-1/6} = 2\frac{d}{dx}\left(\frac{7}{72}\delta\right)$$

or

$$\mathrm{Re}_\delta^{-1/6} = 9.72\frac{d\delta}{dx} = 9.72\frac{d(\mathrm{Re}_\delta)}{d(\mathrm{Re}_x)} \qquad (7.41)$$

6HSDUDWWKHYDULDEOHVDQGLQWHJUDWH DVXPLQJ / DW[

$$\mathrm{Re}_\delta \approx 0.16\,\mathrm{Re}_x^{6/7} \qquad \text{or} \qquad \frac{\delta}{x} \approx \frac{0.16}{\mathrm{Re}_x^{1/7}} \tag{7.42}$$

7KXV WKH WKLFNQHVV RI D WXUEXOHQW ERXQGDU\ OD\HU LQFUHDVHV DV[IDU PRUH UDSLGO\ WKDQ WK ODPLQDU\ LQFUHDVH[(TXDWLRQ LV WKH VRXUFH RI WKH SUREOHP EHFDXVH DOO RWKHU SDUDPHWHUV DU QRZ DYDLODEOH) RU[DP SOH FRP ELQLQJ (TV DQG Z HREWLQ WKH IULFWLRQ YDULDWLRQ

$$c_f \approx \frac{0.027}{\mathrm{Re}_x^{1/7}} \tag{7.43}$$

: ULWLQJ WKLV RXWLQH LQ WHUPV RI DOORZDEOH Z HKDYH

$$\tau_{w,\mathrm{turb}} \approx \frac{0.0135\mu^{1/7}\rho^{6/7}U^{13/7}}{x^{1/7}} \tag{7.44}$$

7XUEXOHQW SODWH IULFWLRQ GUSV VORZ O\ Z LWK[LQFUHDVHV QHDU DV! DQG8 DQG LV UDWKH LQVHQVLWLYH WR YLVFRVLW

Re_δ	10^4	10^5	10^6	10^7
c_f	0.00493	0.00315	0.00217	0.00158

: HFDQ HYDOXDWH WKH GUDJ FRH ILFLHQW E\ LQWHJUDWLQJ WKH Z DOO IULFWLRQ

$$D = \int_0^L \tau_w b\,dx$$

or

$$C_D = \frac{2D}{\rho U^2 bL} = \int_0^1 c_f\,d\left(\frac{x}{L}\right)$$

$$C_D = \frac{0.031}{\mathrm{Re}_L^{1/7}} = \frac{7}{6}c_f(L) \tag{7.45}$$

7KH&, LV RQO SHUFHQW JUHDWHU WKDQ WKH WDLQJ HGJH VNLQ IULFWLRQ FRH ILFLHQW FRP SDUHG Z LW (T IRU ODP LQDU IORZ @

7KH GLVSODFHP HQW WKLFNQHVV FDQ EH HVWLP DWHG IURP WKH RQH V HY HQW SRZ HU DZ DQG (T

$$\delta^* \approx \int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy = \frac{1}{8}\delta \tag{7.46}$$

7KH WXUEXOHQW WDWSODWH VKDSH IDFWRU LV DSSUR[LP DWHO

$$H = \frac{\delta^*}{\theta} = \frac{\frac{1}{8}}{\frac{7}{72}} = 1.3 \tag{7.47}$$

7KH VH DUH WKH EDVLF UHVXOWV R WXUEXOHQW WDWSODWH WKH RU

) LJXUH VKRZ V IODWSODWH GUDJ FRH ILFLHQWV IRU ERWK ODP LQDU DQG WXUEXOHQW 7K VP RRWK Z DOO UHDWHDWRQV DQG DUH VKRZ Q DORQJ Z LWK WKH HIHFWR Z DOURXJKQHVV Z KLFK L

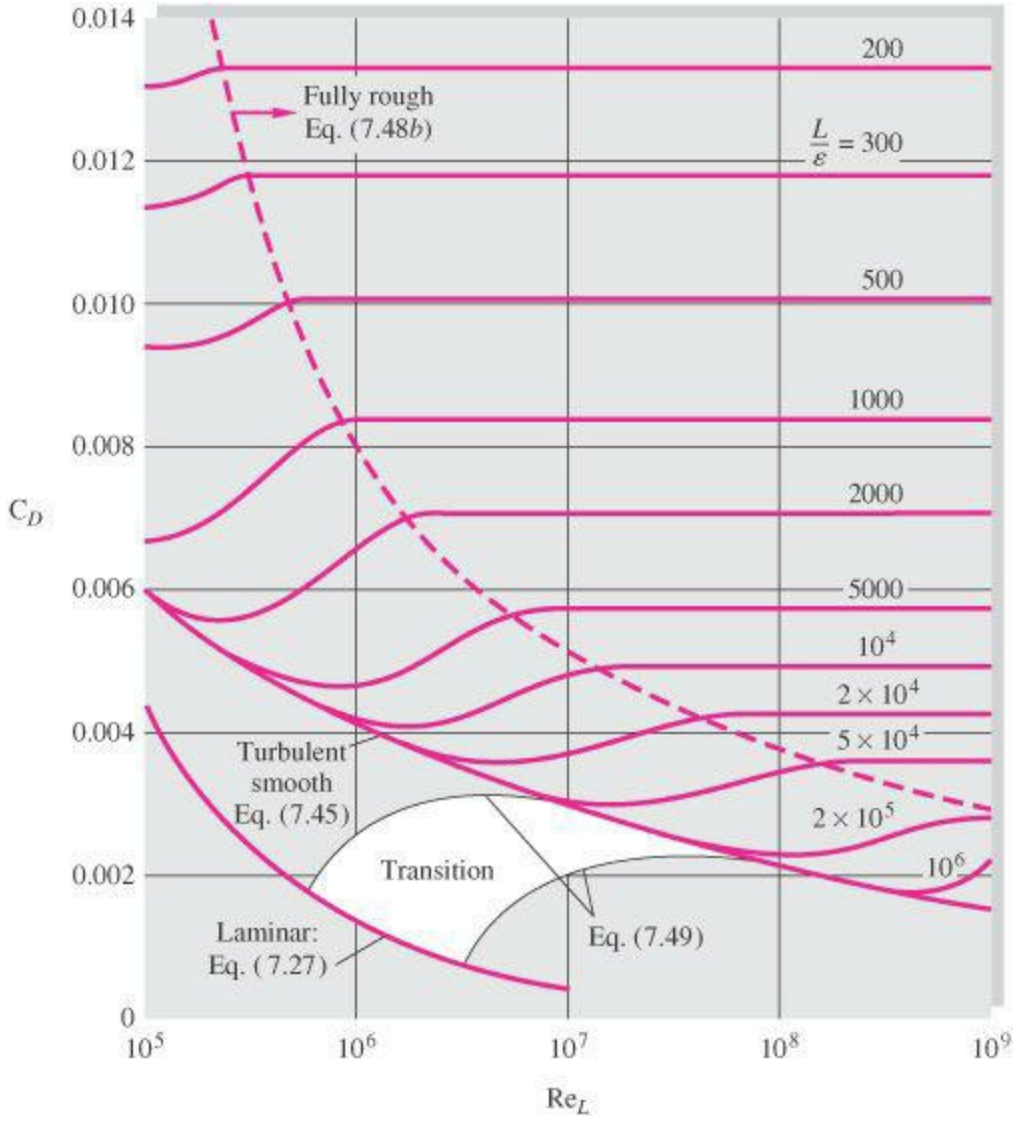
TXLWVWRQ 7KHSURSHUURXJKQHVVSDUDPHWUKHUHLV[ëRU/ ëE\ DQDORJ\ ZLWKWKHSLSHSDUDPHW
ëG ,QWKHIXOO URXJKUHJLPH & LVLQGHSHQGHQWRI WKH5H\QROGVQXPEHU VR WKDWWKHGUDJ YDULH
H[DFWO DV8 DQGLVLQGHSHQGHQWRI 5HHUHQFH SUHVHQWVDWKRU RI URXJKIODWSODWHIORZ DQG5H
JLYHVDFXUYHILWWRUVNQIUFWLRQDQGGUDJ LQWKHIXOO URXJKUHJLPH

$$c_f \approx \left(2.87 + 1.58 \log \frac{x}{\epsilon}\right)^{-2.5} \tag{7.48a}$$

$$C_D \approx \left(1.89 + 1.62 \log \frac{L}{\epsilon}\right)^{-2.5} \tag{7.48b}$$

(TXDWLRQ E LVSORWWHGWRWKHULJKWRI WKHGDVKHGOLQHLQ) LJ 7KHILJXUHDOVR VKRZV WK
EHKDYLRUR WKHGUDJ FRHILFLHQWLQWKHWUDQVLWLRQUHJLRQ î 5H/ î ZKHUHWKHOPLQD
GUDJ DWWKHOHDGLQJ HGJH LV DQ DSSUHFLDEOHIUDFWLRQ RI WKH WRWDOGUDJ 6FKOLFKWLQJ > @VXJJHVWV WK
IROORZLQJ FXUYHILWIRUWKHVHWDQWLWLHVGUDJ FXUYHV GHSHQGLQJ RQWKH5H\QROGVQXPEHU5H WDQV ZKHU
WDQVLWLRQEHJLQV

$$C_D \approx \begin{cases} \dfrac{0.031}{\mathrm{Re}_L^{1/7}} - \dfrac{1440}{\mathrm{Re}_L} & \mathrm{Re}_{\mathrm{trans}} = 5 \times 10^5 \qquad (7.49a) \\ \dfrac{0.031}{\mathrm{Re}_L^{1/7}} - \dfrac{8700}{\mathrm{Re}_L} & \mathrm{Re}_{\mathrm{trans}} = 3 \times 10^6 \qquad (7.49b) \end{cases}$$



)LJ ' UDJ FRHILFLHQWRI ODPLQDUDQGWXUEXOHQWERXQGDU\ OD\HUVRQVPRRWK DQGURXJK IODWSODWHV 7KLVFKDUWLVWKHIODWSODWHDQDORJ RI WKH0RRG\ GLDJUDP RI)LJ

(; $0 3/ (

$ K\GURIRLO IWORQJ DQG IWZLGH HV SDFHG LQ D VHDZDUGIORZ RI IWV ZLWK! VXJVIW DQG IWV D (WXPDWWKHERXQGDU\ OD\HUWKLFNQHVV DWWKH HQG RI WKH SODW (WXPDWWKH IULFWLRQ GUDJ IRU E WXEXOHQWVPRRWKZDOOIORZ IURP WKH OHDGLQJ HGJH F ODPLQ WXEXOHQWORZ ZLWK5H $DQG G WXEXOHQWURXJK ZDOOIORZ ZLWKë IW

6RXWLRQ

3DUWD

7KH5H\QROGVQXPEHULV

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{(40\ \text{ft/s})(1.2\ \text{ft})}{0.000011\ \text{ft}^2/\text{s}} = 4.36 \times 10^6$$

7KXV WKH WXUEXOHQW7KHPDWLPXP ERXQGDU\ OD\HU WKLFNQHVV Z RXO RFFXU IRU WXUEXOHQW ZDWHU VWDUWLQJ DW WKH OHDGLQJ HGJH) URP (T

$$\frac{\delta(L)}{L} = \frac{0.16}{(4.36 \times 10^6)^{1/7}} = 0.018$$

or $$\delta = 0.018(1.2\ \text{ft}) = 0.0216\ \text{ft} \qquad \textit{Ans. (a)}$$

7KLVLV WLPHVWKLFNHUWKDQDIXOO ODPLQDUERXQGDU\ OD\HUDWWKHVDPH5H\QROGVQXPEHU

3DUWE

) RUIXOO WXUEXOHQWWPRRWK Z DOOIORZ WKHGUDJ FRHILFLHQWRQ RQHVLGH RI WKHSODWH LV IURP (T

$$C_D = \frac{0.031}{(4.36 \times 10^6)^{1/7}} = 0.00349$$

7KHQWKHGUDJ RQERWKVLGHVRI WKHIRLOLVDSSUR[LPDWHO

$$D = 2C_D(\tfrac{1}{2}\rho U^2)bL = 2(0.00349)(\tfrac{1}{2})(1.99)(40)^2(6.0)(1.2) = 80\ \text{lbf} \qquad \textit{Ans. (b)}$$

3DUWF

: LWKDODPLQDUERXQGDU\HGJHDQG5H$_{WUDQV}$ î (T D DSSOLHV

$$C_D = 0.00349 - \frac{1440}{4.36 \times 10^6} = 0.00316$$

7KHGUDJ FDQEHUHFRPSXWHGIRUWKLVORZHUGUDJ FRHILFLHQW

$$D = 2C_D(\tfrac{1}{2}\rho U^2)bL = 72\ \text{lbf} \qquad \textit{Ans. (c)}$$

3DUWG

) LQDOO IRUWKHURXJKZDOOZHFDOFXODWH

ZKHUH [LVWKHPRPHQXP WKLFNQHVV DQG+ [/ [[LVWKHVKDSHIDFWRU)URP (T
QHJDWLYHG8 G[LVHTXLYDOHQWWRSRVLWLYHGS G[² WKDWV DQDGYHUVHJUDGLHQW
: HFDQLQWURGXFH(T WRGHWHUPLQH [IRUDJLYHQ8 [LI ZHFRUUHODWHF DQG+ ZLWKWK
PRPHQXP WKLFNQHVV 7KLV KDV EHHQ GRQH E\ H[DPLQLQJ WSLFDOYHORFLW SURILOHV RI ODPLQDU DQ
WXUEXOHQWERXQGDU\ OD\HUIORZ VIRUYDULRXV SUHVWXUHJUDGLHQWV 6RPHH[DPSOHVDUHJLYHQ)LJ
VKRZLQJ WKDWWKH VKDSHIDFWRU+ LV DJRRG LQGLFDWRURI WKH SUHVWXUHJUDGLHQW7KHKLJKHUWKH+ WK
WXUEXOHQWHDGYHUVHJUDGLHQWDQG VHSDUDWLRQRFFXUVDSSUR[LPDWD DW

7KHODPLQDUSURILOHV)LJ D FOHDUO H[KLELWWKH 6 VKDSHDQG D SRLQWRI LQIOHFWLRQ ZLWK D
DGYHUVHJUDGLHQW%XWLQWKHWXUEXOHQWSURILOHV)LJ E WKHSRLQWRI LQIOHFWLRQDUHWSLFDOO EXUL
GHHSZLWKLQWKHWKLQYLVFRXVVXEOD\HU ZKLFKFDQKDUGO EHVHHQRQWKHVFDOHRI WKHILJXUH

)LJ 9HORFLW SURILOHV ZLWK SUHVXUH JUDGLHQW *a* ODPLQDUIORZ *b* WXUEXOHQWIORZ ZLW
DGYHUVHJUDGLHQW

7KHUHDUHVRUHVRI WXUEXOHQWWKRUHVLQWKHOLWHUDWXUH EXWWKH\ DUHDOOFRPSOLFDWHG
DQGZLOOEHRPLWWGKHUH 7KHUHDGHULVUHUHGWRDGYDQFHGWH[WV> ± @
)RUODPLQDUIORZ DVLPSOHDQG HIFLHQWPHWKRGZDVGHYHORSHGE\ 7KZDLWV> @ZKRIRXQ
WKDW(T FDQEHIRUHDWGE\ DVLQJOHGLPHQVLRQOHVVPRPHQXP WKLFNQHVVYDULDEOH GHILQH
DV

$$\lambda = \frac{\theta^2}{\nu}\frac{dU}{dx} \tag{7.53}$$

8VLQJ DVXLWDJKWDQH ILWWR KLV FRUUHODWRQ 7KZDLWV ZDV DEOH WR LQWURGXFH(T LQ FORVH
IRUP ZLWKWKHUHVXOW

$$\theta^2 = \theta_0^2\left(\frac{U_0}{U}\right)^6 + \frac{0.45\nu}{U^6}\int_0^x U^5\,dx \tag{7.54}$$

ZKHUH LV WKH PRPHQWXP WKLFNQHVV DW[XVXDOO WDNHQ WR EH]HUR 6HSDUDWLRQ F[ZD
IRXQG WR RFFXU DW D SDUWLFXODU YDOXH RI

$$\text{Separation:} \qquad \lambda = -0.09 \tag{7.55}$$

)LQDOO 7KZDLWHV FRUUHODWHG YDOXHV RI WKH GLPHQVLRQOHVV VKHDU VWUHVV6 2Z 8 ZLWK DQG KL
JUDSKHG UHVXOWV DQ EH FXUYH ILWWHG DV IROORZV

$$S(\lambda) = \frac{\tau_w \theta}{\mu U} \approx (\lambda + 0.09)^{0.62} \tag{7.56}$$

7KLV SDUDPHWHU LV UHODWHG WR WKH VNLQ IULFWLRQ E\ WKH LGHQWLW\

$$S \equiv \tfrac{1}{2} c_f \mathrm{Re}_\theta \tag{7.57}$$

(TXDWLRQV VR FRQVWUXFW D FRPSOHWH VHW RI IRUP WKH ODPLQDU ERXQGDU\ OD\HU ZLWK
YDULDEOH8 [ZLWK DQ DFFXUDF\ RI " SHUFHQW FRP SDUHG ZLWK FRP SXWHU VROXWLRQV RI WKH ODPLQDU
ERXQGDU\ OD\HU HTXDWLRQV &RPSOHWH GHWDLOV RI 7KZDLWHV¶V DQG RWKHU ODPLQDU WKHRULHV DUH JLYH
LQ5H

$ VLPLODU PRQRJUDSK RQ 7KZDLWHV¶V PHWKRG WDNHV DFFRXQW DQG
(TXDWLRQ LQWURGXFHV

$$\theta^2 = \frac{0.45 \nu x}{U}$$

or

$$\frac{\theta}{x} = \frac{0.671}{\mathrm{Re}_x^{1/2}} \tag{7.58}$$

7KLV LV ZLWKLQ SHUFHQW RI %ODVLXV¶V QXPHULFDO VROXWLRQ (T
: LWK (T SUHGLFWV WKH IODWSODWH WKHDUHWLFDO

$$\frac{\tau_w \theta}{\mu U} = (0.09)^{0.62} = 0.225$$

or

$$c_f = \frac{2\tau_w}{\rho U^2} = \frac{0.671}{\mathrm{Re}_x^{1/2}} \tag{7.59}$$

7KLV LV DOVR ZLWKLQ SHUFHQW RI WKH %ODVLXV UHVXOW(T + RZ HYHU WKH JHQHUDO DFFXUDF\ R
WKLV PHWKRG LV SRRUHU WKDQ SHUFHQW EHFDXVH 7KZDLWHV DFWXDOO ³WXQHG KLV FRUUHODWLRQ FRQVWDQWV
PDNH WKH P DJUHH ZLWK H[DFW WDWSODWH WKHRU\
: HVKDOO QRW FRP SXWH DQ PRUH ERXQGDU\ OD\HU GHWDLOV KHUH EXW DV ZH JR DORQJ LQYHVWLJDWH
YDULRXV LPPHUVHG ERG\ IORZV HVSHFLDOO LQ&KDS ZH VKDOO XVH 7KZDLWHV¶V PHWKRG WR PDN
TXDOLWDWLYH DVVHVVPHQWV RI WKH ERXQGDU\ OD\HU EHKDYLRU

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,Q + RZ DUWKSURSRVHG DQ DSSUR[LPDWH PHWKRG IRU VROYLQJ WKH ERXQGDU\

$$U(x) = U_0\left(1 - \frac{x}{L}\right) \qquad (1)$$

DV D VKHRUHWLFDO PRGHO IRU ODPLQDU ERXQGDU\ OD\HUV WXG\ D 8VH 7KZDLWHV PHWKRG WR FRPSXW
VKHV SDUDPHWRUV SRLQW[VHS IRU DQG FRPSDUH Z LWK WKH H[DFW FRPSXWHU VROXWLRQ[VHS /
JLYHQ E\ + : LSSHUPDQLQ E \$OR FRPSXWH WKH YDOXHV RI F 2Z !8 DW[/

6RXWLRQ

3DUW D

) LUVW QRWH WKDW G[í 8 / FRQWDLQV 9HORFLW GHFUHDVHV SUHVXUH LQFUHDVHV DQG WKH SUHVXUH JUDGLHQW VDGYHUVH WKURXJKRXW1RZ LQWURGXFH(T

$$\theta^2 = \frac{0.45\nu}{U_0^6(1 - x/L)^6}\int_0^x U_0^5\left(1 - \frac{x}{L}\right)^5 dx = 0.075\frac{\nu L}{U_0}\left[\left(1 - \frac{x}{L}\right)^{-6} - 1\right] \qquad (2)$$

7KHQ WKH GLPHQVLRQOHVV SDUDPHWHU LV JLYHQ E\

$$\lambda = \frac{\theta^2}{\nu}\frac{dU}{dx} = -\frac{\theta^2 U_0}{\nu L} = -0.075\left[\left(1 - \frac{x}{L}\right)^{-6} - 1\right] \qquad (3)$$

) URP (T Z HVHH WKLV HTXDOV í IRU VHSDUDWLRQ

$$\lambda_{sep} = -0.09 = -0.075\left[\left(1 - \frac{x_{sep}}{L}\right)^{-6} - 1\right]$$

or $$\frac{x_{sep}}{L} = 1 - (2.2)^{-1/6} = 0.123 \qquad Ans.\ (a)$$

7KLV LV OHVV WKDQ SHUFHQW WKLJKHU WKDQ: LSSHUPDQ[V H[DFW VROXWLRQ DQG WKH FRPSXWDWLRQDO HIRUW YHU PRGHVW

3DUW E

7R FRPSXWH DW[/ NXWWD EH RUHVHSDUDWLRQ Z HILUVW FRPSXW DWKLV SRLQWXVLQJ (T

[/ > @

7KHQ IURP (T WKH VKHDU SDUDPHWHU LV

$$S(x = 0.1L) = (-0.0661 + 0.09)^{0.62} = 0.099 = \tfrac{1}{2}c_f \mathrm{Re}_\theta \qquad (4)$$

: HFDQ FRPSXWH 5H LQWHUPV RI 5H, IURP (T RU

$$\frac{\theta^2}{L^2} = \frac{0.0661}{UL/\nu} = \frac{0.0661}{\mathrm{Re}_L}$$

or $$\mathrm{Re}_\theta = 0.257\,\mathrm{Re}_L^{1/2} \qquad \text{at } \frac{x}{L} = 0.1$$

6XEVWLWXWLQJ(T

$$0.099 = \tfrac{1}{2} c_f\,(0.257\,\mathrm{Re}_L^{1/2})$$

or $$c_f = \frac{0.77}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L = \frac{UL}{\nu} \qquad \textit{Ans. (b)}$$

: HFDQQRWDFFXUDWHO\ FRPSXWH\ ZLWKRXW\ KHYDOXHRI VD\ 8 /

([SHULP HQWDO([WHQGHG) ORZV

%RXQGDU\ OD\HUWKHRU LVYHU LQWHUWZQJ DQG LOOXPLQDWQJ DQG JLYHVXV DJUHDWTXDQWLWY HJUDSR YLVFRXV IORZ EHKDYLRU EXWEHFDXVH RI IORZ VHSDUDWRQ WKH WKHRU GHVQRWJHQHUDO DQRZ I TXDQWLWY HFRPSXWRQRI WKHFRPSOHWIORZ ILHOG ,QSDUWFXODU WKHUHLV DWSUHVHQWQR VDWLVIDFWRU WKHRU H[FHSW&)' UHVXOW IRUWKHIORUHVRQDQDUEWUJ ERG LPPHUVHGLQD VWHDP IORZLQJ DWD DUEWUJ 5HQROGVQXPEU 7KHUHRUH[SHULPHQWRQLVWKHNH\WRWHQWQH[WLQ H[WHQGHGIORZ SURREOPV

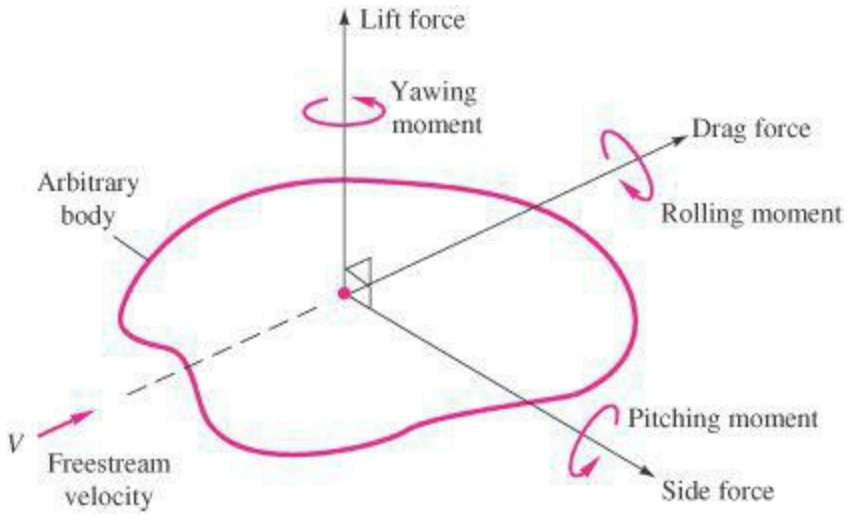
/LWHUDO WKRXVDQGV RI SDSHUV LQ WKH OWHUDWXUH UHSRUWH[SHULPHQWDO GDWD RQ VSHFLILF H[WHQGHG YLVFRXVIORZ V 7KLVVHFWRQJLYHVDEULH GHVFULSWRQRI WKHIRORRZLQJ H[WHQGHGIORZ SUREOPV

' UDJ RI VWR DQG WKUHHGLPHQVLRQDOERGLHV
%OXQWERGLHV
6WHDP OQHGVKDSHV
3HUFRUP DQFH RI OWQJ ERGLHV
$LUFRIVDQGDLUFUDIW
3URMHFWLOHVDQGILQHGERGLHV
%LRGVDQGLQVHFWV

) RUIXUWKHUHDGLQJ VHHWKHJRORPLQHRI GDWFRPSLOHGLQ+ RHUQHU> @,QDWUFKDSWHUVZHVKO VWXG GDWRQ VXSHUVRQLF DLUFRIV &KDS RSHQFKDQQHOIUFWRQ &KDS DQG WXUERP DFKLQHU SHUFRUP DQFH &KDS

' UDJ RI ,P P HUVHG %RGLHV

$Q ERG\ RI DQ\ VKDSHZ KHQLP PHUVHGLQDIOXLG VWUHDP ZLOO[SHULHQFHIRUFHVDQGPRPHQWVIURP
VKHIORZ ,I WKHERG\ KDVDUEWUDU\ VKDSHDQGRULHQWDWLRQ WKHIORZ ZLOO[HUWRUFHVDQGPRPHQWVDERX
DOOWKUHHFRRUGLQDWH[DVVKRZQLQ) LJ ,WLVFXVWRPDU\ WRFKRRVHRQHD[LVSDUDOOHOWR WKHIUH
VWUHDP DQGSRVLWLYHGRZQVWUHDP 7KHIRUFHRQWKHERG\ DORQJWKLVD[LVLVFDOOHGGUDJ DQGWKHPRPHQ
DERXWWKDWD[LV WKHURQQJ PRPHQW7KHGUDJ LVHVVHQWLDOO DIORZ ORVV DQGPXWEHRYHUFRPHLI WK
ERG\ LVWRPRYHDJDLQVWWKHVWUHDP



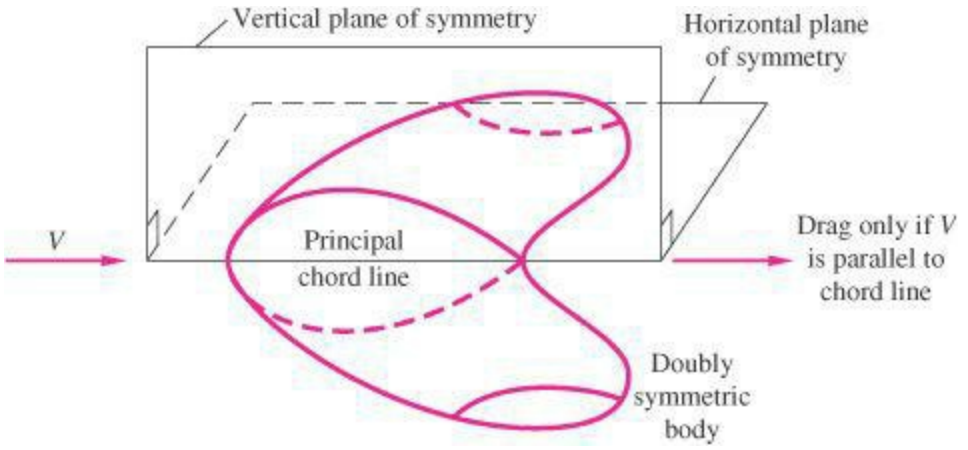
) LJ ' HILQLWLRQRI IRUFHVDQGPRPHQWVRQDERG\ LPPHUVHGLQDXQLIRUP IORZ

$ VHFRQG DQG YHU LPSRUWDQWRUFH LV SHUSHQGLFXODU WR WKHGUDJ DQG XVXDOO SHURPV D XVHX
NWE VXFK DV EHDULQJ WKHZHJKWRI WKHERG\ ,WLV FDOOHG WKHOLW7KHPRPHQWDERXWWKHOLWD[LV L
FDOOHG\DZ

7KH WKLUG FRPSRQHQWLWKHUD GRV QRUD JDLQ LV WKHVLGH IRUFH DQG DERXWWKLV D[LV LV WK
SLWFKLQJ PRPHQW7R GHDOZLWKWKLV WKUHHGLPHQVLRQDOIRUFHPRPHQWWLWXDWLRQ LV PRUH SURSHUO WK
URQHRI DW[WERRN QRQDHURG\QDPLFV>RUH[DPSOH @: HVKDOOOP LWWKHGLVFXVLRQKHUHWOWDQ
GUDJ

: KHQWKHERG\ KDV\PPHWU DERXWWKHOLWGUDJ D[LV DVZLWKDLSODQHV VKLSV DQG FDUVPRYLQ
GLUHFW LQR D VWUHDP WKH VIDFH \DZ DQG URODYDQVK DQG WKH SURECOP UHGXFHV WR D VZR
GLPHQVLRQDOFDVH VZRIRUFHV OWDQG GUDJ DQGRQHPRPHQWSLWFK

$ ILQDOVLPSOLFDWRQRWHQRFFXUZKHQWKHERG\ KDVWZRSODQHVRI \PPHWU DVLQ) LJ $
ZLGHYDULHWRI VKDSHVVXFK DVF\OLQGHUV ZLQJ DQGDOERGLHVRI UHYROXWLRQVDUW\ WKLVUHTXLUHPHQ
,I WKHIUHHVWUHDP LVSDUDOOHOWR WKHLQWHUVHFWLRQRI WKHVHWZRSODQHV FDOOHG WKHSULQFLSDOFKRUGOQHR
WKHERG\ WKHERG\ H[SHULHQFHV GUDJ RQO ZLWK QR OLWVLGH IRUFH RU PRPHQW 7KLV WSH R
GHJHQHUDWHIRUFHGUDJ GDWDLV ZKDWLV PRWWFRPPRQO UHSRUWHG LQ WKHOLWHUDWXUH EXWL WKHIUH
VWUHDP LVQRWSDUDOOHOWR WKHFKRUGOQH WKHERG\ ZLOOKDYHDQXQ\PPHWUF RUHQWDWLRQ DQG DOOWKUH
IRUFHVDQG WKUHPRPHQWFDQDULVHLQSULQFLSOH



) LJ 2 QD VKHGUDJ IRUFHRFFXUVLL VKHIORZ LVSDUDOOHOWRERWKSODQHVRI VLPPHWUL

,QORZ VSHHG IORZ SDVWJHRPHWULFDOO VLPLODUERGLHV ZLWK LGHQWLFDORUHQWDWLRQ DQG UHODWLY URXJKQHVV VKHGUDJFRHILFLHQWVKRXOGEHDIXQFWLRQRI VKHERG\ 5H\QROGVQXPEHU

$$C_D = f(\mathrm{Re}) \qquad (7.60)$$

7KH5H\QROGVQXPEHULVEDVHGXSRQVKHIUHHVWUHDP YHORFLW9 DQGDFKDUDFWHULVWLF/ RI VK ERG\ XVXDOO VKHFKRUGRUERG\OHQJWKSDUDOOHOWRVKHVWUHDP

$$\mathrm{Re} = \frac{VL}{\nu} \qquad (7.61)$$

) RUA\OQGHUV VSKHUHV DQGGLVNV VKHFKDUDFWHULVWLF OHQJWKLVVKHGLDPHWU'

&KDUDFWHULVWLF\$UHD

' UDJFRHILFLHQWVDUHGHILQHGE\ XVLQJ DFKDUDFWHULVWLF DUHD\$ ZKLFK PD\ GLIIHUGHSHQGLQJ RQVK ERG\ VKDSH

$$C_D = \frac{\text{drag}}{\frac{1}{2}\rho V^2 A} \qquad (7.62)$$

7KHIDFWRU$\frac{1}{2}$ LVRXUVWDGLWRQDOWUEXWHVVR(XOHUDQG%HUQRXOOL 7KHDUHD\$ LVXVXDOO RQHRI VKUH VWSHV

) URQWDODUHD VKHERG\ DVVHQIURP VKHVXSHUP VXLWDEOHIRUWKLFN VXEE\ERGLHV VXFK D VSKHUHV FA\OQGHUV FDUV WUXFNV PLVLOHV SURMHFWLOHV DQGVRUSHGRHV

3ODQRUP DUHD VKHERG\ DUHD DVVHQIURP DERYH VXLWDEOHIRUZLGH IODWERGLHV VXFK D ZLQJVDQGK\GURILOV

: HWWHGDUHD FXWWRP DU IRUVXUIDFHVKLSVDQGEDUJHV

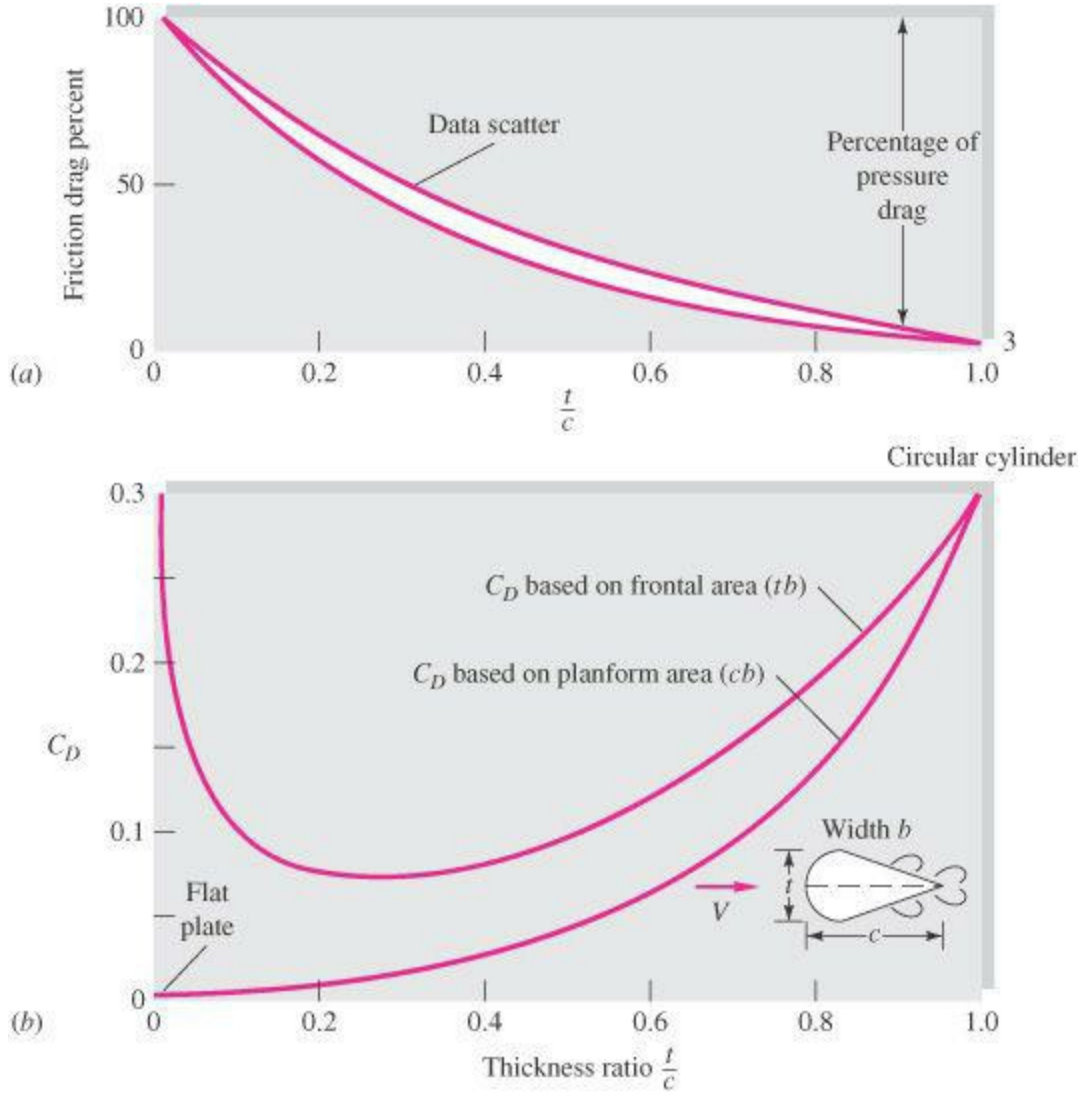
,QXLQJ GUDJ RUVKHUIXLG IRUFHGDWD LWLV LP SRUWDQWRQRWHWKDWDQDUHDUHEHLQJ XVH VRVFDOHVKHPHDXUHGFRHILFLHQWV

) UL FWLRQ ' UDJ DQG 3 UHVVXUH ' UDJ

$ VZ HKDYH PHQWLRQHG WKH WKHRUL RI GUDJ LVZ HDNDQGLQDG HTXDWL H[FHSWRUWKH IODWSODWH 7KLVL EH FDXVH RI IORZ VHSDUDWLRQ %RXQGDU\ OD\HU WKHRUL FDQ SUHGLFWWKH VHSDUDWLRQ SRLQWEXWFDQQ DFFXUDWHO HVWLPDWH WKH XVXDOO ORZ SUHVVXUH GLVWULEXWLRQ LQ WKH VHSDUDWHG UHJLRQ 7KH GLIIHUHQF EHWZHHQ WKH KLJK SUHVVXUH LQ WKH IURQWXDQDWLRQ UHJLRQ DQG WKH ORZ SUHVVXUH LQ WKH UHDU VHSDUDWH UHJLRQ FDXVHV D OLJHGUDJ FRQWULEXWLRQ FDOOHG SUHVVXUH GUDJ 7KLV LV DGGHG WR WKH LQWUDUDG VKHDU VXUIDFH GUDJ RI WKHERG\ ZKLFK LWR WKQH[FHHGV

$$C_D = C_{D,\text{press}} + C_{D,\text{fric}} \qquad (7.63)$$

7KHUHDLY HFRQWULEXWLRQRI IUFWLRQDQG SUHVVXUHGUDJ GHSHQGVXSRQWKHERG\¶V VKDSH HVSHFLDO LW WKLFNQHVV) LJXUH VKRZ V GUDJ GDWD IRUD VWUHDP OQHG FA OQGHURI YHU OLJH GHSWK LQR WK SDSHU $ W]HUR WKLFNQHVV WKHERG\ LV DIODWSODWH DQG H[KLELW SHUFHQWIUFWLRQ GUDJ $ WWKLFNQHV HTXDOWR WKH FKRUG OHQJWK VLP XODWQJ D FLUFXODUA OQGHU WKH IUFWLRQ GUDJ LV RQ DERXW SHUFHQ) UFWLRQ DQG SUHVVXUH GUDJ DUH DERXWHTXDODWWKLFNQHVVWF 1 RWWKDW& LQ) LJ E GRRN TXLWGLIHUHQWZ KHQEDVHGRQIURQDODUHDLQVWDG RI SODQRUP DUHD SODQRUP EHLQJ WKH XVXDOFKRLF IRUWKLVERG\ VKDSH 7KHVZR FXUYHVLQ) LJ E UHSUHVHQW[DFW WKH VDP HGUDJ GDW



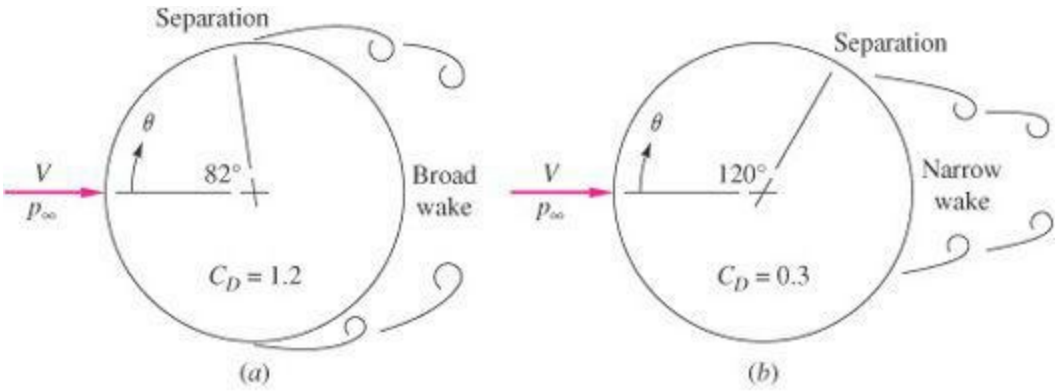
) LJ ' UDJ RI D VWHDP OQHG VZR GLP HQVLRQDO FA OQGHU DW5H $ HIHFWR

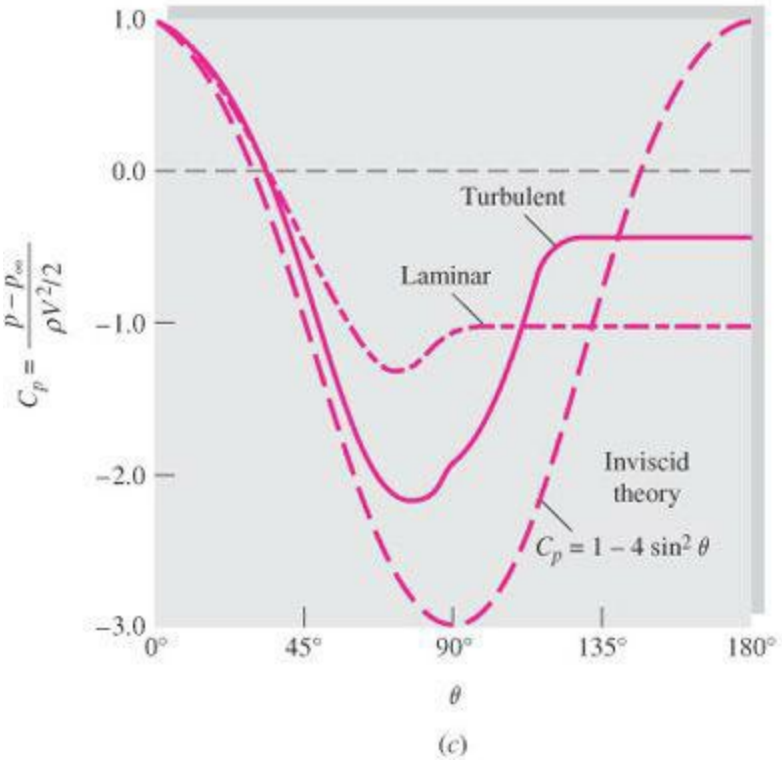
WKLFNQHVV UDWLR RQ SHUFHQWDJH RI IUHHVWUHDP GLDJ *b* WRWDO GUDJ YHUVXV WKLFNQHVV ZKHQ EDVHG RQ VZLRGLIHUHQWDUHDV

) LJXUH LOOXVWUDWHV WKH GUDPDWLF HIIHFWV RI VHSDUDWHG IORZ DQG WKH VXEVHTXHQW GLIIXVH R ERXQGDU\ OD\HUWKHRU\ 7KHWKRUHWLFDOLQYLVFLG SUHVVXUHGLVWULEXWLRQRQD FLUFXODU\OLQGHU &KDS LVVKRZ QDVWKHGDVKHGOLQHLQ) LJ F

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V^2} = 1 - 4\sin^2\theta$$

Z KHUHS DQG9 DUHWKHSUHVVXUHDQGYHORFLW UHVSHFWLYHO LQWKHIUHHVWUHDP 7KHDFWXDOOPLQL DQG WKLEXOHQWERXQGDU\ OD\HUSUHVVXUHGLVWULEXWLRQV LQ) LJ F DUH VDUWDQJO GLIIHUHQWURP WKRV SUHGLFWHG E\ WKHRU / DPLQDUIORZ LV YHU YXOQHUDEOH WR WKH DGYHUVH JUDGLHQWRQ WKH UHDU R WK F\OLQGHU DQG VHSDUDWRQ RFFXUV DW *f* Z KLFK FHUWDLQO FRXOG QRWKDYH EHHQ SUHGLFWHG IURF LQYLVFLG WKHRU 7KH EURDG Z DNH DQG YHU ORZ SUHVVXUH LQ WKH VHSDUDWHG OPLQDUIORZ UHJLRQ FDXVHV WK ODUJHGUDJ &

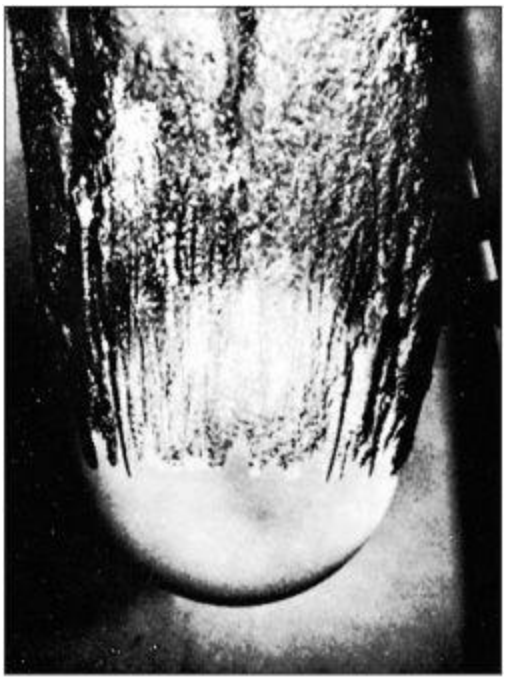




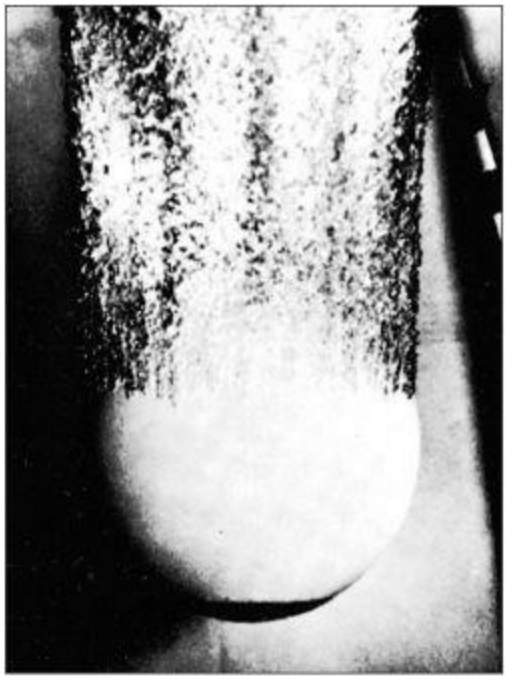
) LJ) GRZ SDWWD FLUFXODU F\OLQGHU *a* OPLQDUVHSDUDWRQ *b* WXUEXOHQWVHSDUDWRQ *c*

VKHRUHWLFDOODQGDFWXDOXUDFHSUHVWXUHGLVWULEXWLRQV

7KHWXUEXOHQWERXQGDU\OD\HULQ)LJ E LVPRUHUHPRYHGIURPWKHVXUIDFHRIWKHGLVN
ƒ ZLWKDUHVXOWLQJ VPDOOHUZDNH KLJKHUSUHVVXUHRQWKHUHDUDQG SHUFHQWDJHGUDJ &
7KLVH[SODLQVWKHVKDUSGURSLQGUDJDWWDQWLWRQLQ)LJ
7KH VDPH VKDUS GLIIHUHQFH EHWZHHQ YXOHUDEOH ODPLQDU VHSDUDWLRQ DQG UHVLVWDQWXEXOH
VHSDUDWLRQ FDQ EH VHHQ IRU D VSKHUH LQ)LJ 7KH ODPLQDUIORZ)LJ D VHSDUDWHV DWDERX
ƒ & ZKLOHWKHWXUEXOHQWIORZ)LJ E VHSDUDWHV DW ƒ & +HUHWKH5H\QROG
QXPEHUV DUH H[DFWO\ WKH VDPH DQG WKH WXUEXOHQWERXQGDU\ OD\HU LV LQGXFHG E\ D SDWK RI VDQ
URXJKQHVV DWWKH QRVH RI WKHEDOO* RO EDOOYIO LQWKLV UDQJHRI 5H\QROGV QXPEHUV ZKLFK LVZK\ WKH
DUHGHOEHUDWH GLP SOHG² WRLQGXFHDWXUEXOHQWERXQGDU\ OD\HUDQG RZHUGUDJ $JDLQZHZRXOGILQ
WKHDFWXDOSUHVWXUHGLVWULEXWLRQRQWKHVSKHUHWR EHTXLWHGLIIHUHQWURP WKDWSUHGLFWHG E\ LQYLVFL
WKHRU\



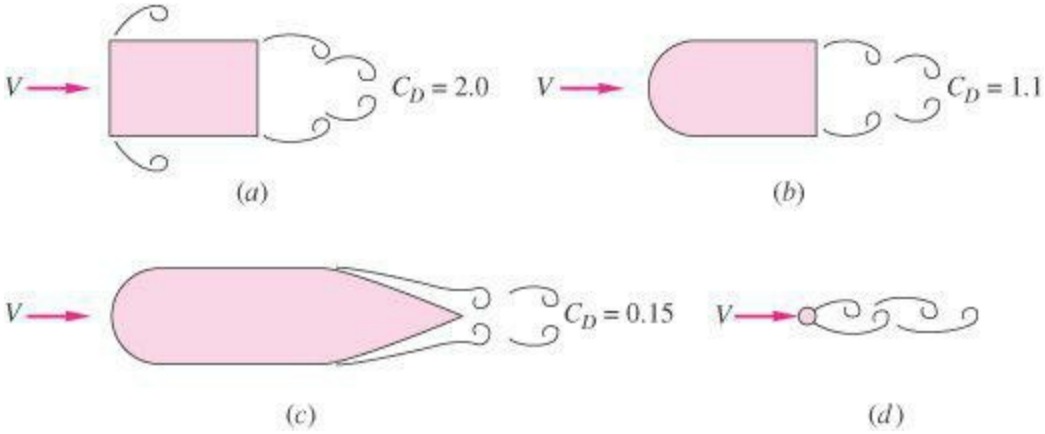
(*a*)



(*b*)

) LJ 6WDQGLIHUHQFHV LQ OLPLQDU DQG WXUEXOHQW VHSDUDWHG RQ DQ LQERZGQJ ED HQGMULQJ ZDWHU IW/ *a* VP RRVK EDOO OLPLQDU ERXQGDU\ OD\HU *b* VDPH HQGL WXUEXOHQ IORZ LQGXFHG E\ SDWFK RI QRVH VDQG URXJKQHVV*(NAVAIR Weapons Division Historical Archives.)*

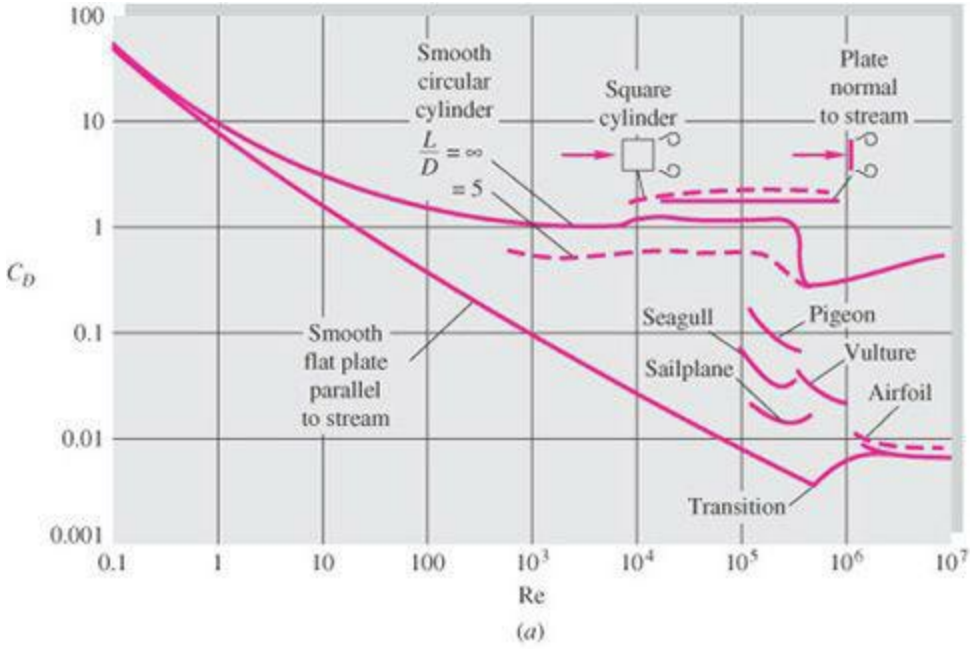
,QJHQHUDO ZHFDQQRWRYHUWHVWHLPSRUWDQFHRI ERG\ VWHDPOLQLQJ WR UHGXFH GUDJ D/6 HQRQ QXPEHUV DERYH DERXW 7KLV LV LOOXVWUDWHG LQ) LJ 7KHUHFDQJXODU\OLQGHU) LJ D KD VHSDUDWRQ DWDOO VKDSH FRQHV DQG YHU KLJK GUDJ 5RXQGLQJ LW QRVH) LJ E UHGXFHV GUDJ E\ DERXW SHUFHQW EXW& LV VWLOO KLJK 6WHDPOLQLQJ LW UHDUWR D VKDUS WDLOLQJ HGJH) LJ F UHGXFHV LW GUDJ DQRWKHU SHUFHQW WR D SUDFWDOO PLQLPXP IRUWKH JLYHQ WKLFNQHVV \$V D GUDP DW FRQGXWWKH FLUFXODU\OLQGHU) LJ G KDV RQH HLJKWK WKH WKLFNQHVV DQG RQH WKUHH KXQGUHGWK WK FURVV VHFWLRQ F) LJ F \HWLWKDV WKH VDPH GUDJ)RUKLJK SHUFRUP DQFH YHKLFOHV DQG RWK-PRYLQJ ERGLHV WKHQDPHRI WKHJDPHLVGUDJ UHGXFWLRQ IRUZKLFK LQWHQVHUHVHDUFK FRQWLQXHVIRUERW DHURG\QDPLF DQGK\GURG\QDPLF DSSOLFDWLRQV>@

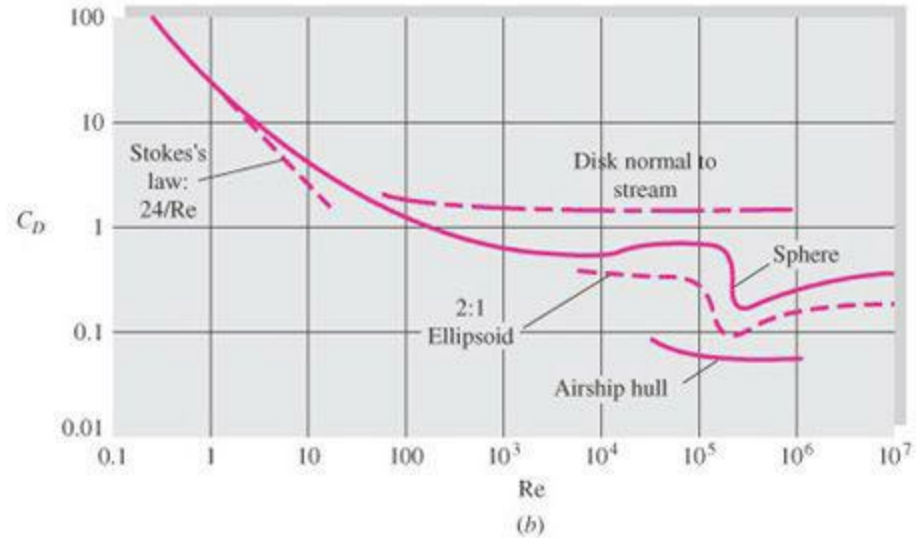


) LJ 7KH LPSRUWDQFH RI VWUHDPOLQLQJ LQ UHGXFLQJ GUDJ RI D ERG\ C_D EDVHG RQ IURQW DUHD *a* UHFWDQJXODU\ OLQGHU *b* URXQGHG QRVH *c* URXQGHG QRVH DQG VWUHDPOLQHG VKDU WDLOLQJ HGJH *d* FLUFXODU\ OLQGHUZLWK WKH VDPH GUDJ DVFDVH *c*

7ZR' LPHQVLRQDO%RGLHV

7KHGUDJ RI VRPH UHSUHVHQWDWLYH ZLGH VSDQ QHDU VWR GLPHQVLRQDO ERGLHV LV VKRZ QYHUVXV VK
5H\QROGVQXPEHULQ) LJ D $ OODERGLHVKDYHKLJK& DVYHU\ORZ FUHSLQJ IORZ 5H" ZKL
VKH VSUHDG DSDUWDWKLJK 5H\QROGVQXPEHUVDFFRUGLQJ WR WKHLUGHJUHH RI VWUHDPOLQLQJ $OOYDOXHVF
& DUHEDVHGRQWKHSODQUP DUHD[FHSWWKHSODWHQRUP DOWR WKHIORZ 7KHELUGVDQGWKHVLSSODQ
DUH RI FRXUVH QRWYHU VWR GLPHQVLRQDO KDYLQJ RQO PRGHVWVSDQ QHQJWK 1 RWH WKDWEHLUGV DUH QF
QHDU DVHILFLHQWDVPRGHUQVDLSSODQHVRUDLUROV> @





) LJ ' UDJ FRHIILFLHQWV RI VPRRWK ERGLHV DWYDU 0 DFK QXPEHUV *a* VZLR GLPHQVLRQDO
ERGLHV *b* WKUHH GLPHQVLRQDOERGLHV 1 RWH WKH5H\QROGVQXPEHULQGHSHQGHQFHRI EOXQWERGLHVD
KLJK5H

&UHHSLQJ) ORZ

,Q * * 6WRNHV VKRZHG WKDW LWKH5H1QROGV QXPEHU VPDOO5H VKHDFFHOHUDWR
WKP VLQWKH1DYLHU6WRNHVHTXDWRQV EF DUHQHJOLJLEOH 7KHIORZ LV WKP HGFUHSLQJ IORZ R
6WRNHV IORZ DQG LV D EDODQFH EHWZHHQ SUHVXUH JUDGLHQWDQG YLVFRXV VWUHVVHV &RQVLGHU DQ
PRPHQWXP UHGXFHWRVWRQH-DUHTXDWRQVIRUYHORFLW DQGSUHVXUH

$$\mathrm{Re} \ll 1: \qquad \nabla \cdot \mathbf{V} = 0 \qquad \text{and} \qquad \nabla p \approx \mu \nabla^2 \mathbf{V}$$

,I WKHJHRPHWW LVVLPSOH IRUH[DPSOH D VSKHUHRUGLVN FORVHGIRUP VROXWRQVFDQEHIRXQG DQ
WKHERG GUDJ FDQEHFRPSXWG> @6WRNHVKLP VHO SURYLGHGWKHVSKHUHGUDJIRUPXOD

$$F_{\text{sphere}} = 3\pi\,\mu U d$$

or

$$C_D = \frac{F}{\frac{1}{2}\rho U^2\,\frac{\pi}{4}d^2} = \frac{24}{\rho U d/\mu} = \frac{24}{\mathrm{Re}_d} \tag{7.64}$$

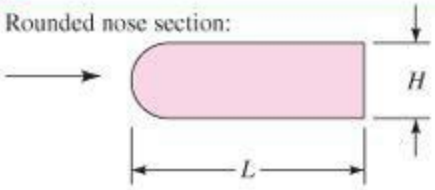
7KLVUHDWRQLVSORWWGLQ) LJ EDQGLVVHQWRHDFFXUDWIRUDERXW5H "
7DEOH JLYHVDIHZ GDWDRQGUDJ EDVHGRQIURQWDODUHD RI VWR GLPHQVLRQDOERGLHVRI YDULRX
FURVVVHFWLRQ DV5H 7KHVKDSHGJHGERGLHV ZKLFKWQGVRFXVHIORZ VHSDUDWRQUHJDGGHVR
WKHFKDUDFWULRI WKHERXQGDU OD\HU DUHLQVHQVLWLYHWR WKH5H QROGV 7KHHOOLSF F\OQGHU
EHLQJ VPRRWKO URXQGHG KDYHWKHORPLQDUWRWXEXOHQWWDQVLWRQHIHFWRI) LJV DQG DQG DU
WKHUHRUHTXLWVHQVLWLYHWRZKHWKHUWKHERXQGDU OD\HULVODPLQDURUWXEXOHQW

7DEOH ' UDJ RI 7Z R' LPHQVLRQDO%RGLHVDW5H

Shape	C_D based on frontal area	Shape	C_D based on frontal area	Shape	C_D based on frontal area
Square cylinder:		Half cylinder:		Plate:	
→ ▭	2.1	→ ◗	1.2	→ ▯	2.0
→ ◆	1.6	→ ◗	1.7	Thin plate normal to a wall: → ◣	1.4
Half tube:		Equilateral triangle:			
→ ◖	1.2	→ ◀	1.6		
→ ◗	2.3	→ ◀	2.0	Hexagon: → ⬢	1.0 ↑ 0.7

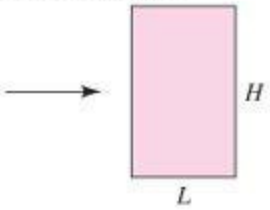
Shape	C_D based on frontal area

Rounded nose section:



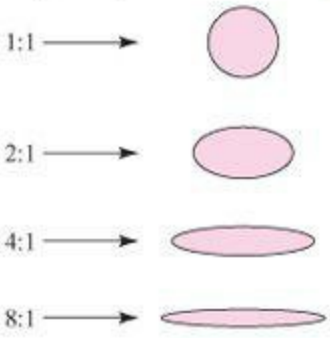
L/H:	0.5	1.0	2.0	4.0	6.0
C_D:	1.16	0.90	0.70	0.68	0.64

Flat nose section:



L/H:	0.1	0.4	0.7	1.2	2.0	2.5	3.0	6.0
C_D:	1.9	2.3	2.7	2.1	1.8	1.4	1.3	0.9

Elliptical cylinder:

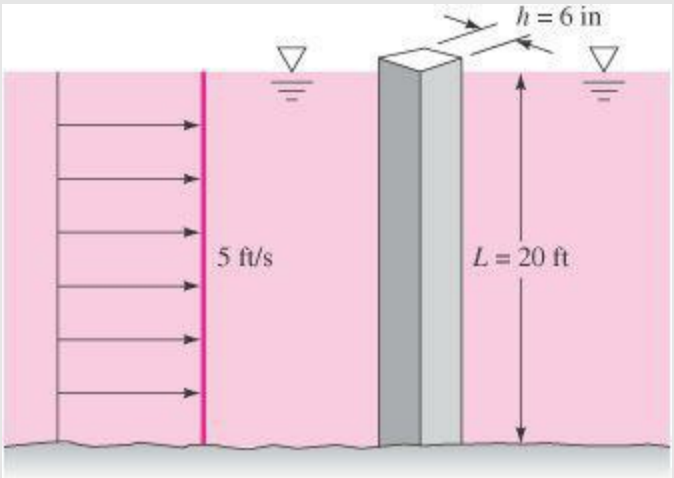


	Laminar	Turbulent
1:1	1.2	0.3
2:1	0.6	0.2
4:1	0.35	0.15
8:1	0.25	0.1

(; $0 3/ (

$ VTXDUH LQSLQQLQJ LVDFWGRQE\ DZDAUIGRZ RI IWVVKDWLV IWGHHS DVVKRZQLQ) LJ ((VWDPDAWKHPDWLPXP EHQGLQJ H[HUWGE\ WKHIGRZ RQWKHERWWRP RI WKHSLQQQJ

(



6RXWLRQ

$VXPHVHDZDMUZLWK! VOXJVIWDQGNLQHPDWF YLVFRVLW IWV : LWKDSLQQ
ZLGWKRI IWZHKDYH

$$\mathrm{Re}_h = \frac{(5\ \text{ft/s})(0.5\ \text{ft})}{0.000011\ \text{ft}^2/\text{s}} = 2.3 \times 10^5$$

7KLVLVWKHUDQJHZKHUH7DEOH DSSOLHV 7KHZRUWFDVHRFFXUZKHQWKHIORZ VWUNHVWKHIO
VLGHRI WKHSLQQ & § 7KHIURQDODUHDLV$ /K IW IW IW7KHGUDJLVHXWP DW
E\

$$F = C_D(\tfrac{1}{2}\rho V^2 A) \approx 2.1(\tfrac{1}{2})(1.99\ \text{slugs/ft}^3)(5\ \text{ft/s})^2(10\ \text{ft}^2) = 522\ \text{lbf}$$

,I WKHIORZ LVXQLRUP WKHFHQWURI WKLVIRUFHVKRXOGEHDWDSSUR[LPDWHO PLGGHSWK 7KHUHRUHWK
ERWWRP EHQGLQJ PRPHQWLV

$$M_0 \approx \frac{FL}{2} = 522(10) = 5220\ \text{ft} \cdot \text{lbf} \qquad \textit{Ans.}$$

$FFRUGLQJ WR WKH IOH[XUH IRUPXOD IURP VWHQJWK RI PDWHULDOV WKH EHQGLQJ VWUHVV DWWKH ERWWR
ZRXOGEH

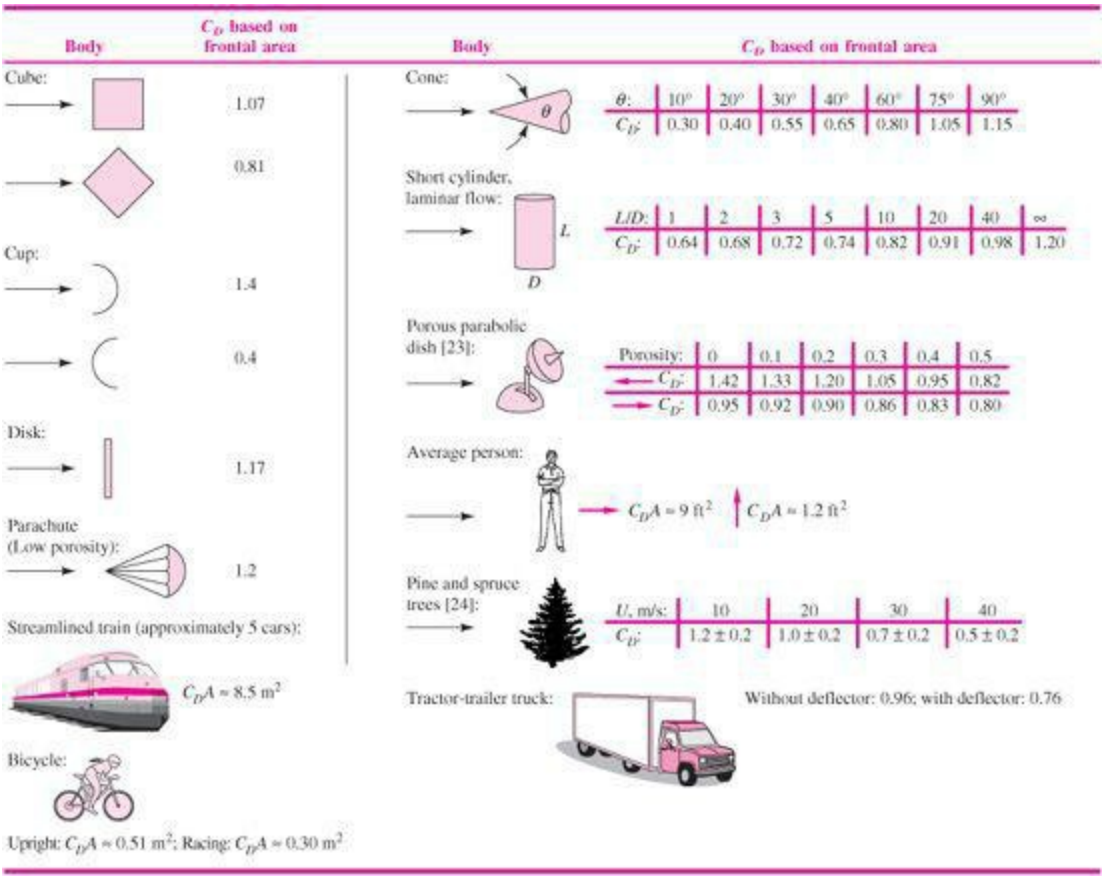
$$S = \frac{M_0 y}{I} = \frac{(5220\ \text{ft} \cdot \text{lb})(0.25\ \text{ft})}{\frac{1}{12}(0.5\ \text{ft})^4} = 251{,}000\ \text{lbf/ft}^2 = 1740\ \text{lbf/in}^2$$

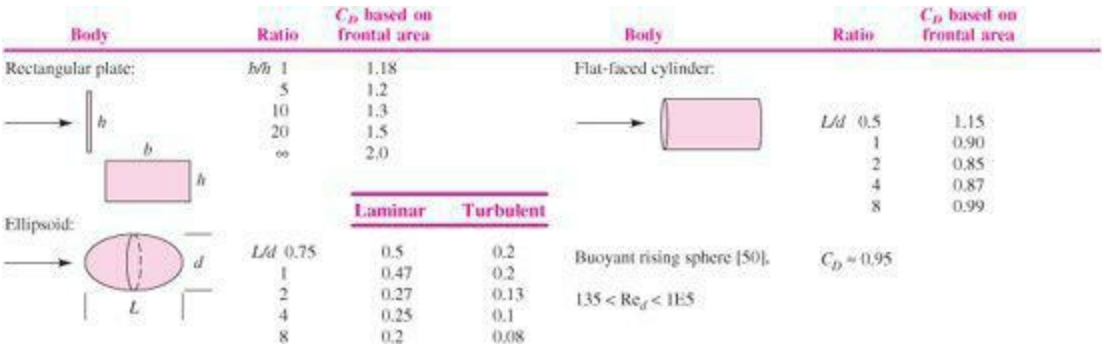
WREHPXOWLSOLHG RI FRXUVH E\ WKHVWUHVVFRQFHQWUDWLRQIDFWRUGXHWRWKHEXLOWLQHQGFRQGLWLRQV

7KUHH' LPHQVLRQDO%RGLHV

6RPHGUDJFRHILFLHQWVRI WKUHHGLPHQVLRQDOERGLHVDUHOLVWHGLQ7DEOH DQG) LJ E $JDL
ZHFDQFRQFOXGHWKDWWKDSHGJHVDZD\ VFDXVHIORZ VHSDUDWLRQDQGKLJKGUDJ WKDWVLQHQWLY HWRWK
5H\QROGV QXPEHU 5RXQGHG ERGLHV OLNH WKH HOOLSVRLG KDYH GUDJ WKDWGHSHQGV RQ WKH SRLQWV
VHSDUDWLRQ VR WKDWERWK WKH5H\QROGV QXPEHUDQG WKHFKDUDFWHULVWLF HQGXUDQFH OHQJWK RI WKHERXQGDU\ OD\HUDUH LPSRUWDQ
%RG\ GHQJWK Z LOOJHQHUDOO GHFUHDVH SUHVVXUH GUDJ E\ PDNLQJ WKHERG UHDLYH PRUH VWUHDPOLQHU EX
VRRQHUURUDWWUWKHIUFWRQGUDJ Z LOOFDWK XS)RUWKHOWDFHG F\OLQGHUL7DEOH SUHVVXUHGUD
GHFUHDVHVZ LWK/ GEXWUFWRQLQFUHDVHV VR WKDWP LQP XP GUDJ RFF XUVDWDERXW G

7DEOH ' UDJ RI 7KUHH' LPHQVLRQDO%RGLHVDW6H





%XR\ DQW5 LVLQJ / LJKW6SKHUHV

7KH VSKHUH GDWD LQ) LJ E DUH IRU UIL[HG PRGHO YLQZ LQG WXQQHO VWHDG\ DQG IURP IDOOLQJ VSKHUH WHVW DQG LQGLFDWH D GUDJ FRHILFLHQW RI DERXW LQWKH UDQJH (5H (,W DV UHFRJQL SRLQWHG RX > @WKDW WKLV VLQJOH YDOXH IRU D IUHO UHJLQ EXR\DQW VSKHUH RUEXEEOH ,I WKH VSKHUH LV OLJKW! VSKHU ! IOXLG DZ DNHLQXDELOLW\ DULVHV LQ WKH UDQJH 5H (7KH VSKHUH WHQGV WR VLJDJ DQ DQJOH RI DERXW ƒ IURP WKH KRUL]RQWDO 7KH GUDJ FRHILFLHQW DV DSSUR[LPDWH GRXEOHG VR D DYHUDJH YDOXH& DV OLVWHG LQ7DEOH > @) RU D KHDYLHU EOD ! VSKHU § ! IOXLG WKH EXR\DQ VSKHUH UHVHUYHUWDOO DQG WKH GUDJ FRHILFLHQW RQOQZ V WKH VDQGGDG FXUYH LQ) LJ E

(; $0 3/ (

$ FRUGLQJ WR 5H WKH GUDJ FRHILFLHQW RI DEOOP S EDVHG RQ VXUIDFH DUHD LV DSSUR[LPDWH

LI 5H/ ! $ FHUWDLQEOPS LV P ORQJ DQG KDV D VXUIDFH DUHD RI P (VWLPDWH WK
SRZHU UHTXLUHG WR SURSHOO WKLV EOPS DW P VDWDWLDQG DUG DOWLWXGH RI P

6ROXWLRQ

- $VVXPSWLRQV : H KRSH WKH 5H\QROGV QXPEHU ZLOO EH KLJK HQRXJK WKDW WKH JLYHQ GDWD DU
YDOLG
- $SSURDFK ' HWHUPLQHLI 5H/ ! DQG LI VR FRPSXWH WKH GUDJ DQG WKH SRZHU UHTXLUHG
- 3URSHUWLHV YDOXHV 7DEOH$ DW P ! NJ P 7 . WKXV § (
NJ P V
- 6ROXWLRQ WKHSV ' HWHUPLQHWKH5H\QROGVQXPEHURI WKHEOPS

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{(1.112\ \text{kg/m}^3)(18\ \text{m/s})(75\ \text{m})}{1.75\text{E-}5\ \text{kg/m-s}} = 8.6\text{E}7 > 10^6 \qquad \text{OK}$$

7KH JLYHQ GUDJ FRHIILFLHQWLV YDOLG &RPSXWH WKH EOPS GUDJ DQG WKH SRZHU GUDJ î
YHORFLWLHV

$$F = C_D \frac{\rho}{2} U^2 A_{\text{wet}} = (0.006)\frac{1.112\ \text{kg/m}^3}{2}(18\ \text{m/s})^2\,(3400\ \text{m}^2) = 3675\ \text{N}$$

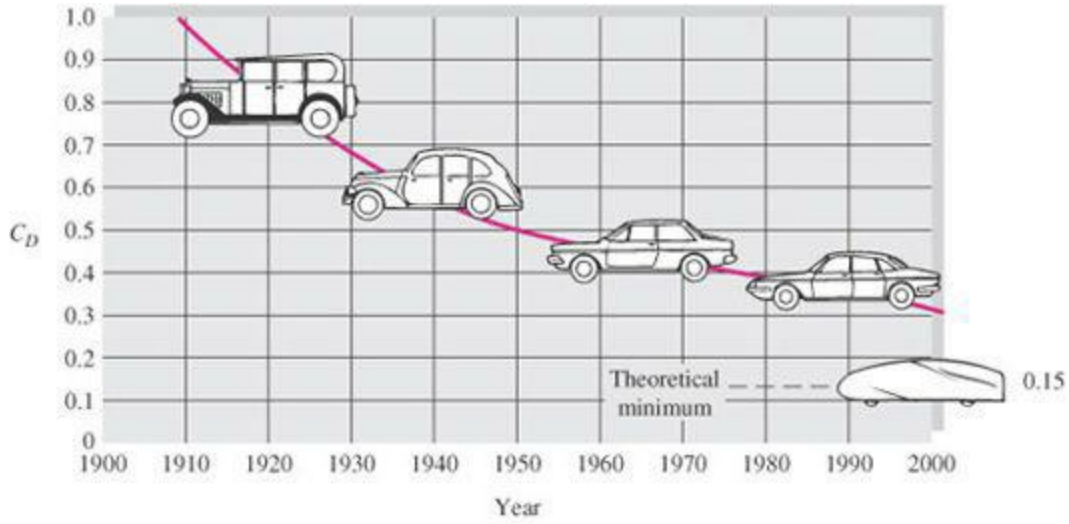
$$\text{Power} = FV = (3675\ \text{N})(18\ \text{m/s}) = 66{,}000\ \text{W} \quad (89\ \text{hp}) \qquad \textit{Ans.}$$

- &RPPHQWV 7KHVH DUH QRP LQDO HWWPDWV ' UDJ LV KLJKO GHSHQGHQWRQ EHWK VKDSH DQ
5H\QROGVQXPEHU DQG WKH FRHIILFLHQW&D KDVFRQVLGHUDEOHXQFHUWDLQWV

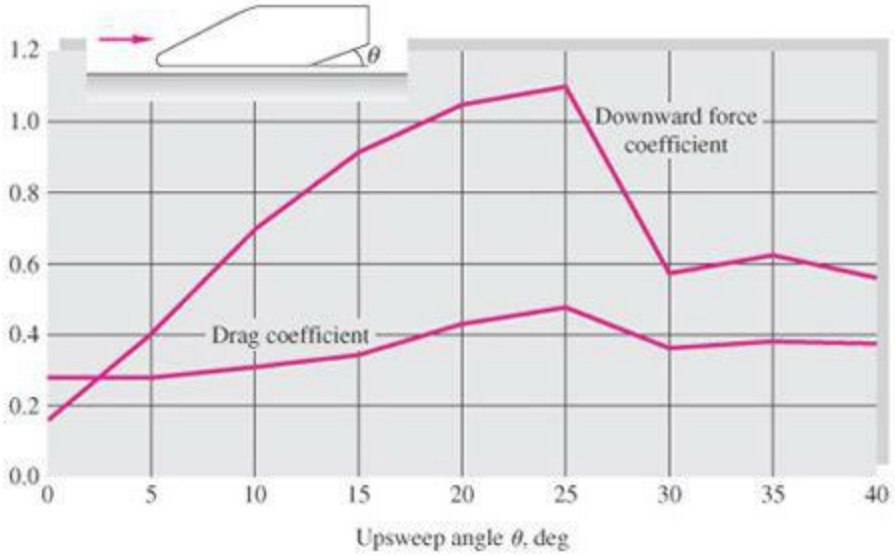
$HURG\QDPLF) RUFHVRQ5RDG9HKLFOHV

$ XVHIXO REOHVDQG WKFNVDUHQRZ WKHVXEMHFWRI PXFKUHVHDUFKRQDHURG\QDPLF IRUFHV ERWKQLWDQ
GUDJ > @$ WWHDWWRQHW[WERRNLVGHYRWHGWR WKHVXEMHFW> @$ YHU UHDGDEOHGHVFULSWRQRI UDF
FDUGUDJ LVJLYHQE\ . DW> @&RQVXPHUPDQXDFWXUHUDQGJRYHUQPHQWWHVWKDVFV\FOHGEHWHH
KLJK VSHHGKLJK KRUVHSRZHUDQG ORZ HU VSHHGORZ HU GUDJ %HWWU WUHDWPOQLQJ RI FDU VKDSHV KD
UHVXOWHGRYHUWKH\HDUVLQDOIUHGFUDVHLQWKHDXWRPRELGUDJ FRHILFLHQWDVVKRZQLQ) LJ D
0 RGHUQ FDU VKDYH DQ DYHUDJH GUDJ FRHILFLHQWRI DERXW EDVHGRQ WKH IURQWDO DUHD 6LQFH WK
IURQWDO DUHD KDV DOVR GHFUHDVHG VKDUSO WKH DFWXDO DUDZ GUDJ IRUFH RQ FDU VKDV GURSSHG HYHQ PRU
WKDQLQGLFDWHG LQ) LJ D 7KHWKRUHWFDOPLQLPXP VKRZQLQWKHILJXUH & LVDERXWWLJKWR
DFRPPHUFLDODXWRPRELOH EXWRZHUYDOXHVDUHSRVVLEOHIRUH[SHULPHQWDOYHKLFOHV VHH3URE 3
1RWWKDWEDVLQJ & RQWKHIURQWDODUHDLVDZNZDUG VLQFHRQHZRXOGQHGDQDFFXUDWGUDZLQJ RI WK
DXWRPRELOHWR HWWPDWLWIURQWDODUHD) RUWKLV UHDVRQ VRPHWHFKQLFDODUWFOHV VLPSO UHSRUWWKHUDZ

GUDJ LQQHZ WRQVRUSRXQG IRUFH RUWKHSURGXFW& $



(*a*)



(*b*)

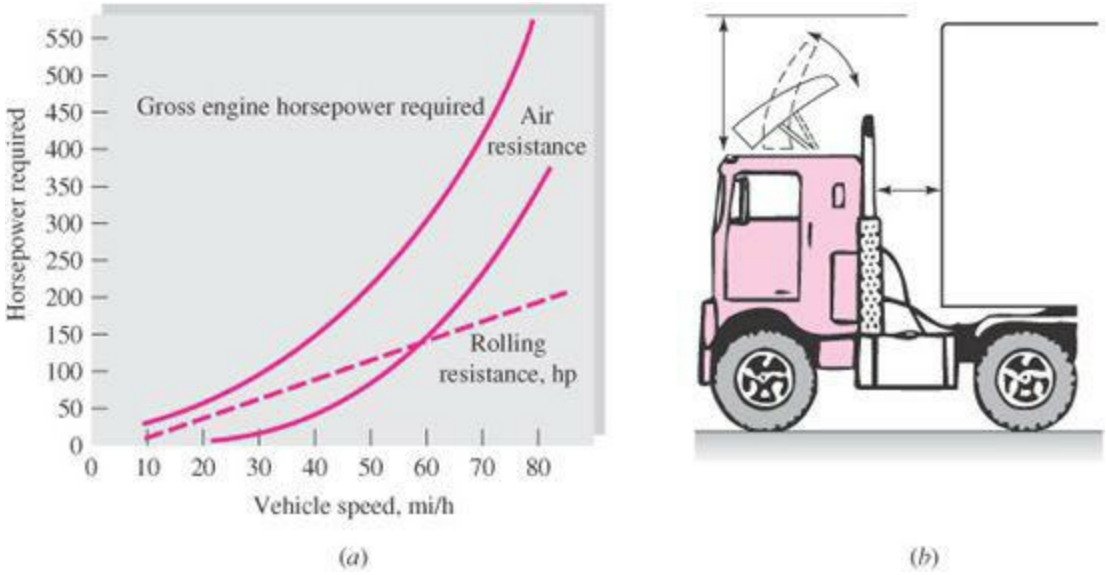
) LJ $ HURG\ QDPLFV RI DXWRPRELOHV *a* WKH KLVWRULFDO WHQGHQFLHV IURP 5H *b* HIHFWV RI ERWWRP UHDUXSVZHHS DQJOH RQ GUDJ DQG GRZQZDUG OLIWRUFH IURP 5H

0DQ\ FRPSDQLHV DQG ODERUDWRULHV KDYH DXWRPRWLYH ZLQG WXQQHOV VRPHWLPHV FDOOHG DQGRUZLQ PRYLQJ IORRUV WR DSSUR[LPDWH DFWXDO RQ URDG FRQGLWLRQV 7KH EOXHVKDSHV RI PRVW DXWRPRELOH VWUHHWHUZLWK WKHLU SUR[LPLW\ WR WKH JURXQG FDXVH D ZLGH YDULHWV RI IORZ DQG JHRPHWULF HIHFW 6LPSOHFKDQJHVLQSDUWRI WKHVKDSHFDQKDYH DODUJHLQIOXHQFHRQDHURG\QDPLFIRUFHV) LJXUH E VKRZV IRUFHGDWDE\ %HDUPDQHWDO > @RUDQLGDQHG VPRRWKDXWRPRELVKDSHZLWKXSVZHHSL WKH UHDUR WKHERWWRP VHFWLRQ : HVHHWKDWE\ VLPSOGGLQJ DQXSVZHHSDQJOHRI *f* ZHFD TXDGUXSOHWKHGRZQZDUGIRUFH JDLQLQJ WUHWDFWLRQDWWKHH[SHQVHRIGRXEOLQJ WKHGUDJ)RUWK VWXG\ WKHHIHFWRI DPRYLQJ IORRUZDV VPDOO DERXWD SHUFHQWLQFUHDVHLQERWKGUDJ DQG GO FRPSDUHGWRDIL[HGIORRU

,WLVGLIILFXOWWRTXDQWLI\ WKHH[DFWHIHFWRI JHRPHWULF FKDQJHVRQDXWRPRWLYHIORZ VLQFH IR H[DPSOH FKDQJHVLQDZLQGVKLHOGVKDSHPLJKWLQIOXHQFHWZLWKGRZQWUDP IORZ RYHUWKHURRI DQGWKQ 1HYHUWKHOHVV EDVHGRQFRUUHODWLRQRI PDQ PRGHODQGIXOOVFDOHWHVWV 5H SURSRVHVDIRUPXODIR

DXWRPREHGUDJ WKDWDGGVVHSDUDWHIHFWVXFKDVIURQWHQGV FRZOVIHQGHUV ZLQGVKLHG URRIV DQG
UHDUHQGV

) LJXUH VKRZVWKHKRUVHSRZHUUHTXLUHGWRGULYHDWSLFDOWDFWRUWUDLOHUWKFNDWSHGVXS
WR PLK IWVRU P V 7KHURQQJUHVLVWDQFHLQFUHDVHVQHDUO DQGWKHDLUGUDJTXDGUDWFDOO
ZLWKVSHHG & § 7KHVWR DUHDERXWHTXDO LPSRUWDQWDW PLK $VVKRZQLQ) LJ E DL
GUDJ FDQEHUHGXFHGE\ DWDFKLQJ DGHOHFWRUWR WKHVRS RI WKHWDFWRU ,I WKHDQJOHRI WKHGHOHFWRUL
DGXVWHGWR FDUU WKHIORZ VPRRWKO RYHUWKHVRS DQG DURXQG WKHVLGHV RI WKHWDLOHU WKHUHGXFWLRQL
& LV DERXW SHUFHQW7KXV DW PLK WKHVRDO UHVLVWDQFH LV UHGXFHG SHUFHQWZLWK
FRUUHVSRQGLQJ UHGXFWLRQLQIXHOFRQVDQG RUWSWPHIRUWKHWUFN) XUWKHUUHGXFWLRQRFFXUZKH
WKHGHOHFWRULV OHQJWKHQHG WR FRYHUWKHJDS EHWZHHQFDE DQG WDLOHU 7KLV WSH RI DSSOHG IOLG
HQJLQHHULQJ FDQEHDOLJHIFWRULQPDQ RI WKHFRQVHUYDWRQ RUHQWDWLRQRUHQWDQSURFOPVRI WK
IXWXUH> @



) LJ ' UDJ UHGXFWLRQ RI D WUDFWRUWUDLOHUWXFN *a* KRUVHSRZHUUHTXLUHGWR RYHUFRPH
UHVLVWDQFH *b* GHOHFWRUDGGHGWR FDE UHGXFHVDLUGUDJ E\ SHUFHQW*Uniroyal Inc.*

3URJUHVV LQ&RPSXWDWLRQDO) OXLG' \QDPLFV &)' PHDQV WKDWFRPSXWFDWLRQDO YHKLFOHLQWHUI
FDQEH SUHGLFWHG ZHOO 5HHUHQFH FRPSDUHV RQH HTXDWLRQ DQG WR HTXDWLRQ WXEXOHQFH
PRGHOV > @ZLWK 1 $ 6 $ GDWD IRU D VLPSOLILHG WUFWRUWUDLOHUPRGHO (YHQ ZLWK WR PLGDRQ PHV
SRLQWV WKH SUHGLFWHG YHKLFOH GUDJ LV IURP WR SHUFHQWKJKHU WKDQ WKH PHDVXUHPHQW 7K
WXEXOHQFH PRGHOV GR QRWUHSURGXFH WKH SUHVXUHV DQG ZDNH WXFWXUH LQ WKH UHDUL WKH YHKLFO
1HZ HUPRGHOV VXFK DV/ DUJH(GG\ 6LPXODWRQ /(6 DQG' LUHFW1 XPHULFDO6LPXODWRQ ' 1 6 Z
QR GRXEWLPSURYHWKHFDOFXODWRQV

(; $0 3/ (

$ KLJK VSHHGFDUZLWKP NJ & DQG$ P GHSOR\V D P SDUDFKXWHWR VORZ
GRZQIURP DQLQLWLDOYHORFLW RI P V) LJ ($VXPLQJ FRQVWDQW& EUDNHVIUHH DQG QR
URQQJ UHVLVWDQFH FDOFXODWWKHGLVWDQFH DQG YHORFLW RI WKHFDUDWU DQG V) RUDL
DWXPH! NJ P DQG QHJOHFWLQJUHUHQFHEHWHQWKHZDNHRI WKHFDUDQGWKHSDUDFKXW

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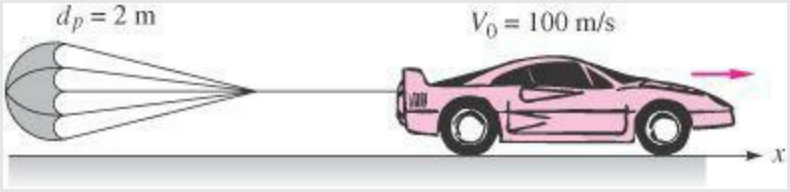


6RXWLRQ

1HZWRQ¶V6HFRQG/DZ DSSOLHGLQWKHGLUHFWLRQRI PRWLRQJLYHV

$$F_x = m\frac{dV}{dt} = -F_c - F_p = -\frac{1}{2}\rho V^2(C_{Dc}A_c + C_{Dp}A_p)$$

ZKHUHVXEVFULSWF GHQRWHVWKHFDUDQGVXEVFULSWS WKHSDUDFKXWH7KLVLVRI WKHIRUP

$$\frac{dV}{dt} = -\frac{K}{m}V^2 \qquad K = \sum C_D A\frac{\rho}{2}$$

6HSDUDWHWKHYDULDEOHVDQGLQWHJUDWH

$$\int_{v_0}^{v}\frac{dV}{V^2} = -\frac{K}{m}\int_0^t dt$$

or
$$V_0^{-1} - V^{-1} = -\frac{K}{m}t$$

5HDUUDQJHDQGVROYHIRUWKHYHORFLW9

$$V = \frac{V_0}{1 + (K/m)V_0 t} \qquad K = \frac{(C_{Dc}A_c + C_{Dp}A_p)\rho}{2} \tag{1}$$

: HFDQLQWHJUDWHWKLVVRIILQGWKHGLVWDQFHWDYHOHG

$$S = \frac{V_0}{\alpha}\ln(1 + \alpha t) \qquad \alpha = \frac{K}{m}V_0 \tag{2}$$

1RZ ZRUNRXWWRPHQXPEHUV)URP 7DEOH & S§ KHQFH

$$C_{Dc}A_c + C_{Dp}A_p = 0.3(1\text{ m}^2) + 1.2\frac{\pi}{4}(2\text{ m})^2 = 4.07\text{ m}^2$$

Then
$$\frac{K}{m}V_0 = \frac{\frac{1}{2}(4.07\text{ m}^2)(1.2\text{ kg/m}^3)(100\text{ m/s})}{2000\text{ kg}} = 0.122\text{ s}^{-1} = \alpha$$

1RZ PDNHDVHEOHRI WKHUHVXOWVIRU9 DQG6 IURP (TV DQG

$ LU UHVLVWDQFH DORQH Z LOO QRW WXUQ D ERG\ FRP SOHWHO ,I \RX GRQ¶W DSSO\ WKH EUDNHV \RX¶OO KDZ D \ WR WKH<XNRQ7HUULWRU DQG VWDUW OLQJ

t, s	1	10	100	1000
V, m/s	89	45	7.6	0.8
S, m	94	654	2110	3940

2 WKHU0 HWKRGV RI ' UDJ 5HGXFWLRQ

6RPH WLPHV GUDJ LV JRRG IRU[DP SOH Z KHQ XVLQJ D SDUDFKXWH ' R QRWXPS RXWR DQ DLUSODQ KRQLQJ DIODWSODWH SDUDFKXWHV\RXUP RWRQ VH3URE 3 0 RWRU WKRXJK GUDJ LVEDGDQG VKRXO EH UHGXFHG 7KH FDWLFDO PHWKRG RI GUDJ UHGXFWLRQ LV WWHDPDQLQJ) LJV DQG) R H[DP SOH QRVH IDLUQJV DQG ERG\ SDQHOV KDYH SURGXFHG PRWRUF\FOHV WKDWFDQ WDY HORY HU PLK 0 RUH UHFQWUHVHDUFK KDV XQFRYHUHG RWKHUPHWKRGV WKDWKRG JUHDWSURPLVH HVSHFLDOO IRUWXEEO IORZ V

2 LOSLSHDQHV LQWRGXFH DQDQXOUWLS RI Z DWUVR UHGXFH WKH SXPSLQJ SRZ HU> @7K ORZ YLVFRVLW Z DWUULGHV WKHZ DOODQGUHGXFHVIULFWRQXS VR SHUFHQW 7XUEOHQWUFWRQLQQDTXLGIORZ VLVUHGXFHGXS VR SHUFHQWE\ GLVWRYLQJ VP DOOP RXQWR D KLJK PRPHXDUZHJKWSRO PHU DGGLWLYH > @: LWKRXWFKDQJLQJ SXPSV WKH 7UDQV$ODN 3LPHDQH6\VWP 7$36 LQFUHDVHG RLOIORZ SHUFHQWE\ LQMHFWQJ VP DOOP RXQWR SRO PH GLVWRYHGLQNURVHQH 6WHP RUHQWG VXUDFHYH JURRYH PLFURUEOWV FDQ UHGXFH WKUEXOHQWUFWRQ XS VR SHUFHQW> @5LEOWKHLJKWVDUHR RUGHU PP DQGZ HUXVHGRQWKH6WDVDQG6WLSV\DFKWKX LQWKH$PHULFDV&XSUDFHV 5LEOWVDUHDOR HIFHQWYHRQDLUFUDWWNQV 6PDOO QHDUZDOODUJHHGG\ EUHDNXS GHYLFHV /(%8V UHGXFHORFDOWKUEXOHQWUFWRQXS V SHUFHQW> @+RZ HYHU RQH PXVWDGG WKHVH VP DOOWXFWXUHV VR WKH VXUDFHDQG/(%8 GUP D\ EHVJQILFDQW

$ LUPLFUREEOHV LQMFWHG DWWKH Z DOORI D Z DWUIORZ FUHDWHD ORZ VKHDUEEOHEODQN> @$ WKLJKYRLGIUDFWRQV GUDJ UHGXFWRQFDQEH SHUFHQW

6SDQZ LVH WDQVYHUVH ZDOORVFLOODWRQP D\ UHGXFHWXEXOHQWUFWRQXS VR SHUFHQW> @ $FWLYHIORZ FRQWRO HVSHFLDOO RI WXEXOHQWORZ V LV WKH Z DY HRI WKH IXWUH DV UHYLHZ HG L 5H 7KHVH PHWKRGV JHQHUDOO UHTXLUH H[SOGLWXUH RI HQHJ\ EWFDQ EH Z RUWK W) R H[DP SOH VDQJHQWDOEORZ LQJ DWKH UHDUR DQ DXR> @HYRNHV WKH&RDQGD HIFWLQZ KLFK WK VHSDUDWG QHDUZ DNHIORZ DWWDFKHV LWHO VR WKH ERG\ WXUDFH DQG UHGXFHV DXR GUDJ XS VR SHUFHQW

' UDJ UHGXFWRQ LV SUHVHQW DQ DUHD RI LQWHUHVWLQJ UHVHDUFK DQG DSSOHV WR P DQ WSHVR DLUORZ V> @DQG Z DWUIORZ VIRUERWKY HKLFOHV DQG FRQGXLW

' UDJ RI 6XUIFH6KLSV

7KH GUDJ GDWJLY HQ VR IDU VXFK DV7DEOHV DQG DUH IRUERGLHV³IXOO LPPHUVHG LQ DIUH VWUHP² WKDWV Z LWK QR IUHH VXUDFH ,I KRZ HYHU WKH ERG\ PRYHV DWRU QHDU D IUHH OTXLG VXUDFH

ZDYHPDNLQJGUDJEHFRPHVLPSRUWDQWDQGLVGHSHQGHQWRQERWKWKH5HQROGVQXPEHUDQGWKH)URXG
QXPEHU7RPRYHWKURXJKDZDWHUVXUIDFHDVKLSPXVWFUHDWHZDYHVRQERWKVLGHV7KLVLPSOLH
SXWWQJHQHUJ\LQRWKHZDWHUVXUIDFHDQGUHTXLUHVDILQLWHGUDJIRUFHWRNHHSWKHVKLSPRYLQJHYHQL
DIULFWLRQOHVVIOXLG7KHVRDOGUDJRDVKLSFDQWKHQEHDSSUR[LPDWHGDVWKHVXP RIIULFWLRQGUDJDQ
ZDYHPDNLQJGUDJ

) §) IUF) ZDYH RU& § & IUF & ZDYH

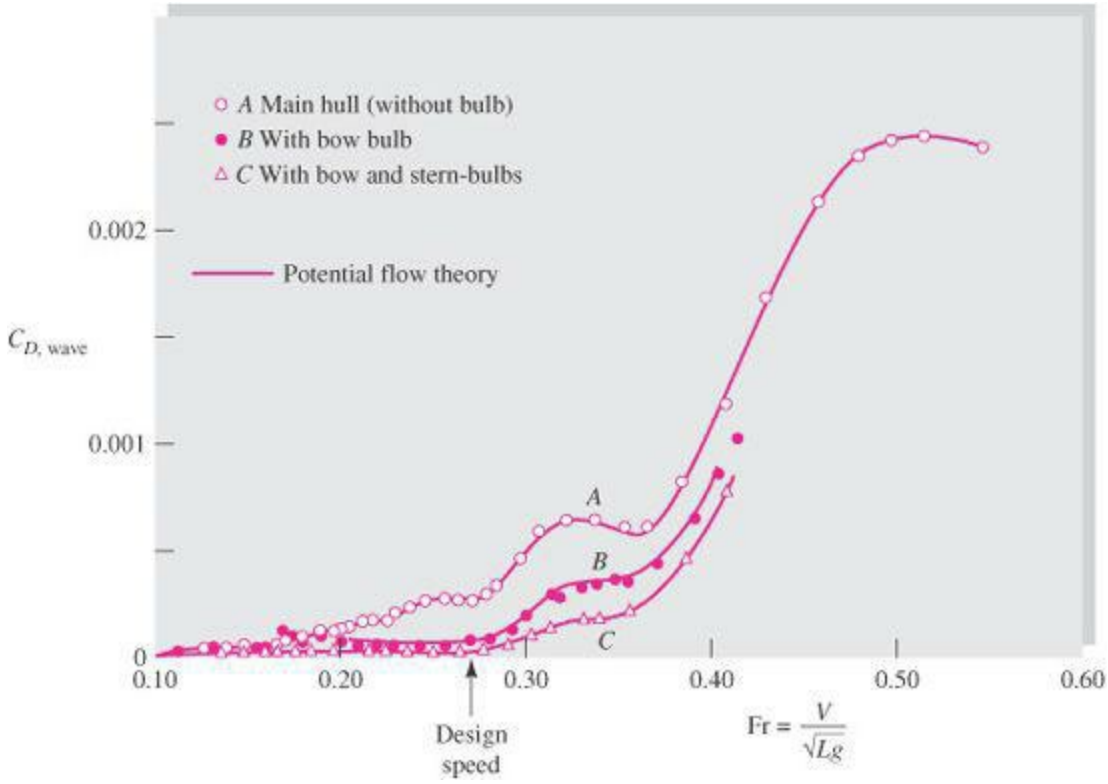
7KHIULFWLRQGUDJFDQEHHVWLPDWHGE\ WKH VKLEXOHQWVIODWSODWHIRUPXOD (T EDVHGRQWK
EHORZ ZDWURUZHWWHGDUHDRI WKHVKLS

5HHUHQFH LVDQLQWHUHVWLQJ UHYLHZ RI ERWKWKHRU DQGH[SHULPHQWRUZDNHPDNLQJ VXUFH
VKLS GUDJ * HQHUDOO VSHDNLQJ WKH ERZ RI WKH VKLS FUHDWHV D ZDYH V\VWHP ZKRVH ZDYHOHQJWK L
UHODWHG WR WKH VKLS VSHHG EXWQRWQHFHVVDULO WR WKH VKLS OHQJWK ,I WKH VWHHS RI WKH VKLS LV D ZDY
WURXJK WKH VKLS LVHVHQWLDOO FOLPELQJ XSKLOODQGKDVKLJK ZDYHGUDJ ,I WKH VWHHS LV D ZDYHFUHVWWK
VKLS LV QHDUO OHYHODQG KDV ORZ HUGUDJ 7KH FULWLFDO RQ IRU WKHVHZR FRQGLWLRQV UHVXOWLQ FHUWDL
DSSUR[LPDW) URXGHQXPEHUV> @

$$\mathrm{Fr} = \frac{V}{\sqrt{gL}} \approx \frac{0.53}{\sqrt{N}} \qquad \begin{array}{l}\text{high drag if } N = 1, 3, 5, 7, \ldots; \\ \text{low drag if } N = 2, 4, 6, 8, \ldots\end{array} \qquad (7.65)$$

ZKHUH9 LV WKH VKLS¶V VSHHG / LV WKH VKLS¶V OHQJWK DORQJ WKH FHQWHUOLQH DQG1 LV WKH QXPEHUR
KDOO OHQJWKV IURP ERZ WR WXUQ RI WKH GUDJ PDNLQJ ZDYHV\VWHP 7KH ZDYHGUDJ ZLOOLFUHDVHZLW
WKH) URXGHQXPEHUDQGRVFLOODWHEHWZHHQORZHUGUDJ) U§ « DQGKLJKHUGUDJ) U§
« ZLWKQHJOLJLEOHYDULDWLRQIRU) U 7KXVLWLVEHWWRGHVLJQDVKLS WRFUXLVHDW
1 6KDSLQJ WKHERZ DQG WXUQFDQIXUWKHUHGXFHZDYHPDNLQJGUDJ
) LJXUH VKRZV WKHGDWD RI ,QL> @IRUDPRGHOVKLS 7KHPDLQKXOO FXUYH$ VKRZV SHDN
DQGYDOOH\ VLQZDYHGUDJ DWWKHDSSURSULDW) URXGHQXPEHUV! ,QWRGXFWLRQRI DEXOE SURWUXVLR
RQ WKHERZ FXUYH% JUHDWO UHGXFHV WKHGUDJ $GGLQJ DVHFRQGEXOE WR WKH WXUQ FXUYH& LV WXL
EHWWU DQG,QL UHFRPPHQGV WKDWWKHGHVLJQ SHGRI WKLV WZR EXOE VKLS EHDW1) U§ ZKLF
LVDQHDU³ZDYHOHV´ FRQGLWLRQ ,QWKLVILJXUH& ZDYH LVGHILQHGDV) ZDYH !9 / LQWKHGRI XUQ
WKHZHWWGDUHD

7KHVROGFXUYHVLQ) LJ DUHEDVHGRQSRWQWLDOIORZ WKHRU IRUWKHEHORZ ZDWUKXOOVKDSH
&KDSWU LV DQ LQWRGXFWRQ WR SRWQWLDOIORZ WKHRU 0 RGHUQ FRPSXWWLV FDQ EH SURJUDPPHG IR
QXPHULFDO&)' VROXWLRQV RI SRWQWLDOIORZ RYHUWKHKXOOV RI VKLSV VXEPDULQHV \DFKWV DQGVDLOERDW
LQFOXGLQJ ERXQGDU OD\HUHIHFWGULYHQ\ WKHSRWQWLDOIORZ > @7KXVWKHUHWFDOSUHGLFWRQ RI IORZ
SDWWXUFHVKLSVLVQRZ DWDIDLU KLJK OHYHO6HHDOR5H



) LJ : DYH P DNLQJ GUDJ RQ D VKLS PRGHO *After Inui [27]* *Note:* 7KH GUDJ FRH ILFLHQW GHILQHG DV C_{DW} F ρV L

%RG\ ' UDJ DW+ LJK 0 DFK 1 XP EHUV

$ FFXUDWH GDWD SUHGLFWLRQ RI VKLS SRLQWDUH IRUQHDU LQFRPSUHVVLEOH IORZ V ZLWK 0 DFK QXPEHU DVVXPHG GHVV VKDQ DERXW %H\RQG VKLS YDULHV FRPSUHVVLELOLW\ FDQ EH YHU LPSRUWDQW ZLWK& IFQ5H 0 D $VKHVWHP 0 DFK QXPEHULQFUHDVHV DWWRPHVXEVRQLF YDOXH0 $FW VKDWGHSHQGV R VKH ERG\¶V EOXQWQHVV DQG VKLFNQHVV VKH ORFDO YHORFLW DWWRPH SRLQWQHDU WKH ERG\ VXUIDFH ZLO EHFRPH VRQLF ,I 0 D LQFUHDVHV EH\RQG 0 D$FW VKRFN ZDYHV IRUP LQWHQVLW\ DQG VSUHDG UDLVLQ VXUIDFH SUHVVXUHV QHDU WKH IURQW VKHEG DQG WKHUHRUHLQFUHDVLQJ WKH SUHVVXUHGUDJ 7KHHIHFWLD EH GUDP DW& LQFUHDVLQJ WQRG DQG \HDV DJR WKLV VKDSS LQFUHDVHZ DV FDOOHG WKH VRQL EDUULHU LPSOLQJ WKDWWKRXGQRWEH VXUPRXQWHG 2I FRXUVH LWFDQ EH? WKHUHVHQ& LV ILQLWH D VXSHUVRQLF EXOEHWKDYHSURYHGIRUFHQXUHV

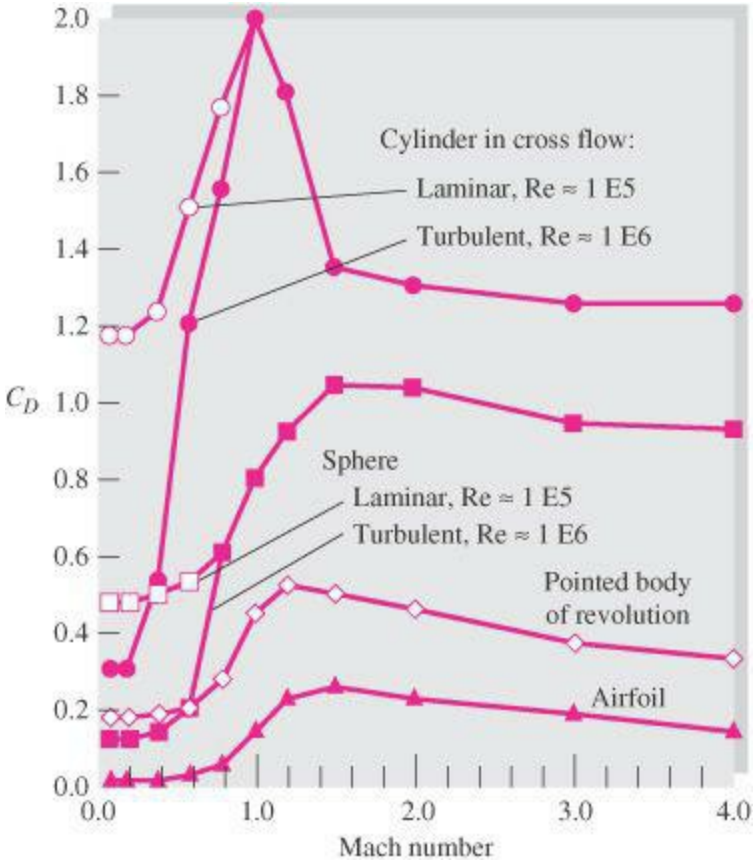
) LJXUH VKRZ V WKHHIHFWRI WKH0 DFK QXPEHURQ WKHGUDJ FRH ILFLHQWRI YDURXVERG\ VKDSH WWWGLQDU : HVHHWKDWFRP SUHVVLELOLW\ DIIHFWVEXQWERGLHVDUHU ZLW0 D$FWHQGQHUV IRUVSKHUHV DQG IRUDLURLOV DQG SRLQG SURMHFWLOHV $OR WKH5HQROGV QXP EH-ODP LQDU YHUXV WKUEXOHQWERXQGDU\ OD\HU IORZ KDV D OLJH HIHFWEHORZ 0 D$FWIRUVSKHUHV DQ FQGHUVEXWEHFRPHVXQLP SRUWDQWERYH0 D§ ,QFRQWDWWKH HIHFWRI WKH5HQROGV QXP EHUL VP DOOIRUDLURLOV DQG SURMHFWLOHV DQG LV QRWWKRZ Q LQ) LJ $ JHQHUDOVWDWHPHQWP LJKWGLYLGH 5HQROGVDQG0 DFK QXPEHUHIHFWDVIROORZ V

Ma ≤ 0.3: Reynolds number important, Mach number unimportant

0.3 < Ma < 1: both Reynolds and Mach numbers important

Ma > 1.0: Reynolds number unimportant, Mach number important

$ WXSHUVRQLF VSHHGV DEURDGEDZ VKRFNZDYH IRUPV LQ IURQWRI WKH ERG\ VHH) LJV E DQ
DQG WKH GUDJ LV PDLQO GXH WR KLJK VKRFN LQGXFHG SUHVVXUHV RQ WKH IURQW0 DNLQJ WKH ERZ I
VKDUS SRLQWHDQ VKDUSO UHGXFHWKH GUDJ) LJ EXWGRHV QRWHOP LQDWWKH ERZ VKRFN &KDSWU
JLYHV DEULH WUHDWHQWRI FRPSUHVVLEOH IORZ 5HHUHQFHV DQG DUH PRUH GYDQFHG WRSLFV
GHYRWHGHQWLUHO WRFRPSUHVVLEOHIORZ

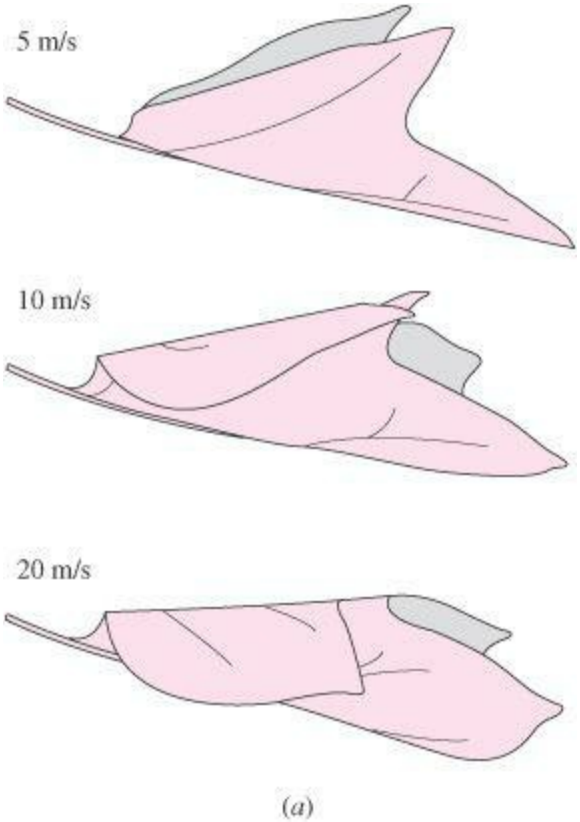


) LJ (IIHFWRI WKH0 DFK QXP EHURQWKH GUDJ RI YDULRXVERG\ VKDSHV ***Data from Refs. 23 and 29.***

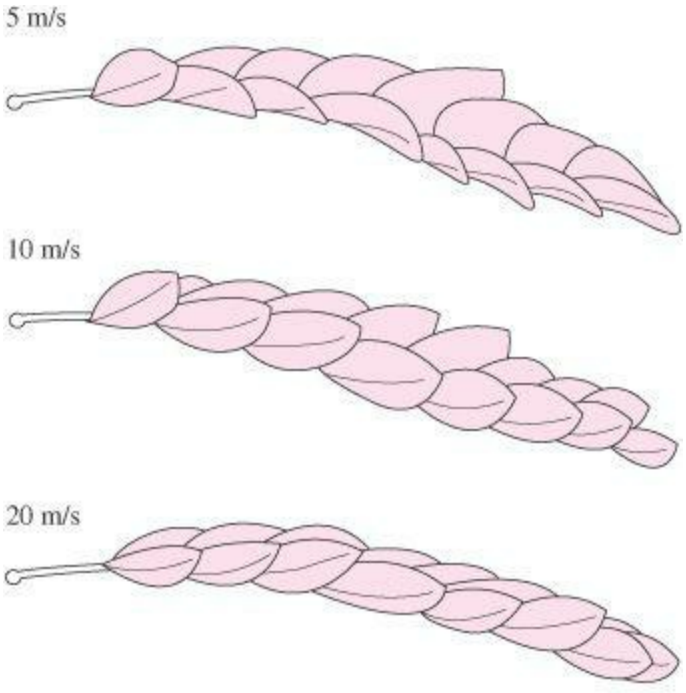
%LRORJLFDO UDJ 5HGXFWLRQ

$ JUHDWGHDO RI HQJLQHHULQJ HIRUWRHV LQWR GHVLJQLQJ PPHUVHGERGLHV WR UHGXFH WKHLUGUDJ 0 R\
VXFK HIRUWFRQFHQWUDWV RQ ULJLG ERG\ VKDSHV $ GLIIHUHQWSURFHVV FFXUV LQ QDWXUH DV RUJDQLVP
DGDSWV WR VXUYLYH KLJK Z LQGV RUFXUHQW DV UHSRUWHG LQ D VHULHV RI SDSHUV E\ 6 9RJHO> @$\$
JRRG H[DP SOH LV D WUHH Z KRVH IOH[LEOH WUXFXUH DOORZ V LWWR UHFRQLJXUH LQ KJK Z LQGV DQG WKX
UHGXFH GUDJ DQG GDP DJH 7UHH URRWX VWMP V KDYH HYROYHG LQ VHYHUDO Z D V WR UHVLWZ LQG LQGXFH
EHQGLQJ PRPHQW DQG WUXQN FURV VHFWRQV KDYH EHFRP H UHVLVWDQWWR EHQGLQJ EXWUHDWLYH HDX V
VZLWDQG UHFRQLJXUH : HVDZ VKLV LQ7DEOH Z KHUH WUHE GUDJ FRHILFLHQW > @UHGXFHG E\
SHUFHQWDV Z LQG YHORFLW LQFUHDVHG 7KHVKDSHRI WKHWUHFKDQJHVWRRFFHUWUHLWDQFH

7KH LQGLYLGXDO EUDQFKHV DQG OHDYHV RI DWWHDQR FXODQG FOXVWHUV UHGXFH GUDJ)LJXUH VKRZV VWHUHVXOWRI ZLQG WXQQHO [SHULPHQWV E\ 9RJHO> @$ WKDS WHHOHDI)LJ D EURDG DQ RSHQLQJ ZLQG FXOYLQR DFRQFDOOR GUDJ VKDSHDV ZLQGLQFUHDVHV $ FRPSRXQG EODFNZDOX OHDI JURXS)LJ E FOXWHUV LQR DOR GUDJ VKDSHDWKJK ZLQG VSHHG $OKRXJK GUDJ FRHILFLHQ ZHUHUHGXFHGXSVR SHUFHQWE\ IOH[LELOLW 9RJHOSRLQWRXWWKDWLJ WUXFWXUHVDUHVRPHWLPHVMX DVHIFLYH $QLQWHUHVWLQJ UHFQWX PSRVLXP > @Z DVGHYRWHGQLHO VR WKHVRQG PHFKDQLFV DQ IOXLGPHFKDQLFVRI EIRORJLFDORUJDQLVPV



(*a*)

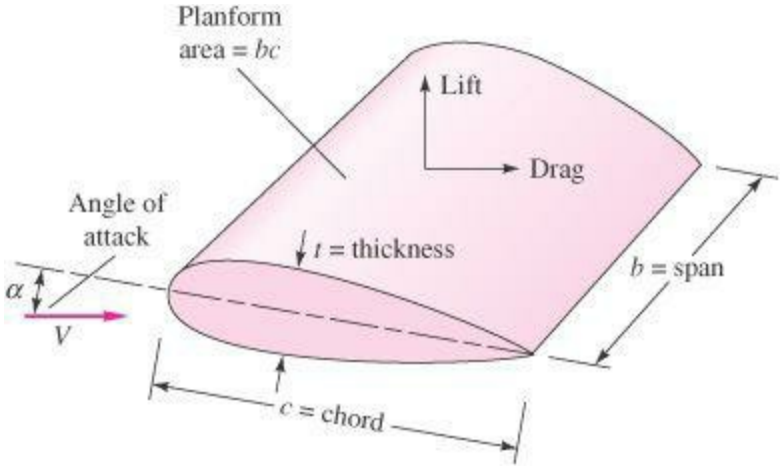


(*b*)

) LJ %LRORJLFDOO\GHVLJQHGVSDQZRQVWR ZLQGIRUFHV *a* DVXFFHVVIXOO\LQWRDFRQFDOVKDSH DWKLJK YHORFLW\ *b* EODFN ZDQXWOHDYHVFXWWULQWRDGRZ GUDJ VKDSHDVZLQGLQFUHDVH ***From Vogel, Ref. 33.***

) RUFHVRQ/ LIWLQJ %RGLHV

/ LIWLQJ ERGLHV DLUFUDIW KXOOV RUYDQHV DUH LQWHQGHG WR SURYLGH DOLJHIRUFHQRUPDOWR WKH IUHH VWUHDP DQG DV OLWWOH GUDJ DV SRVVLEOH &RQYHQWLRQDOO\HPSOR\HG DVKDSHV FRQVLVW RI EODGH¶V ZLQJ² VKDSHV UHDGLO\ YLVXDOL]HG WKLQ VKDUS”ZLWK DURXQGHG OHDGLQJ HGJHDQG DVKDUS WUDLOLQJ HGJH $ VWSLFDOVKDSHLVVKRZQ) LJ



) LJ ' HILQLWLRQVNHWFK IRUDOLIWLQJ YDQH

) RUROUSXUSRVHVZHFRQVLGHUWKHERG\ WREHVXPPHWULF DVLQ) LJ ZLWKWKHIUHHVWUHDP YHORFLW\LQWKHYHUWLFDOSODQH ,I WKHFKRUGOLQHEHWZHHQWKHOHDGLQJDQGWUDLOLQJHGJHVQRWDOLQHF VXPPHWU WKHDLUFROLVVDLG WREHFDPEHUHG 7KHFDPEHUOLQHVWKHOLQHPLGZD EHWZHHQWKHXSSHDQGORZHUVXUIDFHVRI WKHYDQH

7KHDQJOHEHWZHHQWKHIUHHVWUHDP DQG WKHFKRUGOLQHLVFDOOHGWKHDQJOHRI DWWDFN. 7KHOLW DQG WKH GUDJ' YDU ZLWK WKLV DQJOH 7KH GLPHQVLRQOHVV IRUFH GHILQHG ZLWK UHVSHFWWR WK SDQRUP DUHD$S EF

Lift coefficient:

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 A_p} \qquad (7.66a)$$

Drag coefficient:

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A_p} \qquad (7.66b)$$

,I WKHFKRUGOHQWKLVQRWFRQWDQWDVLQWKHVSHUHGZLQJVRI PRGHUQDLUFUDIW$S HF GE

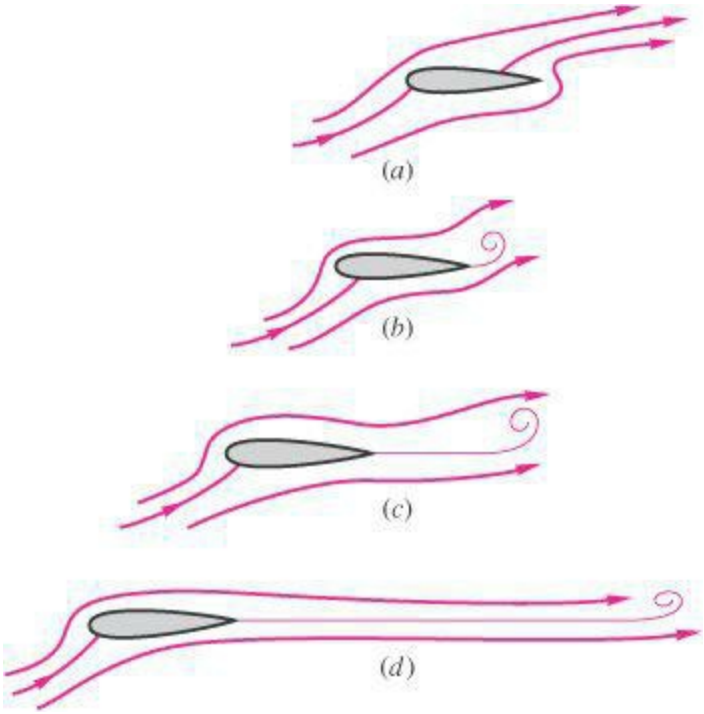
) RUGRV VSHHGIORZ ZLWKDJLYHQURXJKQHVVUDWLR &/ DQG& VKRXOG YDU ZLWK. DQGWKHFKRU 5H\QROGVQXPEHU

&/ I . 5H± RU& I . 5H±

ZKHUH5H± 9F 7KH5H\QROGVQXPEHUVDUHFRPPRQO LQWKHWXUEXOHQWERXQGDU\ OD\HUUDQJHDQ KDYHDPRGHVWHIIHFW

7KHURXQGHG OHDGLQJ HGJH SUHVHQWVIORZ VHSDUDWRQWKHUH EXWWKH VKDUS WUDLOLQJ HGJH FDXVHV WDQJHQWLDOZDNHPRWRQWKDWJHQHUDWHVOLIW) LJXUH VKRZ VZKDWKDSSHQVZKHQDIORZ WDUWXS SDWWDQLWQJ YDQHRUDQDLUFRLO

-XVWDVWU WDUWXS LQ) LJ D WKH VWHDP OLQH PRWRQ LV LUURWDWRQDO DQG LQYLVFLG 7KH UHD WDUJQDWRQSRLQWDXXPLQJ DSRVLWLYH DQJOHRI DWWDFN LVRQWKHXSSHUVXUIDFH DQGWKHUHVQR OLWWEX WKHIORZ FDQQRWBRQJ QHJRWDWWKHVKDUS WXLQDWWKH WDLOLQJ HGJH LWWHSDUDWHV DQGDVWDUWQJ YRUWL IRUPV LQ) LJ E 7KLV WDUWQJ YRUWL LV VKHG GRZQWHDP LQ) LJ F DQGG DQG D VPRRW WXWHDP OLQHIORZ GHYHORSVRYHUWKHZLQJ OHDYLQJ WKHIRLOLQDGLUHFWRQDSSURLPDWO SDUDOOHOWR WK FKRUG OLQH / LWDWWKLV WLPHLVIXOO GHYHORSHG DQGWKHWDUJQ YRUWL LV JRQH 6KRXOGWKHIORZ QRZ FHDVH DWWRSSLQ YRUWL RI RSSRVLWH FORFNZLVH VHQVH ZLOOIRUP DQG EH VKHG ' XULQ IQJK LQFUHDVHVRUGHFUHDVHVLQ QWZLOOFDXVHLQFUHPHQWDOWDUQJ RUWRSSLQ YRUWHV DZD\ZLWK WK HIHFWRI PDLQWDLQLQ DVPRRWKSDUDOOHOIORZ DWWKHWDUJH : HSXUWHWKLVIGHPDWKHPDWFDO LQ&KDS



) LJ 7UDQVLHQWVWDJHVLQWKHGHYHORSPHQWRI OLW *a* WXUWXS UHDUWXJQDWRQSRQWR WKH XSSHUVXUIDFH QR OLW *b* VKDUS WDLOLQJ HGJH LQGXFHVVHSDUDWRQ DQG D WXUWQJ YRUWI IRUPV VDJKWOLW *c* WXUWQJ YRUWI LVVKHG DQG WWHDP OLQHVIORZ VPRRWKO IURP WDLOLQJ HGJH OLWLVQRZ SHUFHQWGHYHORSHG *d* WXUWQJ YRUWI QRZ VKHGIDUEHKLQG WDLOLQJ HGJHQRZ YHU VPRRWK OLWXOO GHYHORSHG

\$WDORZ DQJOHRI DWWDFN WKHUHDUXUFHVKDYHDQ DGYHUVHSUHVXUHJUDGLHQWEXWQRWHQRXJK WR FDXVHVLJQLILFDQWERXQGDU\ OD\HUVHSDUDWRQ 7KHIORZ SDWWKLQVVPRRWK DVLQ) LJ G DQG GUDJ LV VPDOO DQG OLWH[FHOOHQW\$ VWKH DQJOHRI DWWDFN LV LQFUHDVHG WKH XSSHUVXUIDFH DGYHUVH JUDGLHQ EHFRPHVVWURQJHU DQG JHQHUDOO DVHSDUDWRQ EXEEOHEHJLQV WR FUHSI IRUZ DUG RQ WKH XSSHUVXUIDFH

$WDFHUWDLQDQJOH. VR ƒ WKHIORZ LVVHSDUDWHGFRPSOHWHO IURP WKHXSSHUVXUIDFH DVLQ) LJ
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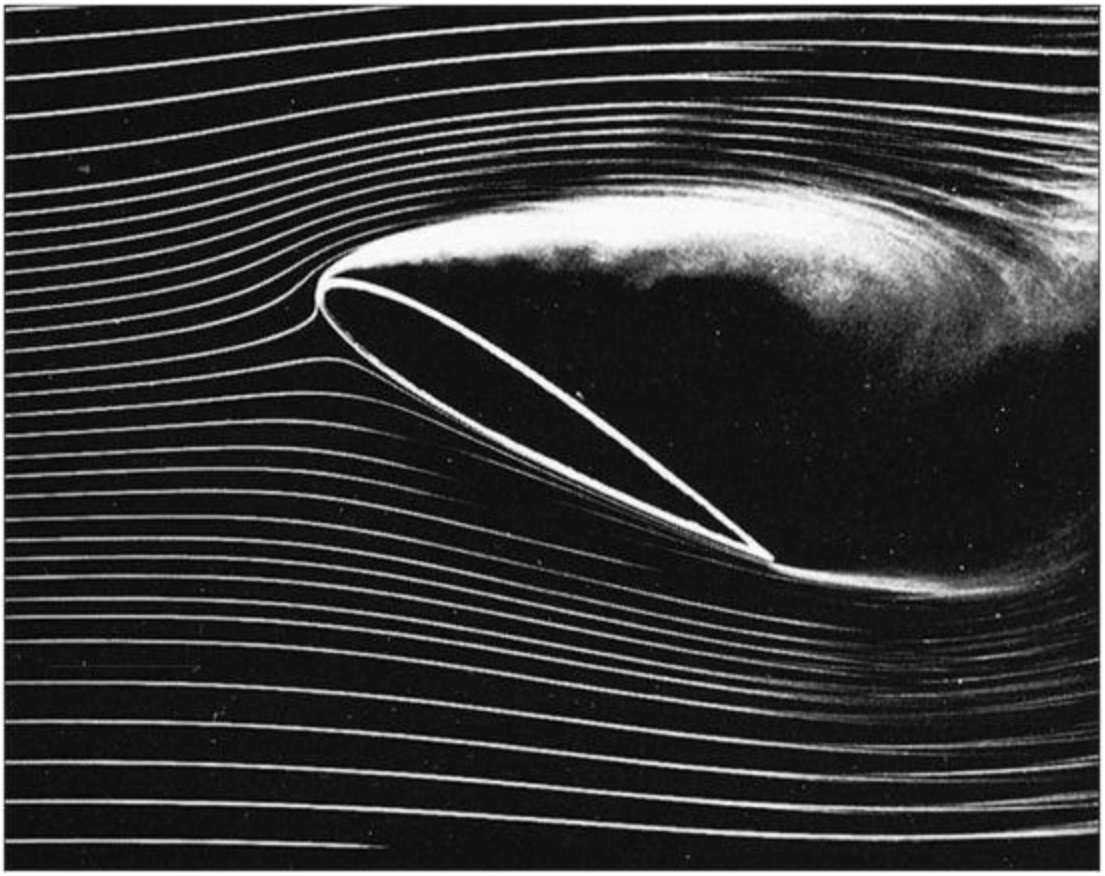
LVQRORQJHUIODEOH



) LJ $WKLJK DQJOHRI DWWDFN VPRNHIORZ YLVXDOL]DWLRQVKRZVWDOOHGIORZ RQWKHXSSH
VXUIDFH RI D QDFWQJ YDQH *National Committee for Fluid Mechanics Films, Educatior
Development Center, Inc., © 1972.*

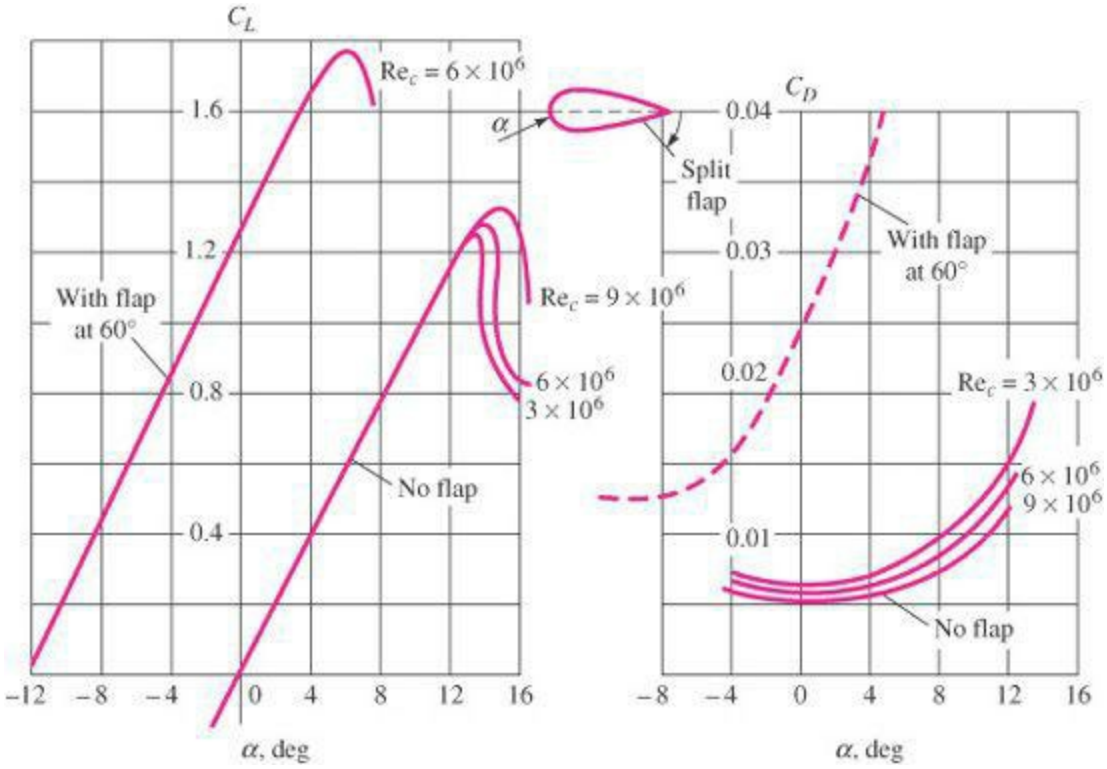
(DUO DLUIOVZHUHWKLQ PRGHOHG DWWUELGV¶ ZLQJV 7KH* HUPDQHQJLQHU2WWR / LOLHQWK
± H[SHULPHQWDG ZLWKIODWDQG FDPEHUHG SODWHV RQ D URWDWQJ DUP +HDQG KLV EURWK
* XWWDY IOHZ WKHZRUGV¶VILUVWJOLGHUQ +RUDWLR) UHGHULFN3KLOOLSV ± EXLOWWKHILUV
ZLQGWXQQHOLQ DQG PHDVXUHG WKHOLWDQG GUDJ RI FDPEHUHG YDQHV 7KHILUVWWKRUR RI QLWZ D
SURSRVHG E\ ) UHGHULFN : / DQFKHVWHU VKRUWO DWWUZDG 0 RGHQDLURLWKRU GDWV IURP
ZKHQWKH5XWLDQK\GURG\QDPLFLWV1 (-RXNRZVN\ ± GHYHORSHG DFLUFXODWRQWKHRUH
&KDS IRUFRPSXWQJ DLURLOQWRUDUEWDU FDPEHUDQG WKLFNQHVV : LWKWKLV EDVLF WKRU D
H[WHQGHGDQG GHYHORSHGE\ 3UDQGWDQG. iUPiQDQGWKHUWXGHQW LWLVQRZ SRVVLEOHWR GHVLJQDQRZ
VSHGDLURLOWR VDWLV\ SDUWFXODUVXUIDFHSUHVVXUHGLVWULEXWQV DQG ERXQGDU\ OD\HUFKDUDFWULVWF\
7KHUHDZKROHIDPLOHVRI DLURLOGHVJQV QRWDEO\ WKRVHGHYHORSHGLQWKH8QLWG6WDWVXQGHUWK
VSRQVRUVKLSRI WKH1$&$ QRZ 1$6$ ([WQVLYHWKRU DQGGDWRQWKHVHDLURLOVDUHFRQWDLQHG
5H : HVKDOOGLVFXVWKLVIXUWKHULQ&KDS 7KHKLVWRU RI DHURQDXWFVLV DUFK DQGHQJDJ
VRSLF DQGKLJKO UHFRPPHQGHGWRWKHUHDGHU> @

) LJXUH VKRZVWKHOWDQGGUDJ RQD\PPHWUF DLURLOGHORWWDVWH1$&$ IRLOWK
ODWWGJLWQGLFDWQJ WKHWKLFNQHVV RI SHUFHQW: LWKQR IODS H[WHQGHG WKLV DLURLO DV H[SHFWHG KD
]HUROWDWZHURQJOHRI DWWDFN 8SWRDERXW ƒ WKHOWFRHILFLHQWQFUHDVHVOQHDU ZLWKDVORSHF
SHUGHJUHH RU SHUUDGLDQ 7KLVLVQDJUHPHQWZLWKWKHWKRU RXWOQHGLQ&KDS

$$C_{L,\text{theory}} \approx 2\pi \sin\left(\alpha + \frac{2h}{c}\right) \qquad (7.67)$$

ZKHUHK F LVWKHPD[LPXP FDPEHUH[SUHVVHGDVDIUDFWLRQRI WKHFKRUG 7KH1\$&\$ KDV]HUR FDPEHUKHQFH&/ CEVLQ. § . ZKHUH. LVLQGHUHQGHQWDUHPHQW

7KHGUDJ FRHILFLHQWRI WKH VPRRWKPRGHODLUFUDIW LQ) LJ LV DVORZ DV ZKLFK LV DFWXDOO\ORZHUWKDQERWKVLGHVRI DIODWSODWH LQWKHEOWORZ 7KLV LV PLVOHDGLQJ LQDVPXFK DV I FRPPHUFLDOFRLOZLOOKDYHURXJKQHVVHIIHFWV IRUH[DPSOH DSDLQWHGZLOGRXEOHWKHGUDJ FRHILFLHQ

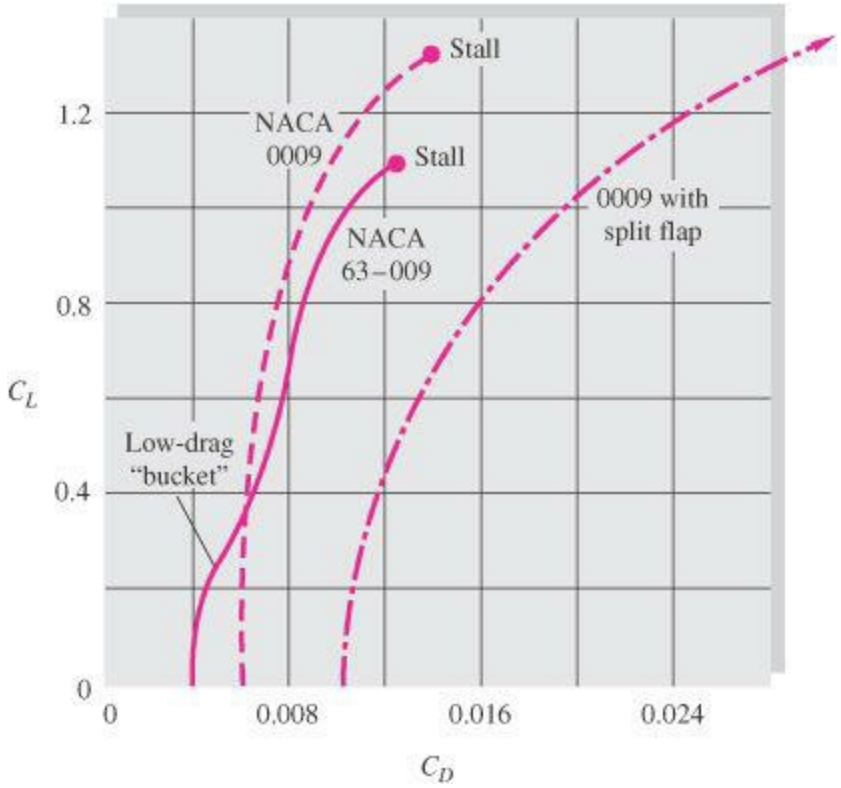


) LJ / LIWDQGGUDJ RI DVPPHWULF1\$&\$ DLUIRLORI LQILQLWHVSDQ LQFOXGLQJ HIHFWV DVSOLWIODS GHIOHFWRQ 1RWHWKDWURXJKQHVVFDQLQFUHDVH C_D IURP VR SHUFHQW

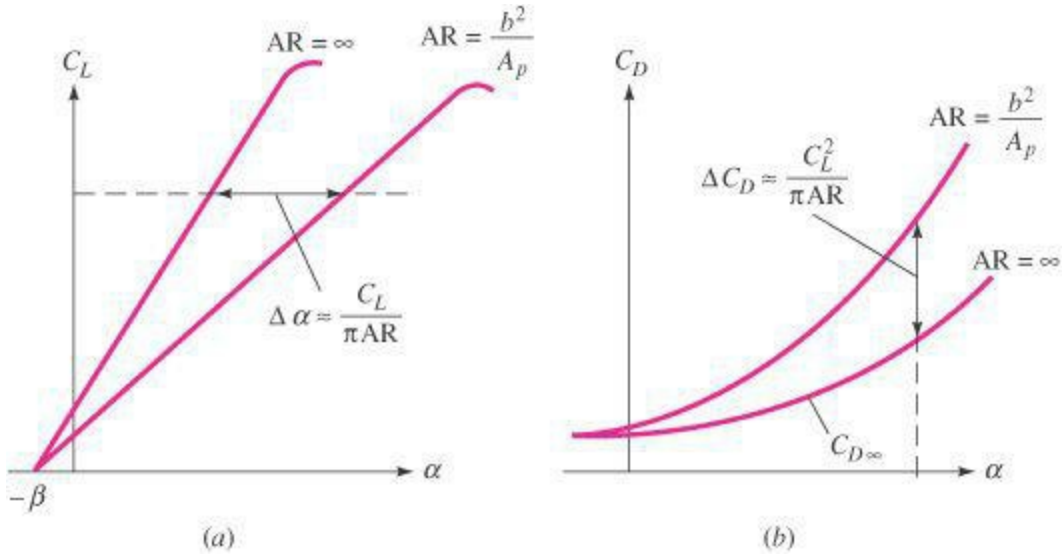
7KHHIIHFWRI LQFUHDVLQJ 5H\QROGVQXPEHULQ) LJ LVWRLQFUHDVHWKHPD[LPXP OLIWDQGVOLJ DQJOH ZLWKRXWFKDQJLQJ WKHVORSHDSSUHFLDEO\ DQGVRUHGXFHWKHGUDJ FRHILFLHQW7KLV LV DVXUH HIHFWLQFHWKHSURRWSHZLOOSUREDEO\ EHDWDKLJKHU5H\QROGVQXPEHUWKDQWKHPRGHO RUPRUH)RUVWDNHRI DQGODQGLQJ WKHOLWLVJUHDWO\ LQFUHDVHG E\ GHOHFWLQJ DSSODWODS DV VKRZQ LQ) LJ 7KLV PDNHVWKH DLUFUOXQ\PPHWUF RUHIIHFWLYHO\ FDPEHUHG DQGFKDQJHVWKH]HUR OLWSRLQWV . í ƒ 7KHGUDJ LV DOVR JUHDWO\ LQFUHDVHG E\ WKHIODS EXWWKHUHGXFWLRQ LQ WDNHRI DQGOLQGLVDQFHLVZRUWKWKH[WDSRZHUHGG

\$ OLWLQJ FUDIWFUXLVHV DWORZ DQJOHRI DWWDFN ZKHUH WKH OLWLV PXFK ODJHU WKDQ WKH GUDJ 0 D[LPXP OLWWR GUDJ UDWLRIRUWKHFRPPRQDLUFUOYOHEHWZHHQ DQG

6RPHDLUFUOY VXFK DV WKH1\$&\$ VHULHV DUHVKDSHGWRSURYLGHIDYRUDEOHJUDGLHQWVRYHUPXFK RI WKHXSSHUVXUIDFH DWORZ DQJOHV 7KXV VHSDUDWLQJ WKH\$&\$ DLUIRLOLQWRWZR VXEHOHPHQWV DOORZV WKHDLUIRLOHOLQV DJRRGHQJWK RI ODPLQDUIORZ HYHQDWKLJK5H\QROGVQXPEHUV 7KHOLWGUDJSRODU) LJ VKRZVWKH1\$&\$ GDWDIURP) LJ DQGDOOPLQDUIORZ DLUIRLO 1\$&\$ RI WKHVDPHWKLFNQHVV 7KHODPLQDUIORZ DLUIRLOKDVDORZ GUDJ EXFNHWDWWPDOODQJHVEXWDOV VXIIHUVORZHUVWDOODQJHDQG ORZHUPD[LPXP OLWFRHILFLHQW7KHGUDJ LV SHUFHQWORZHU LQ WKH EXFNHWEXWWKHEXFNHWGLVDSSHDUVLI WKHUHLVVLJQLILFDQWXUEXOHQFHURXJKQHVV



) LJ / LIWGUDJ SRODUSORWVRUVWDQGDUG DQGDOOPLQDUIORZ 1\$&\$ DLUIRLO



) LJ (IIHFWRI ILQLWH DVSHFWUDWLR RQ OLIWDQG GUDJ RI DQ DLUIRLO *a* HIHFWLYH DQJH LQFUHDVH *b* LQGXFHG GUDJ LQFUHDVH

\$ ODUJHGDWDLQ) LJV DQG DUHIRULQILQLWHVSDQ VKDSHV DVZR GLPHQVLRQDOOLIW SDWWHUQ DERXWZ LQJV ZLWKRXWWLSV 7KH HIIHFWRI ILQLWH VSDQ FDQ EH FRUUHODWHG ZLWK WKH GLPHQVLRQOHVV VSHFLILFDWLR RUDVSHFWUDWLR GHQRWHG \$5

$$\mathrm{AR} = \frac{b^2}{A_p} = \frac{b}{\bar{c}} \tag{7.68}$$

ZKHUH $\bar{c}$ LV WKH DYHUDJH FKRUG GHQJWK) LQLWH VSDQ HIIHFWV DUH VKRZQ LQ) LJ 7KH OLWWOHRS GHFUHDVHV EXWWKH]HUROLIWDQJOHLVWKHVDPH DQGWKHGUDJ LQFUHDVHV EXWWKH]HUROLIWGUDJ LVWKHVDPH

7KH VKRUW RI ILQLWH VSDQ DLUIRLOV> @SUHGLFWV VKDW KH HIIHFWLYH DQJOH RI DWWDFN ILQFUHDVHV DV LQ) LJ E\ WKH DPRXQW

$$\Delta\alpha \approx \frac{C_L}{\pi \mathrm{AR}} \tag{7.69}$$

: KHQ DSSOLHG WR(T VKH ILQLWH VSDQ QDWEH FRPHV

$$C_L \approx \frac{2\pi \sin(\alpha + 2h/c)}{1 + 2/\mathrm{AR}} \tag{7.70}$$

7KH DVRFLDWHG GUDJ LQFUHDVH LVû& §&/ VLQû. §&/ û. RU

$$C_D \approx C_{D\infty} + \frac{C_L^2}{\pi \mathrm{AR}} \tag{7.71}$$

ZKHUH&, LV WKH GUDJ RI WKH ILQLWH VSDQ DLUIRLO DV VNHWFKHG LQ) LJ 7KHVH FRUUHODWLRQV DUH LQJRRG DJUHHPHQW ZLWK H[SHULPHQWDO RQILQLWH VSDQ ZLQJV> @

7KH H[WHQW RI DPD[LPXP CLWFRH ILFLHQWP SOHV WKH H[WHQW RI DPLQLPXP VSHHG RUWKDW VSHHG IRU DFUDLVZ KRVHOHWXSSRUWWLWZ HJKW

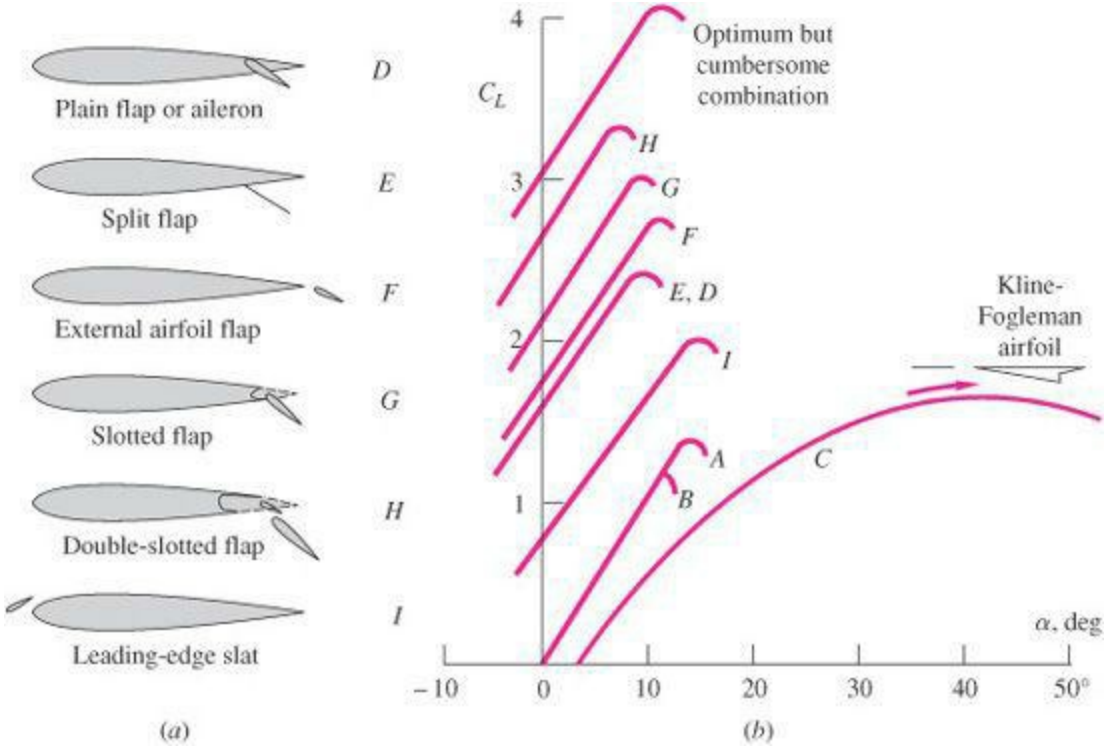
$$L = W = C_{L,\max}\left(\tfrac{1}{2}\rho V_s^2 A_p\right)$$

or

$$V_s = \left(\frac{2W}{C_{L,\max}\rho A_p}\right)^{1/2} \tag{7.72}$$

7KH VDOO VSHHG RI WVSLFDO DLUFUDIWYDULHV EHZHHQ DQG IWV GHSHQGLQJ RQ WKH ZHJKWDQ YDOXH RI &/ PD[7KH SLORWP XWKRGG WKH VPHHG JUHDWU WKDQ DERXW 9V WR DYRLG WKH LQWDELOLW DVWRFLDWHG ZLWK FRP SOHWH VDOO

7KH VSDWWODS LQ) LJ LV RQO RQH RI PDQ GHYLFHV XVHG WR VHFXUH KJK CLWDWRZ VSHHGV) LJXUH D VKRZ V VL[VXFK GHYLFHV ZKRVH CLWSHUFRUP DQFH LV JLYHQ LQ) LJ E DORQJ ZLWK I VWDQGDUG \$ DQG CDP LQDU IORZ % DLUIRLO 7KH GRXEOH VORWWG IODS DFKLHYH&/ PD[§ DQG I FRP ELQDWLRQ RI WKLV SXPS D CHDGLQJ HGJH VODWFDQ DFKLHYH&/ PD[§ 7KHVH DUH QRWWFHQWL FXURVLWLHV IRU LQWDQFH WKH %RHLQJ FRPP HUFLDOMWDLUFUDIWXVHV D WULSOH VORWWG IODS SXPS I CHDGLQJ HGJH VODWGXULQJ ODQGLQJ

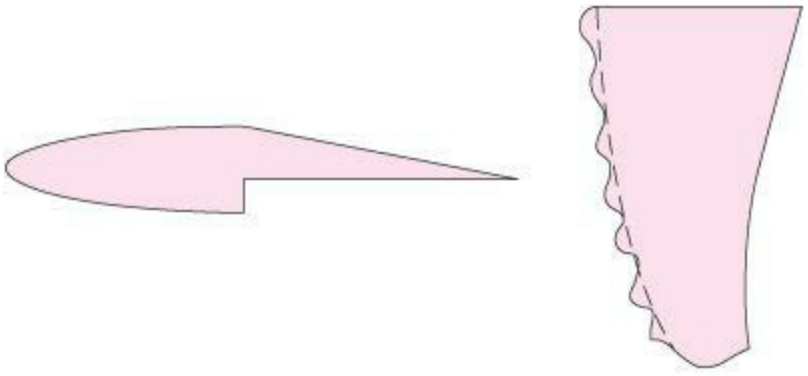


) LJ 3HUFHQWDJH RI DLUIRLO ZLWK DQG ZLWKRXW KLJK OLIW GHYLFHV *A* 1 \$ &\$ *B* 1 \$ &\$ *C* . OQH) RJHP DQ DLUIRLO *from Ref. 17* *D* WRS VKRZQ LQ *a* *a* WSHVRI KLJK OLIW GHYLFHV *b* OLIWFRHIILFLHQWVIRUYDULRXVGHYLFHV

\$ YLRODWRQ RI FRQYHQWRQDOGHUG\QDPLF ZLVGRP LV WKDWP LOLWDU\ DLUFUDIWDUH EHJLQQLQJ WR IO EUHO DERYH WKH VXSHUVRQLF SRLQW) LJKWHUSLORWV DUH GHDOLQJ WR PDNH TXLFN PDQHXYHUV LQ WKH VXSHU UHJLRQDVGHVLGLQ5H 6RPHSODQHVFDQHYHQIO FRQWUROXV ZKLOHVXSHUG WKH* UHPDQ; H[SHULPHQWDODLUFUDIWHFWV VHVDUHFRGE\ IO LQJDW *f*

7KH. OQH) RJHP DQ\$ LUIRLO

) RUGHFDGHV DQ DLUIRLOKDV EHHQ D VP RRWKO YDU LQJ VKDSH ZLWK D URXQGHG OHDGLQJ HGJH DQG VKDUS WUDLOLQJ HGJH ,WSURYLGHV ORZ GUDJ EXWWSLFDO VXSHUV RQ DWWDQJOHV RI DWWDFNIURP VR GHJUHHV ,Q VWR DP DWKUPRGHOSDQH HQWKXVLDVW 5) . OQH DQG))) RJHP DQ GHVLJQHG DQ DLUIRLZ LWK D VKDUS QRVH DQG D UHDUFXUYHGWUDLOLQJ P DQHXYHUDELOLW ZKH IORZ Q> @7KH ZURWWWWWG VKLV VKDSH ZLWKWKH UHVXOWLQJ SDWWHUQ DV FXUYH& LQ) LJ 7KHUHLV QVXSSOR. *f* EXWWKH GUDJ LV YHU KJK) HUW> @URXQGHG WKH OHDGLQJ HGJH DV LQ) LJ D DQG IRXQG ORZ HUGUDJ DQG QR VXSSOX. *f* +HUHSRWWG KJKHUDOWWR GUDJ UDWR VKDQDVXSSGU DLUIRLO) LQDLVK DQG : LWKHUSRRQ> @YHUILHG WKLV ZLWK IXWKHU[SHULPHQW DQG &) ' VLPXODWRQ \$ OWRXJK 1 \$ 6\$ KDV VXJGHG WKH.) FRQFHSWDSSDUHQW WKH GUDJ LV VXSSD EWKJK IRUFRP P HUFL DSSOLFDWRQV +RZ HYHU WKH.) DLUIRLOLVQOZ YHU SRSXODUIRUUDGLR FRQWROOHG PRGHODLUFUDIW



) LJ 7ZR QHZ H[SHULP HQWDOLUHLOY *a* FURVV VHFWLRQ RI WKH . QLQH) RJHOP DQ DLUHLOZ LV D URXQG OHDGLQJ HGJH DQG D WKS FXWRXWDW SHUFHQWFKRUG > @ *b* SODQ YLHZ RI D ZLQ PRGHOGRQWKH KXP SEDFN ZKDOHI OSSHU> @

$: LQJ ,QVSLUHG E\ WKH+ XP SEDFN: KDOH

%LRORJLVWVKDYH ORQJ QRWHG WKHKJK PDQHXYHUDELOLWRI WKHKXP SEDFN ZKDOHZ KHQLWWHNVSUH 8QDNH PRVWZKDOHV WKH KXP SEDFN KDV WXEHUFOHV RU EXMPS RQ WKH OHDGLQJ HGJH RI LWV IOSSHU 0 LNDRVRYLF HWDO> @DWWXAG WKLVLGHD XLQJ DWDQGDUGZ LQJ ZLWKSHURGLF EXMPS JXHGWRLWOHDGLQ HGJH DVLQ) LJ E 7KH\ UHSRUWD SHUFHQWLQFUHDVH LQ WDOODQJOH FRPSDUHG WR WKH VDPH ZLQ ZLWRXWEXMPS SXVKJKHUDWDQGKJKHUDWRGUDJ UDWRV 7KHFRQFHSWKDVSURMLHIRUFRMPHUFL DSSOLFDWRQV WXFK DV ZLQG WXUELQHEODGHV) ORZ YLVXDOL]DWRQ VKRZV WKDWWKH EXMPS FUHDWHQHUJHWL WXUEXOHQFH WKDW SRVLWLYHO\ DOWHUV WKH ZLQJ VXUIDFH KHSLQJ WR GHOD\ VHSDUDWRQ

$ &RP ELQDWRQ &DUDQG $ LUSODQH

(QJLQHHUVKDYH ORQJ GUHDPHG RI DYLDEOHHFDUWKDWFDQIO + RS LQDWKRPH GULYHV WKH DLUSRUWC VRPHZKHUH WKHQGULYHV WKH PRWLO' HVLJQHUHIRUWGDWEDFN WR WKH* OHQQ&XUWLV $XARSDQ ZLWKRWKHUSURMFWLQWKH VDQG V 3HUKDSV WKHPRWLVP RXZDV0 RXQRQ7DLORU¶V$ HURFL LQ 2QD ILYH$ HURFDVZHUHEXLOV7KH\HDU VHPVWKHHHQWKH<HDUR WKH&DU3DQ ZLWK DWOHDWMLYH GLIHUHQWFRPSDQLHV ZRUNLQJ RQ GHVLJQV (QJLQHHUV FDQ QRZ XVH OLGKWZHLJK PDWHULDOV EHWWUHQJLQHV DQG JXLGDQFH\VWHPV 7KHZUWLQJ IDYRUWLWV7UDQVLWRQŠ PDGHE\ 7HUUDXJLD ,QF VKRZQLQ) LJ

) LJ 7KH7UDQVLWRQŠ FDUSDQH*(Courtesy of Terrafugia, Inc.)*

7KH7UDQVLWRQŠ KDVZLQJVWKDWRGRXWRDVSDQRI IW7KHVZLQJVGRQRWRG7KHIURC
FDQDGZLQJGRXEOHVDVDEXPSHUIRUWKHKJKZD\ 7KH KSHQJLQHGULYHVERWKWHUDSURSHOO
IRUIOJKWDQGDORWKHIURQWZKHHOVIRUWKHKJKZD\ * URVVWDNHRI ZHJKWV CEI 2SHUDWLQJQH
RQD /JKW6SRUW$LUSODQHOFHQVH 7KH7UDQVLWRQŠ PDGHDVXFFHVVIXOPDLGHQIOJKWRQ0 DU
7KLVFUDVWWVGDZZLOOFHOHUDWH EHXVHXOIRUVHWXQJHQGRI FKDSWUSUREOPV
) XUWKHULQRUPDWRQRQWKHSHUFRPDQFHRI OLWQJ FUDVWFDQEHIRXQGLQ5HV DQG : I
GLVFXVVWKLVPDWHUDJDLQEUH0 LQ&KDS

(; $0 3/ (

$ QDLUFUDZ HJKV CEI KDVDSDQRUP DUHDRI IW DQGFDQGHOY HUDFRQWDQWKUXWR
CE ,WKDV DQ DVSHFWUDWR RI DQG& ' § 1HJOHFWQJ URODQJ UHVLVWDQFH HWP DWWK
WDNHRI GLVWDQFHDWHDOHYHOLWDNHRI VSHGHTXDOV WPHVWDOOVSHG 7DNH&/ PD[

6ROXWRQ

7KHVDOVSHGIURP (T Z LWKVHDOHYHOGHQVLW! VOXJ IW LV

$$V_s = \left(\frac{2W}{C_{L,\max}\rho A_p}\right)^{1/2} = \left[\frac{2(75{,}000)}{2.0(0.00237)(2500)}\right]^{1/2} = 112.5 \text{ ft/s}$$

+HQFHWDNHRI VSHG9 9_V IW/ 7KHGUDJ LVHWP DWGIURP (T IRU$5 DV

$$C_D \approx 0.02 + \frac{C_L^2}{7\pi} = 0.02 + 0.0455C_L^2$$

$ IRUFHEDODQFHLQWKHGLUHFWRQRI WDNHRI JLYHV

$$F_s = m\frac{dV}{dt} = \text{thrust} - \text{drag} = T - kV^2 \qquad k = \tfrac{1}{2}C_D\rho A_p \qquad (1)$$

6LQFH Z H DUH GRRNLQJ IRU GLVWDQFH QRW WLPH Z H LQWURGXFHG 9 GW 9 G9 GV LQWR (T VHSDUDWH YDULDEOHV DQG LQWHJUDWH

$$\int_0^{s_0} dS = \frac{m}{2}\int_0^{v_0} \frac{d(V^2)}{T - kV^2} \qquad k \approx \text{const}$$

or

$$S_0 = \frac{m}{2k}\ln\frac{T}{T - kV_0^2} = \frac{m}{2k}\ln\frac{T}{T - D_0} \qquad (2)$$

Z KHUH ' kV_0^2 LV WKH VWHDG\ GUDJ (TXDWLRQ LV WKH GHVLUHG WKURWWOH UHODWLRQ IRU VWHDG\ GLVWDQFH) RU WKH SDUWLFXODU QXPHULFDO YDOXHV VWDWH

$$m = \frac{75{,}000}{32.2} = 2329 \text{ slugs}$$

$$C_{L_0} = \frac{W}{\frac{1}{2}\rho V_0^2 A_p} = \frac{75{,}000}{\frac{1}{2}(0.00237)(135)^2(2500)} = 1.39$$

$$C_{D_0} = 0.02 + 0.0455(C_{L_0})^2 = 0.108$$

$$k \approx \tfrac{1}{2}C_{D_0}\rho A_p = (\tfrac{1}{2})(0.108)(0.00237)(2500) = 0.319 \text{ slug/ft}$$

$$D_0 = kV_0^2 = 5820 \text{ lbf}$$

7KHQ (T SUHGLFWV WKDW

$$S_0 = \frac{2329 \text{ slugs}}{2(0.319 \text{ slug/ft})}\ln\frac{12{,}000}{12{,}000 - 5820} = 3650 \ln 1.94 = 2420 \text{ ft} \qquad \textit{Ans.}$$

$ PRUH[DFWDQDOVLV DFFRXQWLQJ IRU YDULDEOH N> @ LYHV WKH VDPH UHVXOW RZ LWKLQ SHUFHQW

(; $0 3/ (

) RU WKH DLUFUDIW RI [([DPSOH LI P D[LP XP WKUXVW V DSSOHG GXULQJ ICLJKW DW P WDQGDUG DOWLWXGH HVWLP DWH WKH UHVXOWLQJ YHORFLW\ RI WKH SODQH LQ PLK

6ROXWLRQ

- $VVXPSWLRQV * LYHQ: GEI $S IW 7 GEI $5 &,
- $SSURDFK 6HWXS WHTXDLRZ HJKWDQG GUDJ HTXDLRWKUXWDQG VRO YHIRUWKHYHORFLW\
- 3URSHUW\ YDOXHV) URP 7DEOH$ DW P ! NJ P VOXJ IW
- 6ROXWLRQ VWHSV : ULWH RXW WKH IRUP XODV IRU DLWDQG GUDJ 7KH XQNQRZQ V Z LOO EH &/ DQG 9

$$W = 75000\ \text{lbf} = \text{lift} = C_L \frac{\rho}{2} V^2 A_p = C_L \frac{0.00128\ \text{slug/ft}^3}{2} V^2 (2500\ \text{ft}^2)$$

$$T = 12000\ \text{lbf} = \text{drag} = \left(C_{D\infty} + \frac{C_L^2}{\pi AR} \right) \frac{\rho}{2} V^2 A_p$$

$$= \left[0.02 + \frac{C_L^2}{\pi(7)} \right] \frac{0.00128\ \text{slug/ft}^3}{2} V^2 (2500\ \text{ft}^2)$$

7KLV[illegible] EXWLQIDFWRPHFKDQLVPDQSXODWRQ GLYLGLQJ: E\ 7 ZRXG UHYHDOD TXDGUDWLF HTXDWRQIRU&/ ,QHLWKHUFDVH WKHILQDOVROXWLRQV

&/ 9 § IWV PLK$QV

- &RPPHQW 7KHVHDUHSUHOLPLQDU\ GHVLJQHWWPDWVKDWGRQRWGHSHQGRQDLUFUDIWKDSH

6XPPDU\

7KLV FKDSWHUKDV GHDOWZLWK YLVFRXV HIHFWLQH[WHQVLYHO\ SDWWERGLHV PPHUVHG LQ D WXEHP :KHQWKH5H\QROGVQXPEHULVODUJH YLVFRXVIRUFHVDUHFRQILQHGWRDWKLQERXQGDU\ OD\HUDQGZDNHL WKH YLFLQLW\ RI WKH ERG\ )RZ RXWLGH WKHVH ³WKHDU OD\HUV´ LV HVVHQWLDOO LQYLVFLG DQG FDQ EH SUHGLFWHGE\ SRWHQWLDOWKHRU DQG%HUQRXOOLVHTXDWLRQ

7KHFKDSWHUEHJDQZLWKDGLVFXVVLRQRI WKHIODWSODWHERXQGDU\ OD\HUDQG WKHXVHRI PRPHQWXP LQWHJUDO HTXDWLRQV WRSUHGLFWWKH ZDOO VKHDU IUHFWLRQ GUDJ DQG WKLFNQHVV RI VXFK OD\HUV 7KHVH DSSUR[LPDWLRQVVXJJHVWWKRZ WRHOPLQDWHFHUWDLQ PDOHPDWLFDO1DYLHU6WRNHVHTXDWLRQV UHVXOWLQ 3UDQGWO¶V ERXQGDU\ OD\HU HTXDWLRQVIRUODPLQDUDQG WXUEXOHQWIORZ 6HFWLRQ WKHQ VROYHG WKH ERXQGDU\ OD\HUHTXDWLRQVWRJLYHYHU DFFXUDWHIRUPXODVIRUFODVSODWHIORZ DWKLJK5H\QROGVQXPEHU5RXJK ZDOOHIHFWZHUHLQFOXGHG DQG6HF JDYHDEULHIGHVFULSWLRQRIWXUEXOHQFHPRGHOLQJDSSURDFKHV $QDGYHUVH GHFHOHUDWLQJ JUDGLHQWZDV VHQ WR FDXVHIORZ VHSDUDWLRQ ZKHUHWKH ERXQGDU\ OD\HU EUHDNVDZD\ IURP WKHVXUIDFHDQGIRUPVDEURDG ORZ SUHVVXUHZDNH

%RXQGDU\ OD\HUWKHRU IDLOVLQVHSDUDWHGIORZ V ZKLFK DUH FRPPRQO VXGLHG E\ H[SHULPHQWR &)' 6HFWLRQ JDYHGDWDRQGUDJ FRHILFLHQWVRI YDULRXVZR DQGWKUHH GLPHQVLRQDOERG\ VKDSHV 7KHFKDSWHUHQGHGZLWKDEULH GLVFXVVLRQRI OLWHUDWXUHVJHQHUDWHG E\ OLWQJ ERGLHV VXFK DV DLUIRLO DQGK\GURIRLOV$LUIRLOVDOR VXIIHUIORZ VHSDUDWLRQRUWXUEXOHQWWKLJK DQJOHVRI LQFLGHQFH

3UREOHPV

0 RVWRI WKHSUREOHPVKHUHQDUHIDLUO VWDLJKWIRUZ DUG 0 RUHGLIILFXOWURSHQHGHGDVLJQPHQ DUHODEHOHGZLWKDQDVWHULVN 3UREOHPVODEHOHGZLWKQ(6 LFRQ ZLOOEHQHILWIURP WKHXVHRI WK (QJLQHHULQJ (TXDWLRQ6ROYHU ((6 ZKLFKSUREOHPVODEHOHGZLWKD FRPSXWHUICRQ PD\ UHTXLU WKH XVHRI D FRPSXWHU 7KH WXDQGDUG HQGRI FKDSWHU SUREOHPV 3 WR3 FDWHJRUL]HG LQWR SUREOHP OLVWKHUH DUHIROORZHGE\ ZRUGSUREOHPV: WR: IXQGDPHQWDOVRI HQJLQHHULQJ H[DP SUREOHPV) (WR) (FRPSUHKHQVLYHSUREOHPV& WR& DQGGHVLJQSURMHFW

3UREOHP ' LVWULEXWLRQ

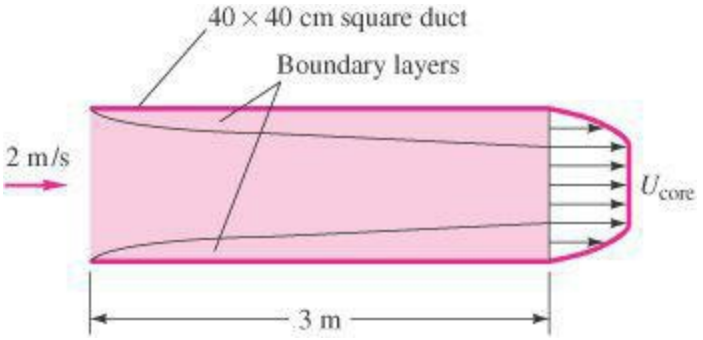
Section	Topic	Problems
7.1	Reynolds number and geometry	P7.1–P7.5
7.2	Momentum integral estimates	P7.6–P7.12
7.3	The boundary layer equations	P7.13–P7.15
7.4	Laminar flat-plate flow	P7.16–P7.29
7.4	Turbulent flat-plate flow	P7.30–P7.47
7.5	Boundary layers with pressure gradient	P7.48–P7.50
7.6	Drag of bodies	P7.51–P7.114
7.6	Lifting bodies—airfoils	P7.115–P7.127

5HIQRQGVQXPEHUDQGJHRPHWU

- 3 $QLGHDOJDV DW *f*& DQG DW IORZ VDW P VSDWDWKLQIODWSODWH $WDSRVLWRQ FP GRZQVWHDP RI WKH OHDGLQJ HGJH WKHERXQGDU\ OD\HUWKLFNQHVV LV PP : KLFK R WKH JDVHVLQ7DEOH$ LVWKLVOLNHO\WREH'
- 3 $ JDV DW *f*& IORZ V DW IWV SDWD VPRRWK VKDUS IODWSODWH $W FP WK ERXQGDU\ OD\HUWKLFNQHVV FP : KLFK RI WKH JDVHVLQ7DEOH$ LVWKLVPRVWOLNHO WR'
- 3 (TXDWRQ E DVXPHV WKDWWKH ERXQGDU\ OD\HUQ WKH SODWH LV WKLQEHQWURP WK OHDGLQJ HGJH RQZDUG ' HYLVH D VFKHPH IRUGHWHUPLQLQJ WKH ERXQGDU\ PRUHDFFXUDWHO\ WKHQWKH IORZ LVOPLQDUXS WRDSRLQW[DQGWKLEHQWWKHUHVW $SSO WKLV VFKHPH WR FRPSXWHRQ RI WKH ERXQGDU\ DW P LQ P V IORZ RI DLU DW *f*& DQG DW SDWD IODWSODWH &RPSDUH \RXUHVXOWZLWK(T E $VXPH5H[FUW§ (
- 3 $ VPRRWKFHUDPLF VSKHUH 6* LVIPHUVHGLQDIORZ RI ZDWUDW *f*& DQG FP V: KDWLVWKHVSKHUHGLDPHWULLWLVHQFRXQWHULQJ D FUHSLQJ PRWRQ 5H G R E WDQLWRQWRWXEXOHQFH 5H G "
- 3 6$(RLODW *f*& IORZ VDW IWVIURP DUHVHUYRLULQWR D LQGLDPHWUSLSH 8VI IODWSODWHWKHRU WR HVWMDWWKHSRVLWRQ[ZKHUHWKHSLSHZDOOERXQGDU\ OD\HUVPHHWV WKHFHQWU &RPSDUHZLWK(T DQGJLYHVRPHH[SODQDWRQVIRUWKHGLVFUHSDF\

0RPHQWXP LQWHJUDOHTXDWLRQV

- 3)RUWKH ODPLQDUSDUDEROF ERXQGDU\ OD\HU SURILOH R(T FRPSXWH WKH VKDSI IDFWRU+ DQGFRPSDUHZLWKWKH H[DFW%ODVLXVUHVXOW(T
- 3 $LU DW *f*& DQG DW HQWHUV D FP VTXDUH GXFWDW LQ)LJ 3 8VLQJ WK ³GLVSODFHPHQWWKLFNQHVV´ FRQFHSWRR)LJ HVWLPDWH D WKH PHDQ YHORFLW DQG E WKH PHDQ SUHVVXUH LQ WKH FRUH R WKH IORZ DWWKH SRVLWRQ[P F : KDWLV WK DYHUDJHJUDGLHQWLQ3DP LQWKLVVHFWLRQ"



- 3 $LU! NJ P DQG (NJ P V IORZ VDW P VSDZQDIODWSODWH $VVK
WKDWLQQJHGJHRI WKHSODWH WKHIRORZLQJ YHORFLW\SURILOHGDWDDUHPHDVXUHG
,I WKH XSSHU VXUIDFH KDV DQ DUHD RI P HTXDO WR PRPHQWXP FRQFHSW WK
IULFWLRQGUDJ LQ1 RQWKHXSSHUVXUIDFH

y, mm	0	0.5	1.0	2.0	3.0	4.0	5.0	6.0
u, m/s	0	1.75	3.47	6.58	8.70	9.68	10.0	10.0

- 3 5HSHDWWKHIODWSODWHPRPHQWXP DQDOVLV RI6HF E\ UHSODFLQJ (T ZLWKWK
VLPSOHEXWXQUHDOLVWLF OLQHDUYHORFLW\SURILOHVXJJHVWHGE\ 6FKOLFKWLQJ > @

$$\frac{u}{U} \approx \frac{y}{\delta} \quad \text{for} \quad 0 \le y \le \delta$$

&RPSXWHPRPHQWXP±LQWHJUDOHWXP DWWKH RI F[/ [DQG+

- 3 5HSHDW3URE 3 XVLQJ WKHSRO\QRPLDOSURILOHVXJJHVWHGE\ . 3RKOKDXVHQLQ

$$\frac{u}{U} \approx 2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}$$

' RHVWKLVSURILOHVDWLV\ WKHERXQGDU\ FRQGLWLRQVRI OPLQDUIODWSODWHIORZ"

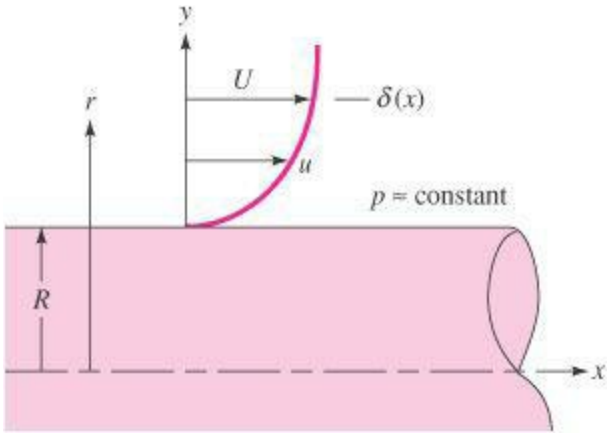
- 3 $LUDW *& DQG DW IORZ VDW P VSDZQDVKDUSIODWSODWH $VVXPLQJ WKDW iUPiQ
SDUDERORF SURILOH DQDOVLV (TV ± LV DFFXUDWH HXQWLDW D WKH ORFD
YHORFLW\X DQG E WKHORFDOVKHDUVWUHVV2DWWKHSRLQW[\ FP PP
- 3 7KHYHORFLW\SURILOHVKDSHX 8 § í H[S í \ / LVDVPRRWKFXUYHZLWKX D
\ DQGX 8 DW / DQGWKXVZRXOGVHHP VRWEHDUHDVRQDEOHVXEVWLWXWIR
WKHSDUDERORF IORWSODWHSURILOHR (T <HWZKHQWKLVQHZ SURILOHLVXVHGLQWK
LQWUDO DQDOVLV RI 6HF ZH JHWWKH ORXV UHVXOW $\delta/x \approx 9.2/\mathrm{Re}_x^{1/2}$, ZKLFK LV
SHUFHQWKLJK : KDWLV WKH UHDVRQ IRU WKH LQDFFXUDF\" +LQW 7KH DQVZHU LV
HYDOXDWLQJ WKH OPLQDUERXQGDU\ OD\HUPRPHQWXP HTXDWLRQ E DWWKH ZDOO\

7KHERXQGDU\ OD\HUHTXDWLRQV

- 3 ' HULYHPRGLILHGIRUPVRI WKHOPLQDUERXQGDU\ OD\HUHTXDWLRQV IRUWKHFDVHR
D[LVYPPHWULF IORZ DORQJ WKHRXWLGHRI DFLUFXODUF\OLQGHUR FRQVWDQWGLXV5 D

LQ) LJ 3 &RQVLGHU WKH VZR VSHFLDO FDVHV D / 5 DQG E / § 5 : KDW DUH WK
SURSHU ERXQGDU\ FRQGLWLRQV"

3



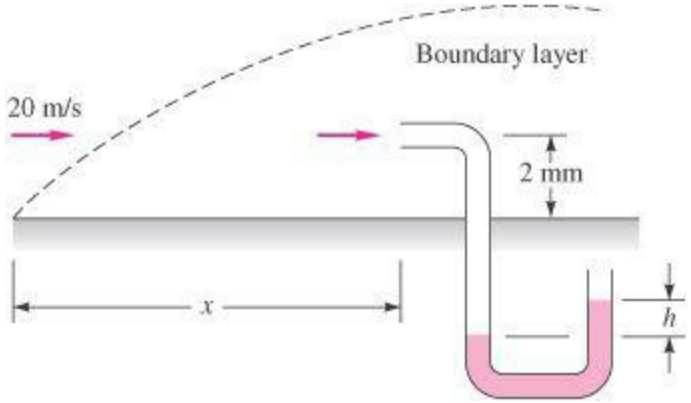
- 3 6KRZ WKDW WKH VZR GLPHQVLRQDOOHVV PLQDUIORZ SDUDPHWHU [

X 8 H&\ # #

LV DQ H[DFW VROXWLRQ WR WKH ERXQGDU\ OD\HU HTXDWLRQV) LQG WKH YDOXH RI WK
FRQVWDQW & LQ WHUPV RI WKH IORZ SDUDPHWHU \$UH WKH ERXQGDU\ FRQGLWLRQV VDWLVILHG' : KD
PLJKW WKLV IORZ UHSUHVHQW
- 3 ' LVFXVV ZKHWKHU IXOO GHYHORSHG ODPLQDU IORZ EHWZHHQ SDUDOOH
SODWHV (T DQG) LJ E UHSUHVHQWV DQ H[DFW VROXWLRQ WR WKH ERXQGDU\
OD\HU HTXDWLRQV DQG WKH ERXQGDU\ FRQGLWLRQV ,QZKDWVHQVH LI DQ DU
GXFWORZ VDOR ERXQGDU\ OD\HU IORZV"

/ DP LQDU IODWSODWH IORZ

- 3 \$ WKLQ IODWSODWH E\ FP LVLPPHUVHG LQD P VWUHDP RI 6\$(RLO DW *f*&
&RPSXWH WKH WRWDO IULFWLRQ GUDJ LI WKH VWUHDP LV SDUDOOHO WR D WKH ORQJ VLGH DQG E
WKH VKRUWVLGH
- 3 &RQVLGHU ODPLQDU IORZ SDVW D VKDUS IODWSODWH RI ZLGWK E DQG OHQJWK / : KD
SHUFHQWDJH RI WKH IULFWLRQ GUDJ RQ WKH SODWH LV FDUULHG E\ WKH UHDUKDOI RI WKH SODWH"
- 3 7KH DSSUR[LPDWH ZHLVWR3URE 3 DUHX § P VDQG2 § 3D DW/
FP DQG\ PP ' RQRWHY DOWKLV WR\RXUIULHQGV ZKR DUH ZRUNLQJ RQ3URE
3 5HSHDW WKH DSSUR[LPDWH E\ XVLQJ WKH H[DFW%ODVLXV IODWSODWH ERXQGDU\ OD\H
VROXWLRQ
- 3 \$LUDW *f* DQG DW IORZ VDW IWW SDWD WKLQ IODWSODWH ZKRVH DUHD E/ LV IW
,I WKH WRWDO IULFWLRQ GUDJ LV OEI ZKDW DUH WKH OHQJWK DQG ZLGWK RI WKH SODWH"
- 3 \$LUDW *f* DQG DW IORZ VDW P VSDWWKH IODWSODWH LQ) LJ 3 \$ SLW
VXUQDPH QDPH SODFHG PP IURP WKH ZDOO GHYHORSV DP DQRP HWHU UKHDGK PP
RI 0 HUDP UHG RLO 6* 8VH WKLV LQRUP DWLRQ WR HVWLP DWH WKH GRZ QVWUHD
SRVLWLRQ[RI WKH SLWRWWXEH \$VXPH DP IORZ

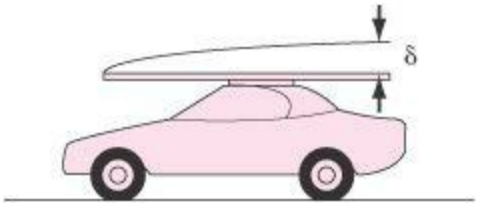


- 3) RUWKH H[SHULPHQWDO DSSDUDWXV RI) LJ 3 VXSSRVH WKH WXEH DP YHORFLWLV XQNQRZ DQG WKH SLWRW WXEH QDUURZV WKH ERXQGDU\ OD\HU RI DLU DW DQ DQG f& 7KH PDQRPHWHU IOXLG LV 0 HUFXU\ UHGRLO DQG WKH IROORZLQJ UHDGLQJV DU PDGH

8VLQJ WKH VHGDWD RQ QRWKH %ODVLXV WKHRU HWXP DW D WKH WXEH DP E WKH ERXQGDU\ OD\HU WKLFNQHVV F WKH ZDOO VKDUWHV DQG G WKH VRLOLFWLRQ GUDJ EHZHH WKH OHDGLQJ HGJH DQG WKH SRVLWRQ RI WKH SLWRW WXEH

y, mm	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
h, mm	1.2	4.6	9.8	15.8	21.2	25.3	27.8	29.0	29.7	29.7

- 3) RU WKH %ODVLXV IODWSODWH SUREOHP (TV VR GRHV D VZR GLPHQVLRQDO WXEH IXQFWLRQ % [\ H[LVW,I VR GHWHUPLQH WKH FRUUHFWLPHQVLRQOHVV IRUP IRU% DVXPLQJ WKDW% DWWKH ZDOO\
- 3 6XSSRVH\RX EX\ D E\ IWWKH HWR SOZ RRG DQG SXWLWRQ\RXUURI UDFN 6HH) LJ 3 <RX GULYH KRPH DW PLK D $ VXPLQJ WKH ERDUG LV SHUHFWO DQJOH ZLWK WKH DLUORZ KRZ WKLFN LV WKH ERXQGDU\ OD\HU DWWKH HQG RI WKH ERDUG' E (VWLPDWWKH GUDJ RQWKH VKHHWRI SOZ RRG LI WKH ERXQGDU\ OD\HUUHPDLQV/DPLQDU F (VWLPDWWKH GUDJ RQWKH VKHHWRI SOZ RRG LI WKH ERXQGDU\ OD\HULV WXUEXOHQWDVXPH WKH ZRRG LV VPRRWK DQG FRP SDUH WKH UHVXOWWR WKDWRI WKH ODPLQDUERXQGDU\ OD\H FDVH



- 3 $LU DW f& DQG DW IORZ V SDVWWKH IODWSODWH LQ) LJ 3 XQGHU GLIIHUHQW FRQGLWLRQV 7KHUH DUH WZR HTXDO VSDFHG SLWRW WXEHV HDFK SODFHG PP IURP WKH ZDOO 7KHP DQRPHWHU IOXLG LV ZDWHU DW f& ,I8 P VDQG/ FP GHWHUPLQH WKH YDOXHV RI WKH P DQRPHWHU UHDGLQJV K DQG K LQPP

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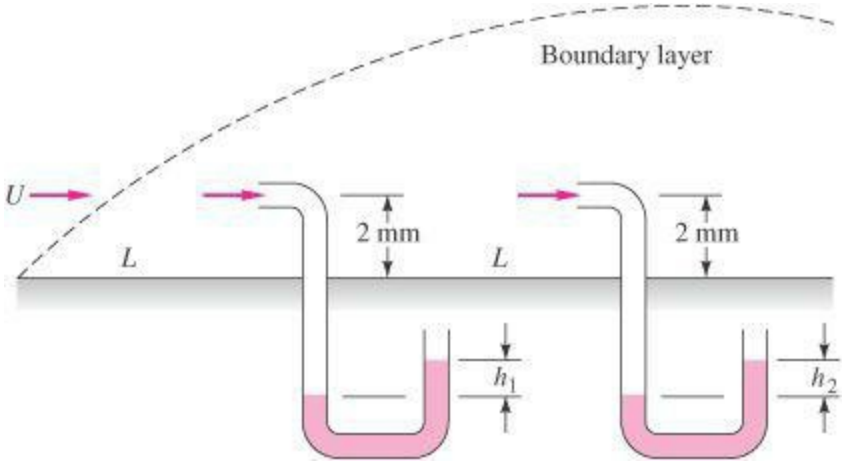
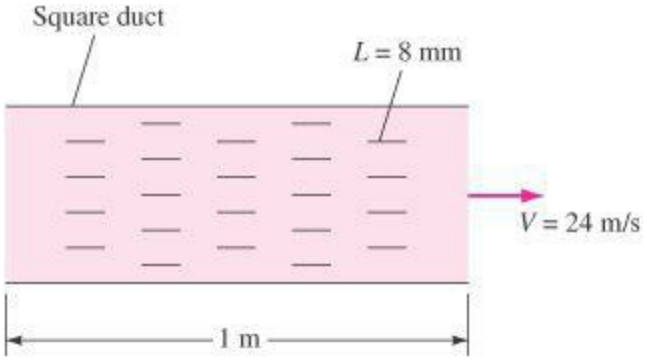


- 3 &RQVLGHU WKH VPRRWK VTXDUH FP E\ FP GXFW Q) LJ 3 7KH IOXLG LV DLU f& DQG DW IORZ LQ DV 9 DYJ P V ,WV GHQVLWHG WR LQFUHDVH WKH SUHVVXUH GUR RYHU WKH P OHQJWK E\ DGGLQJ VKDUS PP ORQJ IODWSODWV DFURVV WKH GXFW D VKRZQ D (VWLPDWH WKH SUHVVXUH GURS L WKH UHDGHU RSODWV E (VWLPDWH KRZ PDQ SODWV DUH QHHGHG WR JHQHUDWH DQ DGGLWLRQDO 3DR SUHVVXUH GURS
- 3 &RQVLGHU D PLQDU ERXQGDU\ OD\HU IORZ SDVWWKH VTXDUH SODWV DUUDQJHPHQW Q) LJ 3 &RPSDUHG WR WKH IULFWLRQ GUDJ RI D VLQJOH SODWH KRZ PXFK GUDJ RI IRXU SODWV WRJHWKHU DV LQ FRQILJXUDWLRQ D DQG E " ([SODLQ \RXU UHVXOW

3



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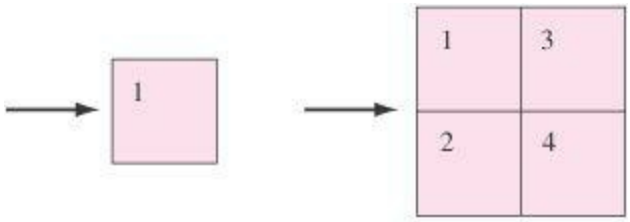
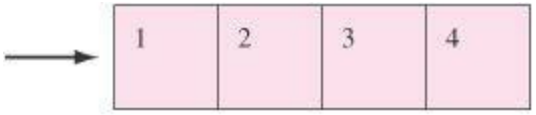


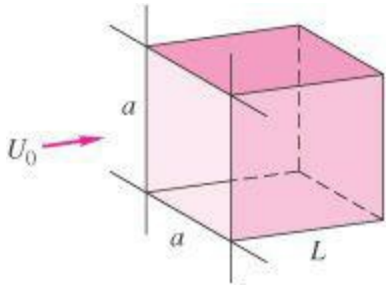
(a)



(b)

- 3 $LUDV f& DQG DVV IORZ VDV P VSDWD VKDSH IODVSODVP P ZLGHDQG P GRQ
  D : KDVLV WKH ZDOO KHDUWUHVV DVWKH HQGRI WKH SODVP E : KDVLV WKH DLUY HORFLV
  DWD SRLQW PP QRUPDO WR WKH HQGRI WKH SODVP F : KDVLV WKH VRXQG IUHTXHQF GU
  RQ WKH SODVP
- 3)RZ VWUDLJKWQHVV DUH DUUDVRI QDUURZ GXFW SDFHG LQ ZLGWKQHOV WR UHPRY
  VZLUODQG RWKHULQSODQH VHFRQGDU YHORFLWLHV 7KH FDQ EH LGHDOL]HG DV VTXDU
  ER[HV FRQWUDFWHG E\ YHUWLFDO DQG KRUL]RQWDO SODVP DV LQ) LJ 3 7KH FURV
  VHFWLRQ LV D E\ D DQG WKH ER[OHQJWK/ $VXPLQJ OPLQDU IORZ SODVP IORZ DQG D
  DUUD RI 1 î 1 ER[HV GHULYH DIRUP XOD IRU D WKH VRXDOGUDJ RQ WKH EXQGOH RI ER[H
  DQG E WKH HIHFWLYH SUHVVXUH GURS DFURVV WKH EXQGOH

3



- 3 /HWWKH IORZ VWUDLJKWQHVV LQ) LJ 3 IRUP DQ DUUD RI î ER[HV RI VL]HD
  FP DQG/ FP ,I WKH DSSURDFK YHORFLWV8 P VDQG WKH IOXLG LV VHD OHYH
  VWDQGDUG DLU HVWP DWH D WKH VRXDODUUD GUDJ DQG E WKH SUHVVXUH GURS DFURVV WK
  DUUD &RPSDUH ZLWK6HF

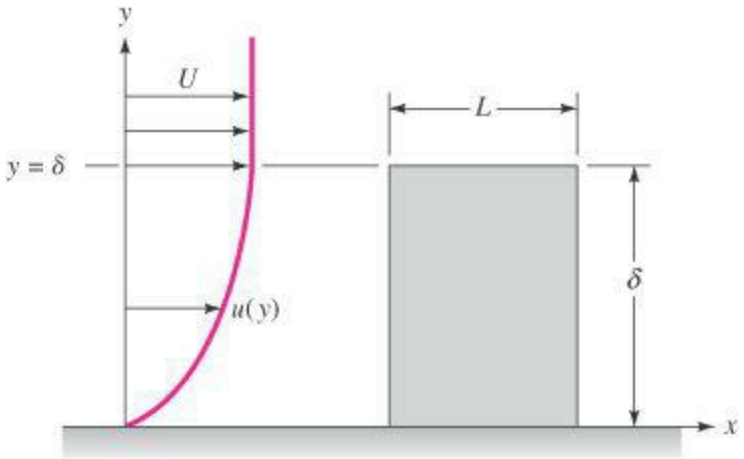
7XUEXOHQWODVSODVPIORZ

- 3 ,Q5H R&KDS 0F. HQHWDO SURSRVHQHZ PRUH DFFXUDWH YDOXHV IRU WK
  VXUEXOHQWORJ ODZ FRQVDQWV DQG% 8VH WKHVH FRQVDQWV DQG WK
  RQH VHYHQWK SRZHU ODZ VR UHSHDWWKH DQDOVLV WKDWOHG WR WKH IRUP XOD IRU WKUEXOHQ
  ERXQGDU OD\HU WKLFNQHVV (T %\ ZKDWSHFHQWLV/ [LQ\RXUQHZ IRUP XO
  GLIIHUHQWURP WKDWQ(T " &RPPHQW
- 3 7KH FHQWHUERUG RQ DVLGERDWLV IWORQJ SDUDOOHOWR WKH IORZ DQG SURWUGHV I
  GRZQEHORZ WKH KXOOLQR VHD ZDWUDW f& 8VLQJ IODVSODVP WKHRU IRUD VPRRV
  VXUDFH HVWP DWLWVGUDJ LI WKHERDWP RYHVDW NQRWV $VXPH5H[W (
- 3 $ IODVSODVP RI OHQJWK/ DQG KHLJKW LV SDFHG DWD ZDOO DQG LV SDUDOOHOWR D
  DSSURDFKLQJ ERXQGDU OD\HU DVLQ) LJ 3 $VXPH WKDWWKH IORZ RYHU WKH SODVP
  IXOO VXUEXOHQWDQG WKDWWKH DSSURDFKLQJ IORZ LV DRQH VHYHQWK SRZHU

$$u(y) = U_0\left(\frac{y}{\delta}\right)^{1/7}$$

8VLQJ VWULS WKHRU GHULYHD IRUPXOD IRU WKH GUDJ FRHILFLHQW RI WKLV SODWH &RPSDUH WKLV UHVXOW ZLWK WKH GUDJ RI WKH VDPH SODWH LPPHUVHG LQ DQ LQFRPS VWUHDP 8

3



- 3 $QDOWLQDWDQDO VLVRI WKXEXOHQWOD\SODWHIORZ ZDVJLYHQE\ 3UDQGWLQ XVLQ DZDOWKHDUWWHVIRUPXODIURP SLSHIORZ

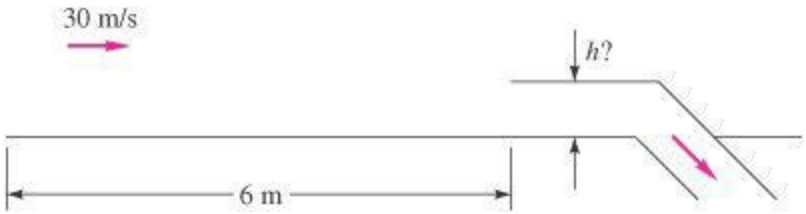
$$\tau_w = 0.0225\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$$

6KRZ WKDWWKLV IRUPXOD FDQ EH FRPELQHG ZLWK(TV DQG WR GHULYH WK IRUOBRZ LQJ UHDWRQVIRUWXEXOHQWOD\SODWIORZ

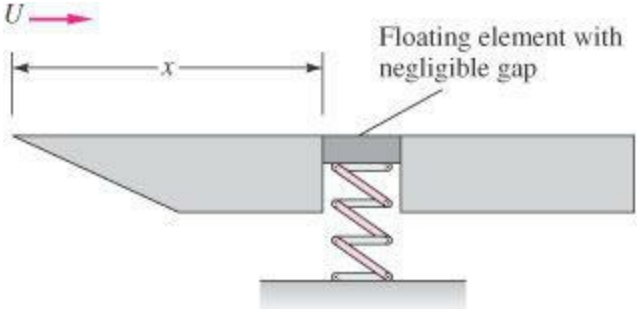
$$\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}} \quad c_f = \frac{0.0577}{\mathrm{Re}_x^{1/5}} \quad C_D = \frac{0.072}{\mathrm{Re}_L^{1/5}}$$

7KHVHIRUPXODVDUHOPLWHGWR5H EHZHHQ î DQG

- 3 &RQVLGHUWXEXOHQWOBRZ SDVWD VKDUS VP RRWKIODWSODWIRI ZLGWKE DQG OHQJWK/ : KI SHUFHQWDJHRI WKHIULFWLRQGUDJRQWKHSODWLVFDUULHGE\ WKHUHDUKDOI WKHSODWW
- 3 7KHVROXWLRQVWR3URE 3 DUH D)) SDWW DQG E)) SDWW ' RQF UHYHDOWKHVH UHVXOWV WR\RXUIULHQGV 5HSHDW3URE 3 DVXPLQJ WKH EOXQGDU OD\HUIORZ LVWXUEXOHQWDQGFRPPHQWRQWKHVWULNLQJ LQFUHDVHLQQXPHULFDOYDOXHV
- 3 EES $ VKLSLV P ORQJ DQGKDVZHWWHGDUHDRI P ,WVSURSHOOHUVFDQGHOLYHD PD[LPXP SRZHURI 0: WR VHDZDWUDW *& ,I DOODGUDJ LV GXHWR IULFWLRQ HVWLPDWWKHPD[LPXP VKLSVSHHG LQNQ
- 3 $LUDW *& DQG DWP IORZ VSDWD ORQJ IODWSODWDWWKHHQG RI ZKLFK LV SDFHG QDUURZ VFRRS DVVKRZQLQ) LJ 3 D (VWLPDWWKHKHLJKWK RI WKHVFRRS LI LWV WR H[WDFW NJ VSHUPHWURI ZLGWKLQWR WKHSDSHU E)LQG WKHGUDJ RQWKHSODWX WR WKHLQDWWR WKHVFRRS SHUPHWURI ZLGWK



- 3 $\WRVSKHUF EROQGDU OD\HUV/DUH YHU VKLFN EXWRQOBZ IRUPXODV YHU VLPLODU V
VKRVH RI IODWSODWH VKRU &RQVLGHU ZLQG EOBZ LQJ DW P V DVD KHJKWRI F
DERYH D VPRRWK EHDFK (VWLPDWH WKH ZLQG VKHDU VWUHVV LQ3D RQ WKH EHDFK LI WKH D
LV VXDQGDUG VHDOHYHOFRQGLWLRQV : KDWZLOOWKH ZLQG YHORFLW VXUQLQJ \RXUQRVHE
LI D \RX DUH VXQQLQJ XS DQG\RXUQRVHLV FP RI WKHJURXQGDQG E \RX DU
OLQJ RQWKHEHDFKDQG\RXUQRVHLV FP RI WKHJURXQG'
- 3 $ K\GURIRLO FP ORQJ DQG P ZLGH PRYHV DW NQLQVHDZDUDW *f& 8VLQ
IODWSODWH WKRU ZLWK5H (HXWP DWLWVGUDJ LQ1 IRU D DVPRRWKZDOODQ
E DURXJKZDOOë PP
- 3 +RHUQHU> S @XDWV/KDWKHGUDJFRHILFLHQWRI DIODJ LQZLQGV EDVHGRQVRV
ZHWWG DUHD E/ LV DSSUR[LPDWG E\ & § / E ZKHUH/ LV WKH IODW
OHQJWKLQWKH IORZ GLUHFWLRQ 7HWW5H QROGV QXPEHU5H ZHUH (RUJUHDWU D
([SODLQZK\ IRU/ E WKHVHGUDJ YDOXHV DUH PXFK KJKHUWKDQ IRUDIODWSODW
$VXPLQJ VHDOHYHOWDQGDUG DLUDW PLK ZLWKDUHDE/ P ILQG E WKHSURSH
IODJGLPHQVLRQVIRUZKFK WKHVRDOGUDJ LVDSSUR[LPDWG 1
- 3 5HSHDW3URE 3 ZLWKWKHVRDFKDQJHWKDWWKHSLWRWSURBHLVQRZ PP IURP WK
ZDOO VWPHVKJKHU 6KRZ WKDWWKHIORZ WKHUHFDQQRWSRXLE) EHDPLQDU DQGXV
VPRRWKZDOOXLEXOHQWORZ WKRU WRHVWPDWWKHSRVLWLRQ[RI WKHSUREH LQP
- 3 $ QJKWDLUFUDWWDHV DW P VLQDLUDW *f& DQG DA ,WZLQJ LVDQ1 $&$
DLURLO ZLWKDFKRUGOHQJWKRI FP DQGDYHU ZLGHVSDQ QHJOHFWDSHFWDW
HIIHFW (VWLPDWWKHGUDJRI WKLVZLQJ SHUXQLWVSDQOHQJWK D E\ IODWSODWWKRU
DQG E XVLQJ WKHGDWDIURP) LJ IRU. *f
- 3 ,QWKHIORZ RI DLUDW *f& DQG DA SDWWDIODWSODWLQ) LJ 3 WKHZDOOVKDULVV
EH GHWHUPLQHG DWSRVLWLRQ[E\ DIORDWLQJ HOHPHQW D VPDOODUHD FRQQHFWHG WR I
WKDLQJDJHIRUFHPHDVXUHPHQW$W[P WKHHOHPHQWLQGLFDWHVDVKDUWUHVR
3D $VXPLQJ WKLBXOHQWORZ IURP WKH OHDGLQJ HGJH HWWP DW D WKH WUHDF
YHORFLW8 E WKHERXQGDU OD\HUWKLFNQHVV/ DWWKH HOHPHQWDQG F WKHERXQGDU
OD\HUYHORFLWX LQP V DW PP DERYHWKHHOHPHQW



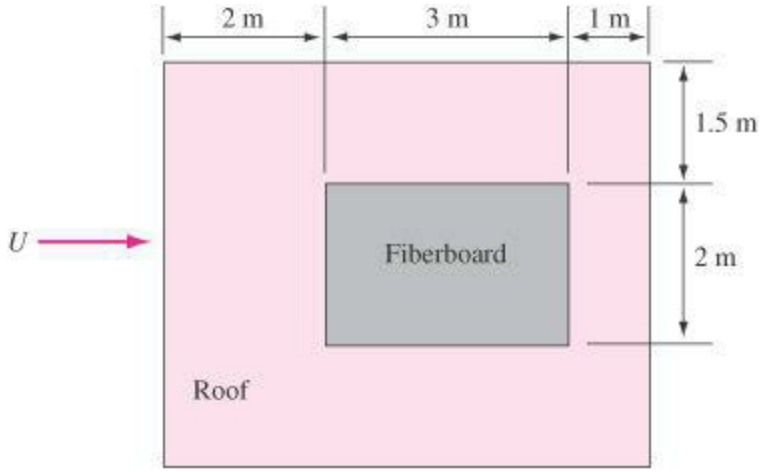
- 3 ([WHQVLYH PHDVXUHPHQWV RI ZDOO VKHDU VWUHVV DQG ORFDO YHORFLW\ IRU WXUEXOHQW IORZ RQ WKH IODW VXUIDFH RI WKH 8QLYHUVLW\ RI 5KRGH ,VODQG ZLQG WXQQHO KDYH EHHQ XVHG WR VK IRUDWRZ LQJ SURSRVHG FRUUHODWLRQ

$$\frac{\rho y^2 \tau_w}{\mu^2} \approx 0.0207 \left(\frac{uy}{\nu}\right)^{1.77}$$

7KXV LI \ DQGX \ DUH NQRZQ DW D SRLQW LQ D IODW SODWH ERXQGDU\ OD\HU WKH ZDOO VKHD PD\ EH FRPSXWHG GLUHFWO\ ,I WKH DQZHU VR SDUW F RI 3URE 3 LVX § P \ GHWHUPLQH WKH VKHDU VWUHVV DQG FRPSDUH ZLWK 3URE 3 ' LVFXV

- 3 $ WKLQ VKHHW RI ILEHUERDUG ZHJKV 1 DQG DHVRQDUROV SDVKRZQLQ) LJ 3 $ VXPH DP EHQWDLUDW *& DQG DW ,I WKH FRHILFLHQW RI VROG IULFWLRQ EHWZHHQ ERDUG DQG URRI LV1 § ZKDWZLQG YHORFLW\ZLOOJHQHUDWHQRXJKIOXLG IULFWLRQ V GLVORGJH WKHERDUG'
- 3 $ VKLS LV P ORQJ DQG KDV D ZHWWHG DUHD RI P ,I LWLV HQFUXVWHG ZLWK EDUQDFOHV WKH VKLS UHTXLUHV KS WR RYHUFRPH IULFWLRQ GUDJ ZKHQ PRYLQJ LQ VHDZDWHU NQDQG *& : KDWLV WKH DYHUDJH URXJKQHVV RI WKH EDUQDFOHV" + RZ IDVWZRXOG WKH VKLS PRYH ZLWK WKH VDPH SRZHU LI WKH VXUIDFH ZHUH VPRRWK" 1HJOHF ZDYHGUDJ

3



- 3 / RFDO ERXQGDU\ OD\HU HIHFWV VXFK DV VKHDU VWUHVV DQG KHDWWUDQVIHU DUH EH FRUUHODWHG ZLWK ORFDO YDULDEOHV UDWKHU XVLQJ GLVWDQFH [IURP WKH OHDGLQJ HGJH 7K PRPHQWXP WKLFNQHVV LV RIWHQ XVHG DV D OHQJWK VFDOH 8VH WKH DQDO\VLV RI WKLV EXOH IODWSODWH ERZ WR ZULWH ORFDO ZDOO VKHDU VWUHVV2 LQWRV RI GLPHQVLRQOHVV DQ FRPSDUH ZLWK WKH IRUPXOD UHFRPPHQGHG E\ 6FKOLFKWLQJ &1 § 5H

%RXQGDU\ OD\HUV ZLWK SUHVVXUH JUDGLHQWV

- 3 ,Q + * | UWHUSUSRVHG WKH GYHUVHJUDGLHQWWAWWDVHV

$$U = \frac{U_0}{(1 + x/L)^n}$$

DQG FRP SXWH VHSDUDWH IRUPXODV IRU ODPLQDU IORZ DQG WXUEXOHQW IORZ DVVXPH WKDW WKH IOXLG LV

- 3 %DVHG RQ \RXU XQGHUVWDQGLQJ RI IODW SODWH ERXQGDU\ OD\HU SURSHUWLHV DQG IDYRUDEOH SUHVVXUH JUDGLHQWV H[SODLQ WKH GLIIHUHQFH EHWZHHQ WKH VHSDUDWLRQ SRLQW IRU D ODPLQDU ERXQGDU\ OD\HU DQG D WXUEXOHQW ERXQGDU\ OD\HU

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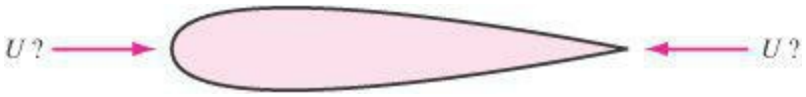

- 3 &RQVLGHU WKH IORZ DURXQG GLIIXVHU LQ)LJ 3 ZKLFK LV VLPLODU WR WKDW LQ)LJ D ZLWK FRQVWDQW ZLGWK E ,I [LV PHDVXUHG IURP WKH LQOHW DQG WKH ZDOO ERXQGDU\ OD\HU WKLQ VKRZ WKDW WKH FRUUHVSRQGLQJ8 [LQ WKH GLIIXVHU LV JLYHQ DSSUR[LPDWHO\ E\

$$U = \frac{U_0}{1 + (2x \tan\theta)/W}$$

ZKHUH: LV WKH LQOHW WKLJKW8 VHW WKLV YHORFLW\ GLVWULEXWLRQ ZLWK 7KZDLWHV PHWKRG V FRPSXWH WKH ZDOO DQJOH IRU ZKLFK ODPLQDU VHSDUDWLRQ ZLOO RFFXU LQ WKH GLIIXVHU OHQJWK/ : 1RWH WKDW WKH UHVXOWLQJ HSHQGHQFH RI WKH5H\QROGV QXPEHU

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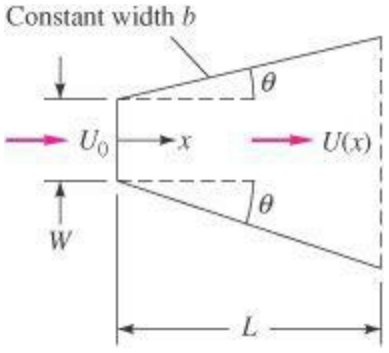


' UDJ RI ERGLHV

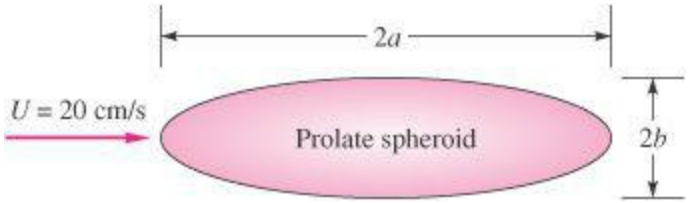
- 3 7KH GHULYDWLRQ RI (T IRU/ [LQ WKH EXOHQW ORZ XVHG YHUV VLPSOH YHORFLW\ FRUUHODWLRQV (TV DQG $ XQLILHG ODZ RI WKH ZDOO ZDV JLYHQ E\ 6SDOGLQJ > @

$$y^+ = u^+ + e^{-\kappa B}\left[e^{\kappa u^+} - 1 - \kappa u^+ - \frac{(\kappa u^+)^2}{2} - \frac{(\kappa u^+)^3}{6}\right]$$

ZKHUHX XX DQG\ \X Y ZKLOH DQG%DUHWKHORJ ODZ FRQVWDQWV LQ(T
7KLV FOHYHULQYHUVH IRUPXOD ILWV DOOWKH GDWD LQ) LJ RXWWR WKH HGJHRI WKHORJDULWKPLF
UHJLRQ +RZ FDQ 6SDOGLQJ¶V IRUPXOD EH XVHG WR LPSURYH(TV DQG DQG WKX
OHDGWR LPSURYHGWKUEXOHQWODVSODWH ORZ UHDWRQ"

- 3 &OLWHWDO > @JLYHWKHIRUPXOD) § CE DE 8E IRUWKHGUDJ RI DSURDO
\SKHURLGLQFUHSLQJ PRWRQ DVVKRZQLQ) LJ 3 7KHKDOWKLFNQHVVE LV PP
,I WKHIOXLG LV6$(: RLODW f& D FKHFNWKDW5H DQG E HVWLPDWHWK
\SKHURLGOHQJWKLI WKHGUDJ LV 1

3



- 3)URP 7DEOH WKH GUDJ FRHILFLHQWVRI D ZLGH SODWH QRUPDO WR D VWHDP LV
DSSUR[LPDWHG / HVWKHVWHDP FRQGLWLRQVE8, DQGS ,I WKHDYHUDJHSUHVVXU
RQ WKH IURQWRI WKH SODWH LV DSSUR[LPDWHG HTXDO WR WKH IUHHVWHDP VWDJQDWLR
SUHVVXUH ZKDWLVWKHDYHUDJHSUHVVXUHRQWKHUHDU
- 3 ,I DPLVWLOHWDNHVRI YHUWLFDOO IURP VHDOHYHODQGGHDYHVWKHDWPRVSKHUH LWKDV]HU
GUDJ ZKHQLWWDUWVDQG]HURGUDJ ZKHQLWILQLVKHV ,WRGRZVWKDWWKHGUDJ PXWEHI
PD[LPXP VRPHZKHUHLQEHWZHHQ 7RVLPSOI\ WKHDQDOVLV DVXPHDFRQVWDQWGUD
FRHILFLHQW& DQGDFRQVWDQWYHUWLFDOFFHOHUDWLRQ D / HVWKHGHQLWYDULDWLRQE
PRGHOHGE\ WKHWRSRVSKHUHUHDWLRQ(T) LQGDQH[SUHVVLRQIRUWKHDOWXG
] ZKHUHWKHGUDJ LVDPD[LPXP &RPPHQWRQ\RXUHVXOW
- 3 $ VKLSWRZ VDVXEPHUJHGF\OLQGHU ZKLFK LV P LQGLDP HWUDQG P ORQJ DW
P V LQIUHVK ZDWUDW f& (VWLPDWHWKHWRZLQJ SRZHU LQN UHTXLUHG LI WK
F\OLQGHULV D SDUDEROLGQG E QRUPDOWR WKHWRZ GLUHFWLRQ
- 3 $ GHOLYHU YHKLFOHIFDUULHVDORQJ VLJQRQWRS DVLQ) LJ 3 ,I WKHVLJQLV YHU WKL
DQG WKH YHKLFOH PRYHV DW PLK D HVWLPDWHWKHIRUFH RQ WKH VLJQ Z LWK Q
FURVVZLQGDQG E GLVFXVVWKHHIHFWRI DFURVZLQG

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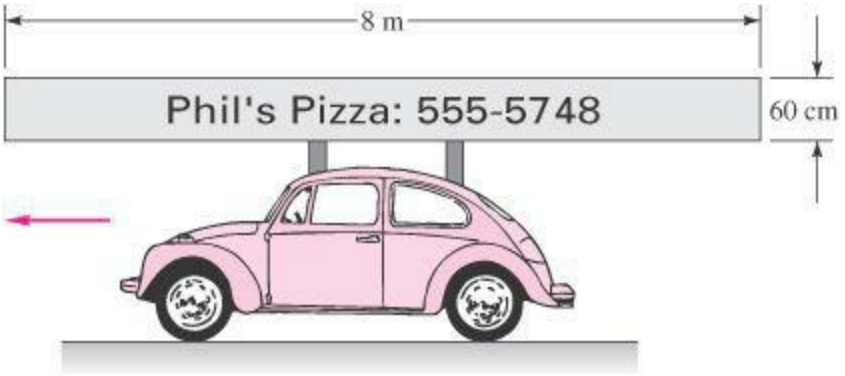

- 3 7KH PDLQ FURVV FDEOH EHWHHQ WZ HLV RI D FRDXDO VXSHQVLRQ EULGJH LV FP L GLDPHWHU DQG P ORQJ (VWLPDWH WKH WRWDO GUDJ IRUFH RQ WKLV FDEOH LQ FURVZ LQGV/ PLK \$UH WKHVH OLPLWDWLRZ FRQGLWLRQV"
- 3 0 RGLI\ 3URE 3 WR EH PRUH UHDOLVWLF E\ DFFRXQWLQJ IRU PLVWLOH GUDJ GXULQ DVFHQW\$VXPH FRQVWDQW WKUXVW7 DQG PLVWLOH ZHLJKW 1HJOHFW WKH YDULDWLRQ RI ZLWK DOWLWXGH 6RLYH IRU WKH DOWLWXGH] LQ WKH VDQG GUDJ DWRVSKHUH WKHQ GUDJ LV DPD[LPXP IRU7 1 : 1 DQG& \$ P 7KHZ ULWUGRHVQ EHOHYHDQDQDOWF VROXWLRQLVSRVVLEOH
- 3 -RHFDQSHGDOKLVELNHDW P VRQDWXDLJKWOHYHOURDGZLWK QR ZLQG 7KHURDG UHVLVWDQFH RI KLVELNHLV 1 VP² WKDWLV 1 RI IRUFHSHUP VRI VSHC 7KHGUDJ DUHD & \$ RI -RHDQGKLVELNHLV P -RH¶VPDVLV NJ DQG WKDWR WKHELNHLV NJ +HQRZ HQFRXQWHUV DKHDG ZLQG RI P V D ' HYHORS D HTXDWRQ IRU WKHVSHHGDWZKLFK-RHFDQSHGDOLQR WKHZLQG +LW\$FXELF HTXDWR IRU9 ZLOOUHVXOW E 6RLYH IRU9 WKDWLV KRZ IDVWFDQ-RHULGHLQR WKHKHDGZLQG F : K\ LV WKHUHVXOWQRWLPSO í P V DVRQHPLJKWLWWXVSHFW
- 3 \$ ILVKQHWFRQLWWR PP GLDPHWUVWULQJVRYHUDSSHGDQGNQRWWG WR IRUP E\ FP VTXDUHV (VWLPDWH WKH GUDJ R P RI VXFK DQHWZKHQWRZHGQRUPDOWR LWVSODQ DW P VLQ *f*& VHZDWU: KDWKRUVHSRZHULVUHTXLUHGWRWRZ IWR WKLVQHW
- 3 \$ ILOWUPD\ EHLGHDOL]HG DVDQDUUD\ RI FA\OQGULFDOILEHUVQRUPDOWR WKHIORZ DVLQ)LJ 3 \$VXPLQJ WKDWWKH ILEHUV DUH XQLRUP0 GLVWULEXWG DQG KDYH GLD FRHILFLHQWJLYHQE\ )LJ D GHULYHDQDSSUR[LPDWH[SUHVVLRQIRUWKHSUHVVXU GURSûSWKURXJKDILOWUR WKLFNQHVV/

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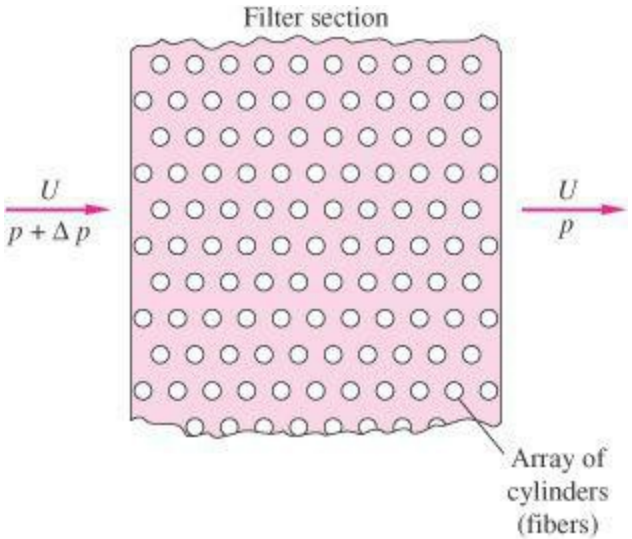


- 3 \$ VHDOHYHOVPRNHWDFNLV P KLJK DQG KDVDVTXDUHFURVVHFWRQ ,WVXSSRUWFD ZLWKWDQG D PDLPXP VLGH IRUFH RI N1 ,I WKH WDFNLV WR VXUYLYH PL KXUULFDQHZLQGV ZKDWLVLWVPD[LPXP SRVVLEOHZLGWK"
- 3)RUWKRVHZKRWKLQNHOHFWULFDUVDUHVLWX . HR 8QLYHUVLWLQ-DSDQKDVWHVWHG IWORQJ SURWRWSHZKRVH HLGKWHOHFWULF PRWRUV JHQHUDWHG RI KRUVHSRZHU 7KH³. D´ FUXLVHV DW PLKU VHH3RSXODU6FLHQFH\$XJXVW S ,I WK

GUDJ FRH ILFLHQWV DQG WKH IURQWDODHDLV IW ZKDWSHUFHQWDJHRI WKLVSRZH
LVH[SHQGHGDJDLQWWHDGHY HODLUGUDJ"

- 3 $ SDUDFKXWWWXMPSVIURP DSODQH XVLQJ DQ P GLDPHWHUFKXWLQWKH WDQGDU
DWRVSKHUH 7KHWRWDOPDVVRI WKHFKXWWDQG WKHFKXWLV NJ $VXPLQJ DQRSH
FKXWDQG TXDLVWDGA PRWRQ HWXP DWWKH WPH WR IDOOIURP WR F
DOWWXGH

- 3 $VVRGLHUVJHWEJJHUDQG SDFNVJHWKHYLHU D SDUDFKXWWDQG GRDG FDQZHJK D
PXFK DV GEI 7KHWDQGDUG IWSDUFKXWPD\ GHVFHQG WRR IDWRUVDHW)R
KHYLHUGRDGV WKH8 6 $UP\ 1DWLN &HQWUKDVGHYHORSHG D IWKJKHUGUD
GHWSRURX; 7 SDUDFKXW VHH KWWS ZZZ QDWLNDUP\ PLO 7KLVSDUFKX
KDVDVHDGHYHOGHVFHQWSHGR IWZLWKD GEI GRDG D : KDWLVWKHGUD
FRHILFLHQWRI WKH; 7 " E +RZ IDWZRXOG WKHWDQGDUG FKXW GHVFHQG DWWH
GHYHOZLWKVXFKDGRDG'

- 3 $ VSKHUHRI GHQVLW!V DQG GLDPHWU' LVGURSSHGIURP UHVWQDIXLGRI GHQVLW! DQ
YLVFRVLW \$VXPLQJ DFRQWDQWGUDJ FRHILFLHQ $ GHULYHDGLIHUHQWDOHTXDWR
IRUWKHIDOOYHORFLW9 WDQG VKRZ WKDWWKHVROXWQLV

$$V = \left[\frac{4gD(S-1)}{3C_{d_0}}\right]^{1/2} \tanh Ct$$

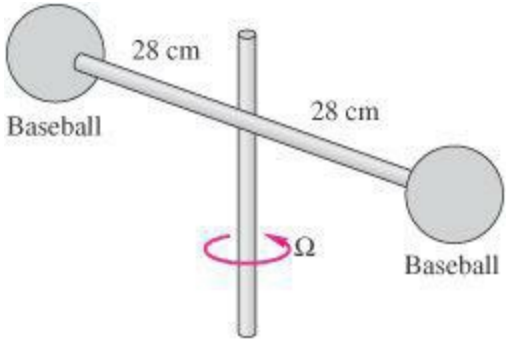
$$C = \left[\frac{3gC_{d_0}(S-1)}{4S^2D}\right]^{1/2}$$

ZKHUH6 !V! LVWKHVSHFLILF JUDYLWRI WKHVSKHUHP DWHULDO

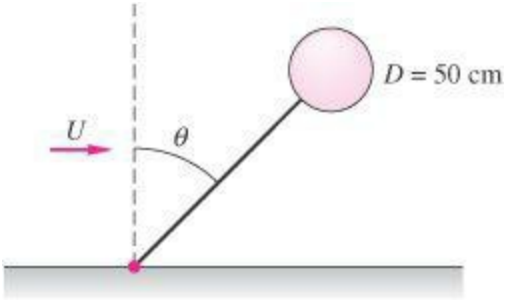
- 3 $ ZRUGG FDVV ELF\FOHULGHUFDQJHQHUDWRQHKDO KRUVHSRZHUIRUGRQJ SHURGV ,
UDFLQJ DWXHDGHYHO HXMP DWWKH YHORFLW WKLV F\FOLWFDQ PDLQWDLQ 1HJOHFWURGQ
IUFWRQ

- 3 7KH 0 DV URYLQJ GERUDWRU SDUDFKXW LQ WKH&KDS RSHQHU SKRWR LV D IV
GLDPHWUGLVNJDS EDQG FKXW ZLWKDPHDXUHGGUDJ FRHILFLHQWRI > @0 DV
KDV YHUV GRZ GHQVLW DERXW (VOXJ IW DQG LWV JUDYLWV LV RQO SHUFHQWR
HDWKJUDYLW ,I WKHPDVVRI SD\GRDG DQG FKXWLV NJ HXMP DWWKHWUPLQDOYHO
YHORFLWRI WKHSDUDFKXW

- 3 7ZR EDVHEDOOVRI GLDPHWU FP DUHFRQQHFWG WR DURG PP LQGLDPHWUDQG
FP GRQJ DV LQ) LJ 3 : KDWSRZHU LQ : LV UHTXLUHG WR NHHS WKH VXVWP
VSLQQLQJ DW UPLQ' ,QFOXGHWKHGUDJ RI WKHURG DQG DXXPHVHDGHYHOWDQGDU
DLU



- 3 7KH $UP\¶V QHZ $736 SHUVRQQHOSDUDFKXWH LV VDLG WR EH DEOH WR EULQJ D GE
  OBDG WRRSHUSOXVSDFN WR JURXQG DW IWWLQ³PLOHKJK ' HQYHU &RORUDGR ,I Z
  DVXPH WKDW7DEOH LV YDOLG ZKDWLV WKH DSSUR[LPDWH GLDPHWU RI WKLV QHZ
  SDUDFKXWH"
- 3 7KH)RUG (VFDSH +\EULGKDVDQHPSWZHLJKWRI GE DGUDJ DUHD& $
  IW> @DQG DURQQQJ UHVLVWDQFHFRHILFLHQW5 & ZLWKRXWEUDNHV RI GE
  SHUPLK RI VSHHG (VWLPDWH WKH PD[LPXP YHORFLW WKLV YHKLFOH FDQ DWWDLQ ZKH
  URQQQJ IUHHO DWWHDOHYHOFRQGLWRQV GRZQD GHJUHHVOSH
- 3 $ VHWWQJ WDQN IRUD PXQLFLSDOZDWUVXSSO LV P GHHS DQG *& ZDWUIORZ
  WKURXJK FRQWXRXV DW FP V (VWLPDWH WKH PLQLPXP GHQJWK RI WKH WDQN WKDWZ LG
  HQVXUH WKDWDOVHGLPHQW6* ZLOOIDOOWR WKH ERWWP IRUSDUWFOHGLPHWU
  JUHDWHUWKDQ D PP DQG E P
- 3 $ EDOORRQLV PLQGLPHWUDQG FRQWLQVKHOXP DW N3DDQG *& %DOORR
  PDWULDO DQG SD\ORDG ZHJK 1 QRWLQFXGLQJ WKH KHOXP (VWLPDW D WK
  WUPLQDODFHQWYHORFLW LQ VHOYHDWDQGDUG DU E WKH ILQDO WDQGDUG DGWWXG
  QHJOHFWQJ ZLQGV DWZKLFK WKH EDOORRQ ZLOOFRP H WR UHVWDQG F WKH PLQLPXP
  GLPHWU P IRUZKLFK WKH EDOORRQ ZLOOMXWWEDUHO EHJLQ WR UL HLQVHOYH
  WDQGDUG DU
- 3 ,WV GLIILFXOWWR GHWHUWKH³IURQDODUHD´ RI DPRWRUF\FOH[VKDSH
  2QH WKHQP HDXUHV WKH GUDJ DUHD WKDWLV & $ LQDUHDXQLW +RHUQHU> @UHSRU
  WKH GUDJ DUHDRI DW\SLFDOPRWRUF\FOH LQFXGLQJ WKH XSULJKWGULYHU DVDERXW IW
  5RQQQJ IUFWRQ LV W\SLFDOO DERXW GE SHUPLK RI VSHHG ,I WKDWLV WKH FDV
  HWUP DW WKH PD[LPXP VHOYHDVSHHG LQPLK RI WKH QHZ +DUOH\ ' DYLGVRQ 9
  5RGGE F\FOH ZKRVHOTXLG FRROHGHQJLQHSURGXFHV KS
- 3 7KH KHOXP ILOOHG EDOORRQ LQ) LJ 3 LV WWKHUHG DW *& DQG DW ZLWK
  VXUQJ RI QHJOJLEOH ZHJKWDQG GUDJ 7KH GLDPHWU LV FP DQG WKH EDOORR
  PDWULDOZHJKV 1 QRWLQFXGLQJ WKH KHOXP 7KH KHOXP SUHVXUHLV N3D
  (VWLPDW DW WKH WLWWDQJOH LI WKH DLUWHP YHORFLW8 LV D P VRU E P V



- 3 7KH PRYLH7KH : RUOGV) DVWHVW, QGLDQ WHOO VWKH VWRU\ RI %XUW0 XQUR D 1HZ =HDODQGHUZKR LQ VHDUFK RI PLKRQVWH%RQHYLOOH6D) ODW 8VLQJ WKH GDWD RI 3UREOHP D HVWLPDWH WKH KRUVHSRZHUQHHGHG WR GULYH WKLV IDVWE : KDWKRUVHSRZHUZRXOGKDYHJRWWQ%XUWSWR PLK"
- 3 7RPHDVXUHWKHGUDJ RI DQXSULJKWSHUVRQ ZLWKRXWYLRODWLQJ KXPDQVXEMHFWSURWHFWLRQ DOLHVL]HGPDQQHTXLQLVDWWDFKHGWR WKH HQG RI D P URGDQGURWDWHG DW UHY PLQ DVLQ) LJ 3 7KHSRZHUUHTXLUHGWRPDLQWDLQWKHURWDWLRQ N % LQFOXGLQJ URGGUDJ SRZHU ZKLFK LV VLJQLILFDQWHWWPDWWKHGUDJ DUHD& \$ RI WK PDQQHTXLQLQP

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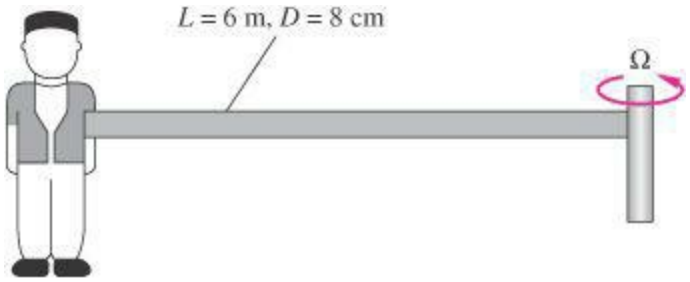

- 3 2Q\$SULO D) UHQFK EXOHWWDLQ VHWD QHZ VSHHG UHFRUG IRU UDLOGULYH-VWDLQV RI PLK EHDWQJ WKHRGUHFRUGE\ SHUFHQW8VLQJ WKHGDWDLQ7DE[HVWLPDWWKHVHDOHYHOKRUVHSRZHUUHTXLUHGWRGULYHWKLVWDLQDWXFKDSHG
- 3 \$VXPHWKDWDUDGLDFWLYHGXWSDUWFHDSSUR[LPDWVDSKHUHRI GHQVLW NJ P +RZ GRQJ LQGD\ ZLOLWNHVXFKDSDUWFHWRVHWWHWRVHDOHYHOIURP DQDOWXG RI NP LI WKHSDUWFHGDPHWUV D P RU E P"
- 3 \$ KHDY\ VSKHUH DWWDFKHGWR D VWULQJ WKRXGKDQJ DWDQDQJOH ZKHQLPPHUVHGLQI VWUHDP RI YHORFLW8 DVLQ) LJ 3 ' HULYHDQH[SUHVLRQIRU DVDIXQFWLRQRI WKH VSKHUHDQG IORZ SURSHUWLHV : KDWLV LI WKH VSKHUHLV VWHO 6* F GLPHWU FP DQG WKHIORZ LV VHDOHYHOVWDQGDUGDLUDW8 P V" 1HJOHFWWK VWULQJ GUDJ

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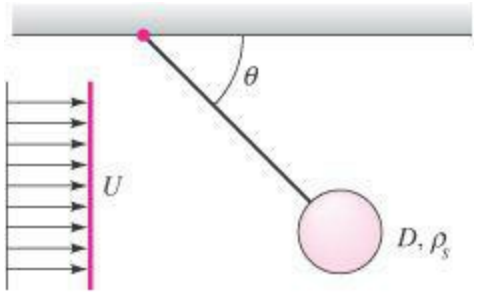

- 3 $ \WSLFDO8 6 $ UP\ SDUDFKXWKDVDSURMHFWLOHGLDPHWURI IW) RUDSD\ORDGPDV RI NJ D ZKDWWRWLQDOYHORFLWZLOOUHVXOWDW P VDQGDUGDOWWKH') RUWK VDPHYHORFLWDQGQHWSD\ORDG ZKDWWLHGUDJ SURGXFLQJ³FKXWH LVUHTXLUHGLI RQ XVHVDVTXDUHIDWSDWKHGE YHUWFDO DQG F KRUL]RQWDO" 1HJOHFWWKHIDF WKDWWDWKDSHVDUHQRWA QDPLFDO VDEOHILQIUHIDO
- 3 6N\GLYHUV IOLQJ RYHUVHDOHYHOJURXQG \WSLFDO MXPS DWDERXW IWDOWWKHDQ IUHIDOVSUHDGHDJOHGXQWWKH\ RSHQWKHLUFKXWHVDWDERXW IW7KH\ WDNHDERX VWRUHFKWUPLQDOYHORFLW (VWP DWKRZ PDQ VHFRQGVRI IUHIDOWKH HQWR LI D WKH IDOVSUHDGHDJOHG RU E WKH IDOIHHWLWW\$VXPHDWRWDOVN\GLYH ZHJKWR GE
- 3 $ KJK VSHHGFDUKDVDGUDJFRHILFLHQWR DQGDIURQWDODUHDR P \$SDUFKXW LVWRE HXVHGWRVORZ WKLVFDUIURP WR P VLQ V: KDWWKRXGWKHFKXW GLDPHWUEH' : KDWGLXDQFH ZLOEHWDYHOHG GXULQJ WKLVGHFHOHUDWRQ' 7DNHP NJ
- 3 $ 3LQJ 3RQJ EDOOZHJKV J DQGKDVDGLDPHWURI FP ,WFDQEHVXSSRUWHGE DQDLUWWRUP DYDFXXP FHDQHURXWHWDVLQ) LJ 3) RUVHDOHYHOWDQGDUGDL ZKDWWWYHORFLWLVUHTXLUHG'

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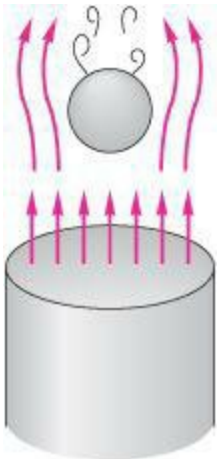

- 3 ,QWKLVHUDR H[SHQLYHIRWLOIXHOV PDQ DQWLQDWVKDYHEHQSXUXHG 2QHLGH IURP 6N\6DLOV ,QF VKRZQLQ) LJ 3 LVWKHDWLWXGSURSXOVLRQRI DVKLS E\ I ODJHWWKHUHGNLW 7KHWRZ IRUFHRI WKHNLWDWLWWKHVKLS¶VSURSHOHUDQGLVDL WRUHGXFHDQQXDOIXHOFRQVXPSWRQE\ ± SHUFHQW) RUDWSLFDO[DPSOH OHWWKH VKLSEH P ORQJ ZLWKDZHWWGDUHDR P 7KHNLWDUHDLV P DQGKD DIRUFHFRHILFLHQWR 7KHNLWFDEOHPDNHVDQDQJOHR *ZLWKWKHKRUL]RQWDO /HW9 ZLQG PLK 1HJOHFWWKLSZDYHGUDJ (VWPDWWKHVKLSVSHHG D GXHWRWK

NLWRQ DQG E LI WKHSURSHOOHUGHOYHUV KSWRWKHZDWU+LQW7KHNLWVH
WKHUHDWYHYHORFLWRI WKHZLQG

3 6KLSSURSXOVLRQDWLWMGE\ DOUJHNLW*(Courtesy of SkySails, Inc*

- 3 +RHUQHU>5H SS ± @XDWVWKDWWKHGUDFRHILFLHQWRI DIODJ RI DVSH
UDWLR EDVHG RQ SODQRUP DUHD 7KH 8QLYHUVLW\ RI 5KRGH ,VODQG KDV D
DXPLQXP IODJSROH P KLJK DQG FP LQGLDPHWU ,WOHVHTXDOV]HG QDWRQ
DQG WDWWHIODJV WRJHWKHU ,I WKHIUDFWXUHWWHVWRI DXPLQXP LV 0 3D ZKDWLV WK
PD[LPXP IODJ VL]H WKDWFDQEHXVHG\HWDYRLGVEUHDNLQJ WKHIODJSROHLQKXUULFDQ
PLK ZLQGV" 1HJOHFWWKHGUDJRI WKHIODJSROH
- 3 $ WDFWRUWDLOHU WUXFN KDV D GUDJ DUHD& $ P EDUH DQG P ZLWK DI
DHURG\QDPLF GHOHFWRU)LJ E ,WURQQJ UHLXDQFHLV 1 IRUHDFK PLOHSH
KRXURI VSHHG &DOFXODWWKHVWDOKRUVHSRZHUHTXLUHGDWHDFKYHOZLWKDQGZLWRX
WKHGHOHFWRUL WKHWUXFNPRYHVDWD PLKDQG E PLK
- 3 $ SLFNXS WUXFN KDV D FOHDQ GUDJ DUHD& $ RI IW (VWLPDWWKH KRUVHSRZH
UHTXLUHGWRGULYHWKHWUXFNDW PLK D FOHDQDQG E ZLWKWKH E\ IWWLJQL
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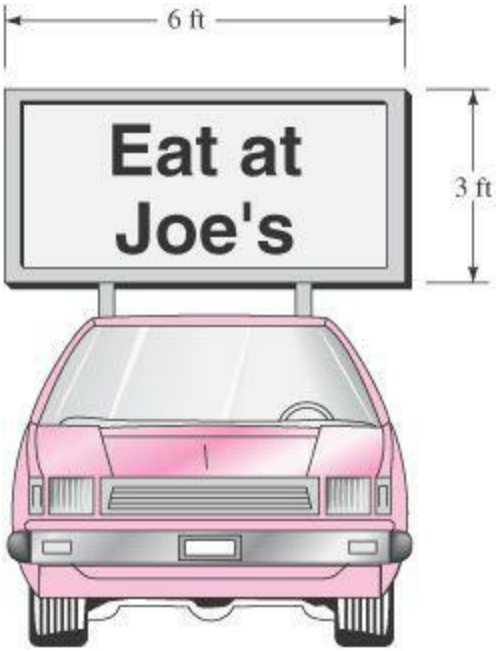

- 3 7KH QHZ $0 75$. KLJK VSHHG $FHOD WDLQFDQUHDFK PLK ZKLFK SUHVHQW VHORP GRHV EHFDXVH RI WKH FXUY FRDWLQH1HZ (QJODQG ,I SHUFHQW WKH SRZHU H[SHQGHG DWWKLV VSHHG LV GXH WR DLUGUDJ HXWP DW WKH WRDO KRUVHSRZH UHTXLUHG E\ WKH$FHOD
- 3 EES ,Q WKH JUHDWKXUFDQH RI ZLQGV RI PLK EOHZ RYHU D ER[FDU L 3URYLGHQFH 5KRGH,VODQG 7KHER[FDUZ DV IWKLJK IWORQ DQG IWZ LGH ZL D IWFOHDUDQFHDERYHWDFN IWDSDUW: KDWZ LQG VSHHGZ RXOG VRSSOHDER[FD ZHJKLQJ GE"
- 3 $ FXS DQHPRPHWUXVHVWR FP GLDPHWUKRORZ KHPLVSKHUHVFRQQHFWHG WR FP URGV DV LQ) LJ 3 5RG GUDJ LV QHJOLJLEOH DQG WKH FHQWDOEHDULQJ KDV I UHVLVWDQFH WRUTXHRI 1 P 0DNLQJ VLPSOLI\LQJ DVXPSWLRQV WR DYHUDJHRXWK WPHYDULQJ JHRPHWU HXWP DWDQGSORWWKHYDUDWRQ DQHPRPHWUUHDGLQ ZLWKZLQGYHORFLW8 LQWKHUDQJH 8 P VIRUVHDOHYHOWDQGDUG DLU

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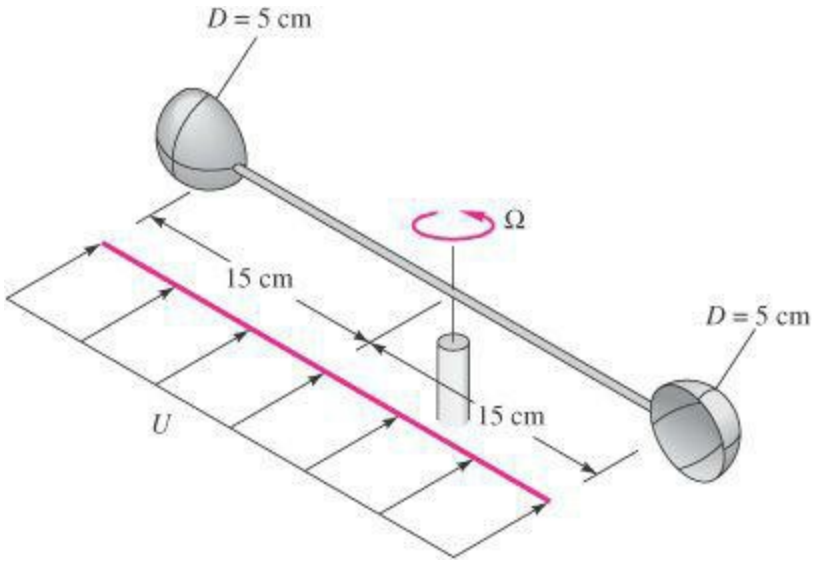

- 3 $ NJ DXWRPRELOHXVHVLWVGUDJ DUHD& $ P SOXVEUDNHVDQGDSDUDFKXW WRVORZ GRZQIURP P V ,WVEUDNHVDSSO 1 RI UHVLVWDQFH $VXPHVHDOHYHOVWDQGDUG DLU ,I WKH DXWRPRELOH PXVWVWRS LQ V ZKDWGLDPHWU SDUDFKXW LV DSSURSULDW
- 3 $ KRWOLP SUREHLVPRXQWHGRQDFRQHDQGURGV\VWHP LQDVHDOHYHOWDLUW RI P V DV LQ) LJ 3 (VWLPDWH WKH PD[LPXP FRQH YHUW[DQJOH DOORZ DEOH LI WK IORZ LQGXFHGEHQGLQJ PRPHQWDWKHURRWR WKHURGLVQRWR H[FHHG 1 FP

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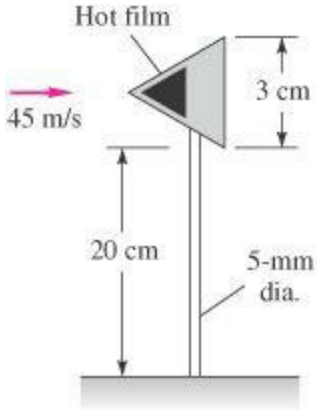

- 3 $ URVDU PL[HUFRQLWWRI VZR P ORQJ KDO WKEHV URVDLQJ DURXQG DFHQWDODU D LQ) LJ 3 8VLQJ WKH GUDJ IURP 7DEOH GHULYHDQH[SUHVVLRQ IRU WKH WRTXH7 UHTXLUHG WR GULYH WKH PL[HUDWDQXOYHORFLW LQDIXLGRI GHQVLW! 6XSSRV WKDWWKHIOXLGLVZDWUDW *& DQGWKHPDLPXP GULYLQJSRZHUDYDLODEOHLV N : KDWLVWKHPDLPXP URVDWRQVSHHG UPLQ'

- 3 $QDLUSODQHZHJKLQJ N1 ZLWKDGUDJDUHD& $§ P ODQGVDWWDOHYHODW P VDQGGHSOR\VDGUDJSDUDFKXW P LQGLDPHWU1RRWKERUEUN HVDUHDSSOHG D +RZ ORQJ ZLOOLWWDNHWKHSODQHWRVORZ GRZQWR P V" E +RZ IDUZLOOLWKDY WDYHOHGLQWKDWWLPH'

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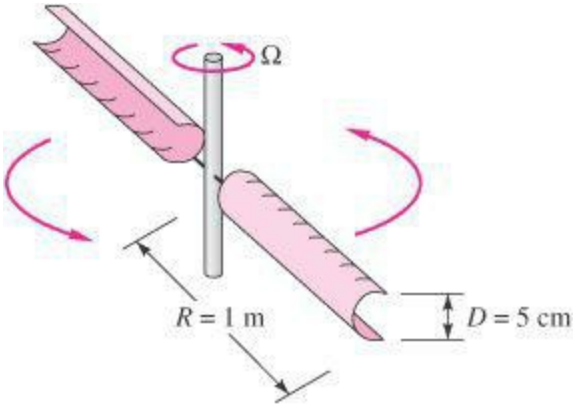

- 3 $ 6DYRQLXVURWRU)LJ E FDQEHDSSUR[LPDWHG E\ WKHVZR RSHQKDO WKEHV LQ)LJ 3 PRXQWHGRQDFHQWDOD[LV ,I WKHGUDJ RI HDFK WKEHV VLPLODUWR WKDWL 7DEOH GHULYHDQDSSUR[LPDWHIRUPXODIRUWKHURWDWRQUDW DVDIXQFWLRQRI 8 ' / DQGWKHIOXLGSURSHUWLHV !

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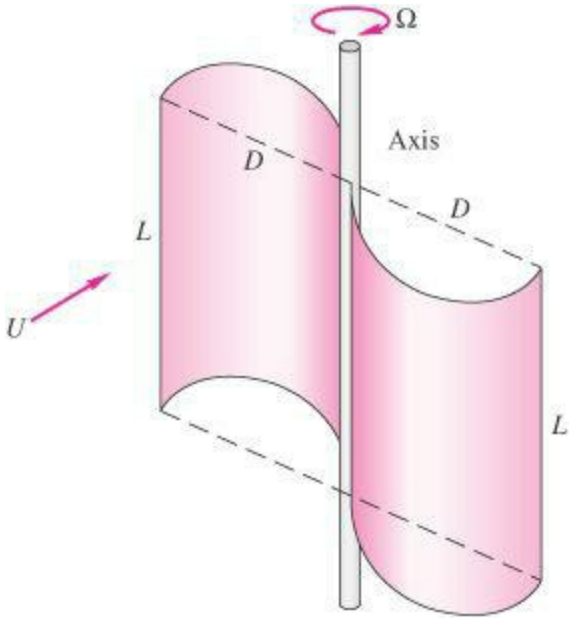


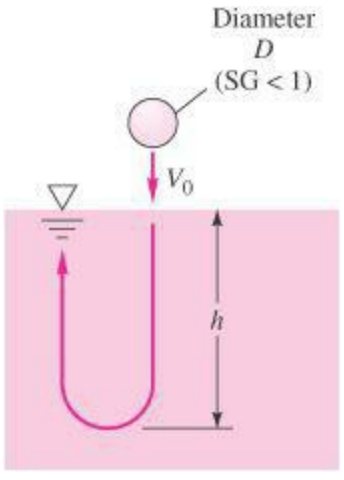
- 3 $ VLPSOHPHDVXUHPHQWRI DXWRPRELOHGUDJFDQEHIRXQGE\ DQXQSRZHUHGFRDVWGRZQ RQ D OHYHO URDG ZLWK QR ZLQG $VXPH FRQVWDQWURQLQJ UHVLVWDQFH)RU D DXWRPRELOHRI PDVV NJ DQGIURQWDODUHD P WKHIROORZLQJ YHORFLWYHUXV WPHGDWDUHREWDLQHGGXULQJDFRDVWGRZQ

(VWLPDWH D WKHURQLQJ UHVLVWDQFHDQG E WKHGUDJFRHILFLHQW7KLVSUREOP LVZH VXLWGIRUFRPSXWDWLRQDOVLVEXWFDQEHGRQHE\ KDQGDOR

t, s	0	10	20	30	40
V, m/s	27.0	24.2	21.8	19.7	17.9

- 3 $ EXR\DQWEDOORI VSHFLILF JUDYLW6* GURSSHGLQWR Z DWUDWLQDWHUYHORFLW9 ZLOOSHQHWUDWHDGLVWDQFHK DQGWKHQSRSRXWDJDLQ DVLQ) LJ 3 0 DNHDG\QDPLL DQDOVLV RI WKLV SUREOP DVXPLQJ D FRQVWDQWGUDJ FRHILFLHQW DQG GHULYH DI H[SUHVVLRQ IRUK DV D IXQFWLRQ RI WKH VXVWP SURSHUWLHV +RZ IDU Z LOOD FP GLDPHWUEDOOZ LWK6* DQG&§ SHQHWUDWLLWHQWVDW P V"

3



- 3 7Z R VWHHOEDOOV 6* DUHFRQQHFWHGE\ DWKLQKLQJHGURGRI QHJOJLEOHZ HLJK DQG GUDJ DVLQ) LJ 3 $ VWRS SUHYHQWVWKHURGIURP URWDWLQJ FRXQWHUFORFNZ LVH

(VWLPDWH WKH VHDOHYHODLUYHORFLW8 IRUZKLFK WKH URG ZLOOILWWEHJLQ WR URWD
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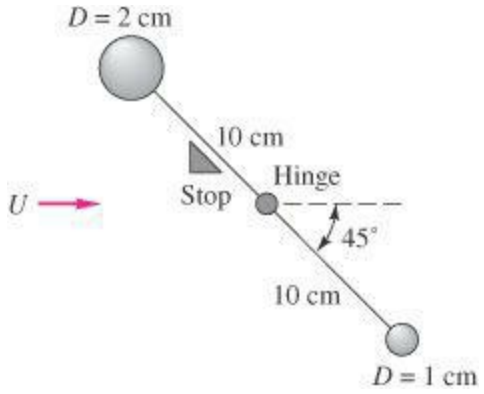

- 3 $ VDFKDUWDLHUWXEHFNLVFRDWLQJ IUHO ZLWKQRE UDNHV GRZQDQ f VORSHDW P VWDQGDUG DQGWKH 5RQDQ UHLVWDQFHLV 1 IRUHYHU P VR VSHHG ,WV IURQW DUHDLV P DQG WKHZHJKWLV N1 (VWLPDWH WKHWHUPLQDOFRDWLQJ YHORFLW LQ PLK IRU D QRGHOFKDUDQG E DGHOFKDUDLQWDOHG
- 3 ,FHEHUJV FDQ EH GULYHQ DWXEXODQWVO VSHHGV E\ WKH ZLQG / HWWKH LFHEHUJ E LGHDOL]HG DV DODUJH IODWA QQGHU' ⩾ / ZLWKRQHHJKWKRI LWVEXONH[SRVHG DV L)LJ 3 / HWWKHVHDZDWUEHDWWW,I WKHXSSHUDQGORZHUGUDJ IRUFHVGHSHQ RQ UHDWLYH YHORFLWWV EHWZHHQ WKH LFHEHUJ DQG WKH IOXLG GHULYH DQ DSSUR[LPDW H[SUHVVLRQIRUWKHWDG LFHEHUJ VSHG9 ZKHQGULYHQE\ ZLQGYHORFLW8
- 3 6DQGSDUWFOHV 6* DSSUR[LPDWO VSKHUFDOZLWKGLDPHWWV IURP VR P DUHLQWRGXFHG LQR DQXSZDUG IORZLQJ VWUHDP RI ZDWUDW f& : KDWLV WK PLQLPXP ZDWUYHORFLWWKDWZLOOFDUU DOOWKHVDQGSDUWFOHVXSZDUG'

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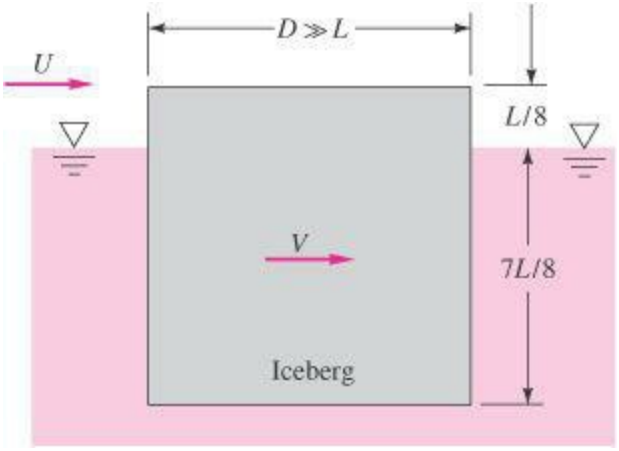

- 3 : KHQLPPHUVHGLQDXQIRUP VWUHDP 9 DKHDY\ URGKLQJHGDV$ ZLOOKDQJ DV3RGH¶ DQJOH DWWUDQ DQDOVLV E\ / 3RGHLQ)LJ 3 $VVXPH WKDWWK FA\OQGHUKDVQRUPDOGUDJ FRHILFLHQW& 1 DQGWDQJHQWLDOFRHILFLHQW& 7 WKDWWHDV WKHGUDJ IRUFHVWR9 1 DQG9 7 UHVSHFWLYHO ' HULYHDQH[SUHVVLRQIRU3RGH¶V DQJOH DVDIXQFWQRQ RI WKHIOXLG DQGURG SDUDPHWWV) LQG IRUDVWHOURG / FP '

FP KDQJLQJ LQVHDOHYHOLDUV P V

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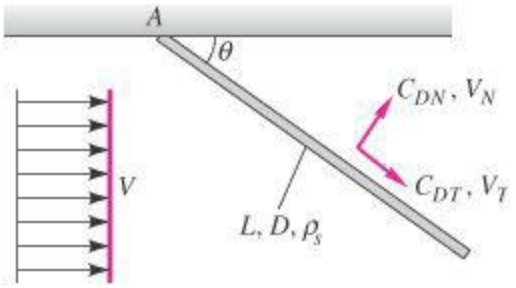

- 3 7KH5XWLDQ7\SKRRQFODVVXEPDULQHLV P ORQJ ZLWKDPDLPXP GLDPHWUF P ,WV SURSXOVRU FDQ GHOLYHU XS VR KS VR WKH VHDZDWU 0RGHOWK VXEPDULQH DV DQ HQJLQHHULQJ SURGXFW RI WKLV VKLS
- 3 $ VKLS P ORQJ ZLWKDZHWWGDUHDRI P KDVWKHKXOOVKDSHWWXGLQ) LJ 7KHUHDUHQR ERZ RUVWULQEXEV 7KHVWDOSURSXOVLYHSRZHUDYDLODEOHLV 0:)RUVHDZDWUDW *& SORWWKHVKLS¶VYHORFLW9 NQYHUVXVSRZHU3 IRU 3 0: : KDWVWKHPRXWHILFLHQWVHWWQJ"
- 3)RUWKHNLWHDXLWXGVKLSRI3URE 3 DJDLQQHJOHFWZDYHGUDJ DQGOHWWKHZLQ YHORFLWVEH PLK (VWLPDWWKHNLWHDUHDWKDWZRXOGWRZ WKHVKLS XQDLGHGE\ WK SURPHOOHUDWDVKLSVSHHGRI NQRWV
- 3 7KHODUJHWWODJ LQ5KRGH,VODQGVDQGVRXWLGH+HE&KDPEHUW¶DXWRGHOOHUKLS R WKHHGJHRI 5RXWH, LQ3URYLGHQFH 7KHIDJ LV IWBRQJ IWZLGH ZHJKV OEI DQGWDNHVIRXUWRQJSHRSOHWRUDLVHLWRUORZHULW8VLQJ3URE 3 IRULQSX HWWPDW D WKHZLQGVSHHG LQPLK IRUZKLFKWKHIDJGUDJ LV OEI DQG E WK IDJ GUDJ ZKHQ WKH ZLQG LV D ORZ HQG FDWJRU KXUULFDQH PLK +LQ 3URYLGHQFHLVDWWHDOHYHO
- 3 7KHGDWDLQ) LJ 3 DUHIRUWKHQDWDQGGUDJ RI DVSLQQLQJ VSKHUHIURP 5H 6XSSRVHWKDWDWWQLVEDOO: § 1 ' § FP LV WUXFN DWWHDOHYHOZLW LQWDOYHOFLW9 P VDQG³WRSVSLQ IURQWR WKHEDOURWDWQJ GRZQZDUG R UV ,I WKHLQWDOKHJKWRI WKHEDOLV P HWWPDW WKHKRUL]RQDOGLVWDQF WDYHOHGEHRUHLVWXUNHVWKHJURXQG
- 3 7KHZRUGUHRUGIRUDXWRPRELOHPLOHDJH PLOHVSHUJDOORQ ZDVHWQ E\ WKH3$& &$5 ,, LQ) LJ 3 EXLOWE\ WXGHQWVDWWKH6ZLW) HGHUDO,QVWWXWR 7HFKQRORJ\ LQ=XULFK > @7KLV OLWWH FDU ZLWK DQHPSW ZHJKWR GEI DQG KHJKWR RQO IWWDYHOHGD NP FRXUHDW NP KUWR VHWWKHUHRUG ,WKDV UHSRUWG GUDJ FRHILFLHQWR FRPSDUEOHWR DQDLURLO EDVHGXSRQDIURQW DUHDR IW D : KDWLVWKHGUDJ R WKLV OLWWH FDU ZKHQRQWKHFRXUVH' E : KD KRUVHSRZHULV UHTXLUHGWSURSHOLW F ' RDELWR UHVDFK DQGH[SODLQZK\ I YDOXHR PLOHVSHUJDOORQLVFRPSOHWD PLVOHDGLQJ LQWKLVSDUWFXODUFDVH
- 3 $ EDVHEDOOSLWFKHUWKURZVDFXUYHEDOOZLWKDQLQWDOYHORFLWVR PLK DQGDVSL R UPLQDERXWDYHUWFDOD[LV $ EDVHEDOOZHJKV GEI DQGKDVDGLDPHW R LQ 8VLQJ WKHGDWDR) LJ 3 IRUVXEVHQWORZ HWWPDWKRZ IDUVXFKE FXUYHEDOOZLOOKDYHGHYLDWGIURP LWVWUDLJKWDQHSDWKZKHQLWUHDFKHVKRPHSODW

IWDZ D\

- 3 $ VDEOH WHQQLV EDOO KDV D PDVV RI J DQG D GLDPHWHU RI FP ,WLV VWUFN KRUL]RQWDOO DWDQ LQLWLDO YHORFLW\ RI P V ZKLOH LWV FP DERYH WKH VDEOH D LQ) LJ 3) RUVH OHYHOLU ZKDWSLQ LQUPLQ ZLOO FDXVH WKH EDOO WR VWUN WKH RSSRVLWH GJHRI WKH VDEOH P DZ D\" 0 DNHQDQDO WFDOHXWP DW XVLQJ) LJ 3 DQG DFFRXQWRUWKHIDFWKDWWKHEDOOGHFHOHUDWHVGXULQJIOJKW

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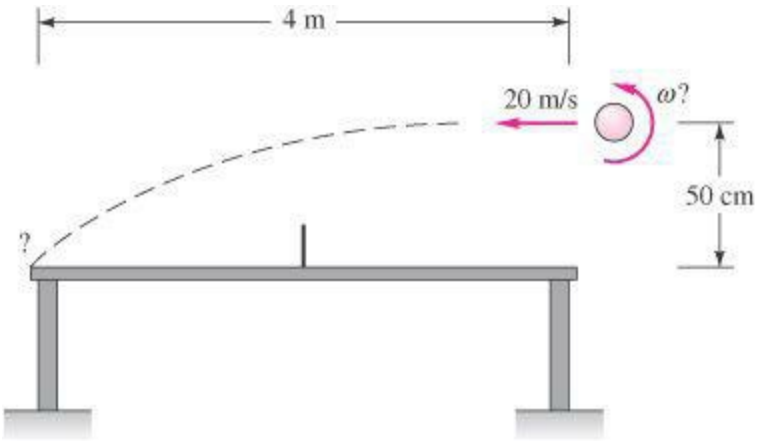

- 3 $ VPRRWKZ RRGHQVSKHUH 6* LVFRQQHFWHGE\ DWKLQULGURGWR DKLQJHL DZLQG WXQQHO DV LQ) LJ 3 $LUDW *DQG DW IORZ V DQG OHYLWDWHV WKVSKHUH D 3OWKHDQJOH YHUXVVSKHUHGLP HWUGLQWKHUDQJH FP " G" FP E &RPPHQWRQWKHIHDVLELOLWRI WKLVFRQLJXUDWRQ 1HJOHFWGUDJ

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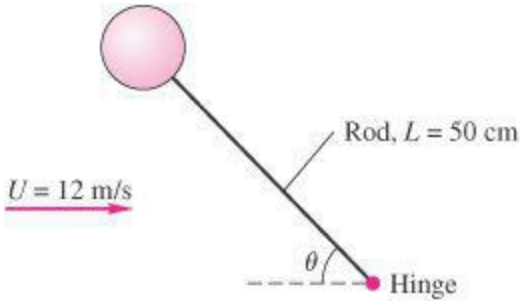
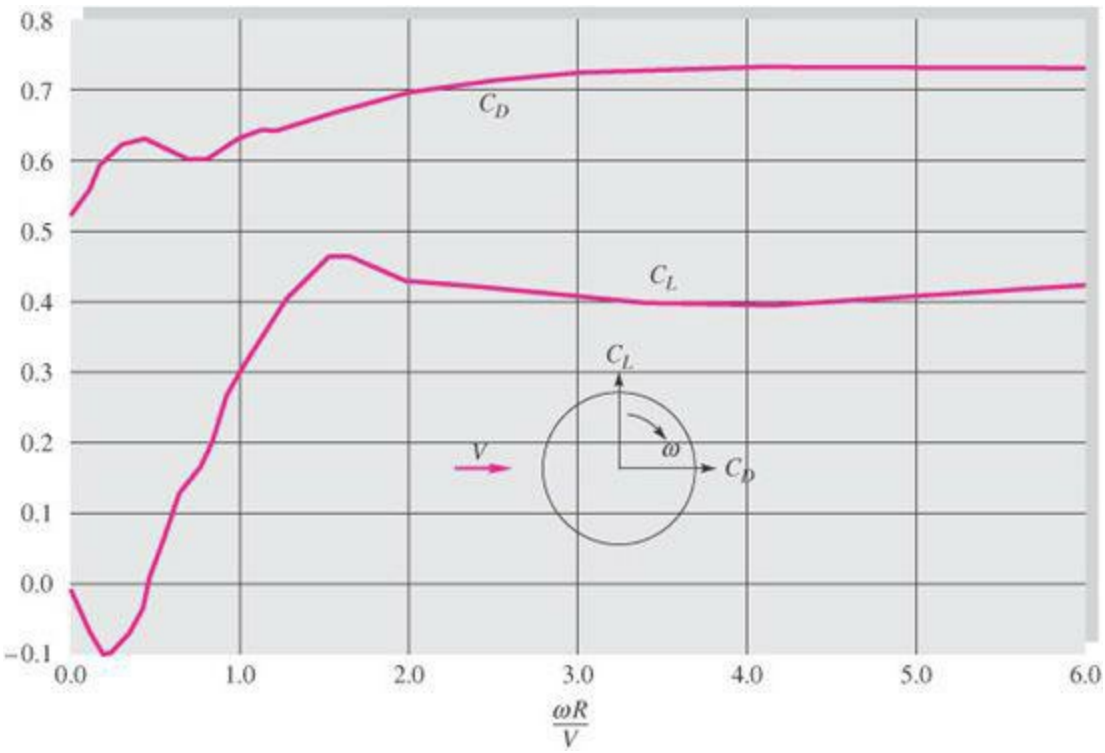


- 3 $QDXWRPRELOHKDV DPDVV RI NJ DQG DGUDJ DUHD& $ P 7KHURQLC UHVLVWDQFHRI 1 LVDSSURPLPDWHO FRQVWDQW7KHFDUL VFRDVWQJ ZLWKRXWEUDNHV NP KDVLWHJLQV WRFOPE DKLOOR SHUFHQWJUDGH VORSH WDQ *f* +RZ IDUXS WKHKLOOZ LOOWKHFDUFRPHWRDVWS'
- 3 7KHGHPSVXEP HUJHQFH YHKLFOH$/9,1 LQWK&KDS RSHQHUSKRWR LV IWORQ DQG IWZ LGH ,WZ HJKV DERXW CBI LQDLUDQG DVFHQGV GHVFHQGV LQWK VHDZ DWUGXHVWR DERXW CBI R SRVLWLYH QHJDWLYH EXR\DQF\ 1 RWQJ WKDWWKHIURQ IDFHRI WKHVKLSLVTXLWGLIIHUHQWRUDVFHQWDQG GHVFHQW D HWWP DW WKHYHORFLWRI HDFKGLUHFWRQ LQPHWHUVSHUPLQXW E +RZ ORQJ GRHVLWWDNHWRDVFHQGIURP LV PDLPXP GHSWKRI P"

/ LIWLQJ ERGLHV² DLUIRLOV

- 3 7KH &HVQD &LWDWLRQ H[HFXWLYH MHWZHLJKV N1 DQG KDV DZLQJ DUHD RI P , FUXLVHV DW NP VWDQGDUG DOWLWXGH ZLWK D OLIWFRHILFLHQWRI DQG D GUD FRHILFLHQWRI (VWLPDWH D VHFXLVHVSHGLQPLK DQG E WKH KRUVHSRZHU UHTXLUHGWRPDLQWDLQFUXLVHYHORFLW
- 3 $QDLUSODQHZHLJKV N1 DQG KDV DZLQJ DUHD RI P DQG DPHDQFKRUGRI P 7KHDLUIRLOSURPHUWLHVDUHJLYHQE\ ) LJ ,I WKHSODQHLVGHVLJQHGWRODQGD 9 9VWDOO XVLQJ DVSDWOODS VHWDW f D ZKDWLV WKHSURPHUDQGLQJ VSHHGL PLK' E : KDWSRZHULVUHTXLUHGIRUWDNHRII DWWKHVDPHVSHHG'
- 3 7KH7UDQVLWRŠ DXWRFDULQ) LJ KDVDZHJKWRI GEI DZLQJVSDQRI IWDQGDZLQJ DUHDRI IW ZLWKDVXPHWUFDOLIRLO&, § \$VXPHWKD WKHIXVHODJHDQGVWDLOVHFWLRQKDYHDGUDJ DUHDFRPSDUDEOHWRWKH7RRWV3LUV> @ &, \$ § IW ,I WKHSXVKHUSURPHOOHUSURYLGHVDWKUXVWRI GEI KRZ IDVWL PLK FDQWKLVFDUSDQHIO DWDQDOWXGHRI IW
- 3 6XSSRVH WKDWWKH DLUSODQH RI 3URE 3 LV IILWWHG ZLWK DOOWKH EHVWKLJK OL GHYLFHV RI) LJ : KDWLV LWV PLQLPXP VWDOOVSHHG LQ PLK' (VWLPDWH WK WRSSLQJ GLVWDQFH LI WKHSODQHODQGVDW 9VWDOOZLWK FRQWDQW&/ DQ &, DQGWKHEUDNLQJ IRUFHLV SHUFHQWRI WKHZHLJKWRQWKHZKHHOV



3 ' UDJ DQG OLWFRHILFLHQWV IRU D URWDWLQJ VSKHUH DW 5H §§ IURP 5H

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3 7KHZRUOG¶VEHVWPLOHDJHVHHWE\ 3\$& &DU,, RI (7+ =XULFK

- 3 $ VWDQVRUWSDQHKDVDPDVV RI NJ DZLQJ DUHD RI P DQG DQ DVSHF UDWLR RI $ VXSHDOOLWDQG GUDJ GXHWRWKHZLQJ DORQH ZLWK&, DQG&/ PD[,I WKHDLUFUDIWVWDHVDW P VDQGDUGDOWLWXGH PDNHDSORWRI GUDJ LQ1 YHUXVV VSHHG IURP VWDOOVR P V DQG GHWHUPLQHWKHRSWPXP FUXLVH YHORFLW PLQPXP GUDJ SHUQQVWSHG
- 3 6KRZ WKDWL (TV DQG DUHYDOLG WKHPD[LPXP OLWRGUDJ UDWLR RFFXU ZKHQ&, &, : KDWDUH / ' PD[DQG. IRUDV\PPHWULF ZLQJ ZKHQ\$5 DQ &, "
- 3 ,QJOGLQJ XQSRZHUHG IOJKWWKHOLWDQG GUDJ DUHLQHTXLOEULXP ZLWKWKHZHJK 6KRZ WKDWL WKHUHLVRZLQG WKHDLUFUDIWVLQVDWDQDQJOH

$$\tan\theta \approx \frac{\text{drag}}{\text{lift}}$$

)RUDVDLSSDQHRI PDVV NJ ZLQJ DUHD P DQG DVSHFWUDWR ZLWKDQ1 \$&\$ DLUIRLO HVWLPDWH D WKH VDOO VSHG E WKH PLQPXP JOGLQJ DQJOH DQG F WK PD[LPXP GLVWDQFHLWFDQJOGHLQWLOODLZKHQLWV P DERYHOHYHJURXQG

- 3 \$ ERDWRI PDV NJ KDVWZR KXGRIRLO HDFK RI FKRUG FP DQG VSDQ P ZLWK&/ PD[DQG&, ,WHQJLQHFDQGHOYHU N WRWKHZDWH)RUVHZDUDW *& HVWLPDWH D WKHPLQPXP VSHHGIRUZKLFKWKHIRLOVXSSRU WKHERDWDQG E WKHPD[LPXP VSHHGDWWDLQDEOH
- 3 ,QSUHZDUGDV WKHUHZDV DFRQWRYHUV SHUKDSV DSRFU SKDO DERXWZKHWKHU WK EXRMHFWLHHH KDV D OHJLWLPDWH DHURG\QDPLF UJKWWR IO 7KH DYHUDJH EXRMEHH %RRPEXVWWUHWWLV ZHJKV J ZLWKDZLQJ VSDQRI FP DQGDZLQJ DUHDR FP ,WFDQ LQGHHG IO DW P V 8VLQJ IL[HG ZLQJ WKHRU ZKDWLV WKH OL

FRQILFLHQWR WKHEHDWKLV VSHG',VWKLVUHDVRQDEOHIRUWSLFDOLURLV"

- 3 7KHEXPEHEHFDQKRYHUDW\HUR VSHGE\ IODSSLQJ LWZ LQJV 8VLQJ WKHGDWDF 3URE 3 GHYLVH D WKHRU IRU IODSSLQJ ZLQJV ZKHUH WKH GRZQWUN DSSUR[LPDWHVDVKRUWODVSODWHQRUPDOWR WKHIORZ 7DEOH DQGWKHXSWUNHL IHDWKHUHGDWQHDU]HURGUDJ +RZ PDQ IODSVSHUVHFRQGRI VXFKDPRGHOZLQ DUHQHHGHG WR VXSSRUWWKHEHHV¶ZHLJKW $FWXDOPHDVXUHPHQWRI EHHVVKRZ I IODSSLQJUDWRI +]
- 3 ,Q DFRPPHUFLDOLUFUDWWRXWDOOSRZHUZKLFKIOLQJDW IWRYHUWKHRSH \$WDQW2FHDQ DERXW PLOHVIURP WKH\$]RUHV,VODQGV 7KHSLORWV ZLWKDGPLUDEO VNLOO SXWWKH SODQH LQRD VKDOORZ JOLGH DQG VXFFHVVIXOO ODQGHG LQ WKH\$]RUHV \$VXP WKDWWKHDLUSODQHVDUWLHV(TV DQG ZLW\$5 &G DQGDVVPHWF DLURLO(VWMDWLWRSWPXP JOLGHGLVWDQFHZLWKDPDWHPDWFO SHUFHQWSLORWV
- 3 8VLQJ WKHGDWDIRUWKH7UDQVLWRŠ DXWRFDUIURP 3URE 3 DQGDPD[LPXP GL FRQILFLHQWRI HVWPDWWKHGLVWDQFHIRUWKHYHKLFOHWRVDNHRI DWD VSHGR 9VWDOO1RWWKDWZHKDYHWRDGGWKHFDUERG GUDJWRWKHZLQJGUDJ
- 3 7KH&KDS RSHQHUSKRWRRI <YHV5RX JLYHVWKHIROORZLQGDWD WKUXW GEI DOWXGH IWDQGZLQJVSDQ IW)XUWKHUDWXPHDZLQJ DUHDRI IWWRWD ZHLJKWRI GEI &, IRUWKHZLQJ DQGDGUDJ DUHDRI IWIRU5RFN0DQ(VWMDWWKHPD[LPXP YHORFLWVRXVLEOHIRUWKLVFRQGLWLRQ LQPLK

: RUG3UREOHPV

- : +RZ GR\RXUHFRJQL]HDERXQGDU\ OD\HU &LWWVRPHSK\VLFDOSURSHUWLHVDQGVRPH PHDVXUHPHQWWKDWWHYHDOSSURSULDWHFKDUDFWULWLF
- : ,Q&KDS WKH5H\QROGVQXPEHUIRUWDQLWRQWXEXOHQFHLQSLSHIORZ ZDVDERX 5H§ ZKHUHDV LQ IODWSODWH IORZ 5H§ (QHDU0 WKUHHRUGHUV R PDJQLWXGHKLJKHU: KDWDFFRXQWVIRUWKHGLIIHUHQFH'
- : : LWKRXWZ UWLQJ DQ HTXDWRQV JLYH D YHUEDO GHVFULSWLRQ RI ERXQGDU\ OD\H GLVSODFHPHQWKLFNQHVV
- : ' HVFUEH LQ ZRUGV RQO WKH EDVLF LGHDV EHKLQG WKH ³ERXQGDU\ OD\H DSSUR[LPDWRQ´
- : : KDWLV DQDGYHUVHSUHVXUHJUDGLHQW* LYHWKUHHH[DPSOHVRI IORZ UHJLPHVZKHU VXFKJUDGLHQWVRFFXU
- : : KDWLV DIDYRUDEOHSUHVXUHJUDGLHQW* LYHWKUHHH[DPSOHVRI IORZ UHJLPHVZKHU VXFKJUDGLHQWVRFFXU
- : 7KHGUDJ RI DQDLURLO)LJ LQFUHDVHVFRQVLGHUDEO LI \RX WKLQWKHVKDSHGJ DURXQG f WRIDFHWKHWUHDP &DQ\RX H[SODLQWKLV"
- : ,Q7DEOH WKHGUDJ FRHILFLHQWRI D VPXFHWUHH GHFUHDVHV VKDUSO ZLWK ZLQ YHORFLWV &DQ\RX H[SODLQWKLV"
- : 7KXWWLV UHTXLUHG WR SURPHODQDLSODQHDWD ILQLWH IRUZ DUG YHORFLWV ' RHVWKLVLP SO DQHQHUJ\ GRVVWKHVXVWHP" ([SODLQWKHFRQFHSWRI WKUXWDQG GUDJ LQWUP VRI WK

ILLWWDZ RI WKH UPRG\QDPLFV

- : +RZ GRHV WKH FRQFHSWRI GUDIWQJ LQDXWRPREOHDQG ELF\FOHUDFLQJ DSSO\ WR WK PDWUDOWXGLHGLQWKLVFKDSWU'
- : 7KHFLUFXODU\QQGHURI)LJ LVGRXE0\PPHWUF DQGWKHUHRUHVKRXOGKDYHQ QW<HWDQWWHQVRUZRXGGHILQWMD UHYHDOILQWURRWPHQVTXDUHYDOXHRI QW &DQ\RXH[SODLQWKLVEHKDYLRU'
- : ([SODLQLQZRUGVZK\ DWKURZQVSLQQLQJ EDOOPRYHVLQDFXUYHGWUDMHFWRU * LY VRPHSK\VLFDOUHDVRQZK\ DVLGHIRUFHLVGHYHORSHGLQDGGLWRQWKHGUDJ

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-)($VPRRWK FP GLDPHWUVSKHUHLVPPHUVHGLQDWXEDP RI *f& ZDWUPRYLQJ DW P V 7KHDSSURSULDWH5H\QROGVQXPEHURI WKLVVSKHUHLVDSSUR[LPDWO
-)(,I LQ3URE)(WKHGUDJ FRHILFLHQWEDVHGRQIURQDODUHDLV ZKDWLVWKHGUD IRUFHRQWKHVSKHUH'
-)(,I LQ3URE)(WKHGUDJ FRHILFLHQWEDVHGRQIURQDODUHDLV DWZKDWWUPLQL YHORFLWZLOOLQDXPLQXP VSKHUH 6* IDOLQWZDWHU DW'
-)( )RUIORZ RI VHDOHYHOWDQGDUGDUDW P VSDUDOHOWR DWKLQIDWSODW H[WP DW WK ERXQGDU\ OD\HUWKLFNQHVVDW[FP IURP WKHOHDGLQJHGJH
-)(,Q3URE)(IRUWKHVDPHIORZ FRQGLWRQV ZKDWLVWKHZDOOVKHDUVWUHVVDW[FP IURP WKHOHDGLQJHGJH'

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-)(: LQGDW f& DQG DW EORZ VDW NP KSDWDIODJSROH P KJK DQG FP L GLDPHWU 7KH GUDJ FRHILFLHQWEDVHG RQIURQDODUHD LV (WWP DW WKH Z LQG LQGXFHGEHQGLQJ PRPHQWDWWKHEDVHRI WKHSROH
  N1 P
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  N1 P
-)(&RQVLGHUZLQGDW f& DQG DW EORZLQJ SDWDFKLPQH\ P KJK DQG FP L GLDPHWU ,I WKHFKLPQH\ PD\ IUDFWXUHDWDEDVHEHQGLQJ PRPHQWRI N1 P DQ LWWGUDJ FRHILFLHQWEDVHG RQIURQDODUHDLV ZKDWLVWKHDSSUR[LPDWPD[LPXP DQORZ DEOHZLQGYHORFLWVWRDYRLGIUDFWXUH"
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-)($ GXVWSDUWFOHRI GHQVLW NJ P VPDOOHQRXJK WR VDLW\ 6WRNHVTV GUDJ CDZ VHWWOHVDW PP VLQDLUDW f& DQG DW : KDWLVLWVDSSUR[LPDWGLDPHWU
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-)($QDLUSODQHKDVDPDVRI NJ DZLQJ VSDQRI P DQGDQDYHUDJHZLQ FKRUGRI P : KHQIOLQJLQDLUR GHQVLW NJ P LWVHQJLQHVSURYLGHDWKUXVWR N1 DJDLQVWDQ RYHUDOO GUDJ FRHILFLHQWRI : KDWLV LWV DSSUR[LPDW YHORFLW"
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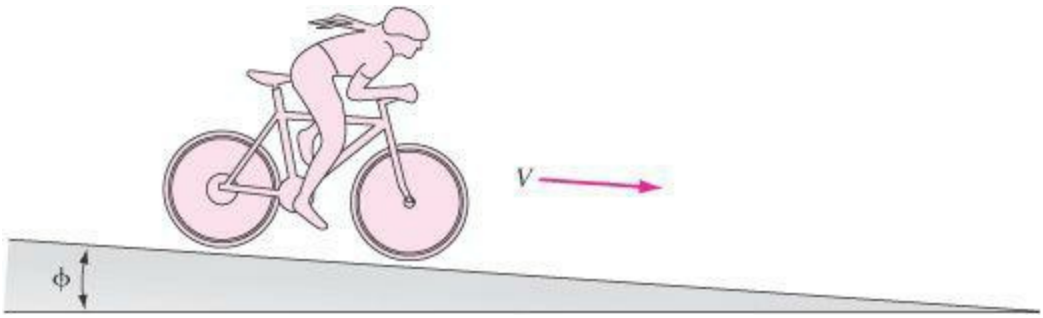
-)( )RU WKH IOJKWFRQGLWRQV RI WKH DLUSODQH LQ3URE)(DERYH Z KDWLV LV DSSUR[LPDWOLWFRHILFLHQW

&RP SUHKHQVLYH3 UREOHPV

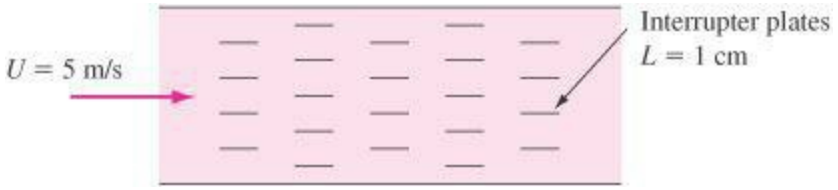
- & -DQHZ DQWVRHWVP DWKHGUDJ FRHILFLHQWRI KHUVHO RQKHUELF\FOH 6KHP HDXUHVWK SURMHFWHGIURQDODUHDWRE PVXS VXSDQGWKHURQQJ UHLVWDQFHWRE 1 VP 7KHPDVRI WKHELNHLV NJ ZKLOHWKHPDVRI -DQHLV NJ -DQHFRDWVGRZ DORQJ KLOOWKDWKDVDFRQVWDQW f VORSH 6HH) LJ & 6KHUHDFKHVDWUP LQDO VWDG VWDW VSHHGRI P VGRZQWKHKLOO(VWP DWWKHDHURG\QDPLF GUDJ FRHILFLHQW& RI WKHULGHUDQGELF\FOHFRPELQDWRQ

&



- & $LUDW f& DQG DW IORZ VDW DYJ P VEHZHHQ RQJ VP RRWSDUDOHOKH H[FKDQJHUSODWV FP DSDUWDV LQ) LJ & ,WLV SURSRVHG WR DGG D QXPEHUR ZLGHO VSDFHG FP ORQJ LQWUXSWUSODWV WR LQFUHDVHWKH KHDWWDQVHU DV VKRZQ $OWRXJK WKHIORZ LQWKHFKDQQHOLV WXUEXOHQWWKHERXQGDU\ OD\HUVRYHUWKHLQWUXSWU SODWVDUHWHQWDWLYH P ZLGHLQWR WKHSDSHU) LQG D WKHSUHVVXUHGURSLQ3DP ZLWKRXWWKHVP DOOSODWVDQG WKHSUHVVXUHGURSLQ3DP ZLWKWKHVHSODWVLQSODFH 7KHQILQG E WKHQXPEHUR VP DOOSODWVSHUPHWHURI FKDQQHOOHQJWK WKDWZLOOFDXVHWKHSUHVVXUHGURS WR ULVHWR 3DP

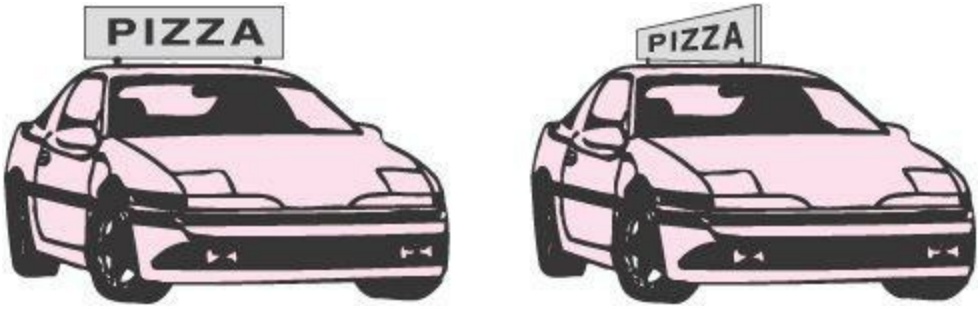
&



- & $ QHZ SL]]D WKURHLVSODQQLQJ WR RSHQ ,WZLOO RI FRXUVH RIHUIUHH GHOLYHU DQ WKHUHRUHQHGDVP DOOGHOLYHU FDUZLWKDOLJHVLQDWWDFKHG 7KHVLJQ DIODVSODWH LV IWKLJKDQG IWZRQJ 7KHERV KDYLQJ QRIHHOIRUIOXLGPHFKDQLFV PRXQWWKHVLJ EXQGV IDFLQJ WKHZLQG 2QHRI KLVGULYHUVLVWDNLQJIOXLGPHFKDQLFV DQGWKROVKLVERV KHFDQVDYHORWVRI PRQH\ E\ PRXQWQJ WKHVLJQSDUDOOHOWR WKHZLQG 6HH) LJ & D &DOFXODWH WKH GUDJ LQ EI RQ WKHVLJQ DORQH DW PLK IW LQERVRUHQWVRQV E 6XSSRVHWKHFDUZLWKRXWDQ VLJQKDVDGUDJ FRHILFLHQWRI DQGI IURQWDO DUHD RI IW) RU9 PLK FDOFXODWH WKHWRWDOGUDJ RI WKH FDU±VLJ FRPELQDWRQIRUERWKRUHQWDWRQV F ,I WKHFDUKDVDURQQJ UHLVWDQFHRI EI DW PLK FDOFXODWH WKHKRUVHSRZHUUHTXLUHG E\ WKHHQJLQHWR GULYHWKHFDUDW PLK L

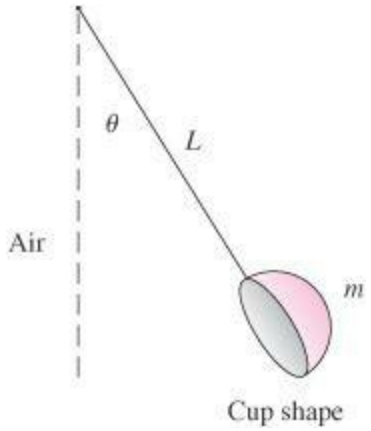
EHWKRUHQDWRQV G)LQDOO LI WKH HQJLQHFDQGHOYHU KS IRU KRQDJDOORQR
JDVROLQH FDOFXODWH WKH IXHO HILFLHQF\ LQPLJDOIRUEHWRUHQDWRQVDW PLK

&



- & &RQVLGHU D SHQGXOXP ZLWK DQ XQVWUHWFKHG DKHPLVSKHULFDO FXS RI GLDPHWHU ' ZKRVH DEVLVVLQWKH SODQH RI RVFLOODWLRQ DV LQ) LJ & 1HJOHFWWKH PDVV DQG GUDJ RI WKH URG/ D 6HWXS WKH GLIIHUHQWLDO HTXDWLRQ IRU WKH RVFLOODWLRQ W LQFOXGLQ GLIIHUHQWFXS GUDJ DLU GHQVLW! LQ HDFK GLUHFWLRQ DQG E QRQGLPHQVLRQDOL]H WKLV HTXDWLRQ F ' HWHUPLQH WKH QDWXUDO IUHTXHQF\ RI RVFLOODWLRQ IRU VPDOO ≪ UDG G) RUWKH VSHFLDOFDVH/ P ' FP P J DQG DLU DW *& DQG DW ZLWK *f ILQG QXPHULFDOO WKH WLPH UHTXLUHG IRUWKH RVFLOODWLRQ DPSOLWXGH WR GURS WR f

&



- & 3URJUDP D PHWKRG RI QXPHULFDO VROXWLRQ RI WKH %DLVXV IODWSODWH UHDWLRQ(T VXEMHFWWR WKH FRQGLWLRQV LQ(TV <RX ZLOO ILQG WKDW RX FDQQRWJH WDUWHG ZLWKRXW QRZ LQJ WKH LQLWLDO VHFRQG GHULYDWLYH ZKLFK OHV EHWZHHQ DQG ' HYLVH DQ LWHUDWLRQ VFKHP H WKDW WDUW DW ZŽ § DQG FRQYHUJHV WR WK FRUUHFWYDOXH 3UHQWRXV X8 I• DQG FRP SDUHZ LWK7DEOH

' HVLJQ3URMHFW

- ' ,WLV GHVLUHG WR GHVLJQ DFXS DQH PRH WUFRUZ LQG VSHHG VLP LODU WR) LJ 3 ZLWK PRUH VRSKLWWFDWHG DSSURDFK WKDQ WKH³DYHUDJH RUTXH PHWKRG RI3URE 3 7K

GHVLJQ VKRXOG DFKLHYH DQ DSSUR[LPDWH OQH DUUHDWRQ EHWZHHQ ZLQG YHORFLWV DQ
URDWRQ UDWLQWKHUDQJH 8 PLK DQG WKH DQHPRPHWHU UHDGLQJ DWZERX
UVDV8 PLK $OOVSHFLFDWRQ² FXSGLDPHWU URGOHQJWK/ URGGLDPHWUG
WKH EHDULQJ WSH DQG DOOPDWHULDO² DUH WR EH VHOHFWHG WKURXJK \RXU DQDOVLV 0 DN
VXLWDEOH DWXPSWRQ/ DERXWKH LQWDQDQH RXGJ RI WKH FXSV DQG URGV DWDQ JLYH
DQJOH WR WKH VXVWP &RPSXWWKH LQWDQDQH RXVRUTXH7 W DQG ILQG DQG LQWUDW
WKH LQWDQDQH RXV DQJXODU DFFHOHUDWRQ RI WKH GHYLFH ' HYHORS DFRPSOHWWKRU IR
URDWRQ UDWLQ YHUXV ZLQG VSHHG LQWKH UDQJH 8 PLK 7U WR LQFOXGH DFWXD
FRPPHUFLDOEHDULQJ IUFWRQ SURSHUWLV

5HIHUHQFHV

+ 6FKODFKWQJHWDO %RXQGDU / D\HU7KHRU 6SULQJHU 1HZ <RUN
) 0 : KLWH 9LVFRXV) OXLG) ORZ GHG 0F* UDZ +LOO 1HZ <RUN
- &RXWXL[0 RGHOLQJ DQG &RPSXWDWLRQ RI %RXQGDU / D\HU) ORZV GHG 6SULQJHU 9HUDJ 1HZ <RUN
- 7X * + <HRK DQG & /LX &RPSXWDWLRQDO) OXLG ' \QDPLFV $ 3UDFWLFDO$SSURDFK %XWWZRUWK+HLQHPDQQ 1HZ <RUN
9 9 6\FKHY HWDO $VSHFWRI 7KHRU RI 6HSDUDWHG) ORZV &DPEULGJH 8QLYHUVLW 3UHVV 1HZ <RUN
, - 6REHL ,QWURGXFWRQ WR,QWHUDFWLYH %RXQGDU / D\HU7KHRU 2[IRUG 8QLYHUVLW 3UHVV 1HZ <RUN
7 YRQ. iUPiQ ³2Q/ DPLQDUDQG7XUEXOHQW UFWRQ´ = $QJHZ 0 DWK 0 HFK YRO SS ±
* % 6FKXEDXHUDQG+ . 6NUDPVWDG ³/ DPLQDU%RXQGDU / D\HU2 VFLOODWRQV DQG6WDELO RI / DPLQDU) ORZ´ 1DW0%U 6WDQG 5HV 3DS $SULO VHHDOVR- $HUR 6FL YRO SS ± DQG1$&$ 5HS
3 6 %HUQDUGDQG- 0 : DOOFH7XUEXOHQW) ORZ $QDOVLV 0 HDVXUHPHQWDQG3UHGLFWRQ : LOH 1HZ <RUN
3 : 5XQWDGHU-U HWDO ³' LIXVHU' DW%RRN´ &UHDWH,QF 7HFK 1RWH + DQRYH 1+ 0D
% 7KZDLWV ³$SSUR[LPDW&DOFXODWRQ RI WKH/ DPLQDU%RXQGDU / D\HU´$HURQDXW4 YR SS ±
6) +RHQHU) OXLG' \QDPLF' UDJ SXEOLVKHGE\ WKH DXWKRU 0 LGDQG3DUN 1-
- ' $QGHUVRQ) XQGDPHQWDOVRI $HURG\QDPLFV WKHG 0F* UDZ +LOO 1HZ <RUN
9 7XFNHUDQG* & 3DUWW³$HURG\QDPLFVRI * OGLQJ) OJKWR) DOFRQV DQG2 WKHU%LRG- ([S %LRO YRO SS ±
(& &7XSSHU,QWURGXFWRQ1DYDO$UFKLWHFWXUH WKHG (OHYLHU 1HZ <RUN
, + $EERWWDQG$ (YRQ' RHQKRI 7KHRU RI : LQJ 6HFWRQV ' RYHU 1HZ <RUN
5 . DQH 7KH8QWP DW3DSHU$LUSODQH 6LPRODQG6FKXWU 1HZ <RUN
6 &KLOGUHVV 0 HFKDQLFVRI 6ZLPPLQJ DQG) OLQJ &DPEULGJH8QLYHUVLW 3UHVV 1HZ <RU

1 DWRQDO &RPPLWWHH IRU) OXLG 0 HFKDQLFV) LOWHUV,QGXVWULDO ([SHULHQFH LQ) OXL
0 HFKDQLFV 0 , 7 3UHVV &DPEULGJH 0 $
' 0 %XVKQHOO DQG - +HQHU (GV 9LVFRXV ' UDJ 5HGXFWLRQ LQ %RXQGDU\ / D\HU
$PHULFDQ,QVWLWXWH $HURQDXWLFV $VWURQDXWLFV 5HVWRQ 9$
³$ XWRPRELOH ' UDJ &RHILFLHQWV 85
KWWS HQZLNLSHGLDRUJ ZLNL$XWRPRELOHBGUDJBFRHILFLHQW
: + +XFKR $HURG\QDPLFV RI 5RDG 9HKLFOHV WK HG 6RF RI $XWRPRWLYH (QJLQHHUV
: DUUHQGDOH 3$
5 ' %OHYLQV $SSOLHG) OXLG ' \QDPLFV + DQGERRN YDQ 1RVWUDQG 5HLQKROG 1HZ <RUN
5 & -RKQVRQ -U * (5DPH\ DQG ' 6 2¶+DJHQ ³: LQG,QGXFHG)RUFHV RQ 7UHHV´
) OXLGV(QJ YRO 0 DUFK SS ±
3 : %HDUPDQ ³7KH (IIHFW RI D 0 RYLQJ) ORRURQ: LQG 7XQQHO6LPXODWRQ RI 5RD
9HKLFOHV´ 3DSHU1R 6$(7UDQVDFWLRQV- 3DVVHQJHU&DUV YRO VHF SS
±
&5& +DQGERRN RI 7DEOHV IRU $SSOLHG (QJLQHHULQJ 6FLHQFH G HG &5& 3UHV %RF
5DWRQ) /
7 ,QL ³: DYHPDNLQJ 5HVLVWDQFH RI 6KLSV´ 7UDQV 6RF 1DY $UFK 0 DULQH(QJUV YRO
SS ±
/ / DUVRQ ³&) ' LQ6KLS ' HVLJQ³ 3URSHOOHUV DQG / LPLWDWLRQV´6KLS 7HFKQRORJ\ 5HVHDUFK
YRO QR -XO SS ±
5 / 6WHHV* = : DWWV DQG- . 9HQDUG(OHPHQWDU) OXLG0 HFKDQLFV WK HG : LOH
1HZ <RUN
- ' $QGHUVRQ -U 0 RGHUQ &RPSUHVVLEOH) ORZ ZLWK +LVWRULFDO3HUVSHFWLYH UG HG
0 F* UDZ +LOO1HZ <RUN
- ' $QGHUVRQ -U +\SHUVRQLF DQG+LJK7HPSHUDWXUH* DV' \QDPLFV $,$$ 5HVRQ 9$
- 5RP +LJK $QJOHRI $WWDFN $HURG\QDPLFV 6XEVRQLF 7UDQVRQLF DQG6XSHUVRQLF) ORZ
6SULQJHU9HUDJ 1HZ <RUN
6 9RJHO³' UDJ DQG5HFRQLJXUDWRQ RI %URDG/ HDYHVLQ+LJK: LQGV´ - ([S %LRO YRO
QR $XJXVW SS ±
6 9RJHO/ LIHLQ0 RYLQJ) OXLGV 3UHVHQWDWLRQ8QLYHUVLW\3UHVV GHG 3UHVHQWDWLRQ1-
- $ &+XPSKUH HG 3URFHHGLQJV G,QWHUQDWLRQDO6\PSRVLXP RQ0 HFKDQLFVRI 3DQW
$QLPDOV DQG7KHLU(QYLURQPHQW(QJLQHHULQJ) RXQGDWLRQ 1HZ <RUN -DQXDU\
' ' -RVHSK 5 %DL . 3 &KHQ DQG< < 5HQDUG ³&RUH$QQXODU) ORZ V´ $QQX 5H
) OXLG0 HFK YRO SS ±
- : +RWWDQG5 + - 6HDQ³6FDOH(IIHFWLQ3RO\PHU6RXWKQ3LSH) ORZ´([S HPHQ
LQ) OXLGV YRO QR -XQH SS ±
6 1DNR ³$ SSOLFDWLRQ RI 9 6KDSH 5LEEHWV WR 3LSH) ORZ V´ -) OXLGV (QJ YRO
' HFHPEHU SS ±
3 7KHHG HG $HURG\QDPLF' UDJ 5HGXFWLRQ7HFKQRORJLHV 6SULQJHU 1HZ <RUN
& / 0 HUNOHDQG6 ' HXWFK ³0 LFURExEEHW' UDJ 5HGXFWLRQ LQ/ LTXLG 7XUEXOHQW%RXQGD
/ D\HUV´ $SSOLHG0 HFKDQLFV5HYLHZV YRO QR SDUW 0 DUFK SS ±
. 6 &KRL DQG* (. DUQLDGDNLV ³0 HFKDQLVP VRQ7UDQVYHUVH0 RWLRQV LQ7XUEXOHQW D
) ORZ V´ $QQXDO5HYLHZ RI) OXLG0 HFKDQLFV YRO SS ±

& - 5R\ - 3D\QH DQG0 0F: KHUWU3D\QH ³5$16 6LPXODWRQVRI D6LPSOLILHG7UDFN
7UDLQHU*HRPHWU´ -)OXLGV(QJLQHHULQJ YRO 6HSW SS ±
(YROXWRQRI)OLJKW,QWHUIHU5/
KWW ZZZ IOLJKW RUJ!
- ' $QGHUVRQ-U$ +LVWRU RI $HURG\QDPLFV &DPEULGJH8QLYHUVLW3UHVV 1HZ <RU

< 7VXL< 0RUULZD DQG2 0L]XQR ³([SHULPHQWDO0 HDVXUHPHQWRI WKH0 DJQXV)RFHRI D5RWDWLQJ6SKHUHDW5H QROGV1XPEHU´ -RXUQDORI)OLGV(QJLQHHULQJ YRO SS ±
5 &ODUN- 5 *UDFH DQG0 (: HEHU%XEEOHV' URSVDQG3DUWFOHV' RYHU1<
0 *DGHD+DN ³)ORZ &RQWURO7KH)XWXUH´ -RXUQDORI $LUFUDIWYRO QR SS
±
' *HURSSDQG+ - 2GHQKDO³' UDJ5HGXFWLRQRI 0RWRU9HKLFOHVE\ $FWLYH)ORZ &RQW
8VLQJWKH&RDQGD(IIHFW´ ([SHULPHQWVLQ)OLGV YRO QR SS ±
= =DSUDQRY DQG6 7DEDNRYD' \QDPLFVRI %XEEOHV' URSV DQG5LJLG3DUWFOHV. OXZHU
$FDGHPLF3XE 1HZ <RUN
' * . DUDP DQHY DQG/ 1 1LNRORY ³)UHH0 5LVLQJ6SKHUHV' R1RW2EH\ 1HZWRQV/D
IRU)UHH6HWWOLQJ´ $,&K(-RXUQDOYRO QR 1RY SS ±
. DJV- 5DFH&DU$HURG\QDPLFV5REHUW%ODNH ,QF &DPEULGJH0 $
$ 6 %URZQ ³0 RUH WKDQ 0 LOHV WR WKH* DOD[0 HFKDQLFDO(QJLQHHULQJ -DQXDU
S
' 0 %XVKQHOO³$ LUFUDIW UDJ5HGXFWLRQ $ 5HYLHZ´3URFHHGLQJV RI WKH,QWHUQDWLRQDO R
0 HFKDQLFDO(QJLQHHUV 3DUW* -RXUQDORI $HURVSDFH(QJLQHHULQJ YRO QR SS
±
' % 6SDOGLQJ ³$ 6LQJOH) RUP XOD IRUWKH/ DZ RI WKH: DOO- $SSO0 HFKDQLFV YRO
QR SS ±
' *)HUW³1HZ $LUFUDO' HVLJQ&RQFHSWZLWK,PSURYHG$HURG\QDPLF&KDUDFWHULVWLFV´-
$HURVSDFH(QJLQHHULQJ YRO QR -XO SS ±
))LDLVK DQG6 :LWKHUVSRRQ ³$HURG\QDPLF3HUIRUPDQFHRI DQ$LUFUDZZLWK6WHS,QGXFH
9RUWH[IRU/LIW$XJPHQWDWLRQ´ - $HURVSDFH(QJLQHHULQJ YRO QR -DQ SS ±
' 6 0 LNDVRYLF HWDO ³/ HDGLQJ (GJH 7XEHUFOHV' HDO 6WUDLQ RQ +XPSEDFN: KDOH
3K\VLFVRI)OXLGV YRO QR 0D\ SS / ±/
5 0F&DQH - 5RV DQG) %URZ DQG7KH $HURG\QDPLFV RI +HDY\ 9HKLFOHV 7UXFN\
%XVHV DQG7UDLQV 6SULQJHU9HUODJ 1HZ <RUN
- 5 &UX] HWDO ³: LQG7XQQHO7HVWLQJ RI 9DULRXV' LVN* DS %DQG3DUDFKXWHV´ $,$$ 3DS
± WK$,$$ $HURG\QDPLF' HFHOHUDWRU6\VWHPV&RQIHUHQFH 0D\

–)RUD FXUYHG ZDOO[FDQ UHSUHVHQWWKH DUF OHQJWK DORQJ WKH ZDOODQG\ FDQ EH HYHUZKHU
QRUPDOWR[ZLWK QHJOLJLEOHFKDQJHLQWKHERXQGDU\ OD\HUTXDOLWRQV DVORQJ DV WKHUDGLXV RI FXUYDWXU
RI WKHZDOOVODUJHFRPSDUHGZLWKWKHERXQGDU\ OD\HUWKLFNQHVV> WR @

–)RUIXUWKHUPDWKHPDWDOGHWDLOV VHH5H 6HF

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– ,QERGLHVZLWKVKHGYRUWHV VXFKDVWKHF\OLQGHULQ) LJ WKHUHPD\ EHRVFLOODWLQJ OLWWOH IRUFH DQGPRPHQW EXWWKHUPHDQYDOXHLV]HUR

– 7KHUHLV DVOJKWHIHFWRI WKH VSHFLILF KHDWDWDRN ZKLFK ZRXOG DSSHDULI RWKHUJDVHV ZHUHWWXWG

–) RU VRPH DLUFRLOV WKH EXEEOH GHDSV QRWFUHHSV IRUZ DUG DQG VWDOO RFFXUV UDSLGO DQGHQJHURXV0

&KDSWHU 3RWHQWLDO) OWZ DQG &RP SXWDWLRQDO) OXLG' \ QDP LFV

8QGHUVWDQGLQJ UHDFK VKDOORZ ZDWHU DQG IHHOLQJ HERWWRP RFHDQ ZDYHV DUH DOO RXW UHVHDUFK RQ HV : DYHV DUH FUHDWHG E\ ZLQGV HSHFLDOO\ VWURQJ ZLQGV² ODUJH GLVWDQFHV EHWZHHQ FUHVWV² WDYHOLQJ IDVWHU DQG GHFD\ VKHOOZHVW6KRUWZDYHV GHFD\ PRUH TXLFNO EXWDUH VWLOQHDU IUHWRQHV 7KH SLFWXUHG ZDYHV EUHDNLQJ RQ WKH EHDFK LQ 1DUDJDQVHWW 5KRGH ,VODQ PLJKWKDYHEHHQIRUPHGIURP DVWRUP RI WKHFRDVWRI \$IULFD 7KHWKHRU RI RFHDQZDYHV> @ EDVHGDORWHQWUHO XSRQIUHWRQHV IROZ ***Photo courtesy of Ellen Emerson White.***

0 RWLYDWLRQ

7KHEDVLF SDUDGLIHUHQWDOHTXDWLRQV RI PDV PRPHQWXP DQGHQHUJ\ ZHUHGLVFXVVHGLQ&KDS \$ IHZ VROXWLRQV ZHUH WKHQ JLYHQ IRU LQFRPSUHVVLEOHYLVFRXV IROZ LQ6HF 7KH YLVFRX VROXWLRQV ZHUH OLPLWHG WR VLPSOHJHRPHWULHV DQG XQLGLUHFWLRQDO IOZ V ZKHUH WKH GLIILFXOWLHVQDQHD FRQYHFWLYH WHUPV ZHUHQHJOHFWHG 3RWHQWLDO IOZ V DUH QRWOLPLWHG E\ VXFK RQDQDO\WLFV 7KHQ L &KDS ZHIRXQG DQ DSSUR[LPDWH SDWKLQJ EHWZHHQ OA\HU IOZ V RQR DQ RXWHULQYLVFLG IOZ SDWWHUQ)RUPRUHFRPSOLFDWHG YLVFRXV IOZ V ZHIRXQG QR WKHRU RUVROXWLRQV IXUWH[SHULPHQWDOGDW 7KHSXUSRVH RI SRWHQWLDOWKHRU DQG VLQGLFDWH E\ FRPSXWDWLRQDO IOXLG G\QDPLFV &)' 7K FRP ELQDWLRQ RI WKHVH WZR JLYHV XV D JRRG SLFWXUH RI LQFRP SUHVVLEOH IOZ WKHRU DQG LWV UHODWLRQV H[SHULPHQW2 QHRI WKHPRWLP SRUWDQWDSSOLFDWLRQV RI SRWHQWLDO IOZ WKHRU LV WR DHURG\QDPLFV DQ PDULQHK\GURG\QDPLFV)LWWKRZHYHU ZHZLOOUHYLHZ DQGH[WQG WKHFRQFHSWRI &K

,QWURGXFWLRQ DQG 5HYLHZ

) LJXUH UHPLQGV XV RI WKH SUREOHPV WR EH IDFHG $ IUHH VWUHDP DSSURDFKHV VZR FORVHO VSDFH
ERGLHV FUHDWLQJ DQ³LQWHUQDO IORZ EHWZHHQ WKHP DQG³H[WHUQDO IORZ V DERYH DQG EHORZ WKHP 7KH
IURQWV RI WKH ERGLHV DUH UHJLRQV RI IDVRUDEOH JUDGLHQWV GHFUHDVLQJ SUHVVXUH DORQJ WKH VXUIDFH DQG WKH
ERXQGDU\ OD\HUV ZLOO EH DWWDFKHG DQG WKLQ ,QYLVFLG WKHRU\ ZLOO JLYH H[FHOOHQW UHVXOWV IRU WKH RXWHU
IORZ LI 5H!) RU WKH LQWHUQDO IORZ EHWZHHQ ERGLHV WKH ERXQGDU\ OD\HUV ZLOO JURZ DQG HYHQWXDOO
PHHW DQG WKH LQYLVFLG FRUH YDQLVKHV ,QYLVFLG WKHRU\ ZRUNV ZHOO LQ D³VKRUW VXFK D
WKH QR]]OH RI DZLQG WXQQHO) RU ORQJHU GXFWV ZHP XVW WAXVP DWWHQG DQ ERXQGDU\ OD\HU JURZ WKDQG EHFDXVH RI
DERXVXVLQJ LQYLVFLG WKHRU\
) RU WKH H[WHUQDO IORZ V DERYH DQG EHORZ WKH ERGLHV LQ) LJ LQYLVFLG WKHRU\ VKRXOG ZRUN HV
IRU WKH RXWAUIORZ V XQLOWKH VXUIDFH SUHVXUH JUDGLHQWV EHFRPH VDGYHUVH LQFUHDVLQJ SUHVXUH DQG WKH
ERXQGDU\ OD\HU VHSDUDWV RU VWDOOV $IWAU WKH VHSDUDWLRQ SRLQW ERXQGDU\ OD\HU WKHRU\ EHFRPH
LQDFFXUDW DQG WKH RXWAUIORZ VWUHDPOLQHV DUH GHIOHFWHG DQG KDYH D WRQJ LQWHUDFWLRQ ZLWK WHYLVFRX
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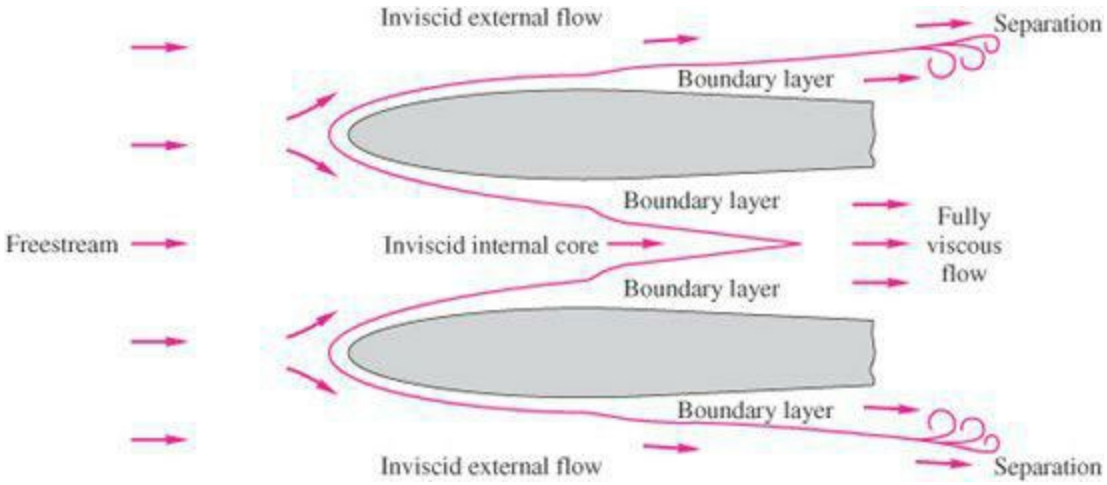
SUHVHQW



) LJ 3DWWKLQJ YLVFRXV DQG LQYLVFLG IORZ UHJLRQV 3RWHQWLDO WKHRU\ LQ WKLV FKDSWHU GHVFULEHV QR
DSSOLFDWLRQ WR WKH ERXQGDU\ OD\HU UHJLRQV

5HYLHZ RI 9HORFLW\ 3RWHQWLDO &RQFHSW

5HFDOO IURP 6HF WKDW YLVFRXV HIIHFWV DUH QHJOHFWHG GRZ VSHHGIORZ V DUH LLURWDWLRQDO î 9
DQG WKH YHORFLW\ SRWHQWLDO Ë H[LWV VXFK WKDW

$$\mathbf{V} = \nabla\phi \quad \text{or} \quad u = \frac{\partial\phi}{\partial x} \quad v = \frac{\partial\phi}{\partial y} \quad w = \frac{\partial\phi}{\partial z} \tag{8.1}$$

7KH FRQWLQXLW\ HTXDWLRQ ∇ 9 UHGXFHV WR/ DSODFH¶V HTXDWLRQ IRU Ë

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{8.2}$$

DQG WKH PRPHQWXP HTXDWLRQ UHGXFHV WR%HUQRXOOL¶V HTXDWLRQ

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2} V^2 + gz = \text{const} \quad \text{where } V = |\nabla \phi| \tag{8.3}$$

7\SLFDOERXQGDU\ FRQGLWLRQV/DUHNQRZQIURHVWUHDP FRQGLWLRQV/

$$\text{Outer boundaries:} \qquad \text{Known } \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \tag{8.4}$$

DQGQRYHORFLWLVQRUPDOWRWKHERXQGDU\ DWWKHERG\ VXUIDFH

$$\text{Solid surfaces:} \qquad \frac{\partial \phi}{\partial n} = 0 \qquad \text{where } n \text{ is perpendicular to body} \tag{8.5}$$

8QDNHWKHQR VOLS FRQGLWLRQ LQ YLVFRXV IORZ KHUH WKHUH LV QR FRQGLWLRQ RQ WKH WDQJHQWLDO VXUIDFH YHORFLWLV9V %Ë %V ZKHUH VLV WKH FRRUGLQDWH DORQJ WKH VXUIDFH 7KLV YHORFLWLVGHWHUPLQHG DV SDUW RI WKH VROXWLRQ WR WKH SUREOHP

2FFDVLRQDOO WKH SUREOHP LQYROYHV DIUHH VXUIDFH IRUZKLFK WKHERXQGDU\ SUHVVXUHLVNQRZ QDQ HTXDOWRS D XVXDOO D FRQVWDQW7KH %HUQRXOOL HTXDWLRQ WKHQ VXSSOLHV DUHODWLRQ DWWKH VXUIDFH EHWZHHQ9 DQG WKHHOHYDWLRQ] RI WKHVXUIDFH)RUVWDG\ IORZ

$$\text{Free surface:} \qquad V^2 = |\nabla \phi|^2 = \text{const} - 2gz_{\text{surf}} \tag{8.6}$$

,WVKRXOG EH FOHDU WR WKH UHDGHU WKDWWKLV XVH RI / DSODFH¶V HTXDWLRQ ZLWK NQRZQ YDOXHV RI WK GHULYDWLYH RI Ë DORQJ WKH ERXQGDULHV LV PXFK HDVLHU WKDQ D GLUHFWDWWDFN XVLQJ WKH IXOO YLVFRX 1DYLHU6WRNHV HTXDWLRQV 7KH DQDO\VLV RI / DSODFH¶V HTXDWLRQ LV YHU ZHOGHYHORSHG DQG LV WUP H SRWHQWLDOWKRU ZLWKZKROHERRNV ZULWWQ DERXWWDSSOLFDWLRQ WR IXLG PHFKDQLFV> WR @7KHU DUH P DQ DQDO\WLFDOWHFKQLTXHV LQFOXGLQJ VXSHUSRVLWLRQ RI HOP HQWDU IXQFWLRQV FRQRUP DOP DSSLQ > @QXP HULFDO LQWHGLIHUHQFHV> @QXP HULFDO LQWHJHP HQW> @QXP HULFDOERXQGDU HOP HQW> @ DQG HOHFWUF RUP HFKDQLFDOODQDORJV> @KDWDUHQZ RXGDWG + DYLQJ IRXQGË [\ ] WIURP VXFK DI DQDO\VLV ZHWKHQFRP SXWH9 E\ GLUHFWGLIHUHQWDWLRQLQ(T DWWUZKLFK ZHFRP SXWSIURP (T 7KH SURFHGXUH LV TXLWH VWUDJKWRUZ DUG DQG P DQ LQWHUWZQJ DBEHLWLGHDO]HG UHVXOWV FDQ E REWDLQHG $ EHDXWLXOFRODFWRQ RI FRP SXWHUJHQHUDWG SRWHQWLDO IORZ VNHWFKHVLVJLYHQE\ . UFKKRI > @

5HYLHZ RI 6WUHDP) XQFWLRQ &RQFHSWV

5HFDOO IURP 6HF WKDWWL DIORZ LV GHVFULEHG E\ RQ VWR FRRUGLQDWV WKH VWUHDP IXQFWRQ DOR H[LWWV DV DQ DOWHUQDWLYH DSSURDFK)RUSODQH LQFRPSUHVLEOH IORZ LQ[\ FRRUGLQDWV WKH FRUUF IRUP LV

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \tag{8.7}$$

7KHFRQGLWLRQ RI LUURWDWLRQDOLWUHGXFHVWR/ DSODFH¶V HTXDWLRQ IRU$ DOR

$$2\omega_z = 0 = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)$$

or

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0} \qquad (8.8)$$

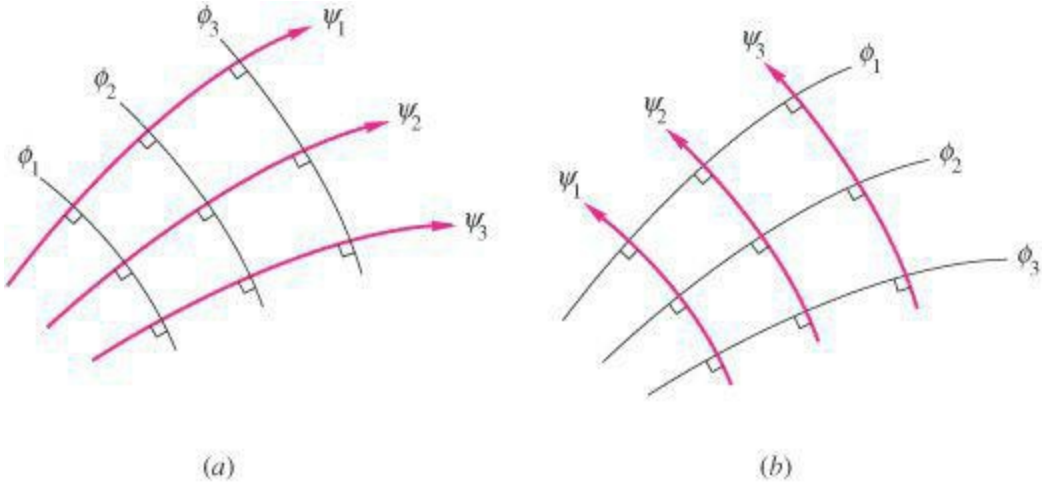
7KH ERXQGDU\ FRQGLWLRQV DJDLQ DUH NQRZQ YHORFLWLHV LQ WKH VWUHDP DQG QR IORZ WKURXJK DQ\ VROLG VXUIDFH

Free stream: Known $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}$ (8.9*a*)

Solid surface: $\psi_{\text{body}} = \text{const}$ (8.9*b*)

(TXDWLRQ E LV SDUWLFXODUO\ LQWHUHVWLQJ EHFDXVH DQ\ QXPEHU RI FRQVWDQW\$ LQ D IORZ FDQ WKHUHIRUH EH LQWHUSUHWHG DV DERG\ VKDSH DQG P D\ HDG WR LQWHUHVWLQJ DSSOLFDWLRQV

) RU WKH DSSOLFDWLRQV LQ WKLV FKDSWHU Z H P D\ FRP SXWH WKH ÏRZ\$ RU ERW DQG WKH VROXWLRQ Z LO EH DQRWKHU JRQDO IORZ QHWZRUV LQ) LJ 2 QFH IRXQG HLWKHU VHWV RI OLQHV P D\ EH FRQVLGHUHG WKH OLQHV DQG WKH RWKHU VHWV Z LOO EH WKH \$ OLQHV %RWK VHWV RI OLQHV DUH OSSOLFDQ DQG FRXOG EH XVHXO



(*a*) (*b*)

) LJ 6WUHDP OLQHV DQG SRWHQWLDO OLQHV DUH RUWKRJRQDO DQG P D\ UHYHUVH UROHV LI UHXQGV DU XVHXO *a* VWSLFDO OQYLVFLG IORZ SDWWHUQ *b* VDP HDV *a* Z LWK URORHV UHYHUVHG

3 OLDQH3 RODU &RRUGLQDWHV

0 DQ\ VROXWLRQV LQ WKLV FKDSWHU DUH FRQYHQLHQWO\ H[SUHVVHG LQ SRODU FRRUGLQDWHV U %RWK WKH YHORFLWLHV FRP SRQHQWV DQG WKH GLIIHUHQWLDO UHODWLRQV IRUË DQG \$ DUH WKHQ FKDQJHG DV IROORZ V

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r} \qquad (8.10)$$

/ DSODFH¶V HTXDWLRQ WDNHV WKH IRUP

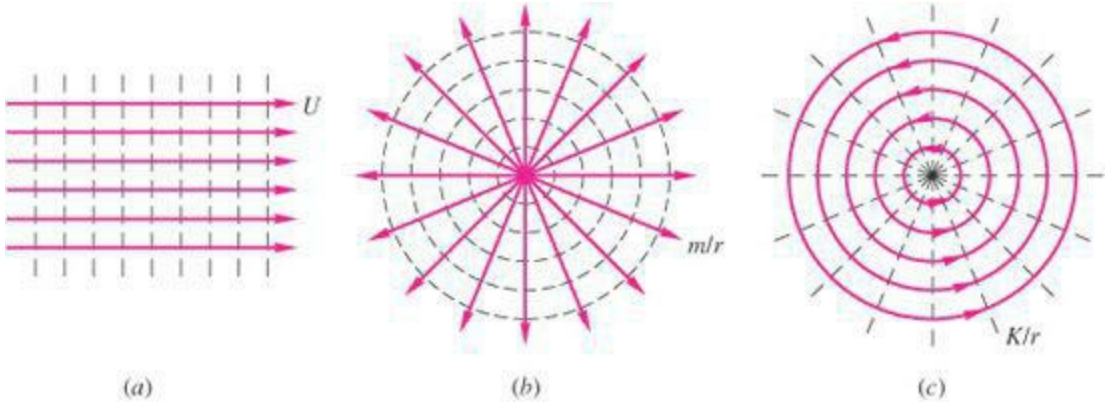
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0 \qquad (8.11)$$

([DFWO WKH VDP H HTXDWLRQ KROGV IRU WKH SRODU FRRUGLQDWH IRUP RI \$ U

$QLQWLXLQJ IDFHWRI SRWHQWLDOIORZ ZLWKQR IUHHVXUIDFHLV WKDWWKHJRYHUQLQJ HTXDWLRQV DQG FRQGLQQR SDUDPHWHUV QRUGR WKHERXQGDU\ FRQGLWLRQV 7KHUHRUHWKHVROXWLRQV DUHSXUHO JHRPHWUF GHSHQGLQJ RQO RQWKHERG VKDSH WKHIUHHVWHDP RUHQWDWLRQ DQG VXSULQJO WK SRVLWLRQRI WKHUDUWDQDWLRQSRLQW 7KHUHLVQR 5H\QROGV)URXGH RU0 DFK QXPEHUWR FRP SOFD WKH G\QDPLF VLPLODULW ,QYLVFLG IORZ V DUH NLQHPDWFDOO VLPLODU ZLWKRXWDGGLWLRQDOSDUDPHWUV UHFDOO) LJ D

(OHP HQWDU\ 3ODQH) ORZ 6RXUFHV

7KHSUHVHQFHRIDGHQVLW GLIIHUHQFH LQYLVFLG LQFRP SUHVVLEOHIORZ V HVSHFLDO WKRVH WKDWSRVHVV ERWK D VWHDP IXQFWLRQ DQG D YHORFLW SRWHQWLDO 0DQ VROXWLRQV PDNHXVH RI WK VXSHUSRVLWLRQ SULQFLSOH VR ZHEHJLQ ZLWKWKHWKUHHHOHP HQWDU EXLOGLQJ EORFNV OLQHDUGLQ) LJ D DXQLIRUP VWHDP LQWKH[GLUHFWLRQ E DQQHVRXUFHRUVLQNDWWKHRULJLQ DQG F DQQHYRUWH[D WKHRULJLQ



) LJ 7KUHHHOHP HQWDU SDQH SRWHQWLDOIORZV 6RDGGDQH VWHDP DQH GDVKHGDQH SRWHQWLDOLQH

8QLIRUP 6WHDP LQ WKH[' LUHFWLRQ

$ XQLIRUP VWHDP 9 L8 DV LQ) LJ D SRVWHVWHV ERWK D VWHDP IXQFWLRQ DQG D YHORFLW SRWHQWLDO ZKLFKPD\ EHIRXQGDVIROORZV

$$u = U = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad v = 0 = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

: HPD\ LQWUDWHDFKH[SUHVLRQDQG GLVFDUGWKHFRQVDQWRI LQWUDWRQ ZKLFKGRQWDIHFWKH YHORFLWLHVLQWKHIORZ 7KHUHVXOWLV

$$\text{Uniform stream } iU: \qquad \psi = Uy \qquad \phi = Ux \tag{8.12}$$

7KHVWHDPOQHV DUHKRUL]RQWDOVWUDLJKWOQHV \ FRQVW DQGWKH SRWHQWLDOOQHV DUH YHUWFDO [FRQVW WKDWLV RUWKRJRQDOWRWKHVWHDPOQHV DVH[SHFWHG

/ LQH6RXUFHRU6LQN DWWKH2 ULJLQ

6XSSRVHWKDWWKH] D[LVZHUHDVRUWRWKLQSLSHPDQIRGWKURXJKZKLFKIOXLGLVXHGDWRDOUDWH4
XQLRUP0 DORQJ LWVOHQJWKE / RRNLQJ DWWKH[\ SODQH ZHZRXOGVHHDFRQGUFDOUDGLDORXWZRZ R
QH VRXUFH DV VNHWKHG LQ) LJ E 3QH SRDUFRRUGLQDWV DUH DSSURSULDWH VHH) LJ DQ
WKHUHLVRFLUFXPIHUHQWLDOYHORFLW\ \$WDQ UDGLXV UWKHYHORFLWLV

$$v_r = \frac{Q}{2\pi rb} = \frac{m}{r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r} \qquad v_\theta = 0 = -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

ZKHUHZHKDYHXVHGWKHSRDUFRRUGLQDWIRUPVRI WKHVWUHDP IXQFWLRQDQG WKHYHORFLW\SRWHQWLDO ,QWUDWLQJ DQG DJDLQGLVFDUGLQJ WKHFRQVWDQWVRI LQWHJUDWLRQ ZHREWLQWKHSURSHUIXQFWLRQVIRUWKH VLPSOHUDGLDOIORZ

Line source or sink: $$\psi = m\theta \qquad \phi = m \ln r \tag{8.13}$$

ZKHUHP 4 CE LVDFRQVWDQWSRVLWLYHIRUDVRXUFH QHJDWLYHIRUDVLQN \$VVKRZQLQ) LJ E
WKHVWUHDPOLQHVDUHUDGLDOVSRNHV FRQVWDQW DQGWKHSRWHQWLDOOLQHVDUHFLUFOHV FRQFHQWU

/ LQH, UURWDWLRQDO9 RUWH[

\$ VZRGLPHQVLRQDO OLQHYRUWH[LVDSXUHO FLUFXODWLQJ VWUHDP PRWLRQ % I U RQO %θ 7KL
VDZLWKVWKHFRQVWUXFWHTXDWLRQLGHQWLFDO DVPDL EHFKFNHG IURP (T E : HPD\ DOVR QRW
WKDWD YDUHWVRI YHORFLW\GLVWULEXWLRQV% U VDZ\ WKH PRPHQWXP HTXDWLRQRI DYLVFRXVIOXLG (T
' : HPD\ VKRZ DVDSUREOHP H[HUFLVH WKDWRQO RQHIXQFWLRQ% U LVLUURWDWLRQDO WKDWV FXO
9 DQG% . U ZKHUH. LVDFRQVWDQW7KLV LV VRPHWLPHVFDOOHGDIUHHYRUWH[IRUZKLFK WK
VWUHDP IXQFWLRQDQGYHORFLWPD\ EHIRXQG

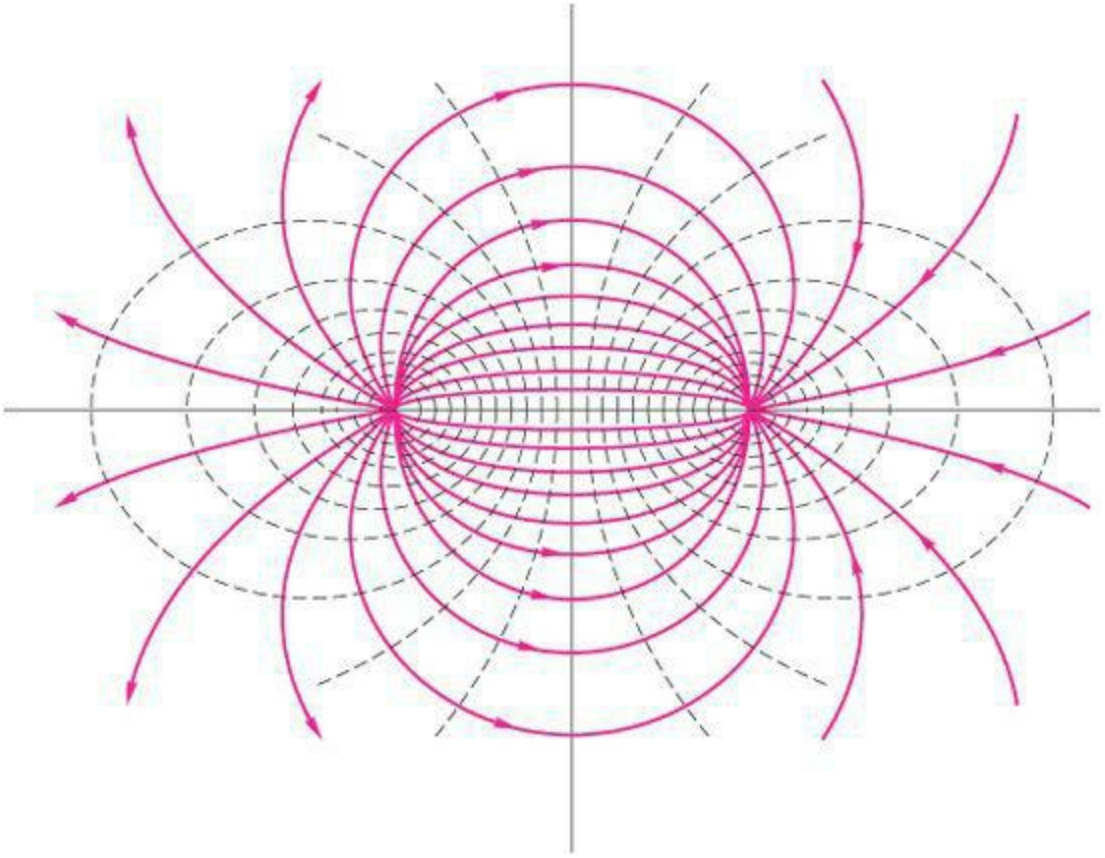
$$v_r = 0 = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r} \qquad v_\theta = \frac{K}{r} = -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

: HPD\ DJDLQLQWHJUDWHWRGHWHUPLQHWKHDSSURSULDWHIXQFWLRQV

$$\psi = -K \ln r \qquad \phi = K\theta \tag{8.14}$$

ZKHUH. LVDFRQVWDQWDQGWKHVWUHQJWKRI WKHYRUWH[\$VVKRZQLQ) LJ F WKHVWUHDPOLQHVDU
FLUFOHV FRQFHQWU DQGWKHSRWHQWLDOOLQHVDUHUDGLDOVSRNHV FRQVWDQW 1RWHWKHVLPLODULWEHWZHH
(TV DQG \$IUHHYRUWH[LVDVRUWRI UHYHUVHGLPDJHRI DVRXUFH 7KH³EDWKWXE YRUWH[´
IRUPHGZKHQZDWHUGUDLQV/WKURXJKDEDWWHP KROHILQDVLQN LVDJRRGDSSUR[LPDWLRQWRWKHIUHH YRUWH[
SDWWHUQ

(DFKRI WKHWKUHHHOHPHQWDU\IORZ SDWWHUQV LQ) LJ LVDQLQFRPSUHVVLEOHLUURWDWLRQDOIORZ DQG
WKHUHRUHVDWLVI\HVERWKSDUW³SRWHQWLDOIORZ´ HTXDWLRQV \$ DQG ¥ 6LQFHWKHVHDUHOLQHDU
SDUWLDO GLIIHUHQWLDO HTXDWLRQV DQ VXP RI VXFK EDVLF VROXWLRQV LV DOVR D VROXWLRQ 6RPH RI WKHV
FRPSRVLWH VROXWLRQV/DUHTXLULQJWKDWWKH QR



) LJ 3RWHQWLDO IORZ GXH WR D OLQH VRXUFH SOXV D QHTXDO OLQH VLQN IURP (T 6RODG OLQH DUH VWUHDPOLQHV GDVKHG OLQHV DUH SRWHQWLDO OLQHV

) RUH[DPSOH FRQVLGHU D VRXUFH P DW[\ í D FRPELQHG ZLWK D VLQN RI HTXDO VWUHQJWK í P SODFHG DW D DV LQ) LJ 7KH UHVXOWLQJ VWUHDP IXQFWLRQ LV VLPSO WKH VXP RI WKH VWR , FDUWHVLDQ FRRUGLQDWHV

$$\psi = \psi_{\text{source}} + \psi_{\text{sink}} = m \tan^{-1} \frac{y}{x + a} - m \tan^{-1} \frac{y}{x - a}$$

6LPLODUO WKH FRPSRVLWH YHORFLW SRWHQWLDO LV

$$\phi = \phi_{\text{source}} + \phi_{\text{sink}} = \frac{1}{2} m \ln [(x + a)^2 + y^2] - \frac{1}{2} m \ln [(x - a)^2 + y^2]$$

%\ XVLQJ WULJRQRPHWULF DQG ORJDULWKPLF LGHQWLWLHV WKHVH PD\ EH VLPSOLILHG WR

Source plus sink:

$$\psi = -m \tan^{-1} \frac{2ay}{x^2 + y^2 - a^2}$$

$$\phi = \frac{1}{2} m \ln \frac{(x + a)^2 + y^2}{(x - a)^2 + y^2} \qquad (8.15)$$

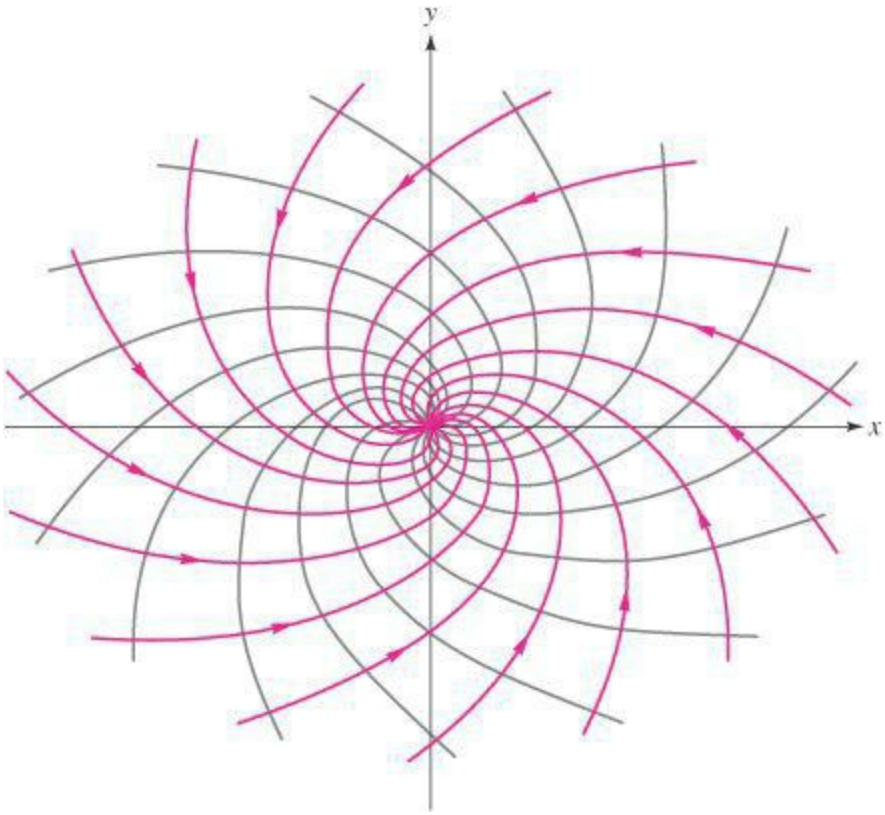
7KHVH OLQHV DUH SORWWHG LQ) LJ DQG DUH VHQVLEOH IRU PLQHV RI RUKRJRQDOLWLHV ZLWK WKH VWUHDPOLQHV SDVVLQJ WKURXJK WKH VRXUFH DQG VLQN DQG WKH SRWHQWLDO OLQHV HQFLUFOLQJ WKH 7KH DU KDUPRQLF ODSODFLDQ IXQFWLRQV WKDW DUH H[DFW DQDORJRXV LQ HOHFWURPDJQHWLF WKRU WR WKH HOHFWU FXUUHQWDQG HOHFWULF SRWHQWLDOV RI DP DJQHWLF SRWHQWLDW" D

6LQN3OXV9RUWH[DVWKH2ULJLQ

$QLQWHUHVWLQJIORZSDWWHUQDSSURSULDWHLQFRPELQDWLRQRFFXUVE\VXSHUSRVLWLRQRI DVLQN DQG I
YRUWH[ERWKFHQWHUHGDWWKHRULJLQ7KHFRPSRVLWHVWUHDPIXQFWLRQDQGYHORFLW\SRWHQWLDODUH

Sink plus vortex: $\psi = m\theta - K \ln r \qquad \phi = m \ln r + K\theta$ (8.16)

: KHQSORWWHG WKHVHIRUP VZR RUWKRJRQDOIDPLOLHVRI ORJDULWKPLF VSLUDOV DV VKRZQ LQ) LJ
7KLV LV DIDLUO UHDOLVWLF VLPXODWRQ RI DWRUQDGR Z KHUHWKH VLQN IORZ PRYHVXS WKH] D[LV LQWR WK
DWPRVSKHUH RUDUDSLGO GUDLQLQJ EDWKXE YRUWH[$VWKHFHQWURI DUHDO YLVFRXV YRUWH[ZKHUH(T
SUHGLFWV LQILQLWH YHORFLW WKH DFWXDOFLUFXODWLQJ IORZ LV KLJKO URWDWLRQDODQG DSSUR[LPDW
VRDG ERG\ URWDWLRQ% § &U



) LJ 6XSHUSRVLWLRQRI DVLQN SOXV DYRUWH[(T VLP XODWMV DWRUQDGR

8QLIRUP6WUHDP3OXV6LQN DWWKH2ULJLQ7KH5DQNLQH+DOI%RG\

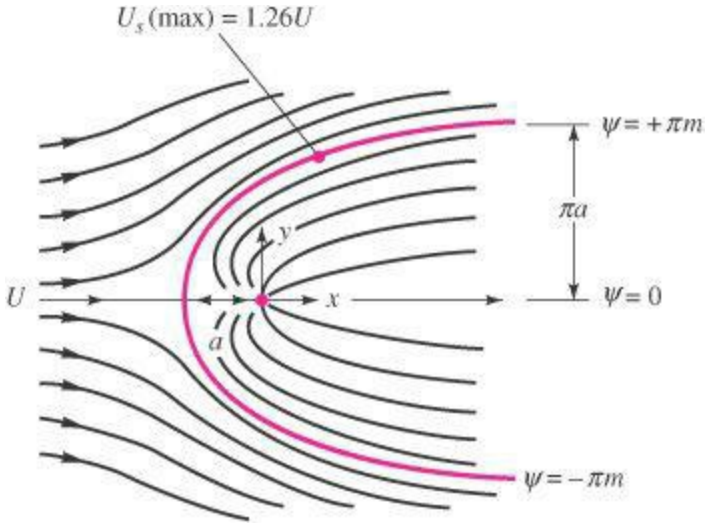
,I Z H VXSHUIP SRVH D XQLIRUP [GLUHFWHG VWUHDP DJDLQVWDQ LVRODWHG VRXUFH D KDOI ERG\ VKDSH
DSSHDUV ,I WKH VRXUFHLV DWWKHRULJLQ WKHFRPELQHG VWUHDP IXQFWLRQLV LQSRODUFRRUGLQDWHV

Uniform stream plus source: $\psi = Ur \sin\theta + m\theta$ (8.17)

: H FDQ VHWWKLV HTXDOWR YDURXV FRQVWDQWV DQG SORWWKH VWUHDPOLQHV DV VKRZQ LQ) LJ $
FXUYHG URXJKO HOOLSWLFDO KDOI ERG\ VKDSH DSSHDUV Z KLFK VHSDUDWHV WKH VRXUFH IORZ IURP WKH VWUHDF
IORZ 7KHERG\ VKDSH Z KLFK LVQDPHG DWWUWKH6FRWWLK HQJLQHHU: - 0 5DQNLQH ± L
IRUPHG E\ WKH SDUWFXOU VWUHDPOLQHV$ “ GE 7KHKDOI ZLGWK RI WKHERG\ IDUGRZQVWUHDP LVGE 8
7KHXSSHUXUFHPD\ EHSORWWHG IURP WKHUHODWLRQ

$$r = \frac{m(\pi - \theta)}{U \sin\theta} \qquad (8.18)$$

,WLVQRWDWWKHHQGSRLQW7KHQRVHRIWKHERG\ZKLFKLVD³VWDJQDWLRQ SRLQWZKHUH9 VDQGVD [\ íD ZKHUHD P8 7KHVWUHDPOLQH$ DOVRFURVVHVWKLVSRLQW UHFDOOWKDWWXUEXOHQFHRFFXUVRQO DWDVWDJQDWLRQSRLQW



)LJ 6XSHUSRVLWLRQRI DVRXUFHSOXVDXQLIRUP VWUHDP IRUPVD5DQNLQHKDOIERG\

7KHFDUWHVLDQYHORFLWFRPSRQHQWVDUHIRXQGE\ GLIIHUHQWLDWLRQ

$$u = \frac{\partial \psi}{\partial y} = U + \frac{m}{r}\cos\theta \qquad v = -\frac{\partial \psi}{\partial x} = \frac{m}{r}\sin\theta \tag{8.19}$$

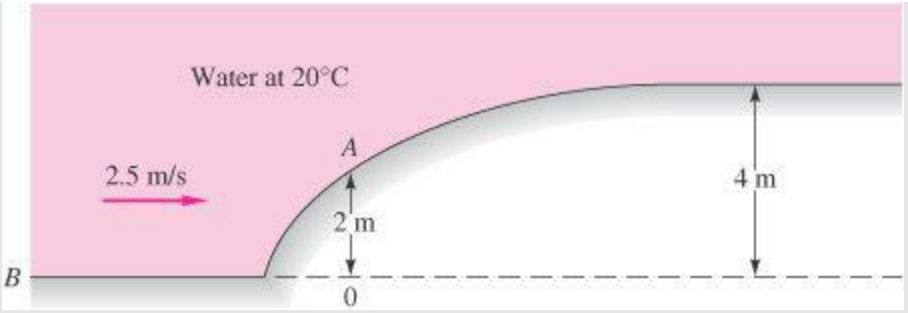
6HWWLQJ X % ZHILQGDVLQJHVDQGQDWRSRLQWDW *DQGU P8 RU[\ íP8 DVVXDWG7KHUHVXOWDQWYHORFLWDWDQ SRLQWLV

$$V^2 = u^2 + v^2 = U^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right) \tag{8.20}$$

ZKHUHZHKDYHVXEVWLWXWHGP 8D ,I ZHHYDOXDWWKHYHORFLWYDORQJ WKHXSSHUVXUIDFH$ GE ZHILQGD PDPLPXP YDOXH8V PD[§ 8 DW *7KLV SRLQWLV ODEHOHG LQ)LJ DQG E\ %HUQRXOOL¶V HTXDWLRQ LV WKH SRLQWRI PLQLPXP SUHVVXUH RQ WKH ERG\ VXUIDFH $IWHUWKLV SRLQWWKH VXUIDFHIORZ GHFHOHUDWHV WKHSUHVVXUHULVHV DQGWKHYLVFRXVOD\HUJURZVWKLFNHUDQGPRUHVXVFHSWLEOH WR³IORZ VHSDUDWLRQ´ DVZHVDZ LQ&KDS

(; $0 3/ (

7KHERWWRP RI DULYHUKDV D PKLJK EXPS WKDWDSSUR[LPDWV D5DQNLQHKDOIERG\ DVLQ)LJ (7KHSUHVVXUHDWSRLQW%RQWKHERWWRP LV N3D DQGWKHULYHUYHORFLWLV P V8VHLQYLVFL WKHRUHPWRHVWLPDWWKHZDWUSUHVVXUHDWSRLQW$ RQWKHEXPS ZKLFKLV P DERYHSRLQW%



6ROXWLRQ

$V LQ DOO LQYLVFLG ZKHRUHV ZH LJQRUH WKH GRZ YHORFLW EHWZHHQ GDU CD\HUV WKDWMRUP RQ VRQ VXUDFHVGXHWRWKHQR VOLSFRQGLWLRQ)URP (T DQG) LJ WKHGRZQWHDP EXPSKDOO KHLJK HTXDOVCE 7KHUHRUH IRUURXUFDVH D P CE P : HKDYHWRILQGWKHVSRWZKHUHWKHEXPS KHLJKWVVKDOWKDWMPXFK K P CED)URP (T ZHPD\ FRPSXWH

$$r = h_A = \frac{a(\pi - \theta)}{\sin\theta} = \frac{\pi}{2}a \qquad \text{or} \qquad \theta = \frac{\pi}{2} = 90^\circ$$

7KXVSRLQWV$ LQ) LJ (LVGLUHFWO DERYHWKH LQLWDOO XQNQRZQ RULJLQRI FRRUGLQDWV CDEHGH 2 LQ) LJ (DQGLV P WRWKHULJKWRI WKHQRVHRI WKHEXPS : LWKU CED DQG CE NQRZQ ZHFRPSXWHWKHYHORFLWDWSRLQW$ IURP (T

$$V_A^2 = U^2\left[1 + \frac{a^2}{(\pi a/2)^2} + \frac{2a}{\pi a/2}\cos\frac{\pi}{2}\right] = 1.405U^2$$

or $$V_A \approx 1.185U = 1.185(2.5\ \text{m/s}) = 2.96\ \text{m/s}$$

)RUZDWUDW *& WDNH! NJ P DQG 1 P 1RZ VLQFHWKHYHORFLWDQGHOHYDWR DUHNQRZQDWSRLQW$ ZHDUHLQDSRVLWRQWRXVH%HUQRXOOLVLQLVFLG LQFRPSUHVVLEOHIORZ HTXDWR WRHVWLP DWS$ IURP WKHNQRZQSURSHUWLHVDWSRLQW% RQWKHVDPHVWUHDPOLQH

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A \approx \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

or $$\frac{p_A}{9790\ \text{N/m}^3} + \frac{(2.96\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 2\ \text{m} \approx \frac{130{,}000}{9790} + \frac{(2.5)^2}{2(9.81)} + 0$$

Solving, we find

$$p_A = (13.60 - 2.45)(9790) \approx 109{,}200\ \text{Pa} \qquad \textit{Ans.}$$

,I WKHDSSURDFKYHORFLW LVXQIRUP WKLVKRXOGEHDSUHWWJRRGDSSUR[LPDWLRQ VLQFHZDWUU UHODWLYHO LQYLVFLGDQGLWVERXQGDU\ CD\HUVDUHWKLQ

8QIRUP6WUHDP DWDQ$QJOH.

,I WKH XQLIRUP VWUHDP LVZ UDWKQLQSODQHSRODUFRRUGLQDWV LWEHFRPHV

Uniform stream iU: $\psi = Ur\sin\theta \qquad \phi = Ur\cos\theta$ (8.21)

7KLV PDNHV LWHDVLHUWR VXSHULPSRVH VDY D VWUHDP DQG D VRXUFH RU YRUWH[E\ XVLQJ WKH VDPH FRRUGLQDWV ,I WKHXQLIRUP VWUHDP LVPRYLQJ DWDQJOH. ZLWKUHVSHFWWRWKH[D[LV² WKDWLV

$$u = U\cos\alpha = \frac{\partial\psi}{\partial y} = \frac{\partial\phi}{\partial x} \qquad v = U\sin\alpha = -\frac{\partial\psi}{\partial x} = \frac{\partial\phi}{\partial y}$$

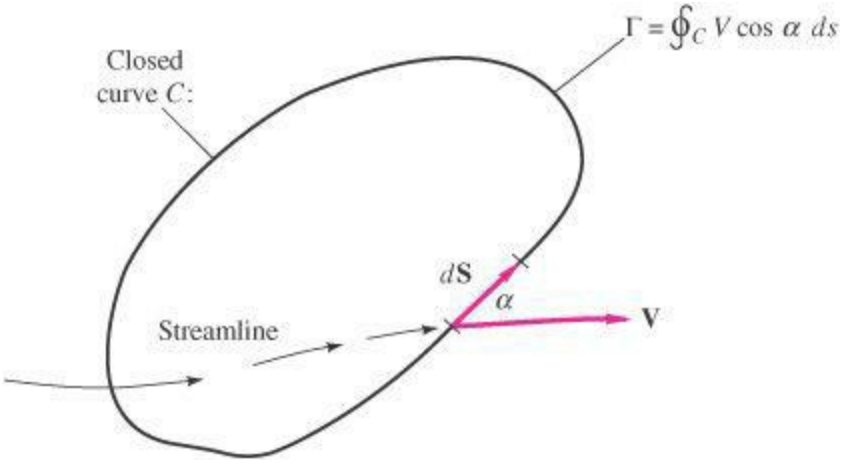
WKHQE\ LQWHJUDWLQJZ HREWDLQWKHFRUUHFWXQFWLRQVIRUIORZ DWDQDQJOH

$$\psi = U(y\cos\alpha - x\sin\alpha) \qquad \phi = U(x\cos\alpha + y\sin\alpha) \qquad (8.22)$$

7KHVHH[SUHVVLRQVDUHXVHXOLQDLUFRORDQJOHRI DWWDFNSUREOHPV 6HF

&LUFXODWLRQ

7KHQDQHYRUWH[IORZ LVLUURWDWLRQDOHYHUZKHUHH[FHSWDWWKHRULJLQ ZKHUHWKHYRUWLFLW\ î 9 LV LQILQLWH 7KLV PHDQVWKDWD FHUWDLQ QXPEHULQWHJUDOFDOHG WKHFLUFXODWLRQ* GRHVQRWYDQLVK ZKHQ WDNHQDURXQGDYRUWH[FRQWRU



)LJ ' HILQLWLRQRI WKHFLUFXODWLRQ*

: LWK UHIHUHQFH WR) LJ WKH FLUFXODWLRQ LV GHILQHG DV WKH FRXQWHUFORFNZLVH QXPEHULQWHJUDO DURXQGDFORVHGFXUYH& RI DUF OHQJWKG V WLPHV WKH YHORFLW\ FRPSRQHQWDQJHQWLWRWKHFXUYH

$$\Gamma = \oint_C V\cos\alpha\, ds = \int_C \mathbf{V}\cdot ds = \int_C (u\,dx + v\,dy + w\,dz) \qquad (8.23)$$

)URP WKHGHILQLWLRQRI - 9 GV ∇ - GV G- IRUDQLUURWDWLRQDOIORZ KHQFHQRUPDO * LQD LUURWDWLRQDOIORZ ZRXOGHTXDOWKHILQDOYDOXHRI Ë PLQXVWKHLQWHJUDOYDOXHRI Ë 6LQFHZHVWDUWDQGHQ DWWKHVDPHSRLQWZHFRPSXWH* EXWQRWRUYRUWH[IORZ :LWKË . IURP (T WKHUHLVI FKDQJHLQË RI DPRXQWCE DVZHPDNHRQHFRPSOHWHFLUFOH

Path enclosing a vortex: $\Gamma = 2\pi K$ (8.24)

$ QWKLQDYHO WKHFDOFXODWLRQFDQEHPDGHE\ GHILQLQJ FLUFXODUSDWKV RI UDGLXVUDURXQG WKHYRUWH[FHQWU IURP (T

$$\Gamma = \int_C v_\theta \, ds = \int_0^{2\pi} \frac{K}{r} \, r \, d\theta = 2\pi K$$

,QJHQHUDO * GHQRWHV WKH QHWDOJHEUDLF VXPTXDQWKRI DOOWKH YRUWH[ILODPHQWV FRQWDLQHG ZLWKLQ WK FORVHG FXUYH ,Q WKH QH[W FRQWZHVKDOOVHH WKDWD UHJLRQ RI ILQLWH FLUFXODWRQ ZLWKLQ D IORZ LQ VWHDP ZLOOEHVXEMHFWHGWRDQLWURSHSURSRUWLRQDOWRERWK8 DQG*

2QHFDQVKRZ E\ XVLQJ (T WKDWD VRXUFH RUVLQN FUHDWHV QR FLUFXODWLRQ ,I WKHUH DUH QY RUWHFHVSUHVHQWWKHFLUFXODWLRQZLOOEH]HURIRUDQ SDWKHQFORVLQJ DQ QXPEHURI VRXUFHVDQGVLQNV

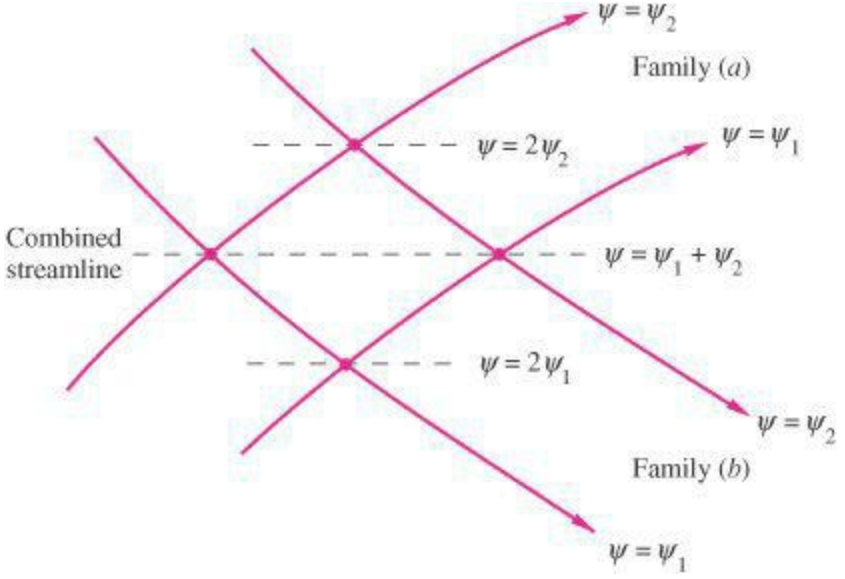
6XSHUSRVLWLRQ RI 3DQH) ORZ 6RXUFHV

: HFDQQRZ IRUP DYDULHWV RI LQWHUHVWLQJ SRWHQWLDOI ORZV E\ VXPPLQJ WKH YHORFLW SRWHQWLDOV DQG VWUHDP IXQFWLRQV RI DXQLIRUP VWUHDP VRXUFHRUVLQN DQG YRUWH[0RVWRI WKH UHVXOW DUH FODVLF F FRXUVH QHGLQJ RQO DEUH WHDWHQWKHUH 6XSHUSRVLWLRQLV YDOGEHFDXVHWKHEDVLF HTXDWLRQV DQG DUHOLQHDU

* UDSKLFDOO HWKRG RI 6XSHUSRVLWLRQ

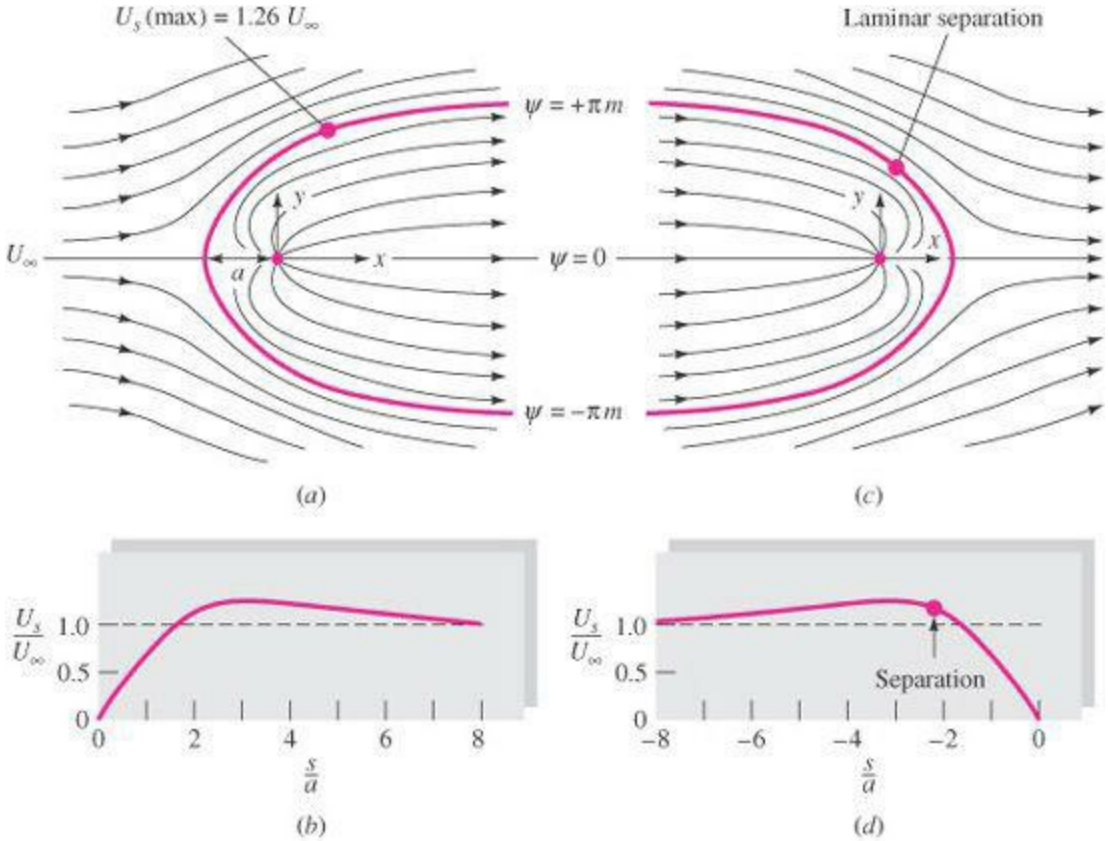
$ VLPSOHPHDQV RI DFFRPSOLVKLQJ $ JUDSKLFDOO LV WR SORWWKHLQGLYLGXDOWUHDP IXQFWLRQ VHSDUDWHO DQGWKHQORRN DWWKHUHVXOWDQWSDWWHUQ7KHYDOXHRI $ DWDFK LQWHUVHFWLRQLV WKH VXP RI WK LQGLYLGXDOYDOXHV$ WKDWFURVV WKHUH &RQVHTXHQWO LQWHUVHFWLRQV ZLWK WKH VDPH YDOXHRI $ FUHDWH WKH GHVLUHG VXSHUPRVHGIORZ WUHDPOLQHV

$ VLPSOH H[DPSOHLV VKRZQ LQ) LJ VXPPLQJ WZR IDPLOLHV RI WUHDPOLQHV$ DQG$ 7KLQGLYLGXDOFRPSRQHQWV DUHSORWWHGVHSDUDWHO DQGIRUWWSLFDOLQWHUVHFWLRQV DUH VKRZQ ' DVKHGOLQH DUHWKHQGUDZQWKURXJK LQWHUVHFWLRQV UHSUHVHQWLQJ WKHVDPHVXP RI $ $ 7KHVHGDVKHGOLQHV DUHWKH GHVLUHG VROXWLRQ 2IWHQ WKLV JUDSKLFDO PHWKRG LV D TXLFN PHDQV RI HYDOXDWLQJ WKH SURSRVH VXSHUSRVLWLRQEHIRUHDIXOOEORZQQXPHULFDOSORWWRXWOLQHLVH[HFXWHG



) LJ ,QWHUVHFWLRQV RI HOHPHQWDU VWUHDPOLQHV FDQ EH MRLQHG WR IRUP D FRPELQH

%RXQGDU\ / D\HU6HSDUDWLRQ RQ D+ DOI %RG\



) LJ 7KH 5DQNLQH KDOI ERG\ SDWWHUQ *c* LV QRW RXGLDU EHFDXVH RI ERXQGDU\ OD\HUVHSDUDWLRQ *a* 8QLRUP VWUHDP SXVD VRXUFH HTXDOV DKDOI ERG\ VWDJQDWLRQ SRLQWDW ía ím *U*, *b* 6DJKWDGYHUVH JUDGLHQWRU*s/a* JUHDWHUWKDQ QR VHSDUDWLRQ *c* 8QLRUP VWUHDP SXVD VLQN HTXDOV WKH UHDUR I DKDOI ERG\ VWDJQDWLRQ SRLQWDW *a m/U*, *d* 6WURQJ DGYHUVH JUDGLHQWRU*s a* ! í VHSDUDWLRQ

$ QXPRXJK WKH LQYLVFLG IORZ SDWWHUQV VHHQ LQ) LJ D DQG F DUH PLUURU LPDJHV WKHLU YLVFRX ERXQGDU\ OD\HU EHKDYLRUV LGHQWLFDO7KH ERG\ VKDSH DQG WKH YHORFLW DORQJ WKH VXUIDFH DUH

$$V^2 = U_\infty^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right) \quad \text{along} \quad r = \frac{m(\pi - \theta)}{U_\infty \sin\theta} \qquad (8.25)$$

7KH FRPSXWHG VXUFH YHORFLWLHV DUH SORWWHG DORQJ WKH KDOI ERG\ FRQWRXUV LQ) LJ E DQG G DV IXQFWLRQ RI DUF GLVWDQFH V DP HDVXUHG IURP WKH VWDJQDWLRQ SRLQW7KH SRWDU DUH DOR P LUURU LPDJHV + RZ HYHU LI WKH QRVH LV LQ IURQW) LJ E WKH SUHVXUH JUDGLHQWWKHUH LV IDYRUDEOH GHFUHDVLQ SUHVXUH DORQJ WKH VXUIDFH ,Q FRQWDVWWKH SUHVXUH JUDGLHQWLVDGYHUVH LQFUHDVLQJ SUHVXUH DORQ WKH VXUIDFH Z KHQWKH QRVH LV LQWKH UHDU) LJ G DQG ERXQGDU\ OD\HUVHSDUDWLRQPD\ RFFXU

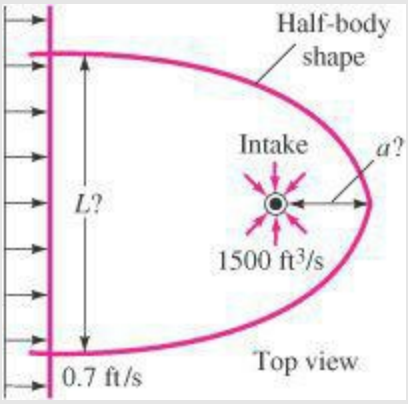
$SSOLFDWLRQ WR) LJ E R 7KZ DWWM¶V ODP LQDU ERXQGDU\ PHWKRG IURP (TV DQG UHY HDOXWHSDUDWLRQGRHVQRWRFFXURQWKH IURQWRI WKH KDOI ERG\ 7KHUHRUH) LJ DLVDYHU UHDOLVWLF SLFWXUH R VWUHDPOLQHV SDWWD KDOI ERG\ QRVH ,QFRQDWZ KHQDSSOLHG WR WKH WDLO) LJ F 7KZ DWWM¶V PHWKRG SUHGLFWV VHSDUDWLRQDWDERXW D§ í RU § *f* 7KXV LI DKDOI ERG\ LVDVRO

VXUIDFH) LJ F LVQRWXHDOXDWHGDWDF DQG DEURDG VHSDUDWHG ZDNH ZLOOIRUP +RZHYHU LI WKH KDOI ERG
WDLOLV DIIXLGQDQH VHSDUDWLQJ WKH VLQNGLUHFWLQJ IORZ IURP WKH RXWHUVWUHDP DVLQ([DPSOH WKH
) LJ F LV TXLWHUHDVRQF DQG XVHXO &RPSXWDWLRQV IRUWKLEXOHQWERXQGDU\ OD\HUWKRV ZRXOG E
VLPLODU VHSDUDWLRQRQWKHWDLOQR VHSDUDWLRQRQWKHQRVH

(; $0 3/ (

$QRIVKRUHSRZHUSODQWFRQVLVWVRIQJ ZDWHULQDN HVXFNVLQ IWV LQZDWHU IWGHHS DVLQ) LJ
(,I WKHVLGDOYHORFLWLVDSSURDFKLQJ WKHLQDN HLV IWV D KRZ IDUGRZQVWUHDP GRHVWKHLQDN
HIIHFWH[WHQGDQG E KRZ PXFKZLGWK/ RI WGDOIORZ LVHQJDLQHGLQRWKHLQDN H'

(



6RXWLRQ

5HFDOOIURP (T WKDWWKHVLQNVWUHQJWKP LVUHDGLOHGWRWKHYROXPHIORZ 4 DQGWKHGHSWKEHLQ
WKHSDSHU

$$m = \frac{Q}{2\pi b} = \frac{1500\ \text{ft}^3/\text{s}}{2\pi(30\ \text{ft})} = 7.96\ \text{ft}^2/\text{s}$$

7KHUHRUHIURP) LJ WKHGHVLUHGGHQJWKVDDQG/ DUH

$$a = \frac{m}{U_\infty} = \frac{7.96\ \text{ft}^2/\text{s}}{0.7\ \text{ft/s}} = 11.4\ \text{ft} \qquad \textit{Ans. (a)}$$

$$L = 2\pi a = 2\pi(11.4\ \text{ft}) = 71\ \text{ft} \qquad \textit{Ans. (b)}$$

) ORZ 3DVWD9 RUWH[

&RQVLGHUDXQIRUP VWUHDP 8, LQWKH[GLUHFWLRQIORZLQJ SDVWD YRUWH[RI VWUHQJWK. ZLWKFHQWUD

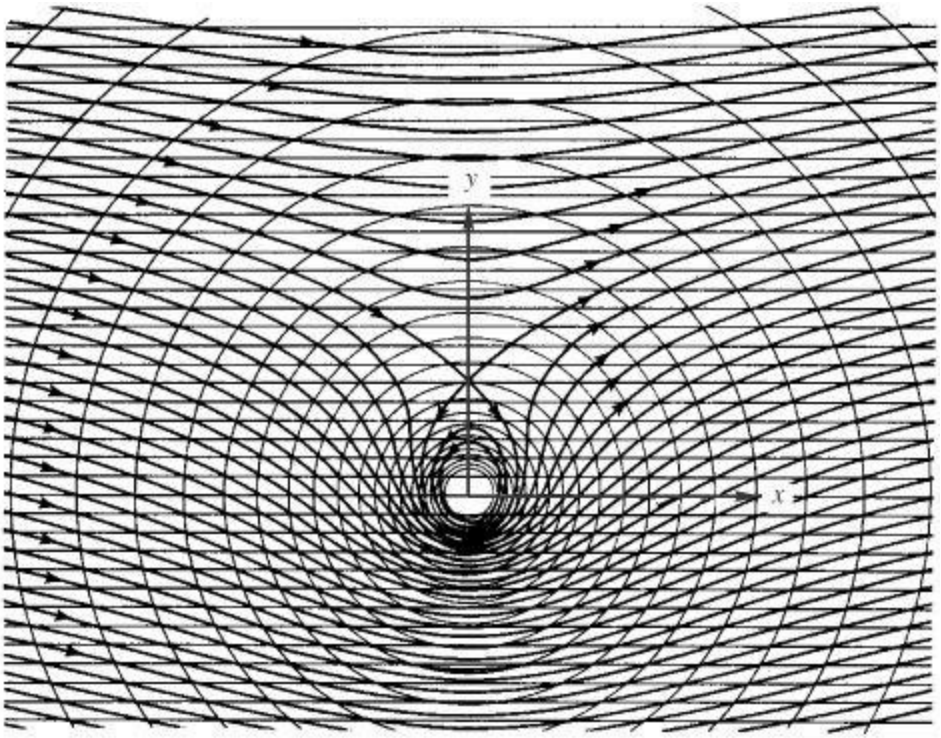
VKHRULJLQ %\ VXSHUSRVLWLRQ WKH FRPELQHG VWUHDP IXQFWLRQ LV

$$\psi = \psi_{\text{stream}} + \psi_{\text{vortex}} = U_\infty r \sin\theta - K \ln r \qquad (8.26)$$

7KH YHORFLW\ FRPSRQHQWV DUH JLYHQ E\

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U_\infty \cos\theta \qquad v_\theta = -\frac{\partial\psi}{\partial r} = -U_\infty \sin\theta + \frac{K}{r} \qquad (8.27)$$

7KH VWUHDP OLQHV DUH SORWWHG LQ) LJ E\ WKH JUDSKLFDO PHWKRG LQWURGXFHG LQ WKH FLUFXODU VWUHDP OLQHV RI WKH YRUWH[ZLWK WKH KRUL]RQWDO OLQHV RI WKH XQLIRUP VWUHDP



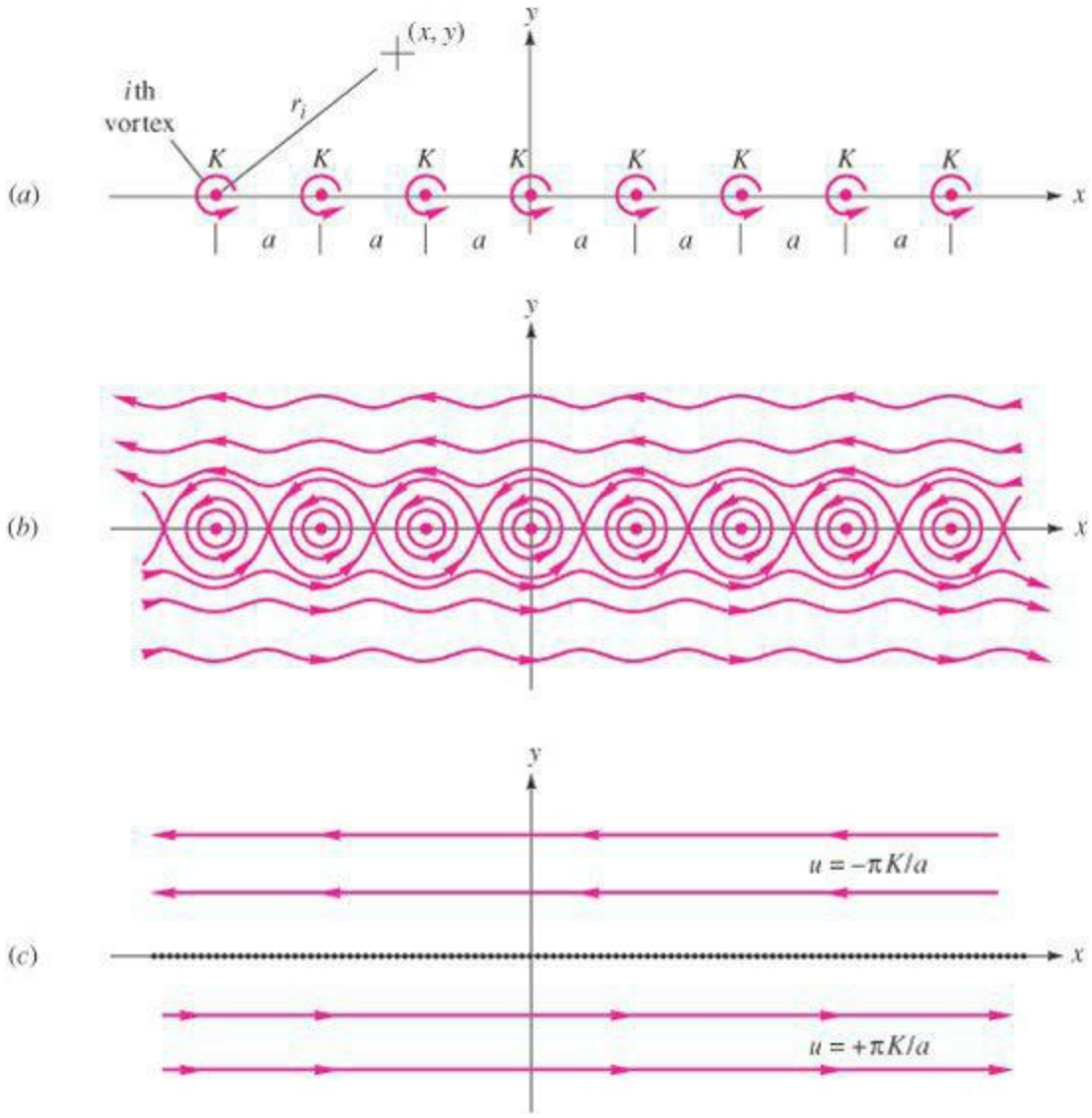
) LJ) ORZ RI DXQLIRUP VWUHDP SDVW D YRUWH[FRQWUXFWHG E\ WKH JUDSKLFDO PHWKRG

%\ VHWWLQJ %\ % IURP ZHILQGDWDQDWRQSRLQWV f U D . 8, RU [\
D 7KLV LV ZKHUH WKH FRXQWHUFORFNZLVH YRUWH[YHORFLW. UH[DFW FDQFHOV WKH VWUHDP YHORFLW
8,

3UREDEO WKHP RVWLQMHUWLQJ WKLQJ DERXWKLV[DPSOHV WKDWKHUHV DQRWHUOLWWRUFHQRUP D
VR WKH VWUHDP RQWKH VXUIDFH RI DQ UHJRQHQFORVLQJ WKH YRUWH[EXWZHSRVSRQHWKLV GLVFXVVLRQ XQW
WKHQH[WVHFWLRQ

$Q,QILQLWH5RZ RI 9RUWLFHV

&RQVLGHUDQ LQILQLWH URZ RI YRUWLFHV RI HTXDO VWUHQJWK. DQG HTXDOVSDFLQJ D DV LQ) LJ D
7KLVFDVHLVLQFXGHGKHUHWROOXVWUDWHWKHLQIUHTXHQF\RI DYRUW[VKHHW



) LJ 6XSHUSRVLWLRQ RI YRUWLFHV a DQ LQILQLWH URZ RI HTXDO VWUHQJWK b VWUHDPOLQH SDWWHUQ IRU SDUW$ c YRUWLFHV DQG VHHWSDUW% YLHZHG IURP DIDU

) URP (T VHHLWK YRUWLFHV LQ) LJ DKDVDVWUHDP IXQFWLRQ$L í. OQU VR WKDWWKHVRXUFH LQILQLWH URZ KDVDFRPELQHGVWUHDP IXQFWLRQ

$$\psi = -K \sum_{i=1}^{\infty} \ln r_i$$

,WFDQEHVKRZQ> 6HF @WKDWWKLVLQILQLWHVXP RI ORJDULWKPV LV HTXLYDOHQWWR DFORVHG IRUP IXQFWLRQ

$$\psi = -\tfrac{1}{2}K \ln\left[\frac{1}{2}\left(\cosh\frac{2\pi y}{a} - \cos\frac{2\pi x}{a}\right)\right] \qquad (8.28)$$

6LQFHWKHSURRI XVHVWKHFRPSOH[YDULDEOH] [L\ L í ZHDUHQRWJRLQJ WR VKRZ WK GHWDLOVYKHUH

7KHVWUHDPOLQHVIURP (T DUHSORWWHG LQ) LJ E VKRZLQJ ZKDWLVFDOOHG DFDWV H\H SDWWHUQ RI HQFORVHG IORZ FHOOV VXUURXQGLQJ WKHLQGLYLGXDOYRUWLFHV $ERYHWKHFDWV H\HVWKHIORZ LV

HQWHUHG WKURXJK HDFK HVWDQG EHORZ WKH FDVH, HV WKH IORZ LV WKH UJKW0 RUHRYHU WKH VHOHFWDQG UJK IORZ VDUH XQLRUP LI X\ X|y| ≫ a, D ZKLFK IROORZV E\ GLIIHUHQWDWLQJ (T

$$u = \frac{\partial \psi}{\partial y}\bigg|_{|y| \gg a} = \pm \frac{\pi K}{a}$$

ZKHUH WKH SOXV VLJQ DSSOLHV EHORZ WKH URZ DQG WKH PLQXV VLJQ DERYH WKH URZ 7KLV XQLRUP CH DQG UJKWWXWHDPLQJ LV VNHWFKHG LQ) LJ F : HVWHV WKDWWKLV HIHFWLV LQGXFHG E\ WKH UROO RYRUWHV 7KHUHLVQR XQLRUP VWUHDP DSSURDFKLQJ WKH UROO LQWKLV H[DPSOH

7KH9 RUWM[6KHHW

: KHQ) LJ E LV YLHZHG IURP DIAR WKH VWUHDPLQJ PRWLQ LV XQLRUP CHWDERYH DQG XQLRUP ULJKWEHORZ DV LQ) LJ F DQG WKH YRUWH[DUH SDFNHG VR FORVHO WRJHWKHUWKDWWKH DUH VPXGJH LQWR DFRQWLQXRXV YRUWM[VKHHW7KHVWUHQJWKRI WKHVKHHWLV GHILQHGDV

$$\gamma = \frac{2\pi K}{a} \tag{8.29}$$

DQG LQWKHJHQHUDOFDVH FDQYDU ZLWK[7KHFLUFXODWLRQDERXWDQ FORVHGFXUYHWKDWHQFORVHV I VKRWWKHQJWKG[RI WKHVKHHWZRXOGEH IURP (TV DQG

$$d\Gamma = u_l\,dx - u_u\,dx = (u_l - u_u)\,dx = \frac{2\pi K}{a}\,dx = \gamma\,dx \tag{8.30}$$

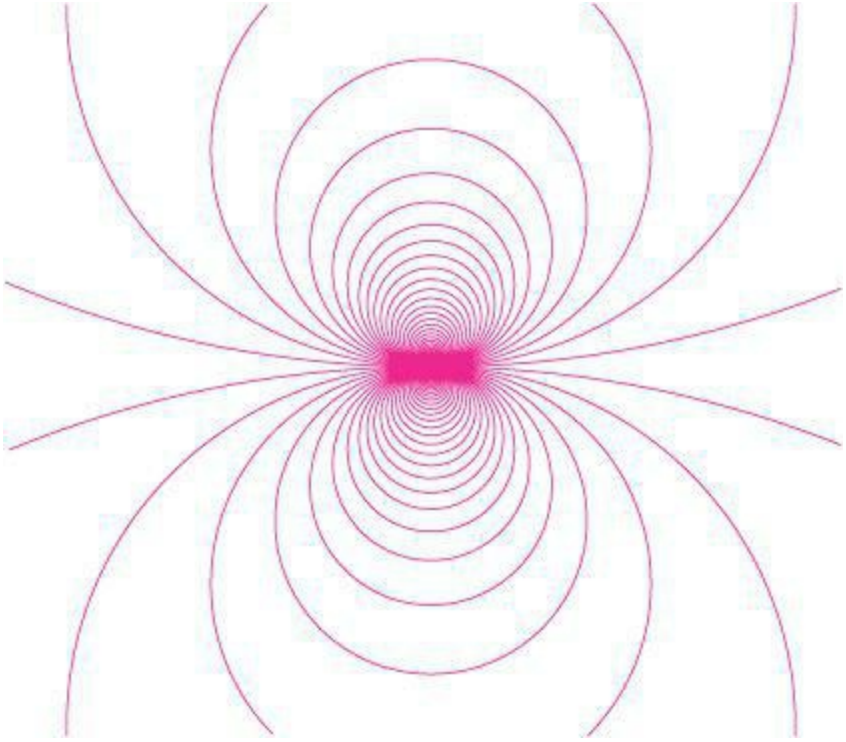
ZKHUH WKHVXEVFULSWVODQG X VDQG IRUORZHUDQG XSSHU UHSHFWLYHO 7KXV WKHVKHHWWXWHQJWK G* G[LV WKH FLUFXODWLRQ SHU XQLWOHQJWK RI WKHVKHW7KXV ZKHQ D YRUWM[VKHHWLV LPPHUVHG LQ I XQLRUP VWUHDP LVSURSRUWLRQDOWR WKHOHQJWKRI DQ VXUIDFHHQFORVLQJ WKHVKHW 1 RWWKDWWKHUHLVQR YHORFLW\ RUP DOWR WKHVKHWDWWKHVKHWXUIDFH 7KHUHRUHDYRUWM[VKH FDQ VLPXODWHDWKLQEHG\ VKDSH OLNHDSODWHRUWKLQDLUIRLO7KLVLV WKHEDVLV RI WKHWKLQDLUIRLOWKHRU PHQWRHGLQ6HF

7KH' RXEOHW

$ VZHPRYHIDUZD\ IURP WKHVRXUFH±VLQNSDLURI) LJ WKHIORZ SDWWLQEHJLQVWRUHVHPEOH IDPLO RI FLUFOHVWDQJHQWWR WKHRULJLQ DV LQ) LJ 7KLVOPLWRI YDQVKLQJO VPDOOGLVWDQFHD L FDOOHG DGRXEOHW7R NHHS WKHIORZ VWHQJWKOLJHHQOYHORFLWLHVDVD EHFRPH VPDOO ZHVSHFI\ WKDWWKHSURGXFW DP UHPDLQFRQVWDQW/HWXVFDOOWKLVFRQVWDQW 7KQWKH VWUHDI IXQFWLRQRI DGRXEOHWV

$$\psi = \lim_{\substack{a\to 0 \\ 2am=\lambda}} \left(-m \tan^{-1} \frac{2ay}{x^2 + y^2 - a^2} \right) = -\frac{2amy}{x^2 + y^2} = -\frac{\lambda y}{x^2 + y^2} \tag{8.31}$$

: HKDYHXVHGWKHIDFWKDWWDQ . §. DV. EHFRPHVVPDOO7KHTXDQWLW LVFDOOHG WKHVWUHQJWKRI WKHGRXEOHW



) LJ $ GRXEOHWRU VRXUFH±VLQN SDLU LV WKH OLPLWLQJ FDVH RI) LJ YLHZHG IURP DIDU 6WHDP OLQHV DUH FLUFOHV WDQJHQWWR WKH[D[LV DWWKH RULJLQ 7KLV ILJXUH ZDV SUHSDUHG XVLQJ WKH *contour* IHDWXUHRI 0 $7/ $%> @

(TXDWLRQ FDQEHUHDUUDQJHGWR\LHOG

$$x^2 + \left(y + \frac{\lambda}{2\psi}\right)^2 = \left(\frac{\lambda}{2\psi}\right)^2$$

VR WKDWDV DGYHUWLVHG WKH VWHDP OLQHV DUH FLUFOHV WDQJHQWWR WKH RULJLQ ZLWK FHQWHUV RQ WKH \ D[LV 7KLV SDWWHUQ LV VHHQ HYHU\GD\) LJ

$ OLWWOH XJK WKH DXWKRU KDV LQ WKH SDVW DERUROXVO VNHWFKHG WUHDP OLQHV E\ KDQG WKLV LV QR ORQJHU QHFHVVDU) LJXUH ZDV FRPSXWHGZQ XVLQJ WKH FRQWRXU IHDWXUH RI WKH VXGHQWYHUVLRQ R 0 $7/ $%> @6LPSO VHWXS DJUHG SRLQW VSHOORXWWKH VWHDP IXQFWLRQ DQG FDOOIRUDFRQWRX) RU) LJ WKHDFWXDOXDWAPHQWZHUH

>; <@ PHWKJULG
36, < ; A < A
FRQWRXU; < 36,

7KLVZRXOG SURGXFH FRQWRXUOLQHVRI $IURP (T ZLWK IRUFRQYHQLHQFH 7KHSOI ZRXOG LQFXGH JULG OLQHV VFDOH PDUNLQJV DQG DVXUIDFH LQJ ER[DQG WKH FLUFOHV PLJKWORRN DEL HODSWFDO 7KHVHEOPLVKHVFDQEHHDVLODWWZLWKWHVWAPHQWRI FRVPHWLF SURYHPHQW

D[LVVTXDUH
JULGRI
D[LVRI

7KH ILQDO SORW) LJ KDV QR PDUNLQJV EXWWKH VWUHDPOLQHV WKHPVHOYHV 0 $7/ $% LV WKXV UHFRPPHQGHG WRROLQG LQDGGLWLRQ KDV VFRUHV RI RWKHUXVHV $ OLWHUDWXUH SUREOHP DWLJQPHQW WKDWFDOOIRU³VNHWFK WKH VWUHDPOLQHV SRWQWLDOOLQHV FDQEHFRPSOHWHG XVLQJ WKLVFRQVWUXFWLRQ)RIXUWKHUGHWDLOV FRQVXOW%H

,QDVLPLODUPDQQHUWKHYHORFLW\SRWQWLDORI DGRXEOHWLVIRXQGE\ VXNLQJ WKHOPLWRI (T DVD'""" DQG DP

$$\phi_{\text{doublet}} = \frac{\lambda x}{x^2 + y^2}$$

or

$$\left(x - \frac{\lambda}{2\phi}\right)^2 + y^2 = \left(\frac{\lambda}{2\phi}\right)^2 \qquad (8.32)$$

7KH SRWQWLDOLQHV DUH FLUFOHV VDQJHQWWR WKH RUJLQ Z LWK FHQWHUV RQ WKH[D[LV 6LPSO WKLQ) LJ FORFNZ LVH f WRYLVXDOL]HWKHË OQHV Z KLFK DUHHYHUZ KHUHRUP DOWR WKHVXUIDFHV 7KHGRXEOHWXQFWLRQVFDQDOR EHZ ULWWQLQSRODUFRRUGLQDWHV

$$\psi = -\frac{\lambda \sin\theta}{r} \qquad \phi = \frac{\lambda \cos\theta}{r} \qquad (8.33)$$

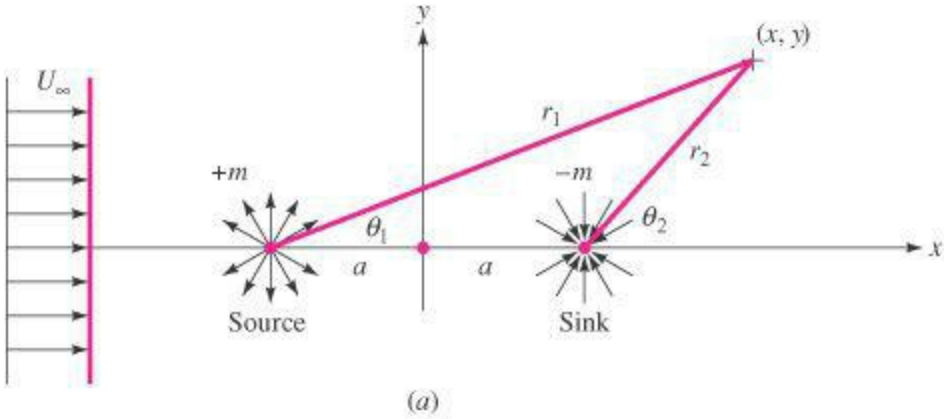
7KHVHIRUPVDUHFRQYHQLHQWRUWKHA\OLQGHUIORZ VR WKHQH[WVHFWLRQ

3ODQH) ORZ 3DVW&ORVHG %RG\ 6KDSHV

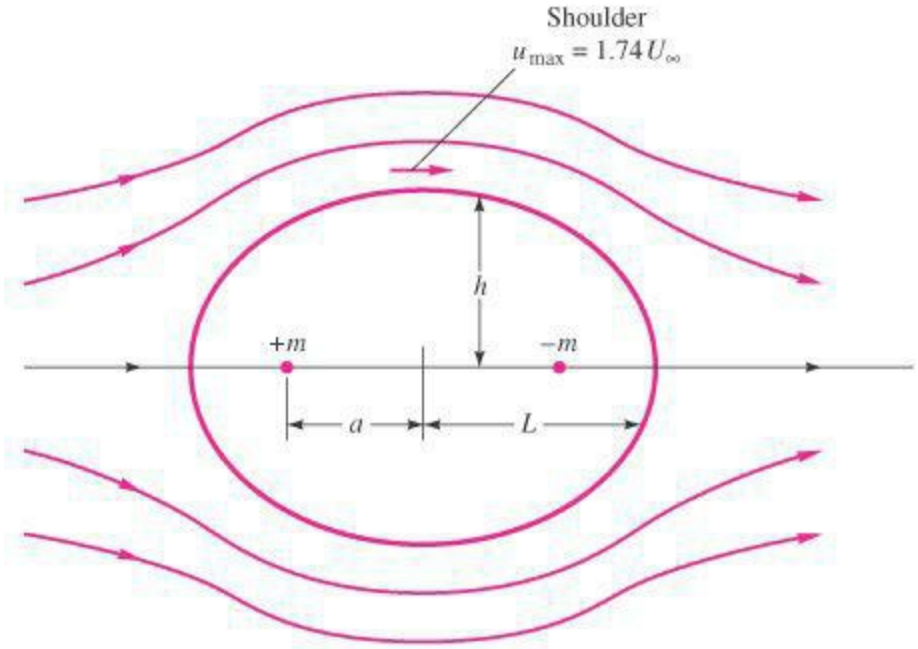
$ YDULHWV RI FORVHG ERG\ H[WHQVLRQ VFDQEHFRQVWUXFWHG E\ VXSHUIP SRVLQJ DXQLRUP VWUHDF Z LWK VRXUFHV VLQNV DQG YRUWHFHV 7KHERG\ VKDSHZ LOOEHFORVHGRQO LI WKHQHWVWUHQJWKHQHWVLQNLQORZ

7KH5 DQNLQH2 YDO

$ FA\OLQGULFDOVKDSHFDOOHG D5DQNLQHRYDO Z KLFK LV GRQJ FRP SDUHG Z LWKLWK HJKWLV IRUPHGE\ I VRXUFHEVLQNSDLUDDQJHGSDUDOOHOWR DXQLRUP VWUHDP DVLQ) LJ D



(a)



(b)

) LJ) ORZ SDVW D 5DQNLQH RYDO *a* XQLRUP VWUHDP SOXV D VRXUFH±VLQN SDLU *b* RYD VKDSHDQG VWUHDP OLQHV IRU*m* *U*, *a*

) URP (TV DQG VKHFRPELQHG VWUHDP IXQFWLRQV

$$\psi = U_\infty y - m \tan^{-1} \frac{2ay}{x^2 + y^2 - a^2} = U_\infty r \sin\theta + m(\theta_1 - \theta_2) \qquad (8.34)$$

: KHQ VWUHDP OLQHV RI FRQVWDQW $ DUH SORWWHG IURP (T DQ RYDO ERG\ VKDSH DSSHDUV DV LO) LJ E 7KH KDOI OHQJWK/ DQG KDOI KHLJKW K RI VKH RYDO GHSHQG RQ VKH UHODWLYH VWUHQJWK RI VRXUF DQG VWUHDP² VKDWLV VKH UDWLR P 8, D ZKLFK HTXDOV LQ) LJ E 7KH FLUFXODWLQJ VWUHDP OLQH LQVLGH VKH RYDO DUH XQLQWHUUXSWHG DQG RQ VXUIDFH Q 7KH RYDOLV VKH OLQH$

7KHUH DUH VWDJQDWLRQ SRLQWV DW WKH IURQW DQG UHDU [“/ DQG SRLQWV RI PD[LPXP YHORFLW DQ PLQLPXP SUHVVXUH DW WKH VKRXOGHUV \ “K RI VKH RYDO $ OWKHVH SDUDPHWHUV DUH D IXQFWLRQ RI VK EDVLF GLPHQVLRQOHVV SDUDPHWUP 8, D ZKLFK ZHFDQ GHWHUPLQH IURP (T

$$\frac{h}{a} = \cot\frac{h/a}{2m/(U_\infty a)} \qquad \frac{L}{a} = \left(1 + \frac{2m}{U_\infty a}\right)^{1/2}$$
$$\frac{u_{max}}{U_\infty} = 1 + \frac{2m/(U_\infty a)}{1 + h^2/a^2} \qquad (8.35)$$

$VZHLQFUHDVHP 8, D IURP]HURWRODUJHYDOXHVWKHRYDOVKDSHLQFUHDVHVLQVL]HDQGWKLFNQHV
IURP DIODWSODWHRI OHQJWK DWRDKXJH QHDUO\ FLUFXODUF\OLQGHU 7KLVLVVKRZQLQ7DEOH ,QWK
OPLWDVP 8, D : ' / K: DQGX PD[8, ZKLFKLVHTXLYDOHQWWRIORZ SDWWD FLUFXOD
F\OLQGHU

$OOWKH5DNLQHRYDOVH[FHSWYHU WKLQRQHVKDYHDOLJHDGYHUVHPUHVXUHJUDGLHQWRQWKH
OHZDUGVXUIDFH 7KXVERXQGDU\OD\HUVHSDUDWRQZLOORFFXULQWKHUHDUZLWKDEURDGZDNHIORZ DQ
WKHIQYLVFLGSDWWKLQLVXQUHDOLVWLF WKDWWHJLRQ

) ORZ 3DVWD &LUFXODU&\OLQGHUZLWK &LUFXODWRQ

)URP 7DEOH DWDOLJHVRXUHVWXQHWKWKH5DNLQHRYDOEHFRPHVDOLJHFLUFOH PXFKJUHDWHUL
GLDPHWHUWKDQWKHVRXUFH±VLQNVSDFLQJ D 9LHZHGRQWKHVFDOHRI WKHF\OLQGHU WKLVLVHTXLYDOHQWWR
XQLRUP VWUHP SOXVDGRXEOHW: HDOR WKURZ LQDYRUWH[DWWKHGRXEOHWF HQWUZKLFKGHVQRWFKDQJH
WKHVKDSHRI WKHF\OLQGHU

$m/(U_\infty a)$	h/a	L/a	L/h	u_{max}/U_∞
0.0	0.0	1.0	∞	1.0
0.01	0.031	1.010	32.79	1.020
0.1	0.263	1.095	4.169	1.187
1.0	1.307	1.732	1.326	1.739
10.0	4.435	4.583	1.033	1.968
100.0	14.130	14.177	1.003	1.997
∞	∞	∞	1.000	2.000

7DEOH 5DQNLQH2 YDO3DUDPHWHUVIURP (T

7KXVWKHVWUHDP IXQFWLRQIRUIORZ SDWWDFLUFXODU\OLQGHUZLWKFLUFXODWLRQ FHQWHUHGDWWKHRULJLQ
LVDXQLRUP VWUHDP SOXVDGRXEOHWSOXVDYRUWH[

$$\psi = U_\infty r \sin\theta - \frac{\lambda \sin\theta}{r} - K \ln r + \text{const} \qquad (8.36)$$

7KHGRXEOHWVWUHQJWK KDVXQLWVRI YHORFLW\WLPHVOHQJWKVTXDUHG)RUFRQYHQLHQFH OHW 8, D
ZKHUHD LVDOHQJWK DQG OHWWKHDUELWUDU\ FRQVWDQWLQ(T HTXDO. OQD 7KHQWKHVWUHDP IXQFWLRQ
EHFRPHV

$$\psi = U_\infty \sin\theta\left(r - \frac{a^2}{r}\right) - K \ln\frac{r}{a} \qquad (8.37)$$

7KHVWUHDPOLQHV DUH SORWWHG LQ) LJ IRUIRXUGLIIHUHQWYDOXHV RI WKHGLPHQVLRQOHVV YRUWH[
VWUHQJWK. 8, D)RUDOOFDVHVWKHOLQH$ FRUUHVSRQGVWRWKHFLUFOH D² WKDWVWV WKHVKDSHRI WK
F\OLQGULFDOERG\ $VFLUFXODWLRQ* CE LQFUHDVHV WKHYHORFLWEHFRPHVIDVWHUDQGIDVWHUEHOZ WK
F\OLQGHUDQGVORZHUDQGVORZHUDERYHLW7KHYHORFLWFRPSRQHQWLQWKHIORZ DUHJLYHQE\

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U_\infty \cos\theta\left(1 - \frac{a^2}{r^2}\right)$$
$$v_\theta = -\frac{\partial \psi}{\partial r} = -U_\infty \sin\theta\left(1 + \frac{a^2}{r^2}\right) + \frac{K}{r} \qquad (8.38)$$

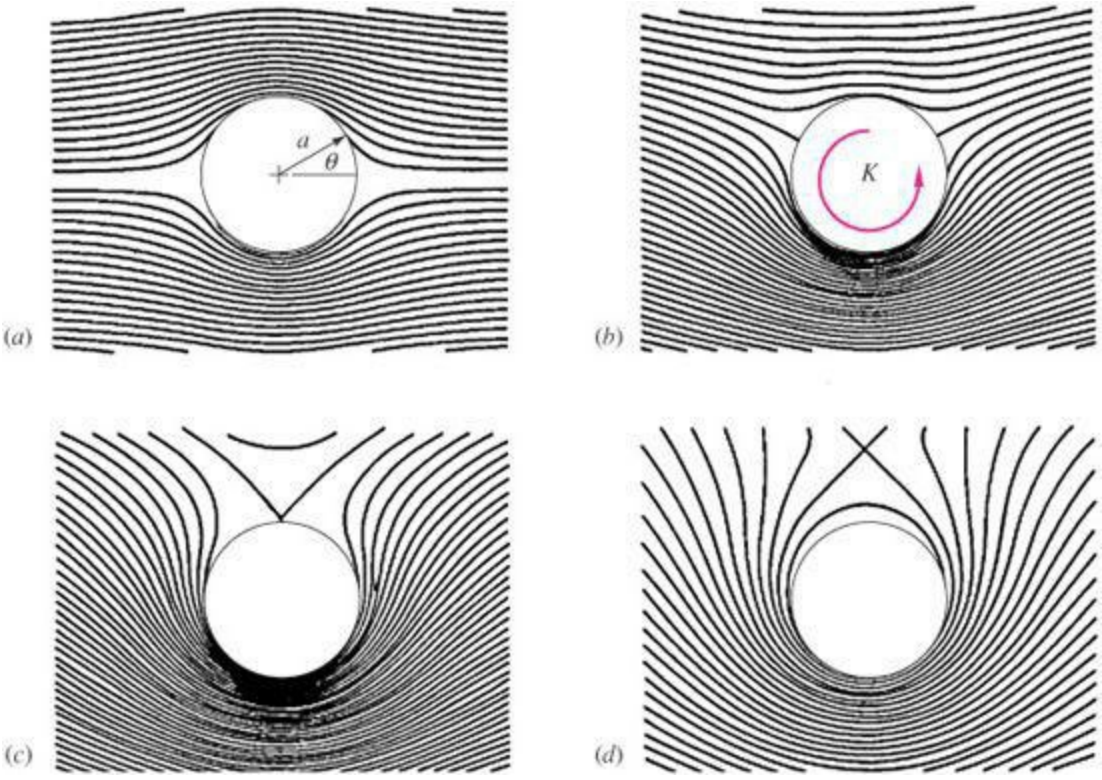
7KHYHORFLWDWKHFRQGHUVXUIDFH DLVSXUHO WDQJHQWLDO DVH[SHFWHG

$$v_r(r = a) = 0 \quad v_\theta(r = a) = -2U_\infty \sin\theta + \frac{K}{a} \qquad (8.39)$$

) RUVPDOO VWR VWDJQDWLRQSRLQWSSHDURQWKHVXUIDFHDWDQJOHV V ZKHUH% RU IURP (T

$$\sin\theta_s = \frac{K}{2U_\infty a} \qquad (8.40)$$

) LJXUH DLV IRU. V DQG f RUGRXEO\ VPPHWULF LQYLVFLG IORZ SDWWDFRQGH ZLWKQR FLUFXODWRQ) LJXUH E LV IRU. 8, D V DQG f DQG) LJ F LV WKHOPLWQ FDVHZKHUHWKHVWR VWDJQDWLRQSRLQWPHHWDWKHSRS . 8, D V f



) LJ) ORZ SDWWDFLUFXODUFRQGHUZLWKFLUFXODWRQIRUYDOXHVRI*K U, a* RI *a b c* DQG *d*

) RU. ! 8, D (T LVLQYDOLG DQGWKHVLQJHWDJQDWLRQSRLQWVDERYHWKHFRQGHUDVLV) LJ G DWDSRLQW KJLYHQE\

$$\frac{h}{a} = \frac{1}{2}\left[\beta + (\beta^2 - 4)^{1/2}\right] \qquad \beta = \frac{K}{U_\infty a} > 2$$

7KH. XWWD-RXNRZVNL/ LIW7KHRUHP

7KHF\OQGHUIORZVZLWKFLUFXODWLRQ) LJV E WRG GHYHORS DQLQYLVFLG GRZQZDUGOLWQRUP D VRVWKHIUHHVWUHDP FDOOHG WKH0DJQXV5RELQVIRUFH 7KLVOLWLSURSRUWLRQDOWR VWUHDP YHORFLW DQ YRUWH[VWUHQJWK ,WVGLVFRYHU E\ H[SHULPHQWKDVORQJ EHHQDWWULEXWHGWR WKH* HUPDQSK\VLFLVW 0DJQXV ZKR REVHUYHG LWLQ ,WLV QRZ NQRZQ> @WKDWWKH EUHDNDQW%UWWK HQJLQH-%HQDPLQ5RELQVILUVWSRLQWHGDOLWWRUFHRQDVSLQQLQJ ERG\ LQ : HVHIURP WKHVWUHDPOQ SDWWHUQVWKDWWKH YHORFLWV RQ WRS RI WKH F\OQGHULV OHVV DQG WKXV IURP %HUQRXOOL¶V HTXDWLRQ WK SUHVXUHLVKLJKHU2QWKERWWRP ZHVHWWKXV SDFNHGVWUHDPOQHV KLJK YHORFLW DQGORZ SUHVXUHYLVFRVLWVLQHJOHFWHG ,QYLVFLGWKRU SUHGLFWWKLVIRUFH

7KHVXUIDFHYHORFLWVLJLYHQE\ (T) URP %HUQRXOOL¶VHTXDWLRQ QHJOHFWLQJ JUDYLW WKHVXUIDFHSUHVVXUHS, LVJLYHQE\

$$p_\infty + \frac{1}{2}\rho U_\infty^2 = p_s + \frac{1}{2}\rho\left(-2U_\infty \sin\theta + \frac{K}{a}\right)^2$$

or
$$p_s = p_\infty + \tfrac{1}{2}\rho U_\infty^2(1 - 4\sin^2\theta + 4\beta\sin\theta - \beta^2) \qquad (8.41)$$

ZKHUH . 8, D DQGS LVWKHIUHHVWUHDP SUHVVXUH ,I E LVWKHF\OQGHUGHSWKLQWR WKHSDSHU WKHGUDJ' LVWKHLQWJUDORYHUWKHVXUIDFHRI WKHKRL]RQWDOFRPSRQHQWRI SUHVVXUHIRUFH

$$D = -\int_0^{2\pi} (p_s - p_\infty)\cos\theta\, ba\, d\theta$$

ZKHUHS, í S LVVXEVWLWXWHGIURP (T %XWWKHLQWJUDORI FRV WPHVDQ SRZHURI VLQ RYHUDIXOOF\FOH CELVLGHQWFDOO]HUR 7KXVZHREWLQWKH SHUKDSVVXUSULVLQJ UHVXOW

$$D(\text{cylinder with circulation}) = 0 \qquad (8.42)$$

7KLVLVD6SHFLDOFDVHRI G¶$OPEHUW¶VSDUDGR[PHQWRQHGLQ6HF

$FFRUGLQJ WR LQYLVFLG WKHRU WKH GUDJ RI DQ ERG\ RI DQ VKDSH IMP PHUVHG LQ D XQLIRUP VWUHDP LVIGHQWFDOO]HUR

' ¶$OPEHUWSXEOLVKHGWKLVUHVXOWQ DQG SRLQWHGRXWKLP VHO WKDWWKGLG QRWTXDUHZLWKWK IDFWIRU UHDOIOXLG IORZV 7KLV XQRUWXQDWH SDUDGR[FDXVHG HYHU\RQH WR RYHUHDFWDQG UHMHFWDO LQYLVFLG WKHRU XQLO ZKHQ 3UDQGWOILUWSRLQWHGRXWWKH SUHVRXG HIIHFWRI WKH VKLQ YLVFRX ERXQGDU OD\HUQRWKHIORZ SDWWHUQLQWKHUHDUDVLQ) LJ E IRUH[DPSOH

7KHOLWRUF/ QRUP DOWR WKHVWUHDP VDN-QSRVLWLYHXSZDUG LVJLYHQE\ VXPPDWRQ RI YHUWFL SUHVXUHIRUFHV

$$L = -\int_0^{2\pi} (p_s - p_\infty) \sin\theta \, ba \, d\theta$$

6LQFH WKH LQWHJUDO RYHU GER DQ RGG SRZHUV RI VLQ LV]HUR RQO WKH WKLUG WHUP LQ WKH SDUHQWKHVH
LQ(T FRQWULEXWHV WR WKH OLIW

$$L = -\frac{1}{2}\rho U_\infty^2 \frac{4K}{aU_\infty} ba \int_0^{2\pi} \sin^2\theta \, d\theta = -\rho U_\infty (2\pi K) b$$

or

$$\boxed{\frac{L}{b} = -\rho U_\infty \Gamma} \qquad (8.43)$$

1 RWH WKDW WKH OLIW WHUP VLQGH SHQGH QWR WKH UDGLXV DR WKH F\OLQGHU $FWXDOO WKRXJK DV ZHVKDO
VHH LQ6HF WKH FLUFXODWLRQ * GHSHQGV RQ WKH ERG\ VL]H DQG RUHQWDWLRQ WKURXJK D SK\VLFD
UHTXLUHPHQW

(TXDWLRQ ZDV JHQHUDO]HG E\ : 0 . XWWD LQ DQG LQGHSHQGHQWO E\ 1 -RXNRZVNL L
DV IROORZV 7KH SURFHVV LQ DLUIRLODQDO VLV 6HF LV VKXV WR GHWHUPLQH WKH FLUFXODWLRQ* DV
IXQFWLRQ RI DLUIRLO VKDSH DQG RUHQWDWLRQ

$FFRUGLQJ WR LQYLVFLG WKHRU WKH OLIWSHU XQLW GHSWK RI DQ F\OLQGHU RI DQ VKDS
LPPHUVHG LQ D XQLIRUP VWUHDP HTXDOV ! X * ZKHUH* LV WKH VWUDQGHWFLUFXODWLRQ FRQGLQH
ZLWKLQ WKH ERG\ VKDSH 7KH GLUHFWLRQ RI WKH OLIW ƒ IURP WKH VWUHDP GLUHFWLRQ URWDWH
RSSRVLWH WR WKH FLUFXODWLRQ

/ LIW DQG ' UDJ RI 5 RWDWLQJ &\OLQGHUV

7KH IORZ LQ) LJ DUH P DWKHP DWLFDO D GRXEOHWSOXV D YRUWH[SOXV D XQLIRUP VWUHDP 7KH
SK\VLFDOUHDOL]DWLRQ FRXOG EH D URWDWLQJ F\OLQGHULQ D IUHH VWUHDP 7KH QR VOLS FRQGLWLRQ ZRXOG FDX
WKH IOXLG LQ FRQWDFW ZLWK WKH F\OLQGHU WR P RYH WDQJHQWLDOO DW WKH F\OLQGHU VXUIDFH 9 D& VHWWLQJ XS D QH
FLUFXODWLRQH 0 HDVXUHPHQWV IRUFHV RQ D VPRRWK F\OLQGHULV YHU GLIILFXOW DG QR UHODEOH GDWD DU
DUH NQRZ Q WR WKH DXWKRU + RZ HYHU 7RNXP DUX DQG ' LPRWNLV> @XVHG D FOHYHU DX[LOLDU\ VFKHP H WR
PHDVXUH FLUFXODWLRQ DQG 6H G

) LJXUH VKRZV VWDQGDUG GUDJ FRHILFLHQW EDVHG RQ IURQWDO DUHD DE IRU D URWDWLQJ F\OLQGH
DW 5H 7KH GUDJ FXUYH LV IURP &)' FDOFXODWLRQV > @5HSRUWHG &)' GUDJ UHVXOWV IUR
VHYHUDO GLIIHUHQW DXWKRUV DUH TXLWH FRQVLVWHQW EHWZHHQ WKH GRQRWDJUHH HYHQ TXDQWLWDWLYH 7K
ZULWHUV IHOW WKDW 5H JLYHV WKH PRVW UHDOLVWLF UHVXOWV 1 RWH WKDW WKH H[SHULPHQWDO& LQFUHDVHV WR
YDOXHV RI DW& 8 7KLV FRQWUDGLFWV WKH RU RI 3UDQGWO LQ WKDW WKH PD[LPXP SRVLWLYH
YDOXH RI &/ ZRXOG EH I § FRUUHVSRQGLQJ WR WKH IORZ FRQGLWLRQV LQ) LJ F 7KH LQYLVFL
WKHRU IRU DQ ZRXOG EH

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 (2ba)} = \frac{2\pi\rho U_\infty K b}{\rho U_\infty^2 ba} = \frac{2\pi v_{\theta s}}{U_\infty} \qquad (8.44)$$

ZKHUH%V . DLVWKHSHULSKHUDOVSHHGRI WKHF\OLQGHU

) LJXUH VKRZV WKDWWKHWKHRUHWLFDOOLIWURP (T LVPXFKWRRKLJK EXWWKHPHDVXUHGOL LVTXLWHVSHFWDEOH PXFKODUJHULQIDFWKDQDWWSLFDOOLIURLORI WKHVDPHFKRUGHQJWK DVLQ) LJ 7KXVURWDWLQJ F\OLQGHUVKDYHSUDFWLFDOSRWHQWLDOV 7KH) OHWQHUURWRUVKLS EXLOWLQ* HUPDQ LQ HPSOR\HG URWDWLQJ YHUWLFDO F\OLQGHUV WKDWGHYHORSHG DWKUXVWGXHWR DQ ZLQGVEORZLQJ SDVWWKHVKLS 7KH) OHWQHUGHVLJQGLGQRWJDLQDQ SRSXODULW EXWXFKLQYHQWLRQVPD\ EHPRUHDWWDFKHGWRWKLVHU RI KLJKHQHUJ\ FRVWV

(; $0 3/ (

7KHH[SHULPHQWDO) OHWQHUURWRUVDLGERDWDWWKH8QLYHUVLWRI 5KRGH,VODQGLVVKRZQLQ) LJ (7KHURWRUV IWLQGLDPHWUDQG IWORQJ DQG URWDWHV DW UPLQ ,WLV GULYHQ E\ D VPDO ODZ QPRZ HUHQJLQH ,I WKHZLQGLV DVWDG\ NQDQGERDWHDVWHPRWRQLVQHJOHFWHG ZKDWLVWK PD[LPXP WKUXVWH[SHFWHGIRUWKHURWRU $VXPHVWDQGDUGDLUDQGZDWHUGHQVLW

6RXWLRQ

&RQVHUWKHURWDWRQUDWHWR& CE UDGV 7KHZLQGYHORFLWLV NQ IWV VRWKHYHORFLWUDWLR

$$\frac{a\omega}{U_\infty} = \frac{(1.25\text{ ft})(23.04\text{ rad/s})}{16.88\text{ ft/s}} = 1.71$$

8VLQJ) LJ ZHUHDG&L § DQG&D §) URP 7DEOH$ WDQGDUGDLUGHQVLWLQ%* XQLV LV VOXJ IW 7KHQWKHWUP DWGURRUQLWDQGGUDJ DUH

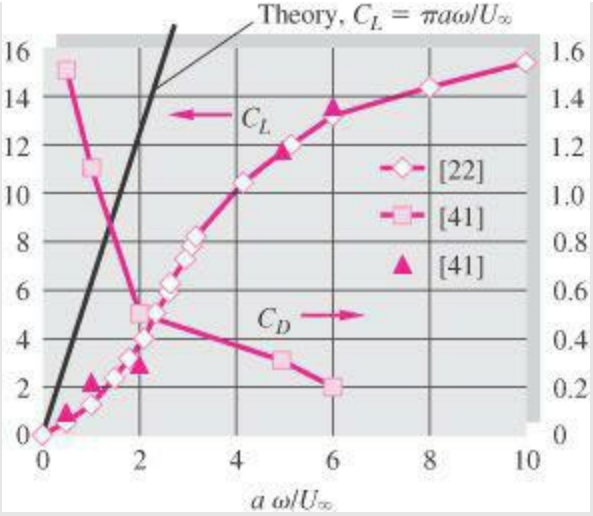
$$L = C_L \frac{1}{2}\rho U_\infty^2 2ba = (2.5)\frac{1}{2}\left(0.00238\frac{\text{slug}}{\text{ft}^3}\right)\left(16.88\frac{\text{ft}}{\text{s}}\right)^2 2(10\text{ ft})(1.25\text{ ft}) = 21.2\text{ lbf}$$

$$D = C_D \frac{1}{2}\rho U_\infty^2 2ba = (0.7)\frac{1}{2}\left(0.00238\frac{\text{slug}}{\text{ft}^3}\right)\left(16.88\frac{\text{ft}}{\text{s}}\right)^2 2(10\text{ ft})(1.25\text{ ft}) = 5.9\text{ lbf}$$

7KHPD[LPXP WKUXVWDYDLODEOHWRWKHVDLGERDWLV WKHUHVXOWDQWRI WKHVHWZR

$$F = [(21.2)^2 + (5.9)^2] = 22.0\text{ lbf} \qquad \textit{Ans.}$$

1RWHWKDWZDWUGHQVLWGLG QRWHQWHULQRXU FDOFXODWLRQ ZKLFK LV DIRUHHGXHWRDLU ,I DQJQH DORQJ WKHERDWV NHHO WKLV WKUXWZLOOGULYHWKHERDWKURXJKWKHZDWUDWD VSHHGRI DERXW NQ



) LJ ' UDJ DQG OLIW RI D URWDWLQJ F\OLQGHU RI OHQJWK DVSHFW UDWLR DW 5H D WR 7RNXP DUK DQG' LP RWDNLV> @DQG 6HQJXSWDHWDO> @

(

&RXUWHV\ RI 5 & / HWW DQG 8QLYHUVLWLV RI 5KRGH, VODQG

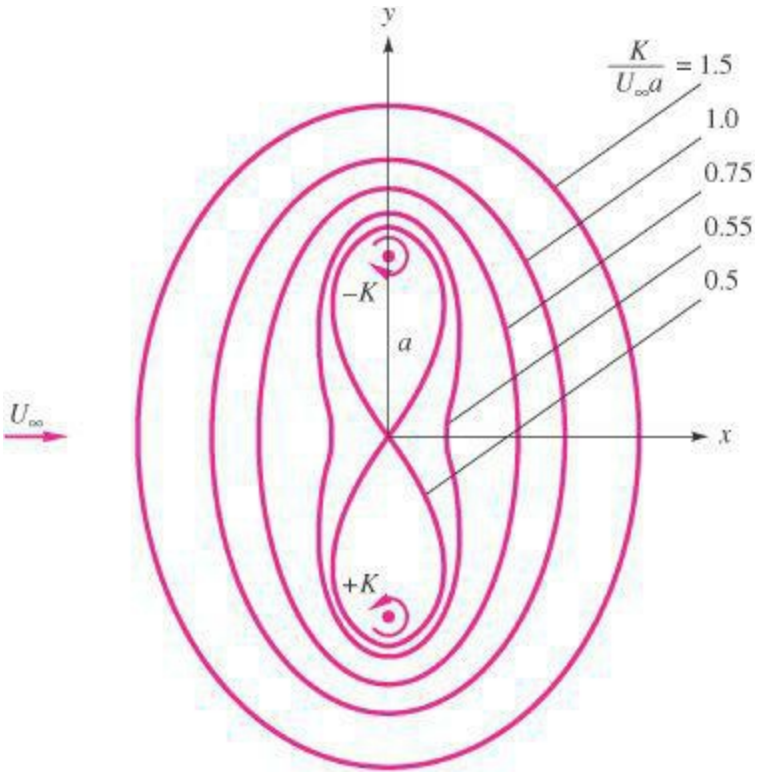
&RPPHQW) RUWKH VDNH RI DQ XPHULFDO[DPSOH ZH KDYH GRQH VRPH WKLQJ LP SURSHUKHUH : H KDYH XVHG GDWD IRU 5H VR HVWLP DWH IRUFHV ZKHQ WKH URWRU 5H § ' RQRWGR WKLV L\RXUHDOWRE DWWU\RXJUDGXDW

7KH. HOYLQ2 YDO

\$ IDPLO RI ERG\ VKDSHV VDOHUWKDQ WKH DUHZLGHFDQEHIRUPHGE\ OHWWQJ DXQLRUP VWUHDP IORZ QRUPDO WR D YRUWH[SDLU ,I8, LV WR WKH ULJKW WKH QHJDWLYH YRUWH[í. LV SODFHG DW< D DQG WK FRXQWHUFORFNZLVH YRUWH[. SODFHG DW< í D DV LQ) LJ 7KHFRPELQHG VWUHDP IXQFWLRQV

$$\psi = U_\infty y - \frac{1}{2} K \ln \frac{x^2 + (y + a)^2}{x^2 + (y - a)^2} \qquad (8.45)$$

7KHERG\ VKDSHLVWKHQQH$ DQGVRPHRI WKHVHVKDSHVDUHVKRZQLQ)LJ) RU. 8, D ! WKHVKDSHLVZLWKLQ SHUFHQWRI D5DQNLQHRYDO)LJ WKLQHG f EXWRUVPDOO. 8, D WK ZDLWEHFRPHVSLQFKHGLQ DQGDILJXUHHJKWKDSHRFFXUVDW) RU. 8, D WKHVWUHDP EODWULJKWEHWZHHQWKHYRUWFHVDQGLVRDWVWRPRUHORVFLFXODUERG\ VKDSHV RQHVXUURXQGLQJ HDFK YRUWH[



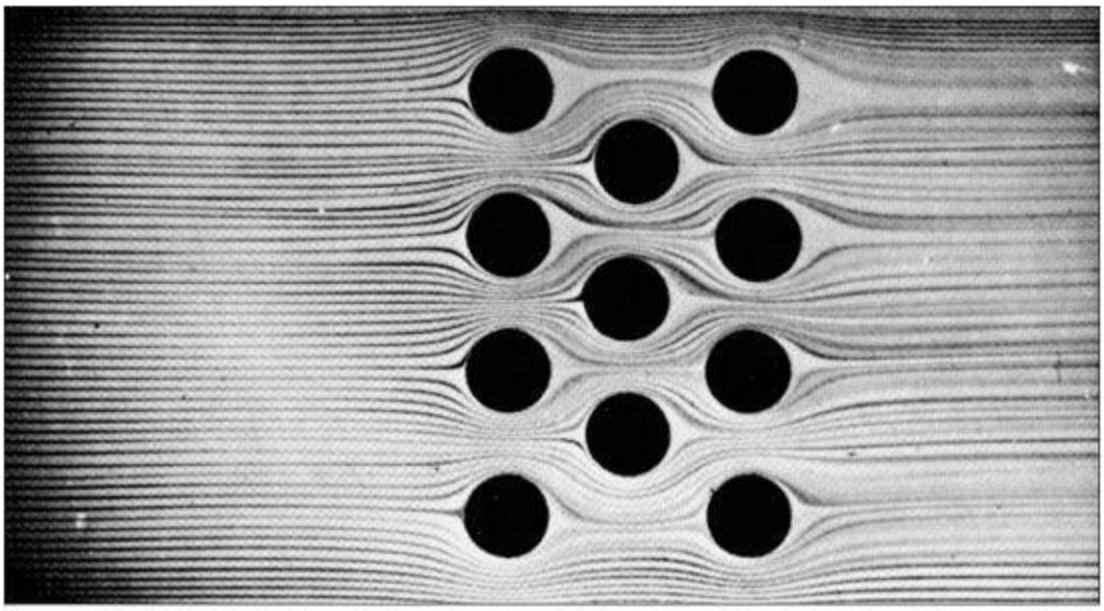
)LJ . HOYLQRYDOERG\ VKDSHVDVDIXQFWLRQRI WKHYRUWH[VWUHQJWK SDUDPHWHU K *U∞a* RXWVLGHWKHVHSDUDWUL[OLQHVRIWKHQRQ

$ FORVHGERG\ RI SUDFWLFDO DQ VKDSHFDQEHFRQVWUXFWHGE\ SURSHUVXSHUSRVLWLRQRI VRXUFHV VLQNV DQG YRUWLFHV 6HH WKH GDVKHG ZRUN LQ 5HV VR IRU IXUWKHU GHWDLOV \$ VXPPDU\ R HOHPHQWDU\ SRWHQWLDOIORZ VLVJLYHQLQ7DEOH

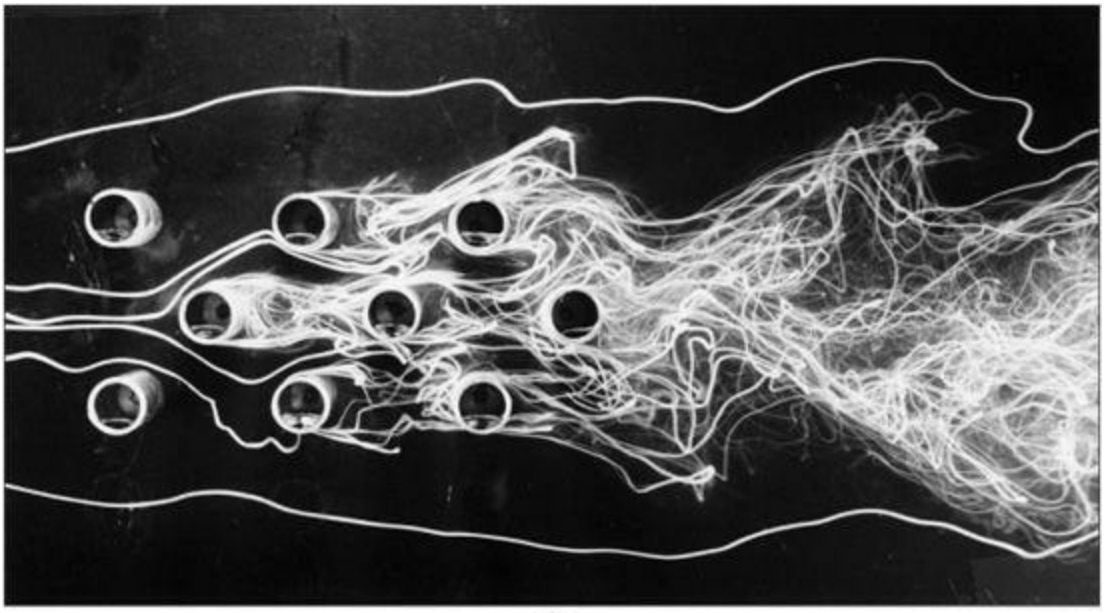
3RWHQWLDO)ORZ \$QDORJV

)RUFRPSUHVVLEOHIORZ JHRPHWULHV RQHFDQUHVRUWRRWKHUPHWKRGVWKDQVXSHUSRVLWLRQR VRXUFHV VLQNV DQG YRUWLFHV \$ YDULHWRI GHYLFHVVLPXODWHVRXUQRQ/DSODFH¶VHTXDWLRQ) URP VR +HOH6KDZ > @GHYHORSHGDWHFKQLTXHZKHUHE\ ODPLQDUIORZ EHWZHHQYHU FORVHO VSDFHG SDUDOOHO SODWHV VLPXODWHG SRWHQWLDO IORZ ZKHQ YLHZHG IURP DERYH WKH SODWH 2EVWDFWRQV VLPXODWHG E\ VKDSHV DQG G\HV WUHSUHVHQWWKH VWUHDPOLQHV 7KH +HOH6KD DSSDUDWXVPDNHVDQH[FHOOHQWGHPRQVWUDWRU GHPRQVWUDWLRQ SRWHQWLDO IORZ > SS ± ± @)LJXUH DLOOXVWUDWHV+HOH6KDZ SRWHQWLDO IORZ WKURXJK DQDUUD\ RI F\OLQGHUV DIORZ SDWWHUQ WKZRXOGEHGLIILFXOWDQDO]HXVLQJ / DSODFH¶VHTXDWLRQ +RZHYHUEHDXVXOWLV DUUD SDWWHUQP DEH LWLV QRWD JRRG DSSUR[LPDWLRQ WR UHDO ODPLQDU YLVFRXV DUUD IORZ)LJXUH E VKRZ

H[SHULPHQWDOVWXGLHVRQDVLPLODUVLWXDWLRQDVLPLODUWRWKH
LQWHUDFWLQJ ZDNHVRI WKHUHDOIORZ)LJ E FDXVHLQWHUIHULQJ GQGWDQYHUVHPRWLRQ QRWWKH
VPRRWKVWUHDPLQJ SDWWHUQ RI WKH SRWHQWLDOIORZ PRGHO)LJ D 7KH PRUDOLV WKDWWKLV LV D
LQWHUQDOIORZ ZLWKPXOWSOHERGLHVDQG WKHUHRUH QRWDJRRGFDQGLGDWHIRUDUHDOWLF SRWHQWLDOIORZ
PRGHO



(*a*)



(*b*)

) LJ) ORZ SDWWDVWUHDPLQHUDUUD\ RI FLOLQGHUV *a* SRWHQWLDOIORZ PRGHOXVLQJ WKH + HO
6KDZ DSSDUDWXV(*TQ Education and Training Ltd.)* *b* H[SHULPHQWDOVWUHDPLQHDQDO\VLV RI DFWXD
VWUHDPLQHUDUUD\ IORZ DW5H *D* § *(From Ref. 36, Courtesy of Jack Hoyt, with the permissio*
of the American Society of Mechanical Engineers.)

7DEOH 6XPPDU\ RI 3DQH,QFRP SUHVVLEOH3RWHQWLDO) ORZV

Type of flow	Potential functions		Remarks
Stream in U	$\psi = Uy$	$\phi = Ux$	See Fig. 8.3*a*
Line source ($m > 0$) or sink ($m < 0$)	$\psi = m\theta$	$\phi = m \ln r$	See Fig. 8.3*b*
Line vortex	$\psi = -K \ln r$	$\phi = K\theta$	See Fig. 8.3*c*
Half-body	$\psi = Ur \sin\theta + m\theta$		
	$\phi = Ur \cos\theta + m \ln r$		See Fig. 8.9
Doublet	$\psi = \dfrac{-\lambda \sin\theta}{r}$	$\phi = \dfrac{\lambda \cos\theta}{r}$	See Fig. 8.12
Rankine oval	$\psi = Ur \sin\theta + m(\theta_1 - \theta_2)$		See Fig. 8.13
Cylinder with circulation	$\psi = U \sin\theta\left(r - \dfrac{a^2}{r}\right) - K \ln \dfrac{r}{a}$		See Fig. 8.14

2VKHUIORZ PDSSLQJ VWUHDPOLQHV DUH GLVFXVVHG LQ 5H (OHFWURPDJQHWF ILHOGV DOVR VDWLV/DSODFH¶V HTXDWLRQ ZLWK YRQOLJH DQDORJRXV VR YHORFLW SRWHQWLDO DQG FXUUHQWV DQDORJRXV VWUHDPOLQHV $WRQHWLPH FRPPHUFLDOODQDORJ ILHGSORWWHUZHUHDYDLODEOH XVLQJ WKLQFRQGXFWLQJ SDSHFXWWRXWKHVKDSHRI WKHIORZ JHRPHWU 3RWHQWLDOOLQHV YRQJUHFRUXV ZHUHSORWWHG E\ SURELQJ WK SDSHU ZLWK D SRWHQWLRPHWU SRLQW + DQG VNHWFKLQJ ³FXUYLOLQHDU VTXDUH WHFKQLTXHV ZHUH DOV SRSXODU 7KH DYLODELOLW DQG WKH VLPSOLFLW RI FRPSXWU SRWHQWLDOIORZ PHWKRGV> WR @KDYH PDGH DQDORJ PRGHOV RE\RODWH

(; $0 3/ (

$. HOYLQRYDOIURP) LJ KDV. 8, D &RPSXWH WKH YHORFLW DW WKH VRS VKRXOGHU RI WK RYDOLQ WHUPV RI 8,

6RO XWLRQ

: H PXVW ORFDWH WKH VKRXOGHU\ K IURP (T IRU$ DQG WKHQ FRPSXWH WKH YHORFLW E\ GLIIHUHQWLDWLQ V DQG\ KDQG[(T EHFRPHV

$$\frac{h}{a} = \frac{K}{U_\infty a} \ln \frac{h/a + 1}{h/a - 1}$$

: LWK. 8, D DQG WKHLQLWLDOJXHVVK D§ IURP) LJ ZHLWHUDWH DQG ILQG WKH ORFDWLR K D

%\ LQVSHFWLQ% DWWKH VKRXOGHUEHFDXVH WKH VWUHDPOLQHV KRU]RQWDO 7KHUHRUH WKH VKRXOGH YHORFLW LV IURP (T

$$u\Big|_{y=h} = \frac{\partial \psi}{\partial y}\Big|_{y=h} = U_\infty + \frac{K}{h-a} - \frac{K}{h+a}$$

,QWURGXFLQJ . 8, DDQG K D ZHREWDLQ

$$u_{\text{shoulder}} = U_\infty(1.0 + 1.84 - 0.39) = 2.45U_\infty \qquad \textit{Ans.}$$

%HFDXVH WKH DUH VKRUWZ DLWHG FRPSDUHG ZLWK D FLUFXODU FRQGHU DOWH . HOLQ RYDOV KDV
VKRXOGHUYHORFLWJUHDWHUWKDQWKHFRQGHUUHVXOW 8, IURP (T

2 WKHU3 ODQH3 RWHQWLDO) ORZV

5HHUHQFHV VR WSHDWP DQ RWKHUSRWHQWLDOIORZVRI LQWHUHVWLQDGGLWRQWRWKHFDVHVSUHVHQWHGL
6HFV DQG ,QSULQFLSOH DQ SODQHSRWHQWLDOIORZ FDQEHVROYHG E\ WKHPHWKRG RI FRQIRUPD
PDSSLQJ E\ XVLQJ WKHFRPSOH[YDULDEOH

] [L\ L

,WWXUQV RXWWKDWDQ DUELWUDU DQDOWF IXQFWRQ RI WKLV FRP SOH[YDULDEOH] KDV WKH UHP DUNDEO
SURSHUWVWKDWERWKLWVUHDODQG LWVLP DJLQDU SDUWDUH VROXWLRQV RI / DSODFH¶V HTXDWLRQ ,I

$$f(z) = f(x + iy) = f_1(x, y) + i f_2(x, y)$$

then

$$\frac{\partial^2 f_1}{\partial x^2} + \frac{\partial^2 f_1}{\partial y^2} = 0 = \frac{\partial^2 f_2}{\partial x^2} + \frac{\partial^2 f_2}{\partial y^2} \tag{8.46}$$

: HVKDOODWLJQWKHSURRI RI WKLVDV3URE : (YHQPRUHUHP DUNDEOHLI \RX KDYHQHYHUVHQI
EHRUHLVWKDWDQHVRI FRQXDQW ZLOOEHHYHUZKHUHSHUSHQGLFXODUWRDQHVRI FRQXDQW

$$\left(\frac{dy}{dx}\right)_{f_1 = C} = -\frac{1}{(dy/dx)_{f_2 = C}} \tag{8.47}$$

7KLV LV WKHIRUWRODO DUELWUDU I] DV ORQJ DV WKLV IXQFWRQ LV DQDOWF WKDWLV LWP XWKDYHI
XQLTXHGHULYDWYHG G] DWHYHU SRLQWQWKHUHJLRQ
7KHQHWUHXOWR (TV DQG LV WKDWWKHIXQFWRQ I DQG I FDQEHLQWHUSUHWG WR EHWK
SRWHQWLDOOLQHV DQG VWUHDP OLQHV RI DQLQYLVFLG ORZ %\ ORQJ FXWWP ZHOHWWKHUHDSDUWR I] EHWK
YHORFLWSRWHQWLDODQG WKHLP DJLQDU SDUWEHWKHVWUHDP IXQFWLRQ

$$f(z) = \phi(x, y) + i\psi(x, y) \tag{8.48}$$

: HWH YDURXV IXQFWLRQV I] DQG VHHZKHUDQ LQWHUHVWLQJ ORZ SDWWLQUHXOW 2 I FRXUH P R\
RI WKHP KDYHDOHG\ EHHQIRXQG DQGZHVLP SO UHSRUWRQWKHP KHUH
: H VKDOO QRWJR LQR WKH GHWDLOV EXWWKHUH DUH H[FHOOHQWWDWHQW RI WKLV FRP SOH[YDULDEO
WKFKQLTXHQERWKDQLQWRGXFWRU > @DQG DP RUHDGYDQFHG> @DYHO 7KHPHWKRGLVHWLP SRUDQ
QRZ EHFDXVHRI WKHSRSXODULWRI FRP SXWUWKFKQLTXHV
$ VDVLP SOHH[P SOH FRQVLGHUWKHOQHDUIXQFWLRQ

I] 8,] 8, [L8, \

,WVROGRZ VIURP (T WKDWË 8, [DQG\$ 8, \ ZKLFK ZHUHFDOOIURP (T UHSUHVHQ
D XQLIRUP VWUHDP LQ WKH[GLUHFWLRQ 2QFH \RX JHWXVHG WR WKH FRPSOH[YDULDEOH WKH VROXWR
SUDFWLFDOO IDOOVLQ\RXUODS

7R ILQG WKH YHORFLWLHV \RX PD\ HLWKHU VHSDUDWHË DQG\$ IURP I] DQG GLIIHUHQWDWH R
GLIIHUHQWDWH GLUHFWO

$$\frac{df}{dz} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} = -i\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial y} = u - iv \qquad (8.49)$$

7KXV WKH UHDO SDUWRI G] G] HTXDOVX [\ DQG WKH LPDJLQDU SDUWHTXDOV í%[\ 7R JHW
SUDFWLFDOUHVXOWWKH GHULYDWLYHG G] PXVWH[LWDQG EH XQLTXH KHQFH WKH UHTXLUHPHQWWKDW EH D
DQDOWLF IXQFWLRQ)RU] 8,] G] G] 8, X VLQFHLWVUHDO DQG% DVH[SHFWHG

6RPHWLPHVLWVFRQYHQLHQWRXVHWKHSRODUFRRUGLQDWIRUP RI WKHFRPSOH[YDULDEOH

] [L\ UH UFRV LUVLQ

ZKHUH

$$r = (x^2 + y^2)^{1/2} \qquad \theta = \tan^{-1}\frac{y}{x}$$

7KLVIRUP LVHVSHFLDOO FRQYHQLHQWZKHQSRZHUVRI] RFFXU

8QLIRUP 6WUHDP DWDQ\$QJOHRI \$WWDFN

\$OOWKH HOHPHQWDU SDQH IGRZ V RI 6HF KDYH D FRPSOH[YDULDEOHIRUPXODWRQ 7KH XQLRUP
VWUHDP 8, DWDQDQJOHRI DWWDFN. KDVWKHFRPSOH[SRWHQWLDO

$$f(z) = U_\infty z e^{-i\alpha} \qquad (8.50)$$

&RPSDUHWKLVIRUP ZLWK(T

/ LQH6RXUFHDWD3RLQW

&RQVLGHUD OLQH VRXUFH RI VWUHQJWKP SDFH GRI WKHRULJLQDWD SRLQW [L\ ,WVFRPSOH
SRWHQWLDOLV

$$f(z) = m \ln (z - z_0) \qquad (8.51)$$

7KLVFDQEHFRPSDUHGZLWK(T ZKLFKLVYDOLGRQO IRUWKHVRXUFHDWWKHRULJLQ)RUDOO
VLQN WKHVWUHQJWKP LVQHJDWLYH

/ LQH9 RUWH[DWD3RLQW

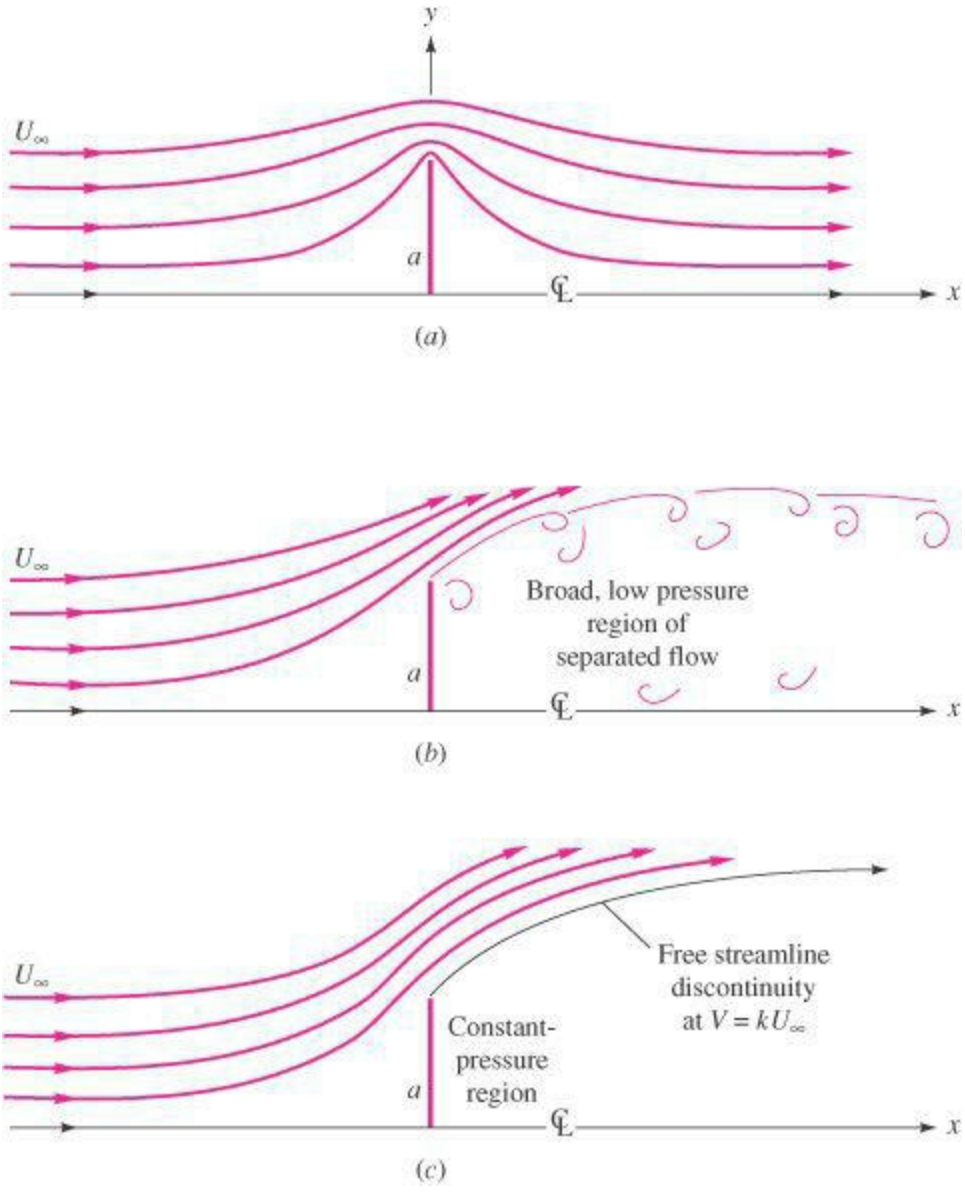
: HFDQVROYHIRU$VRGHWUPLQHWKHVXWDPQHV

(TXDWRQ LVSORWWGLQ) LJ D UHYHDOQJ DGRXEO\ VPPHWUF SDWWHUQ RI VXWDPQHVWKD DSSURDFK YHU FORVHO VR WKHSDWWDQG WKHQGHFWSDQG RYHU ZLWK YHU KLJK YHORFLWLHV DQG ORZ SUHVXUHVQHDUWKHSDWWVV

7KHYHORFLW%, DORQJ WKHSDWWVXUFHLVIRXQGE\ FRPSXWQJ G] GU IURP (T DQGLVRODWQ WKHLPDJLQDU SDUW

6RPHYDOXHVRI VXUFHYHORFLWFDQEHXDWGDVIROORZV

7KHRULJLQLVDWWDQDWRQSRLQWWKHQWKHYHORFLWJURZVOQHDU DWLWWDQGYHU UDSLGO QHDUWKVS ZLWKEORWKYHORFLWDQGDFFHOHUDWRQEHLQJ LQILQLWHDWWKHVS



) LJ 6WUHDPOLQHV LQ XSSHU KDOI SODQH IRU IORZ QRUPDO WR D IODW SODWH RI KHLJKW *a* *a* FRQWUDVW RXV SRWHQWLDO IORZ WKHRU (T *b* DFWXDO PHDVXUHG IORZ SDWWHUQ *c* GLVFRQWLQRXV SRWHQWLDO IORZ WLWK *k* §

$ V \RX PLJKW JXHVV) LJ D LV QRW UHDOLVWLF ,Q D UHDO IORZ WKH VKDUS VDOLHQW HGJH FDXVHV VHSDUDWLRQ DQG DEURDG ORZ SUHVVXUH ZDNH IRUPV LQ WKH OHH DV LQ) LJ E ,QVWHDG RI EHLQJ]HUR WKH GUDJ FRHILFLHQW LV YHU\ ODUJH & § IURP 7DEOH

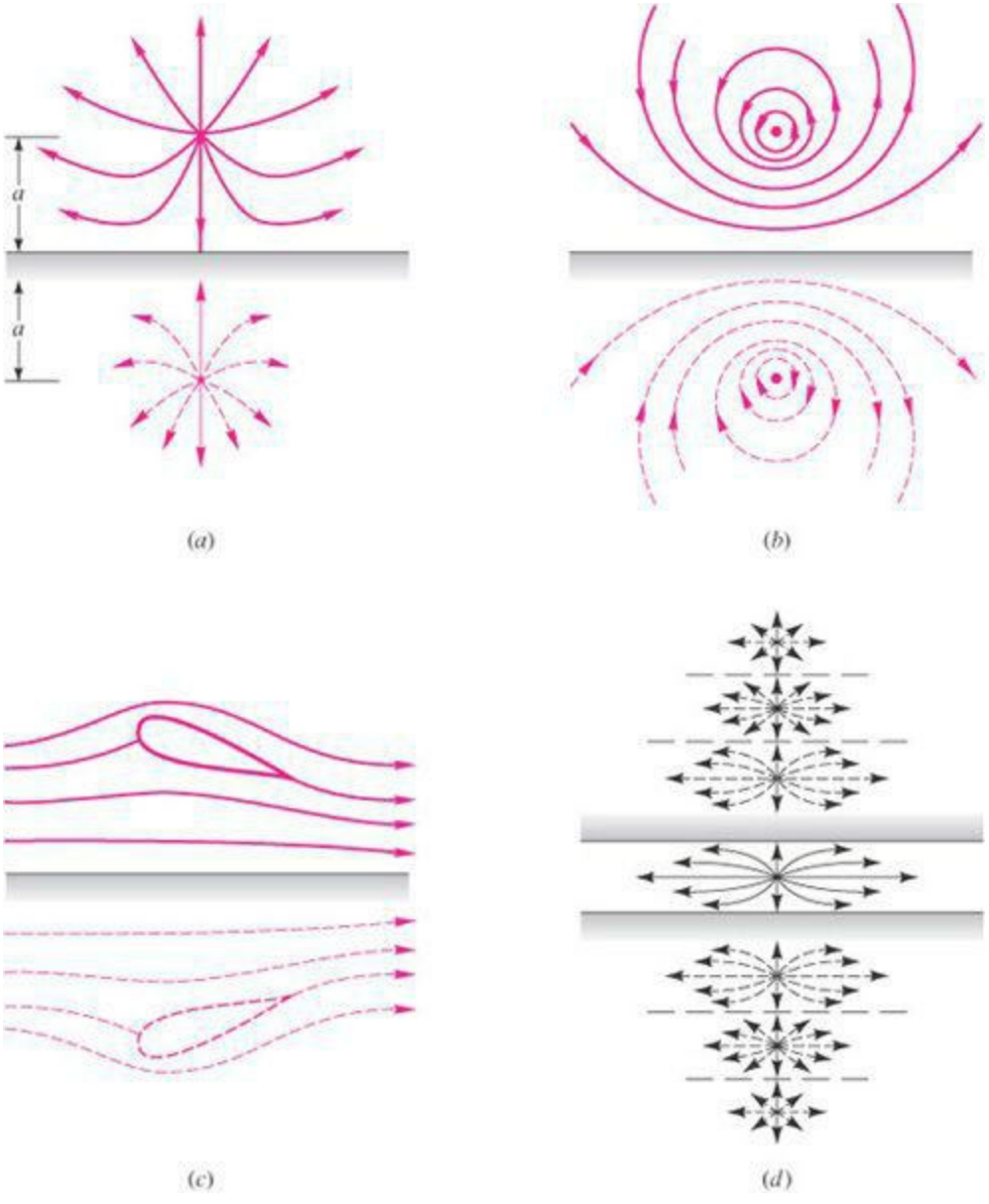
$ GLVFRQWLQRXV SRWHQWLDO IORZ WKHRU WKDW DFFRXQWV IRU IORZ VHSDUDWLRQ Z DV GHYLVHG E + HOPKROW] DQG . LUFKKRII 7KLV IUHH VWUHDPOLQH SRWHQWLDO IORZ LQ) LJ F Z LV WKH VWUHDPOLQH WKDW EUHDNV DZ D IURP WKH VS KDYLQJ D FRQVWDQW Y HORFLW 9 N8) URP %HUQRXOOL HTXDWLRQ WKH SUHVVXUH LQ WKH GHDG Z DWHU UHJLRQ EHKLQG WKH SODWH Z LOO HTXDO & § VR P DWK WKH SUHVVXUH DFRVV WKH IUHH VWUHDPOLQH) RUN WKLV + HOPKROW] . LUFKKRII WKHRU SUHGLFWV S í

!8 DQG DQ DYHUDJH SUHVVXUH RQ WKH IURQW S !8 JLYLQJ DQ RYHUDOO GUDJ FRHILFLHQW LQDJUHHPHQW Z LWK H[SHULPHQW+RZHYHU WKH FRHILFLHQW LV DSURIRXQGO QDQ PXVW EH WKQG WR H[SHULPHQWDO GDW VR IUHH VWUHDPOLQH WKHRU FDQ EH FRQVLGHUHG RQO D TXDOLWDWLH VXFFHVV) RU IXUWKHUGHWDLOV VHH 5H 6HF

,P DJHV

7KH SUHYLRXV VROXWLRQV KDYH DOO EHHQ IRU XQERXQGHG IORZV VXFK DV DFFXPXODWLRQ LQ D EURDG H[SDQVH RI XQLRUP VWHDP LQJ)LJ D +RZHYHU PDQ SUDFWLFDO SUREOHPV LQYROYH DQHDUE\ ULJLG ERXQGDU FRQWDLQLQJ WKH IORZ VXFK DV JURXQGZDWHU QHDUWKHERWWRP RI DGDP DQDLUFRLOHDUWKH JURXQG VLPXODWLQJ OLQHVWRQH ZLQGVWRQHZLWK QDUURZ ZDOOV ,QVXFK FDVHV WKHEDVLF XQERXQGHG SRWHQWLDO IORZ VROXWLRQV FDQ E PRGLILHGIRUZDOOHIIHFWVE\ WKHPHWKRGRI LPDJHV

&RQVLGHUDQQH VRXUFHSODFHGDGLVWDQFHDIURP DZDOODVLQ)LJ D 7RFUHDWHWKHGHVLUH ZDOODQLPDJHVRXUFHRI LGHQWLFDOVWUHQJWKLVSODFHGWKHVDPHGLVWDQFHEHKLQG WKHZDOO%\ V\PPHWU WKHVWR VRXUFHVFUHDWHSDQHVXUIDFHVWUHPOLQHEHWZHHQWKHP ZKLFKLVWDNHQWREHWKHZDOO



)LJ &RQWDLQLQJ ZDOOV FDQ EH FUHDWHG E\ LPDJH IORZV *a* VRXUFH QHDU D ZDOOZLW LGHQWLFDOLPDJHVRXUFH *b* YRUWH[QHDUD ZDOOZLWKLPDJHYRUWH[RI RSSRVLWH VHQVH *c* DLUIRLO JURXQGHIIHFWZLWKLPDJHDLUIRLORI RSSRVLWHFLUFXODWLRQ *d* VRXUFHEHWZHHQWZR ZDOOVUHTXLUQ DQLQLQLWURZ RI LPDJHV

, Q) LJ E D YRUWH[QHDUD Z DOOUHTXLUHV DQ LPDJH YRUWH[WKH VDPH GLVWDQFH EHORZ EXW
RSSRVLWH URWDWRQ : HKDYH VKDGHG LQWKH ZDOO EXWR FRXUHWKH SDWWHUQ FRXOG DOR EH LQWHUSUHWHG D
WKH IORZ QHDUD YRUWH[SDLULQDQXQEHRXQGHG IOXLG
,Q) LJ F DQ DLUROLQD XQIRUP VWHDP QHDUWKH JURXQG LV FUHDWHG E\ DQ LPDJH DLUROL EHORZ
WKH JURXQG R RSSRVLWH FLFXODWRQ DQG GLW7KLV ORRNV HDX EXWDFWXDOO LWLV QRWEHFDXVHWKH DLUROI
DUH VR FORVH VRJHWHU WKDWWKH LQWHUDFWDQG GLVWRUWHDFK RWKHU\V VKDSHV $ UXOH R WKXPE LV WKD
QRQOHJOLEOH VKDSH GLVWRUWLRQ RFFXUV LI WKHERG VKDSHV ZLWKLQ WZR FKRUG OHQJWKV R WKH ZDOO 7R
HDP LQDWHGLVWRUWLRQ DZKROHVHULHV R 3FRUUHFWLYH LPDJHV PXWEHDGGHGWKHIORZ WRUHFDSWXUHWK
VKDSHR WKHRULJLQDOLVRDWHG DLUROI 5HHUHQFH 6HF KDVDJRRGGLVFXVLRQR WKLVSURFHGXU
ZKLFKXVXDOO UHTXLUHVFRP SXWXUVPP DWRQ WKHPXOWSOHLPDJHVQHGG

) LJXUH GVKRZ VDVRXUFHFRQWDLQHGEHWZHHQWZR ZDOV 2QHZDOOUHTXLUHGRQ RQHLPDJ
LQ) LJ D EXWZR ZDOVUHTXLUHQLQILQLWHUD R LPDJHVRXUFHVDERYHDQGEHORZ WKHGHVLUH
SDWWHUQ DVVKRZQ 8VXDOO FRP SXWXUVPP DWRQ LVQHHVWDU EXWWRPHWP HVDFORVHGIRUP VXPPDWR
FDQEHDFKLHYHG DVLQWKHILQLQLWHYRUWH[URZ R(T

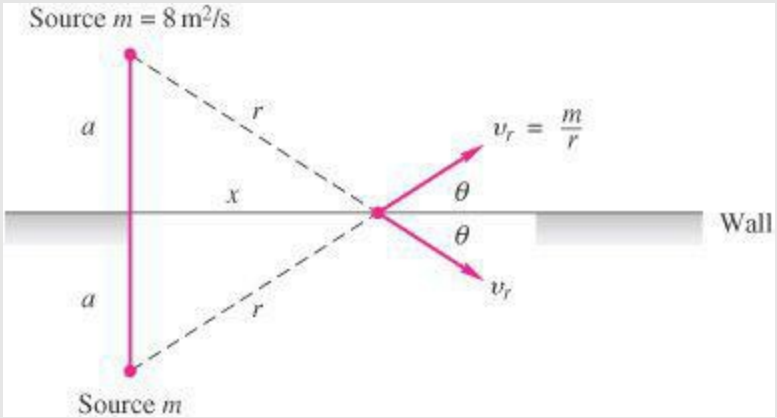
(; $0 3/ (

) RUWKHVRXUFHQHDUD ZDOODVLQ) LJ D WKHZDOOYHORFLWVLV]HUR EHWZHHQWKHVRXUFHV UHVVR
DPD[LPXP PRYLQJ RXWDORQJ WKH ZDOO DQG WKHQ GURSV WR]HUR IDUIURP WKH VRXUFHV ,I WKH VRXUF
WWHQWWKLV P V KRZ IDUIURP WKH ZDOOVKRXOG WKH VRXUFHEHWRHQXUHWKDWWKH PD[LPXP YHORFLW
DORQJ WKHZDOOZLOOEH P V"

6ROXWLRQ

$WDQ SRLQW/ DORQJ WKHZDOO DVLQ) LJ (HDFK VRXUFHLQGXFHVDUDGLDORXWZDUG YHORFLW%U
P U ZKLFKKDVDFRP SRQHQW%U FRV DORQJ WKHZDOO7KHWRWDOZDOOYHORFLWLVWKXV

(



; Z DOO Y U FRV

) URP WKHJHRPHWU R) LJ (U [D DQGFRV [U7KHQWKHWRWDOZDOOYHORFLWFD
EHH[SUHVVHGDV

$$u = \frac{2mx}{x^2 + a^2}$$

7KLV LV]HUR DW[DQG DW["""" 7R ILQG WKH PD[LPXP YHORFLW GLIIHUHQWLDWH DQG VHW WHUP]HUR

$$\frac{du}{dx} = 0 \quad \text{at} \quad x = a \qquad \text{and} \qquad u_{\max} = \frac{m}{a}$$

: H KDYH RPLWWHG D ELW RI DOJHEUD LQ JLYLQJ WKH VH UHVXOWV) RU WKH JLYHQ VRXUFH VWUHQJWK DQG PD[LPXP YHORFLW WKH SURSHUGLWDQFH DLV

$$a = \frac{m}{u_{\max}} = \frac{8 \text{ m}^2/\text{s}}{5 \text{ m/s}} = 1.6 \text{ m} \qquad \textit{Ans.}$$

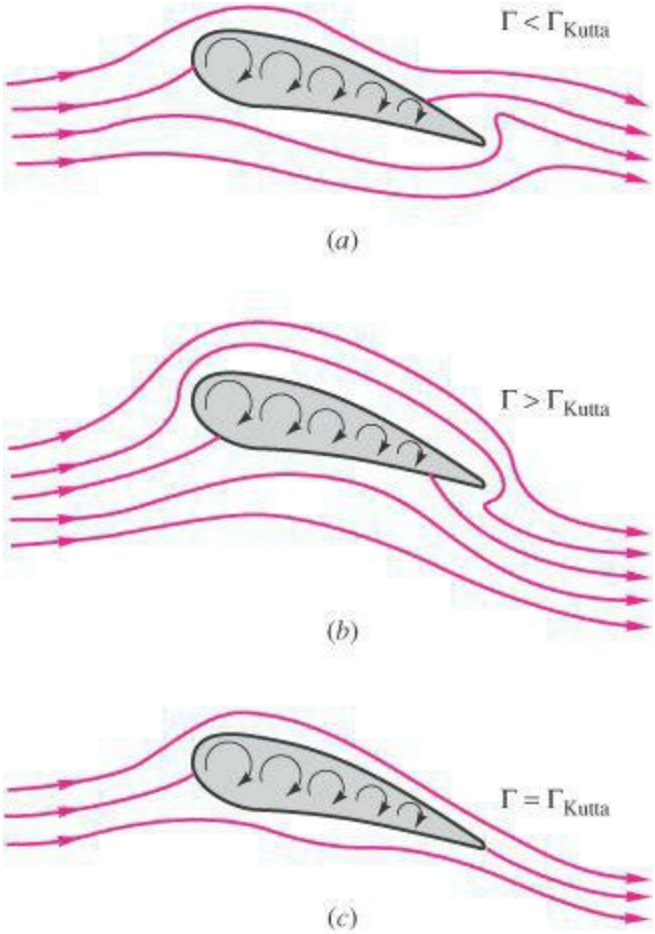
) RU[! D WKHUH LV DQ DGYHUVH SUHVVXUH JUDGLHQW DORQJ WKH ZDOO DQG ERXQGDU\ OD\HU WKRU VKRXOG EH XVHG WR SUHGLFW VHSDUDWLRQ

$ LUIRLO 7KHRU\

$ V PHQWLRQHG LQ FRQQHFWLRQ ZLWK WKH . XWWD - RXNRZVNL OLIW WKHRUHP (T WKH SUREOHP LQ DLUIRLO WKHRU\ LV WR GHWHUPLQH WKH HQHUJHWLF FXODWLRQ* DV D IXQFWLRQ RI DLUIRLO VKDSH DQG IUHH VWUHDP DQJOH RI DWWDFN.

7KH . XWWD &RQGLWLRQ

(YHQ LI WKH DLUIRLO VKDSH DQG IUHH VWUHDP DQJOH RI DWWDFN DUH VSHFLILHG WKH SRVLWLRQ WKHRU\ VROXWLRQ LV QRQXQLTXH $Q LQILQLWH QXPEHU RI VROXWLRQV FDQ EH IRXQG FRUUHVSRQGLQJ WR GLIIHUHQW VSHHGV RI FLUFXODWLRQ*) RXU[DP SOHV RI WKLV QRQXQLTXHQHVV ZHUH VKRZQ IRU WKH FDVH RQ) LJ 7KH VDPH LV WUXH RI WKH DLUIRLO DQG) LJ VKRZV WKUHH DWWDFKHG DFFHSWDEOH³ VROXWLRQV! WR I JLYHQ DLUIRLO IRUVP DOO) LJ D ODUJH) LJ E DQG PHGLXP) LJ F QHWFLUFXODWLRQ <RX FDQ JXHVV ZKLFK FDVH EHWWLP XODWV DUHDO DLUIRLO IURP WKH HDUOLHU GLVFXVLRQ RI WDQLHQWLDO GHYHORSPHQWQ) LJ ,WLV WKH FDVH) LJ F ZKHUH WKH XSSHUDQG ORZHU VPHHWDQG GHDY WKH WDLOLQJ HGJH VP RRWKO ,I WKH WDLOLQJ HGJH LV URXQGHG VOJKWO WKHUH ZLOO EH D WDJQDWRQ SRLQ WKHUH ,I WKH WDLOLQJ HGJH LV VKDUS DSSUR[LPDWQJ PRWDLURLOGHVLJQ/ WKH XSSHU DQG ORZHU VXUIDFH IORZ YHORFLWHVZLOOEHHTXDOO\WKH PHHWDQG GHDYHWKH DLUIRLO



) LJ 7KH . XWWD FRQGLWLRQ SURSHUO VLP XODWHV WKH IORZ DERXW DQ DLUIRLO *a* WKDW OHDG FLUFXODWLRQ WDJQDWLRQ SRLQWRQ UHDUXSSHUVXUIDFH *b* WKDQ PXFK VWDJQDWLRQ SRLQWRQ UHDUORZHU VXUIDFH *c* NXWWDUJKW. XWWD FRQGLWLRQ UHTXLUHV VP RRWK IORZ DWWDLQHG JHGJH

7KLV VWDWHPHQWRI WKH SK\VLFDO SURSHUYDOXHRI * LVJHQHUDO DWWULEXWHG WR: 0 . XWWD KHQF WKHQDPH. XWWD FRQGLWLRQ DOWKRXJK VRP HWH[WVJLYH FUHGLWRR-RXNRZ VNL DQG RU&KDSOJLQ \$OODLUR WKHRUHV XVH WKH. XWWD FRQGLWRQ ZKLFK LV LQJRRG DJUHHPHQWZLWK H[SHULPHQW,WVXLQ/RXWKDWWK FRUUHFWFRFLUFXODWRQ* . XWWG GHSHQGVRQIORZ YHORFLW DQJOHRI DWWDFN DQG DLUIRLOVKDSH

) ODW3 ODW\$ LUIRLO9 RUW[6KHHW7 KHRU\

7KHIODWSODWLV WKH VLP SOHWDLUIRLO KDYLQJ QR WKLFNQHVU³ VKDSH! EXWHYHQLW WKH RU\ LVQRW VLP SOH 7KHSUREOHP FDQEHVROYHG E\ DFRP SOH[YDULDEOHP DSSLQJ > S @EXWKHUHZH VKDOOXV DYRUW[VKHHWDSSURDFK) LJXUH DVKRZV DIODWSODWRI OHQJWK& VLP XODWG E\ DYRUW[VKHHWR YDULDEOH VXUHQJWK [7KHIUHH VWUHDP 8, LV DWDQ DQJOHRI DWWDFN. ZLWK UHVSHFWWR WKH SODWFKRUG OQH

7RPDNHWKHOLW7XS! ZLWKIORZ IURP OHWWRRULJKWDV VKRZQ ZH VSHFL\ KHUHWKDWWKHFLUFXODWRR LVSRVLWLYHFORFNZLVH 5HFDOOIURP) LJ F WKDWWKHUHLV DMXPS LQWDQJHQWLDOYHORFLW DFURVV DVKHHW HTXDOWR WKHORFDOWUHQJWK

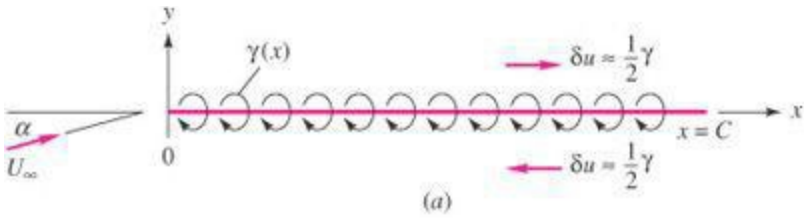
X X [

,I Z HRPLWKHIUHHVWUHDP VKHVKHHWKRXGFDXVHDULJKWZDUGIORZ $u = +\frac{1}{2}\gamma$ RQWKHXSSHUVXUIDFH DQGDQHTXDOODUJRSSRVLWHGIORZ RQWKHORZHUVXUIDFH DVVKRZQLQ) LJ D 7KH. XW FRQGLWLRQIRUWKLVVKDSSWDLOLQJHGJHUHTXLUHVWKDWWKLVYHORFLWGLIIHUHQFHYDQLVKDWWKHWDLOLQJHGJI VRNHHSVWKHH[LWIORZ VPRRWKDQGSDUDOOHO

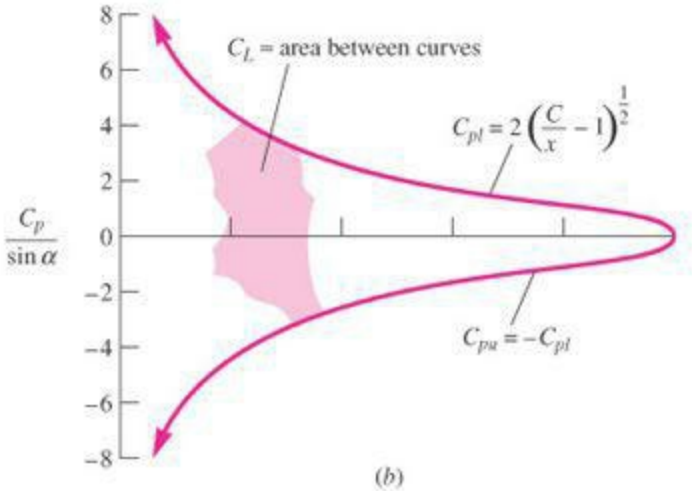
$$\gamma(C) = 0 \tag{8.57}$$

7KH SURSHUVRXUQP XWWDLW\ WKLV FRQGLWLRQ DIWUZKLFK WKH VRXDOOLQWFDQ EH FRPSXWG E VXPPLQJ WKHVKHHWVWUHQJWKRYHUWKHZKROHDLURLO) URP (T IRUDIRLORI GHSWKE

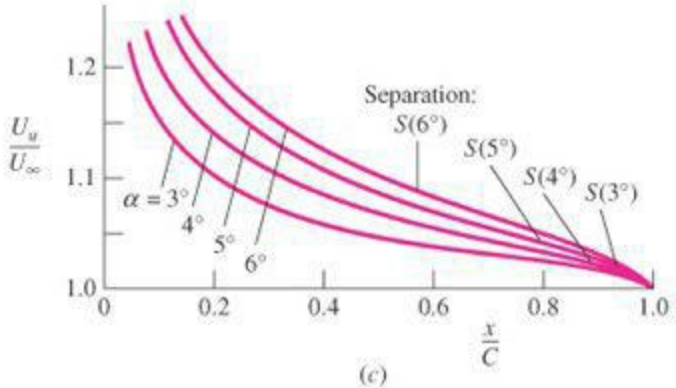
$$L = \rho U_\infty b\Gamma \qquad \Gamma = \int_0^C \gamma(x)\,dx \tag{8.58}$$



(a)



(b)



(c)

) LJ 9RUWLFLW\ VKHHWWRXQGIRUWKHIODWSODWHDLUIRLO *a* VKHHWJHRPHWU *b* WKHRUHWLFDO SUHVVXUH FRHIILFLHQWRQ XSSHU DQG ORZHU VXUIDFHV *c* XSSHUVXUIDFH YHORFLWY ZLWK GLPLQLVKHSDUDWRQSRLQWS

$ QDOWHUQDWLYH ZD\ WR FRPSXWH OLIWLV IURP WKH GLPHQVLRQOHVV SUHVVXUH FRHILFLHQW&S RQ WK

XSSHUDQG OBZ HUVXUIDFHV

$$C_{p_{u,l}} = \frac{p_{u,l} - p_\infty}{\frac{1}{2}\rho U_\infty^2} = 1 - \frac{U_{u,l}^2}{U_\infty^2} \tag{8.59}$$

Z KHUH WKH ODWWH[SUHVVLRQ IRO OBZ V IURP %HUQRXOOL¶V HTXDWLRQ 7KH VXUIDFH YHORFLWLHV VTXDUHG L JLYHQ E\ FRP ELQLQJ WKH XQIRUP VWUHDP DQG WKH YRUWH[VKHHW YHORFLWLHV FRP SRQHQWV IURP) LJ D

$$\begin{aligned} U_{u,l}^2 &= (U_\infty \cos\alpha \pm \delta u)^2 + (U_\infty \sin\alpha)^2 \\ &= U_\infty^2 \pm 2U_\infty \delta u \cos\alpha + \delta u^2 \approx U_\infty^2\left(1 \pm \frac{2\,\delta u}{U_\infty}\right) \end{aligned} \tag{8.60}$$

Z KHUH Z H KDYH P DGH WKH DSSUR[LP DWLRQV // X $\delta u \ll U_\infty$ DQG FRV. § LQ WKH ODWWH[SUHVVLRQ DWWP LQJ D VP DOO DQJOH RI DWWDFN (TXDWLRQV DQG FRP ELQH WR WKH ILLWWRUGH DSSUR[LP DWLRQ

$$C_{p_{u,l}} = \mp \frac{2\,\delta u}{U_\infty} = \mp \frac{\gamma}{U_\infty} \tag{8.61}$$

7KH OLWWRUFH LV WKH LQWJUDO RI WKH SUHVVXUH GLIIHUHQFH RYHU WKH OHQJWK RI WKH DLUIRLO DWXP LQ GHSWKE

$$L = \int_0^C (p_l - p_u) b\, dx$$

or

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 bC} = \int_0^1 (C_{p_l} - C_{p_u})\frac{dx}{C} = 2\int_0^1 \frac{\gamma}{U_\infty} d\left(\frac{x}{C}\right) \tag{8.62}$$

(TXDWLRQV DQG DUH HQWLHO HTXLYDOHQW Z LWK LQ WKH VP DOO DQJOH DSSUR[LP DWLRQV

7KH VKHHWWXUQHQJWK [LV FRP SXWHG IURP WKH UHTXLUHP HQW WKDW WKH QHWQRUP DO YHORFLW\ %[EH]HUR DW WKH VKHHW\ VLQFH WKH VKHHWU SUHVHQWV D VROG SDWK RU WXUEP XXUIDFH &RQVLGHUD VP DO SLHFH RI VKHHW G[OBFDWHG DW SRVLWLRQ[7KH YHORFLW\ % DW SRLQW/ RQ WKH VKHHWLV WKDW RI DLQLQLWLVLP DOOQH YRUW[RI VWUHQJWK G* í G[

$$dv\Big|_x = \frac{d\Gamma}{2\pi r|_{x_0 \to x}} = \frac{-\gamma\, dx}{2\pi\,(x_0 - x)}$$

7KHVRXOQRUP DO YHORFLW\ LQGXFHG E\ WKH HQWLUH VKHHW DW SRLQW/ LV WKXV

$$v_{\text{sheet}} = -\int_0^C \frac{-\gamma\, dx}{2\pi(x_0 - x)} \tag{8.63}$$

0 HDQZ KLOH IURP) LJ D WKH XQIRUP VWUHDP LQGXFHV D FRQWDQWQORUP DO YHORFLW\ DW HYHU SRLQWRQ WKH VKHHWJLYHQE\

Y VWUHDP 8’ VLQ.

6HWWLQJ WKH VXP RI %WKHW DQG %WWDP HTXDO WR]HUR JLYHV WKH LQWHJUDO HTXDWLRQ

$$\int_0^C \frac{\gamma\, dx}{x_0 - x} = 2\pi U_\infty \sin\alpha \qquad (8.64)$$

WR EH VROYHG IRU [VXEVWLWXWH WKH. XWWD FRQGLWLRQ & IURP (T $SSOLHG JK(T LV TXLWH IRUPLGDEOH DQG QRWRQD IRUEHJLQQHUV LQ IDFW WZ DV VROYHG GRQ DJR E\ XVLQJ LQWHJUDO IRUPXODV GHYHORSHG E\ 3RLVRQLQWKHQLQHWHHQWK FHQWXU 7KHVH WHWXUQHG WKWKH VDWLVILHV(T LV

$$\gamma(x) = 2U_\infty \sin\alpha\left(\frac{C}{x} - 1\right)^{1/2} \qquad (8.65)$$

) URP (T WKH VXUIDFH SUHVVXUH FRHILFLHQW DUH WKXV

$$C_{p_{u,l}} = \mp 2 \sin\alpha\left(\frac{C}{x} - 1\right)^{1/2} \qquad (8.66)$$

' HWDLOV RI WKH FDOFXODWRQV DUH JLYHQ LQ DGYDQFHG WH[WV>IRU[DPSOH &KDS @
7KH SUHVXUH FRHILFLHQW IURP (T DUH SORWWHG LQ) LJ E VKRZLQJ WKDWWKH XSSHU VXUIDFH KDV SUHVXUH FRQVWDQWO LQFUHDVLQJ ZLWK[WKDWLV DQ DGYHUVH JUDGLHQW7KH XSSHUVXUIDFH YHORFLW\ $\approx U_\infty + \delta u = U_\infty + \tfrac{1}{2}\gamma$ LVSORWWHGLQ) LJ F IRUYDULRXVDQJOHVRI DWWDFN $ERYH. ƒ WKH VKHWFRQWEXWRQ/ XLVDERXW SHUFHQWRI 8, VR WKDWWKH VP DOOGLVWXUEDQFH DXXP SWLRQLV YLRODWHG) LJXUH F DOR VKRZ V VHSDUDWLRQ SRLQW FRP SXWHG E\ 7KZDLWHV DQG (TV DQG 7KH SUHGLFWLRQ LV WKDWD IODWSODWH ZRXOG EH H[SHULHQFLQJ VLJQLILFDQW VHSDUDWLRQ WKH XSSHUVXUIDFH IRU. ! ƒ ZKLFK LV SSUR[LPDWO FRUUHFW

$$C_L = 2\int_0^1 \frac{\gamma}{U} d\left(\frac{x}{C}\right) = 4\sin\alpha \int_0^1 \left(\frac{C}{x} - 1\right)^{1/2} d\left(\frac{x}{C}\right) = 2\pi \sin\alpha \approx 2\pi\alpha \qquad (8.67)$$

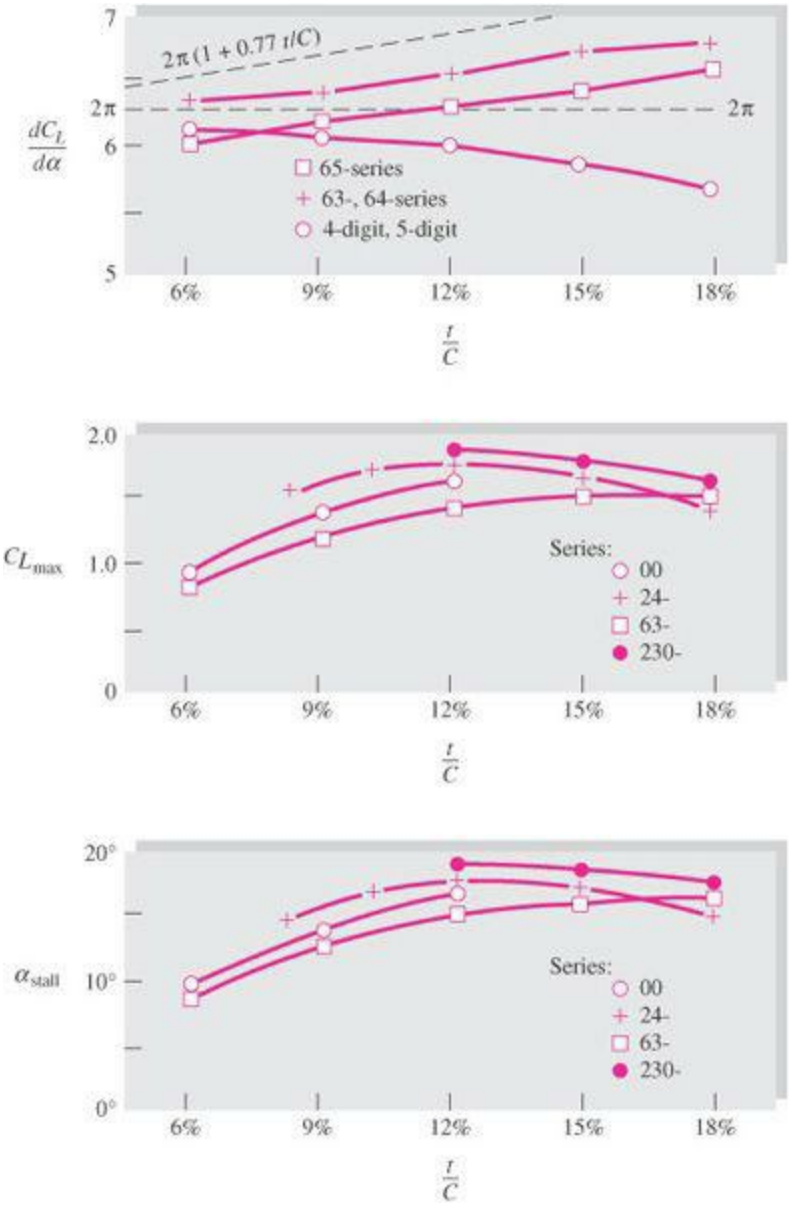
7KH OLWFRH ILFLHQW RI WKH DLUIRLO LV SURSRUWLRQDO WR WKH DUHD EHWZHHQ F_{SO} DQG F_{SX} LQ) LJ E IURP (T
7KLVLVDFODVVLF UHVXOWWKDWZDVGXHGWRHDUO(T ZLWKRXWSURRI
$SOR RI LQWHUHVWLV WKH PRPHQWFRHILFLHQWDERXWWKH OHDGLQJ HGJH /(RI WKH DLUIRLO WDNHQ D SRVLWLYH FRXQWFORFNZLVH

$$C_{M_{LE}} = \frac{M_{LE}}{\frac{1}{2}\rho U_\infty^2 b C^2} = \int_0^1 (C_{p_l} - C_{p_u}) \frac{x}{C} d\left(\frac{x}{C}\right) = \frac{\pi}{2}\sin\alpha = \frac{1}{4}C_L \qquad (8.68)$$

7KXV WKH FHQWUR SUHVVXUH &3 RUSRVLWLRQ RI WKH UHVXOWDQW WR UFH LV DWWKH RQH TXDUWUKRU SRLQW

$$\left(\frac{x}{C}\right)_{CP} = \frac{1}{4} \qquad (8.69)$$

7KLV WKHRUHWLFDO UHVXOW LQGLFDWHV WKDW WKH EHVW DWWDFN
7KH UHVXOWV FDQ EH FRPSDUHG ZLWK [SHULPHQWDO UHVXOWV IRU 1$&$ DLUIRLOV LQ) LJ 7K
WKLQQHVV1$&$ DLUIRLOLVW& DQG WKH WLFNHVWV SHUFHQWRUW& 7KH GLIIXVLYHVSS
G&/ G LV ZLWKLQ SHUFHQW RI WKH WKHRUHWLFDO YDOXH RI CEIRUDOO WKH YDUIRXV DIRIOIDPLQHV DWD
WKLFNQHVVHV ,QFUHDVLQJ WKLFNQHVV WHQGV WR LQFUHDVH EHWW&/ PD[DQG WKH VDOODQJH 7KH VDOODQJH D
W& LV DERXW ƒ DQG ZRXOG EH HYHQ KLJKHU IRU DIODWSODWH YHUVLRQ WKH ERXQGDU\ OD\HUSDUDWR
HWWP DWWV LQ) LJ F %HWWSHUIRUPDQFH LV XVXDOO DWWDLQHG WKH SHUFHQWWKLFNQHVV SRLQWRUDQ
DIIRLO



) LJ / LIWFKDUDFWHULVWLFV RI VPRRWK 1$&$ DLUIRLOV DV D IXQFWLRQ RI WKLFNQHVV UDWLR IR
LQILQLWH DVSHFW UDWLR *(From Ref. 12.)*

3RWHQWLDO7KHRUHWLFDO &DPEHUHG $LUIRLOV

7KH WKHRUHWLFDO PHWKRGV GLVFXVVHG LQ DGYDQFHG WH[WERRNV DUH [W>RUH[DPSOH VR @5H

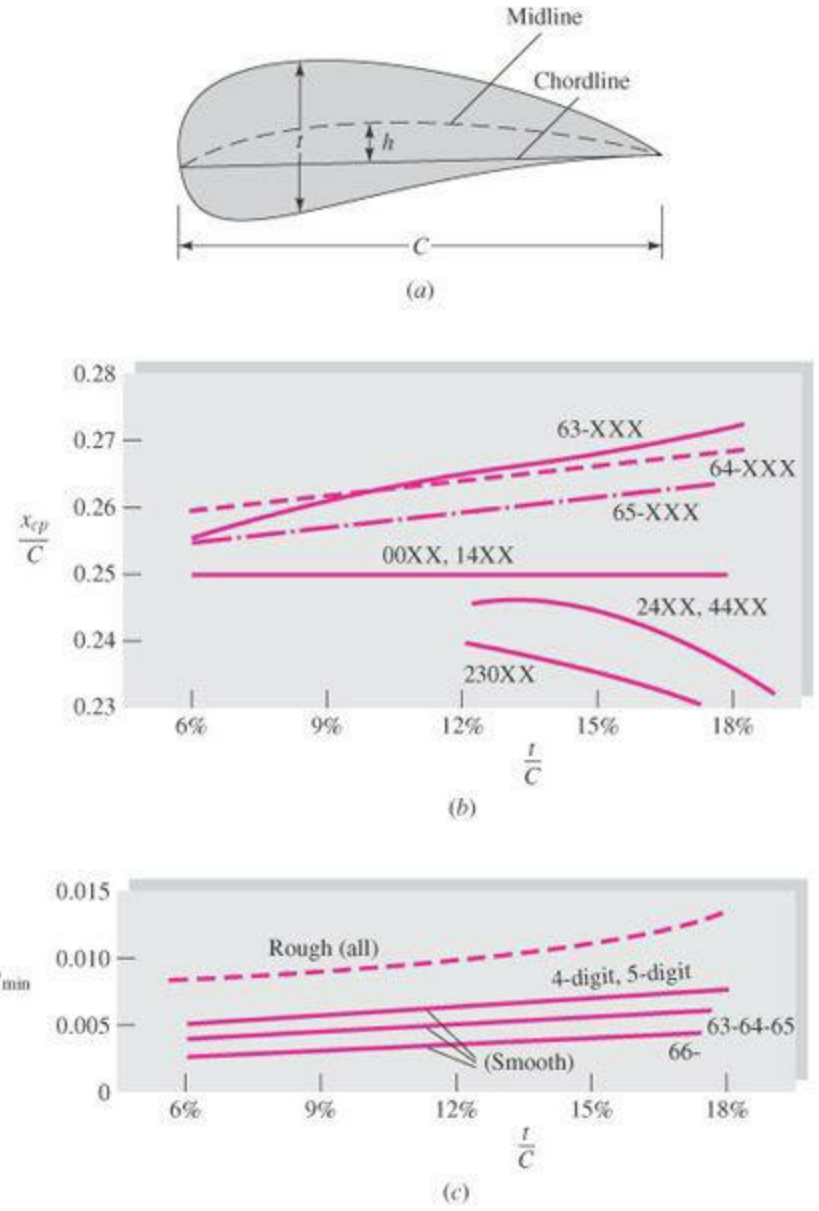
KDVDWKRURXJKDQGFRPSUHKHQVLYHUHYLHZ RI ERXQGLQYLVFLGDQGYLVFRXVDVSHFWVRI DLUIRLOEHKDYLRU

%DVLFDOO WKH WKHRU XVHV DFRPSOH[YDULDEOHPDSSLQJ WKDWWDQVIRUPV WKH IORZ DERXWDFLQGZLWK FLUFXODWLRQ LQ) LJ LQWR IORZ DERXWDIRLOVKDSH ZLWK FLUFXODWLRQ 7KH FLUFXODWLRQ LV WKH DGYXVWHGWRPDWKWH. XWDFRQGLWLRQRI VPRRWKH[LWIORZ IURP WKHWDLOQJHGJH

5HJDUGOHVV RI WKH H[DFWDLUIRLOVKDSH WKH LQYLVFLG PDSSLQJ WKHRU SUHGLFWV WKDWWKH FRUUHFFLUFXODWLRQIRUDQ WKLFNFDPEHUHGDLUIRLO

$$\Gamma_{\text{Kutta}} = \pi b C U_\infty \left(1 + 0.77\frac{t}{C}\right)\sin(\alpha + \beta) \qquad (8.70)$$

ZKHUH WDQ K& DQGK LV WKH PD[LPXP FDPEHU RUPD[LPXP GHYLDWRQ RI WKH DLUIRL PLGOLQHIURP LWVFKRUGOLQH DVLQ) LJ D



) LJ &KDUDFWHULVWLFVRI 1$&$ DLUIRLOV *a* WVSLFDOWKLFN FDPEHUHG DLUIRLO *b* FHQWHUOLQH SUHVVXUHGDWD DQG *c* PLQLP XP GUDJ FRHIILFLHQW

7KHGLVWULEXWLRQRI WKHLQLQWHUVSDQDLUIRLOVVKXV

$$C_L = \frac{\rho U_\infty \Gamma}{\frac{1}{2}\rho U_\infty^2 bC} = 2\pi\left(1 + 0.77\frac{t}{C}\right)\sin(\alpha + \beta) \qquad (8.71)$$

7KLV UHGXFHV WR(T ZKHQ WKH WKLFNQHVV DQG FDPEHU DUH]HUR) LJXUH VKRZV WKDW WKH WKLFNQHVV HIIHFW W& LV QRW YHU\ LPSRUWDQW6RPH DLUIRLOV LQFUHDVH OLIW ZLWK WKLFNQHVV RWKHUV GHFUHDVH DQG QRQH DSSURDFK WKH WKHRU\ YHU\ FORVHO WKH SUHGLFWLRQ EHORZ WR RQ WKH XSSHU VXUIDFH DIIHFWLQJ WKH DLUIRLO³VKDSH! 7KXV LWLV FXVWRPDU\ WR GURS WKH WKLFNQHVV HIIHFWURP WKH WKHRU\

$$C_L \approx 2\pi \sin(\alpha + \beta) \qquad (8.72)$$

7KH WKHRU\ FRUUHFWO SUHGLFWV WKDW D FDPEHUHG DLUIRLOZ LOKDYH ILQLWH OLIW DW]HUR DQJOH RI DWWDF DQG]HUR OLW=/ DWDQ DQJOH

$$\alpha_{ZL} = -\beta = -\tan^{-1}\frac{2h}{C} \qquad (8.73)$$

(TXDWLRQ RYHUSUHGLFWVWKHPHDVXUHG]HUR OLWDQJOHE\ f RUVR DVVKRZQLQ7DEOH 7KH PHDVXUHG YDOXHV DUH HVVHQWLDOO LQGHSHQGHQWRI WKLFNQHVV 7KHGHVLJQDWLRQ; ; LQWKH1 $ &$ VHULHV LQGLFDWHVWKH WKLFNQHVVLQSHUFHQWDQG WKHRWKHUGLJLWVUHIHUWRFDPEHUDQGRWKHUGHWDLOV) RU[DPSOH WKH DLUIRLOKDV SHUFHQWPD[LPXP FDPEHU WKHILUWGLJLWRFF XUULQJ DW SHUFHQWFKRUG WK VHFRQGGLJLWZLWK SHUFHQWPD[LPXP WKLFNQHVV WKHODVWWZRGLJLWV 7KHPD[LPXP WKLFNQHVVQHH QRWRFF XUDWWKHVDPHSRVLWLRQDVWKHPD[LPXP FDPEHU

Airfoil series	Camber h/C, %	Measured α_{ZL}, deg	Theory $-\beta$, deg
24XX	2.0	−2.1	−2.3
44XX	4.0	−4.0	−4.6
230XX	1.8	−1.3	−2.1
63-2XX	2.2	−1.8	−2.5
63-4XX	4.4	−3.1	−5.0
64-1XX	1.1	−0.8	−1.2

) LJXUH E VKRZV WKH PHDVXUHG SRVLWLRQ RI WKH FHQWHU RI SUHVVXUH RI WKH YDULRXV 1 $ &$ DLUIRLOV EHWKV\PPHWULF DQG FDPEHUHG ,QDOOFDVHV[&3 LVZLWKLQ FKRUG OHQJWKRI WKH WKHRUHWLFDO TXDUWHUFKRUG SRLQWSUHGLFWHG E\ (T 7KHVXDQGDUG FDPEHUHG DLUIRLOV DQG VHULHV OH VOJKWO IRUZDUG RI [& DQG WKH ORZ GUDJ VHULHV IRLOV VOJKWO DW7KH V\PPHWULF DIRLOV DUHDW

) LJXUH F VKRZV WKH PLQLPXP GUDJ FRHILFLHQWRI 1 $ &$ DLUIRLOV DV DIXQFWRQ RI WKLFNQHVV $ VPHQWRQHG HDUOHULQ FRQQHFWLRQ ZLWK) LJ WKHVHIRLOV ZKHQ VPRRWKFRQWRXUHG KDYH OHVV GUDJ WKDQ WXEHQWOWORZ SDUDOOHOWR DIIDWSODWH HVSHFLDOO WKH ORZ GUDJ VHULHV + RZ HYHU IRUWDQGDUG VXUIDFHURXJKQHVV DOOIRLOVKDYHDERXWWKHVDPHPLQLPXP GUDJ URXJKO SUHGLFWUHDWUHDQWKDWR DVPRRWKIDWSODWH

: LQJVRI) LQLWH6SDQ

7KHUHVXOWVRI DLUIRLOWKHRU\ DQG H[SHULPHQWLQWKHSUHYLRXVXEVHFWLRQ ZHUHIRUWZR GLPHQVLRQDO

RULQLQLWVSDQ ZLQJV %XWDOUHDOZLQJVKDYHWSVDQGDUHWKHUHRI ILQLWVSDQRUILQLWDVSHF
UDWR$5 GHILQHGE\

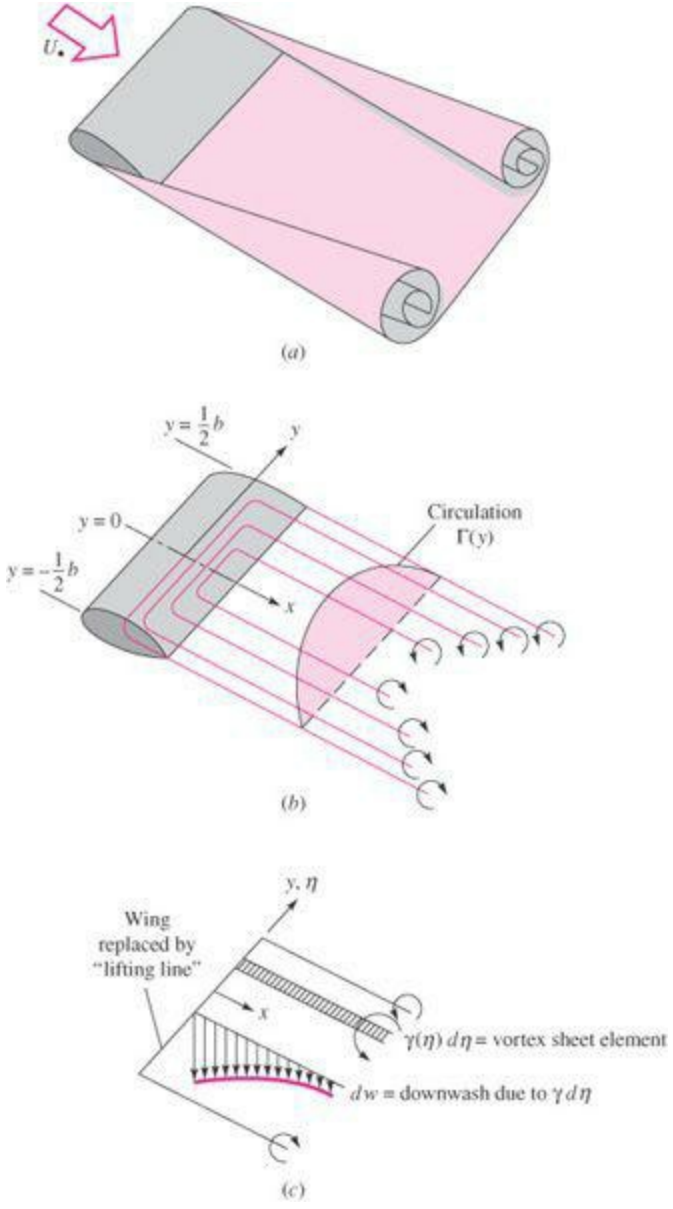
$$\mathrm{AR} = \frac{b^2}{A_p} = \frac{b}{\overline{C}} \tag{8.74}$$

ZKHUHE LVWKHVSDQOHQJWKIURP WS WRWS DQG$S LVWKHSODQRUP DUHDRI WKHZLQJ DVVHQIURF
DERYH 7KHOLWDQGGUDJ FRHILFLHQWVRI DILQLWHDSHFWUDWR ZLQJ GHSHQGVXRQO RQWKHDSHFWUDW
DQGVOLJKWO RQWKHSODQRUP VKDSHRI WKHZLQJ

9RUWFHVFDQQRWHQG LQDIOXLG WKH PXWWHWKHUH[WHQG WR WKHERXQGDU RUIRUP DFORVHGORRS
) LJXUH D VKRZ VKRZ WKHYRUWFHV WKDWSURYLGHWKHZLQJ FLUFXODWLRQ EHQG GRZQWHDP DWLQI
ZLQJ WSV DQG H[WHQG IDUEHKLQG WKHZLQJ WR MRLQ WKHVWDUWQJ YRUWH[ ) LJ GRZQWHDP 7K
VWUQJHWWYRUWFHVDUHVKHGIURP WKHWSV EXWWRP HDUHVKHGIURP WKHERG\ RI WKHZLQJ DVVNHWKH
VFKHPDWFDOO LQ) LJ E 7KHHIHFWLYHFLUFXODWLRQ* \ RI WKHVHVWDLQQJ VHGYRUWFHVLV]HURDWWK
WSV DQG XVXDOO DPD[LPXP DWWKHFHQWUSODQH RUURRWRI WKHZLQJ ,Q 3UDQGWVXFFHVWXO
PRGHOHG WKLVIORZ E\ UHSODFLQJ WKHZLQJ E\ DVLQJOHOLWQJ OLQHDQGDFRQWXRXVVKHHWRI VHPL LQILQL
WDLQJ YRUWFHVRI VWUHQJWK \ G* G\ DVLQ) LJ F (DFKHOPHQWDSLFHFHRI WDLQJ VKHW
G LQGXFHVDGRZQZDVK RUGRZQZDUGYHORFLW GZ \ JLYHQE\

$$dw(y) = \frac{\gamma(\eta)\, d\eta}{4\pi(y - \eta)}$$

DWSRVLWRQ\ RQWKHOLWQJ OLQH 1RWWKHGHORPLQDWRUWUP CEUDKHUWKDQ CEEHFDXVHWKHWDLOLQ
YRUWH[H[WHQGVRQO IURP WR’ UDKHUWKDQIURP í’ WR ’



) LJ / LIWQJ CQH VKHRU IRU D ILQLWH ZLQJ *a* DFVXDOWDLQQJ YRUWH[VKHHWV EHKLQG D ZLQJ *b* VLP XODWRQE\ YRUWH[VKHHWV ³ERXQGI VWR VKHZLQJ *c* GRZQZDVK RQVKHZLQJ GXHVWR D HOHP HQWRI VKHWDLOLQJ YRUWH[VKHHWV

7KHVRDOGRZQZDVKZ \ LQGXFHGE\ VKHHQWLUHWDLOLQJ YRUWH[VKHHWV LVVKRXV

$$w(y) = \frac{1}{4\pi} \int_{-(1/2)b}^{(1/2)b} \frac{\gamma(\eta)\, d\eta}{y - \eta} \qquad (8.75)$$

: KHQ VKH GRZQZDVK LV YHFWRUDOO DGGHG VR VKH DSSURDFKLQJ IUHHVWUHDP 8, VKH HIIHFWLYH DQJO RI DWWDFN DVVHHQ RI VKHZLQJ LVUHGXFHGVR

$$\alpha_{\text{eff}} = \alpha - \alpha_i \qquad \alpha_i = \tan^{-1}\frac{w}{U_\infty} \approx \frac{w}{U_\infty} \qquad (8.76)$$

ZKHUHZ HKDYHXVHGDVP DOODP SOLWGHDSSUR[LP DWLRQZ $w \ll U_\infty$. 8,

7KHILQDOWAS LVWR DVVXP HWKDWWKH ORFDOFLFXODWRQ* \ LVHTXDOWR WKDWR DVZR GLP HQVLRQDO ZLQJ RI VKHVDP HVKDSHDQGVDP HHIIHFWLYH DQJOHR DWWDFN) URP WKLQDLUROWKHRU (TV DQG ZHKDYHWKHHXVP DW

$$C_L = \frac{\rho U_\infty \Gamma b}{\frac{1}{2}\rho U_\infty^2 b C} \approx 2\pi\alpha_{\text{eff}}$$

or

$$\Gamma \approx \pi C U_\infty \alpha_{\text{eff}} \tag{8.77}$$

&RPELQLQJ (TV DQG Z HREWDLQ3UDQGWVODLWQJ OQHWKHRU IRUDILQLWH VSDQZ LQJ

$$\Gamma(y) = \pi C(y) U_\infty \left[\alpha(y) - \frac{1}{4\pi U_\infty} \int_{-(1/2)b}^{(1/2)b} \frac{(d\Gamma/d\eta)\, d\eta}{y - \eta} \right] \tag{8.78}$$

7KLV LV DQ LQWURGXFWRU\ WH[WERRN VR EH VRQYHG IRU* \ VXEVHFWRQV H \ ,WV VLPLODUWR WKH WKLQ DLUIRLOLQWUDOHTXDWRQ DQG HYHQ PRUH IRUPLGDEOH 2QFH LWV VROYHG WK VRXDZ LQJ OLWDQGLQGXFHGGUDJ DUHJLYHQE\

$$L = \rho U_\infty \int_{-(1/2)b}^{(1/2)b} \Gamma(y)\, dy \qquad D_i = \rho U_\infty \int_{-(1/2)b}^{(1/2)b} \Gamma(y)\alpha_i(y)\, dy \tag{8.79}$$

+ HUH LV D FDVH Z KHUH WKH GUDJ LV QRW]HUR LQ DIUFWRQDWV WKH RU EHFDXVH WKH GRZ QZ DVK FDXV WKH OLWWR VDQWEDFNZ DUG E\ DQJOH. L VR WKDWLWKDV D GUDJ FRPSRQHQWSDUDOOHO WR WKH IUHH VWUHDP GUHFWLRQ G L G VLQ. L § G . L

7KHFRPSOHWH VROXWLRQ WR(T IRUDUELWUDU Z LQJ SDQRUP & \ DQG DUELWUDU VZLWW \ LV WUHDWHG LQ DGYDQFHG WH[WV>IRUH[DPSOH @,WXLQV RXWKDWWKHUHLV D VLPSOHUHSUHVHQWDWLYH VROXWR IRUDQXQZ LWHGZ LQJ R HOOLSWFDOSDQRUP

$$C(y) = C_0 \left[1 - \left(\frac{2y}{b} \right)^2 \right]^{1/2}$$

7KHDUHDDQGDVSHFWDWLR RI WKLV Z LQJ DUH

$$A_p = \int_{-(1/2)b}^{(1/2)b} C\, dy = \frac{1}{4}\pi b C_0 \qquad \text{AR} = \frac{4b}{\pi C_0} \tag{8.80}$$

7KH VROXWLRQ WR(T IRUWKLV& \ LV DQ HOOLSWFDOFLUFXODWLRQ GLVWULEXWLRQ R H[DFWO VLPLOD VKDSH

$$\Gamma(y) = \Gamma_0 \left[1 - \left(\frac{2y}{b} \right)^2 \right]^{1/2}$$

6XEVWLWXWLQJ LQWR(T DQG LQWHJUDWLQJ JLYH DUHDWRQEHWZHHQ* DQG&

$$\Gamma_0 = \frac{\pi C_0 U_\infty \alpha}{1 + 2/\text{AR}}$$

Z KHUH. LV DWXP HG FRQVWDQWWFRUVWKHXQZ LWHGZ LQJ

6XEVWLWXWLQJ (T JLYHV WKH HOOLSWLFDO ZLQJ OLIW

$$L = \tfrac{1}{4}\pi^2 b C_0 \rho U_\infty^2 \alpha/(1 + 2/\mathrm{AR})$$

or

$$C_L = \frac{2\pi\alpha}{1 + 2/\mathrm{AR}} \tag{8.81}$$

,I ZH JHQHUDOL]H WKLV WR D WKLFN FDPEHUHG ILQLWH ZLQJ RI DSSUR[LPDWH HOOLSWLFDO SODQIRUP ZI
REWDLQ

$$C_L = \frac{2\pi \sin(\alpha + \beta)}{1 + 2/\mathrm{AR}} \tag{8.82}$$

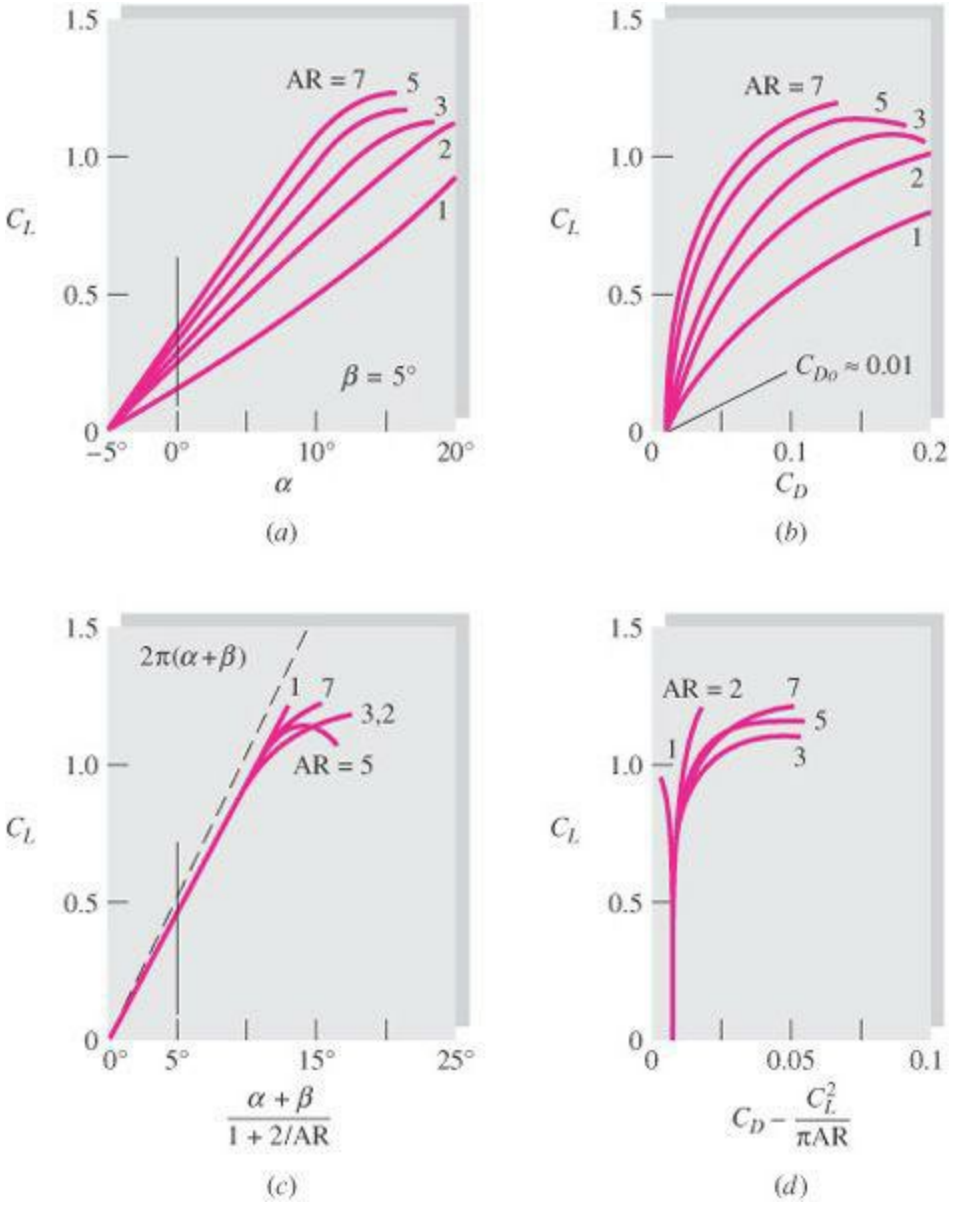
7KLV UHVXOW ZDV JLYHQ ZLWKRXW SURRI DV (T) URP (T WKH FRPSXWHG GRZQZDVK IR
WKH HOOLSWLFDO ZLQJ LV FRQVWDQW

$$w(y) = \frac{2U_\infty \alpha}{2 + \mathrm{AR}} = \text{const} \tag{8.83}$$

)LQDOO WKH LQGXFHG GUDJ FRHILFLHQW IURP (T LV

$$C_{Di} = C_L \frac{w}{U_\infty} = \frac{C_L^2}{\pi \mathrm{AR}} \tag{8.84}$$

7KLV ZDV JLYHQ ZLWKRXW SURRI DV (T
) LJXUH VKRZV WKH HIIHFWLYHQHVV RI WKLV WKHRU ZKHQ WHVWHG DJDLQVW DQRQHOOLSWLFDO FDPEHUH
ZLQJ E\ 3UDQGWO LQ > @) LJXUHV DQG E VKRZ WKH PHDVXUHG OLIW FXUYHV DQG GUDJ SRODUV IR
ILYH GLIIHUHQW DSHFW UDWLRV 1RWH WKH LQFUHDVH LQ VXOODQJH DQG GUDJ DQG WKH GHFUHDVH LQ OLWWOH SHD
WKH DSHFW UDWLR GHFUHDVHV



) LJ &RP SDULVRQ RI WKH RUI DQG H[SHULP HQWDO DIILQLWMI ZLQJ *a* PHDVXUHG GDW> @ *b* PHDVXUHG GUDJ SRODU > @ *c* GDWD UHGXFHG WR LQILQLWMI DVSHFWDWR *d* GUDJ SRODU UHGXFHG WI LQILQLWMI DVSHFWDWR

) LJXUH F VKRZ V WKH GDWD UHSORWWG DJDLQVW HIHFWLYH DQJOH RI DWWDFN. HI . $5 DVSUHGLFWMG E\ (T 7KHVH FXUYHV VKRXOG EH HTXLYDOHQW DQLQILQLWMI DVSHFWDWR ZLQJ DQG WKH GRFRGSVHWRJHWKHU [FHSW DW VDOO 7KHUFRP PRQVOSHG&/ G LVDERXW SHUFHQW DV WKDQ WKH RUHWFDOYDOXH CE EXWKLV LV FRQVLVWHQW ZLWK WKH WKLFNHVV DQG VKDSH IHFWRUMG LQ) LJ

) LJXUH G VKRZ V WKH GUDJ GDWD UHSORWWG ZLWK WKH WKRUHWFDOLQGXFHG GUDJ &L &/ CE$5 VXEWDFWMG RXW$JDLQ H[FHSWQDUXDOO WKH GDWD FRODSVHRQR D VLQJHOQHR Q-HDO FRQWDQWQLQILQLWMI DVSHFWDWR GUDJ & § : HFRQFOXGH WKDWWKH ILQLWMI ZLQJ WKRU LV YHU HIHFWLYH DQG PD\ EH XVHGIRUGHVLJQFDOFXODWRQV

$ LUFUDIW7 UDLOQJ 9 RUWFHV

7KH WDLOQJ YRUWFHV LQ) LJ D DUH UHDO QRWXXWP DWKHP DWFDODEXWDFWRQV 2 QFRPPHUFL

DLUFUDIWVXFK YRUWFHV DUH ORQJ VWURQJ DQG OQJHULQJ 7KH FDQ VWHHK IRUPLQHV EHKLQG D ODUJ
DLUFUDIWDQG HQGDQJHU WKH IROORZLQJ SODQHV E\ LQGXFLQJ GUDWF URQQQ PRPHQW 7KH YRUWH
SHUVLVWHQFH JRYHUQV WKH VHSDUDWLRQ GLVWDQFH EHWZHHQ SODQHV DVDQ DLUSRUWDQG WKXVGHWUPLQHV DLUSRU
FDSDFLW\ $QH[DPSOH VWURQJ WDLOQJ YRUWHVLVVKRZQLQ) LJ 7KHUHLVDFRQWQXLQJ UHVHDUF
HIRUWWR DOOHYLDWH WDLOQJ YRUWHVE\ EUHDNLQJ WKH XS RURWKHUZLVHFDXLQJ WKH VWRGHFN 6HHWK
UHYLHZ DUWFOHE\ 6SDODUW @

) LJ : LQJWS YRUWHVIURP DVPRNH YLVXDOL]DWRQWWXWRI D%RHLQJ 9RUWHVIURP
OUJH DLSODQHVFDQ EH H[WHP HO GDQJHURXV WR DQ IROORZLQJ DLUFUDIWHSHFLDOO VP DOOSODQHV
7KLVWWXWZDVSDUWRI DUHVHDUFK HIRUWWR DOOHYLDWWKHVHZLQJ ZDNHV*(NASA photo.)*

$[LX PPHWULF 3RWQWLDO) ORZ

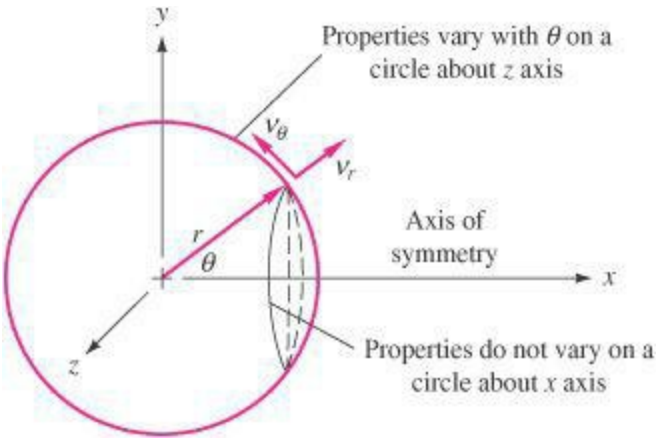
7KH VDPH VXSHUSRVLWLRQ WHFKQLTXH WKDWZRUNHG VR ZHOO IRU SODQH IORZ LQ6HF LV DOW
VXFFHVWXOIRUD[LX PPHWULF SRWQWLDO ORZ : HJLYHVRPHEUH H[DPSOHVKHUH

0 RVWRI WKHEDVLF UHVXOWVFDUU RYHUIURP SODQHVRD[LX PPHWULF ORZ ZLWRQO VOJKWFKDQJH
RZLQJ WRWKHJHRPHWULF GLIHUHQFHV &RQVLGHUWKHIORZLQ UHDGGORZ 6LQFHWKHUHLVQR VXFK WKLQ
DV D SRLQWYRUWH[ZHPXWUJR WKH SODWXUH RI VXG\LQJ FLUFXODWRQ HIHFWLQD[LX PPHWULF ORZ
+RZHYHU DVDQ VPRNHUNQRZV WKHUHLVDQD[LX PPHWULF ULQJ YRUWH[DQGWKHUHDUHDOR ULQJ VRXUFH
DQGULQJ VLQNV ZKFK ZHOHDYHVRGGYDQFHGWH[WV>RUH[DPSOH @

Basic plane flow	Counterpart axisymmetric flow
Uniform stream	Uniform stream
Line source or sink	Point source or sink
Line doublet	Point doublet
Line vortex	No counterpart
Rankine half-body cylinder	Rankine half-body of revolution
Rankine oval cylinder	Rankine oval of revolution
Circular cylinder	Sphere
Symmetric airfoil	Tear-shaped body

6SKHULFDO3RODU&RRUGLQDWHV

$[LVSPPHWULF SRWHQWLDO IORZV DUH FRQYHQLHQWO\ VWXGLHG LQ WKH VSKHULFDO SRODU FRRUGLQDWH V\VWHP)LJ 7KHUH DUH RQO\ WZR FRRUGLQDWHV U DQG I IORZ SURSHUWLHV DUH FRQVWDQW RQ D FLUFOH RI UDGLXV VLQ DERXW WKH [D[LV



)LJ 6SKHULFDOSRODUFRRUGLQDWHVIRUD[LVSPPHWULF IORZ

7KHHTXDWLRQRI FRQWLQXLW\IRULQFRPSUHVVLEOHIORZ LQWKHVHFRRUGLQDWHV

$$\frac{\partial}{\partial r}(r^2 v_r \sin\theta) + \frac{\partial}{\partial \theta}(r v_\theta \sin\theta) = 0 \qquad (8.85)$$

Z KHUH%\ DQG% DUH UDGLDO DQG WDQJHQWLDO YHORFLW\ DV VKRZQ 7KXV D VSKHULFDO SRODU VWUHDP IXQFWLRQ H[LVWVVXFKWKDW

$$v_r = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = \frac{1}{r \sin\theta}\frac{\partial \psi}{\partial r} \qquad (8.86)$$

,QDNHPDQQHUDYHORFLW\SRWHQWLDOÏ U H[LVWVVXFKWKDW

$$v_r = \frac{\partial \phi}{\partial r} \qquad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \qquad (8.87)$$

7KHVH IRUPXODV VHUYH WR GHGXFH WKH$ DQGÏ IXQFWLRQV IRU YDULRXV HOHPHQWDU[LVPPHWULF SRWHQWLDOIORZV

8QLIRUP6WUHDP LQWKH:' LUHFWLRQ

$ VWUHDP 8, LQWKH[GLUHFWLRQKDVFRPSRQHQWV

$$Y_U \quad 8, \quad FRV \quad Y \quad 8, \quad VLQ$$

6XEVWLWXWLRQLQWR(TV DQG DQGLQWHJUDWLQJ JLYH

$$\text{Uniform stream:} \qquad \psi = -\tfrac{1}{2}U_\infty r^2 \sin^2\theta \qquad \phi = U_\infty r\cos\theta \tag{8.88}$$

$ VXVXDO DUELWUDU FRQVWDQWV RI LQWHJUDWLRQKDYHEHHQQHJOHFWHG

3RLQW6RXUFHRU6LQN

&RQVLGHUDYROXPHIOX[4 LVXLQJ IURP DSRLQWVRXUFH 7KHIORZ ZLOOVSUHDG RXWUDGLDOO DQG D UDGLXVUZLOOHTXDO4 GLYLGHGE\ WKHDUHD CEU RI DSSKHUH 7KXV

$$v_r = \frac{Q}{4\pi r^2} = \frac{m}{r^2} \qquad v_\theta = 0 \tag{8.89}$$

ZLWKP 4 CE IRUFRQYHQLHQFH ,QWHJUDWLQJ DQG JLYHV

$$\text{Point source} \qquad \psi = m\cos\theta \qquad \phi = -\frac{m}{r} \tag{8.90}$$

) RUDSRLQWVLQN FKDQJHP VRí P LQ(T

3RLQW' RXEOHWV

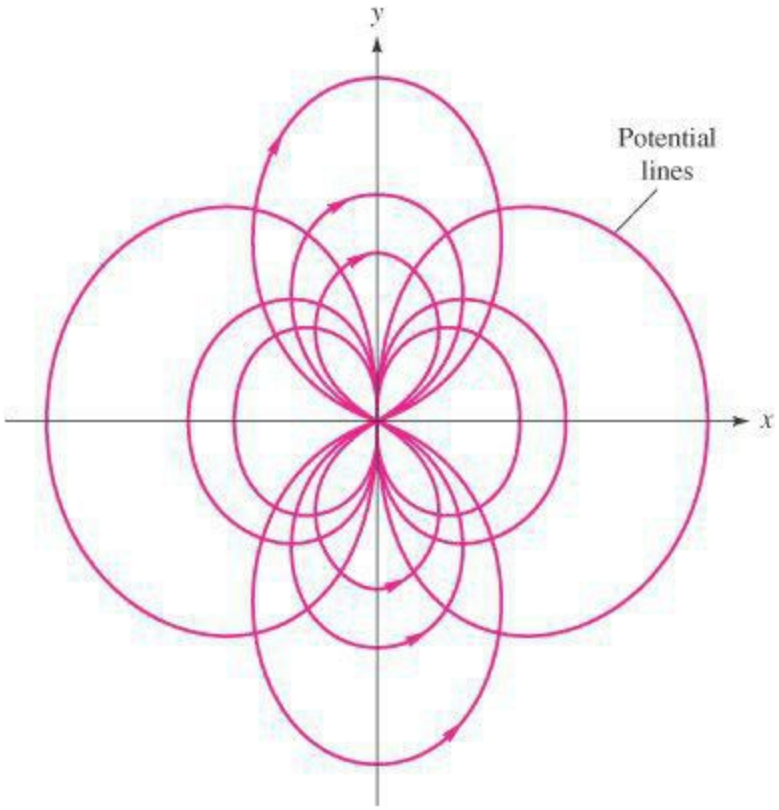
([DFWO DVLQ) LJ SODFHDVRXUFHDW[< í D DQGDQHTXDOVLQNDW D VDNLQJ WK OPLWDVDEHFRPHVVPDOOZLWKWKHSURGXFWDP KHOGFRQVWDQW

$$\psi_{\text{doublet}} = \lim_{\substack{a\to 0 \\ 2am=\lambda}} (m\cos\theta_{\text{source}} - m\cos\theta_{\text{sink}}) = \frac{\lambda \sin^2\theta}{r} \tag{8.91}$$

: HCDYHWKHSURRI RI WKLVCOP LWDVDSURECP 7KHSRLQWGRXECWYHCRFLWSRVQLDOLV

$$\phi_{\text{doublet}} = \lim_{\substack{a\to 0 \\ 2am=\lambda}} \left(-\frac{m}{r_{\text{source}}} + \frac{m}{r_{\text{sink}}} \right) = \frac{\lambda\cos\theta}{r^2} \tag{8.92}$$

7KHVWUHDPOLQHVDQG SRVQLDOOLQHVDUHVKRZ QLQ) LJ 8QDNHWKHSDQHGRXECWCRZ R) LJ QHWKHUHVHWRI OLQHVUHSUHVHQWSHUFHFWFLUFOHV



) LJ 6WUHDPOLQHVDQGSRWHQWLDOOLQHVIRUDSRLQWGRXEOHWDWWKHRULJLQ IURP (TV DQG

8QLIRUP 6WUHDP SOXV D3RLQW6RXUFH

%\ FRPELQLQJ (TV DQG ZH REWDLQ WKH VWUHDP IXQFWLRQ IRU D XQLIRUP VWUHDP SOXV SRLQWVRXUFH DW WKH RULJLQ

$$\psi = -\tfrac{1}{2}U_\infty r^2 \sin^2\theta + m\cos\theta \qquad (8.93)$$

) URP (T WKH YHORFLWY FRPSRQHQWV DUH E\ GLIHUHQWLDWLRQ

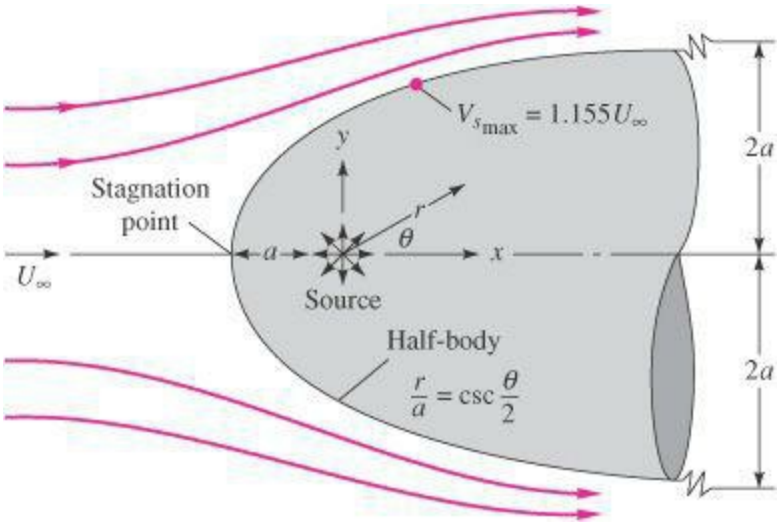
$$v_r = U_\infty \cos\theta + \frac{m}{r^2} \qquad v_\theta = -U_\infty \sin\theta \qquad (8.94)$$

6HWWLQJ WKHVH HTXDOWR]HUR UHYHDOV DVWDJQDWLRQSRLQWDW f DQGU D P 8, DVVKRZ LQ) LJ ,I ZH OHWP 8, D WKHVWUHDP IXQFWLRQFDQEHUHZULWWHQDV

$$\frac{\psi}{U_\infty a^2} = \cos\theta - \frac{1}{2}\left(\frac{r}{a}\right)^2 \sin^2\theta \qquad (8.95)$$

7KHVWUHDP VXUIDFHWKDWSDVVHVWKURXJKWKHVWDJQDWLRQSRLQWU D CE KDVWKHYDOXH$ í 8, D DQGIRUPVDKDOI ERG\ RI UHYROXWLRQHQFORVLQJ WKHSRLQWVRXUFH DVVKRZQLQ) LJ 7KLVKDOI ERG\ FDQEHXVHGWRVLPXODWHDSLRWWXEH) DUGRZQVWUHDP WKHKDOI ERG\ DSSURDFKHVWKHFRQVWDQWVWUHDP VXUIDFHRFFXUV DWWKH[D[LV 7KHPD[LPXP YHORFLWY DQG PLQLPXP SUHVVXUH DORQJ WKHKDOI ERG\ VXUIDFHRFFX DW$\theta = 70.5°,\ r = a\sqrt{3}$ 9V 8, ' RZQVWUHDP RI WKLV SRLQWWKH HLV DQ DGYHUVH JUDGLHQWDV9

VORZ(1) GHFHOHUDWHVWR8, EXWERXQGDU(DHUWKRU LQGLFDWHVQR IORZ VHSDUDWLRQ 7KXV(T LV
DYHU UHDOLVWLF VLP XODWRQ RI DUHDOKDO ERG\ IORZ %XWZKHQWKHXQLRUP VWUHDP LVDGGHGWRDVLQN
IRUP D KDOI ERG\ UHDU VXUIDFH VLP LODU WR) LJ F VHSDUDWLRQ LV SUHGLFWHG DQG WKH UHDU LQYLVFLG
SDWWHUQ VQRWUHDOLVWLF



) LJ 6WUHDPOLQHVIRUD5DQNLQHKDOI ERG\ RI UHYROXWLRQ

8QIRUP 6WUHDP SOXV D3RLQW' RXEOHW

) URP(TV DQG FRPELQDWLRQ RI DXQLRUP VWUHDP DQG DSRLQWGRXEOHWDWWKHRULJLJLYHV

$$\psi = -\frac{1}{2}U_\infty r^2 \sin^2\theta + \frac{\lambda}{r}\sin^2\theta \qquad (8.96)$$

([DPLQDWLRQ RI WKLV UHODWLRQ UHYHDOV WKDWWKH VWUHDP VXUIDFH$ FRUUHVSRQGV WR WKH VSKHUH R
UDGLXV

$$r = a = \left(\frac{2\lambda}{U_\infty}\right)^{1/3} \qquad (8.97)$$

7KLV LV H[DFWO DQDORJRXV WR WKH FRQGXFWRU IORZ RI) LJ D IRUPHG E\ FRPELQLQJ D XQLRUP
VWUHDP DQG DQLQHGRXEOHW
/ HWWLQJ$\lambda = \frac{1}{2}U_\infty a^3$ IRUFRQYHQLHQFH ZHUHZULWH(T DV

$$\frac{\psi}{\frac{1}{2}U_\infty a^2} = -\sin^2\theta\left(\frac{r^2}{a^2} - \frac{a}{r}\right) \qquad (8.98)$$

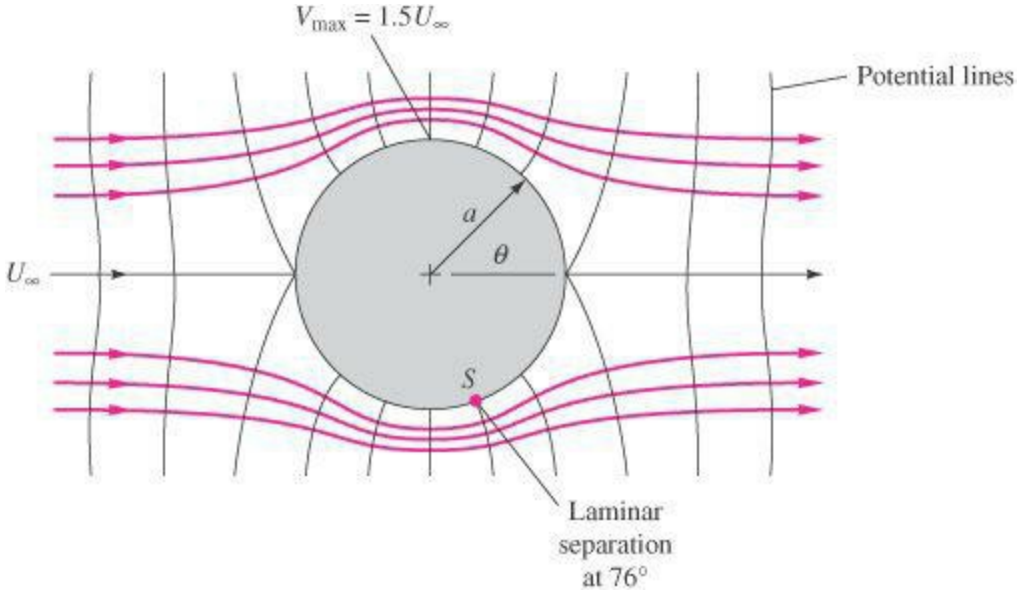
7KH VWUHDPOLQHV IRUWKLV VSKHUH IORZ DUH SORWWHG LQ) LJ %\ GLIIHUHQWDWLQJ IURP(T
WKH YHORFLWLHV FRPSRQHQWVDUH

$$v_r = U_\infty \cos\theta\left(1 - \frac{a^3}{r^3}\right) \qquad v_\theta = -\frac{1}{2}U_\infty \sin\theta\left(2 + \frac{a^3}{r^3}\right) \qquad (8.99)$$

: HVHHWKDWWKHUDGLDOYHORFLWYDQLVKHVDWWKHVSKHUHVXUIDFH D DVH[SHFWHG 7KHUHLV
VWLOODQDNQSRLQWDWWKHIURQWD CE DQGWKHUHDU D RI WKHVSKHUH 7KHPD[LPXP YHORFLWRFFXUVD
WKHVKRXOGHU $a, ±½π) ZKHUH%0 DQG% ∓ 8, 7KHVXUIDFHYHORFLWGLVWULEXWLRQLV

$$V_s = -v_\theta|_{r=a} = \tfrac{3}{2}U_\infty \sin\theta \qquad (8.100)$$

1RWWKHVLPLODULW\WRWKHFRQGXFWLYHVXUIDFHYHORFLWHTXDOWR 8, VLQ IURP (T ZLWK]HU
FLUFXODWLRQ



) LJ 6WUHDPOLQHVDQGSRWHQWLDOOLQHVIRULQYLVFLGIORZ SDVWD VSKHUH

(TXDWLRQ SUHGLFWV DVH[SHFWHG DQDGYHUVHSUHVVXUHJUDGLHQWRQWKHUHDU f RI WK
VSKHUH ,I ZH XVH WKLV GLVWULEXWLRQ ZLWK ODPLQDUERXQGDU OD\HU WKHRU >IRUH[DPSOH S (
VHSDUDWLRQLV FRPSXWHG VR RFFXUDWHERXW f VR WKDWLQ WKH DFWXDOIORZ SDWWHUQ RI) LJ I
EURDGZDNHIRUPVLQWKHUHDU 7KLVZDNHLQWUDFWVZLWKWKHIUHHVWUHDP DQG FDXVHV(T VR E
LQDFFXUDWHYHOLQWKHIURQWRI WKHVSKHUH 7KHPHDXUHGPD[LPXP VXUIDFHYHORFLWLV HTXDOWR D
DERXW 8, DQGRFFXUVDWDERXW f VHH5H 6HF IRUIXUWKHUGHWDLOV

7KH&RQFHSWRI +\GURG\QDPLF0 DVV

: KHQDERG\ PRYHVWKURXJKDIOXLG LWPXVWSXVKDILQLWHPDVVRI IOXLGRXWRI WKHZD ,I WKHERG
LVDFFHOHUDWHG WKHVXUURXQGLQJ IOXLGPXVWDOVR EHDFFHOHUDWHG 7KHERG\ EHKDYHVDVLI LWZHUHKDYL
E\ DQ DPRXQWFDOOHG WKHK\GURG\QDPLF PDVV DOVR FDOOHG WKHDGGHGPDVV RI WKHIOXLG ,
WKHLQWDQJDQHRXVERG\ YHORFLWVLV8 W WKHVXPPDWRQRI IRUFHVPXVWDOVREH HTXDOWR

$$\Sigma \mathbf{F} = (m + m_h)\frac{d\mathbf{U}}{dt} \qquad (8.101)$$

ZKHUHP$K WKHK\GURG\QDPLF PDVV LVDIXQFWLRQRI ERG\ VKDSH WKHGLUHFWLRQRI PRWLRQ DQG WKH
OHWWHU[WQWIORZ SDUDPHWHUVXFKDVWKH5H\QROGVQXPEHU

$FFRUGLQJ WR SRWHQWLDO WKHRU\ > 6HF @P$K GHSHQGV RQO RQ WKH VKDSH DQG GLUHFWLRQ RI PRWLRQ DQG FDQ EH FRPSXWHG E\ VXPPLQJ WKH WRWDO NLQHWLF HQHUJ\ RI WKH IOXLG UHODWLYH WR WKH ERG\ DQG VHWWLQJ WKLV HTXDO WR DQ HTXLYDOHQW ERG\ HQHUJ\

$$\mathrm{KE}_{\text{fluid}} = \int \tfrac{1}{2} dm\, V_{\text{rel}}^2 = \tfrac{1}{2} m_h U^2 \qquad (8.102)$$

7KH LQWUDWLRQ RI IOXLG NLQHWLF HQHUJ\ FDQ DOVR EH DFFRPSOLVKHG E\ D ERG\ VXUDFH LQWUDWLRQ LQYROYLQJ WKH YHORFLWLVSRWHQWLDO> 6HF @

&RQVLGHUWKHSUHYLRXVH[DPSOHRI DVSKHUHLPPHUVHGLQDXQLRUP VWUHDP %\ VXEWUDFWLQJ RXWWK VWUHDP YHORFLWZ HFDQUHSODFWKHIORZ DVLQ) LJ VKRZ LQJ WKHVWUHDPOQHVUHDLYHWRWKHPRYLQ VSKHUH 1RWWKH VLPLOLWLV WR WKH GRXEOHWLQ) LJ 7KH UHDLYH YHORFLWV FRPSRQHQW DU IRXQGE\ VXEWUDFWLQJ 8 IURP (TV

$$v_r = -\frac{Ua^3 \cos\theta}{r^3} \qquad v_\theta = -\frac{Ua^3 \sin\theta}{2r^3}$$

7KHHOPHQWRI IOXLGPDVV LQVSKHULFDOSRODUFRRUGLQDWHV LV

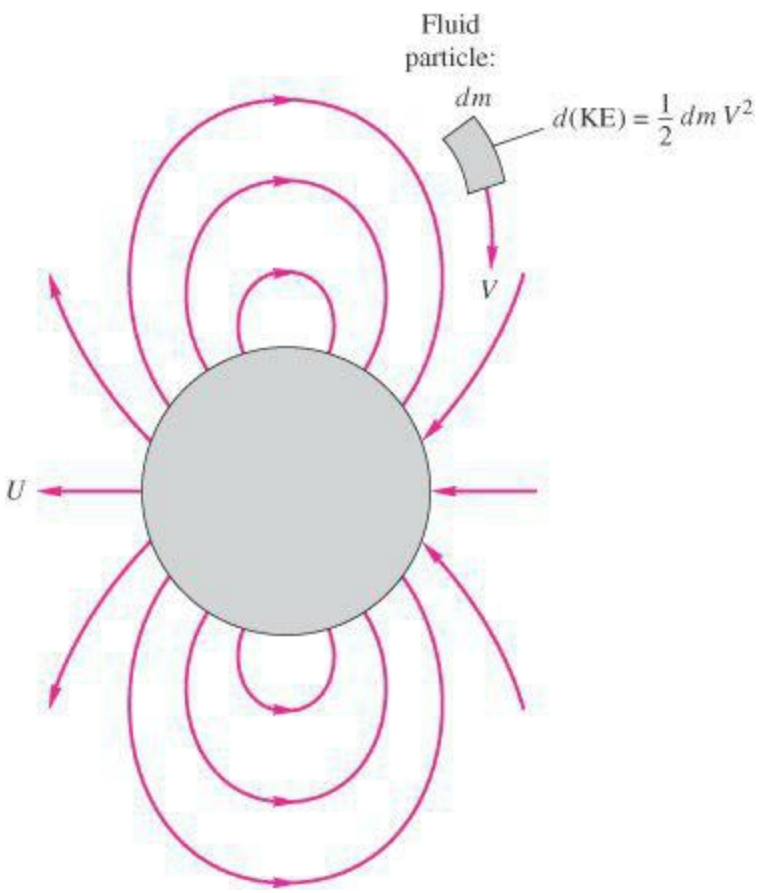
GP ! GEMLQ UGUG

: KHQGP DQG$V_{\text{rel}}^2 = v_r^2 + v_\theta^2$ DUHVXEVWLWXWHGLQWR(T WKHLQWUDWLRFDQEHHYDOXDWHG

$$\mathrm{KE}_{\text{fluid}} = \tfrac{1}{3}\rho\pi a^3 U^2$$

or
$$m_h(\text{sphere}) = \tfrac{2}{3}\rho\pi a^3 \qquad (8.103)$$

7KXV DFFRUGLQJ WR SRWHQWLDO WKHRU\ WKH KLGURG QDPLF PDVV RI D VSKHUH HTXDOV RQH KDOI RI LV GLVSODFHGPDVV LQGHSHQGHQWRI WKHGLUHFWLRQ RI PRWLRQ



) LJ 3RWHQWLDO IORZ VWUHDPOLQHV UHDFKLQJ VSKHUH &RPSDUH ZLWK) LJV DQG

$ VLPLODU UHVXOW RUDFNOHUPRYLQJ QRUPDO WR LWV D[LV FDQEHFRPSXWHG IURP (TV DV VXEWUDFWLQJ RXW WKH VWUHDP YHORFLW\ 7KHUHVXOW V

$$m_h(\text{cylinder}) = \rho\pi a^2 L \qquad (8.104)$$

IRU DFLQGHURI OHQJWK/ DVVXPLQJ VW R GLPHQVLRQDOP RWLRQ 7KHFLQGHU¶V KGURG QDPLF PDV HTXDOV LWV GLVSODFHG PDVV

7DEOHV RI K\GURG QDPLF PDVV IRU YDULRXV ERG\ VKDSHV DQG GLUHFWLRQV RI PRWLRQ DUH JLYHQ E\ 3DWRQ> @6HHDOR5H

1XPHULFDO$QDO\VLV

: KHQ SRWHQWLDO IORZ LQYROYHV FRPSOLFDWHG JHRPHWULHV RU XQXVXDO WUDP FRQGLWLRQV WKHFODVVLFD VXSHUSRVLWLRQ VFKHP H RI 6HFV DQG EHFRPHV OHVV DWWUDFWLYH &RQRUP DO PSSLQJ RI ERG\ VKDSHV E\ XVLQJ WKHFRP SOH[YDULDEOHWHFKQLTXHRI 6HF LVQR ORQJHUSRSXODU1XPHULFDOQDO\VLV LVWKHDSSURSULDWPRGHQDSSURDFK DQGDWDWKUHHGLIHUHQWDSSURDFKHVDUHLQXVH

7KHILQLWHHOPHQWP HWKRG)(0 > @
7KHILQLWHGLIHUHQFHPHWKRG)'0 > ± @
,QWUDOP HWKRGVZ LWKGLVWULEXWHGVLQJXODULWLHV> @
7KHERXQGDU HOPHQWP HWKRG> @

0 HWKRGV D DQG E DUH FRPPRQO\ UHIHUUHG WR DV LWHUDWLYH PHWKRGV EHFDXVH E\
DUHDN\QDPLFDOO\ LQ WKH V > @DQG WKHQ JHQHUDOL]HG LQWR D PXOWLSXUSRVH DSSOLFDWLRQ
VFKQLTXHLQWKH V> @

0 HWKRGV RU)(0 DQG RU)' 0 WKRXJK VWULNLQJO\ GLIIHUHQWLQFRQFHSWXDOFRPSDUDEOH
VFRSH PHWK VL]H DQG JHQHUDOLW\ : H FRQFHQWUDWH RQ WKH ODWWHU PHWKRG IRU LOOXVWUDWLRQ
SXUSRVHV

7KH) LQLWH(OHP HQW0 HWKRG

7KH ILQLWH HOHP HQW PHWKRG > @LV DSSOLFDEOH WR DOO WSHV RI OQHDU DQG QRQOQHDU SDUWLDO
GLIIHUHQWLDO HTXDWLRQV LQ SK\VLFV DQG HQJLQHHULQJ 7KH FRPSXWDWLRQDO GRP DLQ LV GLYLGHG LQWR VP DO
UHJLRQV XVXDOO WULDQJXODU RU TXDGULODWHUDO 7KHVH UHJLRQV DUH GHOQHDWHG Z LWK D ILQLWH QXPEHU RI
QRGHV Z KHUH WKH ILHOG YDULDEOHV VXP SHUDWLRQ YHORFLW\ SUHVVXUH VWHP IXQFWLRQ DQG VR RQ DUH
EH FDOFXODWHG 7KH VROXWLRQ LQ HDFK UHJLRQ LV DSSUR[LPDWHG E\ DQ DOJHEUDLF FRPELQDWLRQ RI ORFDO
QRGDO YDOXHV 7KH DSSUR[LPDWH IXQFWLRQV DUH LQWHUSRODWHG UYHU WKH UHJLRQ DQG WKLU HUURU L
PLQLP L]HG RIWQE\ XVLQJ DZHJKWQJ IXQFWLRQ 7KLV SURFHVV\LHOGV DVHWRI 1 DOJHEUDLF HTXDWLRQV IR
WKH1 XQNQRZ QQRGDOYDOXHV 7KHQRGDOHTXDWLRQV DUH VROYHG VLPXOWDQHRXV\ E\ PDWUL[LQYHUVLRQ R
LWHUDWLRQ) RUIXUWKHUGHWDLOVVHH5H RU

7KH) LQLWH' LIIHUHQFH0 HWKRG

$ OHRXJK VW[WHRRNV RQ QXPHULFDO DQDO\VLV > @DSSO ILQLWH GLIIHUHQFH WHFKQLTXHV WR PDQ
GLIIHUHQWSUREOPV KHUH Z H FRQFHQWUDWH RQSRVQQLDOIORZ 7KH IGHD RI) ' 0 LV WR DSSUR[LPDWH WK
SDUWLDO GHULYDWLYHV LQ D SK\VLFDOHTXDWLRQ E\ ³GLIIHUHQFH´ EHWZHQ QRGDOYDOXHV VSDFHG D ILQLW
GLVWDQFH DSDUW D VRUWR QXPHULFDOFDOFXOXV 7KH EDVLF SDUWDOGLIIHUHQWLDOHTXDWLRQ LV WKXV UHSODFH
E\ D VHWRI DOJHEUDLF HTXDWLRQV IRUWKH QRGDOYDOXHV) RUSRVQQLDO LQYLVFLG IORZ WKHVH DOJHEUDL
HTXDWLRQV DUH OLQHDU EXWWKH DUH JHQHUDOO QRQOLQHDUIRUYLVFRXVIORZ V 7KH VROXWLRQ IRUQRGDOYDOXH
LVREWDLQHG E\ LWHUDWLRQ RUP DWU[LQYHUVLRQ 1RGDOVSDFLQJV QHHGQRWEHHTXDO

+ HUH Z HLOOXVWUDWWKH WZR GLP HQVLRQDO/ DSODFH HTXDWLRQ FKRRVLQJ IRUFRQYHQLHQFHWKH VWHDP IXQFWLRQIRUP

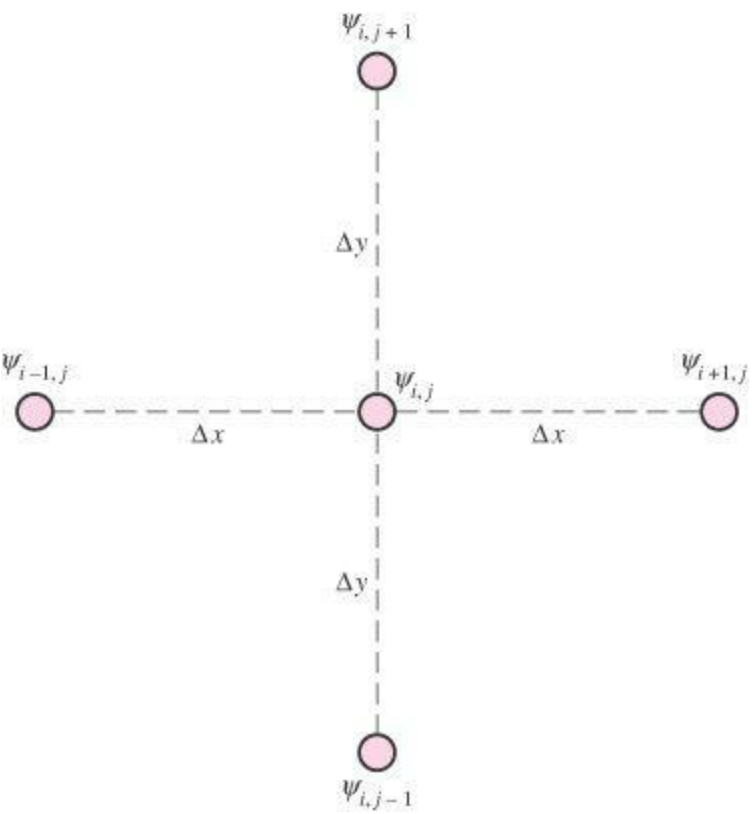
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad (8.105)$$

VXEVFWWR NQRZ QYDOXHV RI $ DORQJ DQ EORG\ VXUFH DQG NQRZ QYDOXHV RI +$ +[DQG+$ +\ LQWK
IUHHVWUHDP

2 XUILQLWH GLIIHUHQFH WHFKQLTXH GLYLGHV WKH IGRZ ILHOG LQWR HTXDO VSDFHG QRGHV DV VKRZ QL
) LJ 7R HFRQRP L]H RQ WKH XVH RI SDUHQWKHVHV RUIXQFWLRQDOQRWDWLRQ VXEVFULSWV DQG SRVLWLRQ RI DQ DUELWUDU\ HTXDO VSDFHG QRGH DQG$ LM GHQRWHV WKH YDOXH RI WKH VWUHDP IXQFWLRQ DWWKL
QRGH

$$\psi_{i,j} = \psi(x_0 + i\,\Delta x,\ y_0 + j\,\Delta y)$$

7KXV$ L M LVMXVWWR WKH ULJKWRI $ LM DQG$ LM LVMXVWDERYH



) LJ ' HILQLWRQVNHWFK IRUDVZR GLP HQVLRQDOUHFWDQJXODUILQLWHGLIIHUHQFHJULG

$QDOJHEUDLF DSSUR[LP DWLRQIRUWKHGHULYDWLYH%$ %[LV

$$\frac{\partial \psi}{\partial x} \approx \frac{\psi(x + \Delta x, y) - \psi(x, y)}{\Delta x}$$

$ VLPLODUDSSUR[LP DWLRQIRUWKHVHFRQGGHULYDWLYHLV

$$\frac{\partial^2 \psi}{\partial x^2} \approx \frac{1}{\Delta x}\left[\frac{\psi(x + \Delta x, y) - \psi(x, y)}{\Delta x} - \frac{\psi(x, y) - \psi(x - \Delta x, y)}{\Delta x}\right]$$

7KHVXEVFULSWQRWDWLRQP DNHVWKHVHH[SUHVVLRQVPRUHFRP SDFW

$$\frac{\partial \psi}{\partial x} \approx \frac{1}{\Delta x}(\psi_{i+1,j} - \psi_{i,j})$$
$$\frac{\partial^2 \psi}{\partial x^2} \approx \frac{1}{\Delta x^2}(\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}) \qquad (8.106)$$

7KHVHIRUPXODVDUHH[DFWLQWKHFDVHXQVOPLWDV û[""" EXWLQQXP HULFDODQDOVLVZHNHHS û[DQG û\ ILQLWH KHQFHWKHWUP ILQLWHGLIIHUHQFHV

,Q DQ H[DFW VLPLODU PDQQHU ZH FDQ GHULYH WKH HTXLYDOHQWGLIIHUHQFH H[SUHVVLRQV IRU WKH GLUHFWLRQ

$$\frac{\partial \psi}{\partial y} \approx \frac{1}{\Delta y}(\psi_{i,j+1} - \psi_{i,j})$$
$$\frac{\partial^2 \psi}{\partial y^2} \approx \frac{1}{\Delta y^2}(\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}) \qquad (8.107)$$

7KH XVH RI VXEVFULSWQRWDWRQ DOORZV WKH VH[SUHVVLRQV WR EH SURJUDPPHG GLUHFWO LQWR D VFLHQWL FRPSXWHUODQJXDJH VXFK DV %$6,& RU) 25 75$1
: KHQ DQG DUH VXEVWLWXWHG LQWR / DSODFH¶V HTXDWLRQ WKH UHVXOWLV WK DOJHEUDLF IRUP XOD

$$2(1 + \beta)\psi_{i,j} \approx \psi_{i-1,j} + \psi_{i+1,j} + \beta(\psi_{i,j-1} + \psi_{i,j+1}) \qquad (8.108)$$

ZKHUH û[û\ GHSHQGV RQ WKH PHVK VL]H VHOHFWHG 7KLV ILQLWH GLIIHUHQFH PRGHO RI / DSODFH¶ HTXDWRQ VXDWHV WKDW WKH WHPS IXQFWLRQ YDOXH$LM LV D OLQHDU FRPELQDWLRQ RI LWV IRXU QHDUHV QHLJKERUV
7KH PRVW FRPPRQ SURJUDPPHG FDVH LV DVTXDUH PHVK IRUZKLFK(T UHGXFH VR

$$\psi_{i,j} \approx \tfrac{1}{4}(\psi_{i,j+1} + \psi_{i,j-1} + \psi_{i+1,j} + \psi_{i-1,j}) \qquad (8.109)$$

7KXV IRUD VTXDUH PHVK HDFK QRGDO YDOXH HTXDOV WKH DULWKPHWLF DYHUDJH RI WKH IRXU QHLJKERU VKRZ Q LQ) LJ 7KH IRUPXOD LV HDVLO UHPHEHUHG DQG HDVLO SURJUDPPHG ,I 3 , - LV VXEVFULSWHG YDULDEOH WKHQ IXQFWLRQ WKH%$6,& RU) 25 75$1 VWDWHPHQW LV

```
P(I, J) = 0.25 * (P(I, J + 1) + P(I, J - 1) + P(I + 1, J) + P(I - 1, J))
```
(8.110)

7KLV LV DSSOHG LQ LWHUDWLYH IDVKLRQ VZHHSLQJ RYHU HDFK RI WKH ILQLWH GLIIHUHQFH PHVK SRLQWV , - ZLWK NQRZ YDOXHV RI 3 VSHFLILHG DW HDFK RI WKH VXUURXQGLQJ ERXQGDU QRGHV $Q LQLWLDO JXHVV HVFDQEH VSHFLILHG IRU WKH ILQLWH GLIIHUHQFH 3 , - DQG WKH LWHUDWLRQ SURFHVV ZLOO FRQYHUJH WR WKH ILQDO DQVZHU D ILQLWH QXPEHU RI VZHHSV 7KH QXPHULFDO HUURU FRPSDUHG ZLWK WKH H[DFW VROXWLRQ RI / DSODFH¶ HTXDWRQ LV SURSRUWLRQDO WR WKH VTXDUH RI WKH PHVK VL]H
&RQYHUJHQFH FDQEH VSHHGHG XS E\ WKH VXFFHVVLYH RYHUUHOD[DWLRQ 625 PHWKRG GLVFXVVHG E &HEHFL> @7KH PRGLILHG625 IRUP RI WKH LWHUDWLRQ LV

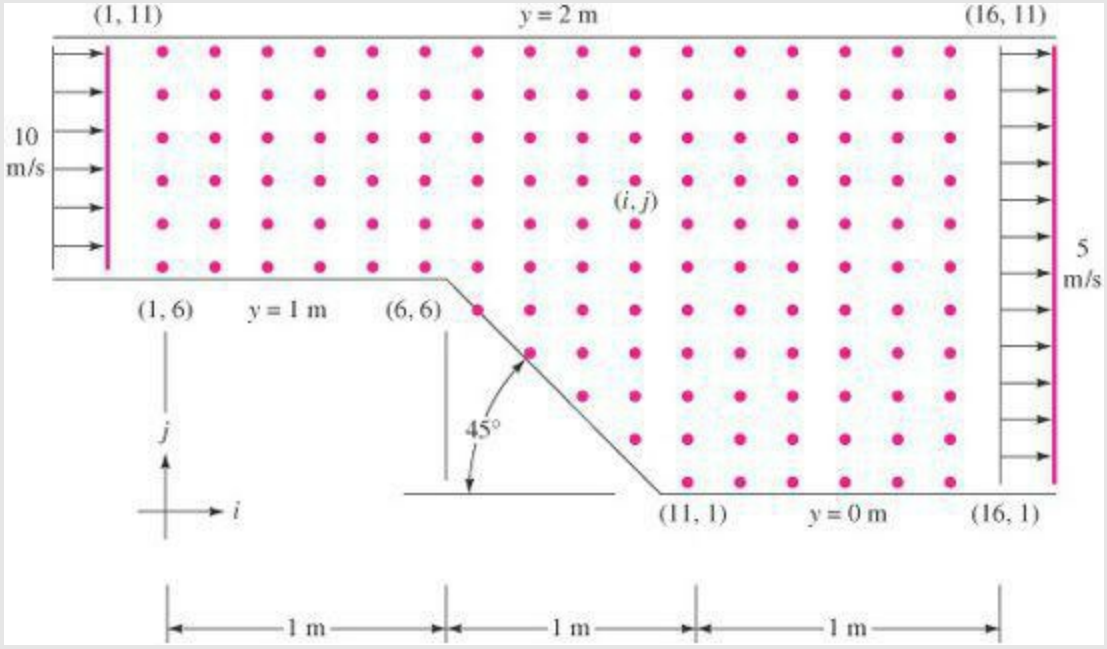
```
P(I, J) = P(I, J) + 0.25 * A * (P(I, J + 1) + P(I, J - 1)
+ P(I + 1, J) + P(I - 1, J) - 4 * P(I, J))
```
(8.111)

7KHUHFRPPHQGHG YDOXH RI WKH625 FRQYHUJHQFH IDFWRU$ LV DERXW 1RWH WKDW WKH YDOXH$ UHGXFHV(T WR
/HVXVLOOXVWUDWH WKH ILQLWH GLIIHUHQFH PHWKRG ZLWK DQH[DPSOH

(; $0 3/ (

0 DNH D QXPHULFDO DQDO VLV XVLQJ û[û\ P RI SRWHQWLDO IRU WKH GXFW[SDQVLRQ VKRZ LQ) LJ 7KH IOOZ HQWV DVVXPH XQLIRUP P V ZKHUH WKH GXFW ZLGWK LV P DQG LV DVVXPHG W

GDYHDWDXQLRUP YHORFLWR P V ZKHUH WKH GXFW ZLGWK LV P 7KHUHLV DVWDJKVWHFWRQ P ORQJ D f H[SDQVLRQ VHFWLRQ DQG DILQDO VWDJKVWHFWLRQ P ORQJ



) LJ 1XPHULFDOPRGHORI SRWHQWLDOIORZ WKURXJK DVZR GLPHQVLRQDO f H[SDQVLRQ 7K QRGDOSRLQWVVKRZQDUH FP DSDUW7KHUHDUH ERXQGDU QRGHVDQG LQWHUQDOQRGHV

6RXWLRQ

8VLQJ WKH PHWK VKRZ QLQ) LJ UHVXOWLQ ERXQGDU QRGHV DQG LQWHUQDOQRGHV ZLWK YDULQJ IURP VR DQGMYDULQJ IURP VR 7KHLQWHUQDOSRLQWVDUHPRGHOHGE\ (T) R FRQYHQLHQFH WHWKHVWUHDP IXQFWLRQEH]HUR DORQJ WKHORZ ZDOO 7KHQVLQFHWKHYROXPHIORZ LV P V P P VSHUXQLWGHSWK WKHVWUHDP IXQFWLRQPXWWHTXDO P VDORQJ WKHXSSHUZDOO 2YHU WKHHQWDQFHDQGH[LWSODQHV WKHVWUHDP IXQFWLRQPXWYDU OLQHDUO VRJLYHXQLRUP YHORFLWLHV

Inlet:	$\psi(1, J) = 2 * (J - 6)$	for J = 7 to 10
Exit:	$\psi(16, J) = J - 1$	for J = 2 to 10

$ OWKHVHERXQGDU YDOXHVPXWEHLQSXWRWKHSURJUDP DQGDUHVKRZQSULQWHGLQ) LJ

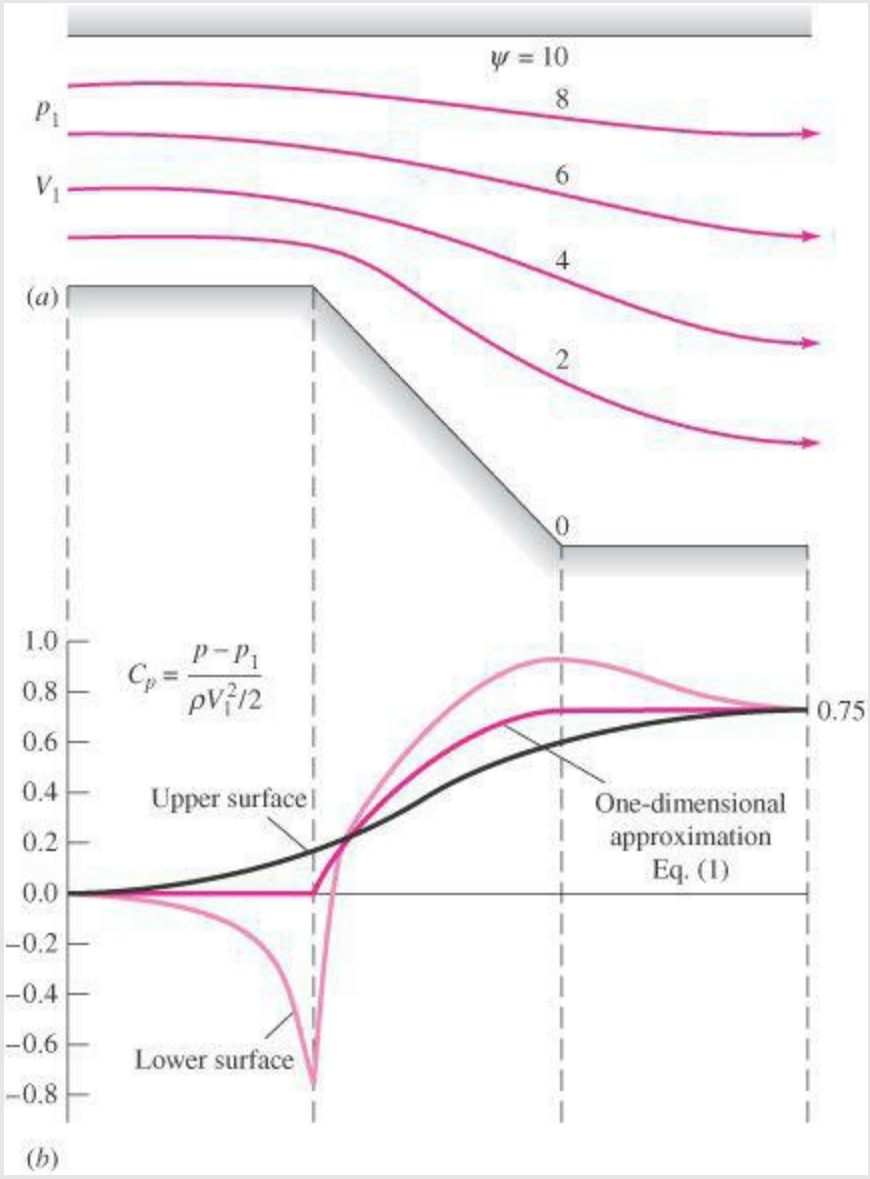
ψ = 10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
8.00	8.02	8.04	8.07	8.12	8.20	8.30	8.41	8.52	8.62	8.71	8.79	8.85	8.91	8.95	9.00
6.00	6.03	6.06	6.12	6.22	6.37	6.58	6.82	7.05	7.26	7.44	7.59	7.71	7.82	7.91	8.00
4.00	4.03	4.07	4.13	4.26	4.48	4.84	5.24	5.61	5.93	6.19	6.41	6.59	6.74	6.88	7.00
2.00	2.02	2.05	2.09	2.20	2.44	3.08	3.69	4.22	4.65	5.00	5.28	5.50	5.69	5.85	6.00
ψ = 0.00	0.00	0.00	0.00	0.00	0.00	1.33	2.22	2.92	3.45	3.87	4.19	4.45	4.66	4.84	5.00
						0.00	1.00	1.77	2.37	2.83	3.18	3.45	3.66	3.84	4.00
							0.00	0.80	1.42	1.90	2.24	2.50	2.70	2.86	3.00
								0.00	0.63	1.09	1.40	1.61	1.77	1.89	2.00
									0.00	0.44	0.66	0.79	0.87	0.94	1.00
										0.00	0.00	0.00	0.00	0.00	0.00

) LJ 6WUHDP IXQFWLRQ QRGDOYDOXHV IRUWKH SRWHQWLDOIORZ RI) LJ %RXQGDU YDOXH

DUH NQRZ QLQSXW , QWHUQDOOQRGHV DUH VRXWRQV WR (T

,QWHJUDWHV DUH VRUWHG IRUWKHLQWHUQDOSRLQWV VD]HUR RUDQ DY HUDJH YDOXH RI P V 7K
SURJUDP WKHQ VWDUWV DWZQ FRQYHQLHQWSRLQWXFKDVWKHXSSHUOHW DQGHYDOXDWHV(T D
HYHU LQWHUQDOSRLQWUHSHDWQJ WKLV VZHHS LWHUDWLYH XQWWKHUHDUHQR IXUWKHUFKDQJHV ZLWKLQVRPH
VHOHFWHG PD[LPXP FKDQJH LQWHQGDYDOHV 7KHUHVXOWV DUH WKHILQWHGLIHUHQFHVLPXODWRQ WK
SRWQQDOIORZ IRUWKLV PHVK VL]H WKH DUH VKRZQ SULQWHG) LJ VR WKUHGLJLWDFFXUDF\ 7K
UHDGHUVKRXOGWWWD IHZ QRGHVLQ) LJ VR YHUL\ WKDW(T LV VDWLVILHGYHUZKHUH 7K
QXPHULFDOFFXUDF\ RI WKHVH SULQWHG YDOXHV LV GLIILFXOWRHWPDW VLQFH WKHUH LV QR NQRZQ H[DF
VROXWQ WR WKLV SUREOP ,QSUDFWH RQH ZRXGNHHS GHFUHDVLQJ WKHPHVK VL]H XQWWKHUHZHUHQ
VLJQLILFDQWFKDQJHVLQQRGDOYDOXHV

$QRWKR) LJ LV WKHFRPSXWHUSORWWQ WKHSURBOP WKHVHQXPEHUVPXWEHPDQSXODW
WR \LHOG SUDFWFDOHQJLQHULQJ UHVXOW) RU H[DPSOH RQH FDQ LQWUSRODW WKHVH QXPEHUV WR VNHWF
YDULRXV VWUHDPOQHV RI WKH IBOZ 7KLV LV GRQH LQ) LJ D : HVHH WKDWWKH VWUHDPOQHV DUH FXUYHG
ERWXSWUHDP DQG GRZQWUHDP RI WKH FRUQHUUHJLRQ HVSHFLDOO QHDU WKH IBOZ HUZ DOO 7KLV LQGLFDW
WKDWWKH IBOZ LVQRWSRQHGLPHQVLRQDO



) LJ 8VHXOUHVXOWV FRPSXWHG IURP) LJ *a* VWUHDPOQHV RI WKH IBOZ *b* SUHVVXU
FRHIILFLHQWGLVWULEXWRQDORQJ HDFK ZDOO

7KH YHORFLWLHV DW DQ SRLQW LQ WKH IORZ FDQ EH FRPSXWHG IURP ILQLWH GLIIHUHQFH IRUPXODV VXFK DV (TV DQG) RU H[DPSOH DW WKH SRLQW , - IURP (T WKH KRUL]RQWDO YHORFLW\ LV DSSUR[LPDWHO

$$u(3, 6) \approx \frac{\psi(3, 7) - \psi(3, 6)}{\Delta y} = \frac{2.09 - 0.00}{0.2} = 10.45 \text{ m/s}$$

DQG WKH YHUWLFDO YHORFLW\ LV]HUR IURP (T ' LUHFW DERYH WKLV RQ WKH XSSHU ZDOO ZKHUH WKH

$$u(3, 11) \approx \frac{\psi(3, 11) - \psi(3, 10)}{\Delta y} = \frac{10.00 - 8.07}{0.2} = 9.65 \text{ m/s}$$

7KH IORZ LV QRW WKH RQH GLPHQVLRQDO LQ WKH HQWUDQFH GXFW 7KH IORZ HUZ DOO ZKLFK FRQWLQV WK GLYHUJLQJ VHFWLRQ DFFHOHUDWHV WKH IOXLG ZKLOH WKH IORZ XSSHUZDOO VHSDUDWHV DQG GHFHOHUDWHV WKH IOXLG $ QRWKHU RXSSXW XQFWLRQ XVHIXO LQ PDNLQJ ERXQGDU\ OD\HU DQDOVHV RI WKH ZDOO UHJLRQ LV WKH SUHVVXUH GLVWULEXWLRQ DORQJ WKH ZDOOV ,I DQG9 DUH WKH SUHVVXUH DQG YHORFLW\ DW WKH HQWUDQFH , FRQGLWLRQV DQG RWKHU SRLQWV DORQJ %HUQRXOOL¶V HTXDWLRQ QHJOHFWLQJ JUDYLW\

$$p + \tfrac{1}{2}\rho V^2 = p_1 + \tfrac{1}{2}\rho V_1^2$$

ZKLFK FDQ EH UHZ ULWWHQ DV DGLPHQVLRQOHVV SUHVVXUH FRHILFLHQW

$$C_p = \frac{p - p_1}{\frac{1}{2}\rho V_1^2} = 1 - \left(\frac{V}{V_1}\right)^2$$

7KLV GHWHUPLQHV S DW U9 LV FRPSXWHG IURP WKH WUHDP IXQFWLRQ GLIIHUHQFH V LQ) LJ) LJXUH E VKRZ V WKH FRPSXWHG ZDOO SUHVVXUH GLVWULEXWLRQV DV FRPSDUHG ZLWK WKH RQH GLPHQVLRQDO FRQWLQXLW\ DSSUR[LPDWLRQ 9 $ § 9 [$ [RU

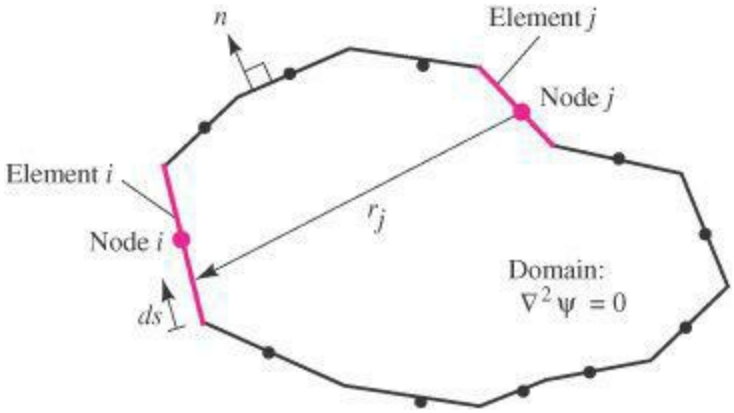
$$C_p(\text{one-dim}) \approx 1 - \left(\frac{A_1}{A}\right)^2 \qquad (1)$$

7KH RQH GLPHQVLRQDO DSSUR[LPDWLRQ ZKLFK LV UDWKHU FUXGH IRU WKLV GUJH f H[SDQVLRQ GH EHWZHHQ WKH XSSHU DQG ORZ HUZ DOO SUHVVXUHV 2 QH GLPHQVLRQDO WKHRU\ ZRXOG EH PXFK PRUH DFFXUDWH IRU D f H[SDQVLRQ $ QDO]LQJ) LJ E ZH SUHGLFW WKDW WKH ERXQGDU\ OD\HU V HSDUDWLQJ ZLOO SURGXFH D VPDOO UHFLUFXODWLQJ UHJLRQ RI ORZ HU ZDOO SUHVVXUH EHWZHHQ WKH FRUQHUV ZKHUH SUHVVXUH LV VWURQJO\ UHGXFHG JUDGLHQW 7KHUHRUH SRVWXODWH D VHSDUDWHG ORZ SUHVVXUH UHJLRQ DV VKRZQ LQ) LJV DQG 3RVWXODWH WKH RUL HQW HG VKDSH ZKHQ ZH UHYHUVH WKH IORZ DURXQG) LJ D WKHQ) LJ E LV VXJJHVWHG DQG ZRXOG UHSUHVHQW D f FRQWUDFWLRQ IORZ 7KH SUHVVXUH ZRXOG IDOO RQ ERWK ZDOOV QR VHSDUDWLRQ IURP [P WR[P %HWZHHQ[P DQG[WKH SUHVVXUH UHPDLQV QHD

GRZ HUVXUFH LQGLFDWLQJ SRVLEOH VHSDUDWLRQ SUREDEO MXVWGRZQVWHP RI VKHFRUQHU
7KLV H[DPSOH VKRXOG JLYH WKH UHDGHU DQ LGHD RI WKH XVHXOQHVV DQG JHQHUDOLW RI QXPHULFD DQDOVLV IOXLG IORZ V

7KH%RXQGDU\ (OHPHQW0HWKRG

$ UHODWLYHO QHZ WHFKQLTXH IRUQXPHULFDOVROXWLRQ SDUWLDOGLIIHUHQWLDHTXDWLRQVLVWKHERXQGDU HOHPHQWPHWKRG %(0 5HHUHFH LV DQ LQWRGXFWRU WH[WERRN RXWOLQLQJ WKH FRQFHSWV RI %(0 7KHUHDUHQR LQWHULRUHOHPHQW 5DWKHU DOOQRGHVDUHSODFHGRQWKEERXQGDU RI WKHGRPDLQ DVLQ) LJ 7KH³HOHPHQW VLV D VPDOOSLHFH RI WKHERXQGDU VXUURXQGLQJ WKHQRGH 7KH³VWHQWK RI WK HOHPHQWDQEHLWKHUFRQVWDQWRUYDULDEOH



) LJ %RXQGDU HOHPHQWV RI FRQVWDQWVWHQWK LQSDQHOSRWHQWLDOFRZ

) RUSDQHSRWHQWLDOFRZ WKHP HWKRGVWDNHVDGYDQWDJH RI WKHSDUWFXODUVROXWLRQ

$$\psi^* = \frac{1}{2\pi} \ln \frac{1}{r} \qquad (8.112)$$

ZKLFK VDWLVILHV/ DSODFH¶VHTXDWLRQ $ (DFKHOHPHQWLVDVXPHGWRKDYHDGLIIHUHQWVWHQWV $L 7KHQUHSUHVHQWWKHGLVWDQFHIURP WKDWHOHPHQWRQ RWKHUSRLQWLQWKHIORZ ILHOG 6XPPLQJ DO WKHVHHOHPHQWDOH IHFW ZLWKSURSHUERXQGDU FRQGLWRQV ZLOOJLYH WKHVROXWLRQWR WKHSRWHQWL IORZ SUREOP

$WDFK HOHPHQWRI WKHERXQGDU ZH WSLFDOO NQRZ HLWKHUWKHYDOXHRI $RUWKHYDOXHRI %$%Q ZKHUHQLVQRUP DOWRWKHERXQGDU 0L[HG FRP ELQDWRQV RI $DQG%$%Q DUHDOR SRVVLEOHEXWDUHQR GLVFXVVHGKHUH 7KHFRUUHFWXWHQJWKV$L DUHVXFK WKDWWKHVHERXQGDU FRQGLWRQV DUHVDWLVILHGDWHYHU HOHPHQW 6XPPLQJ WKHVH HIHFW RYHU1 HOHPHQW UHTXLUHV LQWHUDWRQ E\ SDUW SOXV D FDUHX HYDOXDWRQRI WKH VLQJXODU HIHFWRI HOHPHQWLXSRQLWVHO 7KHPDWKHPDWFDOGHWDLOV DUHJLYHQLQ5H 7KH UHVXOWLV D VHWRI 1 DJHEUDLF HTXDWRQV IRUWKH XQNQRZQ ERXQGDU YDOXHV ,Q WKH FDVH R HOHPHQWVRI FRQVWDQWVWHQWK WKHILQDOH[SUHVVLRQV

$$\frac{1}{2}\psi_i + \sum_{j=1}^{N} \psi_j \left(\int_j \frac{\partial \psi^*}{\partial n} ds \right) = \sum_{j=1}^{N} \left(\frac{\partial \psi}{\partial n} \right)_j \left(\int_j \psi^* ds \right) \qquad i = 1 \text{ to } N \qquad (8.113)$$

7KH LQWURGXFWLRQ RI WKH GRJDULWKPLF SDUWLFXODU VROXWLRQ$ IURP (T DU
HYDOXDWHG QXPHULFDOO\ IRU HDFK HOHPHQW

(TXDWLRQV FRQVLQ 1 HOHPHQWYDOXHV $L DQG %$%Q L RI ZKLFK1 DUH NQRZQ IURP WK
JLYHQ ERXQGDU\ FRQGLWLRQV 7KHUHPDLQLQJ1 DUH VROYHG VLPXOWDQHRXVO IURP (TV * HQHUDO
WKLV FRPSOHWHV WKH DQDOVLV² RQO WKH ERXQGDU\ VROXWLRQ LV FRPSXWHG DQG LQWHUIRU SRLQWV DUH QH
VXEGLG ,QPRVWPDVHV WKHERXQGDU\ YHORFLW DQG SUHVXUH DUH DOWKDWLV QHHGHG

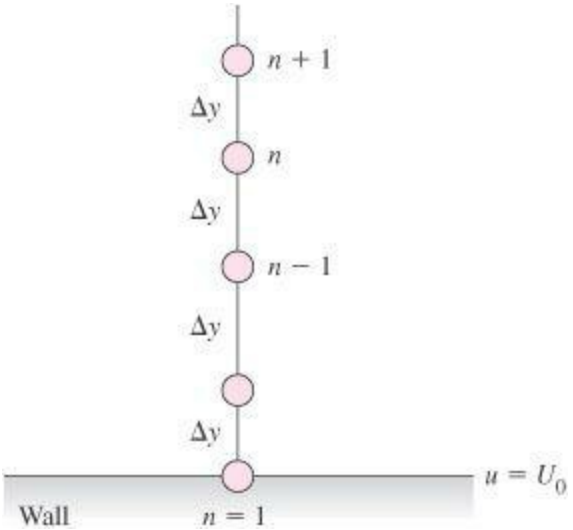
: HLOOXVWUDWH WKH PHWKRG ZLWK VWHDP IXQFWLRQ$ 1 DXUDO WKH HQWHUWFKQLTXH DOR DSSOHV VL
YHORFLW SRWHQWLDOË LI ZH DUH JLYHQ SURSHUFRQGLWLRQV RQË RU%Ë %Q DW HDFK ERXQGDU HOHPHQW7K
PHWKRGLVUHDGLO H[WQGHG WR WKUHHGLPHQVLRQV> @

5HHUHQFH LV D JHQHUDO LQWURGXFWLRQ WR ERXQGDU HOHPHQW ZKLOH 5H HPSKDV]H
SURJUDPPLQJ PHWKRGV 0HDQ ZKLH UHVHDUFK FRQWQX' DUJXWKDQG* UJRUHY> @KDYHGHYHORSHGI
PXOWLYHOERXQGDU HOHPHQW PHWKRG IRUVWDG\ 6WRNHVRUFUHSLQJ IORZV VHH6HF LQLUHJXO
JHRPHWULHV 7KHUVFKHPH DYRLGV WKH KHDY\ PHPRU DQG &38 WPH UHTXLUHPHQW RI PRWERXQGU
HOHPHQWPHWKRGV 7KH\ HWWP DW WKDW&38 WPH LV UHGXFHG E\ DIDFWRU RI DQG UHTXLU
PHPRU LVUHGXFHGE\ DIDFWRUR

9 LVFRXV) ORZ &RP SXWMU0 RGHOV

2 XU SUHYLRXV ILQLWH GLIIHUHQFH PRGHORI / DSODFH¶V HTXDWLRQ DV LQ(T ZDV YHU ZHO
EHKDYHG DQG FRQYHUJHG QLFH0 ZLWK RUZLWKRXW RYHUHDIDWRQ 0 XFK PRUH FDUHLV QHHGHG WR PRG
WKH IXOO1 DYLHU6WRNHV HTXDWLRQV 7KHFKDOOHQJHV DUH TXLWH GLIIHUHQW DQG WKH KDYHEHHQ PHWRDOJ
H[WQVWR WKH UHDUHQW PDQ WH[WERRNV> WR @RQ IXOO YLVFRXV FRPSXWDWLRQDOOIOXL
G\QDPLFV &)' 7KLV LV QRWD WH[WERRN RQ &)' EXW ZH ZLOODGGUHVV VRPH RI WKH LWXHV LQ WK
VHFWLRQ

2 QH' LP HQVLRQDO8 QVWHDG\ ) ORZ



) LJ $ QHTXDOO VSDFHG ILQLWH GLIIHUHQFH PHVK IRU RQH GLPHQVLRQDOYLVFRXV IORZ >(T
@

: HEHJLQ ZLWK D VLPSOLILHG SUREOP VKRZLQJ WKDWHYHQ DVLQJOH YLVFRXV WHUP LQWRGXFHV QHZ
HIHFWDQG SRVWEOHLQXDELOLWLHV 5HFDOO RUUHYLHZ 3URE 3 ZKHUHD ZDOOPRYHVDQGGULYHVI

YLVFRXV IOXLG SDUDGHDWR LWHO * UDYLWV LV QHJOHFWG / HWKH ZDOOEHWKH SDQH< PRYLQJ DW
VSHHG8 W DV LQ) LJ $ XQLIRUP YHUWFDOJUDG RI VSDFLQJ û\ KDV QRGHVQ DWZ KLFK WKH ORFD
YHORFLWV LV WR EH FDOFXODWG ZKHUH VXSHUVFULSWVGHQRWV WKH WLPH VWHS M W7KH ZDOOLVLQ ,I X
X\ WRQD DQG% Z FRQVQXLW ∇ 9 LV VDWLHG DQG ZH HQHGRQ VROVH WKH[
PRPHQXP 1DYLHU6WRNHVHTXDWRQ

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \qquad (8.114)$$

ZKHUH ! 8WDQLQJ WKH VDPH ILQLWHUHQFH DSSUR[LPDWRQV DV LQ(T ZH PD
PRGHQ(T DQJEUDLFDO DVDIRUZDUGWLPHGLIIHUHQFHDQGDFHQWUDOVSDWDOGLIIHUHQFH

$$\frac{u_n^{j+1} - u_n^j}{\Delta t} \approx \nu \frac{u_{n+1}^j - 2u_n^j + u_{n-1}^j}{\Delta y^2}$$

5HDUUDQJHDQGILQGWKDWZHFDQVROYHH[SOLFLWO IRUX DWWKHQH[WWLPHVWHS M

$$u_n^{j+1} \approx (1 - 2\sigma)\, u_n^j + \sigma(u_{n-1}^j + u_{n+1}^j) \qquad \sigma = \frac{\nu \Delta t}{\Delta y^2} \qquad (8.115)$$

7KXVXDWQRGHQDWWKHQH[WWLPHVWHS M LVDZHLJKWHGDYHUDJHRI WKUHHSUHYLRXVYDOXHV VLPLOD
WRWKH³IRXUQHDUHVWQHLJKERUV´ DYHUDJHLQWKHOSODFLDQPRGHORI (T 6LQFHWKHQHZ YHORFLW
LVFDOFXODWGLPPHGLDWO (T LVFDOFXODWHGDQH[SOLFLWPRGHO,WGLIIHUVIURP WKHZHOOEHKDYH
ODSODFLDQPRGHO KRZHYHU EHFDXVHLWPD\ EHXQVWDEOH 7KHZHLJKWQJ FRHIILFLHQWVLQ(T
PXWDOOEHSRVLWLYHWRDYRLGGLYHUJHQFH 1RZ 1 LVSRVLWLYH EXW í 1 PD\ QRWEH 7KHUHRUH RX
H[SOLFLWYLVFRXVIORZ PRGHOKDVDVWDELOLW\UHTXLUHPHQW

$$\sigma = \frac{\nu \Delta t}{\Delta y^2} \le \frac{1}{2} \qquad (8.116)$$

1RUPDOO RQHZRXOGILUWWHWXS WKHPHVK VL]Hû\ LQ) LJ DWWUZKLFK(T ZRXO
OPLWWKHWLPHVWHSûW7KHVROXWRQVIRUQRGDOYDOXHVZRXOGWKQEHEXWQRWQHFHVVDULO WKL
DFFXUDW7KHPHVK VL]HVû\ DQGûWFRXOGEHUHGXFHGWRLQFUHDVHDFFXUDF\ VLPLODUWR WKHFDVHRI WKH
SRWQWDOOIORZ ODSODFLDQPRGHO
)RUH[DPSOH WRVROYH3URE 3 QXPHULFDOO RQHVHWXS DPHVK ZLWKSHQWVRI QRGHV R
PRUH û\ ZLWKLQ WKH H[SHFWHG YLVFRXV OD\HU VHOHFWûWDFFRUGLQJ WR(T DQG VHW VWF
ERXQGDU FRQGLWRQV IRUDOOMX 8 VLQ&WDQGX1 ZKHUH1 LVWKHRXWHUPRVWQRGH)RULQLWL
FRQGLWRQV SHUKDSVDVXPHWKHIOXLGLQLWLDOO DWUHVWX X IRU ” Q” 1 í 6ZHHSLQJ WKHQRGHV ”
Q ” 1 í XVLQJ(T DQ([FHOVSUHDGVKHHWLV H[FHOOHQWRU WKLV RQH JHQHUDWHV QXPHULFD
YDOXHV RI X IRU DV QRJ DV RQH GHVLUHV $IWU DQ LQLWLDO WUDQVLHQW WKH ILQDO³VWDG´ IOXL
RVFLOODWRQ ZLOODSSURDFK WKHFDWLFDOVROXWRQLQYLVFRXVIORZ W[WERRNV> @7U 3URE 3 W
GHPRQXDWWKLV

Q OMUQDWXH,P SOLFLW$ SSURDFK

,QP DQ ILQLWHGLIIHUHQFHSUREOHPV DVWDELOLWLQDOLVDWLRQVXFK DV(T UHTXLUHVDQH[WHPHG VP DOOWLPHVWHS 7RDGGUHVV ODUJHUVWHSV RQHFDQUHFRJQLVHDWWKHPRGHOLQDQLPSOLFLWDWKLRQE\ HYDOXDWLQJ WKHVHFRQG GHULYDWLYHPRGHOLQ(T DWWKHQH[WWLPHVWHS

$$\frac{u_n^{j+1} - u_n^j}{\Delta t} \approx \nu \frac{u_{n+1}^{j+1} - 2u_n^{j+1} + u_{n-1}^{j+1}}{\Delta y^2}$$

7KLVUHDUUDQJHPHQWVXQFRQGLWLRQDOO VWDEOHIRUDQ 1 EXWQRZ ZHKDYHWKUHHXQNQRZQV

$$-\sigma u_{n-1}^{j+1} + (1 + 2\sigma)u_n^{j+1} - \sigma u_{n+1}^{j+1} \approx u_n^j \qquad (8.117)$$

7KLVLVDQLPSOLFLWPHWKRGHOPHDQLQJWKDWRQHPXVWVROYHDOLQHDUV\VWHP RI DOJHEUDLF HTXDWLRQVIR WKHQHZ QRGDOYDOXHVDWWKHM) RUWXQDWHO WKHV\VWHP LVQDUURZ 0 EDQGHG ZLWKWKHXQNQRZQ FRQLQHGWRWKHSUHFHGLQJDQGOLQJ QRGHDUWKHVHFRQG GHULYDWLYHPDNHVWKHFRHILFLHQWPDWUL RI(T LVWULGLDJRQDO D KDSS\ HYHQW$ GLUHFWPHWKRG FDQHG WKHWULGLDJRQDO PDWUL DOJRULWKP 7' 0 $ LV DYDLODEOHDQG H[SODLQHGLQPRVW&)' WH[WV> WR @,I \RX KDYHQ OHDUQHG7' 0 $ \HW(T FRQYHUJHVVDWLVIDFWRULO E\ UHDUUDQJHPHQWDQG LWHUDWLRQ

$$u_n^{j+1} \approx \frac{u_n^j + \sigma(u_{n-1}^{j+1} + u_{n+1}^{j+1})}{1 + 2\sigma} \qquad (8.118)$$

$WHDFKWPHVWHSM VZHHSWKHQRGHV ” Q” 1 í RYHUDQGRYHUXVLQJ(T XQOLWK QRGDOYDOXHVKDYHFRQYHUJHG 7KLVLPSOLFLWPHWKRGLVWDEOHIRUDQ 1 KRZHYHUDGH 7R HQVXU DFFXUDF\ WKRXJK RQHVKRXOGNHHSûWDQGû\ VPDOOFRPSDUHGWRWKHEDLF WPHDQGHQJWKVFDOHVR WKHSUREOP 7KLVDXWKRUJ¶VKDELWVWRNHSûWDQGû\ VPDOOHQRXJKWKDWQRGDOYDOXHVFKDQJHQR PRUH WKDQ SHUFHQWURP RQH Q MWRWKHQH[W

(; $0 3/ (

6$(RLODW *& LVDWUHWWQHDUDZDOOZKHQWKHZDOOVXGGHQO EHJLQVPRYLQJ DWDFRQWDQW P V 8VLQJ WKHH[SOLFLWPHWKRGRI(T HWWPDWWKHRLOYHORFLWDW FP DWU VHFRQGR ZDOOPRWRQ

6RXWLRQ

)RU6$(RLOIURP 7DEOH$ (P V)RUFRQYHQLHQFHLQSXWQJ DQRG H[DFWO DW FP FKRRVHû\ P 7KHWXEHLQWOP LV ûWû\ RUûW V $JDLQIRUFRQYHQLHQFH WR KLW VRQWKHQRVH FKRRVHûW V RU1 DQG í 1 7KHQRXU[SODLWDJEUDLF PRGHO IRUWKLVSUREOP LV

$$u_n^{j+1} \approx 0.3491\, u_n^j + 0.3255(u_{n-1}^j + u_{n-1}^j) \qquad (1)$$

: HDSSO WKLVUHDWRQIURP Q RXWRDWDWQ 1 WRPDNHXUHWKDWWKHGHVLUHGYDOXHR XDWQ LVDFFXUDW 7KHZDOOR VOS ERXQGDU UHTXLUH $u_1^j = 1.0$ P V FRQWDQWWRUDOOM 7KHRXWERXQGDU FRQGLWRQ LV X_1 7KHLQWLDOFRQGLWRQV DUH X$_Q$ IRUQ : HWKHQ DSSO(T

UHSHDWHG IRUQ XQWLOZ HUHDFKM ZKLFK FRUUHVSRQGVWR V 7KLVLVHDVLO SURJUDPPH
RQD VSUHDGVKHHWXFKDV([FHO+HUHZHSULQWRXWRQO M DQG DVIROORZV

j	t	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}
1	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.500	1.000	0.601	0.290	0.107	0.027	0.004	0.000	0.000	0.000	0.000	0.000
11	1.000	1.000	0.704	0.446	**0.250**	0.123	0.052	0.018	0.005	0.001	0.000	0.000

2XUQXPHULFDOHVWLPDWLVX X FP V § P V ZKLFK LV DERXW SHUFHQWKLJK² WKL
SUREOHP KDV DNQRZQ QH[DFWVROXWLRQ X P V> @: HFRXOGLP SURYHWKHDFFXUDF\ LQGHLQLWHO
E\ GHFUHDVLQJ û\ DQGûW

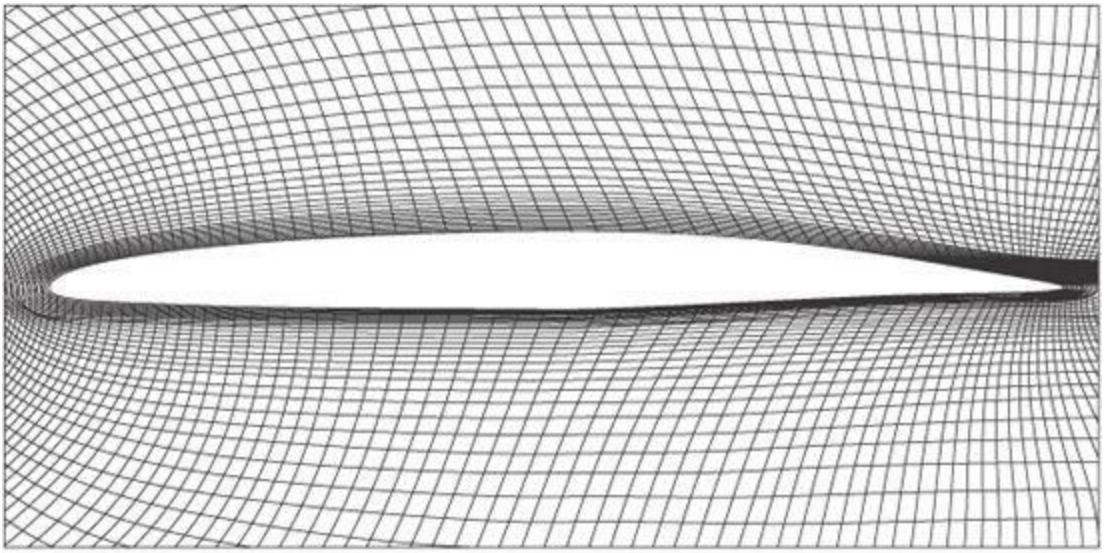
6WHDG\ 7ZR' LP HQVLRQDO DP LQDU) ORZ

7KH SUHYLRXV H[DPSOH XQWHDG\ RQHGLPHQVLRQDO IORZ KDG RQO RQH YLVFRXV WHUP DQG QI
FRQYHFWLYH DFFHOHUDWLRQV / HWXV ORRN EULHI\ DWWKRVH SUHYLRXVO\ GHULYHG HTXDWLRQV
E\ GHFUHDVLQJ ûW DQG û\ DQG ûW

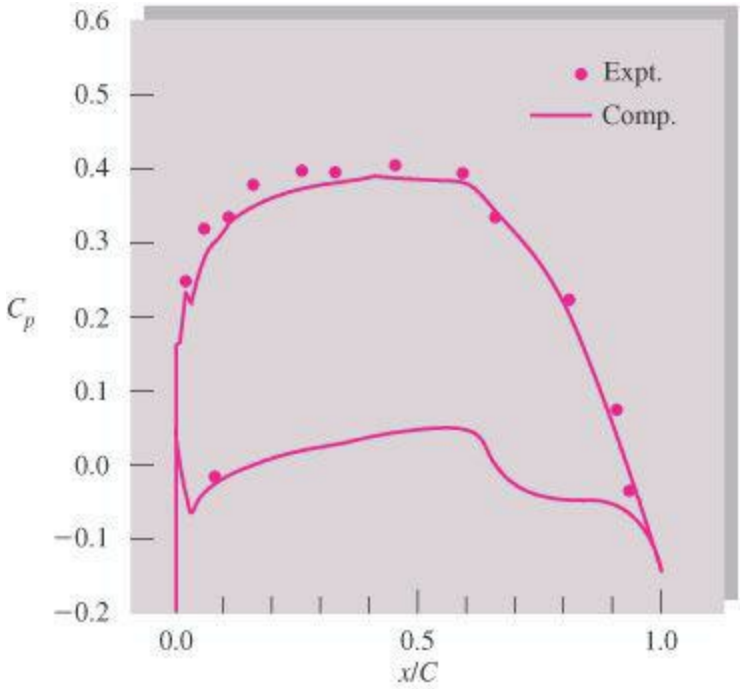
$ VHFRQG GLIILFXOW\ LQ(TV E DQG F LV WKH SUHVHQFH RI QRQOLQHDU FRQYHFWLYH DFFHOHUDWLRQ
VXFK DV X %X % ZKLFK FUHDWHV DX PPHQW LQ YLVFRXV IORZV (DU) DWWKP SW ZKLFK PRGHOHG VXF
WHUPV ZLWK DFHQWUDOGLIIHUHQFH GHG WR QXPHULFDOLQVWDELOLW 7KHUHHG LV WR UHDWFRQYHFWLQ
GLIIHUHQFHV VRHO WR WKHXSZLQGIORZ HQWULQJ WKHFHOO LJRULQJ WKHGRZQZLQGFHOO) RU[DPSOH WK
GHULYDWLYH %X % FRXOGEHPRGHOHG IRUDJLYHQFHOO DV X$SZLQG í X&HOO û 6XFK LPSURYHPHQWVKDY
PDGHIXOO YLVFRXV &)' DQHIFLYHWRO ZLWKYDUXV FRPPHUFLDOXVHUIUHQGO FRGHVDYLODEG
)RUGHWDLOVEHIRUGRUVFRSH VHH5HV DQG WR

0 HWK JHQHUDWRQ DQG JULGGLQJ KDYH DOR EHFRPH TXLWH UHLQHG LQ PRGHUQ &)') LJXUH
LOOXVWUDWHV D &)' VROXWRQ RI WZR GLPHQVLRQDO IORZ SDWDQ 1$&$ 0 2' K\GURIOO> @7I
JULGGLQJ LQ) LJ DLVR WKH& WSH ZKLFK ZUDSV DURXQG WKHOHDGLQJHGJHDQG WDLOV RI EHKLQG WK
IRLO WKXVFDSWXULQJ WKHLP SRUWDQWQHDUZDOODQG ZDNHGHWDLOV ZLWKRXWZDVWLQJ QRGHVLQIURQWRUWK

VLGHV 7KHJULGV]HLV E\



(*a*)



(*b*)

) LJ &)' UHVXOWVIRUZDWHUJUDSDQ1$6$ 0 2' K\GURIRLO(*from Ref. 28, with permission of the American Society of Mechanical Engineers*): *a* & JULGGLQJ E\ QRGHV *b* VXUIDFHSUHVVXUHVDW*α* *f*

7KH&)' PRGHOIRUWKLVK\GURIRLOGUZ LVDOVR TXLWHVRSKLVWLFDWHG DIXOO1DYLHU6WRNHVVROY ZLWKWXUEXOHQFHPRGHOQJ > @DQGDOGRZDQFHIRUFDYLWDWRQEXEEOHIRUPDWRQZKHQVXUIDFHSUHVVXUH GURSEHORZ VKHGRFDOYDSRUSUHVVXUH) LJXUH E FRPSDUHVFRPSXWHGDQGH[SHULPHQWDOVXUIDFH SUHVVXUHFRHILFLHQWIRUDQDQJOHRI DWWDFNRI *f* 7KHGLPHQVLRQOHVVSUHVVXUHFRHILFLHQWVGHILQHGDV &S SVXUIDFH í S !9, 7KHDJUHHPHQWLV H[FHOOHQWDVLQGHGLWLVDOVRIRUFDHVZKHUHWKH K\GURIRLOFDYLWDWHV> @&OHDUO ZKHQSURSHU LPSOHPHQWDWLRQIRUWKHSURSHUIDEHV &)' FDQE DQH[WHPHO HIHFWLYHWRROIRUHQJLQHHUV

&RPP HUFLDO&)' &RGHV

7KH DULYDO RI WKH WKLUG PLOOHQQLXP KDV VHQW DQ HQRUPRXV HPSKDVLV RQ FRPSXWLQJ SRZHU LQ QHDU HYHU\ ILHOG IXQGPHFKDQLFVEHQJ DSUPHH[PSOH ,WV QRZ SRVVLEOH DWWDXWRUPRGHUDWH FRPSOH[JHRPHWULHV DQG IORZ SDWWHUQV WR PRGHO RQ D FRPSXWHU DSSUR[LPDWH WKH HTXDWRQV R PRWRQ RI IXLG IORZ ZLWK GHGLFDWHG &)' WRROVRRNVDYDLODEOH> WR @7KHIORZ UHJLRQL EURNHQ LQWR D ILQH JULG RI HOHPHQWV DQG QRGHV ZKLFK DJHEUDLFDOO VLPXODWH WKH EDVLF SDUWV GLIIHUHQWDO HTXDWRQV RI IORZ : KLOH VLPSOH WZR GLPHQVLRQDO IORZ VLPXODWRQV KDYH ORQJ EHH UHSRUWHG DQG FDQ EH SURJUDPPHG DV WXWRULDOV WKURHGLPHQVLRQDO IORZ V LQYROYLQJ WKRXVDQG RUHYHQPLOOLRQV RI JULG SRLQWV DUH QRZ VROYDEOH ZLWK WHPRGHUQ VXSHUFRPSXWU

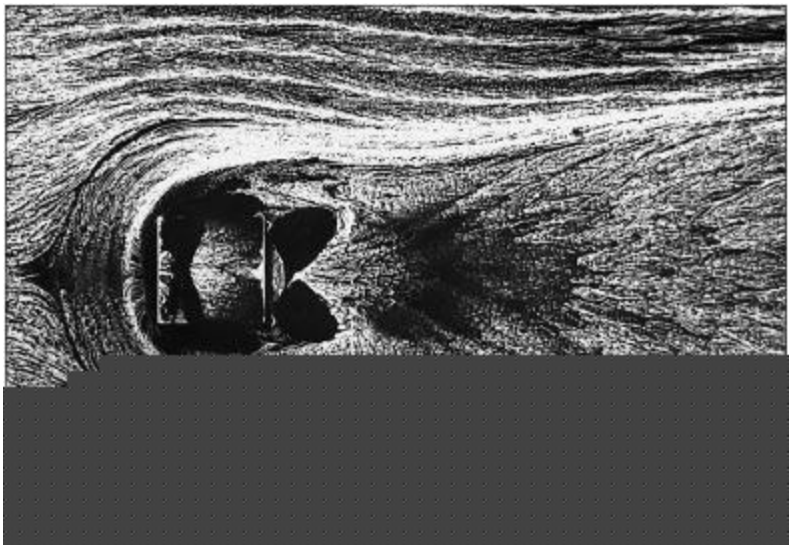
$OWKRXJK HOHPHQWDU FRPSXWUPRGHOLQJ ZDV WHDFKG EUIH WKH JHQHUDOWRSLF RI &)' L HVVHQWLDOO IRUDGYDQFHG WXG RUSURHVVLRQDOSUDFWLFH 7KHEJ FKDQJHRYHUWKHSDWWHUFDGHLVWKL HQJLQHHUV UDWKHUWKDQ DERURXVO SURJUDPPLQJ &)' SUREOPV WKPVHOYHV FDQ QRZ WDNHDGYDQWD RI DQ RI VHYHUDO FRPPHUFLDO&)' FRGHV 7KHVH H[WHQVLYH VRIWZDUH SDFNDJHV DOORZ HQJLQHHUV WR FRQWKFWD JHRPHWU DQG ERXQGDU FRQGLWRQV WR VLPXODWH D JLYHQ YLVFRXV IORZ SUREOP 7K VRIWZDUHWKHQJULGV WKH IORZ UHJLRQ DQG DWWPSWV WR FRPSXWH IORZ SURSHUWLHV DWWDFK JULGHOPHQW7K FRQYHQLHQFH LV JUHDWWKH GDQJHULV DOR JUHDW7KDWV FRPSXWDWRQV DUH QRWPHUHO DXWPDWLF ON ZKHQ XVLQJ DKDQG FDOFXODWRU EXWDKHUUHTXLUH FDUH DQG FRQFHQ IURP WKH XVHU &RQVHUJHQFH DQ DFFXUDF\ DUH UHDOSUREOPV IRUWKH PRGHOU 8VHR WKH FRGHV UHTXLUHV VRP H DWWDQG H[SHUHQFH ,(SDUWFXODU ZKHQ WKH IORZ 5H\QROGV QXPEHU 5H !9/ JRHV IURP PRGHUDW OPLQDU IORZ WKJI VXUEXOHQWIORZ WKH DFFXUDF\ RI WKH VLPXODWRQV QR ORQJHU DWWXHG 7KHUHDVQ WKDWWXUEXOHQWIORZ V DUH QRWFRPSOHWO UHVROYHG E\ WKH IXOOHTXDWRQV RI PRWRQ DQG RQH UHVRUWV WR XVLQJ DSSUR[LPDWWXUEXOHQFHPRGHOV

7XUEXOHQFH PRGHOV> @DUH GHYHORSHG IRUSDUWFXODUJHRPHWULHV DQG IORZ FRQGLWRQV DQG PD EH LQDFFXUDWH RU XQUHDOLVWF IRU RWKHUV 7KLV LV GLVFXVVHG E\ )UHLWDV> @ZKR FRPSDUHG HJK GLIIHUHQWFRPPHUFLDO FRGH FDOFXODWRQV)/2: ')/275$1 67$5 &' 1 6 &)' $)/8(17 &)' 6)/2: ' DQG 1,6$ ')/8,' ZLWK H[SHUPHQWDO UHVXOWVIRU ILYH EHQFKP IORZ H[SHUPHQW &DOFXODWRQV ZHUH PDGH E\ WKH YHQGRUV WKPVHOYHV)UHLWDV FRQFXGHV WKL FRPPHUFLDOFRGHV WKRXJK SURPLVLQJ LQJHQHUDO FDQEHLQDFFXUDWIRUFHUWDLQ ODPLQDUDQG WXUEXOHQ IORZ VLWXDWRQV &KHQJ > @GLVFXVVHV DSRSXODUQHZ FRPPHUFLDOSURJUDP FDOOHG)OZ / DE 5HFH PRGLILFDWRQV WR WKH VXQGDUG WXUEXOHQFH PRGHOV KDYH LPSURYHG WKLU DFFXUDF\ DQG UHOLDELOLW D VKRZQ E\ (OKRXU > @

$QH[DPSOHRI HUDWF &)' UHVXOWKDVDOHDG\ EHHQPHQWRQHG KHUH QDPHO WKHGUDJ DQG OLWF DURXQG FA\OQGHU) LJ 3HUKDSVEHFDXVHWKHIORZ LWHOLVSK\VLFDOO XQXVEH> @UHVXO FRPSXWG E\ GLIIHUHQWZRUNHUV DUH VWULQJO GLIIHUHQW6RPH SUHGLFWG IRUFHV DUH KLJK VRPH OHVV VRPH G FUHDVH 7KH WW[WE\ 6HQJXSWD> @GLVFXVVHV ZK\ VHYHUDO SRSXODU &)' VFKHPHVFDQEHXQUHOLDEOH

,QVSLWRI WKLVZDUQLQJ WRWDW&)' FRGHVZLWKFDUH RQHVKRXGDOR UHDQ]HWDWKHUHVXOWRI JLYHQ&)' VLPXODWRQFDQEHVSHFWFXU) LJXUH LOOXVWUDWV WXUEXOHQWIORZ SDWWFXEH PRXQW RQWKHIORRURI DFKDQQHOZKRVHFHDUDFWHLVWKLFXEH KHJKW&RP SDUH) LJ D DWRS YLHZ R WKH H[SHUPHQWDO VXUFH IORZ > @DV YLVXDOL]HG E\ RLO WWHDN ZLWK) LJ E D &)' VXSHUFRPSXWU UHVXOWXVLQJ WKH PHWRG RI OUJH HGG\ VLPXODWRQ > @ 7KH DJUHPHQWL UHP DUNDEOH 7KH& VKDSHG IORZ SDWWLQ IRQWRI WKH FXEH LV FDXVHG E\ IRUPDWRQ RI DKRUVHVKR YRUWH[DV VHQ LQ D VLGH YLHZ RI WKH H[SHUPHQW> @LQ) LJ F +RUVHVKRH YRUWHV FRPPRQ

UHVXOWZKHQVXUFHVKDUIBRZVPHHWDQREMDFWH: HFRQFXGHWKDW&)' KDVDWWHPQGRXVSRMQWV IRUIBRZSUHGLFWLRQ



(c)

) LJ) OBZ RYHU D VXUIDFH P RXQWHG FXEH FUHDWHV D FRP SOH[DQG SHUKDSV XQ[SHFWH SDWWHUQ *a* H[SHULP HQWDOO\ REWDLQHG VXUIDFH IOBZ DW5H EDVHG RQ FXEH KHLJKW*(Courtesy of Robert Martinuzzi with the permission of the American Society of Mechanical Engineers); (b)* FRP SXWDWLRQDO ODUJH HGG\ VLP XODWLRQ RI WKH VXUIDFH IOBZ LQ*(a) (from Ref. 32, courtesy of Kishan Shah, Stanford University);* DQG*(c)* D VLGH YLHZ RI WKH IOBZ L*a* YLVXDO]HG E\ VP RNH JHQHUDWRQ DQG D ODVHU OJKWVKHHW*(Courtesy of Robert Martinuzzi with the permission of the American Society of Mechanical Engineers)*

6XPP DU\

7KLV FKDSWHU KDV DQDO]HG D KJKO LGHDO]HG EXWYHU XVHXO WSH RI IOBZ LQYLVFLG LQFRP SUHVVLEOH LUURWDWRQDO IOBZ IRUZ KLFK / DSODFH¶V HTXDWRQ KROGV IRUWKH YHORFLW SRWHQWLDO DQG IRUWKH SODQHVWHP IXQFWRQ 7KH P DWK P DWFVLZ HOGHYHORSHG DQG VROXWRQV RI SRWHQWL IOBZ VFDQEH REWDLQHG IRUSUFWFDO DQ\ ERG\ VKSH

6RP H VROXWRQ WHFKQLTXHV RXWOLQHG KHUH DUH VXSHUSRVLWRQ RI HOP HQWDU\ OQH RU SRLQ VROXWRQV LQ ERWK SODQH DQG D[LX\ P P HWUF IOBZ WKH DQDO\WF IXQFWRQ RI D FRP SOH[YDULDEOH XVH RI YDULDEOH VWHQJWK YRUWH[VKHHW DQG QXP HULFDO QDO\VLV RQ D FRP SXWU 3RWHQWLDO WKRU LV HVSHFLDO XVHXO DQG DFFXUDWH IRU WKLQ ERGLHV VXFK DV DLUIRLOV 7KH RQO UHTXLUHP HQWLV WKDW WKH ERXQGDU\ OD\HU EHWKLQ LQRWKHUZRUGV WKH5H\QROGV QXP EHUEH ODUJH) RUEOXQWERGLHV RUKJKO GLYHUJHQWOBZ V SRWHQWLDOWKRU VHUYHV DV D ILUWDSSUR[LP DWRQ W EH XVHG DV LQSXWWR D ERXQGDU\ OD\HU DQDO\VLV 7KH UHDGHU VKRXOG FRQVXOWWKH DGYDQFHG WH[WV >R H[DPSOH VR VR @IRU IXUWKHU DSSOFDWRQV RI SRWHQWLDO WKRU 6HFWRQ GLVFXVH FRP SXWDWRQDOP HWKRGV IRUYLVFRXV QRQSRWHQWLDO IOBZ V

SUREOHP VTXHVWRQV3UREOHP V

0 RVWR WKHSUREOHP VKHUHLQDUH IDLUO VWDLJKWRUZ DUG 0 RUHGLILFXOWURSHQHGDVLJQP HQ DUH OEHOHG ZLWK DQDXWULN 3UREOHP VOEHOHG ZLWK DQ((6 LFRQ ZLOOEHQHILWURP WKH XVHRI WK (QJLQHHULQJ (TXDWRQ6RLYHU ((6 ZKLOH SUREOHP VOEHOHG ZLWK D FRP SXWULFR PD\ UHTXL WKH XVH RI D FRP SXWU 7KH VWDQGDUG HQG RI FKDSWUSUREOHPV3 WR3 FDWRUL]HG LQ WK SUREOHP OWKHUH DUHIOBZ HG E\ ZRUGSUREOHPV: WR: FRP SUHHQVLYH SUREOHPV& W

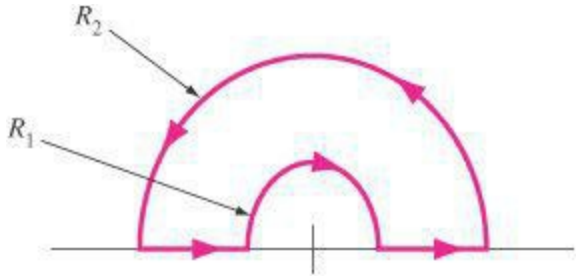
Problem Distribution

Section	Topic	Problems
8.1	Introduction and review	P8.1–P8.7
8.2	Elementary plane flow solutions	P8.8–P8.17
8.3	Superposition of plane flows	P8.18–P8.34
8.4	Plane flow past closed-body shapes	P8.35–P8.59
8.5	The complex potential	P8.60–P8.71
8.6	Images	P8.72–P8.79
8.7	Airfoil theory: two-dimensional	P8.80–P8.84
8.7	Airfoil theory: finite-span wings	P8.85–P8.90
8.8	Axisymmetric potential flow	P8.91–P8.103
8.8	Hydrodynamic mass	P8.104–P8.105
8.9	Numerical methods	P8.106–P8.115

,QWURGXFWRQ DQG UHYLHZ

- 3 3URYH WKDW WKH VXSHUPRVLWLRQ RI WZR SODQH IORZ SROXWLRQ VROXWLRQV LV DOVR D SROXWLRQ VÉ U
- 3 7KH VWUHDP SDWWHUQ IRU) LJ 3 KDV WKH SRODU YHORFLW\ FRPSRQHQWV% 9UDQG%(' HWHUPLQH WKH FLUFXODWLRQ* DURXQG WKH SDWK VKRZQ

3



- 3 8VLQJ FDUWHVLDQ FRRUGLQDWHV VKRZ WKDW WHDFK YHORFLW\ FRPSRQHQW X % Z RI I SRWHQWLDO IORZ VDWLVI\ /DSODFH¶V HTXDWLRQ VHSDUDWHO
- 3 ,V WKH IXQFWLRQ UD GHJLWLP DWH YHORFLW\ SRWHQWLDO LQ SODQH SRODU FRRUGLQDWHV" ,I VR ZKDW LV WKH DVVRFLDWHG VWUHDP IXQFWLRQ$ U "
- 3 $ SURSRVHG KDUPRQLF IXQFWLRQ) [\ ] LV JLYHQ E\

) [\ [] I \

D ,I SRWHQWLDO ILQG D IXQFWLRQ I \ IRU Z KLFK WKH ODSODFLDQ RI) LV]HUR ,I \RX GLQG HG VROYH SDUW D FDQ\RX ILQG D IXQFWLRQ) VHUYH DV E DYHORFLW\ SRWHQWLDO RU F I VWUHDP IXQFWLRQ'

- 3 * LYHQ WKH SODQH SRODU IORZ YHORFLW\ SRWHQWLDL $\phi = Br^2\cos(2\theta)$, Z KHUH% LV I FRQVWDQW D 6KRZ WKDW VWUHDP IXQFWLRQ DOVR H[LVWV E) LQG WKH DJHEUDLF IRUP RI

$\psi(r, \theta)$. (c) F) LQG DQ VDDJQDWRQSRLQWLQWKLVIORZ ILHOG

- 3 &RQVLGHUDIORZ ZLWKFRQWDQWGHQVLWDQG YLVFRVLW ,I WKHIORZ SRVHVWHVDYHORFLV SRAQWDODVGHILQHGE\ (T VKRZ WKDWWH[DFW VDWLVIVWKHIXOO1DYLHU6WRN HTXDWRQV ,I WKLVLVVR ZK\ IRULQYLVFLGWKRU GRZHEDFNDZD\ IURP WKHIX 1DYLHU6WRNHVHTXDWRQV"

(OHPHQWDU\ SODQHIORZ VROXWRQV

- 3)RUWKH YHORFLW GLVWULEXWRQ í %\ % %[Z HYDOXDWH WKH FLUFXODWRQ DERXWWKH UHFWDQJXODUFRVHG FXUYHGHILQHGE\ [\ DQG ,QWHUSUHWRXUUHVXOWHVSHFLDOO YLV j YLVWKHYHORFLWSRAQWDO
- 3 &RQVLGHUWKHVWR GLPHQVLRQDOIORZ X í$[% $\ ZKHUH$ LVDFRQVWDQW(YDOXDW WKHFLUFXODWRQ* DURXQGWKHUHFWDQJXODUFRVHG FXUYHGHILQHGE\ [\ DQG ,QWHUSUHWRXUUHVXOWHVSHFLDOO YLV j YLVWKHYHORFLWSRAQWDO
- 3 $ VWR GLPHQVLRQDO5DQNLQHKDOI ERG\ FP WKLFN LV SODFHG LQ D ZDWUWKQHO *LYHQ WKDWSUHVVXUHIDUXSWUHDP DORQJ WKHERG\ FHQWHOLQH LV NPD: KG LV WKHQRVHUDGLXV RI WKHKDOI ERG\" $VZKDWXQQHOIORZ YHORFLW ZLOOFDXVHWR EXEEOHVEHJLQWRIRUP RQWKHVXUIDFHRI WKHERG\"
- 3 $ SRZHUSODQWGLVFKDUJHVFRROQJ ZDWUWKURXJK WKHPDQIROGLQ)LJ 3 ZKLFK LV FP LQ GLDPHWU DQG P KLJK DQG LV SHUFRUDWHG ZLWK KROHV FP LQ GLDPHWU' RHV WKLV PDQIROGVLPXODWH DQDFHSWDEOH VRXUFH' ,I VR ZKDWLV WKHHTXLYDOHQ VRXUFHVWUHQJWKP"

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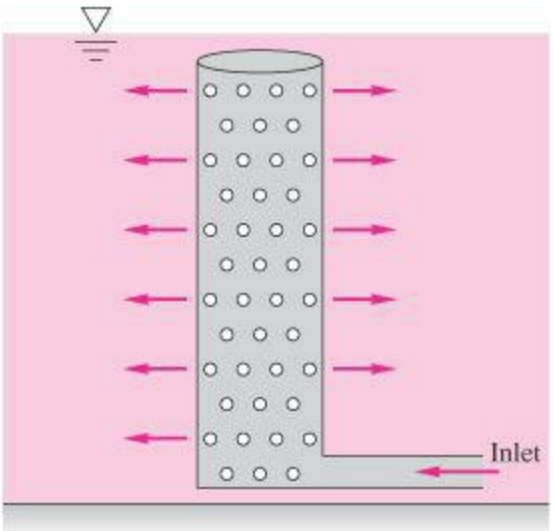

- 3 &RQVLGHUWKHIORZ GXHWR DYRUWH[RI VWUHQJWK. DWWKHRULJLQ (YDOXDWWKHFLUFXODWR IURP (T DERXWWKHFORFNZLVHSDWKIURP U D WR D WR D CE WR D CE DQGEDFNWR D ,QWHUSUHWKHUHVXOW
- 3 6WDUWLQJ DWWKHVWDJQDWRQSRLQWQ) LJ WKHIOXLGDFFHOHUDWRQ DORQJ WKHKDOI ERG VXUIDFHULVHVWR D PD[LPXP DQGHYHQWXDOO GURSVRII WR]HUR IDUGRZQWUHDP D ' RHVWKLVPDWLPXP RFFXUDWWKHSRLQWQ) LJ ZKHUH8PD[8" E ,I QR GRHVWKHPD[LPXP DFFHOHUDWRQRFFXUEHRUHRUDWWUWKDWSRLQW"([SODLQ

- 3 $ VRXUFH RI DQJXODU PRPHQWXP EHFRPHV GHGXFHG IURP VKRZQ LQ)LJ 3 ZLWK %[%\ DQG% U VXFK WKDW

$$v_\theta = \begin{cases} \omega r & r \le R \\ \dfrac{\omega R^2}{r} & r > R \end{cases}$$

' HWHUPLQH ZKHWKHU WKLV IORZ SDWWHUQ LV LUURWDWLRQDO LQ HLWKHU WKH LQQHU RU RXWHU UHJLRQ 8VLQJ WKHUPRPHQXP HTXDWLRQ ' RI $SS ' GHWHUPLQH WKH SUHVVXUH GLVWULEXWLRQ S U LQ WKH VRXQGGR DVVXPLQJ S S DVU"""')LQG WKH ORFDWLRQ DQG PDJQLWXGH RI WKH ORZHVW SUHVVXUH

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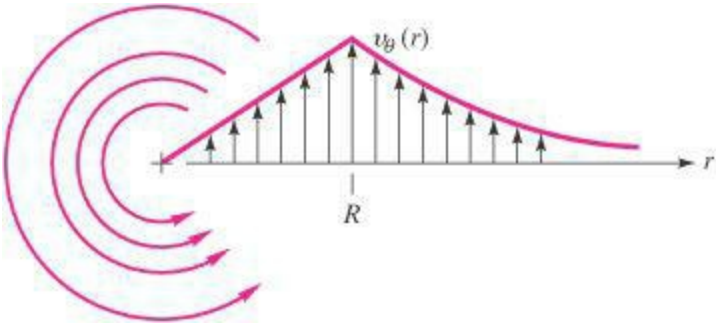

- 3 $ FDYLWDWRU KXUULFDQHRQWKH6DILU6LPSVRQVFDOH ZZZ HQA FRSHGLDFRP! KD DPDJLPXP YHORFLWLRI PLK / HVWKHPDAK SRLQWDGLXVEH5 NP VHH) LJ 3 $VXPLQJ VHDOHYHODVLQGGUG FRQGLWLRQV DWODUJHU D ILQG WKH PLQLPXP SUHVVXUH E ILQG WKHSUHVVXUHDWKHPDAK SRLQW
- 3 $LUIORZ VDW P VDORQJ DIODWXUDFHZKHQLWHQFRXQWHUVDMAWRI DLUIXLQJ IURP WKHKRUL]RQWDOZDOODWSRLQW$ DVLQ) LJ 3 7KHMAWYRXPHIORZ LV P VSHXQLWGHSWKLQRWKHSDSHU ,I WKHMAWLVDSSURLPDAG DV DQLQYLVFLGQHVRXUFH D ORFDAWWKHVWDJQDWLRQSRLQW6 RQWKHZDOO E +RZ IDUYHUWFDO ZLOWKHMAWORZ H[WQGLQR WKHVWHP"

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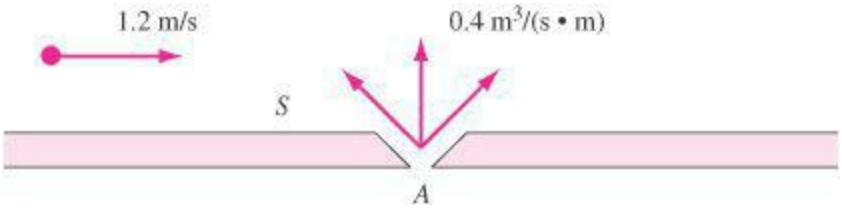

- 3)LQG WKHSRVLWLRQ [\ RQ WKHXSSHUVXUFHRI WKHKDOEERG LQ) LJ DIRUZKLFK WKHORFDOYHORFLWHTXDOV WKHXQLRUP VWHDP YHORFLW: KDWWKRXGGEHWKHSUHVVXU DWWKLVSRLQW

6XSHUSRVLWLRQ RI SODQHIORZV

- 3 3GRVWKHVWUHDPOLQHVDQGSRWHQWLDOIORZ GXHWRDOLQHVRXUFHRI VWUHQJWKP DWD SOXVDVRXUFH P DWí D : KDWLVWKHIORZ SDWWHUQYHZHGIURP DDU
- 3 3GRVWKHVWUHDPOLQHVDQGSRWHQWLDOIORZ GXHWRDOLQHVRXUFHRI VWUHQJWK P DWD SOXVDVLQNí P DWí D : KDWLVWKHSDWWHUQYHZHGIURP DDU
- 3 3GRVWKHVWUHDPOLQHVRI WKHIORZ GXHWRDOLQHYRUWH[. DW D DQGDYRUWH[í . D í D : KDWLVWKHSDWWHUQYLHZHGIURP DDU
- 3 $WSRLQWS LQ) LJ 3 LVDFORFNZLVHOLQHYRUWH[RI VWUHQJWK. P V $WSRLQ %LVDOLQHVRXUFHRI VWUHQJWKP P V ' HWHUPLQHWKHUHVXOWDQWYHORFLW\LQGXFHG E\ WKHVHWZRDWSRLQW&

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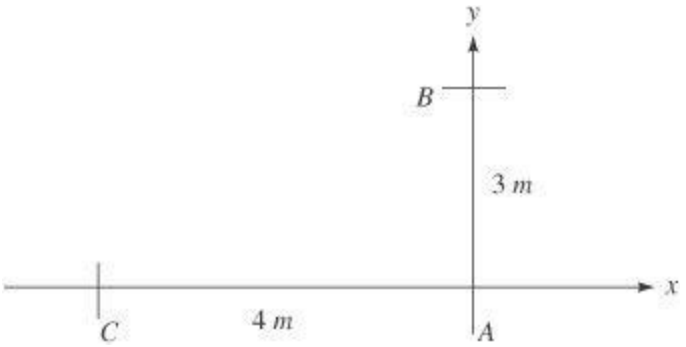

- 3 &RQVLGHULQYLVFLG WDJQDWRQIORZ $. [\ VHH) LJ E VXSHUPRSHG ZLWK VRXUFHDWWKHRULJLQRI VWUHQJWKP 3ORWWKHUHVXOWLQJ VWUHDPOLQHVLQWKHXSSHUKDO SODQH XVLQJ WKHHQJWKVFDOHD P . * LYHDSK\VLFDOLQWHUSUHWDWLRQRI WKHIORZ SDWWHUQ
- 3)LQG WKH UHVXOWDQWYHORFLW\ YHFWRU LQ) LJ 3 E\ WKH XQLRUP VWUHDP YRUWH[DQGOLQHVRXUFH

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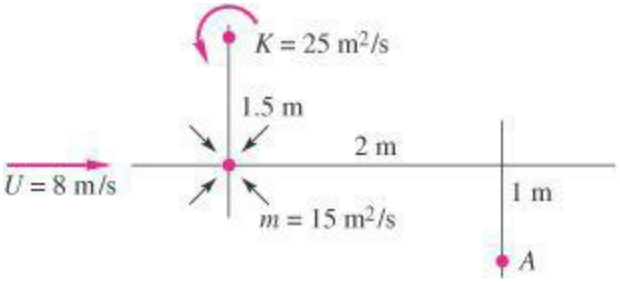

- 3 /LQHVRXUFHVRI HTXDOVWUHQJWKP 8D ZKHUH8 LVDUHIHUHQFHYHORFLW\ DUHSODFHGD [\ D DQG íD 6NHWFK WKH VWUHDP DQGSRWHQWLDOLQHV LQWKHXSSHUKDO SODQH ,V\ D³ZDOO" ,IVR VNHWFK WKHSUHVVXUHFRHILFLHQW

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2}$$

DORQJ WKHZDOOZKHUHS LVWKHSUHVVXUHDW) LQGWKHPLQLPXP SUHVVXUHSRLQWDQ LQGLFDWHZKHUHIORZ VHSDUDWHVRQPLJKWRFFXULQWKHERXQGDU\ OD\HU

- 3 / HVWKHYRUWH[VLQNICRZ RI (T VLPXODWHDVRIQDGRDVLQ) LJ 3 6XSSRV
VKDVWKHFLUFXODWRQDERXWKHVRIQDGRLV* P VDQGWKDVWKHSUHVVXUHDWU
P LV 3DOHWV WKDQ WKHIDUILHOG SUHVXUH \$VXPLQJ LQYLVFLGICRZ DVWHD
OHYHOGHQWLW HVWLPDWH D WKHDSSURSULDWHVLQNVWUHQJWKí P E WKHSUHVWXUHDWU
P DQG F WKHDQJOH DWZKLFK WKHVWUHDPOLQHVFURVVWKHFLUFOHDWU P VH
) LJ 3

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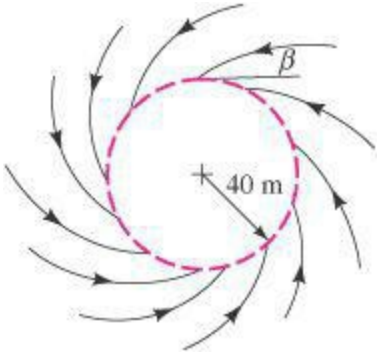

- 3 \$ FRDXDOSRZHUSODQWXVHVQFRROQJ ZDWUKURXJK DYHUWFDOSHURUDWGPDQLROG
DVLQ) LJ 3 7KHVRXOYROXPHICRZ LQDNHLV P V &XUHQWVRI FP VICRZ
SDWWKHPDQLROG DV VKRZQ (VWLPDWH D KRZ IDUGRZQXWHDP DQG E KRZ ID
QRUPDOWR\KHSDSHUWKHHIFWVRI VKHLQDNHDUHIHOWQWKHDPELHQW P GHSZDWJV

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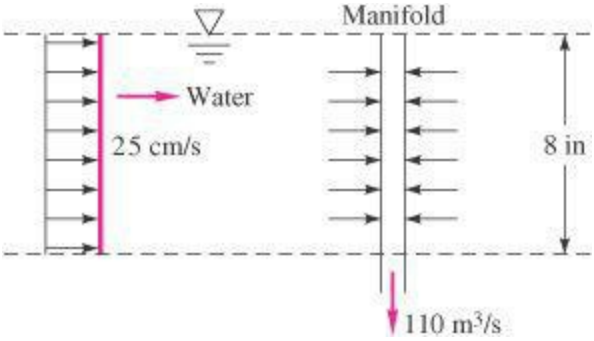

- 3 : DWUDW &ICRZ VSDWWDKDO ERG DVVKRZQLQ) LJ 3 0 HDXUHG SUHVXUHVD
SRLQWV\$ DQG%DUH N3D DQG N3D UHVSHFWLYHO ZLWKXQFHUWDLQWLHVRI N3
HDFK (VWLPDWWKHVXWHDP YHORFLWDQGLWXQFHUWDLQW
- 3 6RXUFHVRI HTXDOVWUHQJWKP DUHSODFHGDWKHIRXUPHWFSRVLWRQV [\ D D
íD D íD íD DQG D íD 6NHWFK WKHVWUHDPOLQHDQGSRWHQWLDOOLQHSDWWHUQV ' I
DQ SODQH³Z DOOV DSSHDU

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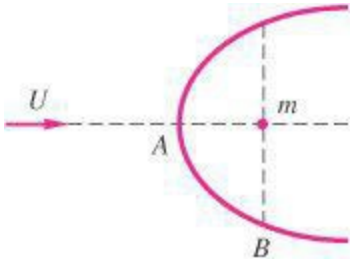

- 3 $ XQLIRUP ZDWUVWHDP 8, P VDQG! NJ P FRPELQHVZLWKDVRXUFHD VWHRULJLQVRIRUP DKDOEERG\ $W[\ P WKHSUHVVXUHLV N3DOHV WKDQS D ,VWKLVSRLQWRXWLGHWKHERG\" (VWLPDWH E WKHDSSURSULDWHVRXUFH VWUHQJWKP DQG F WKHSUHVVXUHDWWKHQRVHRI WKHERG\
- 3 $ VROLGGR LVVLPXODWHGE\ DQQHVLQNP í P VSXVDQQHYRUWH[. P V)LQGWKHDQJOHEHWZHHQDQ VWUHDPOLQH DQGDUDGLDOOLQH DQGVKRZ WKDWWKL LQHSHQGHQWRI ERWKUDQG ,I WKLVVROGR IRUPVLQVHDOHYHOXDQGDUDLDWZKD UDGLXVZLOOWKHRFDOSUHVVXUHEHHTXLYDOHQWR LQ-J"
- 3 $ 5DQNLQHKDOI ERG\ LVIRUPHGDVVKRZQLQ) LJ 3) RUWKHVWUHDP YHORFLWVDQ ERG\ GLPHQVLRQVKRZQ FRPSXWH D WKHVRXUFHVWUHQJWKP LQP V E WKHGLVWDQF D F WKHGLVWDQFHK DQG G WKHVRXOYHORFLWVDVSRLQW$

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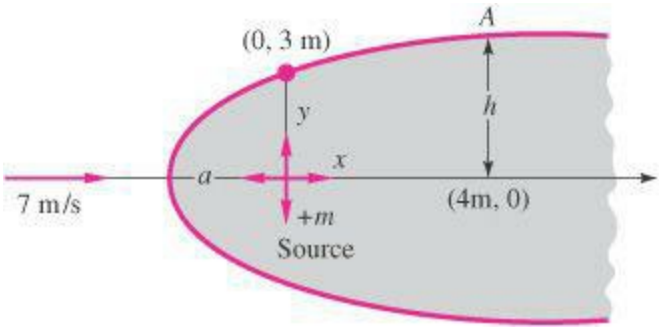

- 3 $ VRXUFHDQGVLQNRI HTXDOVWUHQJWK P P V DUHQHDUDZDOODVLQ) LJ 3) LQGWKHUHVXOWDQWYHORFLWLQGXFHGE\ WKLVSDLUDVSRLQW$ RQWKHZDOO

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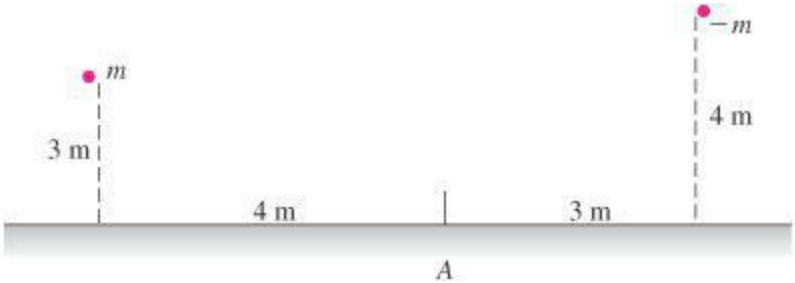

- 3 6NHWFK WKHVWUHDPOLQHV HVSHFLDOO WKHERG\ VKDSH GXHWRHTXDOOQHVRXUFHV P DW D DQG íD SXVDXQLRUP VWUHDP 8, PD
- 3 &RQVLGHUWKUHHHTXDO VSDFHGVRXUFHVRI VWUHQJWKP SDFHGDW[\ D DQG íD 6NHWFK WKHUHVXOWLQJ VWUHDPOLQHV QRWLQJ WKHSRVLWLRQ RI DQ VWDJQDWR

SRLQW: KDWZRXOGWKHSDWKOLQHORRNOLNHIURP DIDU'

3 ODQHICRZ SDWWFORVHG ERG\ VKDSHV

- 3 $ XQLRUP VWUHDP 8, P V DSSURDFKHVD 5DQNLQHRYDODVLQ) LJ ZLWKD FP)LQG WKH VWUHQJWKP RI WKH VRXUFH±VLQN SDLU LQP V ZKLFK ZLOOFDXVH WK VWDJQDQWKRI WKHRYDOWREH FP : KDWLVWKHPD[LPXP ZLGWKRI WKLVRYDO'
- 3 : KHQDQHVRXUFH±VLQNSDLUZLWKP P VLVFRPELQHGZLWKDXQLRUP VWUHDP L IRUPVD5DQNLQHRYDOZKRVHPLQP XP GLPHQVLRQV FP ,ID FP ZKDWDU WKH VWUHDP YHORFLW DQG WKH YHORFLW DWWKH VKRXOGHU' : KDWLV WKH PD[LPXP GLPHQVLRQ'
- 3 $ 5DQNLQHRYDO P ORQJ DQG P KLJK LV PPHUHGLQD VWUHDP 8, P V DVL) LJ 3 (VWLPDWH D WKHYHORFLWDVSRLQW$ DQG E WKHORFDWRQ SRLQW%ZKHU DSDUWFOHDSSURDFKLQJ WKHVDQGDWRQSRLQWZFKLHYHVLWPD[LPXP GHFHOHUDWRQ

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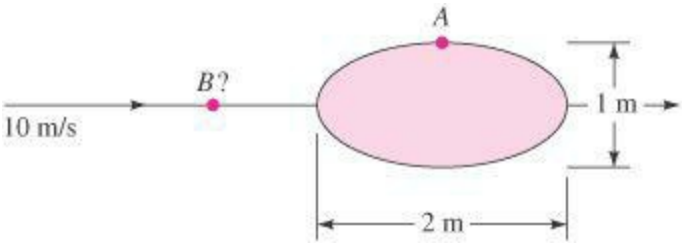

- 3 &RQVLGHU SRWHQWLDO IORZ RI D XQLRUP VWUHDP LQ WKH[GLUHFWLRQ SOXV VZR HTXD VRXUFHV RQHDW[\ D DQG WKHRWKHUDW[\ íD 6NHWFK \RXULGHDV RI WKHERG\ FRQWRXUVWKDWZRXOGDULVHLI WKHVRXUFHVZHUH D YHU ZHDN DQG E YHU VWURQJ
- 3)RUWKH5DQNLQHRYDORI) LJ 3 LI WKHIOXLGLVZDWUDW *& DQGWKHSUHVVXUHID XSVWUHDP DORQJ WKHERG\ FHQWHUOLQH NPD GHWHUPLQHWKHIUHHVWUHDP YHORFLW 8, IRUZKLFK WKHIOXLGZLOOFDYLWDWDVSRLQW$
- 3 0RGLI\ WKH5DQNLQHRYDOLQ) LJ 3 VR WKDWWKHVWUHDP YHORFLWDQGERG\ OHQJWKDU WKHVDPHEXWWKHWKLFNQHVVLVXQNQRZQ QRW P 7KHIOXLGLVZDWUDW *& DQGWK SUHVVXUHIDUXSVWUHDP DORQJ WKHERG\ FHQWHUOLQH NPD) LQGWKHERG\ WKLFNQHV IRUZKLFK FDYLWDWRQZLOORFFXUDVSRLQW$
- 3 $. HOYLQRYDOLVIRUPHGE\ DQQH±YRUW[SDLUZLWK. P V D P DQG8 P V: KDWDUHWKHKHLJKWZLGWK DQGVKRXOGHUYHORFLWRI WKLVRYDO'
- 3 7KHYHUWFDONHORI DVDLGERDWDSSUR[LPDWHV D5DQNLQHRYDO FP ORQJ DQG FP WKLFN 7KHERDWDOVRLQVHZ DWULQWDQGDUDWRSKHUHDW NQRW SDUDOOHOWR WK NHHO$WDVFHQW P EHORZ WKHVXUFH HWXP DWWKHORZHUSRLQWRQWKHVXUFH RI WKHNHHO
- 3 &RQVLGHUZDWUDW *& IORZLQJ DW P VSDWD P GLPHWUFLFXODUF\OQGHU: KD GRXEOHWXQHQWK LQP VLVUHTXLUHGWRVLPXODWWKLVIORZ" ,I WKHVWUHDP SUHVVXUHL NPD XVHLQYLVFLGWKHRU WRHWXP DWWKHVXUFHSUHVXUHDW HTXDOWR D *& E *& DQG F *&

- 3 6XSSRVH WKDW FLUFXODWLRQ LV DGGHG WR WKH FRQGLWLRQ RI 3URE 3 VXFK WKDW SDFHWKH VDWQDWLRQ SRLQW DW HTXDOV ƒ DQG ƒ : KDW LV WKH UHTXLUHG YRUWH VWUHQJWK. LQP V" &RPSXWH WKH UHVXOWLQJ SUHVVXUH DQG VXUIDFH YHORFLW DW WKH VDQDWLRQ SRLQW DQG E WKH XSSHU DQG ORZHU VKRXOHUV : KDW LV WKH OLIW SHU XQLW VSDQ RI FRQGXLW LGWKEH'
- 3 ,I FLUFXODWLRQ. LV DGGHG WR WKH FRQGLWLRQ LQ 3URE 3 D IRUZ KDW YDOXH RI . ZLOO WKH IORZ EHJLQ WR FDYLWDWH DW WKH VXUIDFH" E : KHUH RQ WKH VXUIDFH ZLO FDYLWDWH RQ EHJLQ' F) RUWKLV FRQGLWLRQ ZKHUH ZLOO WKH VDQDWLRQ SRLQW OH'
- 3 $ FRQGHULVIRUPHGE\ EROWLQJ WZR VHPLFRQGUFDOFKDQQHOV WRJHWKHURQ WKHLQLG DV VKRZ QLQ) LJ 3 7KHUHDUH EROWSHUHWURI ZLGWKRQHDFK VLGH DQG WKH LQLGH SUHVXUH LV NPD JDJH 8VLQJ SRWHQWLDOWKRU IRUWKH FRXWLGH SUHVXUH FRP SXWH WKH WHQVLRQIRUFHLQHDFKEROW WKHIOLGRXWLGHLVVHDOHYHOLU

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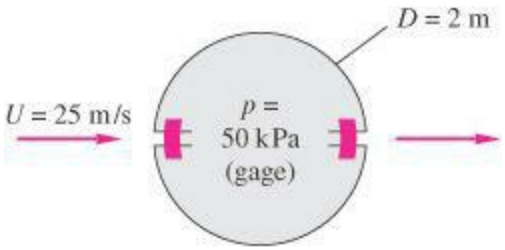

- 3 $ FLUFXODUFRQGHULVILWWHG ZLWK VWR VXUIDFH PRXQWHG SUHVVXUH VHQVRUV WR PHDVXU SD DW ƒ DQG SE DW ƒ 7KH LQHQGLDQ LV WR XVH WKH FRQGHUDV DVWHDG YHORFLPHWU 8VLQJ LQYLVFLG WKRU GHULYH DIRUPXODIRUHVWLPDWQJ 8, LQ WHUP VR SD SE ! DQG WKH FRQGHUUDGLXV D
- 3 : LQG D/8, DQG S IORZ VSDWD 4 XRQH WKXWZ KLFK LV D KDO FRQGHUDGLXV DQG GHQJWK/) LJ 3 7KH LQHQGLDOSUHVXUHLVS_ 8VLQJ LQYLVFLG WKRU GHULY DQ H[SUHVVLRQIRUWKHXSZDUGIRUFHRQWKHGLIIHUHQFHEHWZHHQS_ DQ SV

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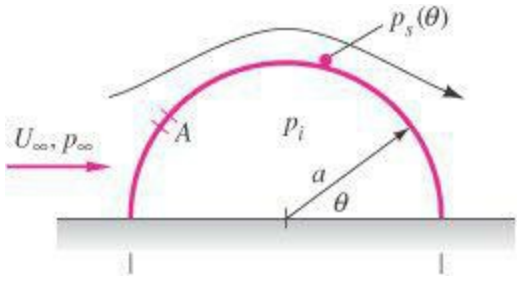

- 3 ,QWRQJ ZLQGV WKH IRUFH LQ 3URE 3 FDQEH TXLWH DJH 6XSSRVH WKDW D KROH LQWRGXFHG LQWKH KXWURRI DWSRLQW$ WR PDNHS_ HTXDOWR WKH VXUIDFH SUHVXUH WKHU $WZ KDWDQJOH VKRXOGKROH$ EHSODFHGWR PDNHWKHQHWZ LQGIRUFH]HUR'
- 3 ,WLV GHVLUHG WR VLPXODWH IORZ SDWD VWR GLPHQVLRQDOLGH RU EXPS E\ XVLQJ WWHDP OQH WKDWSDVVHVDERYH WKH IORZ RYHUDFLQGHU DVLQ) LJ 3 7KHEXPS L

VR EHD KLJK ZKHUHD LV WKH FRQGHUDGLXV : KDWLV WKH HOHYDWRQK RI WKL
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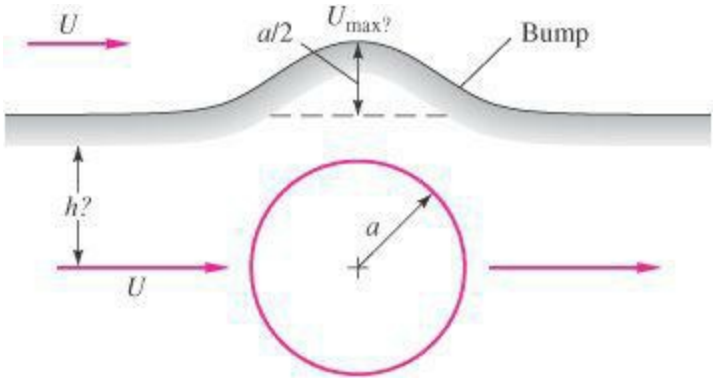

- 3 $ KRFHLV SDFHG LQ WKH IURQWRI DFRQGHUVR PHDVXUH WKH VWUHDP YHORFLW RI VHD OHYHO IUHVK ZDWU 7KH PHDVXUHG SUHVVXUH DWWKH KRFHLV GE IW ,I WKH KRFHL PLVDOJQHGE\ *f* IURP WKH VWUHDP DQG PLVQWSUHVGDVWDQDWRQSUHVXUH ZKL LVWKH HUURULQYHORFLW"
- 3 7KH) OHWQHUURRUVDLGERDWQ) LJ (KDVDZDWUGUDJFRHILFLHQWRI EDVH RQDZHWWGDUHDRI IW ,I WKHURRUVSLQVDW UPLQ ILQGWKHPD[LPXP ERD YHORFLW WKDWFDQ EH DFKLHYHG LQD PLK ZLQG : KDWLV WKH RSWPXP DQJ EHWZHHQWKHERDWDQGWKHZLQG'
- 3 0 RGLI\ 3URE 3 DVIROORZV)RUWKHVDPHVDLOERDWGDWILQGWKHZLQGYHORFLW PLK WKDWZLOOGULYHWKHERDWDWQRSWPXP VSHHGRI NQSDUDOOHDWRLWVNHHO
- 3 7KHRULJLQDO) OHWQHUURRUVKLSZDVSSUR[LPDWD IWDRQJ GLVSODFHG VRQ DQGKDGZHWWGDUHDRI IW$VVXPHWKHGLQ) LJ 3 LWKDGVZRURRRUV I KLJKDQG IWLQGLDPHWHUURRWQJDW UPLQ ZKLFKLVIDURXWLGHWKHUDQJHRI) LJ 7KHPHDVXUHGDWDQGGUDJ FRHILFLHQWVIRUHDFKURRWZHUHDERXW DQG UHVSHFWLYHO ,I WKHVKLSVPRRUHGDQGVXEMHFWHGWRDFURVVZLQGRI IWV DVLQ) LJ 3 ZKDWZLOOWKHZLQG IRUFH SDUDOOHODQG QRUPDOWR WKHVKLS FHQWHUOLQH EH (VWLPDWHWKHSRZHUHTXLUHGWRGULYHWKHURRWV
- 3 $VVXPHWKDWWKH) OHWQHUURRUVKLSRI) LJ 3 KDVDZDWUHVLVWDQFHFRHILFLHQWR +RZ IDVWZLOOWKHVKLSVDLOLQVHZDWUDW *f&* LQD IWZLQGLI WKHNH DQJOV LWHO ZLWK WKH UHVXOWDQWIRUFH RQ WKH URRWV" +LQW 7KLV LV D SUREOP L UHODWLYHYHORFLWHV

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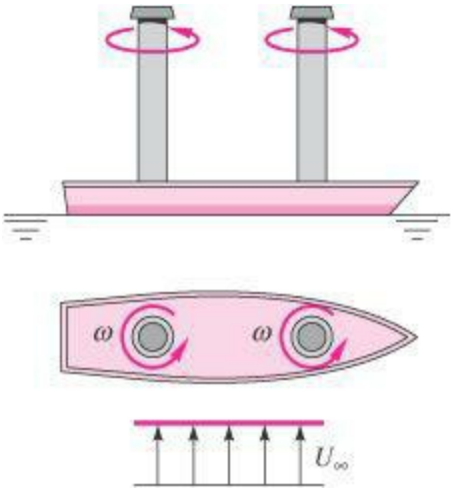

- 3 $ SURSRVHG IUHH VWUHDP YHORFLW\ ZRXOG XVH D FRQGHUZLWK SUHVVXUH WDSV DW ƒ DQG DW ƒ 7KH SUHVVXUH GLIIHUHQFH ZRXOG EH D PHDVXUH RI VWUHDP YHORFLW\ 8, +RZ HYHU WKH FRQGHUPXWEHDQGHG VR WKDW RQH WDS H[DFWO IDFHV WKH IUH VWUHDP / HVWKHPLVDQJOHQDQGEH/ WKDWLV WKH VZRVDSVDUHDW ƒ / DQG ƒ / 0 DNHDSDVFDO WKH SHUFHQWDHHUURULQYHORFLW\PHDVXUHPHQWQWKHUDQJH í ƒ / ƒ DQGFRPPHQWRQWKHLGHD
- 3 ,QSULQFLSOH LWLV SRVVLEOH WR XVH URDWLQJ FRQGHUV DV DLUFUDIWZLQJV &RQVLGHU FRQGHU FP LQGLDPHWU URDWLQJ DW UPLQ ,WLV WR QWD N1 DLUSODQ FUXLVLQJ DW P V : KDWVKRXOG WKH FRQGHUOHQJWK EH' +RZ PXFK SRZHU UHTXLUHG WR PDLQWDLQ WKLV VSHHG' 1HJOHFWHQGHIHFWRQWKHURWDWQJ ZLQJ
- 3 3ORWWKHVWUHDPOLQHVGXHWRWKHFRPELQHGIORZ RI DOQHVLQNíP DWWKHRUJLQSXVOQ VRXUFHV P DWD DQG D +LWW$ FRQGHURI UDGLXV DZLOODSSHDU
- 3 7KH 7UDQVLWRŠ FDUSDQH LQ) LJ OLWV DPD[LPXP RI GEI DWWHD OHYHO 6XSSRVHD IWGLDPHWUURWDWQJ FRQGHU IWORQJ UHSDFHVWKHZLQJ VDWW\LQ) LJ D : KDWURDWRQUDWLQJ LQUPLQ ZRXGLWWKHFDUSDQHDWDWDN RI VSHH RI PLK' E (VWP DWWKHGUDJ RI WKHFRQGHUDWWKLVURDWRQUDWLQJ 1HJOHFWHQ HIHFWV

7KHFRPSOH[SRWHQWLDO

- 3 2QHRI WKHFRQGHUIORZ SDWWHUQVRI) LJ LVJLYHQE\ WKHFDUWHVLDQWHP IXQFWR $ $ \[í \ : KLFKRQH' &QWKHFRUUHVSRQGHQFHEHSURYHGIURP (T "
- 3 3ORWWKHVWUHDPOLQHVRI (T LQWKHXSSHUUJKWXDGUDQWRUQ +RZ GRHVWK YHORFLW\LQFUHDVHZLWK[RXWZDUGDORQJ WKH[D[LVIURP WKHRUJLQ')RUZKDWVRUQ DQJOHDQGYDOXHRI QZRXOGWKLVLQFUHDVHEHOQHDULQ[")RUZKDWVRUQHUDQJOHDQG ZRXOGWKHLQFUHDVHEHDV["
- 3 &RPELQHWKDQGDWRQIORZ) LJ E ZLWKDVRXUFHDWWKHRULJLQ

I] $] P QQ]

3ORWWKHVWUHDPOLQHVIRUP \$/ ZKHUH/ LVDOHQJWKVFDOH ,QWLSUHW

- 3 7KH VXSHUSRVLWRQ LQ3URE 3 OHDGV WR WKDQGQDWRQIORZ QHDUD FXUYHG EXPS L

FRQWDLQVWKHIODZ DOORZ) LJ E ' HWHUPLQHWKHPD[LPXP KHLJKW RI WKHEXPS
DVDIXQFWLRQRI WKHFRQWDLQV\$ DQGP

- 3 &RQVLGHUWKHSRODUFRRUGLQDWHVWHDP IXQFWLRQ \$ %U VLQ Z LWK%HTXDO IR
FRQYHQLHQFH WR IW V D 3RVWKHVWHDPOQH\$ LQWKHXSSHUKDOSODQH E
3RVWKHVWHDPOQH\$ DQG LQWKHIORZ SDWWHUQ F) LQG WKH ORFXV R
SRLQWVDERYH\$ IRUZKLFKWKHUHVXOWDQWYHORFLW IWV
- 3 3RVWQXPEHUIORZ SDWWD ZHGJH RI KDOI DQJOH OHDGV WR DQ LPSRUWDQWSSOLFDWLRQ R
OP LQDUERXQGDU\ OD\HUWKHRU FDOOHG WKH) DONQHU6NDQIORZV> SS ± (
/ HW[GHQRWHGLVWDQFH DORQJ WKHZHGJHZ DOO DVLQ) LJ 3 DQG OHW f 8V
(T WR ILQG WKH YDULDWLRQ RI VXUFH YHORFLW8 [DORQJ WKH ZDOO ,V WK
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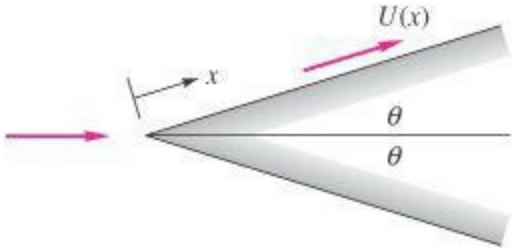

- 3 7KHLQYLVFLG YHORFLW DORQJ WKHZHGJHLQ3URE 3 KDVWKHDQDO\WF IRUP 8 [
&[P ZKHUHP Qí DQGQLVWKHH[SRQHQWQ(T 6KRZ WKDWIRUDQ & DQ
Q FRPSXWDWRQRI WKHERXQGDU\ OD\HUE\ 7KZDLWHVPHWKRG (TV DQG
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VLPLODU> S @
- 3 ,QYHVWLJDWHWKHFRPSOH[SRWHQWLDOIXQFWLRQI] 8,] D] DQGLQYHVWLJDWHIORZ
SDWWHUQ
- 3 ,QYHVWLJDWHWKHFRPSOH[SRWHQWLDOIXQFWLRQI] 8,] P OQ>] D]í D@DQG
LQYHVWLJDWHIORZ SDWWHUQ
- 3 ,QYHVWLJDWHWKHFRPSOH[SRWHQWLDO] \$ FRVK >CE] D@DQG SORWWKHVWUHDPOQH
LQVLGHWKHUHJLRQ VKRZQ LQ) LJ 3 : KDWK\SHQDWHG ZRUG RUJLQDOO\ )UHFK
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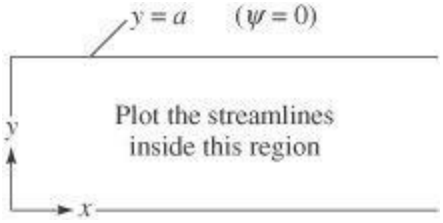

- 3 6KRZ WKDWWKHFRPSOH[SRWHQWLDO $f = U_\infty\{z + \frac{1}{4}a$ FRVK >CE] D@ UHSUHVHQWVIORZ SDWWD
RYDOVKDSHSODFHGPLGZD\ EHWZHHQWZR SDUDOOHOZDOOV $y = \pm\frac{1}{2}a.$: KDWVDSURFWFL
DSSOLFDWLRQ'
- 3) LJXUH3 VKRZ VWKHVWUHDPOLQHVDQGSRWHQWLDOOLQHVRI IORZ RYHUDWKLQSODWHZH

DV FRP SXWHG E\ WKH FRP SOH[SRWHQWLDO PHWKRG &RP SDUH TXDOLWDWLYHO ZLWK) LJ
D 6WDWH WKH SURSHU ERXQGDU FRQGLWLRQV DVVRFLDWHG ZLWK WKH YHORFLW SRWHQWLDO KDV HTXDOO VSDFHG YDOXHV : K\ GR WKH IORZ QHW³VTXDUHV EHFRP VP DOOHULQWKHRYHUGRZ NHV

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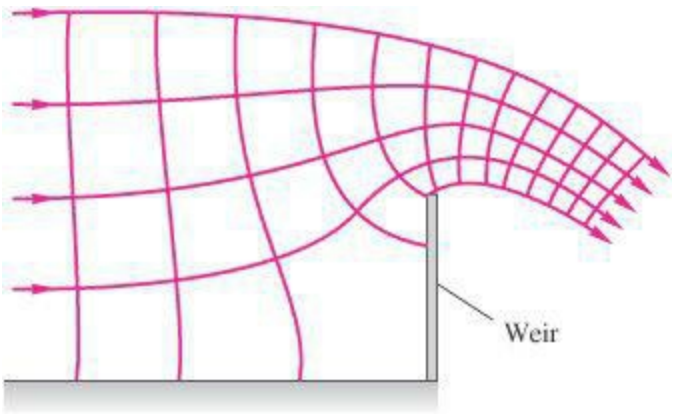

,P DJHV

- 3 8VH WKH PHWKRG RI LPDJHV WR FRQVWUXFW WKH IORZ SDWWHUQ IRU D VRXUFH P QHDU ZDOOV DV VKRZ QLQ) LJ 3 6NHWFK WKH YHORFLW GLVWULEXWLRQ DORQJ WKH IORZ HUZ D \ ,VWKHUH DQ\ GDQJHU RI IORZ VHSDUDWLRQ DORQJ WKLV ZDOO

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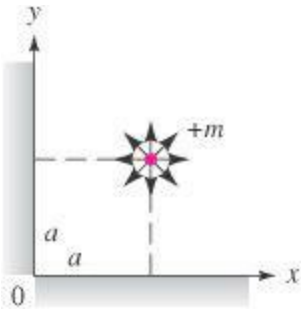

- 3 6HWXS DQ LPDJH V\VWHP WR FRPSXWH WKH IORZ RI D VRXUFH DWXQHTXDOGLVWDQFHV IURP VWUDLJKW ZDOOV DVLQ) LJ 3) LQG WKH SRLQWV PD[LPXP YHORFLW\ RQ WKH\ D[LV

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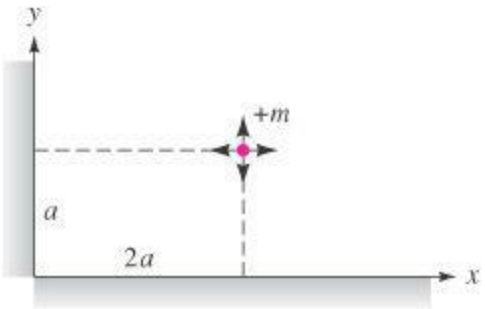

- 3 $ SRVLWLYH FLUFXODWLRQ . LV VWDUWHG LQ D FRUQHU DV LQ) LJ 3 &RPSXWH WKH YRUWLFLW LQGXFHG YHORFLWLHV YHFWRUV DW SRLQWV% [\ D D DQG FRPSDUH ZLWK WKH LQGXFHG YHORFLWLHV ZKHQ RZDOOV DUH SUHVHQW

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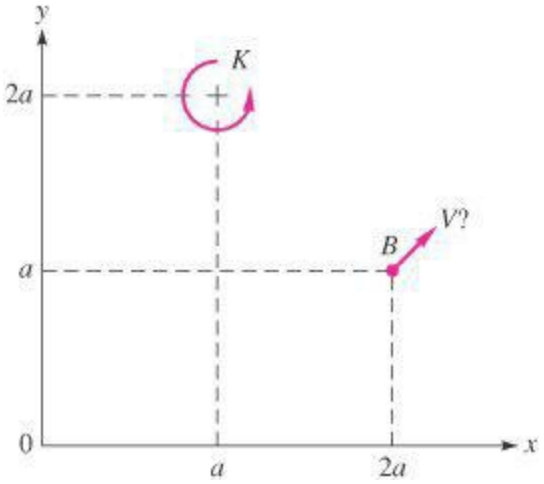

- 3 8VLQJ WKH IRUVRUFHLPDJH SDWKLQG HGGVRFRQWXFWKHIORZ QHDUDFRUQHULQ) LJ 3 ILQG WKH YDOXHRI WKH VRUFHVWUHQJWK WKDWZLOOLQGXFHDZDOOYHORFLWRI P VDWKH SRLQW[\ D MXVWEHORZ WKHVRXUFHVKRZQ LI D FP
- 3 8VHWKHPHWKRGRI LPDJHVWRDSSUR[LPDWHWKHIORZ SDWWLQSDWDF\OQGHUDGLXDQF DIURP DVLQJOHZDOODVLQ) LJ 3 7RLOOXVWDWWKHHIHFWRI WKHZDOOFRPSXW WKH YHORFLWLHV DWFRUUHVSRQGLQJ SRLQWV$ % & DQG' FRPSDULQJ ZLWK DF\OQGHIORZ LQDQLQILQLWHSDQHORI IOXLG

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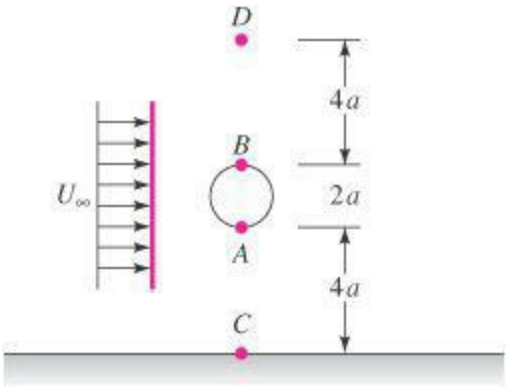

- 3 ' LVFXVV KRZ WKH IORZ SDWWLQRI3URE 3 PLJKWEH LQIOXHQFHG WR EH DQ LPDJH V\VWHP FRQVWUXFWRQIRUFLFXODUZDOOV : K\ DUHWKHUHZ7RLPDJHVLQXWDGRI RQH'
- 3 ,QGLFDWWKH\VWHP RI LPDJHVQHHGHGWRFRQVWUXFWWKHIORZ RI DXQIRUP VWUHDP SDD5DQNLQHKDOI ERG\ FRQVWUXLQHGEHWZHHQW7R SDUDOOHOZDOOV DVLQ) LJ 3) R WKHSDUWFXODUGLPHQVLRQV VKRZQLQWKLVILJXUH HVWLPDWWKHSRVLWRQ RI WKHQRVHR WKHUHVXOWLQJ KDOI ERG\

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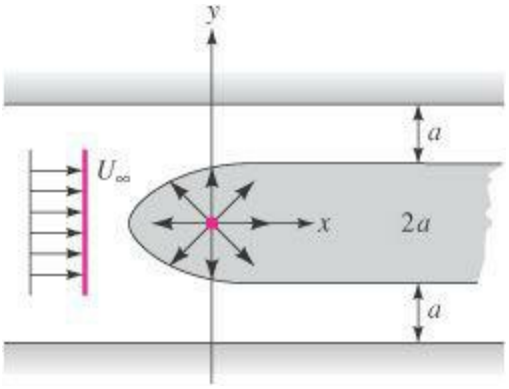

- 3 ([SODLQWKHVLJQLILFDQFHRIWKHIORZRIDQQHVRXUHSODFH XQLIRUPHWUFDOO EHWZHHQWZRSDUDOOHOZDOOVLQ) LJ 3 &RPSXWHWKHYHORFLW RQWKHORZHUZDOODW D +RZ PDQ LPDJHVDUHQHGHGWRHVWDEOLVKXQLIRUPLW ZLWKLQ SHUFHQW

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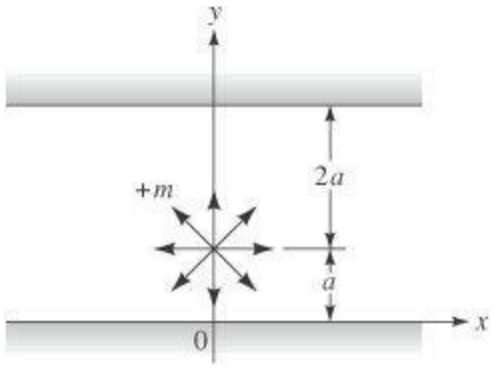

$ LIRLOWKHRU\ VZR GLP HQVLRQDO

- 3 7KHEHDXWXOH[SUHVVLRQIRUGLWR DVZR GLP HQVLRQDOIORZ(T DURV IURP DSSOLQJWKH-RXNRZVNLWDQVIRUPDWRQ] D] ZKHUH] [L\ DQG L 7KHFRQWDQWD LVDOHQJWKVFDOH7KHWKHRU WDQVIRUPVDFHUWDLQFLUFOHLQWKH SODQHLQRDQDLUIRLOLQWKH SODQH7DNLQJ D XQLWRUFRQYHQLHQFH VKRZ WKDWD DFLUFOHZLWKFHQWHUDWWKHRULJLQDQGUDGLXV! ZLOOEHFRPHDQHOOLSVHLQWKH SODQHDQG EDFLUFOHZLWKFHQWHUDW $x = -\epsilon \ll 1, y = 0,$ \ DQGUDGLXV ë ZLO EHFRPHDQDLUIRLOVKDSHLQWKH SODQH+LQW7KH(FHOVSUHGWKHWHVH[FHOOHQWF \RQYLQJWKLVSUREOP

- 3 *LYHQ DQ DLUSODQHRI ZHJKW ZLQJ DUHD$ DVSHFWUDWR $5 DQGIOLQJ DWD DQJOHWR ZKHUHWKHGHQVLW LV! $VVXPHDOOGUDJ DQGOLIWLV GXHWRWKHZLQJ ZKLFK KDVDQLQLWLDWVSDQGUDJ FRHILFLHQW& §) XUWKHUDVVXPHVXILFLHQWWKLQVWUXWWREDODQF ZKDWWHUGUDJ LVFDOFXODWHG D)LQG DQDQJHEUDLF H[SUHVVLRQIRUWKHEHWWFKLV YHORFLW9 $ ZKLFKRFFXUVZKHQWKHUDWR RI GUDJWRVSHGLVDPLQLPXP E $SSO \RXUIRUPXODWRWKHGDWDIQ3URE 3 IRUZKLFK DOOERUIXVJUDSKLQJ SURFHGXU JDYHDQDQZHU9 § P V

- 3 7KHXQLDJKWSODQH* RWDPHU&RQGRULQ ZDVWKHILUWWRFRPSOHWWKH. UHPH

3UHILJXUHHLJKWFRXUHXQGHUKXPDQSRZHU,WZLQJVSDQZDV P ZLWK&DY
P DQGDWRWDOPDVVRI NJ 7KHGUDJFRHILFLHQWZDVDSSUR[LPDWHO 7K
SLQRWZDVDEOHWRGHOYHUó KSWRSURSHOOHGHSDQH $VXPLQJWZRGLPHQVLRQDOIORZ
DWWDFKHGYHOHFLWDW D WKHFUXLVHVSHHGDWWDLQHG E WKHGLWFRHILFLHQWDQG F
WKHKRUVHSRZHUUHTXLUHGWRDFKLHYHDVSHHGRI NQ

- 3 7KH%RHLQJ DLUFUDIWKDVDJURVZHLJKWRI OE ZLQJDUHDRI IV
ZLQJVSDQRI IWDQG&R : KHQFUXLVLQJ DW IWVWDQGDUG
DQGVXGH WKHIRXUHQJLQHVHDFK SURYLGH OE RI WKUXVW$VXPLQJWKDWDOOOGL
DQGGUDJDUHGXHWRWKHZLQJ HWDPDWWKHFUXLVHYHORFLW LQPLK

- 3 5HUHFH FRQVLGLQVLQYLVFLG WKHRU FDOFXODWRQV IRUWKH XSSHUDQG ORZH
VXUIDFH YHORFLW GLVWULEXWLRQV9 [RYHU DQ DLUIRLO ZKHUH[LV WKH FKRUGZLV
FRRUGLQDW $VVLFDOUHVXOWVRUVPDOODQJOHRI DWWDFNLVDVIROORZV 8VHWKHVHGDW
SOXV%HUQRXOOL¶V HTXDWRQ WR HWDPDWD WKHDLUIRLOVXUIDFH DQG F WKHDQJOHR
DWWDFNL WKHDLUIRLOLVV\PPHWULF

x/c	V/U_∞ (upper)	V/U_∞ (lower)
0.0	0.0	0.0
0.025	0.97	0.82
0.05	1.23	0.98
0.1	1.28	1.05
0.2	1.29	1.13
0.3	1.29	1.16
0.4	1.24	1.16
0.6	1.14	1.08
0.8	0.99	0.95
1.0	0.82	0.82

$LUFRLOWKHRU\ ILQLWH VSDQZLQJV

- 3 $ZLQJRI SHUFHQWFDPEHU LQFKRUGDQG LQVSDQLVWHWXGDWDFHUWDLQDQJOHR
DWWDFNLQDZLQGWXQQHOZLWKVHDOHYHOVWDQGDUGDLUDW IWVDQGLVIRXQGWRKDY
OLWRI OE DQGGUDJRI OE (VWLPDWHIURP ZLQJWKHRU D WKHDQJOHRI DWWDFN
E WKHPLQLPXP GUDJRI WKHZLQJDQGWKHDQJOHRI DWWDFNDWZKLFKLWRFFXUV DQG F
WKHPD[LPXP OLWRGUDJUDWLR

- 3 $QDLUSODQHKDVDPDVVRI NJDQGIOHVDW P VDW P WDQGGUDGDOWXG
,WUHFDQJXODUZLQJKDVD P FKRUGDQGDV\PPHWULFDLUIRLODW ƒDQJOHRI DWWDFN
(VWLPDWH D WKHZLQJVSDQ E WKHDVSHFWUDWLR DQG F WKHLQGXFHGGUDJ

- 3 $IUHVKZDWHUERDWRI PDVV NJ LV WXSSRUWHG E\ DUHFDQJXODUK\GURIRLORI DVSHF
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. ƒ HWDPDWD WKHFKRUGOHQJWK E WKHSRZHUUHTXLUHGLI &, DQ
F WKHVSS VSHHGLI WKHERDWLV UHLWWG ZLWK DQHQJLQH WKDWGHOYHUV KS WR WK
ZDWU

- 3 7KH7UDQVLWRŠ FDUSDQHLQ) LJ KDVDJURVZHLJKWRI OE DZLQJVSDQR
IWDQG DZLQJ DUHD RI IW $VXPH WKDWWKH ZLQJ XVHV DQ1$&$

DLUHLO (VWP DWWKH DQJOHV DWDFNRI WKLVFDUSODQHZKHQFUXLVLQJDW PLKDW VWDQGDUGDOWWKGHRI IW

- 3 7KH%HHFKFUDIW & DLUFUDIWKDVDJURVZHJKWRI OE DQGDZLQJDUHDRI IWDQGIOHVDW PLKDW IWXWDQGDUGDOWWKGH ,WLVGULYHQE\ DSURSHOOHU WKDWGHOYHUV KSWRWKHDLU$VXPHIRUWKLVSUREOHP WKDWWKHDLUFUDIWLV WKH1$&$ VHFWLRQGHVFUEHGLQ) LJV DQGQHJOHFWDOOGUDJ H[FHSWWKHZLQJ : KDWLVWKHDSSURSULDWHDVSHFWDWLR IRUWKHZLQJ"

- 3 1$6$ LVGHYHORSLQJ DVZLQJ ZLQJ DLUSODQHFDOOHG WKH%LUG RI 3UH\ > @$VVKRZ LQ) LJ 3 WKHZLQJVSLYRWDNHDSRFNHWWRZDUG D VWDJKWE R EDFNZDUG F ' LVFXVDSRVLEOHDGYDQWDJHIRUHDFKRI WKHVHZLQJSRVLWRQV ,I \R FDQ¶WWKLQNRI RQH UHDGWKHDUWFOH> @DQGUHSRUWWKHFDVH

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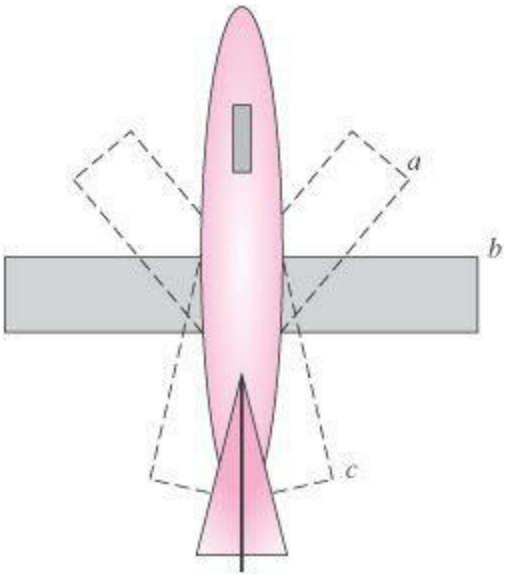

$[LV\PPHWULFSRVWQWDOCIORZ

- 3 ,IË U LQD[LV\PPHWULFIORZ LVGHILQHGE\ (T DQGWKHFRRUGLQDWVDUHJLYH LQ) LJ GHWUPLQHZKDWSDUWDOGLIHUHQWDOHTXDWRQLVVDWLVILHGE\ Ë
- 3 $ SRQWWRXUHZLWKYROXPHIORZ 4 P VLVLPPHUVHGLQDXQIRUP WWHDP R VSHHG P V$ 5DQNLQHKDOEERG\ RI UHYROXWLRQ &RPSXWH D WKHGLVWDQF IURP VRXUFHWRWKHVWDJQDWLRQSRLQWDQG E WKHVZRSRLQW U RQWKHERG\ VXUIDF ZKHUHWKHORFDOYHORFLWHTXDO P V
- 3 7KH5DQNLQHKDOEERG\ RI UHYROXWLRQ) LJ FRXOGVLPXODWWKHVKDSHRI DSLWRW WXEH) LJ $FFRUGLQJ WRLQYLVFLGWKHRU KRZ IDUGRZQWHDP IURP WK QRVHVKRXOGWKHVWDJQ SUHVXUHKROHVEHSODFHGVRWKDWWKHORFDOYHORFLWLVZLWKL “ SHUFHQWRI 8, " &RPSDUH\RXUDQZHUZLWKWKHUHFRPPHQGDWRQ[§ ' LQ) LJ

- 3 ' HWUPLQH ZKHWKHU WKH 6WRNHV WWHDPDQHV IURP(T DUH HYHUZKHUH RUWKRJRQDOWRWKH6WRNHVSRWHQWLDOLQHV IURP(T DVLVWKHFDVHIRU&DUWHVLD DQGSODQHSRWHQWLDOIORZV

- 3 6KRZ WKDWWKH[LVPPHWULFSRWHQWLDOIORZ IRUPHGE\ VXSHUSRVLWLRQRI DSRLQWVRXUF P DW[\ íD DSRLQWVLQNíP DW D DQGDVWUHDP8, LQWKH[GLUHFWLR IRUPVD5DQNLQHERG\ RI UHYROXWLRQDVLQ) LJ 3) LQGDQDOWF H[SUHVVLRQVIR GHWHUPLQLQJWKHOHQJWK / DQGPD[LPXP GLDPHWHU 5 RI WKHERG\ LQWHUPVRI P 8, DQGD

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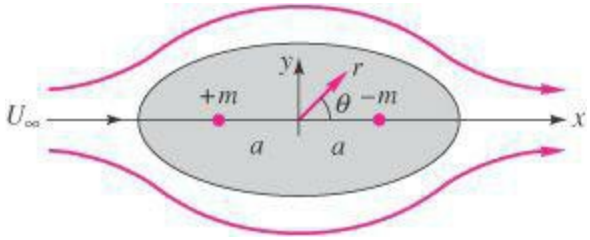

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- 3 7KH5DQNLQHERG\ RI UHYROXWLRQLQ) LJ 3 LV FP ORQJ DQG FP LQGLDPHWHU : KHQLWLV LPPHUVHG LQWKH ORZ SUHVVXUH ZDWU WXQQHODV VKRZQ FDYLWDWLRQ PD DSSHDU DW SRLQW$ &RPSXWH WKH VWUHDP YHORFLW8 QHFHVVDU\ WR FDYLWDWLRQ IRUPDWLRQ IRUZKLFKFDYLWDWLRQRFFXUV

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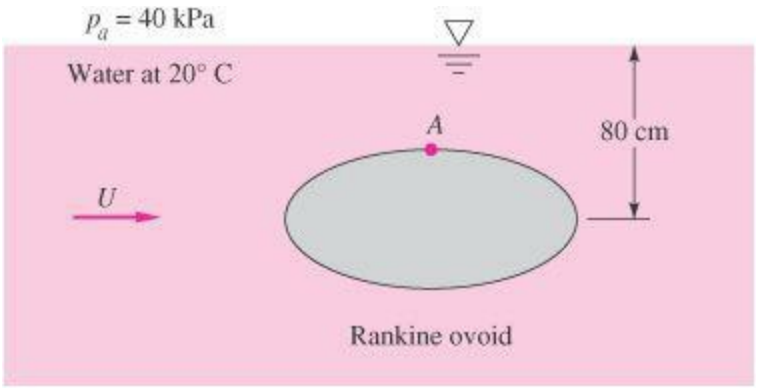

- 3 : HKDYHVXGLHGWKHSRWWRXUH VLQ DQGWKHQHVRXUH VLQ RI LQLQJWHSV LQRWKHSDSHU' RHVLWPDNHDQ VHQVHVRGHIDILQJWKQHVLQ VRXUH D LQ) LJ 3 " ,I VR KRZ ZRXG\RXHVWDEOLVKWKHPDWKHPDWFDOSURPHUWVRI VXFK ILQJWQHVLQ' : KHQ FRPELQHG ZLWK D XQLIRUP VWUHDP DQG D SRWWRXUH R HTXLYDOHQWWXWHQJWKDVLQ) LJ 3 VKRXGDFRVHGERG\ VKDSHEHIRUPHG' 0 DNH JXHVV DQG VNHWFK VRPH RI WKHVH SRVVLEOH VKDSHV IRU YDUIRXV YDOXHV RI WK GLPHQVLRQOHVSDUDPHWHUP 8, /

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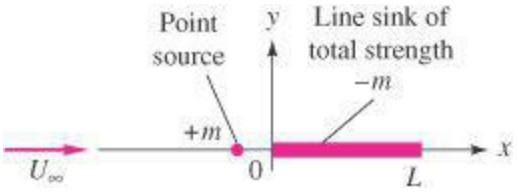

- 3 &RQVLGHUDLUIORZ LQJ SDVWDKHPLVSKHUHUHVWQJ RQDIODWXUDFH DVLQ) LJ 3 , VKH LQWHJUDO SUHVXUH LVS_ ILQG DQ H[SUHVVLRQ IRU VKH SUHVXUH IRUFH RQ VK KHPLVSKHUH %\ DQDORJ\ ZLWK3URE 3 DWZ KDVRLQV$ RQVKHKHPLVSKHUHVKRXG DKRHEHFXWR WKDVKHSUHVXUHIRUFHZ LOOEH]HUR DFFRUGLQJ WRLQYLVFLG WKHRU\"

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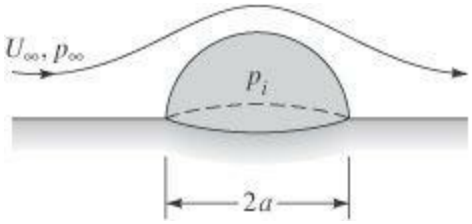

- 3 $ P GLDPHWUVSKHUHLVEHLQJ WRZHGDWSHG9 LQIUHVK ZDWUDW f& DVVKRZ LQ) LJ 3 $VXPLQJ LQYLVFLG WKHRU\ ZLWKDQXQGLVWXUEHGIUHHVXUDFH HVWLP DW WKH VSHG9 LQP V DWZKLFK FDYLWDWRQ ZLOOILUVWDSSHDURQ WKHVSKHUHVXUDFH : KHUHZ LOOFDYLWDWRQDSSHDU) RUWKLVFRQGLWRQ ZKDWZ LOOEHWKHSUHVXUHDWSRLQ $RQWKHVSKHUHZKLFKLV ƒ XSIURP WKHGLUHFWRQRI WUDYHO"

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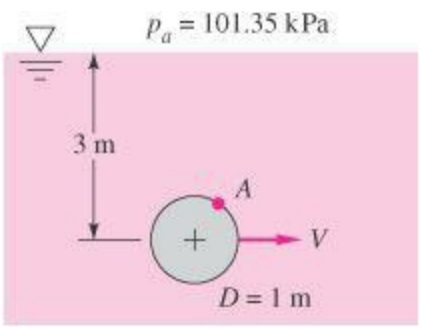

- 3 &RQVLGHUDVWHDGVSKHUH 6* RI GLDPHWU FP GURSSHGIURP UHVWLQZ DW DW f& $VXPHDFRQVWDQWGUDJ FRHILFLHQW&& $FFRXQWQJ IRUWKHVSKHUH K\GURG\QDPLF PDW HULDP DW D LW WUP LQDOYHORFLW DQG E WKH WLPH WR UHDF RI WUP LQDOYHORFLW &RPSDUHWKHVHWRWKHUHVXOWZKHQK\GURG\QDPLF PDVL QHJOHFWHG 9WUPLQDO§ P VDQGW § V DQGGLVFXV
- 3 $ JRO EDOOZ HJKV GE DQG KDV D GLDPHWUR LQ $ SURHVWLRQDOJRO VWULNHV WKHEDOODWLQLWLDOYHORFLW RI IW DQ XSZ DUG DQJOH R f DQG I EDFNVSLQ IURQWR WKHEDOOURWDWQJ XSZ DUG $VXPHWKDWWKHGUDWFRHILFLHQWRQ WKH EDOO EDVHG RQIURQWDODUHD IRGRZ V) LJ 3 ,I WKHJURXQG LV GHYHODQG GUDJ L QHJOHFWHG PDNHDVLPSOHDQDOVLVWR SUHGLFWWKHLPSDFWSRLQWD ZLWKRXWSLQDQ E ZLWKEDFNVSLQR UPLQ
- 3 &RQVLGHULQYLVFLGIORZ SDVWD VSKHUHDVLQ) LJ) LQG D WKHSRLQWRQWKHIURQ

VXUDFHZKHUHWKHIOXLGDFFHOHUDWRQD$PDJ LVPDJLPXP DQG E WKHPDJQLWXGHRI D$PDJ
F ,I WKH VWUHDP YHORFLW LV P V ILQG WKH VPKUHGLDPHWUIRUZKLFKD$PDJ LV
WPHVWKHDFFHOHUDWRQRI JUDYLW &RPPHQW

+\GURG\QDPLFPDVV

- 3 &RQVLGHUDF\OLQGHURI UDGLXVDPRYLQJDWVSHHG8, WKURXJKDWXLOIOLG DVLQ) LJ
3 3ORWWKHVWUHDPOLQHVUHODWLYHWRWKHF\OLQGHUE\ PRGLI\LQJ (T WRJLY
WKHUHODWLYHIORZ ZLWK. ,QWURGXFWRILQG WKHVWUHDOUHODWLYHNLQHWLFHQHUJ\ DQ
YHUL\ WKHK\GURG\QDPLFPDVVRI F\OLQGHUIURP (T

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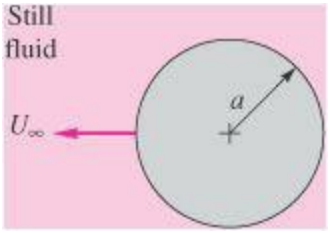

- 3 $ FP GLDPHWUVRQG DOXPLQXP VSKHUH 6* LVDFFHOHUDWQJ DW P V L
ZDWUDW *& D $FFRUGLQJWRSRWQWLDOWKHRU ZKDWVWKHK\GURG\QDPLFPDVVR
WKHVSKHUH' E (VWLPDWWKHIRUFHEHLQJ DSSOLHGWRWKHVSKHUHDWWKLVLQVWDQW

1XPHULFDOPHWKRGV

- 3 /DSODFH¶V HTXDWRQ LQ SRODU FRRUGLQDWV (T LV FRPSOFDWHG E\ WK
YDULDEOHUDGXV &RQVLGHUWKHILQLWHGLIIHUHQFHPHWKLQ) LJ 3 ZLWKQRGHV L M
HTXDO VSDFHG û DQG û UDSDUW' HULYH DILQLWHGLIIHUHQFHPRGHOIRU(T
VLPLODUWRWKHFDUWHVLDQH[SUHVVLRQ

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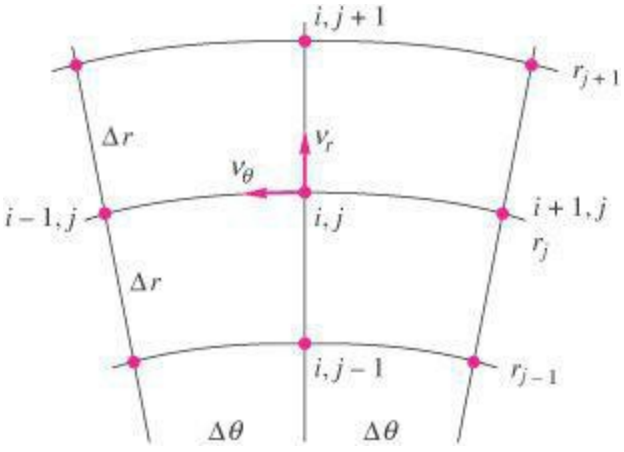

- 3 6HWXS WKHQXPHULFDOSUREOPP R) LJ IRUDQH[SDQVLRQRI f $ QHZ JUL
V\VWP DQG DQRQVTXDUHPHVK PD\ EHQHGHG * LYHWKHSURSHUQRGDOHTXDWRQDQ

EHQGLQJ FRQGLWLRQV ,I SRVVLEOH SURJUDP WKLV f H[SDQVLRQ DQG VROYH RQ I FRPSXWHU

- 3 &RQVLGHU WZR GLPHQVLRQDO SRWHQWLDO IORZ LQWR D VWHS FRQWUDFWLRQ DV LQ) LJ 3 7KH LQOHWYHORFLW8 P V DQG WKH RXOHWYHORFLW8 LV XQLRUP 7K QRGHV L MDUH ODEHOHG LQWKH ILJXUH 6HWXS WKH FRPSOHWH LQWHUIHUHQFH DJHEUDL UHODWLRQV IRU DOO QRGHV 6RON LI SRVVLEOH RQ D FRPSXWHU DQG SORWWKH VWUHDPOLQHV I WKH IORZ

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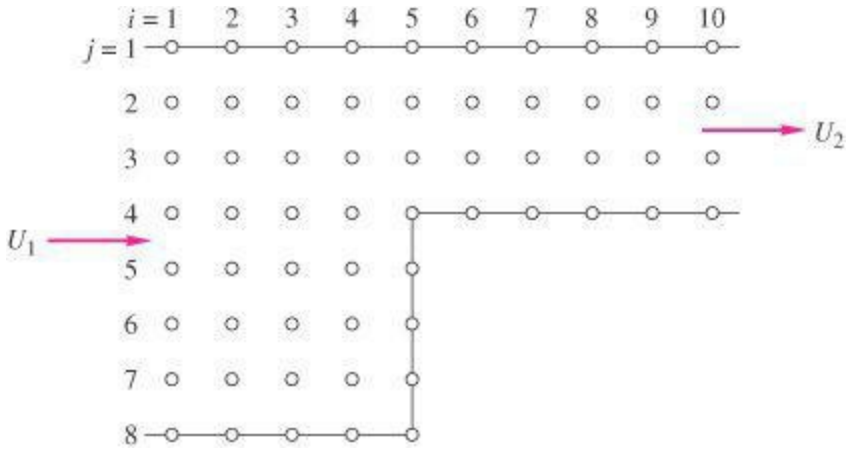

- 3 &RQVLGHU LQYLVFLG IORZ WKURXJK DVWR GLPHQVLRQDO f EHQG ZLWK D FRQWUDFWLRQ D LQ) LJ 3 $VVXPH XQLRUP IORZ DW WKH HQWUDQFH DQG H[LW0 DNH D ILQH GLIIHUHQFH FRPSXWDQDOVLVIRUVPDOOJUGVL]H DWWDDW QRGHV GHWHUPLQHWK GLPHQVLRQOHVV SUHVXUH GLVWULEXWLRQ DORQJ WKH ZDOO DQG VNHWK WKH VWUHDPOLQH <RXPD\ XVHHLWKHUTXDUHRUUHFWDQJXODUJULGV

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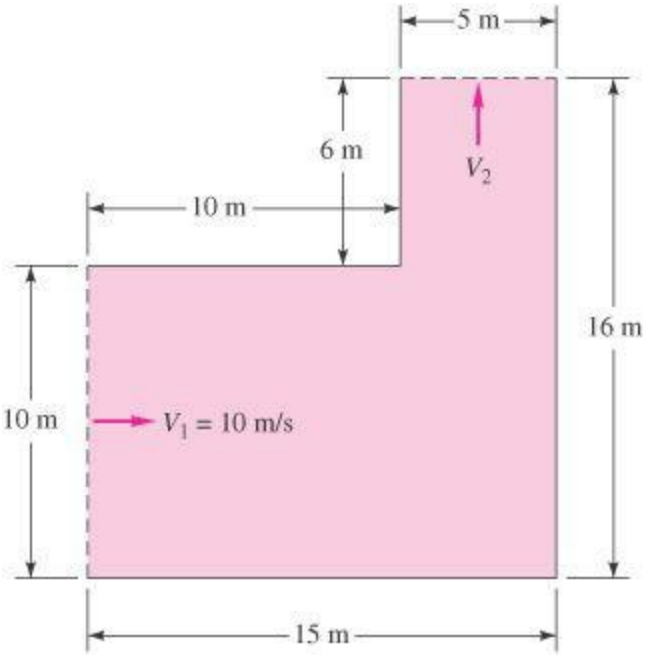


- 3) RUIXOO GHYHORSHG ODPLQDU IRPSUHVVLEOH IORZ WKURXJK D VWDLJKWQRQFLUFXODU GXFWDVLQ6HF WKH1DYLHU6WRNHVHTXDWLRQV UHGXFHWR

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{dp}{dx} = \text{const} < 0$$

ZKHUH \ ] LV WKH SODQH RI WKH GXFW WURVV VHFWLRQ DQG [LV DORQJ WKH GXFW[LV * UDYLWL QHJOHFWHG 8VLQJ DQRQVTXDUH UHFWDQJXODU JULG û[û\ GHYHORS DILQLWH GLIIHUHQFH PRG-IRUPV HTXDWLRQ DQG LQGLFDWH KRZ LWP D\ EH DSSOLHG WR VROYH IRU ICRZ LQ DUHFWDQJXODU GXF RI VLGHOHQJWK D DQG E

- 3 6ROYH 3UREOHP 3 QXPHULFDOO IRU D UHFWDQJXODU GXFW RI VLGHOHQJWK E E\ E XVLQJ DWOHDVW QRGDO SRLQWV (YDOXDWH WKH YROXPH ICRZ UDWH DQG WKH IULFWLRQ IDFWRU DQG FRPSDUH ZLWK WKH UHVXOWLQ 7DEOH

$$Q \approx 0.1143\frac{b^4}{\mu}\left(-\frac{dp}{dx}\right) \qquad f\mathrm{Re}_{D_h} \approx 62.19$$

ZKHUH'K $ 3 E IRU VKLV FDVH &RPPHQW RQ WKH SRVVLEOH WUDGHRIIV RI \RXUPRGHO

- 3 ,Q&)' W[WERRNV> ± @RQH RI WQUHSODFHV WKH OHWWKDQG VLGHV RI (TV E DQG F ZLWK WKH IRORZLQJ ZRH[SUHVLRQV UHVSHFWLYHO

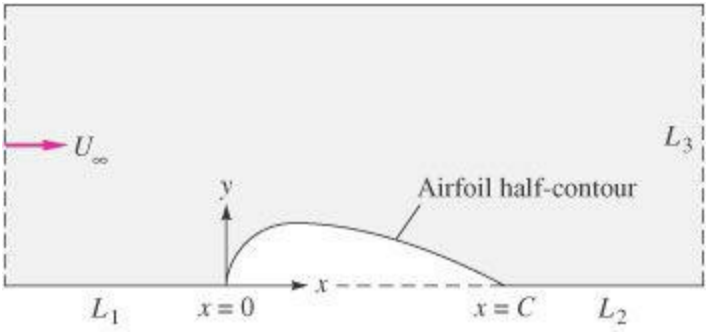
$$\frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(vu) \qquad \text{and} \qquad \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2)$$

$UH WKHVH HTXLYDOHQW H[SUHVLRQV RU DUH WKH\ PHUHO VLPSOLILHG DSSUR[LPDWRQV" (LWK-ZD\ ZK\ PLJKW WKH VHIORUPV EHEHWWU IRUILQLWHGLIIHUHQFH SXUSRVHV"

- 3 5HSHDW 3[DPSOH XVLQJ WKH LPSOLFLW PHWKRG RI (T 7DNH û W DQG û\ P ZKLFK HQVXUHV WKDWDQH[SOLFLW PRGHOZRXOG GLYHUJH &RPSDU \RXUDFFXUDF\ ZLWK 3[DPSOH
- 3 ,I \RXULQWXLWLRQKDVDQRQDHSRAQWDOOHRZ EHRXQGDU HOPHQWVFRP SXWUFRG-FRQVLGHUIICRZ SDVWD VPPHWUF DUURLO DV LQ) LJ 3 7KH EDVF VKDSH RI D 1$&$ VPPHWUF DUURLOVGHILHGE\ WKHIXQFWRQ> @

$$\frac{2y}{t_{\max}} \approx 1.4845\zeta^{1/2} - 0.63\zeta - 1.758\zeta^2 + 1.4215\zeta^3 - 0.5075\zeta^4$$

ZKHUH [& DQG WKH PD[LPXP WKLFNQHVV W/P RFFXUV DW 8VH WKLV VKDSH DV SD RI WKH ICRZ HUEROQGDU IRU]HUR DQJOHRI DWWDFN / HW WKH WKLFNQHVV EH IDLLQ GDUJH VD W/P RU &KRRVH DJHQHURXV QXPEHUR QRGHV DQG FDOFXODWH DQG SORW WK YHORFLW GLVWULEXWLRQ9 8' DORQJ WKH DLUIRLOVXUIDFH &RPSDUH ZLWK WKH WKHRUHWFDOUHVXOWV 5H IRU1$&$ RU DLUIRLOV ,I WPHSHUPLW LQYHVWLJDWWKH HIHFWRI WK ERXQGDU OHQJWK/ / DQG/ ZKLFK FDQLQWURGXFH EHVHWHTXDOWR WKH FKRUG OHQJWK&



- 3 8VH WKH H[SOLFLW PHWKRG RI (T VR VROYH 3URE 3 QXPHULFDOO IR 6$(RLOD W *& ZLWK 8 P VDQG& 0 UDGV ZKHUH0 LV WKH QXPEHUR OHWWHUV LQ \RXU VXUQDPH 7KLV DXWRUZLOO VROYH WKH SUREOHP IRU0 : KH VWDUG\ RVFLOODWRQLVUHDFKHG SORWWKHRLOY HORFLWY HUVXV WLPHDW FP

SUREOHPV TXHVWLRQV: RUG 3UREOHPV

- : : KDWLPSOLFDWLRQV KDYH EHHQ PDGH LQ WKH SRWHQWLDOIRZ WKHRU RI WKLV FKDSWH ZKLFK UHVXOWLQ WKH HOLPLQDWLRQ RI WKH 5H\QROGV QXPEHU) URXGH QXPEHU DQG 0 DF QXPEHUDVLPSOXDQVSDUDPHWHUV"
- : ,Q WKLV FKDSWHU ZH VXSHULPSRVH PDQ EDVLF VROXWLRQV D FRQFHSWDVWRFDWHG ZLWK QDU HTXDWLRQV <HW%HORXQGDU\ HTXDWLRQ LV QRQOLQHDU EHLQJ SURSRUWLRQDOV WKH VTXDUHRI WKHY HORFLW +RZ WKHQ GR ZH HVWXW\ WKH XVHRI VXSHUSRVLWLRQLQLQYLVFL IORZ DQDOVLV"
- : * LYH D SK\VLFDO H[SODQDWLRQ RI FLUFXODWLRQ * DV LWUHDWV VR WKH OLWRUFH RQ D LPPHUVHG ERG\ ,I WKH OQH LQWHJUDO GHILQHG E\ (T LV]HUR LWP HDQV WKDWWK LQWHJUDG LDSHUHFWGLIHUHQWDO² EXWRI ZKDWYDULEOH"
- : * LYH D VLP SOH SURRI RI(T ² QDPH0 WKDWVERWWKH UHDOLQDU SDUWR D IXQFWLRQ I] DUH OSSDFLDQ LI] [L\ : KDWLV WKH VHFUHWRI WKLV UHP DUNDEO EHKDYLRU"
- :) LJXUH FRQWLQV ILYH ERG\ FRUQHUV : LWKRXWFDUU\LQJ RXWDQ FDOFXODWLRQ H[SODLQSK\VLFDO ZKDWWKHYDOXHRI WKHIQYLVFLGIOXLGYHORFLWP XWEHDWWDFK RI WKHM ILYHFRUQHUV ,VDQ IORZ VHSDUDWLRQH[SHFWG'
- : ([SODLQWKH. XWWDFRQGLWLRQSK\VLFDO : K\ LVLWQHFHVDU"
- : : H KDYH EUH0 RXWQG ILQLWH GLIHUHQFH DQG ERXQGDU HOPHQWP HWKRGV IR SRWQWLDOIORZ EXWKDYH QHJOHFWG WKHILQLWHOPHQWWHFKQLTXH ' R VRP HUDGLQJ DQ ZULWDEUH HWD\ RQWKHXVHRI WKHILQLWHOPHQWP HWKRGIRUSRWQWLDOIORZ SUREOHPV

SUREOHPV TXHVWLRQV &RP SUHKHQVLYH 3UREOHPV

- & ' LG\RX NQRZ WKDWVRX FDQ VROYH VLP SOHIOXLGP HFKDQLFV SUREOHPV ZLWK0 LFURVR ([FHO" 7KH VXFFHVVLYH UHDO[DWLRQ WHFKQLTXH IRU VROYLQJ WKH / DSODFH HTXDWLRQ IR SRWQWLDOIORZ SUREOHPV LVHDVLO VHWXS RQD VSUHDGVKHHWWLQH WKHVWHP IXQFWLRQ HDFK LQWHUIRUHOLVVLP SO WKHDYHUDJHRI LWIRXUQHLJKERUV $ VDQH[DP SOH VROYHIR WKHIORZDURXQGDSRWQWLDOIORZ WKURXJK D FRQWDFWLRQ DV JLYHQ LQ) LJ & 1RWH 7F

DYRLGWKH³FLUFXODUUHHUHQFH´HUURU\RXPXVWWKLQRQWKHLWDOLDQRSWLRQ 8VHWKHKHC
LQGH[IRUPRUHLQIRUPDWLRQ)RUIXOOFUHGLWDWDFKDSWHUORXWR\RXUVSUHDGWKHWZLV
WXUHP IXQFWLRQFRQYHUJHGDQGWKHYDOXHRI WKHWXUHP IXQFWLRQDWHDFKQRGHGLVSOD\H
WRIRXUGLJLWVRI DFFXUDF\

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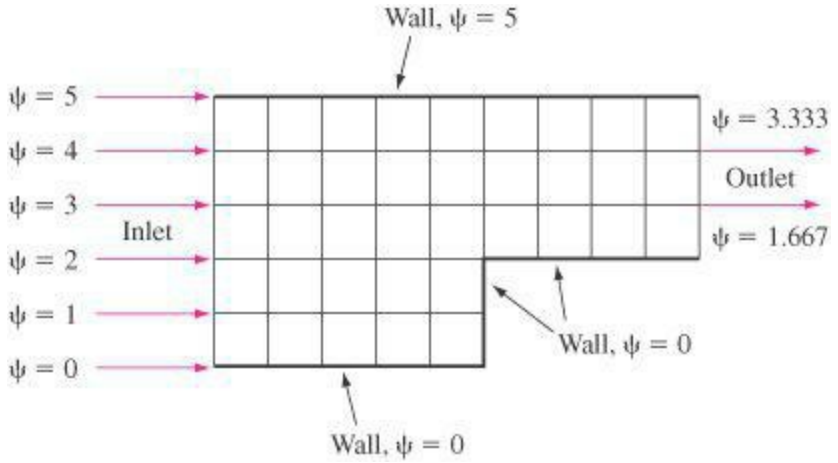

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FDVHRI 6$(RLODW *& VWDUWLQJ IURP UHVWQHDUDI[HGZDOO) DUIURP WKHZDOOWK
RLODFFHOHUDWLQQHDU WKDWLV X X1 DWZKHUHD P V $VW V GHWHUPLQ
D WKHRLOYHORFLW DW FP DQG E WKHLQWDQDQHRXV ERXQGDU\ FRQGLWLRQV
ZKHUHX § X +LQW7KHUHLVDQRQJHURSUHVVXUHJUDGLHQWLQWKHRXWU QHDU
WKHDUIUHH VWUHDP Q 1 ZKLFK PXVWEHLQFOXGHGLQ(T DQG\RXUH[SOL
PRGHO

- & &RQVLGHUSDQHLQYLVFLGIORZ WKURXJKDV\PPHWULF GLIIXVHUDVLQ) LJ & 2QO WK
XSSHUKDOI LVVKRZQ 7KHIORZ LVWR H[SDQGIURP LQOHWKDOI ZLGWK WR H[LWKDOI ZLGW
K DVVKRZQ 7KHH[SDQVLRQDQJOH LV *HVXSDQRQVTXDUHSRVWQW
IORZ PHWKRFRWKLVSUREOP DQGFDOFXODWDQGSORWD WKHYHORFLWGLVWULEXWRQDQG E
WKHSUHVVXUHFRHILFLHQWDORQJ WKHFHQWHOLQH $VXPHXQIRUP LQOHWVDQG H[LWVOH
- & 8VHSRVWQWDOIORZ WR DSSUR[LPDWWKHIORZ RI DLUEHLQJ VXFNHGXS LQWR D YDFXX
FOHDQHUWKURXJK DWZR GLPHQVLRQDOVODWDXDFKPHQWDV LQ) LJ & ,Q WKH[\ SODQ
WKURXJKWKHFHQWHOLQHRI WKHDXDFKPHQWPRGHOWKHIORZ DVDQLQFRPSUHVVLEOHQ
ZLWKLWD[LVLQWKH] GLUHFWLRQDWKHLJKWD DERYHWKHIORU D 6NHWK WKHVXUIDFHQ
DQGORFDWHQ VXLWDEOHSRLQWVLQWKHIORZ E)LQGWKHPDJQLWXGHRI YHORFLW9 [
DORQJ WKHIORRULQWHUVHFWLRQSRLQWVDQGP F /HWWKHSUHVXUHIDUDZD EHS
ZKHUHYHORFLWLV]HUR ' HLQHDYHORFLWVFDOH8 P D ' HWHUPLQHWKHYDULDWLRQR
GLPHQVLRQOHVSUHVXUHFRHILFLHQW&S S í S !8 DORQJ WKHIORU G 7K
YDFXXP FOHDQHULVPRVWHIIHFLYHZKHUH&S LVDPLQLPXP² WKDWLV ZKHUHYHORFLWL
PD[LPXP)LQGWKHORFDWRQVRI PLQLPXP SUHVXUHFRHILFLHQWDORQJ WKH[D[LV H $
ZKLFK SRLQWV DORQJ WKH[D[LV GR \RX H[SHFWWKH YDFXXP FOHDQHUWR ZRUN PRV
HIHFWLYH" ,VLWEHWWDV[GLUHFW EHQHDWKWKHVDWRUDWRPHRWKHU[ORFDWR
DORQJ WKHIORRU' &RQGXFWD VHQVLWLYH[SHULPHQWDWKRPHZLWKDYDFXXP FOHDQHUDQ
VRPHVPDOOSLHFHV RI GXWRUGLWWR WWXRXSURGLFWRQ 5HSRUWRXUHVXOWV DQ

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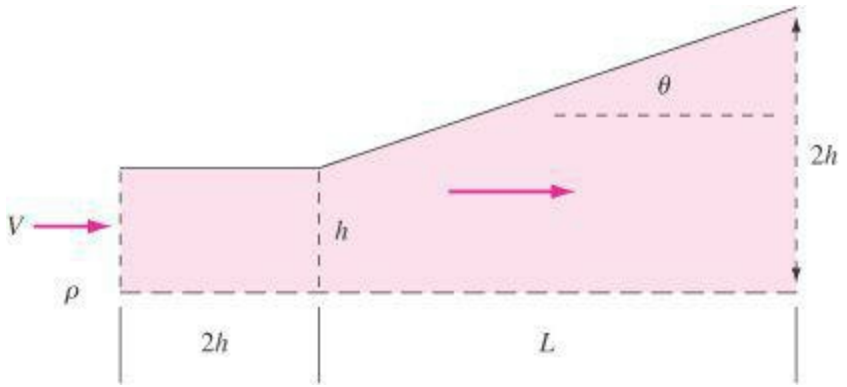

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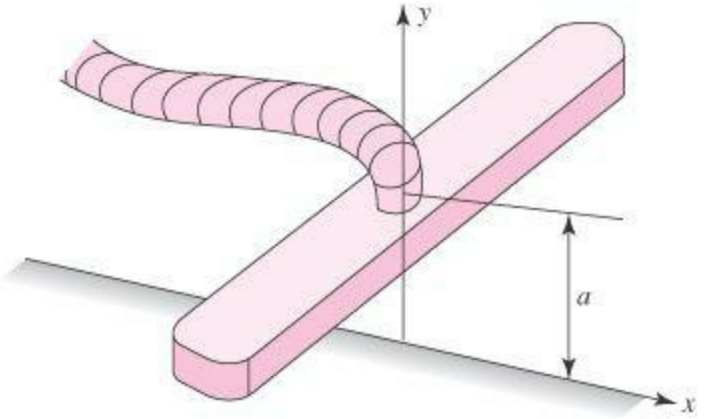

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- & 5HFRQVLGHUWKHQDVGUDJGDWDIRUWKH1$&$ DLURLOIURP3URE 3 D $JDL GUDZ WKHSRODUQDVGUDJ SORWDQG FRPSDUHTXDOWDWYHO ZLWK) LJ E) LQG WK PD[LPXP YDOXHRI WKHQDVWRGUDJUDWR F ' HPRQVWUDWHGWKDWGQHFRQWUFWRQR WKHSRODUSORWWKDWZLOOLPPHGLDWHO\LHOG WKHPD[LPXP / ' LQ E G ,I DQDLUFUD FRXOG XVH WKLV WZR GLPHQVLRQDOZLQJ LQDFWXDOIOJKWQR LQGXFHG GUDJ DQG KDG I SHUHFWSLORWHWWPDWKRZ IDU LQPLOHV WKLV DLUFUDWFRXOG JOGH WR D VHDOHYHUDWH IWDOWWKGH
- &) LQG DIRUPXODIRUWKHVWHDP IXQFWLRQIRUIORZ RI DGRXEOHWRI VWHQJWK DGLVDQFHI IURP DZDOO DV LQ) LJ & D 6NHWK WKHVWHDPOQHV E \$UHWKHUHDQ VWDJQDWR SRLQW F) LQG WKHPD[LPXP YHORFLW\DORQJ WKHZDOODQGLWVSRVLWLRQ

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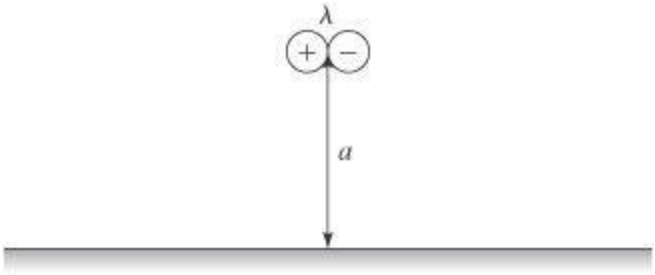
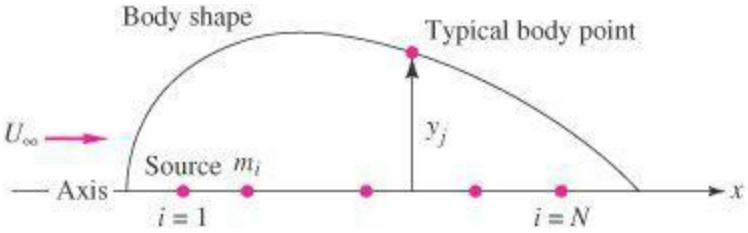


' HVLJQ3URMHFWV

- ' ,Q 7KHRGRUHYRQ. iUPiQGHYHORSHGDVFKHPHWRXVHDXQLRUP VWUHDP SO DURZ RI VRXUFHVDQGVLQNV WRJHQHUDWHDQDUELWUDU\FORVHG ERG\ VKDSH $ VFKHPDWL RI WKH LGHDLV VNHWFKHGLQ) LJ ' 7KHERG\ LV\PPHWULF DQG DW]HUR DQJOH F DWWDFN $ VPDOOR1 VRXUFHVDQGVLQNVDUHGLVWULEXWHGDORQJ WKHD[LVZLWKLQWKHERG\ ZLWK VWUHQJWKV P_L DWSRVLWLRQV [_L IRUL 7KHREMHFWLV WR ILQG WKHFRUUHF GLVWULEXWLRQ RI VWUHQJWKV WKDWDSSUR[LPDWHV D JLYHQ ERG\ VKDSH<[DWD ILQL QXPEHURI VXUIDFHORFDWLRQV DQG WKHQ FRPSXWH WKHDSSUR[LPDWH VXUIDFH YHORFLW DQG SUHVVXUH 7KH VWUHDPOLQHV IRU HLWKHU VWR GLPHQVLRQDO ERGLH GLVWULEXWHGDQHVRUFHV RUERGLHVRI UHYROXWLRQ GLVWULEXWHGSRLQWVRUFHV
-)RUURXUERG\ VKDSHOHWXV VHOHFWWKH1 $&$ DLUIRLO JLYHQE\ WKHIRUPXOD L3URE 3 ZLWKWV_DI ' HYHORS WKH LGHDV VDWAG KHUH LQWR1 VLPXODQHRXV DJHEUDL HTXDWLRQV WKDWFDQ EH XVHG WR VROYH IRU WKH1 XQNQRZQ QDQHVRXUFHVLQN VWUHQJWKV 7KH SURJUDP \RXUHTXDWLRQV IRUD FRPSXWU ZLWK1 VROYH IRUP_L FRPSXWH WKH VXUIDFH YHORFLWV DQGFRPSDUHZLWKWKH WKHRUHWFDOYHORFLWV IRUWKLV VKDSHLQ5H <RXJRD VKRXOGEHWRDFKLHYHFFXUDF\ ZLWKLQ" SHUFHQWRI WKHFDOFXODWHGV ,I QFHVVDU\R VKRXOGDGMXVW1 DQGWKHORFDWRQV RI WKHVRXUFHV

'



- ' 0 RGLI\ 3URE ' WR VROYH IRUWKH SRLQWVRXUFHGLVWULEXWLRQ WKDWDSSUR[LPDWHV D ³ ! ERG\ RI UHYROXWLRQ VKDSH 6LQFH QR WKHRUHWFDOUHVXOWV DUH SXEOLVKHG VLPSO PDNHVXUHWKDWVRXUFHVFRQYHUJHWR" SHUFHQW
- ' &RQVLGHU ZDWHU DW *f& IORZLQJ DW P V LQ D ZDWHUFKDQQHO $ 5DQNLQH RYD FA\OLQGHU FP ORQJ LV WR EH SODFHG SDUDOOHO WR WKH IORZ ZKHUH WKH ZDWHU VWDWH SUHVVXUHLV N3D 7KHRYDO¶V WKLFNQHVV LV DGHVLJQSDUDPHWU 3UHSDUHD SORWRI WK PLQLPXP SUHVVXUH RQ WKH RYDO¶V VXUIDFH DV D IXQFWLRQ RI ERG\ WKLFNQHVV (VSHFLDOO QRWH WKH WKLFNQHVVWV ZKHUH D WKH ORFDO SUHVVXUHLV N3D DQG E FDYLWDWRQ ILUV RFFXUV RQ WKH VXUIDFH

5HIHUHQFHV

- : HUPHU 3RWQDO 7KHRU 6SULQJHU 9HUODJ 1HZ <RUN

- 0 5REHUWRQ+\GURG\QDPLFV LQ 7KHRU DQG $SSOLFDWLRQ 3UHQWLFH+DOO (QJOHZRR &OLIIV 1-

/ 0 0LQH7KRPVRQ 7KHRUHWLFDO+\GURG\QDPLFV VWHG ' RYHU 1HZ <RUN

7 5DQVRUG 3RWQDO 7KHRU LQ WKH &RPSOH[3ODQH &DPEULGJH 8QLY 3UHVV 1HZ <RU

7 &HEHFL &RPSXWDWLRQDO) OXLG ' \QDPLFV IRU (QJLQHHUV 6SULQJHU 9HUODJ 1HZ <RU

5 : /HZLV 3 1LVKLDUDX DQG. 1 6HHKDUDP X) XQGDPHQWDOV RI WKH) LQLWH (OHPHQ 0HWKRGIRU+HDWDQG) OXLG) ORZ : LOH\ 1HZ <RUN

* %HHU, 6PLWK DQG& ' XHQVHU7KH%RXQGDU (OHPHQW0HWKRGZLWK3URJUDPPLQJ)R (QJLQHHUVDQG6FLHQWLVWV 6SULQJHU9HUODJ 1HZ <RUN

$ ' 0RRUH ³) LHOGVIURP) OXLG) ORZ 0 DSSHUV!- $SSO3K\V YRO SS :

+ - 6 +HDH6KDZ ³,QYHVWLJDWLRQRI WKH1DWXUHRI WKH6XUIDFH5HVLVWDQFHRI : DWUDQG I 6WUHDPOLQH0RWRQXQGHU&HUWDLQ(([SHULPHQWDO&RQGLWLRQV! 7UDQV ,QVW1DY $UFKLW YRO S

6 : &KXUFKLOO9LVFRXV) ORZV 7KH3UDFWLFDO8VHRI 7KHRU %XWWHUZRUWK6WRQHKDP 0 $

- ' $QGHUVRQ -U) XQGDPHQWDOV RI $HURG\QDPLFV WKHG 0F* UDZ +LOO 1HZ <RU

, + $EERWWDQG$ (YRQ' RHQKRII 7KHRU RI : LQJ 6HFWLRQV ' RYHU 1HZ <RUN

) 2 6PHDWRQ ,QWURGXFWRU\ $HURG\QDPLFV DQG +\GURG\QDPLFV RI : LQJV DQG %RGLHV S 6RFLHWZDUH%DVHG$SSURDFK $,$$ 5HVWRQ 9$

/ 3UDQGWO ³$SSOLFDWLRQV RI 0RGHUQ +\GURG\QDPLFV WR $HURQDXWLFV! 1$&$ 5HS

) 0 : KLWH 9LVFRXV) OXLG) ORZ GHG 0F* UDZ +LOO1HZ <RUN

& 6 <LK) OXLG0 HFKDQLFV 0 F* UDZ +LOO1HZ <RUN

. 7 3DWDQ ³7DEOHV RI +\GURG\QDPLF 0 DW) DFWRUV IRU 7UDQVODWRQDO0 RWRQ! $60 (: LQWU$QQXDO0 HHWLQJ 3DSHU : $ 817

- / +HW DQG$ 0 2 6PLWK ³&DOFXODWRQ RI 1RQDLWQJ 3RWQDO) ORZ DERXW$UELWUD 7KUHH' LPHQVLRQDO%RGLHV! - 6KLS5HV YRO SS ±

. + +XEQHU7KH) LQLWH(OHPHQW0HWKRGIRU(QJLQHHUV ³HG : LOH\ 1HZ <RUN

- & 7DQQHKLOO' $ $QGHUVRQ DQG5 + 3OHWFKHU&RPSXWDWLRQDO) OXLG0 HFKDQLFV DQ +HDW7UDQVIHU GHG 7D\ORUDQG) UDQFLV %ULWWRO3$

- 1 1HZPDQ0 DULQH+\GURG\QDPLFV 0 ,7 3UHVV &DPEULGJH 0 $

3 7 7RNXPDUX DQG3 (' LPRXOLV ³7KH/ LIWRI D&\OLQGHU([HFXWLQJ 5RWDU 0RWRQVLQ 8QLIRUP) ORZ´ -) OXLG0 HFKDQLFV YRO SS ±

- +) HJLJHUDQG0 3HULF&RPSXWDWLRQDO0 HWKRGVIRU) OXLG' \QDPLFV6SULQJHU9HUODJ 1HZ <RUN

3 - 5RDFKH) XQGDPHQWDOV RI &RPSXWDWLRQDO) OXLG ' \QDPLFV +HUPRVD 3XE
$GGLVRQ:HVOH\ 10

& < &KHQJ ³$ Q,QWURGXFWLRQ WR&RPSXWDWLRQDO) OXLG' \QDPLFVDQG) ORZ / DE´ &KDS I
) OXLG 0 HFKDQLFV IRU &KHPLFDO(QJLQHHUV QG HG -DPHV 2 : LOHV HG 3UHQWLFH+D
8SSHU6DGGOH5LYHU1-

- ' $QGHUVRQ&RPSXWDWLRQDO) OXLG' \QDPLFV 7KH%DVLFVZLWK$SSOLFDWLRQV0 F* UDZ
+LOO1HZ <RUN

7 3 6HQJXSWD) XQGDPHQWDOV RI &RPSXWDWLRQDO) OXLG ' \QDPLFV 2UHQW/ RQJPD
+\GHUDEDG ,QGLD

0 ' HVKSDQGH -) HQJ DQG & / 0 HUNOH ³1 XPHULFDO0 RGHOLQJ RI WKH 7KHUPRG\QDP
(IIHFWVRI &DYLWDWLRQ´ -) OXLGV(QJ -XQH SS ±

3 $ ' XUELQDG5 %$ 3HWHUVRQ6WDWLVWLFDO7KHRU DQG0 RGHOLQJ IRU7XUEXOHQW) ORZ
: LOH\ 1HZ <RUN

& -) UHVDW ³3HUVSHFWLYH 6HDFKDQJ%HQFKPDUNVIURP &RPPHUFLDO&) ' &RGHV´-) OXLG
(QJ YRO -XQH SS ±

5 0 DUWQX]LDQG& 7URSHD ³7KH) ORZ DURXQG6XUIDFH0 RXQWHG 3ULVP DW2 EWDFOHVLQ
) XOO ' HYHORSHG&KDQQHO) ORZ ´ -) OXLGV(QJ YRO 0 DUFK SS ±

. % 6KDKDQG- +) HJLEU ³) OXLG0 HFKDQLFDQV9LHZ RI : LQG(QJLQHHULQJ / DUJH(G
6LPXODWRQRI) ORZ 3DVW D&XEH2 EWDFOH´ - : LQG(QJLQHHULQJ DQG,QGXVWULDO$HURG\QDPLFV
YRO ± SS ±

3 6DJDXW/ DUJH(GG\ 6LPXODWRQ IRU ,QFRPSUHVVLEOH) ORZV $Q ,QWURGXFWRQ UG HG
6SULQJHU1HZ <RUN

: - 3DOP ,QWURGXFWRQ WR0 $7/ $% IRU(QJLQHHUV 0 F* UDZ +LOO1HZ <RUN

$ * LOEW0 $7/ $% $Q,QWURGXFWRQZLWK$SSOLFDWRQV GHG : LOH\ 1HZ <RUN

- : +R\WDQG5 + - 6HOOQ ³) OXLG RYHU7XEH%DQNV² $ 9LVXDOL]DWRQ6WXG´-) OXLG
(QJ YRO -XQH SS ±

6 ' RXJODV ³6ZLWKEDGH) LJKWHU%RPEHU 3RSXODU6FLHQFH1RY SS ±

* %HHU3URJUDPPLQJ WKH%RXQGDU (OHPHQW0 HWKRG $Q ,QWURGXFWRQ IRU (QJLQHHUV
: LOH\ 1HZ <RUN

- ' $QGHUVRQ $+LVWRU RI $HURG\QDPLFVDQG,WV,PSDFWRQ) OLQJ 0DFKLQHV&DPEULGJH
8QLYHUVLW3UHVV &DPEULGJH 8.

%5RELQV0 DWKHPDWLFDO7UDFWV - 1RXUH / RQGRQ

7 . 6HQJXSWD $. DVOZDO6 ' H DQG0 1DU ³7HPSRUDO) OXLG ,QVWDELOLWLVIRU0 DJQX
5RELQV(IIHFWDW+LJK5RWDWLRQ5DWHV´ -) OXLGVDQG6WUXFWXUHV YRO SS ±

*) ' DJXVK DQG0 0 * ULJRULHY ³) DWDQG$FFXUDWH6ROXWLRQV RI 6WDG\ 6WDWHV) OXLG
8VLQJ 0 XOWLGRPDLQ%RXQGDU (OHPHQW0 HWKRGV´ -) OXLGV(QJ YRO -XO SS :

5 + . LUFKKRII 3RWHQWLDO) OXLG &RPSXWDU* UDSKLF 6ROXWLRQV 0 DUFHO' HNNHU 1HZ
<RUN

+ : HUHH ³+\GURG\QDPLF 9LVXDOL]DWLRQ RI WKH) OXLG DURXQG D 6WHDP0 DQG&\OLQGHUZLV
6XFWLRQ &RXWWDX 0 DOOYDUG 7XUELQH 6DLO0 RGHO´ / H5HFKHUFKH $HURVSDWLDOH YRO
SS ±

7 . 6HQJXSWD DQG 6 5 7DOOD ³5RELQV0 DJQXV(IIHFW$ &RQWLQXLQJ 6DJD´&XUUHQ
6FLHQFH YRO QR SS ±

3 5 6SDOGLQJ³$ LLSODQH7UDLOLQJ 9RUWH[´ $QQXDO5HYLHZ) OXLG0 HFKDQLFV YRO

SS ±

0 (OKRXU ³$VHVPHQWDQG0RGLILFDWLRQ RI 2QH(TXDWLRQ0RGHOV RI 7XUEXOHQFH IF

: DOO%RXQGHG)ORZV´ -)OLGV(QJ YRO -XO SS ±

– 7KHUHDUHDQDORJLHVEHWZHHQWKHEDVLFHTXDWLRQV³FLUFXODWLRQ DERXWKHERG\ JLYLQ
ULVHWRDOLIWRUFH2WKHUZLVHWKHVROXWLRQFRXOGQRWEHXQLTXH6HH6HF

– 7KHZULWULVLQGHEWHGWR3URI 7 . 6HQJXSWDRI ,, 7 . DQSXUIRUGDVDQGGLVFXVVLRQIRUWK
VXEVHFWLRQ

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW

– ,WLVRIWHQFDOOHG6WRNHV¶VWUHDP IXQFWLRQ KDYLQJ EHHQXVHGLDSDSHU6WRNHVZURWHLQ
RQYLVFRXVSKHUHIORZ

–) LQLWHGLIIHUHQFHVDUHQRWDOORZHDORQHPXVWVHW8 DQG& HTXDOWRQXPHULFDOYDOXHV

1RWH 8QLWIRUWDQGX¶VDUHVDQGP V UHSHFWLYHO

&KDSWHU &RP SUHVVLEOH) OLZ

7KLVLVDRQHLQDPLOOLRQVKRWRI WKH) +RUQHWLJKWHUSODQHSDVVLQJ WKURXJK WKH VSHHGR
VRXQG (QJLQH-RKQ* D\ DSKRWRJUDSKHUIRUWKH8 6 1DY\ FDXJKWWKHSKRWRMXVWDVWKHDLUFUDI
DSSURDFKHGVRQF VSHHGLQZHWDU 7KH VSHHGLVVOLGKWO EHORZ 0 D DQGYLVLEOHFRQGHQVDWR
VKRFNV IRUP RQWKH VXUIDFHVZKHUHORFDOYHORFLW VVXSHUVRQF ,QDQLQWDQWWKH) ZLOOE
IXOO VXSHUVRQF DQGWKHVHVKRFNVZLOOEHUHSODFHGE\ VKDUS FRQFDOVKRFNV IURP WKH QRVH DQ
RWKHUOHDGLQJHGJHVRI WKHDLUSODQH*(Photo supplied by the U.S. Navy.)*

0 RWLYDWLRQ

$ OOHJKWRI RXUSUHYLRXVFKDSWHUVKDYHEHHQFRQFHQWUDWHG ZLWK³ORZ VSHHG´ RU³LQFRP SUHVVLEOH
IORZ ZKHUHWKHIXLGYHORFLWVLPXFKOHVWVKDQLWVSHHGRI VRXQG ,QIDFWZHGLGQRWHYHQGHYHORS
DQH[SUHVVLRQIRUWKHVSHHGRI VRXQGRI DIXLG 7KDWVGRQHLQWKLVFKDSWHU

: KHQ D IXLG PRYHV DWVSHHGV FRPSDUDEOH WR LWV VSHHG RI VRXQG GHQVLW FKDQJHV EHFRP
VLJQLILFDQWDQG WKH IORZ LV WHUPHG FRP SUHVVLEOH 6XFK IORZ V DUH GLIILFXOWWR REWDLQ LQ ODTXLGV VLQF
KLJK SUHVVXUHV RI RUGHU DW DUH QHHGHG WR JHQHUDWH VRQF YHORFLWLHV ,Q JDVHV KRZHYHU I
SUHVVXUHUDWLRV RI RQO ZLOOOLNHO FDXVHVRQF IORZ 7KXVFRP SUHVVLEOHJDVIORZ LVTXLWFRP P RQ
DQGWKLV VXEMHFWLV RIWHQFDOOHGJDVG\QDPLFV

3UREDEO WKH VWR PRWLP SRUWDQWDQG GLVWLQFWLYH HIHFW RI FRP SUHVVLELOLW RQ IORZ DUH
FKRNLQJ ZKHUHQWKHGXFWORZ UDWH LV VKDUSO OP LWHG E\ WKH VRQF FRQGLWLRQ DQG VKRFN ZDYH
ZKLFK DUHQHDU0 GLVFRQWLQXRXV SURSHUWFKDQJHVLQDVXSHUVRQF IORZ 7KHSXUSRVH RI WKLVFKDSWUL
VR H[SODLQ VXFK WWLNJ SKHQRP HQD DQG WR IDPLOLDUL]H WKH UHDGHU ZLWK HQJLQHULQJ FDOFXODWRQV R

FRP SUHVVLEOHIORZ

6SHDNLQJ RI FDOFXODWRQV WKH SUHVHQFH DSSUR[LPDWH (QJLQHHULQJ (TXDWL
6RONHU ((6 &RP SUHVVLEOHIORZ DQDO\VLV ILOHG ZLWK VFRUHV RI FRP SOHFDWHG DQJHEUDLF HTXDWLRQ
PRWLRQ ZKLFK DUH YHU GLIILFXOW PDQLSXODW RU LQYHUW &RQVHTXHQW IRU QHDU D FHQWU
FRP SUHVVLEOHIORZ WK[WERRNVKDYHUHGRQH[WQLYHWEHVR 0 DFK QXPEHUHDWRQ VHH$SS %
IRUQXPHULFDOZRUN : LWK((6 KRZHYHU DQ VHWRI HTXDWLRQVLQWKLV FKDSWUFDQEHWSHGRXDQ
VROYHGIRUDQ YDULDEOH VHHSDUWE RI[[DPSOH IRUDQHVSHFLDO LQWFDWH[DPSOH : LWKVXF
DWRO$SS %VHUYHVRQ DVDEDFNXSDQGLQGHGPD\ VRQYDQLVKIURP WK[WERRNV

,QWURGXFWLRQ 5HYLHZ RI 7KHUPRG\QDPLFV

: HWRRNDEUH ORRNLQ&KDS >(TV WR @RVHHZKHQZHPLJKWDHO QHJOHFWWK
FRP SUHVVLEOHIORZ LQKHUHQWLQ HYHU UHDO IOXLG : H IRXQG WKDWWKH SURSHU FULWHULRQ IRU D QHDU
LQFRP SUHVVLEOHIORZ ZDVDVP DOOD DFK QXPEHU

$$\mathrm{Ma} = \frac{V}{a} \ll 1$$

ZKHUH9 LVWKHIORZ YHORFLW DQGD LVWKHVSHHGRI VRXQGRI WKHIOXLG 8QGHUVP DOO0 DFK QXPEH
FRQGLWLRQV FKDQJHV LQ IOXLG GHQVLW DUH HYHUZKHUH VP DOOLQ WKHIORZ ILHOG 7KH HQHUJ\ HTXDWR
EHFRP HVXQFRXSOHG DQG WKP SHUDWXUH HIHFW FDQ EH HLWKHUHGRUSXWDVLGHIRUDQDUWXG 7KH
HTXDWRQRI VWDWHGHQHUDWVLQRWKHVLP SOHVWDWP HQWKDWGHQVLW VLQHDU FRQVWDQW7KVP HDQ/WK
DQLQFRP SUHVVLEOHIORZ UHTXLUHVRQO DPRP HQWXP DQGFRQWLQXLW DQDO\VLV DVZHVKRZHGZLWKP DQ
H[DP SOHVLQ&KDSV DQG

7KLVFKDSWUWDWFRP SUHVVLEOHIORZ V ZKLFK KDYH0 DFK QXPEHUV JUHDWHUWKDQ DERXW DQ
WKXV H[KLELWQRQOLQHDUJLEOHGHQVLW FKDQJHV ,I WKHGHQVLW FKDQJHV DUH VLJQLILFDQWLWRORRZV IURP WK
HTXDWRQRI VWDWWKDWWKHWMP SHUDWXUHDQG SUHVVXUHFKDQJHV DUH DOR VXEVWDQWLO / DJHWMP SHUDWXU
FKDQJHV LP SO WKDWWKHHQHUJ\ HTXDWRQFDQ QR ORQJHUEHQHJOHFWG 7KHUHRUHWKHZRUN LV GRXEOH
IURP VZR EDVLF HTXDWRQV WR IRXU

&RQWLQXLW HTXDWRQ
0 RP HQWXP HTXDWRQ
(QHUJ\ HTXDWRQ
(TXDWRQRI VWDWH

WR EHVROYHG VLP XOWDQHRXV IRUIRXUXQNRZQ QV SUHVVXUH GHQVLW WMP SHUDWXUH DQG IORZ YHORFLW
S ! 7 9 7KVWKHJHQHUDOWKHRU RI FRP SUHVVLEOHIORZ LV TXLWFRP SOFDWG DQG ZHWL KHUHV
PDNHIXWKHUVP SOLFDWRQV HVSHFLDOO E\ DVXP LQJ DUHYHUVLEOHDGLDEDWF RULVHQWRSLF IORZ

7KH0 DFK 1XP EHU

7KH0 DFK QXP EHULV WKHGRP LQDQWSDUDP HWULQFRP SUHVVLEOHIORZ DQDO\VLV ZLWKGLIIHUHQWHIFV
GHSHQGLQJ RQ LW P DJQLWXGH $HURG QDP LFLWW HVSHFLDOO PDNH D GLVWLQFWLRQ EHWZHHQ WKH YDULRX
UDQJHVRI 0 DFK QXP EHU DQG WKHIORZ LQJ URXJK FODVVLILFDWLRQV DUHFRP P RQO XVHG

$Ma < 0.3$:	*incompressible flow*, where density effects are negligible.
$0.3 < Ma < 0.8$:	*subsonic flow*, where density effects are important but no shock waves appear.
$0.8 < Ma < 1.2$:	*transonic flow*, where shock waves first appear, dividing subsonic and supersonic regions of the flow. Powered flight in the transonic region is difficult because of the mixed character of the flow field.
$1.2 < Ma < 3.0$:	*supersonic flow*, where shock waves are present but there are no subsonic regions.
$3.0 < Ma$:	*hypersonic flow* [11], where shock waves and other flow changes are especially strong.

7KHQXPHULFDOYDOXHVOLVWHGDUHRQO\URXJKJXLGHV7KHYHILYHFDWHJRULHVRI IORZ DUHDSSURSULDWH IRUWKHVSHHGDUHDQDORJRXVWRWKHSRSXODUQRWLRQVRI VXEVRQLFVXSHUVRQLFDQGKLJKVSHHGDHURG\QDPLFV)RULQVWDQFHDLUIORZV WKHPRVWSRSXODUDSSOLFDWLRQLVVLPSO\ ZKHWKHUWKHIORZ LV VXEVRQLF 0 D RU VXSHUVRQLF 0 D! EHFDXVHWKHHIIHFWV DUHDFKDQJH UHYHUVHV DVZHVKRZ LQ6HF 6LQFHVXSHUVRQLF IORZ HIIHFWVPD\ JR DJDLQWLQWLQWLRQ \RX VKRXG VWXG\ WKHVHGLIIHUHQFHVFDUHXOO\

7KH6SHFLILF+HDW5DWLR

,QDGGLWLRQ WR JHRPHWU DQG 0DFK QXPEHU FRPSUHVVLEOH IORZ FDOFXODWLRQV DOVR GHSHQG RQ I VHFRQGGLPHQVLRQOHVVSDUDPHWU WKHLVSHFLILF KHDWDWR RI WKHJDV

$$k = \frac{c_p}{c_v} \tag{9.1}$$

(DUOLHU LQ&KDSV DQG ZH XVHG WKH VDPH VPERON WR GHQRWH WKHUPDOFRQGXFWLYLW RI I IOXLG : HDSRORJL]HIRUWKHGXSOLFDWLRQ WKHUPDOFRQGXFWLYLW GHVQRWDSSHDULQWKHVHOLVWRI RI WKHVW[W

5HFDOOIURP) LJ WKDWWKIRUWKHFRPPRQJDVHV GHFUHDVHV VORZ0 ZLWK WMP SHUDWXUH DQG GH EHWZHQ DQG 9DULDWRQVLQNKYHRQ DVOJKWHIIHFWRQFRP SUHVVLEOHIORZ FRP SXWDWRQV DQ DLU N § LV WKH GRPLQDQWOLG RI LQWHUW7KHUHRUH DOWKRXJK ZH DVLJQ VRP H SUREOP LQYROYLQJ RWKHUJDVHV QH VWMP DQG &2 DQG KHOXP WKHFRP SUHVVLEOHIORZ VDEOHV LQ$SS %DU EDVHG VROO RQWKHVLQJOHYDOXHN IRUDLU

7KLVWWRQDLQVRQ DVLQJHFKDSWURQFRP SUHVVLEOHIORZ EXWDVXDO ZKROHERRNVKDY EHHQZULWWQRQWKHVXEMHFW+HUHZHOLVWRQ FULQHQWRUFOLFDOWW5HHUHQFHV WR DU LQWURGXFWRU RU LQWUHGLDWWHDWHQW ZKLOH5HV DUHGYDQFHG ERRNV 2QHFDQ DOV EHFRPHVSHFLDOL]HGZLWKLQWKLVSHFLDOW RI FRP SUHVVLEOHIORZ 5HHUHQFH FRQFHQWV KSHURQL IORZ² WKDWV DWYHU KJK0 DFKQXPEHUV 5HHUHQFH H[SODLQVWKH[FLWQJ QHZ WHFKQLTXHRI GLUHF VLPXODWRQRI JDVIORZ VZLWKDPRGHFXODUG\QDPLFVPRGHO&RPSUHVVLEOHIORZ LVDOR ZHOOVXLWHGIR FRPSXWDWRQDOIOXLG G\QDPLFV &)' DVGHVFULEHGLQ5H) LQDOO DVKRUWWKRURXJKO UHDGDEO QR FDOFXOXV 5H GHVFULEHVWKH SULQFLSOHVDQG SURPLVHRI KJK VSHHG VXSHUVRQLF IOJKW) UF WMP HWRWMP HZ HVKDOOGHUVRP HVSHFLDO]HGWRSLF WRWKHVHRWKHUWW[W

: H QRWH LQ SDVLQJ WKDWDWOHDWWZR IORZ SDWWUQV GHSHQG VWURQJO RQ YHU VP DOO GHQVLW GLIIHUHQFHV DFRXVWF DQG QDWXUDO FRQYHFWRQ $FRXVWF > @LV WKH VXG RI VRXQG Z DY

SURSDJDWRQ ZKLFK LV DFFRPSDQLHG E\ H[WHPHO VP DOOFKDQJHV LQ GHQVLW SUHVVXUH DQG WHPSHUDWU
1 DXUDOFRQVHUYDWLQ LV WKH JHQHUDWLQ SDWKLQ VHWXS E\ EXR\DQF\ IRUFHV LQ DIOXLG VWDWLFH E
XQHYHQ KHDWLQJ RU XQHYHQ FRQGLWLRQV RI GLVWRUWHG PDWHULDO + HUH ZH DUH FRQFHUQHG RQO ZL
VWDG\ FRPSUHVLEOH IORZ ZKHUH WKH IOXLG YHORFLW LV RI PDJQLWXGH FRPSDUDEOH WR WKDW RI WKH VSHGR
VRXQG

7KH3HUHFW DV

,QSULQFLSOH FRPSUHVLEOH IORZ FDQ XQDWLQ VFDQEHP DGHIRUDQ IOXLG HTXDWLRQ RI VWDW DQG ZI
VKDOODWLQ SURECHP V LQYROYLQ WKH VWDP VDEOHV> @WKH JDV VDEOHV> @DQG GDTXLGV>(T @
%XWLQ IDFWP RXWHP HQDU WUDW WHDWHQW DUH FRQLQHG WR WKH SHUHFWJDV ZLWK FRQXDQWSHFLFK HDW

$$p = \rho RT \qquad R = c_p - c_v = \text{const} \qquad k = \frac{c_p}{c_v} = \text{const} \tag{9.2}$$

) RUDOOUHDOJDV HV F$ F% DQG NYDU ZLWK WP SHUDWXUHEXWRQ P RGHUDWH IRUH[DP SOH F$ RI DL
LQFUHDVHV SHUFHQWDV WP SHUDWXUHLQFUHDVHV IURP WR *) 6LQFHZ HUHO GHDOZ LWK VXFK OJI
WP SHUDWXUHFKDQJHV LWV TXLWUHDVRQDEOHWR DXXP HFRQXDQWSHFLFK HDW

5HFDOOIURP 6HF WKDWWKH JDV FRQXDQWV UHDWG WR DXQLYHUVDO FRQXDQW GLYLGHGE\ WKH JD
PROHFXDU ZHJKW

$$R_{\text{gas}} = \frac{\Lambda}{M_{\text{gas}}} \tag{9.3}$$

ZKHUH

$$\Lambda = 49{,}720 \text{ ft-lbf/(lbmol} \cdot {}^\circ\text{R)} = 8314 \text{ J/(kmol} \cdot \text{K)}$$

) RU DLU0 DQG ZH VKDOO DGRSWWKH IROORZLQJ SURSHUW YDOXHV IRU DLU WKURXJKRXWWKL
FKDSWU

$$R = 1716 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) = 287 \text{ m}^2/(\text{s}^2 \cdot \text{K}) \qquad k = 1.400$$

$$c_v = \frac{R}{k-1} = 4293 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) = 718 \text{ m}^2/(\text{s}^2 \cdot \text{K}) \tag{9.4}$$

$$c_p = \frac{kR}{k-1} = 6009 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) = 1005 \text{ m}^2/(\text{s}^2 \cdot \text{K})$$

([SHULP HQWDO YDOXHV RI NIRUHJKWRP P RQJDVHV Z HUH VKRZ QLQ) LJ) URP WKLV ILJXUH DQG WK
PROHFXDU ZHJKWWKH RWKHUSURSHUWLHV FDQEH FRP SXWG DV LQ(TV
7KH FKDQJHV LQ WKH LQWHUQDO HQHUJ\ € DQG HQWKDOS\ K RI D SHUHFWJDV DUH FRP SXWG IRUFRQXDQ
VSHFLFK HDWDV

$$\hat{u}_2 - \hat{u}_1 = c_v(T_2 - T_1) \qquad h_2 - h_1 = c_p(T_2 - T_1) \tag{9.5}$$

) RUYDULDEOHVSHFLFK HDWRQHP XWLQWHJUDWH $\hat{u} = \int c_v\,dT$ DQG $h = \int c_p\,dT$ RUXVHWKH JDV WDEOHV> @
0 RXWP RGHUQWKHUP RG\QDP LFVWH[WVQRZ FRQWDLQ VRIWZDUH IRUHYDOXDWLQJ SURSHUWHV RI QRQLGDOJDV

,VHQWURSLF3URFHVVV

7KHLVHQWURSLF DSSUR[LPDWLRQ LV FRPPRQLQFRPSUHVVLEOHIORZ VKHRU : HFRPSXWHVKHHQWURS\ FKDQJHIURP VKHILUVWDQG VHFRQGODZ VR VKHUPRG\QDPLFVIRUDSXUHVXEVWDQFH> RU @

$$T\,ds = dh - \frac{dp}{\rho} \tag{9.6}$$

,QWURGXFLQJ GK F$S G7 IRUD SHUHFWJDV DQG VROYLQJ IRUGV ZH VXEVWLWXWH! 7 S5 IURP VKH SHUHFWJDVODZ DQGREWLQ

$$\int_1^2 ds = \int_1^2 c_p \frac{dT}{T} - R\int_1^2 \frac{dp}{p} \tag{9.7}$$

,I F$S LV YDULDEOH VKHJDV WDEOHVRU((6 ZLOOEHQHHGHG EXWRUFRQVDQW$S ZHREWLQ VKHDQDOWL UHVXOW

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = c_v \ln\frac{T_2}{T_1} - R\ln\frac{\rho_2}{\rho_1} \tag{9.8}$$

(TXDWLRQV DUHXVHGVRFRPSXWHVKHHQWURS\ FKDQJHDFURVVDVKRFNZDYH 6HF ZKLFKL DQLUHYHUVLEOHSURFHVV

) RULVHQWURSLF IORZ ZH VHW V DQG REWLQ VKHVH LQWHUPHGLDWH SRZHUODZ UHODWLRQV IRUD LVHQWURSLF SHUHFWJDV

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{\rho_2}{\rho_1}\right)^k \tag{9.9}$$

7KHVHUHODWLRQVDUHXVHGLQ6HF

(; $0 3/ (

$UJRQIORZ VWKURXJK DVHHVXFK WKDWWKH LQLWLDOFRQGLWLRQVS 0 3DDQG! NJ P DQ LWILQDOFRQGLWLRQVS N3DDQG7 . (VWLPDWH D VKHLQLWLDOWHPSHUDWXUH E VKHILQDO GHQVLW F VKHFKDQJHLQHQWKDOS\ DQG G VKHFKDQJHLQHQWURS\ RI VKHJDV

6RROXWLRQ

) URP 7DEOH$ IRUDUJRQ 5 P V . DQGN 7KHUHRUHHWWHPDWLWVSHFLILF KHDWV FRQVWDQWVPSHUHWXUHIURP (T

$$c_p = \frac{kR}{k-1} = \frac{1.67(208)}{1.67-1} \approx 519 \text{ m}^2/(\text{s}^2 \cdot \text{K})$$

7KHLQLWLDOWHPSHUDWXUHDQGILQDOGHQVLWLHVPDNHIURP WKHLGHDOJDVODZ (T

$$T_1 = \frac{p_1}{\rho_1 R} = \frac{1.7\text{ E6 N/m}^2}{(18\text{ kg/m}^3)[208\text{ m}^2/(\text{s}^2 \cdot \text{K})]} = 454\text{ K} \qquad Ans.\ (a)$$

$$\rho_2 = \frac{p_2}{T_2 R} = \frac{248\text{ E3 N/m}^2}{(400\text{ K})\,[208\text{ m}^2/(\text{s}^2 \cdot \text{K})]} = 2.98\text{ kg/m}^3 \qquad Ans.\ (b)$$

)URP (T WKHHQHUJ\ FKDQJHLV

$$h_2 - h_1 = c_p(T_2 - T_1) = 519(400 - 454) \approx -28{,}000\text{ J/kg (or m}^2/\text{s}^2) \qquad Ans.\ (c)$$

7KHDJRQWHPSHUDWXUHDQGHQHUJ\ GHFUHDVHDVZHPRYHGRZ QWKHWEH $FWXDOO WKHUHPDLQ QR EHDQ H[WUDFWHGRQJ WKDWLV WKHIXLGHQHUJ\ PDLEHFRQYHUWHGE\ IULFWLRQ WRLQFUHDVHGNLQHWHQHUJ\ 6HF

)LQDOO WKHHQWURS\ FKDQJHLVFRPSXWHGIURP (T

$$\begin{aligned} s_2 - s_1 &= c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \\ &= 519\ln\frac{400}{454} - 208\ln\frac{0.248\text{ E6}}{1.7\text{ E6}} \\ &= -66 + 400 \approx 334\text{ m}^2/(\text{s}^2 \cdot \text{K}) \qquad Ans.\ (d) \end{aligned}$$

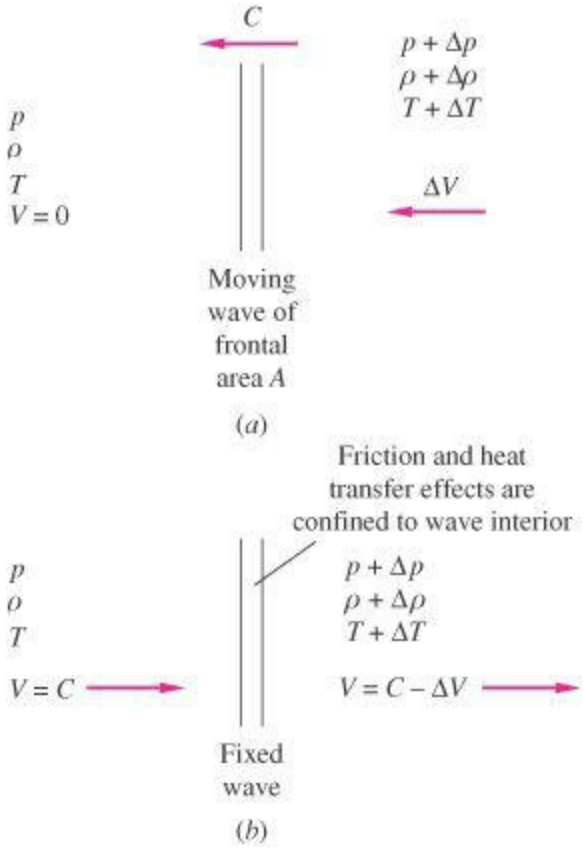
7KHIULGHQWRS\ KDVLQFUHDVHG ,I WKHUHLVQR KHDWWDQVIHU WKLVLQGLFDWHVDQLUUHYHUVLEOHSURFHVV 1RWHWKDWHQWURS\ KDV WKHVDPHXQLWVDVWKHJDVFRQVWDQWV6HFLF KHDW

7KLVSUREOHP LVQRWWUXHDELWDU\ QXPEHUV ,WFRUUHFWO VLPXODWHV WKHEHKDYLRUR DJRQPRYLQ VXEVRQLFDOO WKURXJK DWXEHZLWKDUHJIULFWLRQDOHIIHFWV 6HF

7KH6SHHG R6RXQG

7KH VR FDOOHG VSHHG R VRXQG LV WKH UDWH RI SURSDJDWLRQ R D SUHVVXUH SXOVH R LQLQWHVLW WKURXJK D VXLGIOXLG ,WLV D WKHUPRG QDPLF SURSHUW R DIOXLG / HWXV DQDO]H LWE\ IULFRQVLGHULQJ DSXOVHR IILQWHVLW DVLQ) LJ ,Q) LJ DWKHSXOVH RUSUHVVXUHZDYH PRYHVD VSHHG& WRZ DUG WKH VWDWLRQDU IOXLG S ! 7 9 DWWKH HHWOHDYLQJ EHKLQG DWWKH ULJKWD IOXLG R LQFUHDVHG SURSHUWLHV S ûS ! û! 7 û7 DQG DIOXLGYHORFLWû9 WRZ DUG WKHOHWRQBRZ LQJ WK ZDYHEXWP XFK VORZ HU: HFDQGHWHUPLQHWKHVHHIFWE\ PDNLQJ DFRQWROYROXPHDQDOVLV DFURV WKHZDYH 7R DYRLG WKHXQWDG\ WHUPVWKDWZRXOG EHQHFHVDU LQ) LJ D ZHDGRSWLQWDG WK FRQWROYROXPHR) LJ E ZKLFK PRYHVDWZDYHVSHHG& WRWKHOHW7KHZDYHDSSHDUVIL[HGIURP WKLV YLHZSRLQWDQG WKHIOXLG DSSHDUVWR KDYH YHORFLWV& RQ WKHOHWDQG& í û9 RQ WKHULJKW7K

VKHUPRG\QDPLF SURSHUWVS ! DQG7 DUHQRWDIHFWHGE\ WKLVFKDQJHRI YLHZSRLQW



) LJ & RQWURO YROXP H DQDO VLVRI DILQLWH VWHQJWK SUHVXUHZDYH *a* FRQWURO YROXP HIL[H VR VWKDWOLGDWDHW *b* FRQWURO YROXP HP RYLQJ WHWDWZDYHVSHHG*C*

7KHIORZ LQ) LJ E LVVWHDG\ DQGRQH GLP HQVLRQDOFRVWKHZDYH 7KHFRQWUROYROXP HIL[HG VR WKDWOLGDWDHW VXUIURP (T

$$\rho AC = (\rho + \Delta\rho)(A)(C - \Delta V)$$

or

$$\Delta V = C\frac{\Delta\rho}{\rho + \Delta\rho} \tag{9.10}$$

7KLVSURYHVRXUFRQWQOWRQWKDWWKHLQGXFHGIOXLG YHORFLW RQWKHULJKWLV P XFK VP DOOHUWKDQ WK ZDYH VSHHG& ,Q WKH OP LWRI LQILQLWHVLP DO ZDYH VWUHQJWK VRXQG ZDYH WKLV VSHHG LV LWH LQILQLWHVLP DO

1RWHWKDWWKHUHDUHQR YHORFLW JUDGLHQWV RQHLWKHUVLGHRI WKHZDYH 7KHUHRUH HYHQLI IXL YLVFRVLW LV ODUJH IULFWLRQDOHIHFW DUHFRQILQHG WR WKHLQWHULRU RI WKHZDYH \$GYDQFHG WHVW>R H[DPSOH @VKRZ WKDWWKHWKLFNQHVV RI SUHVXUHZDYHVLQJDVHV LV RI RUGHU í IWDWDWRVSKHUL SUHVXUH 7KXV ZH FDQ VDIHO QHJOHFWIULFWLRQ DQG DSSO WKH RQHGLP HQVLRQDOP RP HQWXP HTXDWR DFURVVWKHZDYH

$$\sum F_{\text{right}} = \dot{m}(V_{\text{out}} - V_{\text{in}})$$

or

$$pA - (p + \Delta p)A = (\rho AC)(C - \Delta V - C) \tag{9.11}$$

$ JDLQ WKH DUHD FDQ FHO DQG Z HFDQ VROYH IRU WKH SUHVVXUH FKDQJH

$$\Delta p = \rho C\,\Delta V \tag{9.12}$$

,I WKH Z DYH VWUHQJWK LV YHU VP DOO WKH SUHVVXUH FKDQJHV VP DOO
) LQDOO FRP ELQH (TV DQG VR JLYH DQ[SUHVLRQ IRU WKH Z DYH VSHHG

$$C^2 = \frac{\Delta p}{\Delta \rho}\left(1 + \frac{\Delta \rho}{\rho}\right) \tag{9.13}$$

7 KH ODUJHU WKH VWUHQJWK û! ! R WKH Z DYH WKH IDVWHU WKH Z DYH VSHHG WKDWLV SRZ HUXOH[SORVLR Z DYHV P RYH P XFK P RUH TXLFNO WKDQ VRXQG Z DYHV ,Q WKH OP LWR LQILQLWHVLP DO VWUHQJWK û! Z HKDY Z KDWLV GHILQHG WR EH WKH VSHHG R VRXQG D R DIOXLG

$$a^2 = \frac{\partial p}{\partial \rho} \tag{9.14}$$

%XW WKH HYDOXDWLRQ R WKH GHULYDWLYH UHTXLUHV NQRZ OHGJH R WKH WKHUP RG\ QDP LF SURSHUWLHV XQGHUJRQ E\ WKH IOXLG DV WKH Z DYH SDVVHV 6LU, VDF 1 HZ WRQLQ PDGH DIDP RXV HUURU E\ GHULYLQJ DIRUP XO IRU VRXQG VSHHG WKDW Z DV HTXLYDOHQW WR DVVXP LQJ DQ LVRWKHUP DO SURFHVV WKH UHVXOWEHLQJ SHUFH VRR GRZ IRUDLU IRU[DPSOH + HUDLRQDO]HG WKH GLVFUHSDQF\ DV EHLQJ GXH WR WKH³FUDWLXGH GX SDUWFOHV DQG VR RQ LQ WKH DLU WKH HUURUV FHUWDLQ XQGHUWDQGDEOH Z KHQ Z HUH OFWKDWWZ DV P DG \HDUV EHIRUH WKH SURPHUEDVLV Z DV OLG IRU WKH VHFRQG OD Z R WKH UP RG\ QDP LFV : H QRZ VHH WKDW WKH FRUHFWSURPHWV P XWEH DGLDEDWF EHFDXVH WKH UH DUH QR WKP SHUDWU JUDGLHQW[FHSWLQ LGH WKH Z DYH LWWHO) RU YDQLVKLQJ WUXQGW KV RXQG Z DYHV Z H WKHUHRUH KDYH D LQILQLWHVLP DOGLDEDWF RULVHQWRSLF SURPHWV 7 KHFRUUHFW[SUHVLRQ IRU WKH VRXQG VSHHGLV

$$\boxed{a = \left(\left.\frac{\partial p}{\partial \rho}\right|_s\right)^{1/2} = \left(k\left.\frac{\partial p}{\partial \rho}\right|_T\right)^{1/2}} \tag{9.15}$$

IRUDQ LGHDO JDV RUQLTXLG (YHQD VROGKDVD VRXQG VSHHG
) RUD SHUHFWJDV) URP (T RU Z HGHGXFHWKDWWKH VSHHG R VRXQGLV

$$a = \left(\frac{kp}{\rho}\right)^{1/2} = (kRT)^{1/2} \tag{9.16}$$

7 KH VSHHG R VRXQG LQFUHDVHV DV WKH VTXDUHURRWR WKH DEVROXWWP SHUDWXUH) RUDLU Z LWKN DQHDVLO P HP RU[HGGLP HQVLRQDOLRUP XODLV

$$\begin{aligned} a(\text{ft/s}) &\approx 49[T(^\circ\text{R})]^{1/2} \\ a(\text{m/s}) &\approx 20[T(\text{K})]^{1/2} \end{aligned} \tag{9.17}$$

$ WWD OHYHO WKDQGDUG WP SHUDWXUH *f) *f5 D IW/ 7 KLV GHFUHDVHV LQ WKH XSSH DWPRVSKHUH Z KLFK LV FRROHU DW IWWXDQGDUG DOWLWXGH 7 í *f) *f5 DQGD IW/ RU SHUFHQWPHV

7DEOH 6RXQG6SHHGR 9DUR XV0 DWPULDOVDW *f) *f& DQG DW

Material	a, ft/s	a, m/s
Gases:		
H_2	4,246	1,294
He	3,281	1,000
Air	1,117	340
Ar	1,040	317
CO_2	873	266
CH_4	607	185
$^{238}UF_6$	297	91
Liquids:		
Glycerin	6,100	1,860
Water	4,890	1,490
Mercury	4,760	1,450
Ethyl alcohol	3,940	1,200
Solids:*		
Aluminum	16,900	5,150
Steel	16,600	5,060
Hickory	13,200	4,020
Ice	10,500	3,200

*Plane waves. Solids also have a *shear-wave speed.*

6RPH UHSUHVHQWDWLYH YDOXHV RI VRXQG VSHHG LQ YDULRXV PDWHULDOV DUH JLYHQ LQ7DEOH) R
OLTXLGV DQG VROLGV LWV FRPSUHVVLELOLW\ RU EXONPRGXOXV. RI WKH PDWHULDO

$$K = -\mathcal{V}\frac{\partial p}{\partial \mathcal{V}}\bigg|_s = \rho\frac{\partial p}{\partial \rho}\bigg|_s \qquad (9.18)$$

,QWUP VRI EXONPRGXOXV WKHQ D . !) RUH[DPSOH DWXDQGDUGFRQGLWLRQV WKHEXONPRGXOX
RI OLTXLG FDUERQWHWUDFKORULGHLV * 3DDEVROXWH DQGLWVGHQVLW\LV NJ P ,WV VSHHG RI VRXQ
LVWKHUHRUHD (3D NJ P P V IW/ 6WHOKDV DEXONPRGXOXV RI DERX
(3DDQGZDWUDERXW (3D VHH7DEOH$ RU WPHVOHWVWKDQWKHD
) RU VROGV LWLV VRPHWPHV DWXPHG WKDWWKH EXONPRGXOXV LV DSSUR[LPDWHO HTXLYDOHQWW
<RXQ¶VPRGXOXV HDWWFLW(EXWLQ IDFWKHUHUDWHGSHQGVRQ3RLVVRQ¶VUDWLR1

$$\frac{E}{K} = 3(1 - 2\sigma) \qquad (9.19)$$

7KH VZR DUH HTXDO IR $\sigma = \frac{1}{3}$ ZKLFK LV DSSUR[LPDWO WKH FDVH IRUPDQ FRPPRQ PHWDOV VXFK D
VWHODQG DOXPLQXP

(; $0 3/ (

(WUP DWWKH VSHHG RI VRXQG RI FDUERQ PRQR[LGHDW N3D SUHVVXUHDQG *& LQP V

6RXWLRQ

) URP 7DEOH$ IRU&2 WKHPROHFXDUZHJKWV DQGN§ 7KXVIURP (T 5&2

P V . DQG WKH JLYHQ WHPSHUDWXUH LV f& . 7KXV IURP (T
ZH HVWLPDWH

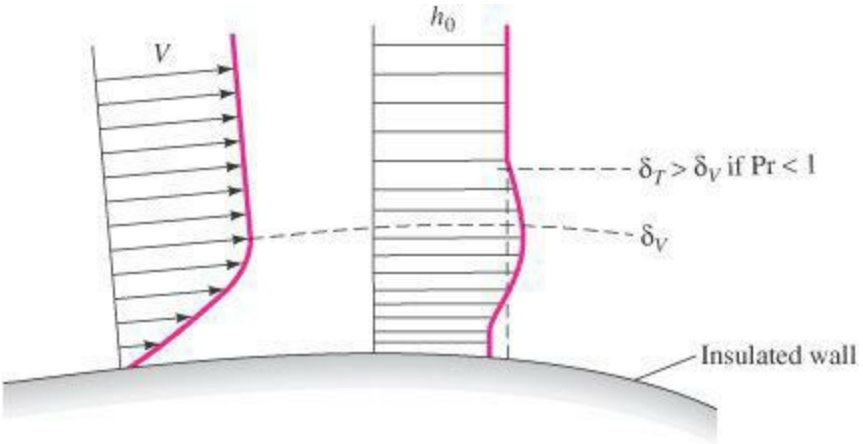
$$a_{CO} = (kRT)^{1/2} = [1.40(297)(573)]^{1/2} = 488 \text{ m/s} \qquad Ans.$$

$GLDEDWLF DQG ,VHQWURSLF 6WDJQ) ORZ

$V PHQWLRQHG LQ6HF VWKH LVHQWURSLF DSSUR[LPDWLRQ JUHDWO VLPSOLILHV D FRPSUHVVLEOH IORZ FDOFXODWLRQ 6R GRHV WKH DVVXPSWLRQ RI DGLDEDWLF IORZ HYHQ LI QRQLVHQWURSLF &RQVLGHUKJK VSHHG IORZ RI DJDV SDVWDQ LQXODWHG ZDOO DV LQ) LJ 7KHUHLV QR VKDIWZRU GHOLYHUHG WR DQ SDUW RI WKH IOXLG 7KHUH RUH HYHU VWHDP WXEH LQ WKH IORZ VDWLVILHV WKH VWDG\ IORZ HQHUJ\ HTXDWLRQ LQ WKH IRUP RI (T

$$h_1 + \tfrac{1}{2}V_1^2 + gz_1 = h_2 + \tfrac{1}{2}V_2^2 + gz_2 - q + w_v \qquad (9.20)$$

ZKHUH SRLQW LV XSWUHDP RI SRLQW <RX PD\ ZLVK WR UHYLHZ WKH GHILQLWLRQ RI (T DQG LV GHYHORSPHQW: HVDZ LQ([DPSOH WKDW SRWHQWLDO HQHUJ\ FKDQJHV RI DJDV DUH H[WUHPHO VPDO FRPSDUHG ZLWK NLQHWLF HQHUJ\ DQG HQWKDOS\ WHUPV : HVKDOO HQFRXQWHU WKH WHUPV J] DQGJ] LQ DOOJD G\QDPLF DQDOVHV



) LJ 9HORFLW DQG WHPSHUDWXUH HQWKDOS\ GLVWULEXWLRQV QHDU DQ LQVXODWHG ZDOOLQ D WSLFDO KLJK VSHHGJDV IORZ

,QVLGH WKH WHUPDO DQG YHORFLW ERXQGDU\ OD\HUV LQ) LJ WKH KHDWDQVIHUDQG YLVFRXV ZRUN WHUPV T DQGZ% DUH QRW]HUR %XW RXWVLGH WKH ERXQGDU\ OD\HUT DQGZ% DUH]HUR E\ GHILQLWLRQ VR WKDW WKH RXWIORZ VDWLVILHV WKH VLPSOHU HDUQ

$$h_1 + \tfrac{1}{2}V_1^2 = h_2 + \tfrac{1}{2}V_2^2 = \text{const} \qquad (9.21)$$

7KH FRQGDQVLQ(T LV HTXDO WR WKH PD[LPXP HQWKDOS\ WKDW WKH IOXLG ZRXOG DFKLHYH L EURXJKW WR UHVWDGLDEDWLFDOO : HFDOOWKLV YDOXHK WKH VWDJQDWLRQ HQWKDOS\ RI WKH IORZ 7KXV ZI

UHZ ULWW(T LQWKHIRUP

$$h + \tfrac{1}{2}V^2 = h_0 = \text{const} \qquad (9.22)$$

7KLVVKRXOGKROGIRUVWDG\ DGLDEDWF IORZ RI DQ FRPSUHVVLEOHIOXLG RXWLGHWKHERXQGDU\ OD\H 7KHZDOOLQ) LJ FRXOGEHHLWKHUWKHVXUIDFHRI DQLPPHUVHGERG\ RUWKHZDOORI DGXFW: HKDY VKRZQWKHGHDOLQJRI) LJ \WSLFDOO WKHWKHUPDOOD\HUWKLFNQHVV/7 LVJUHDWHUWKDQWKHYHORFLWOD\H WKLFNQHVV/9 EHFDXVHPRVWJDVHVKDYHD GLPHQVLRQOHVV3UDQGWOQXPEHU3U WKDQXQLW\ VHH IR H[DPSOH 5H VHF 1RWWKDWWKHVXDQDORJRXVQDPHV\ YDULHVLQWLGHWKHWHUPDOERXQGDU\ OD\HU EXWWDYHUDJHYDOXHLVWKHVDPHDVWKDWDWWKHRXWHUHGJHRWKHLQXODWHG ZDOO) RUORQSHUFWDVHVZHPD\ KDYHWRXVH((6 RUWKHVWDP \DEOHV> @RUWKHJDVDEOHV> @ LPSOHPHQW/(T %XWRUDSHUFWDVK F\7 DQG(T EHFRPHV

$$c_pT + \tfrac{1}{2}V^2 = c_pT_0 \qquad (9.23)$$

7KLV HVDEOHVWKHV WKH VDQDORJRXVWHPSHUDWXUH7 RI DQ DGLDEDWF SHUFWDV IORZ² WKDWLV WK WHPSHUDWXUHLVDFKLHYHVZKHQGHFHOHUDWHGWRUHVWDGLDEDWFDOO

$Q DOWHUQDWLQWHUSUHWDWRQ RI(T RFFXUV ZKHQ WKH HQHUJ\ DQG WHPSHUDWXUH GURS V DEVROXWH]HUR VR WKDWWKHYHORFLWDFKLHYHVDPD[LPXP YDOXH

$$V_{\max} = (2h_0)^{1/2} = (2c_pT_0)^{1/2} \qquad (9.24)$$

1R KJKHUIORZ YHORFLWV FDQ RFFXUXQGHUV DGGLWLRQDOHQHUJ\ LV DGGHG WR WKH IOXLG WKURXJK VKD ZRUNRUKHDWWDQVIHU6HF

0DFK1XPEHU5HDWLRQV

7KHGLPHQVLRQOHVVIORUP RI(T EULQJVLQWKH0DFKQXPEHU0 DDVDSDUDPHWU E\ XVLQJ(T IRUWKHVSHHGRI VRXQGRI DSHUFWDV' LYLGHWKURXJKE\ F\7 WRREWLQ

$$1 + \frac{V^2}{2c_pT} = \frac{T_0}{T} \qquad (9.25)$$

%XWURP WKHSHUFWDVODZ F\7 >N5 Ní @ D Ní VR WKDW(T EHFRPHV

$$1 + \frac{(k-1)V^2}{2a^2} = \frac{T_0}{T}$$

or

$$\boxed{\frac{T_0}{T} = 1 + \frac{k-1}{2}\text{Ma}^2 \qquad \text{Ma} = \frac{V}{a}} \qquad (9.26)$$

7KLVUHDWLRQVSORWWHGLQ) LJ YHUXVWKH0DFKQXPEHUIRUN $W0 D WKHWHPSHUDWXU KDVGURSSHGWR$\frac{1}{6}T_0$.

6RXWLRQ

- $VXPSWLRQV / HWDLUEHDSSUR[LPDWHGDVDQLGHDOJDVZLWKFRQVWDQW 7KHIORZ LVDGLDEDWL EXWQRWLVHQWURSLF ,VHQWURSLF IRUPXODVDUHXVHGRQO WKFRPSXWHORFDOS DQG! ZKLFKYDU
- $SSURDFK 8VHDGLDEDWF DQGLVHQWURSLF IRUPXODVWRILQGWKHYDURXVSURSHUWLHV
- ,GHDOJDVSDUDPHWUV)RUDLU 5 P V . N DQGF$ P V .
- 6RXWLRQVWHSV D E F G : LWK7 S DQG9 NQRZ Q RWKHUSURSHUWLHVDVSRLQW IRORZ

2 QFH WKH 0 DFK QXPEHU LV IRXQG IURP (T OHFDO VWDJQDWLRQ SUHVXUH DQG GHQVLW IRORZ

$$\mathrm{Ma}_1 = \sqrt{5\left(\frac{T_{01}}{T_1} - 1\right)} = \sqrt{5\left(\frac{349\ \mathrm{K}}{320\ \mathrm{K}} - 1\right)} = \sqrt{0.448} = 0.67 \qquad \textit{Ans. (b)}$$

$$p_{01} = p_1(1 + 0.2\ \mathrm{Ma}_1^2)^{3.5} = (170\ \mathrm{kPa})[1 + 0.2(0.67)^2]^{3.5} = 230\ \mathrm{kPa} \qquad \textit{Ans. (d)}$$

$$\rho_{01} = \frac{p_{01}}{RT_{01}} = \frac{230{,}000\ \mathrm{N/m^2}}{[287\ \mathrm{m^2/(s^2 \cdot K)}](349\ \mathrm{K})} = 2.29\ \frac{\mathrm{N \cdot s^2/m}}{\mathrm{m^3}} = 2.29\ \frac{\mathrm{kg}}{\mathrm{m^3}} \qquad \textit{Ans. (c)}$$

- &RPPHQW1 RWWKDWZ HXVHGGLPHQVLRQDO QRQ0 DFK QXPEHU IRUPXODVZKHUHFRQYHQLHQW
- 6RXWLRQVWHSV H I %RWK9$ DQG9 DUHGLUHFW UHDGLQJVWR VDQDORJRXWSXWVIURP (TV DQG

$$V_{\max} = \sqrt{2c_pT_0} = \sqrt{2[1005\ \mathrm{m^2/(s^2 \cdot K)}](349\ \mathrm{K})} = 837\ \frac{\mathrm{m}}{\mathrm{s}} \qquad \textit{Ans. (e)}$$

$$V^* = \sqrt{\frac{2k}{k+1}RT_0} = \sqrt{\frac{2(1.4)}{(1.4+1)}\left(287\ \frac{\mathrm{m^2}}{\mathrm{s^2 \cdot K}}\right)(349\ \mathrm{K})} = 342\ \frac{\mathrm{m}}{\mathrm{s}} \qquad \textit{Ans. (f)}$$

- $WSRLQW GRZQVWUHDP WKHVWDJQDWLRQ EXWVLQFHWKHIORZ LVDGLDEDWF WKH VWDJQDWLRQVWDJQDWLRQVFRQVWDQW7 7 . 7KXV IURP (T

$$T_2 = T_{02} - \frac{V_2^2}{2c_p} = 349 - \frac{(290\ \mathrm{m/s})^2}{2[1005\ \mathrm{m^2/(s^2 \cdot K)}]} = 307\ \mathrm{K}$$

+HQFH IURP (T D VHFWLRQDO DUHDV DW SRLQWV LV

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{k/(k-1)} = (135\ \text{kPa})\left(\frac{349\ \text{K}}{307\ \text{K}}\right)^{1.4/0.4} = 211\ \text{kPa} \qquad Ans.\ (g)$$

- &RPPHQWV 3DUWJ DUDWR WSHIGHDOJDV IRUPXOD LV PRUH GLUHFWXDOQILQGLQJ WKH0 DF QXPEHU ZKLFK WXUQV RXWRE H0 D DQG XVLQJ WKH0 DFK QXPEHUIRUPXOD (T IR S 1RWH WKDWS LV SHUFHQWOHVV WKDQS 7KH IGRZ LV QRQLVHQWRSLF (QWRS\ ULVH GRZQWUHDP DQG VXSHUVRQLF DQG VXEVRQLF DUH GHQLWGURS GXHLQWKLVFDVHWRIULFWLRQDOORVVHV

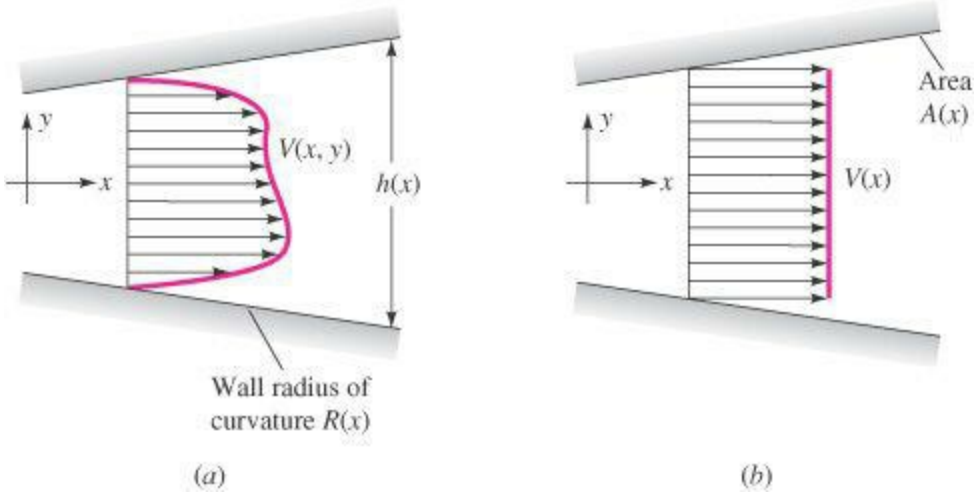
,VHQWURSLF)ORZ ZLWK $UHD&KDQJHV

%\ FRPELQLQJ WKH LVHQWURSLF DQG RUDGLDEDWF IGRZ UHDWLRQV ZLWK WKH HTXDWLRQ RI FRQWLQXLW ZH FDQ VWXG\ SUDFWDO FRP SUHVLEOH IGRZ SUREOPV 7KLV VHFWLRQ WHDW WKH RQHGLPHQVLRQDO IGRZ DSSUR[LPDWLRQ

) LJXUH LOOXVWDWV WKH RQHGLPHQVLRQDO IGRZ DVXPSWLRQ $ UHDO IGRZ) LJ D KDV QR VOS D WKH ZDOOV DQG DYHUDFLW SURILOH9 [\ WKDWYDULHV DFURVV WKH GXFWWFWLRQ FRP SDUHZ LWK) LJ ,I KRZ HYHU WKH DUHDFKDQJHLV VP DOODQG WKH ZDOOUDGLXV RI FXUYDWXUHOUJH

$$\frac{dh}{dx} \ll 1 \qquad h(x) \ll R(x) \qquad (9.36)$$

WKHQ WKH IGRZ LV DSSUR[LPDWD RQHGLPHQVLRQDO DV LQ) LJ E ZLWK9 § 9 [UHDFWLQJ WR DUH FKDQJH$ [&RPSUHVLEOHIGRZ QR]]OHVDQG GLIXVHUVGRQRWDOZD\V DQG\ FRQGLWLRQV EXWZH XVHWKHRQHGLPHQVLRQDOWHRU DQZD\ EHFDXVHRI LWVVLPSOFLW



) LJ &RPSUHVLEOHIGRZ WKURXJK DGXFW*a* UHDOIXLG YHORFLW SURILOH *b* RQHGLPHQVLRQDO DSSUR[LPDWLRQ

) RUVWADG RQH GLPHQVLRQDO IORZ WKHHTXDWLRQ RI FRQWLQXLWLV IURP (T

$$\rho(x)V(x)A(x) = \dot{m} = \text{const} \tag{9.37}$$

%HRUHDSSOLQJ WKLVWR GXFWWKHRU Z HFDQ GHDQD OWURP WKHGLIHUHQWLDOIRUP RI (T

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \tag{9.38}$$

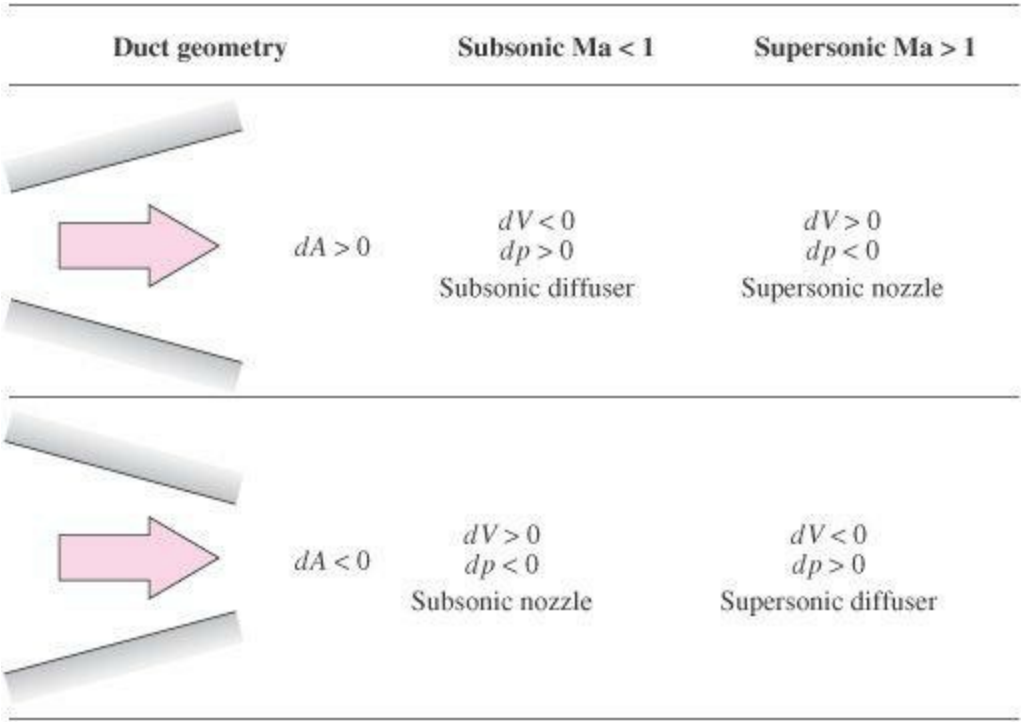
7KHGLIHUHQWLDOIRUPVRI WKHIULFWLRQOHVVPRPHQXP HTXDWLRQ DQG WKHVRXQG±VSHHGUHODWLR DUHUHFDOOHGKHUHIRUFRQYHQLHQFH

$$\begin{aligned} &\text{Momentum} && \frac{dp}{\rho} + V\,dV = 0 \\ &\text{Sound speed:} && dp = a^2\,d\rho \end{aligned} \tag{9.39}$$

1RZ HDPLQDWGSDQGG EHWHHQ(TV DQG WKREWLQWKHIRGOZLQJ UHDWRQEHWHH YHORFLWFKDQJHDQG DUHDFKDQJHLQLVHQWURSLF GXFWORZ

$$\frac{dV}{V} = \frac{dA}{A}\frac{1}{\text{Ma}^2 - 1} = -\frac{dp}{\rho V^2} \tag{9.40}$$

,QSHFWRQ RI WKLV HTXDWLRQ ZLWKRXWDFWXDOO VROYLQJ LW UHYHDOV D IDVFLQDWQJ DSHFWRF FRPSUHVLEOHIORZ 3URPHUWFKDQJHV DUH RSSRVLWHVLJQIRUVXEVRQLF DQG VXSHUVRQLF IORZ EHFDXV RI WKH WUP 0 D í 7KHUHIRUFRPELQDWRQV RI DUHDFKDQJHDQG 0 DFK QXPEHU VXPPDUJHG L) LJ

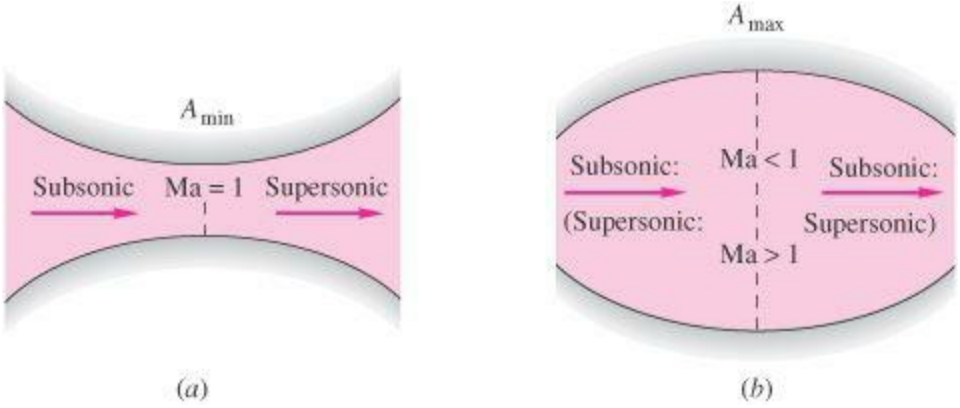


) LJ (IIHFWRI 0 DFK QXP EHURQSURSHUWFKDQJHVZLWKDUHDFKDQJHLQGXFWORZ

) URP HDULHUFKDSWUV Z H DUH XVHG WR VXEVRQLF EHKDYLRU 0 D : KHQ DUHD LQFUHDVH

YHORFLWLHV DQG SUHVVXUHV DFURVV VKRFNV ZKLFK LV GHQRWHG D VXEVRQLF GLIIXVHU %XWLQVSHUVRQ
IORZ 0 D! WKH YHORFLWLHV DFWXDOO\ LQFUHDVH ZKHQ WKH DUHDLQFUHDVHV DVXSHUVRQLF QR]]OH 7KHVH P
RSSRVLQJ EHKDYLRUHFXUVIRUDQ DUHDGHFUHDVH ZKLFK VSHHGV XS DVXEVRQLF IORZ DQG VORZV GRZQ
VXSHUVRQLF IORZ

: KDVDERXWKH VRQLF SRLQW0 D " 6LQFH LQLQWDFFHOHUDWRQ LV SK\VLFDOO LP SRVLEOH(T
LQGLFDWHV WKDWV9 FDQ EH ILQWRQ ZKHQG$ ² WKDWLV D PLQLP DUHD WKURDW RI
P[LP DUHD EXOH ,Q) LJ ZH SDWK WKURWHUD WKURDWH FWRQ DQG D EXOH VHFWRQ XLQJ WK
UXOHV IURP) LJ 7KH WKURDW RUFRQYHUJLQJ±GLYHUJLQJ VHFWRQ FDQ VP RRWKO DFFHOHUDWH D VXEVRQL
IORZ WKURXJK VRQLF WR VXSHUVRQLF IORZ DV LQ) LJ D 7KLV LV WKH RQO ZD\ DVXSHUVRQLF IORZ FDQ E
FUHDWHG E\ H[SDQGLQJ WKH JDV IURP D VXEVRQLF FRQGLWLRQ UDWKHU WKDQ WR7KH EXOH VHFWRQ IDLOV WKH EXOH0 DF
QXP EHUP RYHVDZ D\ IURP D VRQLF FRQGLWLRQ UDWKHU WKDQ WR DGLW



) LJ) URP(T LQ IORZ WKURXJK D WKURDW*a* WKH IOXLG FDQ DFFHOHUDWH VP RRWKO
WKURXJK VRQLF DQG VXSHUVRQLF IORZ ,QIORZ WKURXJK WKH EXOH*b* WKH IORZ DWWKH EXOH FDQQRWEH
VRQLF RQSK\VLFDOJURXQGV

$QWKRXJK VXSHUVRQLF IORZ GRZQWUHDP RI D QR]]OH UHTXLUHV D VRQLF WKURDWWKH RSSRVLWH LV QR
WUXH $ FRP SUHVVLEOHJDVFDQSDVWKURXJK DWKURDWH[SDQVLRQZLWKRXWEHFRP LQJ VRQLF

3HUIHFW* DV$ UHD&KDQJH

: HFDQXVHWKHSHUIHFWJDV DQGLVHQWURSLF IORZ UHODWLRQV WR FRQVWUXWKH FRQWUDFWLQJ
LQWR DQ DJHEUDLF H[SUHVLRQ LQYROYLQJ RQO DUHD DQG0 DFK QXP EHUD VIRQ IORZ V (TXDWWKHP DVWIORZ
DWDQ VHFWRQ WR WKHP DVWIORZ XQGHUVRQLF FRQGLWLRQV ZKFK P D\ QRWDFWXDOO RFFXULQWKHGXFW

$$\rho V A = \rho^* V^* A^*$$

or

$$\frac{A}{A^*} = \frac{\rho^*}{\rho}\frac{V^*}{V} \qquad (9.41)$$

%RWK WKH WUP V RQ WKH ULJKWDUH IXQFWLRQ V RQO RI 0 DFK QXP EHUIRU LVHQWURSLF IORZ) URP(TV
DQG

$$\frac{\rho^*}{\rho} = \frac{\rho^*}{\rho_0}\frac{\rho_0}{\rho} = \left\{\frac{2}{k+1}\left[1 + \frac{1}{2}(k-1)\,\mathrm{Ma}^2\right]\right\}^{1/(k-1)} \qquad (9.42)$$

) URP (TV DQG Z HREWDLQ

$$\frac{V^*}{V} = \frac{(kRT^*)^{1/2}}{V} = \frac{(kRT)^{1/2}}{V}\left(\frac{T^*}{T_0}\right)^{1/2}\left(\frac{T_0}{T}\right)^{1/2}$$

$$= \frac{1}{\mathrm{Ma}}\left\{\frac{2}{k+1}\left[1 + \frac{1}{2}(k-1)\,\mathrm{Ma}^2\right]\right\}^{1/2} \qquad (9.43)$$

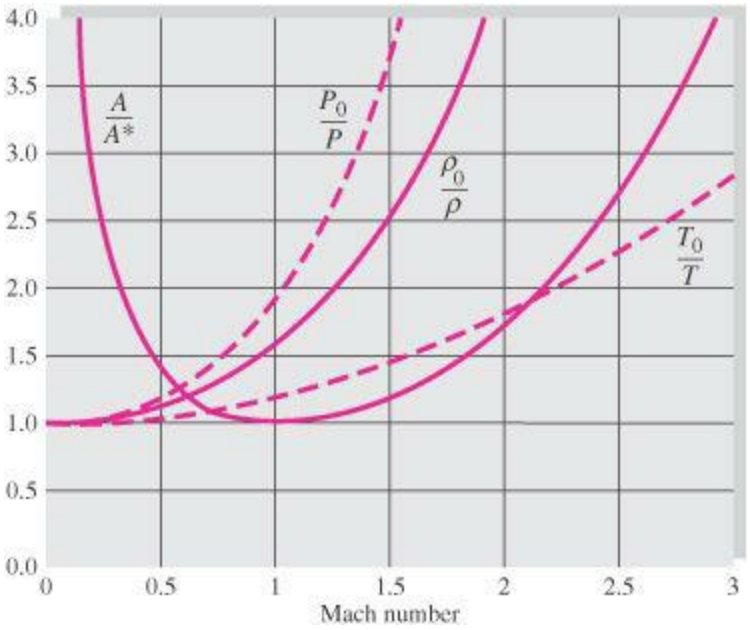
&RPELQLQJ (TV WR Z HJHWKHGHVLUHGUHVXOW

$$\boxed{\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\frac{1 + \frac{1}{2}(k-1)\,\mathrm{Ma}^2}{\frac{1}{2}(k+1)}\right]^{(1/2)(k+1)/(k-1)}} \qquad (9.44)$$

) RUN (T WDNHVWKHQXPHULFDOIRUP

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\frac{(1 + 0.2\,\mathrm{Ma}^2)^3}{1.728} \qquad (9.45)$$

Z KLFK LV SORWWHG LQ) LJ (TXDWLRQV DQG HQDEOHV XV WR VROYH DQ RQHGLPHQVLRQDO LVHQWURSLF DLUIORZ SUREOHP JLYHQ VD WKH VKDSH RI WKH GXFW$ [DQG WKH VWDJQDWLRQ FRQGLWLRQV DQ DVVXPLQJ WKDWWKH UHDUHQR VKRFNZDYHVLQWKHGXFW



) LJ $ UHD UDWLR DQG IOXLG SURSHUWLHV YHUVXV 0 DFK QXPEHU IRU LVHQWURSLF IORZ RI D SHUIHFW JDVZLWKk

) LJXUH VKRZV VWKDWWKH PLQLPXP DUHD WKDWFDQ RFFXULQ D JLYHQ LVHQWURSLF GXFWWKRZ LV WKH VRQLF RUFULWLFDO WKURDWDUHD $ OORWKHUGXFWWHFWRQV PXWKDYH$ JUHDWHUWKDQ$,QPDQ IORZ VI FULWLFDOVRQLF WKURDWV QRWDFWXDOO SUHVHQWDQG WKH IORZ LQWKH GXFWLV HLWKHUHQWLUHO VXEVRQLF RU PRUHUDUHO HQWLUHO VXSHUVRQLF

&KRNLQJ

) URP (T VWH LQYHUVH UDWLR $ $ HTXDOV ! 9 ! 9 VWH PDVV IORZ SHU XQLW DUHD DW DQ
VHFWLRQ FRPSDUHG ZLWK WKH FULWLFDO PDVV IORZ SHU XQLW DUHD) URP) LJ WKLV LQYHUVH UDWLR UHVIURF
]HUR DW 0 D WR XQLW DW 0 D DQG EDFN GRZQ WR]HUR DW DQ DJH 0 D 7KXV IRU JLYHQ VWDJQDWLR
FRQGLWLRQV WKH PD[LPXP SRVVLEOH PDVV IORZ SDVVHV WKURXJK DGXFWZKHQ LWV WKURDW DVSHFW
VRQF FRQGLWLRQ 7KH GXFW V WKH QVDLG WR EH FKRNHG DQG FDQ FDUU QR DGGLWLRQDO PDVV IORZ XQDHV WK
WKURDW VZLGHQHG ,I WKH WKURDW V FRQWUDFWHG IXUWKHU WKH PDVV IORZ WKURXJK WKH GXFW P XWZ FHUDH
) URP (TV DQG WKH PD[LPXP PDVV IORZ LV

$$\dot{m}_{\max} = \rho^* A^* V^* = \rho_0 \left(\frac{2}{k+1}\right)^{1/(k-1)} A^* \left(\frac{2k}{k+1} RT_0\right)^{1/2}$$
$$= k^{1/2} \left(\frac{2}{k+1}\right)^{(1/2)(k+1)/(k-1)} A^* \rho_0 (RT_0)^{1/2} \qquad (9.46a)$$

) RUN WKLV UHGXFHV WR

$$\dot{m}_{\max} = 0.6847 A^* \rho_0 (RT_0)^{1/2} = \frac{0.6847 p_0 A^*}{(RT_0)^{1/2}} \qquad (9.46b)$$

) RUL VHQWRSLF IORZ WKURXJK DGXFW WKH PD[LPXP PDVV IORZ SRVVLEOH LV SURSRUWLRQDO WR WKH WKURD
DUHD DQG WDJQDWLRQ SUHVVXUH DQG LQYHUVHO SURSRUWLRQDO WR WKH VTXDUH URRW WKH WDJQDWLRQ
WHPSHUDWXUH 7KHVH DUH VRPHZKDW VXUSULVLQJ UHVXOWV EHFDXVH VR OHVV XVXDO LQ WHPSOHV

7KH/ RFDO0 DW/) ORZ) XQFWLRQ

(TXDWLRQV JLYH WKH PD[LPXP PDVV IORZ ZKLFK RFFXUV DW WKH FKRNLQJ FRQGLWLRQ VRQH
H[LW 7KH\ FDQEHP RGLILHG WR SUHGLFW WKH DFWXDO QRQP D[LPXP PDVV IORZ DW DQ VHFWLRQ ZKHUH
DUHD$ DQG SUHVVXUH S DUH NQRZ Q 7KH DOJHEUD IRQYROXWHG VR KHUH ZH JLYH RQO WKH ILQDO UHVXO
H[SUHVVHG LQGLPHQVLRQOHVV IRUP

$$\text{Mass flow function} = \frac{\dot{m}}{A} \frac{\sqrt{RT_0}}{p_0} = \sqrt{\frac{2k}{k-1} \left(\frac{p}{p_0}\right)^{2/k} \left[1 - \left(\frac{p}{p_0}\right)^{(k-1)/k}\right]} \qquad (9.47)$$

: HVWHVV WKDVS DQG $ LQWKLV UHDWLRQ DUH WKH RFDOYDOXHV DVSRVLWLRQ[$VS S IDQV WKLV IXQFWLR
ULVHV UDSLGO DQG WKHQ GHYHOVRXWDWKH PD[LPXP RI (TV $ IHZ YDOXHV PD\ EH WDEXODWHG KHU
IRUN

p/p_0	1.0	0.98	0.95	0.9	0.8	0.7	0.6	≤0.5283
Function	0.0	0.1978	0.3076	0.4226	0.5607	0.6383	0.6769	0.6847

(TXDWLRQ LVKDQG LI WDJQDWLRQ FRQGLWLRQV DUH NQRZ QDQG WKH IORZ LV QRW FKRNHG
: KHQ $ $ LV NQRZ Q DQG WKH 0 DFK QXPEHU LV GHWHUPLQHG QR DOJHEUDLF VROXWLRQ LV NQRZ Q WR WK
ZULWH U + DQG LWHUDWLRQ ZLWK D JRRG JXHVV ZRXOG VXIILFH EXW (6 VIGHDOIRUWKLV VLWXDWLRQ $VXP
WKDW$ DQG$ DQG NDUHNQRZ Q 7KHQDOOZ HQHGLV WKH((6 WDWHPHQW

$A/A^* = (1/\mathrm{Ma})*((1 + 0.5*(k-1)*\mathrm{Ma}^2)/0.5/(k + 1))^{(0.5*(k + 1)/(k-1))}$ (9.48)

((6ZLOOSURYLGHWKH0DFKQXPEHULQDIODVKIRUDQUHDOJDVNEHWZHHQ DQG
1RWWKDWZR VROXWLRQV DUH SRVVLEOH IRU D JLYHQ$ $ RQH VXEVRQLF DQG RQH VXSHUVRQLF 7K
SURSHU VROXWLRQ FDQQRWEH VHOHFWHG ZLWKRXWIXUWKHU LQIRUPDWLRQ VXFK DV NQRZQ SUHVWXUH R
WHPSHUDWXUHDWWKHJLYHQGXFWVHFWLRQ

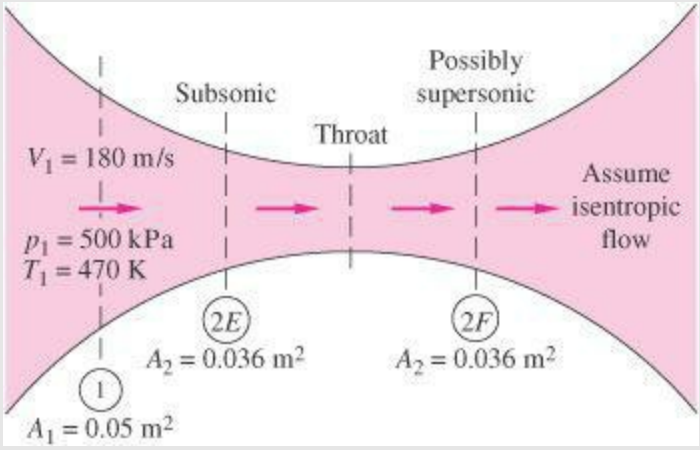
(;$03/(

$LUIORZV LVHQWURSLFDOO WKURXJK DGXFW$ WXHFWLRQ WKH DUHDLV P DQG9 P V S
N3D DQG7 . &RPSXWH D 7 E 0 D F S DQG G EHWK$ DQG ,I DWWHFWLRQ WK
DUHDLV P FRPSXWH0 D DQGS LI WKHIORZ LV H VXEVRQLF RU I VXSHUVRQLF $VWXPHN

6RROXWLRQ

3DUWD

$ JHQHUDOVNHWFK RI WKHSUREOP LVVKRZQLQ) LJ (: LWK9 DQG7 NQRZQ WKHHQHUJ\ HTXDWR
JLYHV



(

$$T_0 = T_1 + \frac{V_1^2}{2c_p} = 470 + \frac{(180)^2}{2(1005)} = 486\ \mathrm{K} \qquad \textit{Ans. (a)}$$

3DUWE

The local sound speed $a_1 = \sqrt{kRT_1} = [(1.4)(287)(470)]^{1/2} = 435$ m/s. Hence

$$\text{Ma}_1 = \frac{V_1}{a_1} = \frac{180}{435} = 0.414 \qquad \textit{Ans. (b)}$$

3DUWF

: LWK0 D NQRZ Q WKHVWDJQDWRQSUHVVXUHIRGRZ VIURP (T

$$p_0 = p_1(1 + 0.2\,\text{Ma}_1^2)^{3.5} = (500\text{ kPa})[1 + 0.2(0.414)^2]^{3.5} = 563\text{ kPa} \qquad \textit{Ans. (c)}$$

3DUWG

6LPLODUO IURP (T WKHFULWLFDOVRQLF WKURDWDUHDLV

$$\frac{A_1}{A^*} = \frac{(1 + 0.2\,\text{Ma}_1^2)^3}{1.728\,\text{Ma}_1} = \frac{[1 + 0.2(0.414)^2]^3}{1.728(0.414)} = 1.547$$

or

$$A^* = \frac{A_1}{1.547} = \frac{0.05\text{ m}^2}{1.547} = 0.0323\text{ m}^2 \qquad \textit{Ans. (d)}$$

7KLVWKURDWPXWDFXDOO EHSUHVHQWLQWKHGXFWL WKHIORZ LVWKEHFRPHVXSHUVRQLF : HQRZ NQRZ $ 6R WKFRPSXWHWKHPDVIORZ ZHFDQXVH(TV ZKLFK UHPDLQ YDOG EDVHGRQWKHQXPHULFDOYDOXHRI $ ZKHWKHURUNRWDWKURDWDFXDOO H[LWV

$$\dot{m} = 0.6847\frac{p_0 A^*}{\sqrt{RT_0}} = 0.6847\frac{(563{,}000)(0.0323)}{\sqrt{(287)(486)}} = 33.4\text{ kg/s} \qquad \textit{Ans. (d)}$$

2UZHFRXOGIDUHHTXDOO ZHOOZLWKRXUQHZ ³GRFDOPDVIORZ´ IRUPXOD(T XVLQJ VD WKHSUHVXUHDQGDUHDDWWHFWRQ * LYHQS S (T \LHOGV

$$\dot{m}\frac{\sqrt{287(486)}}{563{,}000(0.05)} = \sqrt{\frac{2(1.4)}{0.4}}(0.889)^{2/1.4}[1 - (0.889)^{0.4/1.4}] = 0.444 \quad \dot{m} = 33.4\frac{\text{kg}}{\text{s}} \qquad \textit{Ans. (d)}$$

3DUWH

EES $VXPHVXEVRQLF IORZ FRUHVSRQGVVRVHFWRQ (LQ)LJ (7KHGXFWFRQWDFWVWRDQDUHUDWR$ $ ZKLFKZHILQGRQWKHOHWWLGHRI)LJ RUWKHVXEVRQLF SDUWR 7DEOH % 1HLWKHUWKHILJXUHQRUWKHWDEOHVWKDWDFFXUDWH ((6 ZLOOJLYHDQDUELWUDULO DFFXUDWH VROXWLRQZLWKRQO WKUHHVLJQLILFDQW LQ6, XQLWV

```
A2 = 0.036
Astar = 0.0323
A2/Astar = (1 + 0.2*Ma2^2)^3/1.2^3/Ma2
```

6SHFLI\ WKDWRXW DQWLDVXEVRQLF VROXWLRQ HJ OPLW0 D DQG((6 UHSRUW

$$Ma_2 = 0.6758 \qquad Ans.\ (e)$$

>$ VNIRUD VXSHUVRQLF VROXWLRQ UHTXLUH0 D ! DQG \RX UHFLYH0 D ZKLFK LV WK
DQWZHUVRSDWWI @7KHSUHVXUHLVJLYHQE\ WKHLVHQWRSLF UHDWLRQ

$$p_2 = \frac{p_0}{[1 + 0.2(0.676)^2]^{3.5}} = \frac{563\ \text{kPa}}{1.358} \approx 415\ \text{kPa} \qquad Ans.\ (e)$$

3DUWH GRHVQRWUHTXLUH D WKURDWVRQLF RU RWKHUZLVH WKH IORZ FRXOG VLPSO EH FRQWDFWQ
VXEVRQLFDOO IURP $ WR$

3DUWI

EES 7KLV WPH DWXPH VXSHUVRQLF IORZ FRUHVSRQGLQJ WR VHFWLRQ) LQ) LJ ($JDLQ WKH DUH
UDWLRLV$ $ DQG ZHORRNIRQWKHUJKWWLGHRI) LJ RUWKHVXSHUVRQLF SDU
RI 7DEOH% ² WKH ODWWUFDQEHUHDG TXLWDFFXUDWDO DV0 D § ((6 ZLOOJLYH DYHU DFFXUD
VROXWRQ ZLWK WKH VDPH WKUHH VLJQLFDQW IURP SDUWH 6SHFLI\ WKDW\RX ZDQWDVXSHUVRQLF VROXWR
HJ OPLW0 D ! DQG((6 UHSRUW

$$Ma_2 = 1.4001 \qquad Ans.\ (f)$$

$JDLQ WKH SUHVXUHLVJLYHQE\ WKHLVHQWRSLF UHDWLRQ DWWKH QHZ 0 DFK QXPEHU

$$p_2 = \frac{p_0}{[1 + 0.2(1.4001)^2]^{3.5}} = \frac{563\ \text{kPa}}{3.183} = 177\ \text{kPa} \qquad Ans.\ (f)$$

1 RWH WKDW WKH VXSHUVRQLF IORZ SUHVXUH GHYHOLV PXFK GHV WKDQS LQSDUWH DQG D VRQLF WKURD
PXWWKDY HRFFXUHG EHWZH HQVHFWLRQV DQG)

(; 0 3/ (

,WLV GHVLUHG WR H[SDQG DLU IURP S N3D DQG7 . WKURXJK D WKURDWR R DQH[LW0 DF
QXPEHUR ,I WKH GHVLUHG PDVI IORZ LV NJ V FRPSXW D WKH WKURDWDUHDDQG WKH H[LWE SUHVXUH
F WHPSHUDXUH G YHORFLW DQG H DUHD DVXPLQJ LVHQWRSLF IORZ ZLWK

6RXWLRQ

7KH VKURDWDUHD IRUGRZ VIURP (T EHFDXVH WKH VKURDWGRZ PXWWEH VRQLF WR SURGXFH H VXSHUVRQLF H[LW

$$A^* = \frac{\dot{m}(RT_0)^{1/2}}{0.6847p_0} = \frac{3.0[287(500)]^{1/2}}{0.6847(200{,}000)} = 0.00830\ \text{m}^2 = \frac{1}{4}\pi D^{*2}$$

or $$D_{\text{throat}} = 10.3\ \text{cm} \qquad \textit{Ans. (a)}$$

: LWK WKH H[LW0 DFK QXPEHU NQRZQ WKH LVHQWURSLF IORZ UHODWLRQV JLYH WKH SUHVVXUH DQ WHPSHUDWXUH

$$p_e = \frac{p_0}{[1 + 0.2(2.5)^2]^{3.5}} = \frac{200{,}000}{17.08} = 11{,}700\ \text{Pa} \qquad \textit{Ans. (b)}$$

$$T_e = \frac{T_0}{1 + 0.2(2.5)^2} = \frac{500}{2.25} = 222\ \text{K} \qquad \textit{Ans. (c)}$$

7KHH[LWYHORFLWIRGRZ VIURP WKHNQRZQ0 DFKQXPEHUDQGWHPSHUDWXUH

$$V_e = \text{Ma}_e(kRT_e)^{1/2} = 2.5[1.4(287)(222)]^{1/2} = 2.5(299\ \text{m/s}) = 747\ \text{m/s} \qquad \textit{Ans. (d)}$$

7KHH[LWDUHDIRGRZ VIURP WKHNQRZQWKURDWDUHDDQGH[LW0 DFKQXPEHUDQG(T

$$\frac{A_e}{A^*} = \frac{[1 + 0.2(2.5)^2]^3}{1.728(2.5)} = 2.64$$

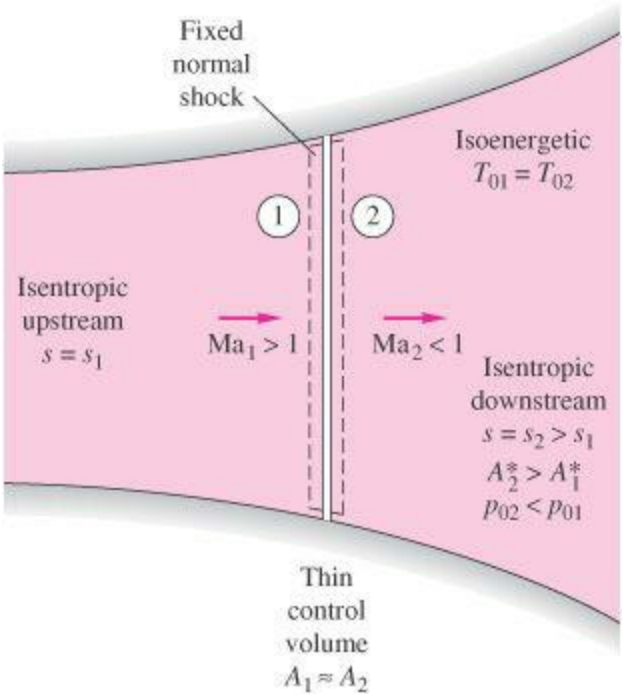
or $$A_e = 2.64A^* = 2.64(0.0083\ \text{m}^2) = 0.0219\ \text{m}^2 = \tfrac{1}{4}\pi D_e^2$$

or $$D_e = 16.7\ \text{cm} \qquad \textit{Ans. (e)}$$

2QHSRLQWPLJKWEHQRWHG 7KHFRPSXWDWLRQRI WKHWKURDWDUHD$ GLGQRWGHSHQGLQDQ ZD RQWK QXPHULFDOYDOXHRI WKHH[LW0 DFKQXPEHU7KHH[LWZDVVXSHUVRQLF WKHUHRUH WKHWKURDWLV VRQLF DQ FKRNHG DQGQR IXUWKHULQFUHDVHLQ0DVQRQLVQHGHG

7KH1RUP DO6KRFN: DYH

$ FRPPRQLUHYHUVLELOLW\RFFXUULQJ LQVXSHUVRQLF LQWHUQDOORUH[WHUQDOIORZ VLV WKHQRUP DOVKRFN Z DYH VN HWKHG LQ) LJ ([FHSWDWQHDUYDFXXP SUHVXUHV VXFK VKRFN Z DYHV DUH YHU WKLQ DIHZ PLFURP HWHUV WKLFN DQG DSSUR[LP DWH GLVFRQWLQXRXV FKDQJH LQ IORZ SURSHUWLHV : HVHOHFWD FRQWR YROXP HIXWWEHRUHDQG DWWUWKH Z DYH DVLQ) LJ



) LJ) OHZ VKRXJK DIL[HG QRUPDO VKRFN ZDYH

7KH DQDO VLV LV LGHQWLFDO WR WKDW RI) LJ WKDW LV DVKRFN ZDYH LV DIL[HG WR QJ SUHVXUHZDY-
7R FRPSXWH DOO SURSHUWLHV FKDQJHV UDWKHU WKDQ MXVW WKH ZDYH VSHHG ZH XVH DOO RXU EDVLF RQH
GLPHQVLRQDOXVDG IORZ UHDWLRQV OHWWQJ VHFWLRQ EHXSVWUHDP DQG VHFWLRQ EHGRZQVWUHDP

Continuity: $$\rho_1 V_1 = \rho_2 V_2 = G = \text{const} \qquad (9.49a)$$

Momentum: $$p_1 - p_2 = \rho_2 V_2^2 - \rho_1 V_1^2 \qquad (9.49b)$$

Energy: $$h_1 + \tfrac{1}{2}V_1^2 = h_2 + \tfrac{1}{2}V_2^2 = h_0 = \text{const} \qquad (9.49c)$$

Perfect gas: $$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \qquad (9.49d)$$

Constant c_p: $$h = c_p T \qquad k = \text{const} \qquad (9.49e)$$

1 RWH WKDW ZH KDYH FDQFHOHG RXW WKH DUHDV$ §$ ZKLFK LV MXVWLILHG HYHQ LQ D YDULDEOH GXF-
VHFWLRQ EHFDXVH RI WKH WKLQQHVV RI WKH ZDYH 7KH IILWWXFFHWWXOODQDOVHV RI WKHVH QRUPDO VKRFN
UHDWLRQV DUH FUHGLWHG WR : - 0 5DQNLQH DQG$ +XJRQLRW KHQFH WKH PRGHUQ WHU
5DQNLQH+XJRQLRWUHDWLRQV ,I ZH DVVXPH WKDW WKH XSWUHDP FRQGLWLRQV S 9 ! K 7 DUH NQRZ Q
(TV DUH ILYH DOJHEUDLF UHDWLRQV LQ WKH ILYH XQNQRZQV S 9 ! K 7 %HFDXVH RI WK
YHORFLW VTXDUHG WHUP WZR VROXWLRQV DUH IRXQG DQG WKH FRUUHFWLRQ LV GHWHUPLQHG IURP WKH VHFRQ
ODZ RI WKHUPRG\QDPLFV ZKLFK UHTXLUHV WKDW W ! V

7KH YHORFLWLHV9 DQG9 FDQ EH HDVLO\ REWDLQHG IURP (TV D WR F WR REWDLQ WKH 5DQNLQH-
+XJRQLRWUHDWLRQ

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right) \qquad (9.50)$$

7KLV FRQGLWLRQ RQO VKHUPRG\QDPLF SURSHUWLHV DQG LV LQGHSHQGHQWRI VKH HTXDWLRQ RI VWDWH ,QWURGXFLQJ WKHSHUHFWJDVODZ K F_S7 NS>Ní !@ZHFDQUHZULWWKLVDV

$$\frac{\rho_2}{\rho_1} = \frac{1 + \beta p_2/p_1}{\beta + p_2/p_1} \qquad \beta = \frac{k+1}{k-1} \qquad (9.51)$$

: HFDQFRPSDUHWKLVZLWKWKHLVHQWURSLF IORZ UHODWLRQIRUDYHU Z HDNSUHVVXUHZDYHLQDSHUHFJDV

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{1/k} \qquad (9.52)$$

$OR WKHDFWXDOFKDQJHLQHQWURS\ DFURVVWKHVKRFNFDQEHFRPSXWHGIURP WKHSHUHFWJDVUHODWLRQ

$$\frac{s_2 - s_1}{c_v} = \ln\left[\frac{p_2}{p_1}\left(\frac{\rho_1}{\rho_2}\right)^k\right] \qquad (9.53)$$

$VXPLQJ DJLYHQZDYHVWUHQJWKS S Z HFDQFRPSXWHWKHGHQVLW UDWLR DQGWKHHQWURS\ FKDQJH DQGOD\WKHP DVIRORZVIRUN

: HVHHWKDWWKHHQWURS\ FKDQJHLVQHJDWLYHLI WKHSUHVXUHGHFUHDVHVDFURVVWKHVKRFN ZKFK YLRODWHVWKHVHFRQGODZ 7KXVDUDUHDFWLRQVKRFNLVLPSRVVLEOHLQDSHUHFWJDV : HVHHDOVR WKDW ZHDNVKRFNZDYHV S S ” DUHYHU QHDU LVHQWURSLF

$\frac{p_2}{p_1}$	ρ_2/ρ_1		$\frac{s_2 - s_1}{c_v}$
	Eq. (9.51)	Isentropic	
0.5	0.6154	0.6095	−0.0134
0.9	0.9275	0.9275	−0.00005
1.0	1.0	1.0	0.0
1.1	1.00704	1.00705	0.00004
1.5	1.3333	1.3359	0.0027
2.0	1.6250	1.6407	0.0134

0 DFK1XPEHU5HODWLRQV

)RUDSHUHFWJDVDOWKHSURSHUWUDWLRVDFURVVWKHQRUPDOVKRFNDUHXQLTXHIXQFWLRQVRI NDQG WKH XSWUHDP 0 DFKQXPEHU0 D)RUH[DPSOH LI ZHHDPLQDW! DQG9 IURP (TV D WR F DQ LQWRGXFHK NS>Ní !@ZHREWLQ

$$\frac{p_2}{p_1} = \frac{1}{k+1}\left[\frac{2\rho_1 V_1^2}{p_1} - (k-1)\right] \qquad (9.54)$$

%XWRUDSHUHFWJDV $\rho_1 V_1^2/p_1 = kV_1^2/(kRT_1) = k\,\mathrm{Ma}_1^2$, VR WKDW(T LVHTXLYDOHQWWR

$$\frac{p_2}{p_1} = \frac{1}{k+1}\left[2k\,\mathrm{Ma}_1^2 - (k-1)\right] \tag{9.55}$$

) URP VKLV HTXDWLRQ ZH VHH WKDW IRU DQ N S ! S RQH LI 0 D ! 7KXV IRU IORZ VKURXJK I QRUPDO VKRFN ZDYH WKH XSVWUHDP 0 DFK QXPEHU PXVW EH VXSHUVRQLF WR VDWLV\ WKH VHFRQG ODZ R WKHUPRG\QDPLFV

: KDWHERXWWKHGRZQVWUHDP 0 DFK QXPEHU) URP WKH SHUHFWJDVLGHQWLW 9 NS 0 D ZHFDQ UHZ ULWH(T E DV

$$\frac{p_2}{p_1} = \frac{1 + k\,\mathrm{Ma}_1^2}{1 + k\,\mathrm{Ma}_2^2} \tag{9.56}$$

ZKLFK UHDGV WKH SUHVVXUH UDWLR ERVV0 DFK QXPEHUV %\ HTXDWLQJ (TV DQG ZHFDQ VROYH IRU

$$\mathrm{Ma}_2^2 = \frac{(k-1)\,\mathrm{Ma}_1^2 + 2}{2k\,\mathrm{Ma}_1^2 - (k-1)} \tag{9.57}$$

6LQFH0 D PXVWEH VXSHUVRQLF WKLV HTXDWRQ SUHGLFWV IRU DOO N! WKDW0 D PXVWEH VXEVRQLF 7KX D QRUPDO VKRFN ZDYH GHFHOHUDWHV D IORZ DOP RVWGLVFRQWLQXRXV) IURP VXSHUVRQLF WR VXEVRQL FRQGLWLRQV

) XUWKHUPRQLSXODWLRQ RI WKH EDVLF UHDWLRQV IRUD SHUHFWJDV JLYHV DGGLWLRQDOHTXDWRQ UHDWLQJ WKHFKDQJHLQSURSHUWLHVDFURVVDQRUPDOVKRFNZDYHLQDSHUHFWJDV

$$\frac{\rho_2}{\rho_1} = \frac{(k+1)\,\mathrm{Ma}_1^2}{(k-1)\,\mathrm{Ma}_1^2 + 2} = \frac{V_1}{V_2}$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)\,\mathrm{Ma}_1^2\right]\frac{2k\,\mathrm{Ma}_1^2 - (k-1)}{(k+1)^2\,\mathrm{Ma}_1^2} \tag{9.58}$$

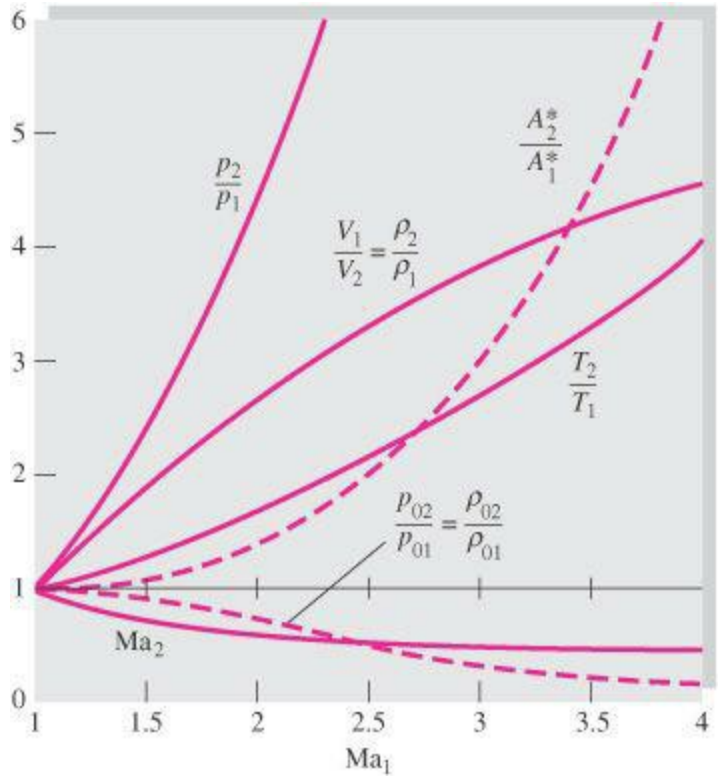
$$T_{02} = T_{01}$$

$$\frac{p_{02}}{p_{01}} = \frac{\rho_{02}}{\rho_{01}} = \left[\frac{(k+1)\,\mathrm{Ma}_1^2}{2 + (k-1)\,\mathrm{Ma}_1^2}\right]^{k/(k-1)}\left[\frac{k+1}{2k\,\mathrm{Ma}_1^2 - (k-1)}\right]^{1/(k-1)}$$

2I DGGLWLRQDOLQIRUPDWLRQ LV WKH IDFWWKDW WKH FULWLFDO RUVRQLF VKURDWDUHD$ LQDGXFWLQFUHDVHV DFURVV DQRUPDO VKRFN

$$\frac{A_2^*}{A_1^*} = \frac{\mathrm{Ma}_2}{\mathrm{Ma}_1}\left[\frac{2 + (k-1)\,\mathrm{Ma}_1^2}{2 + (k-1)\,\mathrm{Ma}_2^2}\right]^{(1/2)(k+1)/(k-1)} \tag{9.59}$$

$ OOWKHVH UHDWLRQV DUH JLYHQ LQ7DEOH % DQG SORWWG YHUVXV XSWUHDP 0 DFK QXPEHU0 D LQ) LJ IRUN : H VHH WKDWSUHVXUH LQFUHDVHV JUHDWO ZKLOH WKP SHUDWXUH DQG GHQVLW LQFUHDV PRGHUDWO 7KH HIFWYH WKURDWDUHD$ LQFUHDVHV VORZ O DWLWWDQG WKHQ UDSLGO 7KH IDLOXUH R VXGHQWWR DFFRXQWRUWKLVFKDQJHLQ$ LVDFRP PRQVRXUHRI HUURULQVKRFNFDOFXODWLRQV

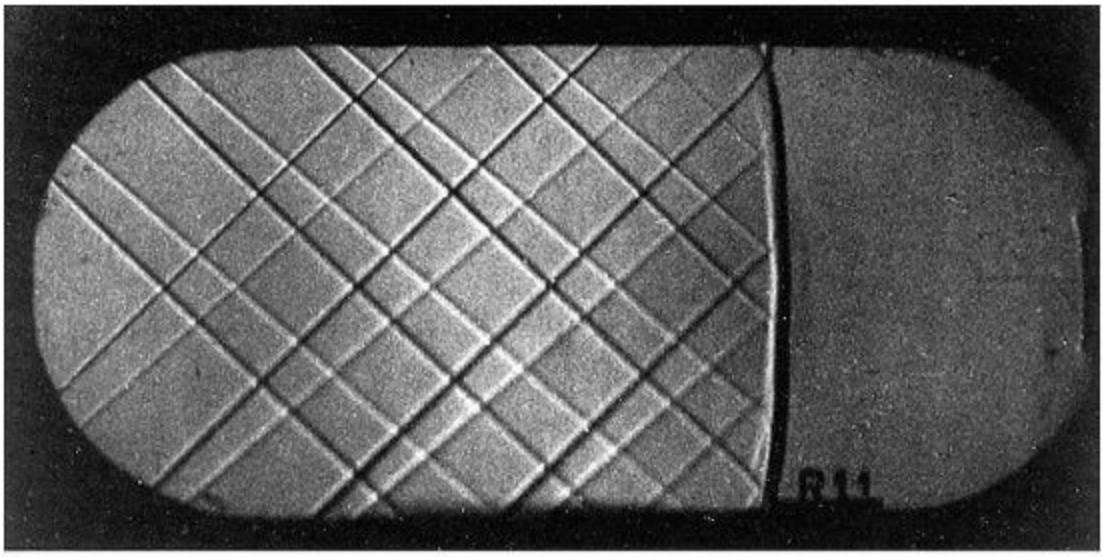


) LJ & KDQJH LQ IORZ SURSHUWLHV DFURVV D QRUPDO VKRFN Z DYH IRU k

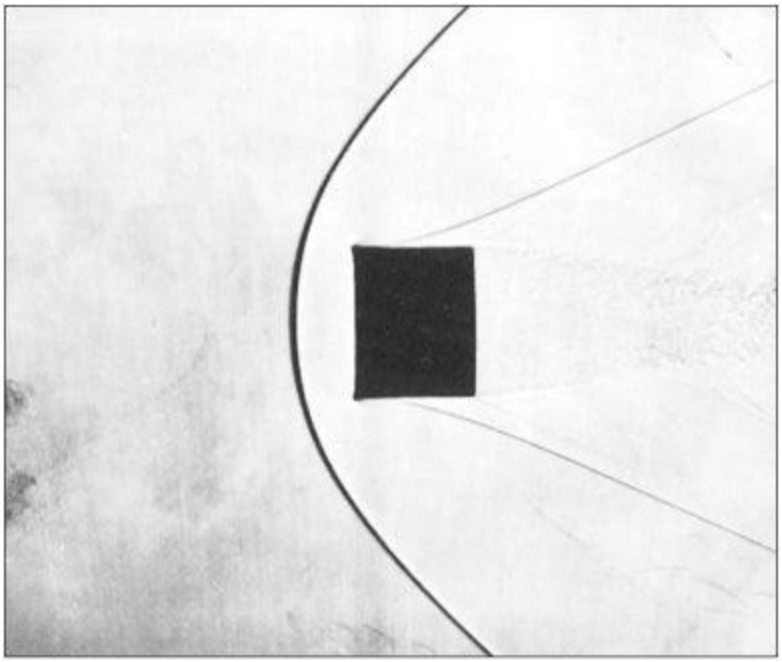
7KH VDQH QDWXUH UHP DLQV WKH VDP H EXW WKH VDQH QDWXUH SUHVXUH DQG GHQVLW\ GHFUHDVHV WKH VDP HUDWR LQ RWKHU Z RUGV WKH IORZ DFURVV WKH VKRFN LV DGLDEDWLF EXW QRQ-LVHQWURSLF 2WKHUEDVH SUHQFLSOHV JRYHUQLQJ WKH EHKDYLRUR VKRFN Z DYHV FDQ EH VXP P DUL]HG DV IROORZ V

7KH XSWUHDP IORZ LV VXSHUVRQLF DQG WKH GRZQWUHDP IORZ LV VXEVRQLF
) RUSHUHFWDVHV DQG DOR IRUUHDOIOXLGV H[FHSWXQGHUEL]DUH WKHUPRG\QDPLF FRQGLWLRQV UDUH DFWRQVKRFNVDUHLPSRVVLEOH DQGRQO DFRPSUHVVLRQVKRFNFDQH[LVW
7KH HQWRS\ LQFUHDVHV DFURVV D VKRFN Z LWK FRQVHTXHQWGHFUHDVHV LQ VWDJQDWLRQ SUHVXUH DQ VWDJQDWLRQ GHQVLWLV DQG DLQFUHDVHLQWKHIIHFWLYHVRQLF VKURDWUHD
: HDN VKRFN Z DYHV DUH YHU Q HDU0 LVHQWURSLF

1 RUP DO VKRFN Z DYHV IRUP LQ GXFW XQGHUWDQLHQWRQV VXFK DV LQ VKRFN WXEHV DQG L WWDG\ IORZ IRUFHUDLQUDQJHV RI WKHGRZQWUHDP SUHVXUH) LJXUH D VKRZ V D QRUP DO VKRFN LQI VXSHUVRQLF QR]]OH) ORZ LV IURP HIWWR ULJKW 7KH REQTXH Z DYH SDWWLQ WR WKH HIWLV IRUP HG E URXJKQHVV HOHP HQWV RQ WKH QR]]OH Z DOOV DQG LQGLFDWV WKDWWKH XSWUHDP IORZ LV VXSHUVRQLF 1 RWWK DEVHQFH RI WKHVH0 DFK Z DYHV VHH6HF LQWKH VXEVRQLF IORZ GRZ QWUHDP
1 RUP DO VKRFN Z DYHV RFFXUQRWRQ LQ VXSHUVRQLF GXFWORZ V EXWDOR LQ D YDULHWY RI VXSHUVRQL H[WHUQDO IORZ V \$ Q H[DP SOH LV WKH VXSHUVRQLF IORZ SDWWD EXQGHUG VKRZ Q LQ) LJ E 7KH EHRZ VKRFN LV FXUYHG Z LWK D SRUWRQ LQ IURQWRI WKH EHRG WKDWLV HVWHQGHG QRUP DOWR WKH RQFRP LQJ IORZ 7KLV QRUP DOSRUWRQ RI WKH EHRZ VKRFN VDUWLHV WKH SURPHUWFKDQJHFRQGLWRQV IXUWDVRXQGHG LQWKH VHFWRQ 7KH IORZ LQVLGH WKH VKRFN QHDU WKH EHRG QRVH LV WKXV VXEVRQLF DQG DWUHDWLYH0 KLJ WWP SHUDXUH7 ! 7 DQG FRQYHFWLYH KHDWWDQVIHUHVHSHFLDOO KJK LQWKLV UHJLRQ



(*a*)



(*b*)

) LJ 1 RUP DOVKRFNVI RUP LQEHWKLQMHFWHGDODQGH[WHUQDOOREVV *a* 1 RUP DOVKRFNLQDGXFW QRWH WKH0DFK ZDYHSDWWHUQVRWKHOHWWXSWHDP LQGLFDWLQJ VXSHUVRQLF IORZ. *(Courtesy of U.S. Air Force Arnold Engineering Development Center b* 6XSHUVRQLF IORZ SDWWD EOXQWHGG FUHDWHV D QRUP DOVKRFN DWWKH QRVH WKH DSSDUHQWVKRFN WKLFNQHVV DQG ERG\ FRQHU FXUYDWXU DUHRSWFDOGLVWRUWLRQV*(Courtesy of U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground.)*

(DFK QRQORUP DOSRUWRQ RI WKHERZ VKRFNLQ) LJ E VDWLVILHV WKHREOTXHVKRFNUHDWLRQVW EHRXWLQGLQG6HF 1 RWHDOOR WKHREOTXHUHFRP SUHVLRQVKRFNRQWKHVLGHVRI WKHERG\ : KDWKD KDSSHQHGLVWKDWWKHVXEVRQLF QRVHIORZ KDVDFFHOHUDWHGURXQGWKHFRQHUVEDFNWRVXSHUVRQLF IORZ D ORZ SUHVVXUH ZKLFK PXVWWKHQ SDV WKURXJK WKH VHFRQG VKRFN WR PDWK WKH KJKHU GRZQWHDP SUHVVXUHFRQGLWLRQV

1 RWWKHILQH JUDLQHGWXEXOHQWZDNHWUXFWXUHLQWKHURWR WKHERG\ LQ) LJ E 7KHWXEXOHQ ERXQGDU\ OD\HUDORQJ WKHVLGHVRI WKHERG\ LVDOR FOHDUO YLVLEOH

7KH DQDO VLV RI DFRPSOH[PXOWLPHQVLRQDO VXSHUVRQLF IORZ VXFK DV LQ) LJ LV EH\RQG WK
VFRSH RI WKLV ERRN) RU IXUWKHU LQIRUPDWLRQ VHH HJ 5H &KDS RU5H &KDS

0 RYLQJ 1 RUP DO6KRFNV

7KH SUHFHGLQJ DQDO VLV RI WKH IL[HG VKRFN DSSOLHV HTXDOO Z HOO WR WKH PRYLQJ VKRFN LI Z HUH YHU
WKH WDQVRUP DWLRQ XVHG LQ) LJ 7R PDNH WKH XSWUHDP FRQGLWLRQV VLPXODWH DWPRVSKHULF Z HPRY
WKH VKRFN RI) LJ WR WKH HIIHFWLYH9 WKDW V Z HIL[RXU FRRUGLQDWH V WR DFRQYROXPHPRYLQ
Z LWK WKH VKRFN 7KH GRZQWUHDP IORZ WKHQ DSSHDUV WR PRYH WR WKH HIIHFWLYH9 í 9
IROORZLQJ WKH VKRFN 7KH WKHUPRG\QDPLF SURSHUWLHV DUH GHWHUPLQHG E\ WKLV WDQVRUP DWLRQ VR WKDW D
RXU(TV WR DUH VWLOO YDOLG

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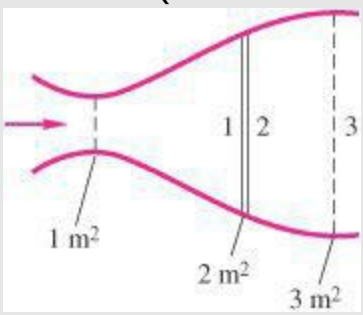
$ LUIORZ VI URP DUHV HUYRLU Z KHUH S N3D DQG 7 . WKURXJK D WKURDWR VHFWLRQ LQ) LJ
(Z KHUH WKH UHLV DQRUP DO VKRFN Z DV & RP SXW D S E S F S GA_2^* H S IA_3^* J S
DQG K 7

(



6RO XWLRQ

- 6\ VWHP VNHWFK 7KLV LV VKRZ QLQ) LJ (%HZH HQV HFWLRQV DQG LV DQRUP DO VKRFN
- $VVXPSWLRQV ,VHQWURSLF IORZ EH IRUH DQG DIWHU WKH VKRFN / RZ HUS DQG! DIWHU WKH VKRFN
- $SSURDFK $IWHU ILLWWLQJ WKDW WKH WKURDW V VRQLF Z RUN\ RXU Z D\ IURP VR VR
- 3URSHUWLHV) RU DLU 5 P V . N DQG F$_S$ P V . 7KH LQG
  VWDJQDWLRQ SUHVVXUH R N3D LV FRQVDQW XS VR SRLQW
- 6RO XWLRQ VWHS D $ VKRFN Z DV H[DQG HG DW WKH WKURDW LV VRQLF DQG FKRNHG$A_{throat} = A_1^* = 1\ m^2$ 7KH DUHD UDWLR JLYHV0 D IURP(T IRUN

$$\frac{A_1}{A_1^*} = \frac{2\ m^2}{1\ m^2} = 2.0 = \frac{1}{Ma_1}\frac{(1 + 0.2\,Ma_1^2)^3}{1.728} \qquad \text{solve for} \qquad Ma_1 = 2.1972$$

6XFK IRXUGHFLPDOSODFH DFFXUDF\ PLJKWUHTXLUH LWHUDWLRQ RU WKH XVH R ((6 / LQH

LQWHUSRODWRQ LQ7DEOH% ZRXOG JLYH0 D § TXLWH JRRG DOVR 7KH SUHVVXUH DWWHFWLRQ VKHQ IROORZV IURP WKH LVHQWURSLF UHODWLRQ (T

$$p_1 = \frac{p_{01}}{(1 + 0.2\text{Ma}_1^2)^{3.5}} = \frac{300 \text{ kPa}}{[1 + 0.2(2.194)^2]^{3.5}} = 28.2 \text{ kPa} \qquad \textit{Ans. (a)}$$

- 6WHSV E F G 7KH SUHVVXUH S LV IRXQG IURP WKH QRUPDO VKRFN(T RU7DEOH%

$$p_2 = \frac{p_1}{k+1}[2k\,\text{Ma}_1^2 - (k-1)] = \frac{28.2 \text{ kPa}}{(1.4+1)}[2(1.4)(2.194)^2 - (1.4-1)] = 154 \text{ kPa} \qquad \textit{Ans. (b)}$$

6LPLODUO IRU 0 D § 7DEOH % JLYHV S S § ((6 JLYHV DQG $A_2^*/A_1^* \approx 1.592$ ((6 JLYHV 7KXV WKH JRRG DFFXUDF\

$$p_{02} \approx 0.628 p_{01} = 0.628(300 \text{ kPa}) \approx 188 \text{ kPa} \qquad \textit{Ans. (c)}$$

$$A_2^* = 1.59 A_1^* = 1.59(1.0 \text{ m}^2) \approx 1.59 \text{ m}^2 \qquad \textit{Ans. (d)}$$

- &RPPHQWV 7R FDOFXODWH A_2^* GLUHFWO\ ZLWKRXW7DEOH % \RX ZRXOG QHHG WR SDXVH DQG FDOFXODWH0 D § IURP (T VLQFH(T LQYROYHV ERWK0 D DQG0 D
- 6WHS H I 7KH IIGRZ IURP WR LV LVHQWURSLF EXW DWKLJKHU HQWURS\ WKDQ XSVWUHDP RI WK VKRFN WKHUHRUH

$$p_{03} = p_{02} \approx 188 \text{ kPa} \qquad \textit{Ans. (e)}$$

$$A_3^* = A_2^* \approx 1.59 \text{ m}^2 \qquad \textit{Ans. (f)}$$

- 6WHSV J K 7KH IIGRZ LV DGLDEDWF WKURXJKRXW WR WKH WDQDQWP SHUDWXUHLVFRQVWDQW

$$T_{03} = T_{02} = T_{01} = 500 \text{ K} \qquad \textit{Ans. (h)}$$

1 H[W WKH DUHD UDWLR XVLQJ WKH QHZ VRQLF DUHD JLYHV WKH0 DFK QXP EHUDWH FWLRQ

$$\frac{A_3}{A_3^*} = \frac{3 \text{ m}^2}{1.59 \text{ m}^2} = 1.89 = \frac{1}{\text{Ma}_3}\frac{(1 + 0.2\,\text{Ma}_3^2)^3}{1.728} \qquad \text{solve for} \qquad \text{Ma}_3 \approx 0.33$$

((6 ZRXOG\LHOG0 D) LQDOO ZLWK NQRZQ (T \LHOGV S

$$p_3 = \frac{p_{02}}{(1 + 0.2\,\text{Ma}_3^2)^{3.5}} \approx \frac{188 \text{ kPa}}{[1 + 0.2(0.33)^2]^{3.5}} \approx 174 \text{ kPa} \qquad \textit{Ans. (g)}$$

- &RPPHQWV ((6 ZRXOGJLYHS NBD VR ZHVHHWKDW7DEOH% LV VDLWDFWRU IRUWKL
WSHRI SUREOHP $ GXFWORZ ZLWKDQRUPDOVKRFNZDYHUHTXLUHVWXDLJKWRUZDUDSSOLFDWRQR
DJHEUDLF SHUFHFWDVUHDWRQV/ FRXSOHGZLWKDOWHWKRXJKWDVWRZKLFKIRUPXODLVDSSURSUDW
IRUWKHJLYHQSURSHUW

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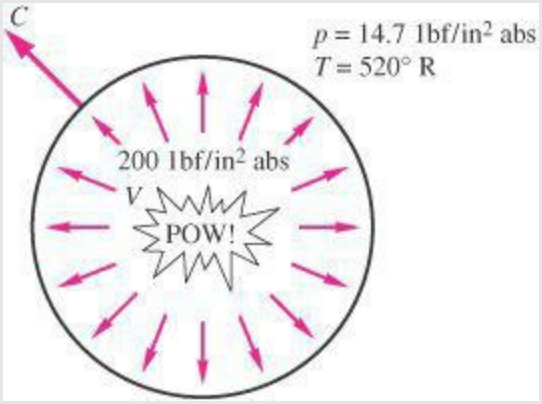
$QH[SORVLRQLQDLUN FUHDWHVDVSKHULFDOVKRFNZDYHSURSDJDWQJ UDGLDOO LQRXWZDUGLUD
WDQGDUGFRQGLWRQV/ $WWKHLQWDQWWKRZQLQ) LJ (WKHSUHVXUHMXWWLQJLGHWKHVKRFNLV GEI LQ
DEVROXW(WWP DW D WKHVKRFNVSHG& DQG E WKHDLUYHORFLW9 MXWWLQGLGHWKHVKRFN

(



6RXWLRQ

3DUWD

,QVSLWRI WKHVSKHULFDOJHRPHWU WKHIORZ DFURVWKHVKRFNPRYHVQRUPDOWR WKHVSKHULFDOZ DY
IURQWKHQHH WKHQRUPDOVKRFN UHODWLRQV WR DSSO) L[LQJ RXU FRQWUROYROXPH WR WK
PRYLQJ VKRFN ZHILQGWKDWWKHSURSHUFRQGLWLRQV/WRXVHLQ) LJ DUH

$$C = V_1 \qquad p_1 = 14.7 \text{ lbf/in}^2 \text{ absolute} \qquad T_1 = 520°\text{R}$$
$$V = V_1 - V_2 \qquad p_2 = 200 \text{ lbf/in}^2 \text{ absolute}$$

7KH VSHHG RI VRXQG RXWLGH WKH VKRFN LV $a_1 \approx 49T_1^{1/2} = 1117 \text{ ft/s}.$: H FDQ ILQG 0 D IURP WKH NQRZ
SUHVVXUHUDWLR DFURVWKHVKRFN

$$\frac{p_2}{p_1} = \frac{200 \text{ lbf/in}^2 \text{ absolute}}{14.7 \text{ lbf/in}^2 \text{ absolute}} = 13.61$$

) URP (T RU 7 DEOH %

$$13.61 = \frac{1}{2.4}(2.8\,\mathrm{Ma}_1^2 - 0.4) \qquad \text{or} \qquad \mathrm{Ma}_1 = 3.436$$

7KHQ E\ GHILQLWLRQ RI WKH 0 DFK QXPEHU

$$C = V_1 = \mathrm{Ma}_1\, a_1 = 3.436(1117\ \mathrm{ft/s}) = 3840\ \mathrm{ft/s} \qquad \textit{Ans. (a)}$$

3 DUW E

7R ILQG 9 Z HQHHG WKH WHPSHUDWXUH RU VRXQG VSHHG LQ WKH VKRFN 6LQFH 0 D LV NQRZ Q IURP (T RU 7 DEOH % IRU 0 D Z H FDQ SXW 7 7 7KHQ

7 7 f5 f5

$ WWXFK D KLJK WHPSHUDWXUH Z H VKRXOG DFFRXQW RU QRQ SHUIHFW JDV HIIHFWV RU DVVXPH WKH JD VEHKDYHV> @EXW Z H RQ\ W+ HHG WR WHPSHUDWXUH DW IURP WKH SHUIHFW JDV HQHUJ\ HTXDWLRQ WKDW

$$V_2^2 = 2c_p(T_1 - T_2) + V_1^2 = 2(6010)(520 - 1679) + (3840)^2 = 815{,}000$$

or $$V_2 \approx 903\ \mathrm{ft/s}$$

1 RWH WKDW Z H GLG WKLV Z LWKRXW EHWHULQ WR FRPSXWH 0 D Z KLFK HTXDOV RU $a_2 \approx 49T_2^{1/2} = 2000\ \mathrm{ft/s}.$

) LQDOO WKH DLUYHORFLW\ EHKLQG WKH VKRFN LV

$$V = V_1 - V_2 = 3840 - 903 \approx 2940\ \mathrm{ft/s} \qquad \textit{Ans. (b)}$$

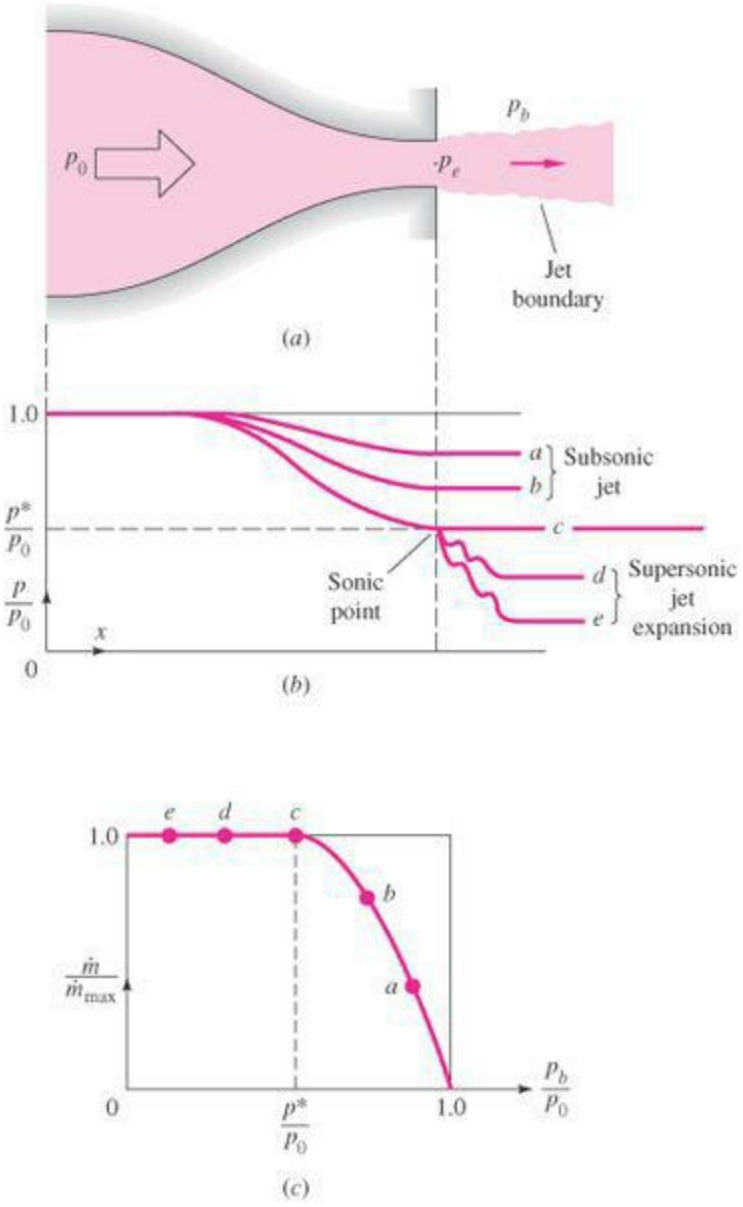
7KXV D SRZ HUIXO [SORVLRQ FUHDWHV D EULH EXW LQWHQVH EODVW Z LQG DV LW SDVVHV

2 SHUDWLRQ RI &RQYHUJLQJ DQG ' LYHUJLQJ 1R]]OHV

%\ FRP ELQLQJ WKH LVHQWURSLF IORZ DQG QRUPDO VKRFN UHODWLRQV SUHV WKH FRQFHSWV RI VRQLF WKURDW FKRNLQJ Z HFDQ RXWOLQH WKH FKDUDFWHULVWLFV RI FRQYHUJLQJ DQG GLYHUJLQJ QR]]OHV

&RQYHUJLQJ 1R]]OH

) LUVW FRQVLGHU WKH FRQYHUJLQJ QR]]OH VKRZQ LQ) LJ D 7KHUH LV DQ XSVWUHDP UHVHUYRLU D VXEVRQLF SUHVXUH S 7KH IORZ LV LQGXFHG E\ ORZ HULQJ WKH GRZQVWUHDP RXWOHW SUHVXUH S EHORZ S UHVXOWLQJ LQ WKH VHTXHQFH RI VXEVRQLF IORZ VKRZQ LQ) LJ E DQG F



)LJ 2SHUDWRQ RI D FRQYHUJLQJ QR]]OH *a* QR]]OH JHRPHWU\ VKRZLQJ FKDUDFWHULVWLF SUHVVXUHV *b* SUHVVXUH GLVWULEXWLRQ FDXVHG E\ YDULRXV EDFN SUHVVXUHV *c* PDVV IORZ YHUVXV EDFN SUHVVXUH

)RU D PRGHUDWH SUHVVXUH UDWLR S_E VR VXEVRQLF DQG E WKH VKURDW SUHVVXUH LV KLJKHU WKDQ WKH FULWLFDO YDOXH S WKDW ZRXOG PDNH WKH VKURDW VRQLF 7KH IORZ LQ WKH QR]]OH LV VXEVRQLF WKURXJKRXW DQG WKH PDVV IORZ UDWH S_H HTXDOV WKH EDFN SUHVVXUH S_E 7KH PDVV IORZ LV SUHGLFWHG E\ VXEVRQLF VHQWURSLF WKHRU\ DQG LV GHWHUPLQHG WKDQ WKH FULWLFDO YDOXH $\dot{m}_{max}$, DV VKRZ QLQ)LJ F

)RU FRQGLWLRQ F WKH EDFN SUHVVXUH H[FHHGV HTXDOV WKH FULWLFDO SUHVVXUH S RI WKH VKURDW 7KH VKURDW EHFRPHV VRQLF WKH MHW H[LWV DW VRQLF S_H S_E DQG WKH PDVV IORZ HTXDOV LWV PD[LPXP YDOXH IURP (TV 7KH IORZ XSVWUHDP RI WKH VKURDW LV VXEVRQLF HYHUZKHUH DQG SUHGLFWHG E\ LVHQWURSLF WKHRU\ EDVHG RQ WKH ORFDO DUHD UDWLR $ [$ DQG7DEOH%

)LQDOO\ LI S_E LV ORZ HUHG IXUWKHU WKH FRQGLWLRQV GRUHEHOZ S WKH QR]]OH FDQQRW HVSRQG IXUWKHU EHFDXVH LWV FKRNHG DW WKH PD[LPXP VKURDW PDVV IORZ 7KH VKURDW PDVV IORZ UDWH S_H S DQG WKH QR]]OH SUHVVXUH GLVWULEXWLRQ LV WKH VDPH DV LQ VWDJH F DV VNHWFKHG LQ)LJ E 7KH H[WHUQDO H[SDQVLRQ FDQ VR WKDW WKH MHW SUHVVXUH FDQ EH UHGXFHG IURP S GRZQ WR S_E 7KH MHW WKXV H[

FRP SOH[DQG PXOWLPHGLD RQOLQH UHVRXUFHV WKDW QHHG %HQJ VXSHUVRQLF WKHRU\ DQG VLJQL
XSWUHDP WR LQFOXGH WKH FKRNHG IORZ FRQGLWLRQV LQ WKH QR]]OH
,I WKH VWDJQDWLRQ SUHVVXUH FKDQJH EHWZHHQ DFRPSUHVVRU DQG LWV HGLVFKDUJH
FKDQJHV DUH VPDOO DQG LVHQWURSLF WKH FRQYHUJLQJ QR]]OH ZLOO EH VWDG R
QHDU 2 VR 2 WKHUH LV WKH QR]]OH ZLOO EH EORZLQJ GRZQ ZLWK S GHFUHDVLQJ DQG S LQFUHDVLQJ DQG WK
IORZ VWDWHV ZLOO EH FKDQJLQJ IURP VXE VWDWH EDFN WR VWDWH % ORZ GRZQ FDOFXODWRQV DU
XVXDOO PDGH E\ DTXDL VWDG DQDO VLV EDVHG RQ LVHQWURSLF VWDG IORZ WKHRU IRUWKHLQWDQDQHRX
SUHVXUHV S WDQGS W

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$ FRQYHUJLQJ QR]]OH KDV DWKURDW DUHD FP DQG VXEJQDWLRQDU RQ DLUFRQGLWLRQV RI N3D DQG
. &RPSXWH WKH H[LWSUHVXUH DQG PDVV IORZ LI WKH EDFN SUHVXUH LV D N3D DQG E N3D
$VXPHN

6RXWLRQ

) URP (T IRUN WKHFULWLFDO VRQLF WKURDWSUHVXUHLV

$$\frac{p^*}{p_0} = 0.5283 \quad \text{or} \quad p^* = (0.5283)(120\ \text{kPa}) = 63.4\ \text{kPa}$$

,I WKHEDFNSUHVXUHLVOHVVWKDQWKLVPRXQWWKHQR]]OHIORZ LVFKRNHG

3DUWD

) RUS N3D! S WKHIORZ LV VXEVRQLF QRWFKRNHG 7KH H[LWSUHVXUHLVS S 7KHWKURD
0 DFK QXPEHULVIRXQGIURP WKHLVHQWURSLF UHODWLRQ RU7DEOH%

$$\text{Ma}_e^2 = 5\left[\left(\frac{p_0}{p_e}\right)^{2/7} - 1\right] = 5\left[\left(\frac{120}{90}\right)^{2/7} - 1\right] = 0.4283 \qquad \text{Ma}_e = 0.654$$

7RILQGWKHPDVVIORZ ZHFRXOGSURFHHGZLWKDVHULDOODXDFNRQ0 D 7H D 9H DQG! H KHQFHV
FRPSXWH! $9 + RZ HYHU VLQFH WKH ORFDO SURPWLV NQRZQ WKLV SDUWLV LGHDOO VXLWHG IRU WK
GLPHQVLRQOHVVPDVVIORZ IXQFWLRQLQ(T : LWKS S FRPSXWH

$$\frac{\dot{m}\sqrt{RT_0}}{Ap_0} = \sqrt{\frac{2(1.4)}{0.4}(0.75)^{2/1.4}[1 - (0.75)^{0.4/1.4}]} = 0.6052$$

$$\text{hence} \qquad \dot{m} = 0.6052\frac{(0.0006)(120{,}000)}{\sqrt{287(400)}} = 0.129\ \text{kg/s} \qquad \textit{Ans. (a)}$$

for $p_e = p_b = 90$ kPa *Ans. (a)*

3DUWE

)RUS$ NBD S WKHIORZ LVFKRNHG VLPLODUWRFRQGLWLRQGLQ) LJ E 7KHH[LWSUHVVXUHL VRQLF

$$p_e = p^* = 63.4 \text{ kPa} \qquad \textit{Ans. (b)}$$

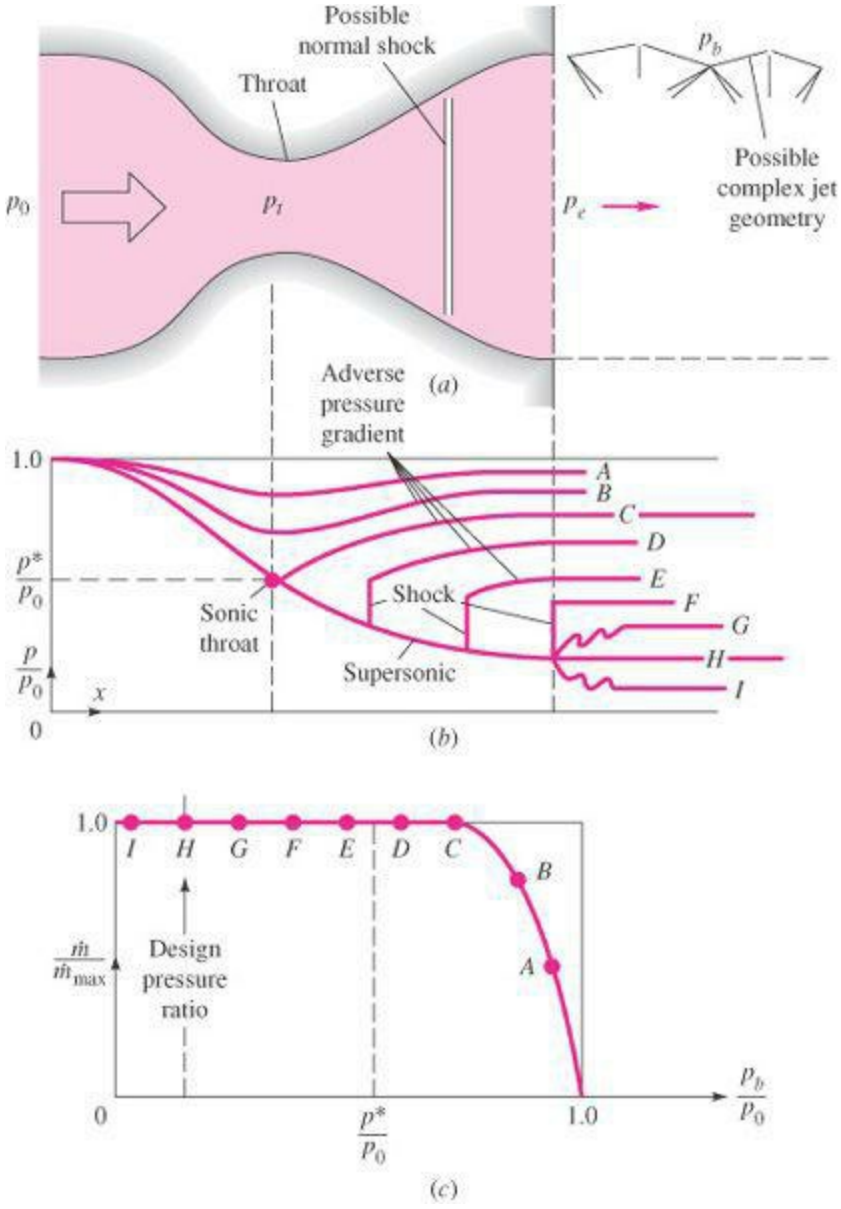
7KH FKRNHG PDVVIORZ LVDPD[LPXP IURP (T E

$$\dot{m} = \dot{m}_{\max} = \frac{0.6847 p_0 A_e}{(RT_0)^{1/2}} = \frac{0.6847(120{,}000)(0.0006)}{[287(400)]^{1/2}} = 0.145 \text{ kg/s} \qquad \textit{Ans. (b)}$$

$Q EDFNSUHVVXUHOHWVWKDQ NBDZRXOGFDXVHWKLVVDPHFKRNHGPDVVIORZ 1RWWKDWWKH SHUFHQWLQFUHDVHLQH[LW0DFK QXPEHU IURP WR KDVLQFUHDVHGWKHPDVVIORZRQO SHUFHQWIURP WR NJV

&RQYHUJLQJ ' LYHUJLQJ 1R]]OHV

1RZ FRQVLGHUWKHFRQYHUJLQJ±GLYHUJLQJ QR]]OHVNHWFKHGLQ) LJ D ,I WKHEDFNSUHVVXUHS$ L OHVV HQRXJK WKHUH ZLOOEH VXSHUVRQLF IORZ LQWKHGLYHUJLQJ SRUWRQ DQG D YDULHW RI VKRFN ZDYH FRQGLWLRQV PD\ RFFXU ZKLFK DUH VNHWFKHG LQ) LJ E /HWWKH EDFN SUHVVXUH EH JUDGXDOO GHFUHDVHG



) LJ 2 SHUDWRQ RI D FRQYHUJLQJ±GLYHUJLQJ QR]]OH *a* QR]]OH JHRPHWUL ZLWK SRVVLEOH IORZ FRQLJXUDWLRQV *b* SUHVVXUH GLVWULEXWLRQ DORQJ WKH ZDOOV *c* P DW IORZ YHUVXV EDFN SUHVVXUH

) RUFXUYHV $ DQG % LQ) LJ E WKH EDFN SUHVVXUH LV TXLWH ORZ HQRXJK WR LQGXFH VRQLF IORZ LQ WKH WKURDW DQG WKH IORZ LQ WKH QR]]OH LV VXSHUVRQLF WKURXJKRXW 7 KH SUHVVXUH GLVWULEXWLRQ IRU VXSHUVRQLF LV HQWLUHO\ DHURG\QDPLF DQG FKDQJHV RQO\ VXFK DV LQ7DEOH% 7KH[LWSUHVVXUHS E DQG WKH M LV VXSHUVRQLF

) RU FXUYH & WKH DUHD UDWLR $ H[FHHGV HTXDOV WKH FULWLFDO UDWLR $ IRU D VXSHUVRQLF 0 D LQ 7DEOH % 7KH WKURDWHFRPHV VRQLF DQG WKH PDVV IOX[UHDFKHV D PD[LPXP LQ) LJ F 7KH UHP DLQGHU RI WKH QR]]OH IORZ LV VXSHUVRQLF LQFOXGLQJ WKH H[LWSORZ DQG S E 7KLV LV FDOOHG WKH GHVLJQ SUHVVXUH UDWLR RI WKH QR]]OH DQG LV WKH EDFN SUHVVXUH VXLWDEOH IRU SURSHU DQG VXSHUVRQLF ZLQG WXQQHO RSHUDWLRQ

1 RZ EDFN XS DQG VXSSRVH WKDW S E GHV EHZ H HQFRXQWHU & DQG + ZKLFK LV P SRVVLEOH DFFRUGLQ

VR SXUH) LVHQWURSLF IORZ FDQ EH XQGHUVWRRG 7KH EDFN SUHVVXUH' VR) RFFXUL) LJ E 7KH VKURF
UHP DLQV FKRNHG DW WKH VRQLF YDOXH DQG ZH FDQ PDNH S_H S_E E\ SODFLQJ D QRUPDO VKRFN DW WKLV WKR
ULJKW SODFH LQ WKH GLYHUJLQJ VHFWLRQ VR FDXVH DVXEVRQLF GLIIXVHU IORZ EDFN VR WKH EDFN SUHVVXU
FRQGLWLRQ 7KH PDVV IORZ UHP DLQV DW P[LPXP LQ) LJ F $WEDFN SUHVVXUH) WKH UHTXLUHG QRUP D
VKRFN VWDQGV LQ WKH GXFW[LW$WEDFN SUHVVXUH* QR VLQJOH QRUPDO VKRFN FDQ GR WKH NRE DQG VR WK
IORZ FRP SUHVVHV RXWLGH WKH H[LWQDFRP SOH[VHULHV RI REOLTXH VKRFNV XQWLO LWV DWKHV S_E
) LQDOO DWEDFN SUHVVXUH, S_E LV GRZ HUWKDQ WKH GHVLJQ SUHVVXUH+ EXW WKH QR]]OH LV FKRNHG DQ
FDQQRW UHSRQG 7KH H[LW IORZ H[SDQGV LQ D FRP SOH[VHULHV RI VXSHUVRQLF ZDYH PRWRQV XQWLO
PDWKHV WKH GXFW EDFN SUHVVXUH 6HH 5H 6HF IRU IXUWKHU GHWDLOV RI WKH VHRI GHVLJQ MWWGXZ
FRQLJXUDWLRQV
1RWH WKDW WRUS_E GHV WKDQ EDFN SUHVVXUH& WKHUHIRUH QR VXSHUVRQLF IORZ LQ WKH QR]]OH DQG WKH WKURD
FDQ UHFHLYH QR VLJQDOI IURP WKH H[LWEHKDYLRU 7KH IORZ UHP DLQV FKRNHG DQG WKH WKURDWKDV QR LGH
ZKDWVWKH H[WFRQGLWLRQ/DUH
1RWH DOVR WKDW WKH QRUP DOVKRFN SDWKLQJ LGHDLV LGDQ] ' RZ QXWHP RI WKH VKRFN WKH QR]]G
IORZ KDV DQ DGYHUVH SUHVVXUH JUDGLHQW XVXDOO\ HDGLQJ WR ZDOOERXQGDU\ OD\HUVHSDUDWLRQ %\ RFNDJ
E\ WKH JUHDW WKLFNHQHG VHSDUDWHG OD\HU LQWHUDFW WRQJ\ ZLWK WHFRUHIORZ UHFDO) LJ DQ
XVXDOO\ LQGXFHV D VHULHV RI ZHDN VZR GLPHQVLRQDO FRP SUHVVLRQ VKRFNV UDWKHU WKDQ D VLQJOH RQ-
GLPHQVLRQDO QRUP DO VKRFN VHH 5H SS DQG IRUIXUWKHUGHWDLOV

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$ FRQYHUJLQJ±GLYHUJLQJ QR]]OH) LJ D KDV DWKURDWDUHD R P DQG DQ H[WDUHD R
P $LUWDQDWRQFRQGLWLRQV/DUHS N3DDQG7 . &RP SXWHWKH H[WSUHVVXUHDQ
PDVVIORZ IRU D GHVLJQFRQGLWLRQDQG WKH H[WSUHVVXUHDQGPDVVIORZ LI E S_E § N3DDQG F S_E
§ N3D $VXPHN

6RXWLRQ

3DUWD

7KH GHVLJQ FRQGLWLRQ FRUUHVSRQGV WR VXSHUVRQLF LVHQWURSLF IORZ DWWKH JLYHQ DUHD UDWLR $_H $_W
: HFDQ ILQG WKH GHVLJQ0 DFK QXPEHUE\ LWHUDWLQJ RI WKH DUHD UDWLR IRUP XOD
RUXVLQJ ((6

$$Ma_{e,design} \approx 2.95$$

7KH GHVLJQSUHVVXUHUDWLR IRQOWZ VIURP (T

$$\frac{p_0}{p_e} = [1 + 0.2(2.95)^2]^{3.5} = 34.1$$

or $$p_{e,\text{design}} = \frac{1000 \text{ kPa}}{34.1} = 29.3 \text{ kPa} \qquad \textit{Ans. (a)}$$

6LQFH WKH WKURDW LV FKRNHG WKH VRQLF GLVFKDUJH FRQGLWLRQV (T E DSSOLHV

$$\dot{m}_{\text{design}} = \dot{m}_{\max} = \frac{0.6847 p_0 A_t}{(RT_0)^{1/2}} = \frac{0.6847(10^6 \text{ Pa})(0.002 \text{ m}^2)}{[287(500)]^{1/2}} \qquad \textit{Ans. (a)}$$

$$= 3.61 \text{ kg/s}$$

3DUW E

) RU S E NBD ZH DUH GHILQLQJ IDUEHOW WKH VXEVRQLF LVHQWURSLF FRQGLWLRQ & LQ) LJ E EXW ZH PD\ HYHQ EHEHOW FRQGLWLRQ) ZLWK D QRUPDO VKRFN LQ WKH H[LW WKDW LV FRQGLWLRQ* ZKHUH REOTXHVKRFNV RFFXUR XWLGH WKH H[LWSODQH ,I LWLV FRQGLWLRQ* WKHQ S H GHJQ NBD EHFDXV QR VKRFN KDV \HWRFFXUUHG 7R ILQG RXWFRP SXWH FRQGLWLRQ) E\ DVXP LQJ DQ H[LWQRUP DO VKRFN ZLW 0 D ² WKDW LV WKH GHJQ0 DFK QXP EHU UXWXSWUHDP RI WKH VKRFN) URP (T

$$\frac{p_2}{p_1} = \frac{1}{2.4}[2.8(2.95)^2 - 0.4] = 9.99$$

or $$p_2 = 9.99 p_1 = 9.99 p_{e,\text{design}} = 293 \text{ kPa}$$

6LQFH WKLV LV GHVV WKDQ WKH JLYHQ S E NBD WKHUHLV D QRUP DO VKRFN IXUWXSWUHDP RI WKH H[L SODQH FRQGLWLRQ(7KH H[LWOW LV VXEVRQLF DQG HTXDO WKH EDFN SUHVVXUH

$$p_e = p_b = 300 \text{ kPa} \qquad \textit{Ans. (b)}$$

Also $$\dot{m} = \dot{m}_{\max} = 3.61 \text{ kg/s} \qquad \textit{Ans. (b)}$$

7KHWKURDWVXWOOVRQLF DQG FKRNHGDWWPD[LPXP PDVIORZ

3DUW F

) LQDOO IRUS E NBD ZKLFK LVXS QHDUFRQGLWLRQ& ZH FRPSXW0 D H DQG S H IRUFRQGLWLRQ& DVDFRP SDULVRQ $JDLQ$H $W IRUWKLVFRQGLWLRQ ZLWKDVXEVRQLF0 D H HVWLP DWHGIURP (T

$$0 D_H \approx \S \qquad \text{H[DFW}$$

7KHQWKHLVHQWURSLF H[LWSUHVVXUHUDWLR IRUWKLVFRQGLWLRQLV

$$\frac{p_0}{p_e} = [1 + 0.2(0.147)^2]^{3.5} = 1.0152$$

or $$p_e = \frac{1000}{1.0152} = 985 \text{ kPa}$$

7KH JLYHQ EDFN SUHVVXUH RI N3D LV GHVWDQWLYH FRUUHVSRQGLQJ URXJKO VR FRQGLWLRQ LQ) LJ E 7KXV IRUWKLV FDVH WKH HLVDQRUP DOVKRFN NXWGRZQWUHP RI WKH WKURDWDQG WKH WKURD LVFKRNHG

$$p_e = p_b = 900 \text{ kPa} \qquad \dot{m} = \dot{m}_{max} = 3.61 \text{ kg/s} \qquad Ans.\ (c)$$

) RUWKLV ODUJH H[WDUHD UDWR WKH H[WSUHVVXUH ZRXOG KDYH WR EH ODUJHU WKDQ N3D WR FDXVH VXEVRQLF IORZ LQWKH WKURDWDQG DP DW IORZ GHVWDQP D[LP XP

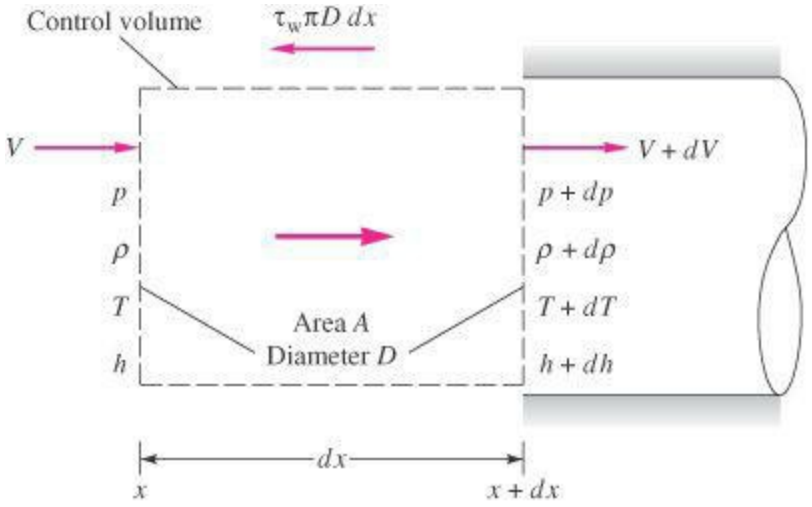
&RP SUHVVLEOH ' XFWJ ORZ ZLWK) UL FWLRQ

6HFWLRQ VKRZHG WKH HIHFWV RI DUHD FKDQJHRQ D FRP SUHVVLEOH IORZ ZKLHQHJOHFWQJ IULFWLRQDQ KHDWWDQVIHU : HFRXOG QRZ DGG IULFWLRQDQG KHDWWDQVIHUWR WKH DUHD FKDQJH DQG FRQVLGHUFRXSOHG HIHFW ZKLFK LV GRQH LQ DGYDQFHG WH[W WJ >IRU H[DP SOH &KDS @,QWKDG DV DQ HOEP HQDV LQWRGXFWLRQ WKLV VHFWLRQ WHDWV RQ WKH HIHFWV IULFWLRQ QHJOHFWQJ DUHD FKDQJH DQG KHDWWDQVIHU 7KHEDVLF DWXP SWLRQV/DUH

6WDG\ RQH GLP HQVLRQDODGLDEDWF IORZ
3HUHFWJDV ZLWK FRQVDQWVSHFLILF KHDW
&RQVDQWDUHDVWDJKVGXFW
1 HJOLJLEOH VKDIWZRUNDQG SRWHQWLDOHQHUJ\ FKDQJHV
: DOWKHDUWWHV FRUUHODWHGE\ D' DUF\ IULFWLRQIDFWRU

,QHIHFWZ HDUH WXG\LQJ D0 RRG\ \SH SLSH IULFWLRQ SUREOP EXWZ LWK DUJH FKDQJHV LQ NLQHW HQHUJ\ HQWKDOS\ DQG SUHVXUHLQWKHIORZ

7KLV \SH RI GXFWIORZ² FRQVDQWDUHD FRQVDQWWDJQDWRQ HQWKDOS\ FRQVDQWP DW IORZ EX YDULDEOH P RP HQWXP GXH WR IULFWLRQ² LV RIWQ WUP HG) DQQR IORZ DWU* LQR) DQQR DQ ,WDDL HQJLQHHUBRQLQ ZKR ILUWWXGLHG WKLV IORZ) RUD JLYHQP DW IORZ DQG VDJQDWRQHQWKDOS\ D SORWWHQWKDOS\ YHUXVHQWRS\ IRUDOOSRVLEOHIORZ VDWWV VXEVRQLF RUVXSHUVRQLF LVFDOHGD) DQQ QH 6HH3URE V 3 DQG3 IRUH[DP SOHVRI D) DQQR QH



) LJ (OHP HQWDO FRQWURO YROXP HIRUI ORZ LQ D FRQYHUJHQW DUHD GXFW Z LWK IULFWLRQ

&RQVLGHU WKH HOHP HQWDO GXFW FRQWURO YROXP HR DUHD $ DQG OHQJWK G[LQ) LJ 7KH DUHD L FRQYDQW WKURXJKHUI ORZ SURSHUWLHV S ! 7 K 9 PD\ YDU Z LWK [$SSOLFDWLRQ RI WKH WKUH FRQVHUYDWLRQ ODZ V WR WKLV FRQWURO YROXP HJLYHV WKH GLIHUHQWLDO HTXDWLRQV

Continuity: $$\rho V = \frac{\dot{m}}{A} = G = \text{const}$$

or $$\frac{d\rho}{\rho} + \frac{dV}{V} = 0 \qquad (9.60a)$$

x momentum: $$pA - (p + dp)A - \tau_w \pi D\, dx = \dot{m}(V + dV - V)$$

or $$dp + \frac{4\tau_w dx}{D} + \rho V\, dV = 0 \qquad (9.60b)$$

Energy: $$h + \tfrac{1}{2}V^2 = h_0 = c_p T_0 = c_p T + \tfrac{1}{2}V^2$$

or $$c_p\, dT + V\, dV = 0 \qquad (9.60c)$$

6LQFH WKH VH WKUH HTXDWLRQV KDYH ILYH XQNQRZQV S ! 7 9 DQG2 Z H QHHG WZR DGGLWLRQDO UHDWLRQV 2 QH LV WKH SHUIHFW JDV ODZ

$$p = \rho RT \quad \text{or} \quad \frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \qquad (9.61)$$

7R HOP LQDWH 2 DV DQ XQNQRZQ LWLV DXXPHG WKDW Z DOO WKH UHDWLRQV FRUUHDWHG E\ D ORFDO' DUF IULFWLRQ IDFWRU I

$$\tau_w = \tfrac{1}{8} f \rho V^2 = \tfrac{1}{8} f k p\, \mathrm{Ma}^2 \qquad (9.62)$$

Z KHUH WKH OD WWHU IRUP IROORZV IURP WKH SHUIHFW JDV VSHHG RI VRXQG H[SUHVVLRQ D NS ! , SUDFWH I FDQ EH UHDWHG WR WKH ORFDO5 H\QROGV QXPEHUDQG Z DOO URXJKQHVV IURP VD WKH 0 RRG FKDUW) LJ

(TXDWLRQV DQG DUHILWWRUGHUGLIIHUHQWLDOHTXDWLRQVDQGFDQEHLQWHJUDWHG E\ XVLQ
IUFWLRQIDFWRUGDWD IURP DQ LQGHVWHFWLRQ ZKHUHS 7 9 DQGVRRQDUHNQRZQ WKHGHVLUHGS[
7[DQGWKHQLNHDBRQJ WKHGXFW,WLVSUDFWLFDOLPSRVVLEOHWRHOLPLQDWHOHQJWKYDULDEOHWRJLYH
VD\ D VLQJOHGLIIHUHQWLDOHTXDWLRQIRUS[EXWDOOHTXDWLRQVFDQEHZULWWQLQWHUPVRI WKH0DF
QXPEHU0D[DQGWKHIULFWLRQIDFWRUE\ XVLQJWKLVGHILQLWLRQRI0DFKQXPEHU

$$V^2 = \mathrm{Ma}^2\, kRT$$

or

$$\frac{2\,dV}{V} = \frac{2\,d\,\mathrm{Ma}}{\mathrm{Ma}} + \frac{dT}{T} \tag{9.63}$$

$GLDEDWLF)ORZ

%\ HOLPLQDWLQJ YDULDEOHVEHWZHHQ(TV VR ZHREWDLQWKHZRUNLQJUHODWLRQV

$$\frac{dp}{p} = -k\,\mathrm{Ma}^2\,\frac{1 + (k-1)\mathrm{Ma}^2}{2(1-\mathrm{Ma}^2)}\,f\frac{dx}{D} \tag{9.64a}$$

$$\frac{d\rho}{\rho} = -\frac{k\,\mathrm{Ma}^2}{2(1-\mathrm{Ma}^2)}\,f\frac{dx}{D} = -\frac{dV}{V} \tag{9.64b}$$

$$\frac{dp_0}{p_0} = \frac{d\rho_0}{\rho_0} = -\frac{1}{2}\,k\,\mathrm{Ma}^2\,f\frac{dx}{D} \tag{9.64c}$$

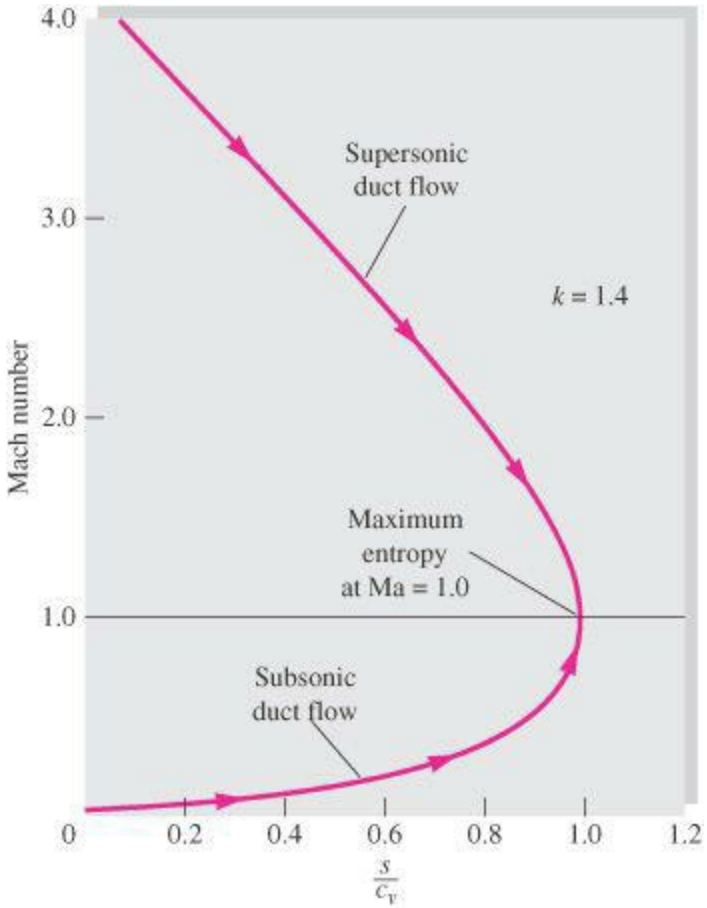
$$\frac{dT}{T} = -\frac{k(k-1)\,\mathrm{Ma}^4}{2(1-\mathrm{Ma}^2)}\,f\frac{dx}{D} \tag{9.64d}$$

$$\frac{d\,\mathrm{Ma}^2}{\mathrm{Ma}^2} = k\,\mathrm{Ma}^2\,\frac{1 + \frac{1}{2}(k-1)\,\mathrm{Ma}^2}{1-\mathrm{Ma}^2}\,f\frac{dx}{D} \tag{9.64e}$$

$\OWKHVHH[FHSWGS S KDYHWKHIDFWRU í 0 D LQWKHGHQRPLQDWRUVRWKDWWKHWDUHDFKDQJH
IRUPXODVLQ)LJ VXEVRQLFDQGVXSHUVRQLFIORZ KDYHRSSRVLWHHIHFW
: HKDYHDGGHG WR WKLV OLWWKDWHQWRS\ PXWLQFUHDVH DORQJ WKH GXFWIRU HLWKHU VXEVRQLF R
VXSHUVRQLF IORZ DV D FRQVHTXHQFH RI WKH VHFRG ODZ IRU DGLDEDWLF IORZ)RU WKH VDPH UHDVRQ
WKDWQDWRSUHVXUHDQGGHQVLWPXWWERWKGHFUHDVH

Property	Subsonic	Supersonic
p	Decreases	Increases
ρ	Decreases	Increases
V	Increases	Decreases
p_0, ρ_0	Decreases	Decreases
T	Decreases	Increases
Ma	Increases	Decreases
Entropy	Increases	Increases

7KH NH\ SDUDPHWHU LQ WKLV GLVFXVVLRQ LV WKH 0DFK QXPEHU : KHWKHU WKH LQGHSHQGHQW LV VXEVRQLF R VXSHUVRQLF WKH GXFW 0DFK QXPEHU DZD\ IURP XQLW\ GRZQVWUHDP WKH SDWWHUQ ZKLFK WKH HQWURS\ LQFUHDVHV ,I WKH SUHVVXUH DQG GHQVLW\ DUH FRPSXWHG IURP (TV D DQG E DQG WKH HQWURS\ IURP (T WKH UHVXOWDQW EHSORWWHG LQ) LJ YHUVXV 0DFK QXPEHU IR N 7KH PDLP XP HQWURS\ RFFXUV DW 0D VR WKDW WKH VHFRQG ODZ UHTXLUHV WKDW WKH GXFW SURPHUWHV FRQVXPDQG DSSURDFK WKH VRQLF SRLQW6LQFH! FRQVXPDQG GHFUHDVHV DORQJ WKH GXFW GXH WR WKH IULFWLRQDO QRQLVHQWURSLF ORVVHV WKH\ DUH QRWXVHUODUHUHQFH SURPHUWHV ,QWKDG WK VRQLF SURPHUWHV p^*, ρ^*, T^*, p_0^*, and ρ_0^* DUH WKH DSSURPULDWH FRQVWDQWUHUHQFH TXDQWLWHV LQ DGLDEDWL GXFWORZ 7KH WKRU WKHQFRP SXWHV WKH UDWLR p/p^*, T/T^*, DQG VR IRUWK DV D IXQFWLRQ RI ORFDO0 DF QXPEHUDQG WKH LQWURGXFHG IULFWLRQHIHFW



) LJ $ GLDEDWF IULFWLRQDO IORZ LQ D FRQVWDQWDUHD GXFWDOZD\V DSSURDFKHV 0 D VDWXU\ WKH VHFRQG ODZ RI WKHUPRG\QDPLFV 7KH FRPSXWHG FXUYH LV LQGHSHQGHQW RI WKH YDOXH R WKH IULFWLRQ IDFWRU

7R GHULYH ZRUNLQJ IRUPXODV ZH IILWWDWDWDFN(T H ZKLFK UHODWHV WKH 0DFK QXPEHUV IULFWLRQ 6HSDUDWWKH YDULDEOHV DQG LQWUDWH

$$\int_0^{L^*} f\frac{dx}{D} = \int_{\mathrm{Ma}^2}^{1.0} \frac{1-\mathrm{Ma}^2}{k\,\mathrm{Ma}^4[1+\frac{1}{2}(k-1)\,\mathrm{Ma}^2]}\,d\,\mathrm{Ma}^2 \qquad (9.65)$$

7KH XSSHU OLPLW VKH VRQLF SRLQW ZKHUH UHQRWLWLV DFWXDOO\ UHDFKHG LQ WKH GXFWORZ 7K ORZHU OLPLW LV DUELWUDULO\ SODFHG DW WKH SRVLWLRQ[ZKHUH 0DFK QXPEHUV 0 D 7KHUHIXOWV WK LQWHUPHGLDWH SRVLWLRQV

$$\frac{\bar{f}L^*}{D} = \frac{1-\mathrm{Ma}^2}{k\,\mathrm{Ma}^2} + \frac{k+1}{2k}\ln\frac{(k+1)\,\mathrm{Ma}^2}{2+(k-1)\,\mathrm{Ma}^2} \qquad (9.66)$$

ZKHUH $\bar{f}$ LV WKH DYHUDJH IULFWLRQ IDFWRU EHWZHHQ DQG/ ,Q SUDFWLFH DQ DYHUDJH LV DOZD\V DVVXPHG DQG QR DWWHPSW LV PDGH WR DFFRXQW IRU WKH VOLJKW FKDQJH LQ 5H\QROGV QXPEHU DORQJ WKH GXFW) RU QRQFLUFXODU GXFWV ' LV UHSODFHG E\ WKH K\GUDXOLF GLDPHWHU ' K î DUHD SHULPHWUDV LQ(T

(TXDWLRQ LV VDWLVIDFWRULO\ YHULILHG 0DFK QXPEHUV LQ7DEOH % 7KH OHQJWK/ LV WKH OHQJWK R GXFWUHTXLUHG WR GHYHORS D GXFWORZ IURP 0 DFK QXPEHU 0 D WR WKH VRQLF SRLQW0 DQ SURFHGP LQYROYH VKRUWGXFW WKDWQHYHU EHFRPH VRQLF IRU ZKLFK WKH VRXWRQ XVHV WKH GLIIHUHQFHV LQ WK VDWXUDWHG3PD[LPXP ´ RUVRQLF OHQJWK) RUH[DPSOH WKH OHQJWKû/ UHTXLUHG WR GHYHORS IURP 0 D W 0 D LVJLYHQE\

$$\bar{f}\frac{\Delta L}{D} = \left(\frac{\bar{f}L^*}{D}\right)_1 - \left(\frac{\bar{f}L^*}{D}\right)_2 \qquad (9.67)$$

7KLVDYRLGVWKHQHHGIRUVHSDUDWHWDEXODWRQVIRUVKRUWGXFW

,WLVUHFRPPHQGHGWKDWWKHIULFWLRQIDFWRU $\bar{f}$ EHHVWLPDWHGIURP WKH0RRG\ FKDUW) LJ IRUWK DYHUDJH 5H\QROGV QXPEHUDQG ZDOOURXJKQHVVUDWLR RI WKHGXFW$YDLODEOHGDWD> @RQGXFWULFWLRQ IRU FRPSUHVVLEOH IORZ VKRZ JRRG DJUHHPHQWZLWK WKH 0RRG\ FKDUWIRU VXEVRQLF IORZ EXWWK PHDVXUHG GDWDIQ VXSHUVRQLF GXFWORZ DUHXS WR SHUFHQWGHVWKDQWKHHTXLYDOHQW0RRG\ IULFWLRQ IDFWRU

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$ LUIORZ V VXEVRQLFDOO LQDQDGLDEDWF FP GLDPHWUGXFW7KHDYHUDJHIULFWLRQIDFWRULV : KDWWHQJWKRI GXFWLV QHFHVVDU\ WR DFFHOHUDWWKHIORZ IURP 0 D WR0 D " : KDWZDGGLWLRQD OHQJWKZLOODFFHOHUDWLWWR0 D " $VXPHN

6RROXWLRQ

(TXDWLRQ DSSOLHV ZLWKYDOXHVR $\bar{f}$ / ' FRP SXWHGIURP (T RUUHDGIURP 7DEOH %

$$\bar{f}\frac{\Delta L}{D} = \frac{0.024\,\Delta L}{0.02\text{ m}} = \left(\frac{\bar{f}L^*}{D}\right)_{\text{Ma}=0.1} - \left(\frac{\bar{f}L^*}{D}\right)_{\text{Ma}=0.5}$$

$$= 66.9216 - 1.0691 = 65.8525$$

Thus
$$\Delta L = \frac{65.8525(0.02\text{ m})}{0.024} = 55\text{ m} \qquad \textit{Ans. (a)}$$

7KHDGGLWLRQDOOHQJWKû/•VRJRIURP 0 D VR0 D LVWDNHQGLUHFWO IURP 7DEOH%

$$f\frac{\Delta L'}{D} = \left(\frac{fL^*}{D}\right)_{\text{Ma}=0.5} = 1.0691$$

or
$$\Delta L' = L^*_{\text{Ma}=0.5} = \frac{1.0691(0.02\text{ m})}{0.024} = 0.9\text{ m} \qquad \textit{Ans. (b)}$$

7KLVLVWKHDOHQJWKRIWKHVHFDOHGXDOWRQV ,WDNHV P VRDFFHOHUDWXS VR0 D DQGWKHQRQH P PRUHVRJHWDOOWKHZD\ XS VRWKHVRQLF SRLQW

)RUPXODVIRUURWKHUIORZ SURSHUWLHVDORQJ WKHGXFWFDQEHGHULYHGIURP (TV (TXDWR H FDQEHXVHGWRHOLPLQDWH G[' IURP HDFKRI WKHRWKHUHODWLRQV JLYLQJ IRUH[DPSOH GS S D DIXQFWLRQRQO RI 0 DQGG0 D 0 D)RUFRQYHQLHQFHLQWDEXODWLQJ WKHUHVXOWV HDFK H[SUHVVLRQL WKHQLQWURGXFHGDOOWKHZD\IURP S 0 D VRWKHVRQLF SRLQWS 7KHLQWURGXFHGUHVXOWDUH

$$\frac{p}{p^*} = \frac{1}{\text{Ma}}\left[\frac{k+1}{2+(k-1)\,\text{Ma}^2}\right]^{1/2} \qquad (9.68a)$$

$$\frac{\rho}{\rho^*} = \frac{V^*}{V} = \frac{1}{\text{Ma}}\left[\frac{2+(k-1)\,\text{Ma}^2}{k+1}\right]^{1/2} \qquad (9.68b)$$

$$\frac{T}{T^*} = \frac{a^2}{a^{*2}} = \frac{k+1}{2+(k-1)\,\text{Ma}^2} \qquad (9.68c)$$

$$\frac{p_0}{p_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{\text{Ma}}\left[\frac{2+(k-1)\,\text{Ma}^2}{k+1}\right]^{(1/2)(k+1)/(k-1)} \qquad (9.68d)$$

$\OCDWKHVHUDWRVDUHDOVRXVHGLQ7DEOH%)RUILQGLQJ FKDQJHVEHWZHHQSRLQWV0 D DQG0 D WKDWDUHQRWVRQLF SURGXFWVRI WKHVHUDWRVDUHXVHG)RUH[DPSOH

$$\frac{p_2}{p_1} = \frac{p_2}{p^*}\frac{p^*}{p_1} \qquad (9.69)$$

VLQFHS LVDFRQVWDQWWHUPFHYDOXHIRUWKHIORZ

(; $0 3/ (

) LJ %HKDYLRUDO GXFWLOHZ ZLWK DQRPLQDOVXSHUYLVLQJ0 D *a L D* "
IOHZ LV VXSHUYLVLQJ VKURXJKRXWGXFW *b L D* ! *L D* QRUP DOVKRFN DW0 D ZLW
VXEVRQLF IOHZ WKHQ DFFHOHUDWLQJ WR VRQLF H[LWSRLQW *c L D* VKRFN PXVWQRZ RFFXUDW0 D
d L D ! IOHZ PXVWEHHQWUHO VXEVRQLF DQG FKRNHGDW H[LW

(; \$0 3/ (

\$ LUHQWLV D FP GLDPHWUGXFWDVS N3D 7 . DQG9 P V 7KHIUFWLR
IDFWRULV &RPSXWH D WKHPD[LPXP GXFWOHQJWKIRUWKHVHFRQGLWLRQV E WKHPDVIOHZ LI WKHGXF
OHQJWKLV P DQG F WKHUHGXFHGPDVIOHZ LI / P

6RXWLRQ

3DUWD

) LUVWFRPSXWH

$$T_1 = T_0 - \frac{\frac{1}{2}V_1^2}{c_p} = 500 - \frac{\frac{1}{2}(100\ \text{m(s)}^2}{1005\ \text{m}^2/\text{s}^2 \cdot \text{K}} = 500 - 5 = 495\ \text{K}$$

$$a_1 = (kRT_1)^{1/2} \approx 20(495)^{1/2} = 445\ \text{m/s}$$

Thus $$\text{Ma}_1 = \frac{V_1}{a_1} = \frac{100}{445} = 0.225$$

) RUWKLV0 D IURP (T RULQWHUSRODWLRQLQ7DEOH%

$$\frac{\bar{f}L^*}{D} = 11.0$$

7KHPD[LPXP GXFWOHQJWKSRVVLEOHIRUWKHVHLQLWLDWLRQVLV

$$L^* = \frac{(\bar{f}L^*/D)D}{\bar{f}} = \frac{11.0(0.03\ \text{m})}{0.02} = 16.5\ \text{m} \qquad \textit{Ans. (a)}$$

3DUWE

7KHJLYHQ/ P LVGHWHUPLQHG DQGVRWKHGXFWLVQRWFKRNHGDQGWKHPDVVIORZ IRGHWHUPLQHGLQSDUW D VIURP WKHLQLWLDWLRQV

$$\rho_{01} = \frac{p_{01}}{RT_0} = \frac{200{,}000\ \text{Pa}}{287(500\ \text{K})} = 1.394\ \text{kg/m}^3$$

$$\rho_1 = \frac{\rho_{01}}{[1 + 0.2(0.225)^2]^{2.5}} = \frac{1.394}{1.0255} = 1.359\ \text{kg/m}^3$$

whence $$\dot{m} = \rho_1 A V_1 = (1.359\ \text{kg/m}^3)\left[\frac{\pi}{4}(0.03\ \text{m})^2\right](100\ \text{m/s})$$

$$= 0.0961\ \text{kg/s} \qquad \textit{Ans. (b)}$$

3DUWF

6LQFH/ P LVJUHDWHUWKDQ WKHGXFWP XWFKRNHEDFNXQWLO / FRUUHVSRQGLQJ WRDGHH

LQDHV0 D

$$L^* = L = 30 \text{ m}$$

$$\frac{\bar{f}L^*}{D} = \frac{0.02(30 \text{ m})}{0.03 \text{ m}} = 20.0$$

EES ,WLVGLIILFXOWRLQWHUSRODWHIRUI ' LQ7DEOH% DQGLPSRVWLEOHWRLQYHUW(T IRU VKH0DFKQXPEHUZLWKRXWDERRNXVLWDQRQ%XWLWVDEUHQGHUHUXVLQJWKHIROORZLQJWKUHHVWDWHPHQWV

N

I/ '

```
fLD = (1 − Ma^2)/k/Ma^2 + (k + 1)/2/k*LN((k + 1)*Ma^2/(2 + (k − 1)*Ma^2))
```

6LPSOI\VSHFLI\ 0 D LQWKH9DULDEOH,QIRUP DWRQPHQX DQG((6 UHSRUWV

$$\text{Ma}_{\text{choked}} = 0.174 \quad \text{(23 percent less)}$$

$$T_{1,\text{new}} = \frac{T_0}{1 + 0.2(0.174)^2} = 497 \text{ K}$$

$$a_{1,\text{new}} \approx 20(497 \text{ K})^{1/2} = 446 \text{ m/s}$$

$$V_{1,\text{new}} = \text{Ma}_1 a_1 = 0.174(446) = 77.6 \text{ m/s}$$

$$\rho_{1,\text{new}} = \frac{\rho_{01}}{[1 + 0.2(0.174)^2]^{2.5}} = 1.373 \text{ kg/m}^3$$

$$\dot{m}_{\text{new}} = \rho_1 A V_1 = 1.373\left[\frac{\pi}{4}(0.03)^2\right](77.6)$$

$$= 0.0753 \text{ kg/s} \quad \text{(22 percent less)} \qquad \textit{Ans. (c)}$$

0 LQRU/ RVVHVLQ&RP SUHVVLEOH) ORZ

) RULQFRP SUHVVLEOHSLSHIORZ DVLQ(T WKHORVVFRHILFLHQW LVWKHUDWLRI SUHVVXUHKHDG ORVV ûS!J WRWKH YHORFLW KHDG 9 J LQWKHSLSH 7KLV LVLQSSURSULDWHLQFRP SUHVVLEOHSLSH IORZ ZKHUH! DQG9 DUHQRWFRQVWDQW%HQHGLFW> @XJJHVWVWKDWKHVXDF SUHVXUHORVV S í S EH UHDDWGWRGRZQWHDP FRQGLWRQVDQGDLXDF ORVVFRHILFLHQW V

$$K_s = \frac{2(p_1 - p_2)}{\rho_2 V_2^2} \qquad (9.70)$$

%HQHGLFW> @JLYHV H[DPSOHV RI FRPSUHVVLEOH ORZ-HV LQ VXGGHQ FRQWDFWRQV DQG H[SDQVLRQV ,
GDWD DUH XQDYDLODEOH DIILWWDSSUR[LPDWRQ ZRXOG EH ZRXVH. V§. IURP 6HFWLRQ

,VRWKHUPDO) ORZ ZLWK) UFWLRQ / RQJ 3LSHOLQHV

7KH DGLDEDWLF IULFWLRQDO IORZ DVXPSWLRQ LV DSSURSULDWH WR KLJK VSHHG IORZ LQ VKRUW GXFWV) F
IORZ LQ ORQJ GXFWV VXFK DV QDWXUDO JDV SLSHOLQHV WKH JDV VWDWH PRUH FORVHO DSSUR[LPDWHV DI
LVRWKHUPDO IORZ 7KH DQDO\VLV LV WKH VDPH H[FHSW WKDW WKH LVHQWURSLF HQHUJ\ HTXDWLRQ F L
UHSODFHG E\ WKH VLPSOHU HQHUJ\ HTXDWLRQ

7 FRQWG7

$ JDLQLWV SRVVLEOH WR ZULWH DOO SURSHUWLHV FKDQJHV LQ WHUPV RI WKH 0 DFK QXPEHU ,QWURGXFWLRQ RI WK
0 DFK QXPEHU IULFWLRQ UHODWLRQ\LHOGV

$$\frac{\tilde{f}L_{\max}}{D} = \frac{1 - k\,\mathrm{Ma}^2}{k\,\mathrm{Ma}^2} + \ln\left(k\,\mathrm{Ma}^2\right) \qquad (9.71)$$

ZKLFK LV WKH LVRWKHUPDO DQDORJ RI (T IRU DGLDEDWLF IORZ
7KLV IULFWLRQ UHODWLRQ KDV WKH LQWHUHVWLQJ UHVXOW WKDW WPD[EHFRPHV]HUR QRW DW WKH VRQLF SRLQW EXW
0 D$W N LI N 7KH LQDFWORZ ZKHUH WKH VXEVRQLF RUVXSHUVRQLF WHQGV GRZ QXWHP
VRZ DUG WKLV OLPLWLQJ 0 DFK QXPEHU N ,I WKH VXEHOHQJWK/ LV JUHDWHU WKDQ/ PD[IURP (T I
VXEVRQLF IORZ ZLOO FKRNH EDFN WR D VPDOOHU 0 D DQG PDV IORZ DQG D VXSHUVRQLF IORZ ZLO
H[SHULHQFH DQRUPDO VKRFN GXWHQVLPLODUW) LJ

7KH H[WUHPH DFKRNHG IORZ LV QRW VRQLF DQG VR WKH XVH RI WKH DXWRULVLQ DSSURSULDW / H
S• ! • DQG 9• UHSUHVHQWSURSHUWLHV DW WKH FKRNLQJ SRLQW/ /PD[7KHQ WKH LVRWKHUPDO DQDORJ VL OHDGV W
WKH IROORZLQJ 0 DFK QXPEHUUHODWLRQ/IRUWKH IORZ SURSHUWLHV

$$\frac{p}{p'} = \frac{1}{\mathrm{Ma}\,k^{1/2}} \qquad \frac{V}{V'} = \frac{\rho'}{\rho} = \mathrm{Ma}\,k^{1/2} \qquad (9.72)$$

7KH FRPSOHWH DQDO\VLV DQG VRP H H[DPSOHV DUH JLYHQ LQ DGYDQFHG WH[WV>IRU H[DPSOH 6HF @

0 DV) ORZ IRU D* LYHQ 3UHVVXUH' URS

$ QLQWHUHVWLQJ E\ SURGXFW RI WKH LVRWKHUPDO DQDORJ V LV DQ H[SUHVVLRQ E\ZH HQ WKH SUHVVXU
GURS DQG GXFWP DW IORZ 7KLV FRPPRQ SUREOP UHTXLUHV QXPHULFDO LWUDWLRQ IRU DGLDEDWLF IORZ D
RXWOLQHG KHUH ,Q LVRWKHUPDO IORZ ZH PD\ VXEVWLWXWH G9 9 íGS S DQG9 * >S 57 @ LQ(T
VR REWLQ

$$\frac{2p\,dp}{G^2RT} + f\frac{dx}{D} - \frac{2\,dp}{p} = 0$$

6LQFH* 57 LVFRQVWDQWRULVRWKHUPDOIORZ WKLVPD\ EHLQWHJUDWHGLQFORVHGIRUP EHWZHHQ [S S DQG / S

$$G^2 = \left(\frac{\dot{m}}{A}\right)^2 = \frac{p_1^2 - p_2^2}{RT[\bar{f}L/D + 2\ln(p_1/p_2)]} \qquad (9.73)$$

7KXVPDVVIORZ IRORRZVGLUHFWO\ IURP WKHNQRZQHQGSUHVVXUHV ZLWKRXWDQ XVHRI 0DFKQXPEHU RUVDEOHV

7KHZULWUGHVQRWNQRZ RI DQ GLUHFWDQDORJ\ WR(T IRUDGLDEDWF IORZ +RZHYHU D XVHXOGLDEDWF UHDWRQ LQYROYLQJ YHORFLWLHVLQWHGRI SUHVVXUHV LV

$$V_1^2 = \frac{a_0^2[1 - (V_1/V_2)^2]}{k\bar{f}L/D + (k+1)\ln(V_2/V_1)} \qquad (9.74)$$

ZKHUHD N57 LVWKHVDQGQDWRQVSHHGRI VRXQG FRQVWDQWRUDGLDEDWF IORZ 7KLVPD\ EH FRPELQHG ZLWK FRQWLQXLW IRUFRQVWDQWGXFWDUHD9 9 ! ! SXV WKHIRORZLQJ FRPELQDWRQ R DGLDEDWF HQHUJ\ DQGWKHSHUHFWDVUHDWRQ

$$\frac{V_1}{V_2} = \frac{p_2}{p_1}\frac{T_1}{T_2} = \frac{p_2}{p_1}\left[\frac{2a_0^2 - (k-1)V_1^2}{2a_0^2 - (k-1)V_2^2}\right] \qquad (9.75)$$

,I ZHDUHJLYHQWKHHQGSUHVVXUHV QHLWKHU9 QRU9 ZLOOQHO EHNQRZQLQDGYDQFH +HUH LI ((6 LV QRWDYDLODEOH ZH VXJJHVWWRQ WKHIRORZLQJ VLPSOH SURFHGXUH %HJLQ ZLWKD § D DQG WK EUDFNHWHG WUP LQ(T DSSUR[LPDWH HTXDOWR 6ROYH(T IRUD ILUVWHVWPDWR 9 9 DQGXVHWKLVYDOXHLQ(T WRJHWDEHWWHUHVWPDWRI 9 8VH9 WRLPSURYH\RXUHVWPDWH RI D DQGUHSHDWWKHSURFHGXUH 7KHSURFHVVKRXOGFRQYHUJHLQDIHZ LWHUDWRQV

(TXDWRQV DQG KDYHRQHIODZ : LWKWKH0DFKQXPEHUHODWHGWRWKH IUFWRQ FKRNLQJ SKHQRPHQRQ LV QRWGLUHFWO HYLGHQW7KHUHRUH DVXPLQJ DVXEVRQLF LQOHWFRQG FKHFN WKH H[LW0DFKQXP0 D WR HQVXUH WKDWWLV QRWJUHDWUWKDQ N IRULVRWKHUPDOIORZ R JUHDWU WKDQ IRU DGLDEDWF IORZ : H LOOXVWUDWH ERWK DGLDEDWF DQG LVRWKHUPDO IORZ ZLWK WKH IRORZLQJ H[DPSOH

(; $0 3/ (

$ LUHQWLV DSLSHRI FP GLDP HWUDQG P OHQJWKDWS N3DDQG7 . ,I DQG WKH H[LWSUHVVXUH LVS N3D HVWLPDWH WKH PDVV IORZ IRU D LVRWKHUPDO IORZ DQG E DGLDEDWF IORZ

6ROXWLRQ

3DUWD

) RUOLVRWKHUPDOIORZ (T DSSOLHVZLWKRXWWXUDWRQ

$$\frac{\bar{f}L}{D} + 2\ln\frac{p_1}{p_2} = \frac{(0.025)(1.2\text{ m})}{0.01\text{ m}} + 2\ln\frac{220}{140} = 3.904$$

$$G^2 = \frac{(220{,}000\text{ Pa})^2 - (140{,}000\text{ Pa})^2}{[287\text{ m}^2/(\text{s}^2\cdot\text{K})](300\text{ K})(3.904)} = 85{,}700 \quad \text{or} \quad G = 293\text{ kg/(s}\cdot\text{m}^2)$$

6LQFH$ CE P (P WKHLVRWKHUPDOPDVVIORZ HWWPDWLV

$$\dot{m} = GA = (293)(7.85\text{ E-}5) \approx 0.0230\text{ kg/s} \qquad \textit{Ans. (a)}$$

&KHFNWKDWWKHH[LW0DFKQXPEHULVORZHQRXJK

$$\rho_2 = \frac{p_2}{RT} = \frac{140{,}000}{(287)(300)} = 1.626\text{ kg/m}^3 \qquad V_2 = \frac{G}{\rho_2} = \frac{293}{1.626} = 180\text{ m/s}$$

$$\text{or} \qquad \text{Ma}_2 = \frac{V_2}{\sqrt{kRT}} = \frac{180}{[1.4(287)(300)]^{1/2}} = \frac{180}{347} \approx 0.52$$

7KLVLVZHOOEHORZ FKRNLQJ DQGWKHLVRWKHUPDOVRXWRQLVDFFXUDWH

3DUWE

EES) RUDGLDEDWLFIORZ ZHFDQLWHUDWHE\ KDQG LQWKHWPHKRQRUHGIDVKLRQ XVLQJ (TV DQG SOXVWKHGHILQLWLRQRI VWDJQDWLRQVSHHGRI VRXQG $ IHZ \HDUVDJR WKHDXWKRUZRXOGKDYHGRQH MXVWWKDWODERURXV + RZHYHU ((6 PDNHVKDQGZRUN DQG PDQSXODWLRQRI HTXDWLRQV XQQHFHVVDU DQWKRXJK FDUHXOSURJUDPPLQJ DQGJRRGJXHVVHVDUHUHTXLUHG ,I ZHLJQRUHVXSHUORXVRXSXWWXFK D 7 DQG9 VWDWPHQWVDUHDSSURSULDWH) LUVWVHOHFWWKHJLYHQSK\VLFDOSURSHUWLHV LQ6, XQLWV

N
3
3
7

1H[WDSSO\WKHDGLDEDWLFIUFWLRQUHDWLRQV (TV DQG WKRERWKSRLQWV DQG
I/' ±0 D A N0 D A N N/1 N 0 D A N± 0 D A
I/' ±0 D A N0 D A N N/1 N 0 D A N± 0 D A
' HOWD/'
I/' I/' ' HOWD/'
7KHQDSSO\WKHSUHVVXUHUDWLRIRUPXOD D WKERWKSRLQWV DQG

3 3WDU N N± 0 D A A 0 D

3 3VWDU N N± 0 D A A 0 D

7KHVH DUH DGLDEDWF UHDWLRQV VR ZH QHHG QRW WXUN HU VSHOOR XW TXDQWLWLHV VXFK DV7 RU D XQLW
ZHZ DQWWKHP DVDGGLWLRQDOR XWSXW

7KH DERYH VWDWHPHQWV DUH DFWXDOO\ DQG((6 ZLOOVROYH WKHP IRU0 D DQ
0 D +RZ HYHU WKHSUREOHP DVVIRUPDWIORZ VR ZHFRPSDUHWKHV\VWHP

9 0 D VTUW 7

5KR 3 7

0 GRW 5KR SL A 9

,I ZHDSS0 QR FRQWDLQW ((6 UHSRUW³FDQQRVROYH EHFDXVHLWGHDXOWDORZ VDOOYDULDEOHV
QHEHZHHQí ' DQG ' 6RZHQHU9DULDEOH,QRUPDWRQDQGFRQWDLQ0 D DQG0 D WRQHEHZHH
DQG VXEVRQF IORZ ((6 VWDWFRPSDLQVWKDWW³FDQQRVROYH EXWKLQWWKDW³EHWWUJXHVVDU
QHGG´ ,QGHG WKH GHDXOWJXHVVIRU((6 YDULDEOHV DUH QRUPDOL WRR ODUJHIRUWKH0 DF
QXPEHUV * XHVVWKH0 DFKQXPEHUVHTXDOWR RUHYHQ DQG((6 VWDWFRPSDLQV IRUDVXEV
UHDVRQ 6LQFHIû/ ' 0 D FDQ EH QR ODUJHUWKDQ VHH7DEOH %
)LQDOO WKHQ ZHJXHVV0 D DQG0 D RU DQG((6 UHSRUWWKHVROXWLRQ

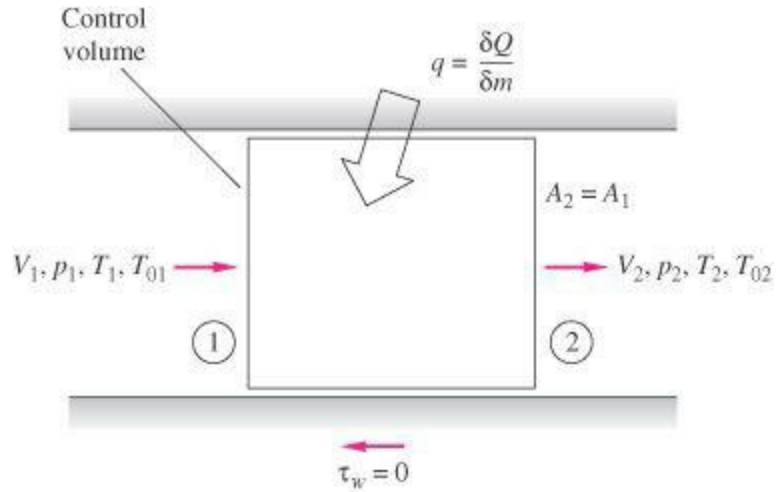
$Ma_1 = 0.3343 \quad Ma_2 = 0.5175 \quad \frac{fL}{D_1} = 3.935 \quad \frac{fL}{D_2} = 0.9348$$

$$p^* = 67,892 \text{ Pa} \quad \dot{m} = 0.0233 \text{ kg/s} \qquad \textit{Ans. (b)}$$

7KRXJK WKHSURJUDPPLQJ LVFRPSOLFDWHG WKH((6 DSSURDFK LV VXSHULRUWR KDQGLWHUDWLRQ DQG F
FRXUVH ZHFDQVDYHWKLVSURJUDP IRUXVHDJDLQZLWKQHZ GDWD

) ULFWLRQOHVV' XFWJ ORZ ZLWK + HDWUDQVIHU

+HDWDGGLWLRQRUUHPRYDOKDV DQLQIXHQFH RQ DFRPSUHVVLEOHIORZ $GYDQFHG WHWWR>R
H[DPSOH &KDS @FRQVLGHUWKH FRP ELQHG HIHFWRI KHDWWDQVIHUFRXSOHG ZLWKIULFWLRQ DQG DUH
FKDQJHLQDGXFW+HUHZHFRQLQHWKHDQDOVLVWRKHDWDQVIHUZLWKQRIULFWLRQLQDFRQVWDQWDUHDGXFW
7KLVWSHRI GXFWIORZ² FRQVWDQWDUHD FRQVWDQWPRPHQWXP FRQVWDQWDWIORZ EXWYDULDEO
VWDJQDWLRQHQWKDOS\ GXHWR KHDWWDQVIHU² LVRIWQWUHG5D\OHLJKIORZ DWMu-RKQ: LOOLP 6WK
/ RUG5D\OHLJK ± DIDPRXVSK\VLFLVWDQGHQJLQHHU)RUDJLYHQPDWIORZ DQGPRPHQWP
D SDUWWR HQWKDOS\ YHUXV HQWURS\ IRUDOOSRVVLEOHIORZ WDWHV VXEVRQF RU VXSHUVRQF IRUPV I
5D\OHLJKQH 6HH3URE V 3 DQG3 IRUH[DPSOHVRI D5D\OHLJKQH



) LJ (OPH HQWDOFRQWROYROXPH IRU IULFWLRQOHVV IORZ LQ D FRQVWDQWDUHD GXFWZLWK KHD WUDQVIHU 7KH OHQJWK RI WKH HOHPHQWV LQGLFDWHV PLQLPXP LQWHUVHFWLQJ VLSSOLHG WKHRU

&RQVLGHUWKH HOHPHQWDOFRQWROYROXPH LQ) LJ %HWZHHQVHFWLRQV DQG DQDPRXQW KHDW4 LVDGGHG RUUHPRYHG WKURXJK : LWKQR IULFWLRQRUDUH FKDQJH WKHFRQWROYROXPHFRQVHUYDWLRQUHODWLRQVDUHTXLWHVLPSOH

Continuity:	$\rho_1 V_1 = \rho_2 V_2 = G = \text{const}$	(9.76*a*)
x momentum:	$p_1 - p_2 = G(V_2 - V_1)$	(9.76*b*)
Energy:	$\dot{Q} = \dot{m}(h_2 + \frac{1}{2}V_2^2 - h_1 - \frac{1}{2}V_1^2)$	
or	$q = \frac{\dot{Q}}{\dot{m}} = \frac{\delta Q}{\delta m} = h_{02} - h_{01}$	(9.76*c*)

7KH KHDWWUDQVIHUUHVXOWV LQ D FKDQJH LQ VWDJQDWLRQ HQWKDOS\ RI WKH IORZ : H VKDOOQRWVSHFLI\ H[DFWO\ KRZ WKHKHDWLVWUDQVIHUUHG? FRP EXWLRQ QXFOHDUUHDFWLRQ HYDSRUDWLRQ FRQGHQVDWLRQ RUZDO KHDWH[FKDQJH? EXWLPSO\ WKDWWKSSHQHG LQ PRXQW E HWZHHQ DQG : HUHPDUN KRZ HYHU WKD ZDOO KHDWH[FKDQJH LV QRWD JRRG FDQGLGDWH IRU WKH WKRU EHFDXVH ZDOO FRQYHFWLRQ LV LQHYLWDEO FRXSOHG ZLWKZDOOIULFWLRQ ZKLFK ZHQHJOHFWHG

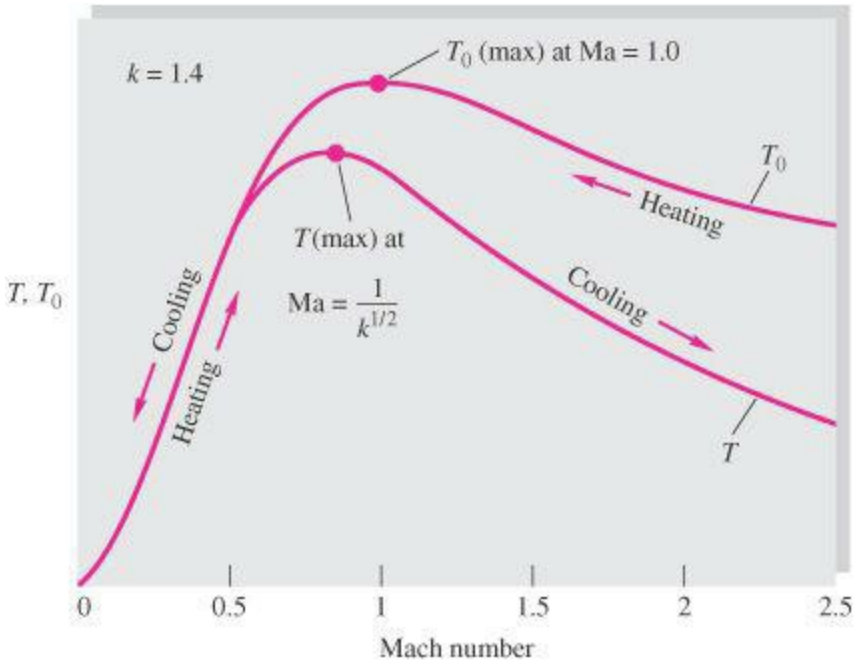
7R FRPSOHWHWKHDQDOVLV ZHXVHWKHSHUHFWJDVDQG0DFKQXPEHUHODWLRQV

$$\frac{p_2}{\rho_2 T_2} = \frac{p_1}{\rho_1 T_1} \qquad h_{02} - h_{01} = c_p(T_{02} - T_{01})$$
$$\frac{V_2}{V_1} = \frac{\text{Ma}_2\, a_2}{\text{Ma}_1\, a_1} = \frac{\text{Ma}_2}{\text{Ma}_1}\left(\frac{T_2}{T_1}\right)^{1/2} \tag{9.77}$$

) RUDJLYHQKHDWWDQVIHUT /4 /P RUHTXLYDOHQWD JLYHQFKDQJHK í K (TV DQ FDQEHVROYHGDOJHEUDLFDOO IRUWKHSURSHUWUDWLRVS S 0D 0D DQGVR RQEHZHHQLQOHW DQGRXWOHW1 RWHWKDWEHFDXVHWKHKHDWWDQVIHUDOORZVWKHHQWURS\ WR HLWKHULQFUHDVHRUGHFUHDVH WK VHFRQGODZ LPSRVHVQR UHVWULFWLRQVRQWKHVHVROXWLRQV

%HIRUHZULWQJ GRZQWKHVHSURSHUWUDWR IXQFWLRQV ZHLOOXWDWWKH HIHFWRI KHDWWDQVIHUL) LJ ZKLFK VKRZV7 DQG7 YHUVXV0 DFK QXPEHULQWHGXFW+HDWQJ LQFUHDVHV7 DQGFRROQ GHFUHDVHVLW7KHPD[LPXP SRVLEOH7 RFFXUVDW0 D DQGZHVHHWKDWKHDWQJ ZKHWKHUWKHIORQ LVVXEVRQLF RUVXSHUVRQLF GULYHVWKHGXFW0 DFK QXPEHUWRZDUGXQLW 7KLVLVDQDORJRXVWRWKHHIHF RI IULFWLRQLQWKHSUHYLRXVVHFWLRQ 7KHWHPSHUDWXUHRI DSHUHFWJDVIURP 0 D XS WR0 D N DQGWKHQGHFUHDVHV 7KXVWKHUHLV DSHFXOD3 RUDWKDWXQH[SHFWHG UHJLRQ ZKHUHKHDWQ LQFUHDVLQJ7 DFWXDOO GHFUHDVHV WKH JDVWHPSHUDWXUH WKH GLIIHUHQFH EHWQ UHOHFWHG LQ D ODJJL

LQFUHDVHV WKH JDVNLQHWLF HQHUJ\ ) RUN WKLV SHFXOLDUDUHDOHVEHWZHHQ0 D DQG0 D LQWHUWLQJ EXWQRWVWHU XVHXOLQRUPDWRQ



) LJ (IIHFWRI KHDWWUDQVIHURQ0 DFK QXP EHU

7KHFRRPSOHWHOXWR WKHHIIHFWRI VLPSOH7 FKDQJHRQGXFWORZ SURSHUWLHVLVDVIROORZV 3URBEDE0 WKH PRWWVLJQLILFDQWLWP RQ WKLV OLWWLV WKH VWDQGDUQ SUHVXUHS ZKLFK DQZ DV GHFUHDVHVGXULQJ KHDWQJ ZKHWKHUWKHIORZ LV VXEVRQF RUVXSHUVRQF 7KXVKHDWQJ GRHVLQFUHDVHWK 0 DFK QXP EHUR DIORZ EXWHQGLVDORVVLQHIIHFWLYHSUHVXUHUHFRYHU

	Heating		Cooling	
	Subsonic	Supersonic	Subsonic	Supersonic
T_0	Increases	Increases	Decreases	Decreases
Ma	Increases	Decreases	Decreases	Increases
p	Decreases	Increases	Increases	Decreases
ρ	Decreases	Increases	Increases	Decreases
V	Increases	Decreases	Decreases	Increases
p_0	Decreases	Decreases	Increases	Increases
s	Increases	Increases	Decreases	Decreases
T	*	Increases	†	Decreases

*Increases up to Ma = $1/k^{1/2}$ and decreases thereafter.
†Decreases up to Ma = $1/k^{1/2}$ and increases thereafter.

0 DFK 1 XP EHU5 HODWLRQV

(TXDWLRQV DQG FDQ EH UHDUUDQJHG LQ WHUPV RI WKH 0 DFK QXP EHU DQG WKH UHVXOWLQJ) RUFRQYHQLHQFH ZH VSHFLI\ WKDWWKH RXOHWVWDWLRQ LV VRQLF 0 D ZLWK UHHUHQFH SURSHUWLHV DQGT_0^*, T^*, p^*, ρ^*, V^*, and p_0^* 7KH LQDHWLV DWXPHG WR EH DWDUELWUDU 0 DFK QXP EHU0 D (TXDWLRQV DQG WKHQWDNHWKHIROORZLQJ IRUP

$$\frac{T_0}{T_0^*} = \frac{(k+1)\,\mathrm{Ma}^2\,[2+(k-1)\,\mathrm{Ma}^2]}{(1+k\,\mathrm{Ma}^2)^2} \qquad (9.78a)$$

$$\frac{T}{T^*} = \frac{(k+1)^2\,\mathrm{Ma}^2}{(1+k\,\mathrm{Ma}^2)^2} \qquad (9.78b)$$

$$\frac{p}{p^*} = \frac{k+1}{1+k\,\mathrm{Ma}^2} \qquad (9.78c)$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(k+1)\,\mathrm{Ma}^2}{1+k\,\mathrm{Ma}^2} \qquad (9.78d)$$

$$\frac{p_0}{p_0^*} = \frac{k+1}{1+k\,\mathrm{Ma}^2}\left[\frac{2+(k-1)\,\mathrm{Ma}^2}{k+1}\right]^{k/(k-1)} \qquad (9.78e)$$

7KH VH IRUPXODV DUH DOO VDWLVILHG YHUVXV 0DFK QXPEHU LQ7DEOH % 7KH VDEOHV DUH YHU FRQYHQLHQW LQDVPXFK DV 9 DQG WKH ONH DUH JLYHQ EXW DUH VRPHZ KDWFXPEHUVRPH LI WK JLYHQ LQRUP DWRQF HQWUVRQ7 DQG7 / HWXVLOOXVWUDWHZ LWKDQH[DPSOH

(; $0 3/ (

$ IXHOHGDLUPL[WXUH DSSURDFKLQJ DWGDVDLUZ LWKN HQWUVDGXFVWRP EXWRQFKDPEHUDW9 P V S N3D DQG7 . 7KHKDWDGGLWRQE\ FRPEXWRQV N- NJ RI P L[WXUH &RPSX D WKH H[LWSURSHUWV9 S DQG7 DQG E WKH VRDOKHDWDGGLWRQWKDWZRXOG KDYH FDXVHG DVRQL H[LWWRZ

6RXWLRQ

3DUWD

) LUVWFRP SXW $ $ $ 7KHQFRP SXW WKHFKDQJHLQVWDJQDWLR WWP SHUDWXUHRI WKHJDV

$$T \quad FS\,7 \quad \pm 7$$

$$q = c_p(T_{02} - T_{01})$$

$$\text{or} \qquad T_{02} = T_{01} + \frac{q}{c_p} = 303\ \mathrm{K} + \frac{900{,}000\ \mathrm{J/kg}}{1005\ \mathrm{J/(kg\cdot K)}} = 1199\ \mathrm{K}$$

: HKDYHHQRXJKLQIRUP DWRQWRFRP SXW WKHLQLWDO0 DFK QXP EHU

PRUHDQG PRUHDVZHDGG PRUHKHDWZLWKWKHLQGLYLGXDOWYHOFLWDSSURDFKLQJ]HUR)RUVXSHUYROFIGRZ
HYHQLI 0D LVLQLQLWM WKHUHLV DILQLWHUDLR IRUN 7KXVLI KHDWL
DGGHG ZLWKRXWOPLWWR D VXSHUYROF IGRZ D QRUPDO VKRFN ZDYH DGMXVWHQWLV UHTXLUHG WK
DFFRPPRGDWWKHUHTXLUHGSURSHUWFKDQJHV
,QVXEVRQF IGRZ WKHUHLVQRWKRUHWFDOOPLWWRWKHDPRXQWRI FRRQQJ DGRZHG 7KHH[WGRZ NX
EHFRPHVVGRZHUDQG VGRZHU DQG WKHWMPSHDXUHDSSURDFKHV]HUR ,QVXSHUYROF IGRZ RQO DILQLW
DPRXQWRI FRRQQJ FDQEHDGGHG EHRUHWKHH[WGRZ DSSURDFKHVLQLQLWM0DFK QXPEHU ZLWK7
DQG WKH H[WMPSHDXUH HTXDO WR]HUR 7KHUH DUH YHU IHZ SUDFWFD
DSSOFDWRQVIRUVXSHUYROF FRRQQJ

(; $0 3/ (

: KDWKDSSHQVWRWKHLQGHWGRZ LQ([DPSOH LI WKHKHDWGGLWRQLVLQFUHDVHGWR N- NJ DQ
WKHLQGHWSUHVXUHDQG VDQGQDWRQWMPSHDXUHDUHIL[HG' : KDWZLOOEHWKHVXEVHTXHQWGHFUHDVHLQPDV
IGRZ"

6ROXWLRQ

)RUT N- NJ WKHH[WZLOOEHFKRNHGDWWKHVXDJQDWRQWMPSHDXUH

7KLVLVKJKHUWKDQWKHYDOXH . LQ([DPSOH VRZHNQRZ WKDWFRQGLWLRQ ZLOOKDY
VR FKRNH GRZQ WR D GRZHU 0DFK QXPEHU 7KH SURPHU YDOXH LV IRXQG IURP WKH UDWL
) URP 7DEOH% RU(T D IRUWKLVFRQGLWLRQ ZHUHDG WKHQHZ GRZHUH
HQJDQH0DFKQXPEHU0D $ § : LWK7 DQGS NQRZQ WKHRWKHULQLWMSURPHUWLVIRGRZ
IURP WKLV0DFKQXPEHU

)LQDOO WKHQHZ GRZHUHGPDWIGRZ SHUXQLWDUHDLV

7KLVLV SHUFHQWWKWVWKDQLQ([DPSOH GXHWRFKRNLQJ E\ H[FHVVKHDWGGLWRQ

5 HODWRQVKLS VR WKH 1 RUP DO 6KRFN : DYH

7KH QRUP DO VKRFN ZDYH UHODWRQ VRI 6HF DFWLRQ OXUNZLWKLQ WKH VLP SOH KHDWQJ UHODWRQ VDV VSHFLDOFDVH) URP 7DEOH % RU) LJ ZHVHH WKDW RUDJLYHQ VDJQDWRQ WHP SHUDWXUH GHW/WKDQ WZR IORZ VDWDW/VDWW\ WKH VLP SOH KHDWQJ UHODWRQ/ RQH VXEVRQF DQG WKH RWKHU SXSHUVRQF 7KH VWI VDWDW/KDYH WKH VDP HYDOXHRI 7 WKH VDP HP DW IORZ SHUXQLWDUHD DQG WKH VDP HYDOXHRI S ! 9 7KHUHRUH WKH VHWR VDWDW/DUH H[DFW HTXLYDOHQWWR WKH FRQGLWRQ/RQ HDFK VLGHRI D QRUP DO VKRFN ZDYH 7KH VHFRQG ODZ ZRXOG DJDLQ HTXLUHWKDW WKH XSWUHDP IORZ 0 D EH VXSHUVRQF 7R LOOXVWUDWH WKLV SRLQWLQ 0 D DQG IURP 7DEOH % UHDC $ DQG S S 1 RZ IRU WKH VDP H YDOX $ XVH 7DEOH % RU (T D WR FRP SXW 0 D DQG S S 7KH YDOXHRI 0 D LV H[DFW ZKDWZ HUHDG LQ WKH VKRFN WDEOH 7DEOH % DV WKH GRZ QWUHDP 0 DFK QXP EHUZ KHQ0 D 7KH SUHVXUH UDWLR IRU WKH VHWR VDWDW/LS S S S S S S ZKLFK DJDLQ LV MXWZ KDWZ HUHDG LQ 7DEOH % IRU0 D 7KLV LOOXVWUDWR QLVP HDQWRQ0 WR VKRZ WKH SK\VLFDO EDFNJURXQG RI WKH VLP SOH KHDWQJ UHODWRQ/ L ZRXOG EH VLOO WR P DNH DSUDFWLFH RI FRP SXWQJ QRUP DO VKRFN ZDYH VLQWKLV P DQQHU

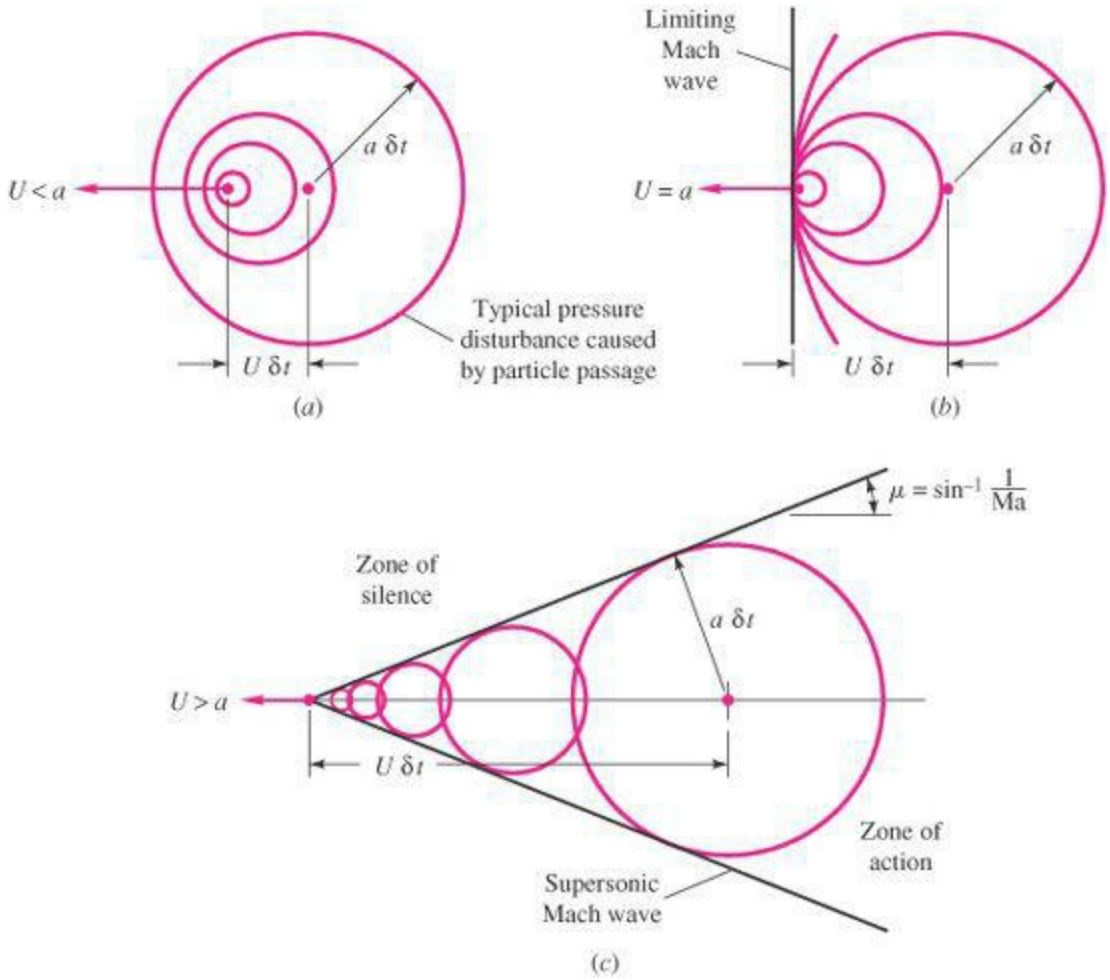
7 ZR ' LP HQVLRQDO 6XSHUVRQLF) ORZ

8 S WR WKLV SRLQWZ H KDYH FRQVLGHUHG RQ0 RQH GLP HQVLRQDO FRP SUHVVLEOH IORZ WKRUHV 7KL LOOXVWUDWG P DQ LP SRUWDQWH IHFW EXW D RQH GLP HQVLRQDO ZRUOG FRP SOHWO ORVHV VLJKWR WKH ZDY P RWRQ/ WKDWDUH VR FKDUDFWHULVWLF RI VXSHUVRQF IORZ 7KHRQ³ ZDYH P RWRQ ZHFRXOG P XVWULQI RQH GLP HQVLRQDO WKRU ZDV WKH QRUP DO VKRFN ZDYH ZKLFK DP RXQWG RQ0 WR DIORZ GLVFRQWQXLW L WKH GXFW

0 DFK : DYHV

: KHQZ HDGG DV FRQG GLP HQVLRQ WR WKH IORZ ZDYH P RWRQ/LP P HGLDWO EHFRP HDSSDUHQW WK IORZ LV VXSHUVRQF) LJXUH VKRZ VDFKDEUDWG JUDSKLFDOFRQWKFWRQ WKDWDSSHDUV LQ HYHU IOL P HFKDQFV WH[WERRN DQG ZDV ILUVWSUHVHQWG E\ (UQVW0 DFK LQ 7KH ILJXUH VKRZ V WKH SDWWHUQ F SUHVXUH GLVWXUEDQFHV VRXQG ZDYHV VHQWRXWE\ D VP DOOSDUWFOH P RYLQJ DWSHHG8 WKURXJK D WXE IOLG ZKRVH VRXQG YHORFLWLVD

$ V WKH SDUWFOH P RYHV LWFRQWQXDOO FUDVKHV DJDLQWWOLG SDUWFOHV DQG VHQGV RXWSKHUFL VRXQG ZDYHV P DQDWQJ IURP HYHU SRLQWDORQJ LWV SDWK \$ IHZ RI WKHVH VSKHUFDOGLWXUEDQFH IURQ DUH VKRZ Q LQ) LJ 7KH EHKDYLRUR WKHVH IURQW LV TXLWH GLIHUHQWDFFRUGLQJ WR ZKHWKHU WK SDUWFOHVSHHGLV VXEVRQF RUVXSHUVRQF



) LJ : DYH SDWWHUQV VHWXS E\ D SDUWLFOH PRYLQJ DW VXSHUVRQLF VSHHG U LQWR VWLOO DLU RU VRXQG YHORFLW\ a a VXEVRQLF b VRQLF DQG c VXSHUVRQLF PRWLRQ

,Q) LJ D WKH SDUWLFOH PRYHV VXEVRQLFDOO 8 D 0 D 8 D 7KH VSKHULFDO GLVWXUEDQFH PRYHV RXWZDUGV DQG GRZQVWUHDP RI WKH SDUWLFOH EXW WKH SDUWLFOH DOR EHFDXVH WKH VWDWH DGMXVWV WKH WLPH LQWHUYDO WR ZKLFK WKH SDUWLFOH KDV PRYHG RQO 8 /W7KHUHRUH D VXEVRQLF ERG\ PRWLRQ PDNHV LWV SUHVHQFH IHOW HYHUZKHUH LQ WKH IORZ ILHOG <RX FDQ³KHDÚ RU³IHHÓ WKH SUHVXUH ULVH RI DQRQFRPLQJ ERG\ EHIRUH LW UHDFKHV \RX 7KLV LV DSSDUHQW ZK\ WKDVSLJHRQ LQ WKH URDG ZLWRXW QLQJ DURXQG WR ORRN DW\RX WDNHV WR WKH DLU DQ DYRLGV EHLQJ KLWE\ \RXUFDU

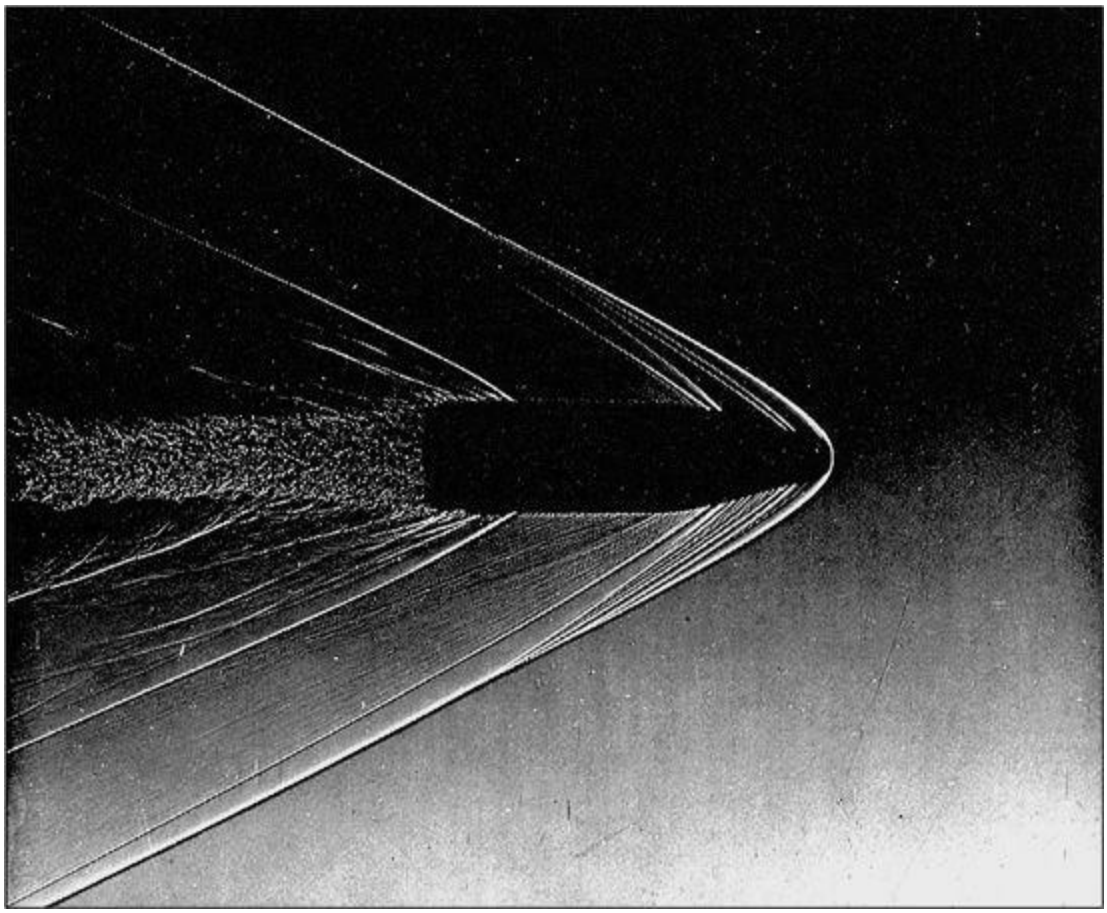
$WWRQLF VSHHG 8 D) LJ E WKH SUHVXUH GLVWXUEDQFHV PRYH DW[DFWO WKH VSHHG RI WKH SDUWLFOH DQG WKXV SLOHXS RQ WKH HWDWKH SRVLWLRQ RI WKH SDUWLFOH LQR D VRUWR ³IURQWORFXV´ ZKLFK LV QRZ FDOOHG D0 DFK ZDYH DWU(UQWW0 DFK 1 RGLVWXUEDQFHU UHDFKHV EH\RQG WKH SDUWLFOH ,I \RX DV WDWRQHG WR WKH HWRI WKH SDUWLFOH \RX FDQRV³KHDÚ WKH RQFRPLQJ PRWRQ ,I WKH SDUWLFOHEHZ LV KRUQ \RX FRXOGQ¶WKHDUWKDWHLWKHU $ VRQLF FDUFDQVQHDNXSRQDSJHRQ

,Q VXSHUVRQLF PRWRQ 8 ! D WKH ODFN RI DGYDQFH ZDUQLQJ LV HYHQ PRUH SURQRXQFHG 7KH GLVWXEDQFHV FDQQRW FDWFK XS ZLWK WKH IDWP RYLQJ SDUWLFOH WKDWFUHDWG WKHP 7KH DOOWDO EHKLQG WKH SDUWLFOH DQG DUH WUQJHQWWR D FRQLFDOOHG0 DFK FRQH) URP WKH JHRPHWU RI) LJ F WKH DQJOH RI WKH0 DFK FRQHLV VHQWR EH

$$\mu = \sin^{-1}\frac{a}{U}\frac{\delta t}{\delta t} = \sin^{-1}\frac{a}{U} = \sin^{-1}\frac{1}{\mathrm{Ma}} \qquad (9.79)$$

7KH KLJKHU WKH SDUWLFOH0 DFK QXPEHU WKH PRUH VHQQHU WKH0 DFK FRQH IRUH[DPSOH LV f D

0 D DQG ƒ DW0 D) RUWKHOPLWQJ FDVHRI VRQLF IORZ 0 D ƒ WKH0 DFK FRQ
EHFRPHVDSSDQHIURQWPRYLQJ ZLWKWKHSDUWFOH LQDJUHPHQWZ LWK) LJ E
<RX FDQQRW?KDÚ WKHGLVWXUEDQFHFDXVHGE\ WKHVXSHUVRQLF SDUWFOHLQ) LJ F XQWO\RX DU
LQWKH]RQHRI DFWRQLQIOXHWKH0 DFKFRQH 1 RZDUQLQJ FDQUHDFK\RXUHDUVL\RX DUHLQWKH]RQHR
VLOHQFHRXWLGHWKHFRQH 7KXVDQREVHUYHURQWKHJURXQGEHQDWKDVXSHUVRQLF DLSODQHGRHVQRWKD
WKHVRQLF ERRP RI WKHSDVVLQJ FRQHXQWLOWKHSODQHLVZHOOSDVW
7KH0 DFK ZDYHQHG QRWEH DFRQH 6LPLODUZDYHVDUHIRUPHGE\ DVPDOOGLVWXUEDQFHR DQ
VKDSHPRYLQJ VXSHUVRQLFDOO ZLWKUHVSHFWWR WKHPHLHQWXLG) RUH[DPSOH WKH³SDUWFOH LQ) LJ
F FRXOGEHWKHHDGLQJHGJHR DVKDUSIODWSODW ZKLFK ZRXOGIRUP D0 DFK ZHGJHR H[DFW WK
VDPH DQJOH 0 DFK ZDYHVDUHIRUPHGE\ VPDOOURXJKQHVVRUERXQGDU\ OD\HULUUHJXODULWLHV LQ
VXSHUVRQLF ZLQGWXQQHORUDWKHVXUIDFHR DVXSHUVRQLF ERG\ / RRNDJDLQDW) LJ 0 DFK ZDYH
DUHFHDUO\ YLVLEOHDORQJ WKHERG\ VXUIDFHGRZQWUHDP RI WKHUHFRPSUHVVLRQVKRFN HVSHFLDOO DWWK
UHDUFRUQHU 7KHUDQJHOHVDERXW ƒ LQGLFDWLQJ D0 DFK QXPEHUR DERXW DORQJ WKLV VXUFH $
PRUHFRPSOHWDWG\VWXP R0 DFK ZDYHVHPDQDWVIURP WKHVXSHUVRQLF SURWHFWOHLQ) LJ 7K
0 DFKDQJOHVFKDQJH LQGLFDWLQJ DYDULDEOHVXSHUVRQLF0 DFKQXPEHUDORQJ WKHERG\ VXUFH 7KHUDQ
DOVRVHYHUDOWXUEXOHQFHVKRFNZDYHVIRUPHGDORQJ WKHVXUFH



) LJ 6XSHUVRQLF ZDYH SDWWHUQ HPDQDWLQJ IURP D SURWHFWOHP RYLQJ DW0 D § 7K
KHDY\ OLQHV DUH REOTXH VKRFN ZDYHV DQG WKH OLJKWOLQHV 0 DFK ZDYHV*(Courtesy of U.S. Army*
Ballistic Research Laboratory, Aberdeen Proving Ground.)

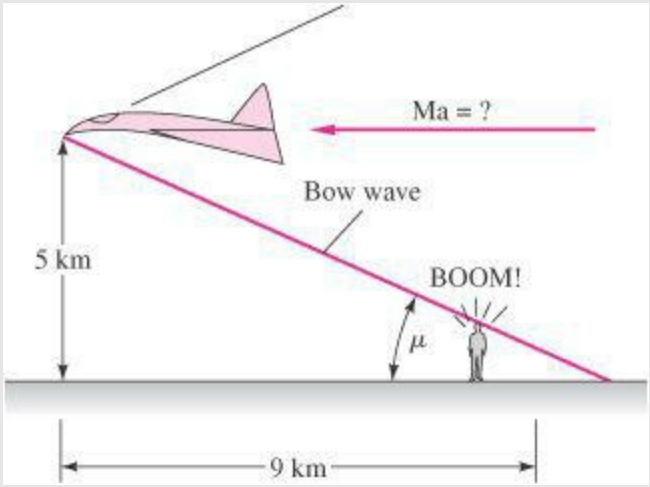
(; $0 3/ (

$QRE VHUYHUURQWKHJURXQGGRHVQRWKHDUWKHVRQLF EDQJ FDXVHGE\ DQDLUSODQHPRYLQJ DW NP DQGWKGHXQLOLWV NP SDWWKHU: KDWLVWKHDSSUR[LPDWH DFKQXPEHURI WKHSODQH' $VXPHDVPD GLVWXUEDQFHDQGQHJOHFWWKHYDULDWLRQ RI VRXQGVSHHGZLWKDOWLWXGH

6RXWLRQ

$ ILQLWHGLVWXUEDQFHQHDQDLUSODQHZLOOFUHDWHDILQLWHVWUHQJWKVKRFNZDYHZKRVHDQJO ZLOOEHVPHZKDWDUJHUWKDQWKH0 DFKZDYHDQJOH DQGZLOOFXUYHGRZQZDUGGXHWRWKH YDULDWRQLQDWPRVSKHULF VRXQGVSHHG ,I ZHQHJOHFWWKHVHHIHFWV WKHDQWXGHDQGGLVWDQFHDPHDVXUHG DVVHQLQ) LJ (7KXV

(



$$\tan \mu = \frac{5 \text{ km}}{9 \text{ km}} = 0.5556 \quad \text{or} \quad \mu = 29.05^\circ$$

+HQFH IURP (T

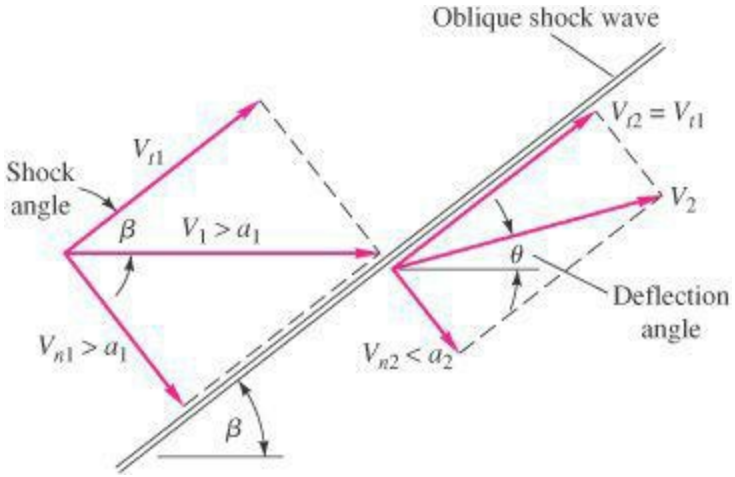
$$\text{Ma} = \csc \mu = 2.06 \qquad \textit{Ans.}$$

7KH2 EQXTXH6KRFN: DYH

) LJXUHV DQG DQGRXUDUHUGLVFXVVLRQDOLQGLFDWWKDWDVKRFNZDYHFDQIRUP DWD REQXTXHDQJOHVR WKHRQHSRLQW VXSHUVRQLF VWUHDP 6XFKDZDYHZLOOGHFHOHUDWHWKHVWUHDP WKURXJK DQJOH XQLNHWKHQRUPDOVKRFNZDYH IRUZKLFKWKHGRZQVWUHDP IORZ LVLQWKHVXEVRQLF GLUHFWLRQ ,(HVWHQFH DQREQXHVKRFNLVFDXVHGE\ WKHQHFHVVLWLIRUDVXSHUVRQLF VWUHDP WRWXUQWKURXJK VXFK DQJOH ([DPSOHVFRXOGEHDILQLWHZHGJHDWWKHOHDGLQJHGJHRI DERG\ DQGDUDPSLQWKHZDOORI VXSHUVRQLF ZLQGWXQQHO

7KHIORZ JHRPHWU RI DQREQXHVKRFNLVVKRZQLQ) LJ $VIRUWKHQRUPDOVKRFNRI) LJ

VWDWH GHQRWHV WKH XSVWUHDP FRQGLWLRQV DQG VWDWH LV GRZQVWUHDP 7KH VKRFN DQJOH KDV D DUELWUDU\ YDOXH DQG WKH GRZQVWUHDP IORZ 9 WKLQV DVVXPHG JUHDWHU WKDQ RQH ZKLFK LV D IXQGDPHQWDO RQH DQG VWDWH FRQGLWLRQV 7KH XSVWUHDP IORZ LV DOZD\V VXSHUVRQLF EXW WKH GRZQVWUHDP 0DFK QXPEHU0 D 9 D PD\ EH VXEVRQLF VRQLF RUVXSHUVRQLF GHSHQGLQJ RQ WKH FRQGLWLRQV



) LJ * HRP HWU RI IORZ WKURXJK DQ REOLTXH VKRFN ZDYH

,W LV FRQYHQLHQW WR DQDO\]H WKH IORZ E\ EUHDNLQJ LWV SLQR RUP DO DQG WDQJHQWLDO FRP SRQHQWV ZL UHVSHFW WR WKH ZDYH DV VKRZ QLQ) LJ) RU D WKLQ FRQWURO YROXP HIXUWHQFRP SDVLQJ WKH ZDYH ZI FDQ WKHQ GHULYH WKH IROORZLQJ LQWHJUDO UHODWLRQV FDOFHOQJ RXV$ $ RQH DFK VLGH RI WKH ZDYH

$$\text{Continuity:} \qquad \rho_1 V_{n1} = \rho_2 V_{n2} \tag{9.80a}$$

$$\text{Normal momentum:} \qquad p_1 - p_2 = \rho_2 V_{n2}^2 - \rho_1 V_{n1}^2 \tag{9.80b}$$

$$\text{Tangential momentum:} \qquad 0 = \rho_1 V_{n1}(V_{t2} - V_{t1}) \tag{9.80c}$$

$$\text{Energy:} \qquad h_1 + \tfrac{1}{2}V_{n1}^2 + \tfrac{1}{2}V_{t1}^2 = h_2 + \tfrac{1}{2}V_{n2}^2 + \tfrac{1}{2}V_{t2}^2 = h_0 \tag{9.80d}$$

: HVHHIURP (T F WKDWWKHUHLVQR FKDQJHLQWDQJHQWLDOYHORFLW\DFURVVDQREOLTXHVKRFN

$$V_{t2} = V_{t1} = V_t = \text{const} \tag{9.81}$$

7KXVWDQJHQWLDOYHORFLW\KDVDVLWVRQO\ HIHFWWKHDGGLWLRQRI DFRQVWDQWWLQHWLF HQHUJ\ $\frac{1}{2}V_t^2$ WR WKHDF VLGH RI WKH HQHUJ\ HTXDWLRQ G : HFRQFOXGHWDW(TV DUHLGHQWLFDOWR WKH QRUP DOVKRFN UHODWLRQV ZLWK9 DQG9 UHSODFHG E\ WKH QRUP DOFRP SRQHQW9$ DQG9 $OWKH YDULRX UHODWLRQVIURP 6HF FDQEH XVHG WR FRP SXWHSURSHUWLHV RI DQREOLTXH VKRFN ZDYH 7KHWDFNLVV XHWKH³QRUP DO 0DFK QXPEHUVLQSODFHRI 0 D DQG0 D

$$\boxed{\mathrm{Ma}_{n1} = \frac{V_{n1}}{a_1} = \mathrm{Ma}_1 \sin\beta}$$

$$\boxed{\mathrm{Ma}_{n2} = \frac{V_{n2}}{a_2} = \mathrm{Ma}_2 \sin(\beta - \theta)} \qquad (9.82)$$

7KHQ IRUD SHUIHFWJDV ZLWKFRQVWDQWVSHFLILF KHDWV WKH SURSHUWUDWRV DFURVV WKH REOLTXHVKRFDUHWKHDQDORJVRI (TV VR ZLWK0 D UHSODFHGE\ 0 DQ

$$\frac{p_2}{p_1} = \frac{1}{k+1}\left[2k\,\mathrm{Ma}_1^2 \sin^2\beta - (k-1)\right] \qquad (9.83a)$$

$$\frac{\rho_2}{\rho_1} = \frac{\tan\beta}{\tan(\beta-\theta)} = \frac{(k+1)\,\mathrm{Ma}_1^2 \sin^2\beta}{(k-1)\,\mathrm{Ma}_1^2 \sin^2\beta + 2} = \frac{V_{n1}}{V_{n2}} \qquad (9.83b)$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)\,\mathrm{Ma}_1^2 \sin^2\beta\right] \frac{2k\,\mathrm{Ma}_1^2 \sin^2\beta - (k-1)}{(k+1)^2\,\mathrm{Ma}_1^2 \sin^2\beta} \qquad (9.83c)$$

$$T_{02} = T_{01} \qquad (9.83d)$$

$$\frac{p_{02}}{p_{01}} = \left[\frac{(k+1)\,\mathrm{Ma}_1^2 \sin^2\beta}{2 + (k-1)\,\mathrm{Ma}_1^2 \sin^2\beta}\right]^{k/(k-1)} \left[\frac{k+1}{2k\,\mathrm{Ma}_1^2 \sin^2\beta - (k-1)}\right]^{1/(k-1)} \qquad (9.83e)$$

$$\mathrm{Ma}_{n2}^2 = \frac{(k-1)\,\mathrm{Ma}_{n1}^2 + 2}{2k\,\mathrm{Ma}_{n1}^2 - (k-1)} \qquad (9.83f)$$

$ OOWKHVH DUH VDEXODWHGLQWKHQRUPDOVKRFN7DEOH% ,I \RX ZRQGHUHG ZK\ WKDWWDEOHDVNHG WK 0 DFK QXPEHUVDV0 DQ DQG0 DQ LWWKRXOGEHFOHDUQRZ WKDWWKHVDEOHLV DOR YDQG IRUWKHREOLTX VKRFNZDYH

7KLQNLQJ DOWKLV RYHU ZHUHDQH ZLWK KLQGVLJKWWKDWDQREOLTXHVKRFNZDYHLV WKHIORZ SDWWL RQHZRXOGREVHUYHE\ UXQQLQJ DORQJ DQRUPDOVKRFNZDYH) LJ DWD FRQVDQWGQHQDOSHG9\ 7KXVWKHQRUPDOGQREOLTXHVKRFNVDUHUHDWGE\ DJDQGHDQ RULQHUWDO YHORFLW\WDQJHQWLDO WR WKHUHRUHVDWW\ WKHVDPHEDVLF HTXDWRQV

,I ZH FRQVLGHU ZLWK WKLV UXQDORJ WKHVKRFN DQDORJ\ ZH ILQG WKDWWKH GHOHFWRQ DQJOH LQFUHDVHV ZLWK SHHG9W XS WR D P[LPXP DQG WKHQ GHFUHDVHV) URP WKH JHRPHWU RI) LJ WK GHOHFWRQDQJOHLVJLYHQE\

$$\theta = \tan^{-1}\frac{V_t}{V_{n2}} - \tan^{-1}\frac{V_t}{V_{n1}} \qquad (9.84)$$

,I ZHGLIIHUHQWDM ZLWKUHVSHFWR9W DQG VHWWKHUHVXOWHTXDOWR]HUR ZHILQG WKDWWKHPD[LPXP GHOHFWRQDQJOHRFFXUVZKHQ9W9Q 9Q 9Q : HFDQVXEVWLWXWHWKLVEDFNLQWR(T VR FRPSXWH

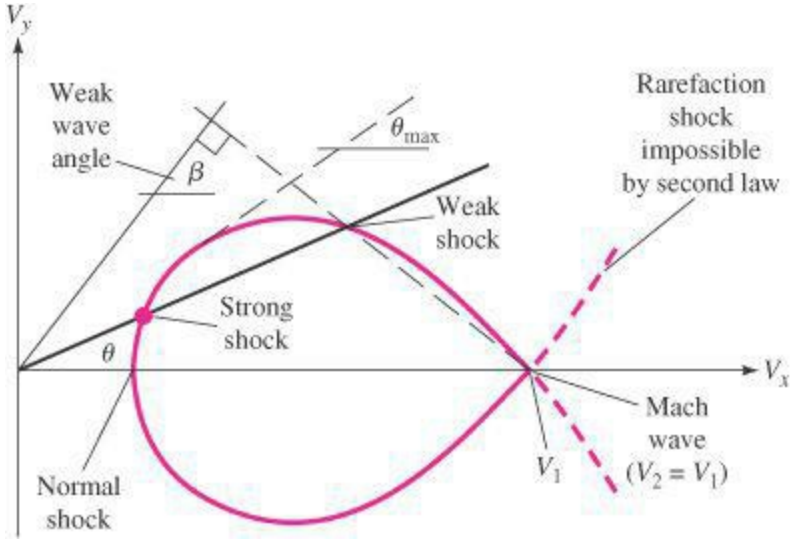
$$\theta_{\max} = \tan^{-1} r^{1/2} - \tan^{-1} r^{-1/2} \qquad r = \frac{V_{n1}}{V_{n2}} \tag{9.85}$$

) RUH[DPSOH LI 0 D IURP 7DEOH % ZH ILQG WKDW 9 9 VKH VTXDUH URRWV ZKLFK LV 7KHQ (T SUHGLFWV DP D[LPXP GHOHFWLRQ RI VDÓ í VDÓ

ƒ 7KH GHOHFWLRQ LV TXLWH OPLWHG IRU LQLQLWH 0 D) URP 7DEOH % IRU WKLV FDVH 9 9 DQG ZH FRPSXWH IURP (T WKDW P D[ƒ

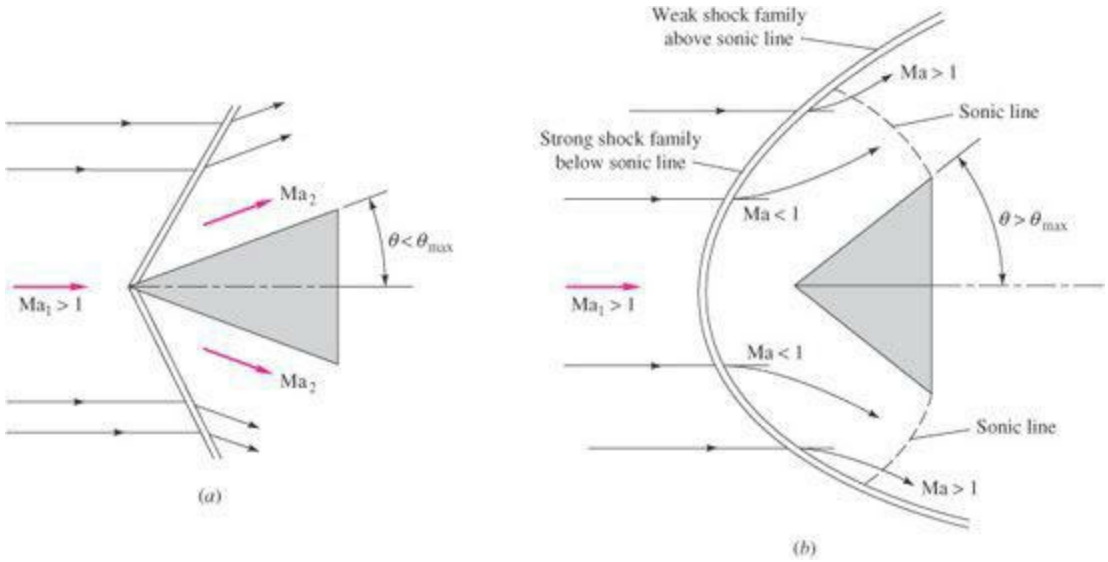
7KLV OPLWDWLRQ LGHDOL]DWLRQ LGHD DQG RWKHU IDFWV EHFRPH PRUH HYLGHQW LI ZH SORW WKH VRXUFH VWUHQJWK RI (TV) RUJLYHQ YDOXHV RI 9 DQG D DV[PLQJ DV XSSDOWKDW ZH FDQ SORW SRVLEOH VROXWLRQV IRU 9 GRZQVWUHDP RI WKH VKRFN) LJXUH GRHV WKLV LQ YHORFLW\ FRPSRQHQW FRRUGLQDWHV 9[DQG 9\ ZLWK[SDUDPHWHUV 9 6XFK D SORW LV FDOOHG D KRGRJUDSK 7KH HDV\ GDUN OLQH WKDW ORRNV OLNH DQ DIDWLQH WUDQVIRUPDWLRQ RI DOO SK\VLFDO SRVLEOH VROXWLRQV IRU WKH JLYHQ 0 D 7KH VZR GDVKHG OLQH ILVKWDLOV DUH VROXWLRQV WKDW LQFUHDVH 9 WKH DUH SK\VLFDO LP SRVVLEOH EHFDXVH WKH\ YLRODWH WKH VHFRQG ODZ

([DPLQLQJ WKH VKRFN SRODUL) LJ ZH VHH WKDW JLYHQ GHOHFWLRQ OLQH RI VP DOO DQJOH FURVVHV WKH SRODUDWZR SRVLEOH VROXWLRQV WKH VWURQJ VKRFN ZKLFK JUHDW GHFHOHUDWV WKH IORZ DQG WKH ZHDN VKRFN ZKLFK FDXVHV D PXFK PLOGHU GHFHOHUDWLRQ 7KH IORZ GRZQVWUHDP RI WKH VWURQJ VKRFN LV DOZD\V VXEVRQLF ZKLOH WKDW RI WKH ZHDN VKRFN LV XVXDOO\ VXSHUVRQLF EXW RFFDVLRQDOO\ VXEVRQLF L WKH GHOHFWLRQ LV ODUJH %RWK WSHV RI VKRFN RFFXULQ SUDFWLFH 7KH ZHDN VKRFN LV P RUH SUHYDOHQW EHFDXVH WKH VWURQJ VKRFN ZLOO RFFXUL WKH UHLV DEORFN DJH RU KJK SUHVVXUH FRQGLWLRQ GRZQVWUHDP



) LJ 7KH REOTXH VKRFN SRODU KRGRJUDSK VKRZ LQJ GRXEOH VROXWLRQV WWRQJ DQG ZHDN IR VP DOOGHOHFWLRQ DQJOH DQG QR VROXWLRQV DWDOO RU ODUJH GHOHFWLRQ

6LQFH WKH VKRFN SRODUL VRQO RI ILQLWH VL]H WKHUH LV P D[LPXP GHOHFWLRQ P D[VKRZ Q LQ) LJ WKDW WXUQ UJH V WKH XSSHU JHR WKH SRODU FXUYH 7KLV YHULILHV WKH NLQHP DWF GLVFXVVLRQ WKDW GH VR (T : KDW KDSSHQV LI D VXSHUVRQLF IORZ LV IRUFHG WR GHOHFWWKURXJK DQ DQJOH JUHDWHU WKD P D[" 7KH DQZHU LV LOOXWUDWHG LQ) LJ IRU IORZ SDWWD ZHGJH VKDSHG ERG\



) LJ 6XSHUVRQLF IORZ SDVWD ZHGJH *a* VP DOO ZHGJH DQJOH DW WDFKHG REOLTXH VKRFN IRUP V *b* ODUJH ZHGJH DQJOH DW WDFKHG VKRFN QRW SRVVLEOH EURDG FXUYHG GHWDFKHG VKRFN IRUP V

, Q) LJ D WKH ZHGJH KDOI DQJOH LV OHVV WKDQ PD[DQG WKXV DQ REOLTXH VKRFN IRUP V DW WKH QRVH RI ZDYH DQJOH IXQFWLRQ FDXVH WKH RQFRPLQJ VXSHUVRQLF VWUHDP WR GHFHOHUDWH WKURXJK WKH ZHGJH DQJOH ([FHSW RU WKH XVXDO VP DOO HIIHFWV RI EORXQGDU\ OD\HU JURZ WK VHH IRU H[DPSOH 5H 6HF ± WKH 0DFK QXPEHU 0 D LV FRQVWDQW DORQJ WKH ZHGJH VXUIDFH DQG LV JLYHQ E\ WKH VROXWLRQ RI (TV 7KH SUHVVXUH GHQVLW\ DQG WHPSHUDWXUH DORQJ WKH VXUIDFH DUH DOVR FRQVWDQW DV SUHGLFWHG E\ (TV : KHQ WKH IORZ UHDFKHV WKH FRUQHU RI WKH ZHGJH LWH[SDQGV V KJKHU 0 DFK QXP EHUDQG IORP VDZ DNH QRVWKRZ Q VLP LODU WR WKDW Q) LJ

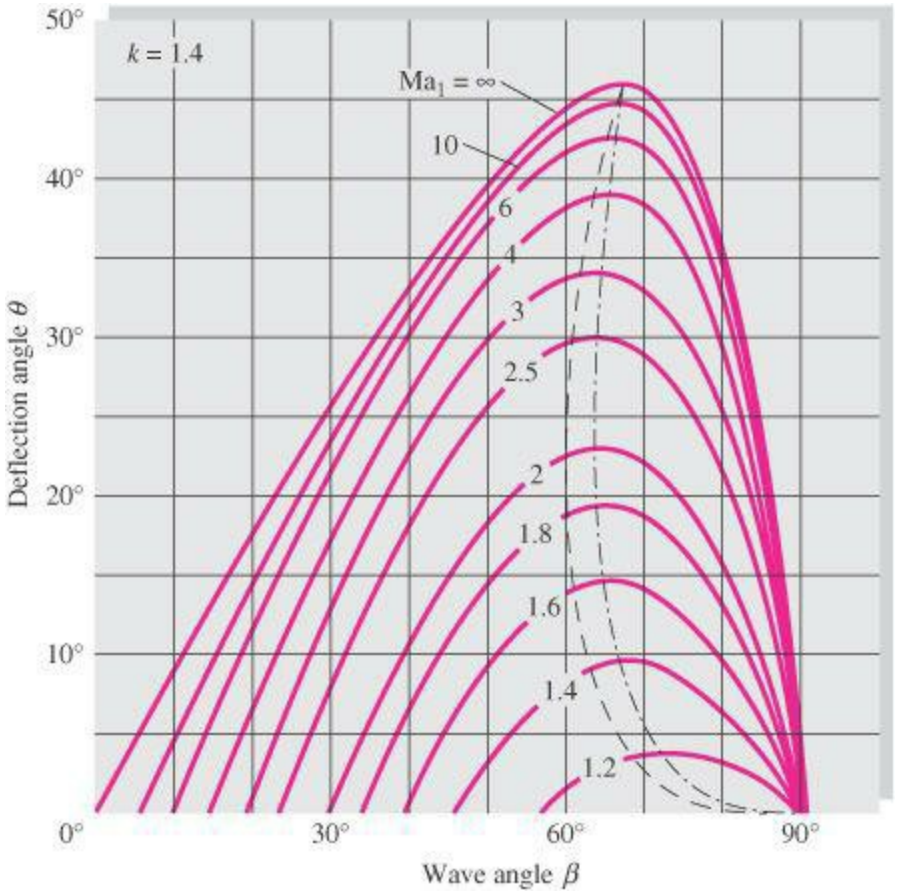
, Q) LJ E WKH ZHGJH KDOI DQJOH LV JUHDWHU WKDQ PD[DQG DQ DWWDFKHG REOLTXH VKRFN L LP SRVVLEOH 7KH IORZ FDQQRW GHFHOHUDWH WKURXJK WKH HQWLUH DQJOH PD[\HW WRP HKRZ WKH IORZ PX JHWDURXQG WKH ZHGJH $ GHWDFKHG FXUYHG VKRFN ZDYH IRUP V LQ IURQW RI WKH EOXQ GLVFRQWLQXRXV GHFHOHUDWLQJ WKH IORZ WKURXJK DQJOHV VP DOOHU WKDQ PD[7KH IORZ WKHQ FXUYHV H[SDQGV DQG GHFHV VXEVRQLF DOO DURXQG WKH ZHGJH EH FRP LQJ VRQLF DQG WKHQ VXSHUVRQLF DV LW VSDVVHV WKH FRUQHU UHGLUHF 7KH IORZ IXUWLQLVLGH HDFK SRLQW RQ WKH FXUYHG VKRFN H[FHSW VDWV WLVI WKH REOLTXH VKRFN UHDWRQ IRU WKDWSDUWFXODUYDOXHRI DQG WKH JLYHQ0 D (YHU FRQGLWRQ DORQJ WKH FXUYHG VKRFN LV SRLQWRQ WKH VKRFN SRODUR) LJ 3RLQWV QHDU WKH IURQW RI WKH ZHGJH DUH LQ WKH VXRQ JVKRFN IDP LO DQG SRLQWV DW WKH VRQLF OLQH DUH LQ WKH ZHDN VKRFN IDP LO 7KH DQDO VLV RI GHWDFKHG VKRFN ZDYHV L H[WHP HO FRP SOH[> @ DQG H[SHUP HQWDWRQ LV XVXDOO QHGHG VXFK DV WKH VKDGRZ JUDSK RSWFD VWFKQLTXH RI) LJ

7KH FRP SOHWH IDP LO RI REOLTXH VKRFN VROXWRQV FDQ EH SORWWHG RU FRP SXWHG IURP (TV) RU D JLYHQ N WKH ZDYH DQJOH YDUH V ZLWK 0 D DQG IURP (T E %\ XVLQJ D WULJRQRP HWU LGHQWLW IRUDQ í WKLV FDQEHUHZ ULWWQLQWKHP RUHFRQYHQLHQWIRUP

$$\tan\theta = \frac{2\cot\beta\,(\mathrm{Ma}_1^2\sin^2\beta - 1)}{\mathrm{Ma}_1^2\,(k + \cos 2\beta) + 2} \qquad (9.86)$$

$ OOSRVVLEOH VROXWRQV RI (T IRUN DUH VKRZ Q LQ) LJ) RUGHWDFKRQV PD[WKHUH DUH WZR VROXWRQV DZHDN VKRFN VP DOO DQG DWURQJ VKRFN ODUJH DV H[SHFWHG $ OOSRLQW DORQJ WKH GDVK±GRWWHG LQH IRU PD[VDWLV\ (T $ GDVKHG OLQH KDV EHHQ GUDZQ WR VKRZ ZKHUH 0 D

LVH[DFWO VRQF : HVHHWKDWKHUHLVDQDUURZ UHJLRQQHDUPD[LPXP GHOHFWLRQZKHUHWKHZHDNVKRFN
GRZQWUHDP IORZ LVVXEVRQLF



) LJ 2 EOTXH VKRFN GHOHFWRQ YHUVXV ZDYH DQJOH IRU YDULRXV XSVWUHDP 0 DFK QXPEHUVk
GDVK±GRWWHG OHFXUYH OHFXV RI $θ PD[GLYLGHV VWRQJ ULJKW IURP ZHDN OHW VKRFNV GDVKH
FXUYH OHFXV RI VRQLF SRLQW GLYLGHV VXEVRQLF 0 D ULJKW IURP VXSHUVRQLF 0 D OHW

) RU]HURGHOHFWRQV VWHZHDNVKRFNIDPLO VDWWLHVWKHZDYHDQJOHUHDWLRQ

$$\beta = \mu = \sin^{-1}\frac{1}{\mathrm{Ma}_1} \qquad (9.87)$$

7KXVZHDNVKRFNVRI YDQLVKLQJ GHOHFWRQDUH HTXLYDOHQWWR 0 DFK ZDYHV 0 HDQZKLOHWKHVWURQ
VKRFNVDOOFRQYHUJHDWWHURGHOHFWRQWRWKHQRUPDOVKRFNFRQGLWLRQ *f*
7ZRDGGLWLRQDOREOTXHVKRFNFKDUWDUHJLYHQLQ$SS %IRUN ZKHUH) LJ % JLYHVWK
GRZQWUHDP 0 DFK QXPEHU0 D DQG) LJ % WKH SUHVVXUHUDWLRS S HDFK SORWWG DV D IXQFWLRQ R
0 D DQG $GGLWLRQDOJUDSKV WDEOHV DQGFRPSXWHUSURJUDPVDUHJLYHQLQ5HV DQG

9HU : HDN6KRFN: DYHV

) RUDQ ILQLWH WKHZDYHDQJOH IRUDZHDNVKRFNLVJUHDWHUWKDQWKH0 DFK DQJOH) RUVP DOO
(T FDQEH H[SDQGHG LQ D SRZHUVHULHV LQ WDQ ZLWK WKH IRGBWLQJ QQH-DU]HG UHVXOWRUWKH
ZDYHDQJOH

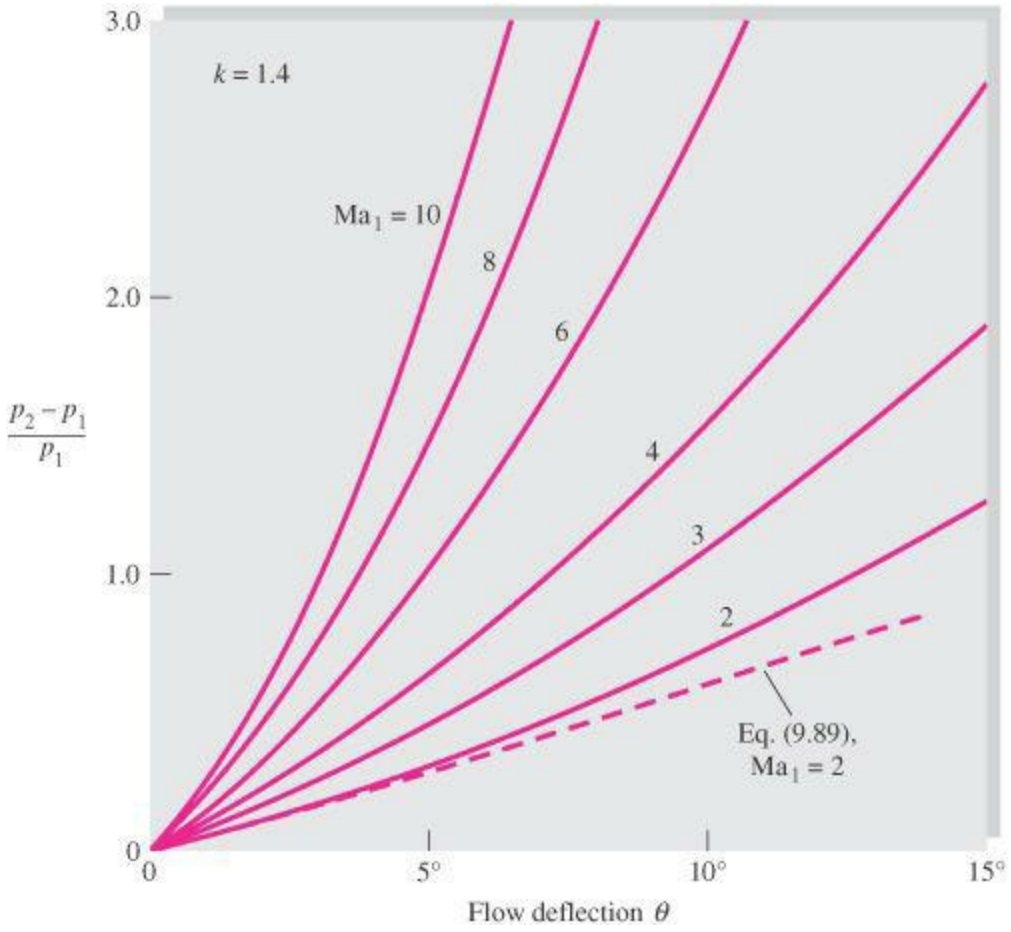
$$\sin\beta = \sin\mu + \frac{k+1}{4\cos\mu}\tan\theta + \cdots + \mathcal{O}(\tan^2\theta) + \cdots \qquad (9.88)$$

)RU0 D EHZHHQ DQG DQGGHOHFWLRQVOHVVKDQ ƒ WKLVUHDWLRQSUHGLFWV WRZLWKLQ ƒ IR DZHDNVKRFN)RUODUJHUGHOHFWLRQVLWFDQEHXVHGDVDXVHXOLQLWLDOJXHVVIRULWHUDWLYHVROXWLRQR (T

2WKHUSURSHUWLHVFKDQJHVDFURVVWKHREOTXHVKRFNFDQDOVREH[SUHVVHGLQDSRZHUVHULHVIR VPDOOGHOHFWLRQDQJOHV 2I SDUWLFXODULQWHUHVWLVWKHSUHVVXUHFKDQJHIURP (T D IRUZKLFKWK QHDU]HURUHVXOWRUDZHDNVKRFNLV

$$\frac{p_2 - p_1}{p_1} = \frac{k\,\mathrm{Ma}_1^2}{(\mathrm{Ma}_1^2 - 1)^{1/2}}\tan\theta + \cdots + \mathcal{O}(\tan^2\theta) + \cdots \qquad (9.89)$$

7KHGLIIHUHQWDOIRUP RI WKLVUHDWRQLVXVHGLQWKHQ[WVHFWLRQWRGHYHORSDWKHRU IRUVXSHUVRQL H[SDQVLRQVWKURXJK) LJXUH VKRZVWKH[DFWZHDNVKRFNSUHVXUHMXPS FRPSXWHGIURP (T D $WVPDOOGHOHFWLRQVWKHFXUYHVDUHQHDUZLWKVORSHVJLYHQE\ (T



)LJ 3UHVVXUHMXPSDFURVVDZHDNREOTXHVKRFNZDYHIURP (T D IRU k)R YHUVPDOOGHOHFWLRQV (T DSSOLHV

)LQDOO LWLVHGXFDWLRQDOWR[DPLQHWKHHQWURS\ FKDQJHDFURVVDYHU ZHDNVKRFN 8VLQJ WK VDPH SRZHU VHULHV H[SDQVLRQ WHFKQLTXH ZH FDQ REWDLQ WKH IROORZLQJ UHVXOWIRU VPDOO GHOHFWLRQV

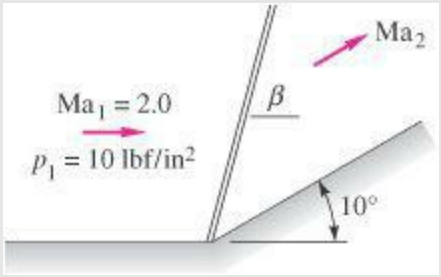
$$\frac{s_2 - s_1}{c_p} = \frac{(k^2 - 1)\mathrm{Ma}_1^6}{12(\mathrm{Ma}_1^2 - 1)^{3/2}} \tan^3\theta + \cdots + \mathcal{O}(\tan^4\theta) + \cdots \qquad (9.90)$$

7KH HQWURS\ FKDQJH LV FXELF LQ WKH GHIOHFWLRQ DQJOH 7KXV Z HDN VKRFN Z DYHV DUH YHU\ QHDU LVHQWURSLF DIDFWKDWLV DOVR XVHGLQWKHQH[WVHFWLRQ

(; $0 3/ (

$ LUDW0 D DQGS OEH LQ DEVROXWHLV IRUFHG WR WKLQ WKURXJK ƒ E\ DUDP SDWKH EROG VXUDFH $ Z HDN REOLTXH VKRFN IRUP VDVLQ) LJ ( ) RUN FRP SXWHIURP H[DFWREOLTXH VKRFN WKHRU D WKH Z DYHDQJOH E 0 D DQG F S $OR XVHWKHOQH DU]HGWKHRU WR HWUP DW G DQ H S

(



6RXWLRQ

EES : LWK0 D DQG ƒ NQRZ Q Z HFDQHWUP DW § ƒ " ƒ IURP) LJ) RUPRU KDQGFDOFXODWG DFFXUDF\ Z HKDYHWR VROYH (T E\ LWHUDWLRQ 2UZ HFDQSURJUDP (T LQ((6 Z LWKVL[VWDWHP HQW LQ6, XQLW Z LWKDQJOHVLQGHJUHHV

0 D
N
7KHWD
1XP 0 DA 6,1 %HWDA ± 7$1 %HWD
' HQRP 0 DA N &26 %HWD
7KHWD $5&7$1 1XP ' HQRP

6SHFLI\ WKDW%HWD! DQG((6 SURPSWV UHSRUWDQDFFXUDWUHVXOV

$$\beta = 39.32^\circ \qquad \textit{Ans. (a)}$$

7KHQRUP DO0 DFK QXP EHUXSVWUHDP LVWKXV

$$0\,D_Q \quad 0\,D\,\mathrm{VLQ} \quad \mathrm{VLQ} \quad f$$

: LWK0 D$ Z HFDQXVHWKHQRUP DOVKRFNUHODWRQV 7DEOH% RU) LJ RU(TV WR

VRFRP SXWH

$$\mathrm{Ma}_{n2} = 0.8031 \qquad \frac{p_2}{p_1} = 1.707$$

7KXVWKHGRZQVWUHDP 0DFK QXPEHUDQG SUHVVXUHDUH

$$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{n2}}{\sin(\beta - \theta)} = \frac{0.8031}{\sin(39.32^\circ - 10^\circ)} = 1.64 \qquad \textit{Ans. (b)}$$

$$p_2 = (10\ \text{lbf/in}^2\ \text{absolute})(1.707) = 17.07\ \text{lbf/in}^2\ \text{absolute} \qquad \textit{Ans. (c)}$$

1 RWHWKDWWKHFRP SXWHGSUHVVXUHUDWLRDJUHHVZLWK) LJV DQG%

) RUWKH OQHDUL]HG WKHRU WKH 0DFK DQJOH LV VLÓ ƒ (TXDWLRQ WKH HXLP DWHVWKDW

$$\sin\beta \approx \sin 30^\circ + \frac{2.4\tan 10^\circ}{4\cos 30^\circ} = 0.622$$

or $$\beta \approx 38.5^\circ \qquad \textit{Ans. (d)}$$

(TXDWLRQ HXLP DWHVWKDW

$$\frac{p_2}{p_1} \approx 1 + \frac{1.4(2)^2\tan 10^\circ}{(2^2-1)^{1/2}} = 1.57$$

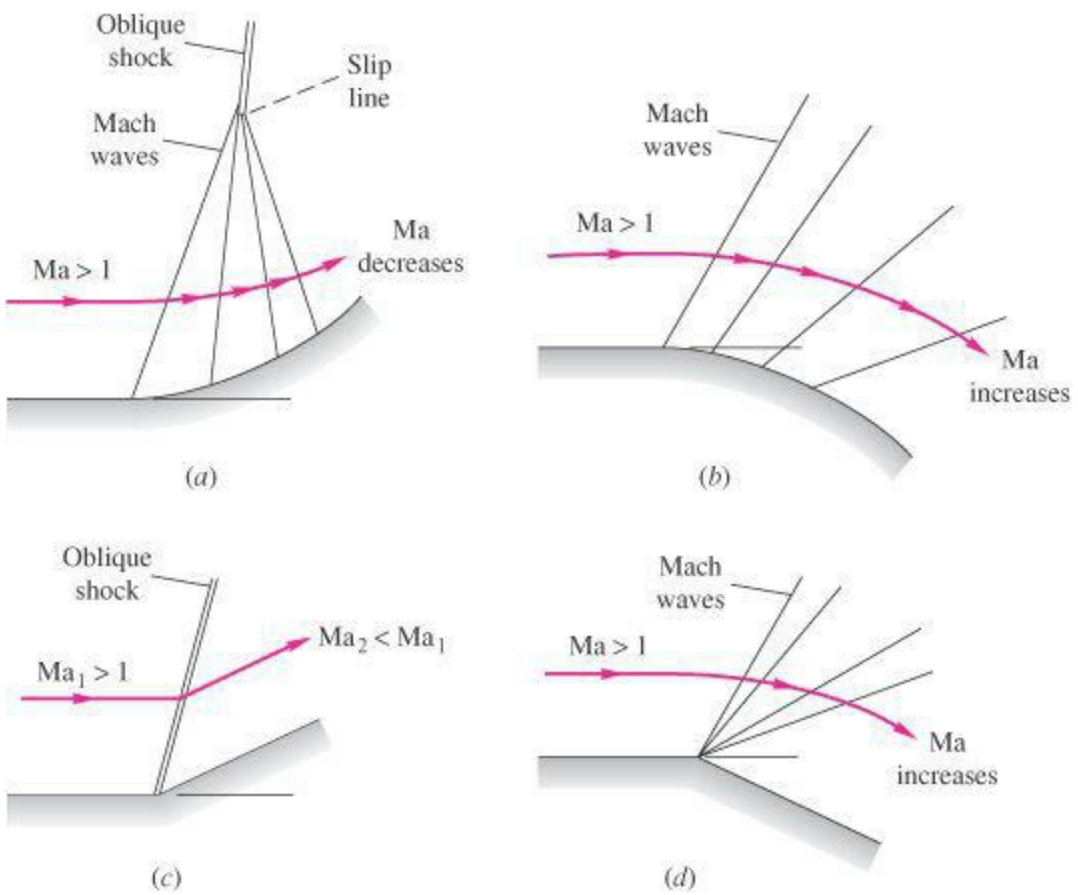
or $$p_2 \approx 1.57(10\ \text{lbf/in}^2\ \text{absolute}) \approx 15.7\ \text{lbf/in}^2\ \text{absolute} \qquad \textit{Ans. (e)}$$

7KHVHDUHUDWRQDEOHHXLP DWHVLQVSLWHRI WKHIDFWWKDW ƒ LVUHDOO QRWVPDOO IORZ GHOHFWLRQ

3UDQGWDOO H\HU([SDQVLRQ: DYHV

7KH REOTXH VKRFN VRXUFH RI 6HF LV IRUD ILQLWH FRP SUHVVLYH GHOHFWLRQ WKDWREVWUFWV VXSHUVRQLF IORZ DQGWKXVGHFUHDVHVLWV0DFK QXPEHUDQG YHORFLW 7KH SUHVHQWH[WRQWHDWJUDGXDO FKDQJHVLQIORZ DQJOHWKDWDUH SUP DULO H[SDQLYH WKH ZLGHQWKHIORZ DUHD DQG LQFUHDVHWKH0DFK QXPEHUDQG YHORFLW 7KH SURSHUWFKDQJHVDFFXPXODWLQLQILQLWHVLPDOLQFUHPHQW DQGWKHQH[W UHDWRQV DQG DUHXVHG 7KHORFDOIORZ GHOHFWRQVDUHLQILQLWHVLPDO VR WKDWWKHIORZ LV QHDUO LVHQWURSLF DFFRUGLQJ WR(T

) LJXUH VKRZ VIRXUH[DPSOHV RQHRI ZKLFK) LJ F IDLOVWKHWHVWVRUJUDGXDOFKDQJHV 7KHJUDGXDOFRP SUHVVLRQ RI) LJ DLVHXHQWDO LVHQWURSLF ZLWK D VPRRWKLQFUHDVHLQSUHVVXU DORQJ WKH VXUIDFH EXWWKH 0DFK QXPEHULQFUHDVHVDORQJ WKH VXUIDFH DQG WKHZDYHVWUQG WR FRDOHVF IDUWKHUDZD\ DQ REOTXH VKRFN ZDYH 7KH JUDGXDO H[SDQVLRQ RI) LJ E FDXVHV D VPRRWLVHQWURSLF LQFUHDVHRI 0DFK QXPEHUDQG YHORFLW DORQJ WKH VXUIDFH ZLWK GLYHUJLQJ 0DFK ZDYHIRUPHG



) LJ 6RPH H[DPSOHV RI VXSHUVRQLF H[SDQVLRQ DQG FRPSUHVVLRQ *a* JUDGXDOLVHQWRSLF FRPSUHVVLRQRQD FRQFDYH VXUIDFH 0DFK ZDYHV FRDOHVFH IDUWKHU RXWWR IRUP REOLTXH VKRFN *b* JUDGXDOLVHQWRSLF H[SDQVLRQ RQ FRQYH[VXUIDFH 0DFK ZDYHV GLYHUJH *c* VXGGHQ FRPSUHVVLRQ QRQLVHQWRSLF VKRFN IRUPV *d* VXGGHQ H[SDQVLRQ FHQWUHG LVHQWRSLF IDQ RI 0DFK ZDYHV IRUPV

7KH VXGGHQ FRPSUHVVLRQ RI) LJ F FDQ QRW EH DFFRPSOLVKHG E\ 0DFK ZDYHV \QREOLTX VKRFN IRUPV DQG WKH IORZ LV QRQLVHQWRSLF 7KLV FRXOG EH ZKDW RX ZRXOG VHH LI \RX ORRNHG DW) LJ D IURP IDUDZD D\ ) LQDOO WKH VXGGHQ H[SDQVLRQ RI) LJ GLVLVHQWRSLF DQG IRUPV D IDQ R FHQWUHG0DFK ZDYHV HP DQDWQJ IURP WKH FRUQHU 1RWWKDWWKH IORZ RQDQ VWHDPOLQH SDVWLQJ WKURXJ WKH IDQ FKDQJHV VPRRWKO WKKLJKHU0DFK QXPEHUDQG YHORFLW ,QWKH OLPLWDV Z HQHDUWKH FRUQHUWK IORZ H[SDQGV DOPRWGLVFRQQQXV DWWKH VXUIDFH 7KH FDVHV LQ) LJ D E DQG FDQ DOOEH KDQGOHG E\ WKH 3UDQGWOO H\HU VXSHUVRQLF Z DYH WKHRU RI WKLV VHFWRQ ILUWWRUPXODWG E\ / XGZ L 3UDQGWODQG KLV WXGHQW7KHRGRU0H\HULQ WR

1RWWKDWQRQH RI WKLV GLVFXVLRQ PDNHV VHQVH LI WKH XSWUHDP 0DFK QXPEHULV VXEVRQLF VLQF 0DFK ZDYHDQG VKRFN ZDYHSDWWHUQV FDQQRW[LWWQ VXEVRQLF IORZ

7KH3UDQGWOO0H\HU3HUH*DV)XQFWLRQ

&RQVLGHUD VPDOO QHJDWLQLQWHQVLW0DOIORZ GHOHFWRQG VXFK DV RFFXUV EHWZHQ WKH ILUWWZR 0DFK ZDYHVLQ) LJ D) URP (TV DQG ZHKDYH LQWKHOLPLW

$$\beta \approx \mu = \sin^{-1}\frac{1}{\mathrm{Ma}} \qquad (9.91a)$$

$$\frac{dp}{p} \approx \frac{k\,\mathrm{Ma}^2}{(\mathrm{Ma}^2-1)^{1/2}}\,d\theta \qquad (9.91b)$$

6LQFH WKH IORZ LV QHDUO LVHQWURSLF WKH IULFWLRQOHVV GLIIHUHQWLDO UHODWLRQ IRU SHUIHFW JDV

$$dp = -\rho V\,dV = -kp\,\mathrm{Ma}^2\frac{dV}{V} \qquad (9.92)$$

&RPELQLQJ (TV D DQG VR HOLPLQDWHV GS ZH REWDLQ D UHODWLRQ EHWZHHQ YHORFLW\ DQG GHQVLW\ FKDQJH

$$d\theta = -(\mathrm{Ma}^2-1)^{1/2}\frac{dV}{V} \qquad (9.93)$$

7KLV FDQ EH LQWHJUDWHG LQWR DQ DXQFWLRQ RI 0D DORQH IRU LQWHUQDO IORZV ZKHUH WKH GHQVLW\ IURP WKH GHILQLWLRQ RI 0D FKQP EHU

$$V = \mathrm{Ma}\,a$$

or

$$\frac{dV}{V} = \frac{d\,\mathrm{Ma}}{\mathrm{Ma}} + \frac{da}{a} \qquad (9.94)$$

) LQDOO ZH FDQ HOLPLQDWH GD E\ FRPELQLQJ (TV DQG KHQFH D LV FRQWDQW IRU SHUIHFW JDV

$$a = a_0[1 + \tfrac{1}{2}(k-1)\,\mathrm{Ma}^2]^{-1/2}$$

or

$$\frac{da}{a} = \frac{-\tfrac{1}{2}(k-1)\,\mathrm{Ma}\,d\,\mathrm{Ma}}{1 + \tfrac{1}{2}(k-1)\,\mathrm{Ma}^2} \qquad (9.95)$$

(OLPLQDWLQJ G9 9 DQG GD IURP (TV VR ZH REWDLQ D UHODWLRQ VROHO\ EHWZHHQ YHORFLW\ DQG 0D

$$d\theta = -\frac{(\mathrm{Ma}^2-1)^{1/2}}{1 + \tfrac{1}{2}(k-1)\,\mathrm{Ma}^2}\frac{d\,\mathrm{Ma}}{\mathrm{Ma}} \qquad (9.96)$$

%HFDXVH LQWHJUDWLQJ WKLV H[SUHVVLRQ ZH QRZ WKDWWKH SUIP DU DSSOLFDWLRQ LV VR H[SDQVLRQ LQFUHDVLQJ 0 D DQG GHFUHDVLQJ 7KHUHIRUH IRU FRQYHQLHQFH ZH GHILQH WKH 3UDQGWO0 H\HU DQJOH & 0 D ZKLFK LQFUHDVHV ZKHQ GHFUHDVHV DQG LV]HUR DW WKH VRQLF SRLQW

$$d\omega = -d\theta \qquad \omega = 0 \qquad \text{at} \qquad \mathrm{Ma} = 1 \qquad (9.97)$$

7KXV ZH LQWHJUDWH (T IURP WKH VRQLF SRLQW WR DQ Y DOXHRI 0 D

$$\int_0^{\omega} d\omega = \int_1^{\mathrm{Ma}} \frac{(\mathrm{Ma}^2-1)^{1/2}}{1 + \tfrac{1}{2}(k-1)\,\mathrm{Ma}^2}\frac{d\,\mathrm{Ma}}{\mathrm{Ma}} \qquad (9.98)$$

7KH LQWHJUDO YDUHHYDOXDWHG LQFORVHG IRUP ZLWK WKH UHVXOWLQ UDGLDQV

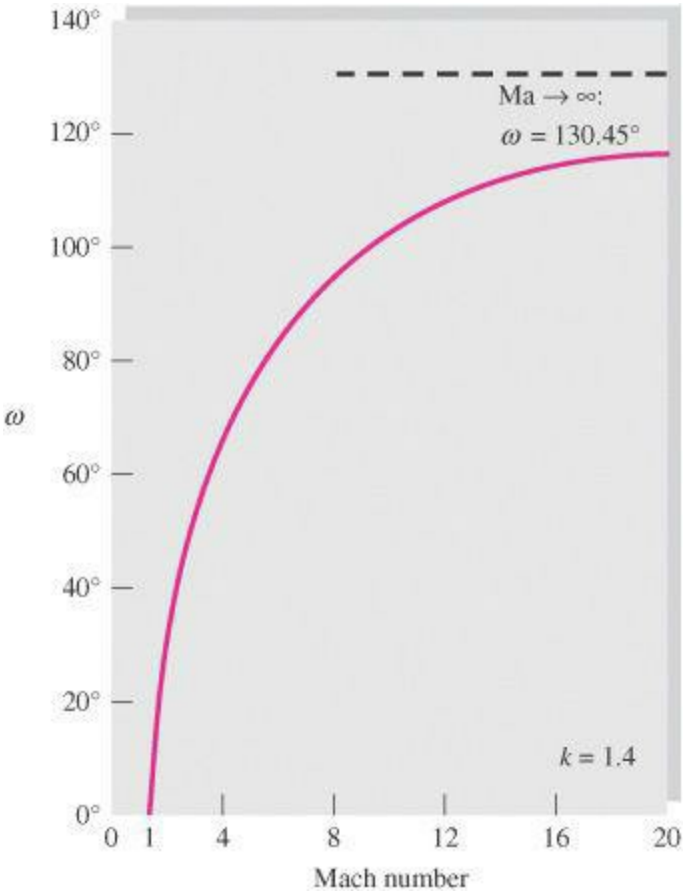
$$\omega(\mathrm{Ma}) = K^{1/2}\tan^{-1}\left(\frac{\mathrm{Ma}^2-1}{K}\right)^{1/2} - \tan^{-1}(\mathrm{Ma}^2-1)^{1/2} \qquad (9.99)$$

where $$K = \frac{k+1}{k-1}$$

7KLV LV WKH3UDQGWO0 H\HU VXSHUVRQLF H[SDQVLRQ IXQFWLRQ ZKLFK LV SORWWHG LQ) LJ DQ
VDEXODWHG LQ7DEOH% IRUN . 7KHDQJOH& FKDQJHVUDSLGO DWLWVWDQGWKHQOHYHOVRI D
KLJK0 DFKQXPEHUWRDOPLWQJ YDOXHDV0 D: ’

$$\omega_{\max} = \frac{\pi}{2}(K^{1/2}-1) = 130.45^\circ \quad \text{if} \quad k = 1.4 \qquad (9.100)$$

7KXV DVXSHUVRQLF IORZ FDQH[SDQGRQO WKURXJK DILQLWHWKLQJ DQJOHEHIRUHLWUHDFKHVLQLQV
0 DFKQXPEHUPD[LPXP YHORFLW DQG]HURWHPSHUDWXUH



) LJ 7KH3UDQGWO0 H\HUVXSHUVRQLF H[SDQVLRQIURP (T IRUk

* UDGXDOH[SDQVLRQRUFRPSUHVVLRQEHWZHHQILQLWO0 DFKQXPEHUV0 D DQG0 D QHLWKHUR ZKLFK
LVXQLW LVFRPSXWHGE\ UHDUUDQJ WKHVXUQLQJ DQJOHû & WRWKHGLIHUHQFHLQ3UDQGWO0 H\HUDQJOHVIRUWK
VWZR FRQGLWLRQV

$$\Delta\omega_{1\to 2} = \omega(\mathrm{Ma}_2) - \omega(\mathrm{Ma}_1) \qquad (9.101)$$

7KHFKDQJHû& PD\ EHHLWKHUSRVLWLYH H[SDQVLRQ RUQHJDWLYH FRPSUHVVLRQ DVGRQJ DVWKHHQ
FRQGLWLRQVOLHLQWKHVXSHUVRQLF UDQJH / HVXVLOXWLDWZLWKDQH[DPSOH

([DPSOH

$LU N IORZVDW0 D DQGS N3D &RPSXWHWKHILQDOGRZQWUHDP 0 DFKQXPEHU
DQGSUHVVXUHIRU D DQH[SDQVLRQWKURXJK f DQG E DJUDGXDOFRPSUHVVLRQWKURXJK f

6RROXWLRQ

3DUWD

7KHLVHQWURSLF VWDJQDWLRQSUHVVXUHLV

$$S \quad S > \qquad @ \qquad N3D$$

DQGWKLVZLOOEHWKHVDPHDVWKHGRZQWUHDP SRLQW) RU0 D ZHILQGIURP 7DEOH% RU(T
WKDW& f 7KHIORZ H[SDQGVWRDQHZ FRQGLWLRQVXFKWKDW

$$\& \quad \& \quad \ddot{} \& \qquad f \quad f \qquad f$$

,QHQWURQRI(T WRILQG0 DZKHQ& LVJLYHQ LVLPSRVVLEOHZLWKRXWLWHUDWLRQ 2QFH
DJDLQ RXUIULHQG((6 HDVLO\ KDQGOHV(T ZLWKIRXUVLJQLILFDQW DQGHVVSHFLILHGLQGHJUHHV

N
& N N± A
2PHJD
2PHJD & $5&7$1 0 DA ± A & ±$5&7$1 0 DA ± A
6SHFLI\ WKDW0 D! DQG((6 UHDGLO UHSRUWVDQDFFXUDWHUHVXOW

$$Ma_2 = 4.32 \qquad Ans.\ (a)$$

7KHLVHQWURSLF SUHVVXUHDWWKLVQHZ FRQGLWLRQLV

$$p_2 = \frac{p_0}{[1 + 0.2(4.32)^2]^{3.5}} = \frac{7347}{230.1} = 31.9\ \text{kPa} \qquad Ans.\ (a)$$

3DUWE

7KHIORZ FRPSUHVVHVWRDORZHU3UDQGWO0 H\HUDQJH

$$\& \qquad f \pm \quad f \qquad f$$

$JDLQIURP (T 7DEOH% RU((6 ZHFRPSXWHWKDW

$$\mathrm{Ma}_2 = 2.125 \qquad \textit{Ans. (b)}$$

$$p_2 = \frac{p_0}{[1 + 0.2(2.125)^2]^{3.5}} = \frac{7347}{9.51} = 773\ \mathrm{kPa} \qquad \textit{Ans. (b)}$$

6LPLODUO ZHFRPSXWHGHQVLWDQGWHPSHUDWXUHFKDQJHVE\ QRWLFLQJWKDW7 DQG! DUHFRQVWDQWVR LVHQWURSLFIORZ

$SSOLFDWLRQVWR6XSHUVRQLF$LUFRLOV

7KHREQTXHVKRFNDQG3UDQGWO0 H\HU[SDQVLRQWKHRULHVFDQEHXVHGWRSDWKWRJHWKHUDQPE-RI LQWHUHVWLQJ DQGSUDFWLFDOXSHUVRQLF IORZ ILHOGV 7KLVPDUULDJH FDOHGVKRFNH[SDQVLRQWKHRU LV OPLWHGE\ VWR FRQGLWLRQV ([FHSWLQUDUHLQVWDQFHVWKHIORZ PXWEHVXSHUVRQLF WKURXJKRXWDQ WKHZDYHSDWWLQPXWQRWVXIHULQWHUHQFHIURP ZDYHVIRUPHGLQRWKHUSDUWVRI WKHIORZ ILHOG

$ YHU VXFFHVVIXODSSOFDWLRQ RI VKRFN H[SDQVLRQ WKHRU LV WR VXSHUVRQLF DLUFRLOV) LJXUH VKRZ VWR H[PSOHV DIODWSODWH DQG DGLPRQG VKDSHG IRLO ,QFRQWDVWWR VXEVRQLF IORZ GHVLJQ) LJ WKHVH DLUFRLOV PXWKDYH VKDUS OHDGLQJ HGJHV ZKLFK IRUP DWWDFKHG REQTXHVKRFNV R H[SDQVLRQIDQV 5RXQGHGVXSHUVRQLF OHDGLQJ HGJHVZRXOGFDXVHGHWDFKHGERZ VKRFNV DVLQ) LJ RU E JUHDWO LQFUHDVLQJ WKHGUDJ DQGORZHULQJ WKHOLW

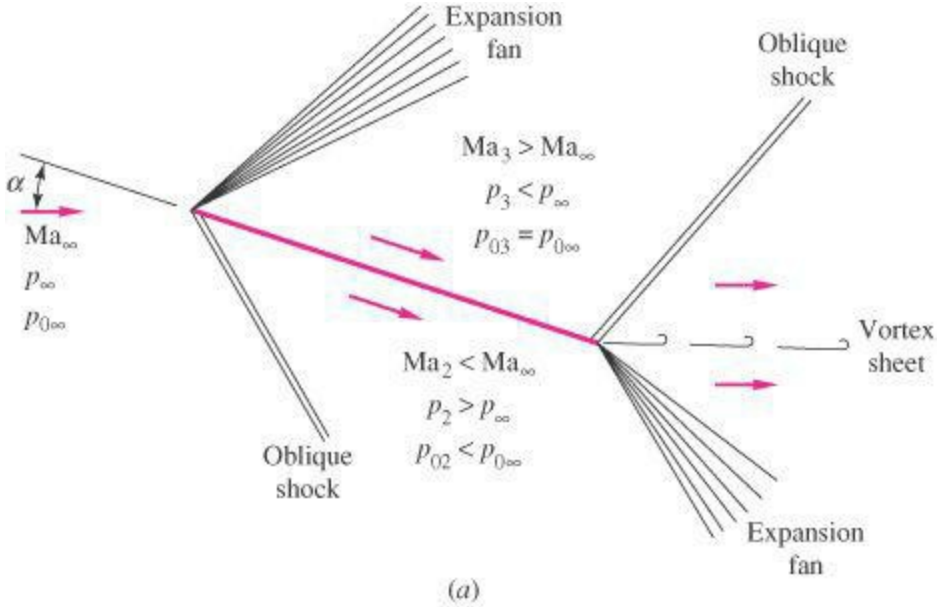
,QDSSOLQJ VKRFNH[SDQVLRQWKHRU RQH[PLQHVHFK VXUFHVWKLQJ DQJOHWRVHHZKHWHUWKH DQH[SDQVLRQ ³RSHQLQJ XS´ RUFRPSUHVVLRQ REVWUFWRQ WKWKHVXUFHIORZ) LJXUH DVKRZ VI IODWSODWH IRLO DWDQ DQJOH R DWWDFN 7KHUH LV D OHDGLQJ HGJH VKRFN RQ WKH ORZ HUHGJH ZLWK IORZ GHOHFWLRQ . ZKLOHWKHXSSHUHGJHKDVDQH[SDQVLRQIDQZLWKLQFUHDVLQJ 3UDQGWO0 H\HUDQJHû& . : HFRPSXWHS ZLWKH[SDQVLRQWKHRU DQGS ZLWKREQTXHVKRFNWKHRU 7KHIRUFHRQWKHSODWHL WKXV) S í S &E ZKHUH& LVWKHFKRUGOHQJWKDQGEWKHVSDQZLGWK DVXVXPLQJ QR ZLQJWLSHIHFWV 7KLVIRUFHLVQRUPDOWR WKHSODWH DQGWKXVWKH OLWRUFHQRUPDOWR WKHXWUHDP LV/) FRV. DQGWKH GUDJ SDUDOOHOWR WKHXWUHDP LV') VLQ. 7KHGLPHQVLRQOHVFRHILFLHQWV&/ DQG& KDYHWKHVDPH GHILQLWLRQVDVLQOBW VSHHGIORZ (TV H[FHSWWKDWWKHSUHFWDVODZ LGHQLW\ $\frac{1}{2}\rho V^2 \equiv \frac{1}{2}kp\,\mathrm{Ma}^2$ LV YHU XVHXCKHUH

$$C_L = \frac{L}{\frac{1}{2}kp_\infty \mathrm{Ma}_\infty^2 bC} \qquad C_D = \frac{D}{\frac{1}{2}kp_\infty \mathrm{Ma}_\infty^2 bC} \qquad (9.102)$$

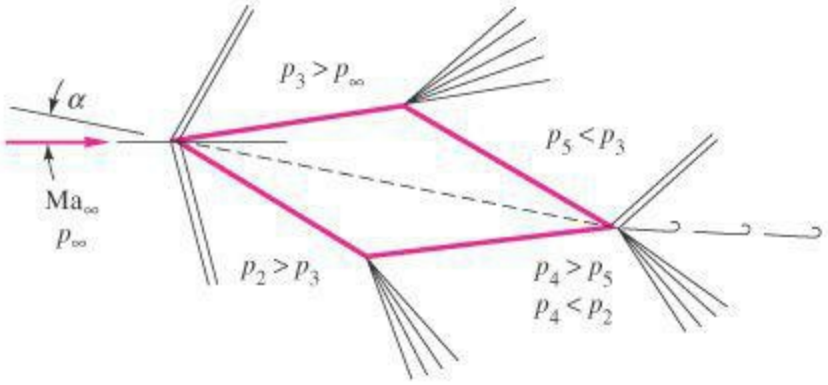
7KHWSLFDOXSHUVRQLF QLWFRHILFLHQWV P XFK VP DOOHUWKDQWKHVXEVRQLF YDOXH&/ § CE EXWWK QLWFDQEHYHU ODUJHEHFDXVHRI WKHOLJHYDOXHR $\frac{1}{2}\rho V^2$ DVXSHUVRQLF VSHHGV

$ WKHWDLOLQJ HGJHLQ) LJ D DVKRFNDQG IDQSSHDULQUHYHUVHG SRVLWLRQV DQGEHQGWKHVZF IORZ VEDFNVR WKDWWKH UHSDUDOOHOLQWKHZDNHDQGKDYHWKHVDPH SUHVXUH 7KH GRQWKDYHTXL

VKH VDPH YHORFLW EHFDXVH RI VKH XQHTXDO VKRFN VWUHQJWKV RQ WKH XSSHU DQG ORZHU VXUIDFHV KHQFH I
YRUWH[VKHHWV DUH NLQG WKH ZLQJ 7KLV LV YHU LQWHUHVWLQJ EXW QRW WKH WDLOORQJ
HGJH SDWWLQ HQWLUHO VLQFH LWV GRVDJHV QRVDIHFWKH VXUIDFH SUHVXUHV 7KH VXSHUVRQLF VXUIDFH IORZ
FDQQRW?KDÚ WKHZ DNHGLWXUEDQFHV



(a)



(b)

) LJ 6XSHUVRQLF DLUIRLOV *a* IODWSODWH KLJKHU SUHVVXUH RQ ORZHU VXUIDFH GUDJ GXH W
VP DOOGRZ QWWHDP FRP SRQHQWV RI QHWSUHVVXUH IRUFH *b* GLDPRQG IRLO KLJKHU SUHVVXUHV RQ ERW
ORZHU VXUIDFHV DGGLWLRQDOGUDJ GXHWR ERG\ WKLFNQHVV

7KH GLDPRQG IRLOLQ) LJ E DGGV VZR PRUH ZDYH SDWWHUQV WR WKH IORZ $WWKLV SDUWFXODU.
OHWV WKDQ WKH GLDPRQG KDO DQJOH WKHUH DUH OHDGLQJ HGJH VKRFNV RQ ERWK VXUIDFHV WKH XSSHU VKRF
EHLQJ PXFK ZHDNHU 7KHQ WKHUH DUH H[SDQVLRQ IDQV RQ HDFK VKRXOGHU RI WKH GLDPRQG 7KH 3UDQGW
0 H\HUDQJOH FKDQJHû& HTXDOV WKH VXP RI WKH OHDGLQJ HGJH DQG WUDLOLQJ HGJH GLDPRQG KDO DQJOH
) LQDOO WKH WUDLOLQJ HGJH SDWWHUQ LV VLPLODU WR WKDW RI WKH IODWSODWH D DQG FDQ EH LJQRUHG LQ WK
FDOFXODWLRQ %RWK ORZHU VXUIDFH SUHVXUHV S DQG S DUH JUHDWHU WKDQ WKH XSSHU FRQVWDQW DQG WK
QIWLVQHDU WKDW WKH IODWSODWH 7KHUHLV DQ DGGLWLRQDOGUDJ GXH WR WKLFNQHVV EHFDXVH S DQG S RQ WK
WUDLOLQJ VXUIDFHV DUH ORZHU WKDQ WKH XSSHU VXUIDFH S DQG S 7KH GLDPRQG GUDJ LV JUHDWHU WKDQ WKH ID
SDWWHUQ GUDJ EXW WKLV PXVW EH HQG XUHG LQ SUDFWLFH WR DFKLHYH D ZLQJ VWUXFWXUH VWURQJ HQRXJK WR KROG WKH P
IRUFHV

7KH WKHRU VNHWFK HG LQ) LJ LV LQJRRG DJUHHPHQW ZLWK PHDVXUHG VXSHUVRQLF QLWDQG GUDJ D

ORQJ DV WKH 5H\QROGV QXPEHUV QRW WRR VPDOO WKLFN EHXQGDU OD\HUV DQG WKH 0DFK QXPEHUQRW WRR ODUJH K\SHUVRQLF IORZ ,WWKLQV RXW WKDW UDJH 5H& DQG PRGHUDWH VXSHUVRQLF 0D WKH EHXQGDU OD\HUV DUH WKLQ DQG VHSDUDWH RQ HDFK RWKHU RFFXU VR WKDW WKH VKRFN IURP HDFK RI DOWHUQDWH RU DOWHUQDWH RU LVTXLWH VXFFHVVIXO/ HWXV ORRN QRZ DW DQH[DPSOH

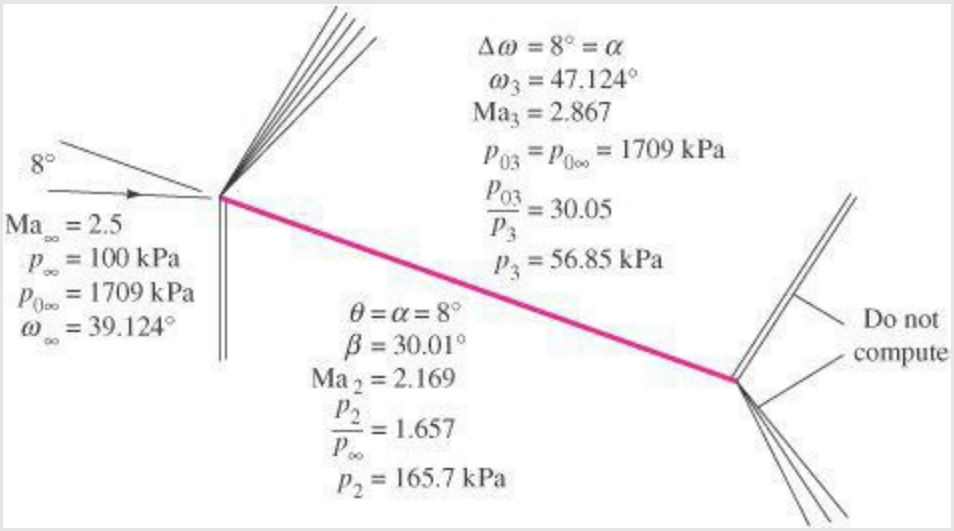
(; $0 3/ (

$ IODWSODWH DW DLUROOZ LW& P LVLPPHUVHG DW f LQD VXUHDP ZLW0 D DQGS NBD &RP SXWH D &/ DQG E & DQG FRP SDUHZ LWK OHZ VSHHG DLUROOV &RP SXWH F OLWDQG G GUD LQQHZ WRQ/SHUXQLWSDQZ LGWK

6RO XWLRQ

,QWRDGR XLQJ D ORWR VSDFH RXOQLQJ WKH GHVLOHG REOTXHVKRFN DQG 3UDQGWO0 H\HUH[SDQVLR FRP SXDWLRQ/ ZH OLWDOO SHUWLQHQWUHXOW LQ) LJ (RQ WKH XSSHUDQG OHZ HUVXUFHV 8VLQJ WK WKHRUHVRI 6HFV DQG \RX VKRXOG YHUI\ HYHU VLQJOHRQHRI WKHFDOFXODWRQVLQ) LJ (V PDNHVXUH WKDWDOOGHVLOVRI VKRFNH[SDQVLRQ WKHRU DUHZ HOXQGHUWRRG

(



7KHLPSRUWDQWLQDOUHVXOWDUHS DQGS IURP ZKLFK WKHVRODOIRUFHSHUXQLWZ LGWKRQ WKHSODWHLV

) S ± S E& ± NBD P P NL

7KHOLWDQGGUDJ SHUP HWUZ LGWKDUHWKXV

$$L = F\cos 8° = 216\ \text{kN} \qquad \textit{Ans. (c)}$$

$$D = F\sin 8° = 30\ \text{kN} \qquad \textit{Ans. (d)}$$

7KHVHDUHYHU\ODUJHIRUFHVIRUDQD P RI Z LQJ DUHD
) URP (T WKHOLWHUDWXUHILFLHQWV

$$C_L = \frac{216\ \text{kN}}{\frac{1}{2}(1.4)(100\ \text{kPa})(2.5)^2(2\ \text{m}^2)} = 0.246 \qquad \textit{Ans. (a)}$$

7KHFRPSDUDEOHORZ VSHHGFRHILFLHQWURP (T LV&/ CEVLQ f ZKLFK LV
VPHVODUJHU
) URP (T WKHGUDJFRHILFLHQWV

$$C_D = \frac{30\ \text{kN}}{\frac{1}{2}(1.4)(100\ \text{kPa})(2.5)^2(2\ \text{m}^2)} = 0.035 \qquad \textit{Ans. (b)}$$

) URP) LJ IRUWKH1 $&$ DLUIRLO&. DW f LVDERXW RUDERXW VPHVVPDOOHU
1 RWH WKDWWKLV VXSHUIRQLF WKHRU SUHGLFWV DILQLWH GUDJ LQ VSLWH RI DXXPLQJ IUFWLRQOHVV IORZ
ZLWKLQILQLWHZLQJ DSHFWDWLR 7KLVLVFDOOHGZDYHGUDJ DQGZHVHHWKDWWKHG¶$OPEHUWSDUDGR[F
]HURERG\ GUDJGRHVQRWRFFXULQVXSHUVRQLF IORZ

7KLQ $LUIRLO7KHRU\

,Q VSLWH RI WKH VLPSOLFLW RI WKH IODWSODWH JHRPHWU WKH FDOFXODWLQ([DPSOH ZHU
ODERULRXV ,Q $FNHUHW> @GHYHORSHG VLPSOH\HWHIHFWLYH H[SHUWLRQ IRU WKH OLWGUDJ DQ
FHQWUL SUHVXUHR VXSHUVRQLF DLUIRLOV DXXPLQJ VPDOOWKLFNQHVDQGDQJOHRI DWWDFN
7KH WKHRU LV EDVHG RQ WKH OLQHDUL]HG H[SHUWLRQ ZKHUH VDq § VXUDFH GHOHFWLR
UHODWLYHWRWKHIUHHVWUHDP DQGFRQGLWLRQ LVWKHIUHHVWUHDP 0 D 0 D) RUWKHIODWSODWHDLUIRLOWK
VWUDLJKWRFRUH) LVEDVHGRQ

$$\frac{p_2 - p_3}{p_\infty} = \frac{p_2 - p_\infty}{p_\infty} - \frac{p_3 - p_\infty}{p_\infty}$$
$$= \frac{k\,\text{Ma}_\infty^2}{(\text{Ma}_\infty^2 - 1)^{1/2}}[\alpha - (-\alpha)] \qquad (9.103)$$

6XEVWLWXWLQJQR(T JLYHVWKHOLQHDUL]HGOLWFRHILFLHQWRUDVXSHUVRQLFIODWSODWHDLUIRLO

$$C_L \approx \frac{(p_2 - p_3)bC}{\frac{1}{2}kp_\infty \text{Ma}_\infty^2 bC} \approx \frac{4\alpha}{(\text{Ma}_\infty^2 - 1)^{1/2}} \qquad (9.104)$$

&RPSXWDWLRQV IRU GLDPRQG DQG RWKHU ILQLWHWKLFNQHVV DLUIRLOV VKRZ QR ILWWRUGHU HIIFWR
WKLFNQHVVRQOLW7KHUHRUH(T LVYDOLGIRUDQ VKDSHGJHGVXSHUVRQLF WKLQDLUIRLOGWD VPDOO
DQJOHRI DWWDFN
7KHIODWSODWHGUDJ FRHILFLHQWV

$$C_D = C_L \tan\alpha \approx C_L\alpha \approx \frac{4\alpha^2}{(\mathrm{Ma}_\infty^2 - 1)^{1/2}} \qquad (9.105)$$

+RZ HYHU WKH WKLFNHU DLUIRLO VKDYH DGGLWLRQDO WKLFNQHVV GUDJ / HVWKHFKRUG GQHR WKH DLUIRLOE WKH[D[LV DQG GHWHUPLQHV WKH XSSHU VXUIDFH VKDSHEHGHQRWHG E\ \X [DQG WKH ORZHU SURILOHE\ \O[7KHQ WKH FRPSOHWH \$FNHUHWGUDJ WKRU VHH 5H 6HF IRU GHWDLOV VKRZV WKDWWKH DGGLWLRQDO GUDGHSHQGVRQWKHPHDQVTXDUHRI WKHVORSHVRI WKHXSSHUDQG ORZHUVXUIDFHV GHILQHGE\

$$\overline{y'^2} = \frac{1}{C}\int_0^C \left(\frac{dy}{dx}\right)^2 dx \qquad (9.106)$$

7KHILQDOH[SUHVVLRQIRUGUDJ > S @LV

$$C_D \approx \frac{4}{(\mathrm{Ma}_\infty^2 - 1)^{1/2}}\left[\alpha^2 + \frac{1}{2}(\overline{y_u'^2} + \overline{y_l'^2})\right] \qquad (9.107)$$

7KHVHDUHDOLQHDUSRQDEOHDJUHPHQWZLWKPRUHH[DFWPRPXDWRQV DQG WKHLUH[WHPHVLPSOLFLV PDNHVWKHP DWWDFWLYHDOWHUQDWLYHVWRWKHOEERURXVEXWDFFXUDWVKRFNH[SDQVLRQWKRU &RQVLGHUWK IRQOWRZLQJH[DPSOH

(; \$0 3/ (

5HSHDWSDUW D DQG E RI ([DPSOH XVLQJ WKHOQHDUL]HG\$FNHUHWKHRU

6RXWLRQ

)URP (TV DQG ZHKDYH IRU0 D DQG. *f* UDG

$$C_L \approx \frac{4(0.1396)}{(2.5^2 - 1)^{1/2}} = 0.244 \qquad C_D = \frac{4(0.1396)^2}{(2.5^2 - 1)^{1/2}} = 0.034 \qquad \textit{Ans.}$$

7KHVHDUHOWVWKDQ SHUFHQWORZHUWKDQWKHPRUHH[DFWPRPXDWRQVRI ([DPSOH

\$ IXUWKHU UHYLVLRQ WKH\$FNHUHWOQHDUL]HG WKRU LV DQ H[SUHVVLRQ IRUWKH SRVLWLRQ[&3 RI WK FHQWHURI SUHVVXUH &3 RI WKHIRUFHGLVWULEXWLRQRQWKHZLQJ

$$\frac{x_{CP}}{C} = 0.5 + \frac{S_u - S_l}{2\alpha C^2} \qquad (9.108)$$

ZKHUH6X LVWKH FURVVVHFWLRQDODUHDEHWZHHQWKHXSSHUVXUIDFHDQG WKHFKRUDQG6O LVWKHDUH EHWZHHQWKHFKRUDQG WKHORZHUVXUIDFH) RUDV\PPHWULF DLUIRLO6O 6X ZHREWDLQ[&3 DWWKHKDOI FKRUG SRLQWLQFRQQHFWZLWKWKHORZ VSHHGDLUIRLOUHVXOWV (T ZKHUH[&3 LV DWWKH TXDUWL

FKRUG

7KH GLIIHUHQFH LQ GLIILFXOW\ EHWZHHQ WKH VLPSOH $FNHUPDQQ DQG VKRFN H[SDQVLRQ WKHRU L
HYHQJUHDWHU IRU DWKLFNDLURLO DV WKH IRQRZ LQJ H[DPSOHVKRZ V

(; $0 3/ (

%\ DQDORJ\ ZLWK([DPSOH DQDO]HG GLDPRQG RUGRXEOH ZHGJH DLUIRLORI ƒ KDO DQJOH DQ
& P DW ƒ DQG0 D &RPSXW&/ DQG& E\ D VKRFN H[SDQVLRQ WKHRU DQG E $FNHU
WKHRU 3LQSRLQWWKHGLIIHUHQFHIURP ([DPSOH

6RXWLRQ

3DUWD

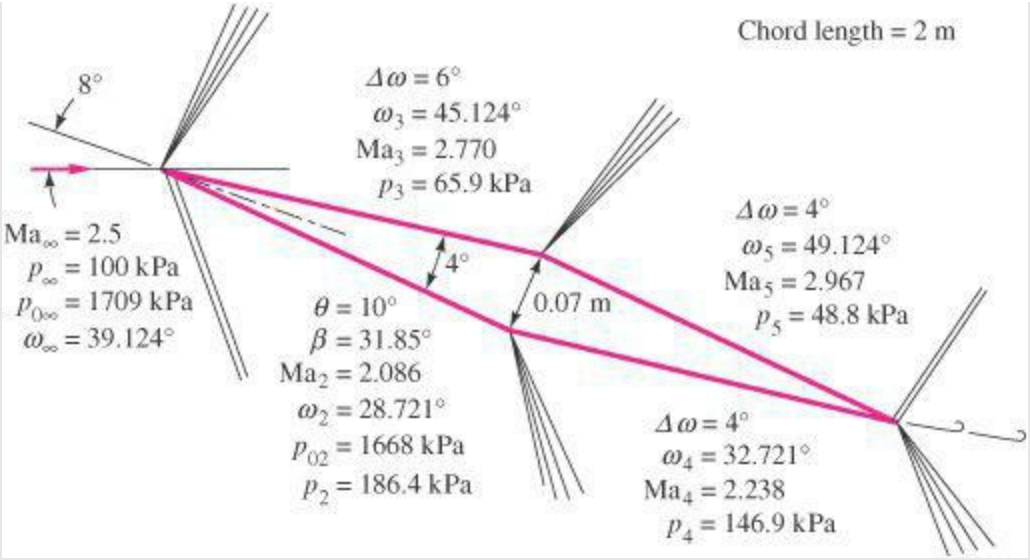
$JDLQZHRPLWWKHGHWDLOVRI VKRFN H[SDQVLRQ WKHRU DQG VLPSO OLWWKH SURSHUWLHV FRPSXWHG R
HDFKRI WKHIRXUDLUIRLOVXUIDFHVLQ) LJ ($VXPHS N3D 7KHUHEHWKDIRUFH) QRUPD
WR WKHFKRUGOLQHDQGDIRUFH3 SDUDOOHOWR WKHFKRUG) RUWKHQRUPDOIRUFHWKHSUHVVXUHGLIIHUHQFHR
WKHIURQWKDOLVS í S í N3D DQGRQWKHUHDUKDOLWLVS í S í

N3D 7KHDYHUDJHSUHVVXUHGLIIHUHQFH½ N3D VRWKDWWKHQRUPDOIRUFHLV

) N3D P N1

) RUWKHFKRUGZLVHIRUFH3 WKHSUHVVXUHGLIIHUHQFHRQWKHVRS KDOLVS í S í
N3D DQGRQWKHERWWRP KDOLWLVS í S í N3D 7KHDYHUDJHGLIIHUHQFH½
N3D ZKLFKZKHQPXOWLSOLHGE\ WKHIURQWDODUHD PD[LPXP WKLFNQHVVWLPHV P ZLGWK
JLYHV

3 N3D P P N1



(

%RWK) DQG3 KDYH FRP SRQHQWV LQ WKH GLUDJ GLUHFWLRQV 7KH GLVWRUWHG RUP DOWR WKH IUH VWUHDP LV

/) FRV $f \pm 3$ VLQ f N1

') VLQ f 3 FRV f N1

DQG

) RU FRP SXWLQJ WKH FRHILFLHQWV WKH GHQRPLQDWRU (T LV WKH VDPH DV LQ ([DPSOH 9.19: $\frac{1}{2}kp_\infty \mathrm{Ma}_\infty^2 bC = \frac{1}{2}(1.4)(100\text{ kPa})(2.5)^2(2\text{ m}^2) = 875\text{ kN}$. 7KXV ILQDOO VKRFNH[SDQVLRQ WKHRU SUHGLFWV

$$C_L = \frac{216.2\text{ kN}}{875\text{ kN}} = 0.247 \qquad C_D = \frac{32.4\text{ kN}}{875\text{ kN}} = 0.0370 \qquad \textit{Ans. (a)}$$

3DUW E

0 HDQZ KLOH E\ $FNHUHWKRU & LV WKH VDPH DV LQ ([DPSOH

$$C_L = \frac{4(0.1396)}{(2.5^2 - 1)^{1/2}} = 0.244 \qquad \textit{Ans. (b)}$$

7KLV LV SHUFHQWOHVV WKDQ WKH VKRFNH[SDQVLRQ UHVXOWDERYH) RUWKH GLUDJ Z HQHG WKH PHDQ VTXDUHVORSHV IURP (T

$$\overline{y_u'^2} = \overline{y_l'^2} = \tan^2 2° = 0.00122$$

7KHQ (T SUHGLFWVWKLVQDU]HGUHVXOW

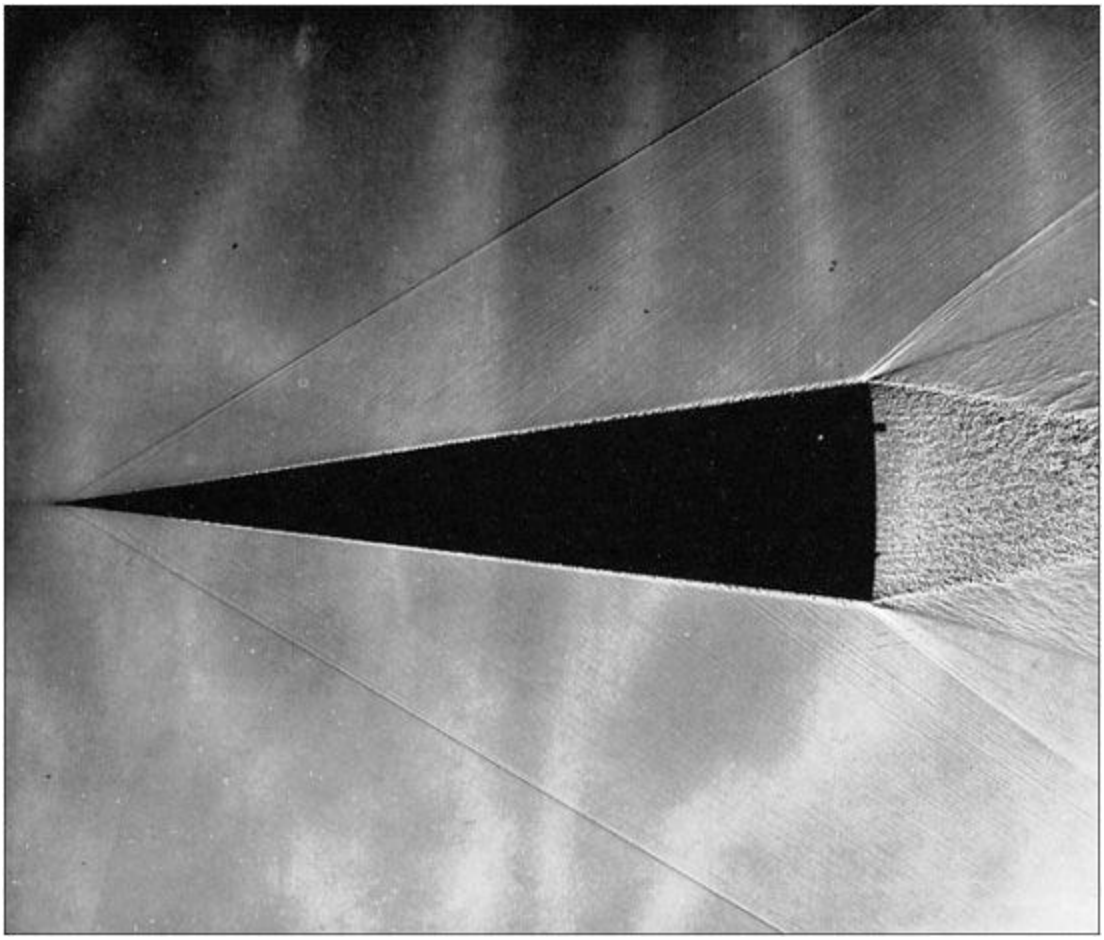
$$C_D = \frac{4}{(2.5^2 - 1)^{1/2}}\left[(0.1396)^2 + \tfrac{1}{2}(0.00122 + 0.00122)\right] = 0.0362 \qquad Ans.\ (b)$$

7KLV LV SHUFHQWDJH HUURU EHFDXVH WKH FRUUHFW SURFHGXUH : HFRXOG MXGJH$FFHSWDEOH EH³VDWLVIDFWRU\´$FFHSWDEOH SURFHGXUHS NBD í SHUFHQWS NBD í SHUFHQWS NBD í SHUFHQWDQGS NBD í SHUFHQW

7KUHH' LPHQVLRQDO6XSHUVRQLF)ORZ

: HKDYH JRQH DERXW DV IDU DV ZH FDQ JR LQ DQ LQWURGXFWRU\ WH[WERRN FRPSUHVVLEOH IORZ 2 FRXUVH WKHUH LV PXFK PRUH DQG \RX DUH LQYLWHG WR VWXG\ IXUWKHU LQ WKH UHIHUHQFHV DVWKH HQG RI WK FKDSWHU

7KUHH GLPHQVLRQDO VXSHUVRQLF IORZ VDUHKJKO FRPSOH[HVSHFLDOO\ LI WKH\ FRQFHQWUDWH RQ ZKFK WKHUH RUH FRQGLQHPEHGGHG UHJLRQV RI VXEVRQLF DQG VWDQRQLF IORZ DV LQ) LJ 6RP IORZ V KRZ HYHU \LHOG WR DFFXUDWH WKHRUHWFDOWHQWXFK DVIORZ SDWWHUQ DW\HRLQFLGHQFH D VKRZ Q LQ) LJ 7KH H[DFWWKHRU RI FRQH IORZ LV GLVFXVVHG LQ DGYDQFHG WH[WV>RUH[DPSH &KDS @DQG H[WHQVLYH WDEOHV RI VXFK VROXWLRQV KDYH EHHQ SXEOLVKHG > @7KHUH DUH VLPLODULW EHWZHHQ FRQH IORZ DQG WKH ZHGJH IORZ VLQXXDDG LQ) LJ DQ DWWDFKHG REOTXH VKRFN D WKL VXUIXOHQWERXQGDU\ FD\HU DQG DQH[SDQVLRQ IDQ DWWKHUHDUFRUQHU +RZ HYHU WKH FRQFDOVKRFNGHOHFV WKH IORZ WKURXJK DQDQJOHOWVWKDQ WKH FRQHKDO DQJOH XQDNHWKHZHGJHVKRFN $VLQWKHZHGJHIORZ WKHUHLV DPD[LPXP FRQHDQJOHIRUYHZKFK WKHVKRFNPXWGHWDFK DVLQ) LJ E)RUN DQ 0D ' WKH PD[LPXP FRQH KDO DQJOH IRU DQ DWWDFKHG VKRFN LV DERXW ƒ FRPSDUHG Z LWK WK PD[LPXP ZHGJHDQJOHRI ƒ VHH5H



) LJ 6KDGRZJUDSK RI IORZ SDWWDQ ƒ KDOI DQJOH FRQH DW0 D 7KH VXUEXOHQ
ERXQGDU\ OD\HU LV FOHDUO\ YLVLEOH 7KH 0DFK QXPEHUV VOJKWO\ DQG WKH 0DFK QXPEHU YDULH
IURP MXVWLQVLGH WKH VKRFN WR DWWKH VXUIDFH*(Courtesy of U.S. Army Ballistic Research*
Laboratory, Aberdeen Proving Ground.)

7KH XVH RI FRPSXWDWLRQDO IOXLG G\QDPLFV &)' LV QRZ YHU SRSXODU DQG VXFFHVVIXO L
FRPSUHVVLEOH IORZ VWXGLHV> @) RU[DP SOH D VXSHUVRQLF FRQH IORZ VXFK DV) LJ HYHQ DW
DQJOHV RI DWWDFN FDQ EH VROYHG E\ QXPHULFDO PXOWLRQ RI WKH IXOO WKUHH GLPHQVLRQDO YLVFRXV 1DYLH
6WRNHV HTXDWLRQV> @
) RUP RUH FRP SOLFDWHG ERG\ VKDSHV RQH XVXDOO\ UHVRUWV WR H[SHULPHQWDWLRQ LQ D VXSHUVRQLF ZLQ
VXQQHO) LJXUH VKRZV D ZLQG VXQQHO WXG RI VXSHUVRQLF IORZ SDWWD P RGHORI DQ LQWHUFHSWR
DLUFUDIW7KHP DQ MXQFWLRQV DQG ZLQJVSDQG VKDSHFKDQJHV P DNH WKH RUHWLFDODQDOVLV YHU GLIILFXO
+ HUH WKH VXUIDFH IORZ SDWWHUQV ZKLFK LQGLFDWH ERXQGDU\ OD\HU GHYHORSP HQWDQG UHJLRQV RI IORZ
VHSDUDWLRQ KDYH EHHQ YLVXDOL]HG E\ WKH VP HDULQJ RI RLOGURSV SODFHG RQWKH P RGHOVXUIDFH EHRUH WK
WHVW

$ VZ HVKDOOVHH LQ WKH QH[WFKDSWU WKHUH LV DQ LQWHUHVWLQJ DQDORJ\ EHWZHHQ JDV G\QDP LF VKRF
ZDYHV DQG WKH VXUIDFH ZDWHU ZDYHV WKDW RFFXU LQ DQ RSHQ FKDQQHO IORZ &KDSWU R 5H H[SODLQ
KRZ DZ DWUFKDQQHOFDQ EH XVHG LQ DQ LQH[SHQVLYH VXSHUVRQLF IORZ H[SHULP HQW

1HZ 7UHQGV LQ $HURQDXWLFV

7KH SUHYLRXV HGLWLRQ RI WKLV WH[WGLVFXVVHG WKH ; -RLQW6WUNH) LJKWHU DQG VWR DLUDQHV WK
$ LUEXV$ DQG WKH %RHLQJ ' UHDP OLQHU 7KH %RHLQJ KDV EHHQ GHD\HG E\ DZ RUN HUVWUN

DQG E\ ZHJKWDQG FRPSRVLWH DLUFUDIW OLPPPDV 7KH ZLOOQRWXDUWHDYHUHVXQO DWWK
HDUDHW7KH$LUEXV$ KDVDOR HQFRQWHUHG SURGXFWLRQGHDV EXWKDVDWWDW FXWRPHUVDQG
DVR WKLVGDW KDVGHDYHUHG SDQHV)LQDOO WKH; -RLQW6WULNH)LJKWHULFDXJKWQDFRWEOQ
H[DFHUEDWGE\ WKHZRUOGZLGHFRQF VORZGRZQ 7KH2EDPDDGPLQLVWDWLRQZLOOKDYHWRGHFLG
KRZ WR FRQSURPLVH EHWZHHQ WKH; DQG WKH VSHGLHU FRXQWHU DQG PRUH PDQHXYHUDEOH)
ILJKWHU

1$6$ LVH[SORULQJ DQDHURQDXWDOGUHDP² WRIO IRUH[DPSOH IURP 1HZ <RUN WR7RN\RLQVWF
KRXUV ,QRWKHUZRUGV WKH DUHGHVLJQLQJ DK\SHUVRQLF 0D DLUSODQH FDOOHG WKH; $ > (
&RQVHQVRQDO WXUERMHWDQG UDPMHWHQJLQHV DUH YHU LQHILFLHQWDWXFK KJK VSHHGV 7KH SURSRVH
VROXWLRQ LV DVFUDPMHWVXSHUVRQLF FRPEXWLRQ UDPMHW$QDUWLVWVNHWFK RI WKH DLUFUDIWLV VKRZQ LQ
) LJ

) LJ 7KH SURSRVHG; $ K\SHUVRQLF DLUSODQH 7KH VFUDPMHWHQJLQHLV XQGHUQHDWK $I
SDVLQJ WKURXJK WKHHQJLQHUHP DLQVIXOO VXSHUVRQLF *(Courtesy of the X-43A Development Team,
DFRC, NASA.)*

8QDNH FRQYHQWLRQDO MHWHQJLQHV WKH VFUDPMHWDGP LWV VXSHUVRQLF DLU ZLWKRXWFDXLQJ VKRFN
ZDYHV DQGDOR DFKLHYHVVXSHUVRQLF FRP EXWLRQDQG H[KDXVWDWXSHUVRQLF VSHHGV 7KHGUDZEDFNL
WKDWVFUDPMHWDUHH[WUHPHO LQHILFLHQWDWORZ VSHHGV VXFK DVWDNHRI DQG FOLPELQJ 7HVWVWRGDWR
WKH; $ KDYHEHHQDFFRPSOLVKHGE\ GURSSLQJ WKHSDQHIURP D% ERPEHUDQG WKHQDFFHOHUDWQ
LWZ LWKD 3HJDVXV URFNHW: KHQWKHVFUDPMHWWLFNVLQ VSHHGV JUHDWHUWKDQ0 DFK KDYHEHHQUHDFKHG
7KLVLVWZLFHWKHVSHHGRI WKHUFRUGVHWE\ WKH$LU)RUFH65 %DFNELGGLQ

6XPP DU\

7KLV FKDSWHU EHJLQV WR GHYHORS EURDG VXEMHFW FRPSUHKHQVLRQ VR PDQ\ PHWKRGV FDQ EH GLVFXVVHG LQ D ZKLFK LV DJHDQGFDXVHVWKH0 DFKQPEHU0 D 9 D ZKLFK LV DJHDQGFDXVHVWKHIOXLG GHQVLWVRYDU VLJQLILFDQWO 7KLVPHDQVWKDWWKHFRQWLQXLWDQGPRPHQWXP HTXDWLRQVPXWEHFRXSOHG VRWKHHQHUJ\ UHDWLRQDQGWKHHTXDWLRQ VWDWVRVROYHIRUWKHIRXUXQNQRZQV S ! 7 9

7KHFKDSWHU UHYLHZHGWKHWKHUPRG QDPLF SURSHUWLHV DQ LGHDOJDV DQG GHULYHG DIRUPXODIR WKHVSHGRI VRXQG DIOXLG 7KHDQDOLVZDVWKQVLPSOLILHGWRQHGLPHQVLRQDOWADG DGLDEDW IORZ ZLWKRXWWKDWZRUN IRUZKLFK WKH VWDJQDWLRQ HQWKDOS\ RI WKH JDV LV FRQVWDQW\ IXUWKH VLPSOLFDWLRQ WR LVHQWURSLF IORZ HQDEOHV IRUPXODV WR EH GHULYHG IRUKJK VSHG JDV IORZ LQ YDULDEOH DUHDGXFW7KLVUHYHDOVWKHSKHQRPHQRQRI VRQLF IORZ FKRNLQJ PD[LPXP PDVIORZ LQWK WKURDWRI DQR]]OH\$VXSHUVRQLF YHORFLWHVWKHUHLVWKHSRVVLELOLWRI DQRUPDOVKRFNZDYH ZKHUHWK JDV GLVFRQWLQXRXV0 UHYHUVH VXEVRQLF FRQGLWLRQV 7KHQRUPDOVKRFN H[SODLQVWKH HIFWRI EDFN SUHVVXUHQWKSHUPDQFHRI FRQYHUJLQJ±GLYHUJLQJ QR]]OHV

7RLOOXVWUDWHQQLVHQWSLF IORZ FRQGLWLRQV WKHFKDSWHU EHJLQ0 IRFXVHG RQ FRQWDQWDUHD GXF IORZ ZLWKIULFWLRQDQGZLWKKHDWWUDQVIHU ERWKRI ZKLFKOHDGWRFKRNLQJ RI WKH[LWIOW

7KHFKDSWHUHQGHGZLWKDGLVFXVVLRQRI WZR GLPHQVLRQDOXSHUVRQLF IORZ ZKHUHEQTXHVKRFN ZDYHVDQG3UDQGWO0 H\HU LVHQWURSLF H[SDQVLRQZDYHVSSHDU: LWKDSURPHUFRPELQDWRQRI VKRFN DQGH[SDQVLRQVRQHFDQDQDO]HVXSHUVRQLF DLUIRLOV

SUREOPV TXHVWLRQV3UREOPV

0 RVWRI WKHSUREOPVKHUHQDUHIDLU VWDLJKWRUZDUG 0 RUHGLIILFXOWURSHQHGHGDVLJQPHQ DUHODEHOHGZLWKDQDVWHULN 3UREOPVODEHOHGZLWKQ((6 LFRQ ZLOOEHQHILWURP WKHXVHRI WK (QJLQHHULQJ (TXDWLRQ6ROYHU ((6 ZKLOHSUREOPVODEHOHGZLWKDFRPSXWHULFRQ PD\ UHTXLU WKHXVHRI DFRPSXWU 7KHVWDQGDUG HQGRI FKDSWHUSROPV3 VR3 FDWURU]HG LQ WK SUREOPP OLWKHUDUHIRGHG E\ ZRUGSROPV: VR: IXQGDPHQWDOVRI HQJLQHHULQJ H[DF SROPV)(VR)(FRPSUHKHQVLYHSROPV& VR& DQG GHVLJQ SURMHFW' DQ '

3UREOPP ' LVWULEXWLRQ

Section	Topic	Problems
9.1	Introduction	P9.1–P9.9
9.2	The speed of sound	P9.10–P9.18
9.3	Adiabatic and isentropic flow	P9.19–P9.33
9.4	Isentropic flow with area changes	P9.34–P9.53
9.5	The normal shock wave	P9.54–P9.62
9.6	Converging and diverging nozzles	P9.63–P9.85
9.7	Duct flow with friction	P9.86–P9.106
9.8	Frictionless duct flow with heat transfer	P9.107–P9.115
9.9	Mach waves	P9.116–P9.121
9.9	The oblique shock wave	P9.122–P9.139
9.10	Prandtl-Meyer expansion waves	P9.140–P9.148
9.10	Supersonic airfoils	P9.149–P9.157

,QWURGXFWRQ

- 3 $QLGHDOJDV IORZ VDGLDEDWFDOO WKURXJK DGXFW$ WWHFWRQ S NBD 7 *f*& DQG9 P V)DUKHUGRZQWUHDP S NBD DQG7 *f*& &DFXOD 9 LQP VDQGV í V LQ- NJ . LI WKHJDVLV D DLU N DQG E DUJRQ N

- 3 6RQH3URE 3 LI WKHJDVLVWDP 8VHWRDSSURDFKHV D DQLGDOJDVIURP 7DEO $ DQG E UHDOJDVGDWDIURP WKHWDP WDEOHV> @
- 3 ,I NJ RI R[\JHQLQDFORVHGWDQNDW *f& DQG N3DLVKHDWGXQWOWKHSUHVVU ULVHVWR N3D FDOFXODWH D WKHQHZ WHPSHUDWXUH E WKHWRWDONHDWWDQVHU DQG F WKHFKDQJHLQHQWURS\
- 3 &RQVLGHUWDG\ DGLDEDWF DLUBRZ LQDGXFW\$VWHFWRQ% WKHSUHVXUHLV N3DDQ WKHGHQLWLV NJ P \$VWHFWRQ' WKHSUHVXUHLV N3DDQGWKHWHPSHUDWU LVí *f& D)LQGWKHHQWURS\ FKDQJH LI DQ E: KLFKZD\ LVWKHDLUIRZ LQJ"
- 3 6WDP HQWLV D QR]]OH DW *f& 0 3D DQG D WDUG\ VSHHG RI P V DQ DFFHOHUDWHV LVHQWURSLFDOO XQWLOWH[WDWWDLQPHQW FRQGLWLRQV (WKP DWWKH H[L YHORFLWDQGWHPSHUDWXUH
- 3 8VH((6 RWKHUVRIWZDUH RUWKHJDVWDEOHVWRHWKDQH DQGF\ WKHLUUDWR DQGWKHL GLIHUHQFH IRU&2 DW . DQG FRPSDUH ZLWK LGHDOJDV HWKDQH VLPLODUWR(TV
- 3 \$LUIORZ VWKURXJK DYDULDEOHDUHDGXFW\$VWHFWRQ \$ FP S N3D! NJ P DQG9 P V \$VWHFWRQ WKHDUHDLV H[DFWO WKHVDPH EXWWK GHQLWLVPXFKORZHU! NJ P DQG7 . 7KHUHLVQRWDQ\HURI ZRU RUKHDW\$VXPHRQHGLPHQVLRQDOVWDG IORZ D +RZ FDQ\RX UHFRQFLOH WKHV GLIHUHQFHV" E)LQGWKHPDVIORZ DVWHFWRQ &DOFXODWH F 9 G S DQG H V í V +LQW7KLVSUREOP UHTXLUHVWKHFRQWQXLWHTXDWRQ
- 3 \$VVRVSKHULF DLUDW *f& HQWHUV DQG ILOOV DQLQXODUGVDQNWKDWV LQLWDOO HYDFXDWHG 8VLQJ DFRQWROYROXPHDQDOVLV IURP(T FRPSXWWKHVDQN DLUWWPSHUDWU ZKHQLWVIXOO
- 3 /LTXLG K\GURJHQDQG R[\JHQ DUHEXUQHGLQDFRPEXWRQFKDPEHUDQG IHGWKURXJK URFNHWQR]]OHWKDW[KDXWV DV9 H[LW P VWRDQDPELHQWSUHVVXUHRI N3D 7K QR]]OH[LWGLDPHWHULV FP DQGWKHNWW[LWGHQLWLV NJ P ,I WKHH[KDXWJD KDVDPRODFXODUZHJKWRI HWXP DW D WKH[LWJDVWPSHUDWXUH E WKHPDVIORZ DQG F WKHWKUXVWGHYHORSHGE\ WKHURFNHW

7KHVSHHGRI VRXQG

- 3 \$ FHUWDLQDLUFUDIWDWOHVDWWKHVDPH0 DFKQXPEHUUHJDUGOHVVRI LWWDOWLWXGH &RPSDUH VR LWV VSHHG DW P VDQGDUG DOWLWXGH LWOHV NP K IDVWHUDWWHDOHYHO ' HWHUPLQHLWV0 DFKQXPEHU
- 3 \$W *f& DQG DW HWXP DWWKHVSHHGRI VRXQGRI D QLWURJHQ E K\GURJHQ F KHOXP G VWDP DQG H 8) N§
- 3 \$VXPHWKDWZDWIRROBRZ V(T ZLWKQ§ DQG%§ &RPSXWWKHEXO PRGXOXV LQN3D DQGWKHVSHHGRI VRXQG LQP V DWD DW DQG E DW WK GHSHQGSDUWRI WKHRFHDQ F &RPSXWWKHVSHHGRI VRXQGDW *f& DQG DW DQGFRPSDUHZLWKWKHPHDVXUHGYDOXHRI P V \$ + 6PLWKDQG\$: /DZVRQ - &KHP 3K\V YROO S
- 3 &RQVLGHUVWDP DW . DQG N3D (VWP DWLWVSHHGRI VRXQGE\ WKUHHGLIHUH

PHWKRGV D XVLQJ WKH KDQG QHZ ((6 WKHUPRSK\VLFDO IXQFWLR
6281' 63((' 6WDPS S 7 7 E DVXPLQJ DQLGHDOJDVIURP 7DEOH%
RU F XVLQJ ILQLWHGLIIHUHQFHVIRUVHQVRSLF GHQVLWLHVEHWZHHQ N3D DQG
N3D

- 3 $W DW DQG *f& WKH GHQVLWLV RI PHWKODOBRKROLV GEP IW $W DW LV
GHQVLW LQFUHDVHV WR GEP IW 8VH WKLV GDWD WR HVWLPDWH WKH VSHHG RI VRXQG
&RPPHQWRQ WKH SRVVLEOHXQFHUWDLQWLHV LQ WKLVHVWLPDWH
- 3 7KH SUHVVXUH GHQVLW UHODWLRQ IRUHWKDQROLV DSSUR[LPDWHG E\ (T ZLWK%
DQGQ 8VH WKLV UHODWLRQ WR HVWLPDWH WKH VSHHG RI VRXQG RI HWKDQROLW
DW RSHUHV
- 3 $ ZHDN SUHVXUHSXOVH ûS SURSDJDWHV WKURXJK WKH DLU' LVFXVV WKH WSHRI UHOFWH
SXOVH WKDW RFFXUV DQG WKH EHQGDU FRQGLWLRQ WKDW PXVW EH VDWLVILHG ZKHQ WKH ZDY
VWULNHV QRUPDO WR DQG LV UHOFWHG IURP D D VROG ZDOO DQG E D IUHH OTXL
VXUDFH
- 3 $ VXEPDULQH DW D GHSWK RI P VHQGV D VRQDU VLJQDO DQG UHFHLYHV WKH UHOFWH
ZDYHEDFN IURP DVLP LODUVXEPHUJHG REMHFWQ V 8VLQJ 3URE 3 DV D JXLGH
HVWLPDWH WKH GLVWDQFH WR WKH RVWHURBMHFW
- 3 5DFH FDUV DW WKH ,QGLDQDSROV 6SHHGZD\ DYHUDJH VSHHGV RI PLK $IW
GHWHUPLQLQJ WKH DQJXODU RI ,QGLDQDSROV ILQG WKH 0DFK QXPEHUV WKHVH FDUV DQ
HVWLPDWH WKHWUFRPSUHVVLELOLW\ FRHIFLHQW WIHFWKLUDHURG\QDPLFV

$GLDEDWF DQG LVHQWURSLF IORZ

- 3 ,Q WKH 65 $ IOLQJ DW NP VDQGDUG DOWXGH VHWD MHWSRZHUHG DLUFUD
\SHHGUHFRUG RI NP K (VXP DW WKH WRP SHUDWXUH LQ*f& DWLV IURQWXDQDWR
SRLQW$W WKDW0 DFK QXPEHUZRXOG LWKDYH DIURQWXDQDWRQ SRLQW WP SHUDWXUH F
*f&"
- 3 $LUIORZ VLVHQWURSLFDOO LQDFKDQQHO 3URPHUWLV DW WHFWRQ DUH9 P V7
. DQGS N3D $WWHFWRQ GRZQWUHP WKH WP SHUDWXUH KDV GURSSHG W
*f&) LQG D WKHSUHVXUH E YHORFLW DQG F 0 DFK QXPEHUDW WHFWRQ
- 3 &2 H[SDQGV LVHQWURSLFDOO WKURXJK DGXFWURP S N3D DQG7 *f& WRS
N3D DQG9 P V &RPSXW D 7 E 0 D F 7 G S H 9 DQG I
0 D
- 3 * LYHQ WKH SLWRWVWDJQDWRQ WP SHUDWXUH DQG SUHVXUH DQG WKH WDWF SUHVXU
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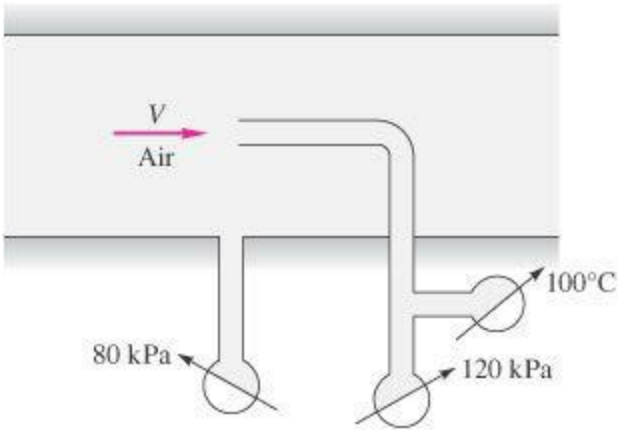


- 3 $ JDV DVVXPHG LGHDO IORZ V LVHQWURSLF IURP SRLQW ZKHUH WKH YHORFLW LV QHJOLJLEOH WKH SUHVVXUHLV NPD DQG WKH WHPSHUDWXUHLV f& VR SRLQW ZKHU WKH SUHVVXUHLV NPD : KDWLV WKH0 DFK QXPEHU0 D LI WKHJDVLV D DLU E DUJRQ RU F &+ " G &DQ\RX WKLQN ZLWKRXW FDOFXODWLQJ ZKLFK JDV ZLOOEH WKH FRROHVW SRLQW"
- 3)RU ORZ VSHHG QHDU0 LQFRPSUHVVLEOH JDV IORZ WKH VWDJQDWLRQ SUHVVXUH FDQ EH FRPSXWHG IURP %HUQRXOOL¶V HTXDWLRQ

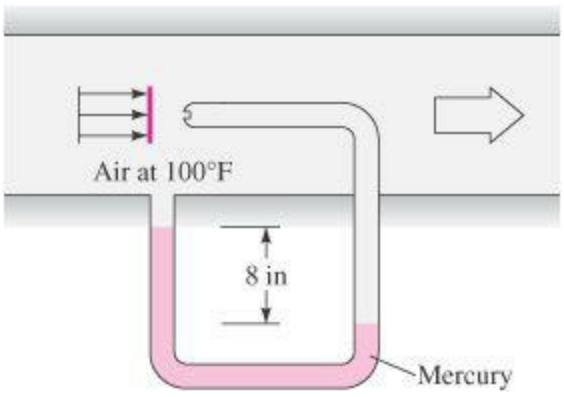
$$p_0 = p + \frac{1}{2}\rho V^2$$

D)RU KLJKHU VXEVRQLF VSHHGV VKRZ WKDWWKH LVHQWURSLF UHODWLRQ D FDQ EH H[SDQGHGLQD SRZHUVHULHVDVIROORZV

$$p_0 \approx p + \frac{1}{2}\rho V^2\left(1 + \frac{1}{4}\mathrm{Ma}^2 + \frac{2-k}{24}\mathrm{Ma}^4 + \cdots\right)$$

E 6XSSRVH WKDWD SLWRW WXEH LQ DLUP HDVXUHV WKH SUHVVXUH GLIIHUHQFH íS DQ XVHV WKH %HUQRXOOL UHODWLRQ ZLWK VDJQDWLRQ GHQVLW\ RI HXWP DW WKH JDV YHORFLW\ $WZ KDW0 DFK QXP EHUZ LOOWKHUHUROEH SHUFHQW

- 3 ,I LWLV NQRZ Q WKDWWKH DLUYHORFLW LQ WKH GXFWLV IWV XVH WKH P HUFXU P DQRP HWH P HDVXUHP HQWLQ) LJ 3 RI HXWP DW WKH VDJQDWLF SUHVVXUH LQ WKH GXFWLQ CEI LQ DEVROXWH



- 3 6KRZ WKDWRULVHQWRSLFIORZ RI DSHUHFWJDVLDSLWRWXDWF SUREHPHDVXUHVS S DQG7 WKHJDVYHORFLWFDQEHFDOFXODWHGIURP

$$V^2 = 2c_pT_0\left[1 - \left(\frac{p}{p_0}\right)^{(k-1)/k}\right]$$

: KDWZRXOGEHDVRXUFHRI HUURULDVKRFNZDYHZHUHIRUPHGLQIURQWRI WKHSURBH'

- 3 $ SLWRWXEH PRXQWHG RQ DQ DLUSODQH IO LQJ DW P WDQGDUG DOWLWXGH UHDGV I WDQGDUQDWRQ SUHVXUH RI N3D (WLPDW WKH SODQH¶V D YHORFLW DQG E 0DF QXPEHU
- 3 $ CDUJHYDFXXP WDQN KHOGDW N3D DEVROXWH VXFNVVHDOHYDWDQGDUGDUWKURXJK FRQYHUJLQJ QR]]OHZKRVHWKURDWDUHD IS FP (WLPDW D WKHPDVIORZ UDWH WKURXJKWKHQR]]OHDQG E WKH0DFKQXPEHUDWWKHWKURDW
- 3 6WDUW IURP DCDUJHWDQN ZKHUH7 *f& DQGS 0 3D H[SDQGVLVHQWURSLFDOO WKURXJKDQR]]OHXQWLDWDVHFWRQRI FP GLDPHWUWKHSUHVXUHLV N3D 8VLQ ((6 RUWKHVWDP WDEOHV> @HVWLPDWH WKHWHPSHUDWXUH E WKHYHORFLW DQG F WKHPDVIORZ DWWKLVVHFWRQ ,VWKHIORZ VXEVRQLF'
- 3 : KHQ GRHV WKH LQFRPSUHVVLEOHIORZ DVVXPSWRQ EHJLQ WR IDLO IRU SUHVXUH &RQVWUXFWD JUDSK RI S SIRULQFRPSUHVVLEOHIORZ RI DSHUHFWJDVDVFRPSDUHGW (T D 3ORWWERWKYHUVXV0DFKQXPEHUIRU " 0D" DQGGHFLGHIRUXRXWHC ZKHUHWKHGHYLDWRQLVWRR JUHDW
- 3 $LUIORZ VDGLDEDWFDOO WKURXJKDGXFW$WRQHVHFWRQ9 IW7 *f) DQ S CEI LQ DEVROXW ZKLHIDUWKHUGRZQWUHP 9 IW7DQGS CEI LQ DEVROXW &RPSXW D 0D E 8PQ DQG F S S
- 3 7KH CDJH FRPSUHVVHG DLU WDQN LQ) LJ 3 H[KDXVW IURP D QR]]OH DWDQ H[L YHORFLWV P V 7KHPHUFXU PDQRPHWUHG FP $VXPLQJ LVHQWRSLF IORZ FRPSXWWKHSUHVXUH D LQWKHWDQNDQG E LQWKHDWPRVSKHUH F : KDWLV WKH H[LV0DFKQXPEHU'

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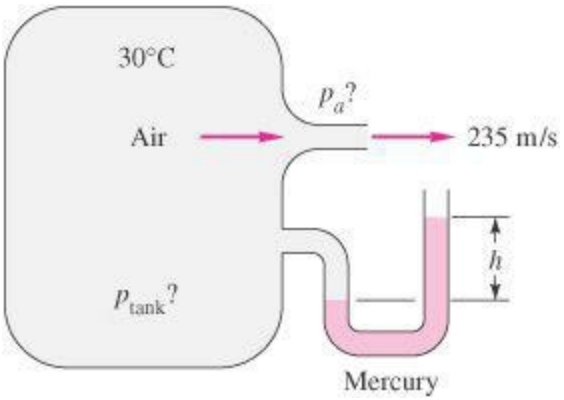

- 3 $LUIORZ VLVHQWURSLFDOO IURP DUHVHUYRLU ZKHUHS N3D DQG7 . V VHFWRQ LQDGXFWZKHUH$ P DQG9 P V &RPSXWH D 0D E 7 F S G $ DQG H $,VWKHIORZ FKRNHG'

,VHQWURSLF IORZ ZLWK DUHD FKDQJHV

- 3 &DUERQGLR[LGH LQDOODUJHVDQNDW *& DQG N3D H[KDXVWVWR DW WKURXJK FRQYHUJLQJ QR]]OH ZKRVH WKURDWDUHD LV FP 8VLQJ LVHQWURSLF LGHDOJDV WKHRU FDOFXODWH D WKH H[LWMAP SHUDWXUHDQG E WKHPDVVIORZ
- 3 +HDXP DW . HQWHUVDQR]]OHLVHQWURSLFDOO \$WWHFWRQ ZKHUH\$ P DSLWRWXDWF DUDQJHPHQWVH) LJ 3 PHDVXUHVWDQDQSUHVXUHRI N3DDQGWDWF SUHVXUHRI N3D (WWPDW D 0D E PDVIORZ À F 7 DQ G \$
- 3 \$QDLUVDQNRI YROXPH P LVLQLWDOO DW N3DDQG *& \$VV LWEHJLQ H[KDXVWLQJ WKURXJK DFRQYHUJLQJ QR]]OHWR VHDOHYHOFRQGLWLRQV 7KHWKURDWDUHD L FP (WWPDW D WKHLQLWDOPDVIORZ LQNJ V E WKHWPHUHTXLUHGWR EORZGRZQWR N3DDQG F WKHWPHDZKLFKWKHQR]]OHFHDVHVEHLQJFKRNHG
- 3 0DNHDQ H[DFWFRQWROYROXPH DQDOVLVRI WKHEORZGRZQSURFHVV LQ) LJ 3 DVXPLQJ DQ LQXODWHGWDQN ZLWKQHJOLJLEOH NLQHWF DQG SRWHQWLDOHQHUJ\ ZLWKLQ \$VXPHFULWFDOIORZ DWWKH H[LWDQG VKRZ WKDWHRWS DQG7 GHFUHDVHGXULQ EORZGRZQ 6HWXSILWWRUGHUGLIIHUHQWLDOHTXDWRQVIRUS W DQG7 W DQGUHGXF DQGVROYHDVIDUDV\RXFDQ
- 3 3UREOHP PDNHVDQLGHDOVHQRUSURWFWRUFRPELQHG GDERUDWRU DQG FRPSXWA SUREOHP DVGHVFULEHGLQ5H 6HF ,Q%REHUDQG. HQRQ]VGDE H[SHUPHQ WKHWDQNKDGDYROXPHRI IWDQGZDVLQLWLDOO ILOHGZLWKDLUDW GE LQ JDJI DQG f) \$WPRVSKHUF SUHVXUH ZDV GE LQ DEVROXWH DQG WKH QR]]OH H[L GLDPHWHUZDV LQ \$IWU VRI EORZGRZQ WKHPHDVXUHGWDQNSUHVXUHZDV GE LQ JDJH DQG WKHWDQNWPSHUDWXUHZDVí f) &RPSDUH WKHVH YDOXHVZLWK WK WKRUHWFDOODQDOVLVRI 3UREOHP 3

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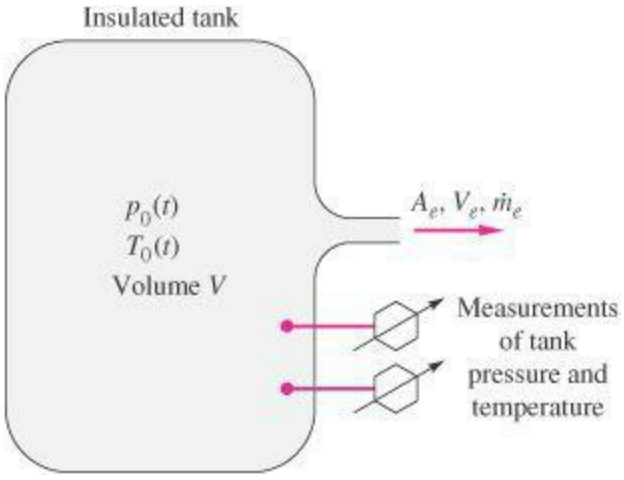

- 3 &RQVLGHUVHQWRSLF IORZ LQDFKDQQHORI YDULQJ DUHD IURP VHFWLRQ WR VHFWLR : HNQRZ WKDW0 D DQG GHVLUHWKDWWKHYHORFLWUDWR9 9 EH (WWPDW D 0D DQG E \$ \$ F 6NHWFK ZKDWWKLVFKDQQHOORRNVOLNH) RUH[DPSOH GHVI FRQYHUJHRUGLYHUJH' ,VWKHUHDWKURDW

- 3 6WDP LQD VDQN DW N3D DQG . GLVFKDUJHV LV HQWRS LFDOO VR D ORZ SUHVVXU DW WRS K HUH WKURXJK D FRQYHUJLQJ QR]]OH Z LWK H[LW DUHD FP D 8VLQJ DQ LGHDO JD DSSUR[LPDWLRQ IURP 7DEOH % HVWLPDWH WKH PDVV IORZ E : LWKRXW DFWXDO FDOFXODWLRQV LQGLFDWH KRZ \RX Z RXOG XVH UHDO SURSHUWLHV RI VWDP IURP ((6 V ILQG WKH PDVV IORZ
- 3 $ LU Z LW K D VDQGQDWRQ SUHVVXUH RI N3D IORZV WKURXJK WKH QR]]OH LQ) LJ 3 Z KLFK LV P ORQJ DQG KDV DQ DUHD YDULDWRQ DSSUR[LPDWHG E\

$ § ± [[

Z LW K $ LQ FP DQG [LQ P ,W LV GHVLUHG WR SORW WKH FRPSOHWH LDPLO RI LVHQWRSL SUHVVXUH S [LQ WKLV QR]]OH IRU WKH UDQJH RI LQOHW SUHVVXUHV S N3D ,QGLFDW Z KLFK LQOHW SUHVVXUHV DUH QRW SK\VLFDO SRVLEOH DQG GLVFXVV EULHO ,I \RXU FRPSXWHU DQ RQOLQH JUDSKLFV URXWLQH SORWV DVFHQGLQJ SUHVVXUH SURILOHV RWKHUZLVH MXVW KLWW KLJKOLJKW DQG H[SODLQ

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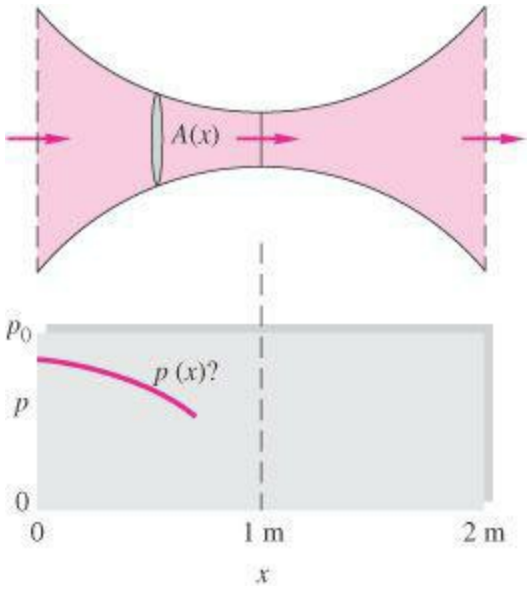

- 3 $ ELF\FOH WUH LV ILOOHG Z LW K DLU DW DQ DEVROXW SUHVVXUH RI N3D DQG WK VWHDG\ PDVV IORZ UDWH RI *f& 6XSSRVH WKH YDOYH EUHDNV DQG DLU VWDUWV WR H[KDXVW RXW RI WKH WUH LQWR WKH DWPRVSKHUH S$ N3D DEVROXW DQG 7$ *f& 7KH YDOY H[LWV PP LQ GLDPHWHU DQG LV WKH VP DOO HVW FURVV VHFWLRQDO DUHD RI WKH HQWL \VWHP) UHFWDQJXODU VORWV FDQ EH LJQRUHG KHUH RQH GLPHQVLRQDO LV HQWURSLF IORZ LV UHDVRQDEOH DVVXPSWLRQ D) LQG WKH 0DFK QXPEHU YHORFLW DQG WHPSHUDWXUH DW WKH H[LWSODQH RI WKH YDOYH LQ WKH LQLWLDO P DW IORZ UDWH RXW WKH WUH F (VWLP DWH WKH YHORFLW DW WKH H[LWSODQH XVLQJ WKH LQFRPSUHVVLEOH %HUQRXO HTXDWLRQ + RZ Z HOO GR HV WKLV H[WP DWH DJUH Z LW K WKH ³H[DFW DQVZ HU RI SDUW D' ([SODLQ
- 3 $ LUIORZ VLVHQWURSLF DOO WKURXJK DYDULDEOH DUHD GXFW $VWHFWLRQ $ FP S N3D! NJ P DQG 0 D $VWHFWLRQ WKH DUHD LV H[DFW WK VDPH EXW WKH IORZ LV PXFK IDVWHU &RPSXW D 9 E 0 D F 7 DQG G WKH PDV

ICRZ H ,V\WHUHDVRQF\KURDWEHWZHHQVHFWRQV DQG " ,I VR ILQGLWDUHD

- 3 ,Q3URE 3 ZHNQHZ QRWKLQJ DERXWFRPSUHVVLEOHICRZ DVWKHWLPH VR ZHPHUH DVXPHG H[LWFRQGLWRQV S DQG7 DQG FRPSXWHG9 DV DQ DSSOLFDWLRQ RI WK FRQWLQXLWHTXDWRQ 6XSSRVHWKDWWKHWKURDWGLPHWULV LQ)RUWKHJLYHQVXJQDWR FRQGLWRQV LQ WKH URFNHWFKDPEHU LQ) LJ 3 DQG DVXPLQJ N DQG I PRDFXODUZHJKWRI FRPSXWWKHDFWXDOH[LWYHORFLW SUHVXUH DQG WMPSHUDWU DFFRUGLQJ WR RQHGLPHQVLRQDO WKHRU ,I S$ CEI LQ DEVROXW FRPSXW WK WKUXWURP WKH DQDOVLVRI 3URE 3 7KLV WKUXWLV HQWHUHO LQGHSHQGHQWRI WK VXDQDWMPSHUDWU FKHFNWKLVE\ FKDQJLQJ7 WR *f*5 LI \RXOLNH : K\"

- 3 $LUICRZ VLVHQWRSLFDOO IURP DODUJHVDQNWKURXJK DYDULDEOHDUHDQR]]OH $VWHFWR ZKHUH$ FP FRQGLWRQVDUHS N3D7 . DQG9 P\ )LQG D WKHPDVVICRZ E 0D F WKHWKURDWDUHD DQG G WKHSUHVXUH DQ WMPSHUDWXUHLQWKHVDQN

- 3 $ RQHGLPHQVLRQDOLVHQWRSLF DLUICRZ KDV WKHIROORZLQJ SURSHUWLV DWRQH VHFWR ZKHUHWKHDUHDLV FP S N3D! NJ P DQG9 P V' HWHUPLQ D WKHWKURDWDUHD E WKHVXDQDWMPSHUDWXUH DQG F WKHPDVVICRZ

- 3 ,QZLQGXQQHOWHVWLQJ0DFK DVP DOODUHDGHFUHDVHGE\ PRGHOEORFNDJ FDQ EH LPSRUWDQW6XSSRVH WKH WHVWVHFWLRQ DUHD LV P ZLWK XQEORFNHG WM FRQGLWRQV0 D DQG7 *f*& : KDWPRGHODUHD ZLOOILLWWFDXVH WKH WM VHFWLRQ WR FKRNH' ,I WKHPRGHOFURVVVHFWLRQLV P SHUFHQWEORFNDJH ZKDWSHUFHQWDJHFKDQJHLQWWVHFWLRQYHORFLWUHXOW

- 3 $ IRUFH) 1 SXVKHVDSLVWRQRI GLDPHWU FP WKURXJK DQLQVXODWGFYOLQGH FRQWDLQLQJ DLUDW *f*& DVLQ) LJ 3 7KHH[WGLDPHWULV PP DQGS$ DW (VWLPDW D 9H E 9S DQG F À[H]

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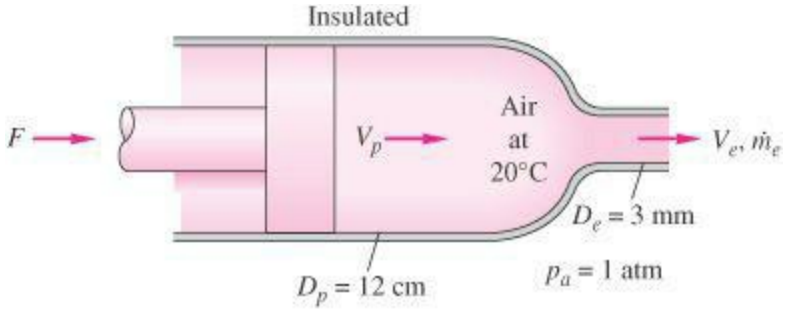

- 3 &RQVLGHUWKHYHQWXULQR]]OHRI) LJ F ZLWK' FP DQGG FP 6WDWLRQDWR WMPSHUDWXUHLV . DQGWKHXSWUHDP YHORFLW9 P V ,I WKHWKURDWSUHVXUL LV N3D HWWPDW ZLWKLVHQWRSLF ICRZ WKHRU D S E 0D DQG F WKHPDV ICRZ

- 3 &DUERQGLR[LGHLV VXRUHGLQDWDQNDW N3DDQG . ,WGLVFKDUJHVWRDVHFRQ WDQNWKURXJKDFRQYHUJLQJ QR]]OHZKRVHH[LWDUHDLV FP : KDWLV WKHLQLWDOPDV ICRZ UDWH WKHVHFRQGWDQNKDVDSUHVXUHRI D N3DRU E N3D'

- 3 7KHVFUDPMHWHQJLQHRI) LJ LVVXSHUVRQF\KURXJKRXW\$ VNHWFK LVVKRZQLQ) LJ & 7HWWKHIROORZLQJ GHVLJQ 7KHICRZ HQWHULV DW0 D DQG DLUSURSHUWLV IR

P DQGWXGH ,QDWDUHDLV P WKHPLQLPXP DUHDLV P DQGWKHH[LWDUH
LV P ,I WKHUHLVQR FRPEXWLRQ D ZLOOWKHIORZ WLOOEHVXSHUVRQLF LQWK
WKURDW$OR GHWHUPLQH E WKHH[LW0 DFK QXPEHU F H[LWYHORFLW DQG G H[L
SUHVVXUH

- 3 $ FRQYHUJLQJ±GLYHUJLQJ QR]]OHH[LWVP RRVKO WRVHDOHYHOWDQGDUDWRSKHUH ,WL
VXSSOLHGE\ D P WDQNLQWRLO DW N3DDQG *f& $VXPLQJ LVHQWURSLF IORZ
LQWKH QR]]OH HVWLP DWH D WKH WKURDWDUHD DQG E WKH WDQN SUHVVXUH DWU VR
RSHUDWRQ 7KHH[LWDUHDLV FP

- 3 $LUIORZ VWUDLJLO IURP DUHVHUYRLUDW *f& WKURXJK DQR]]OHRI H[LWDUHD FP DQ
VXUNHVDYHUWFDOSODWHDVLQ) LJ 3 7KHIORZ LVVXEVRQLF WKURXJKRXW$ IURFHR
1 LVUHTXLUHGWRKROGWKHSODWHVWDWLRQDU\ &RPSXWH D 9H E 0D H DQG F S
LI S' N3D

7KHQRUP DOVKRFN ZDYH

- 3 7KH DLUIORZ LQ3URE 3 XQGHUJRHV D QRUPDOVKRFN MXVWSDVWWKH VHFWRQ ZKHU
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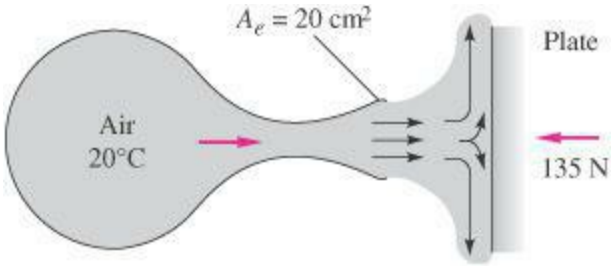

- 3 $LU VXSSOLHGE\ DUHVHUYRLUDW N3D IORZV WKURXJK D FRQYHUJLQJ±GLYHUJLQ
QR]]OHZKRVHWKURDWDUHDLV FP $ QRUPDOVKRFN VWDQGVZKHUH$ FP D
&RPSXWHWKHSUHVVXUHMXVWGRZQVWUHDP RI WKLVVKRFN 6WDWLDWKHUGRZQVWUHDP DW$
FP HVWLPDWH E S F $ DQG G 0D

- 3 $LUIURP DUHVHUYRLUDW *f& DQG N3DIORZV WKURXJK DGXFWDQGIRUPV DQRUPDO
VKRFNGRZQVWUHDP RI DWKURDWDUHD FP %\ DQRGGFRLQFLGHQFHWLWV IRXQGWKDW
WKHVWDWQDWRQSUHVVXUHGRZQVWUHDP RI WKLVVKRFNH[FHHGV HTXDOV WKHWKURDWSUHVVXUH
: KDWLVWKHDUHDZKHUHWKHVKRFNZDYHVWDQGV"

- 3 $LUIORZ VIURP DWDQN WKURXJK DQR]]OHLQRWKHVWDQGDUGDWPRSKHUH DV) LJ 3
$ QRUPDOVKRFN VWDQGVLQWKH H[LWPRVWFDVVDQ (VXPDUL]H WKHSUHVVXU
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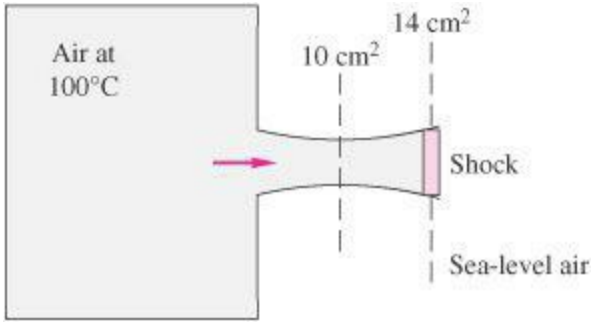
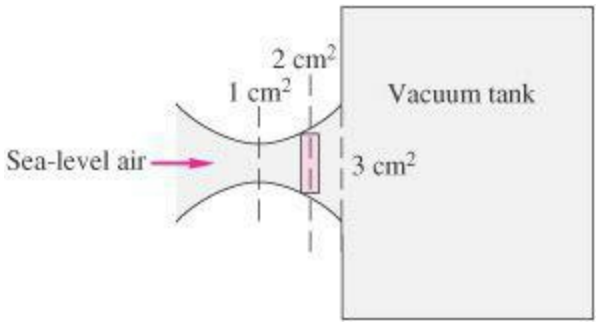


- 3 7KHGDWDRI3URE 3 UHSUHVHQWFRQGLWLRQVEHRUHDQGDIWUDQRUPDOVKRFNZDYH, VKH YHORFLW DWV HFWLRQ % LV P V ZKDWDUH D VKH YHORFLW DW E VK VWDJQDWLRQVPSHUDXUH F VKHVWDJQDWLRQSUHVXUHDW DQG G VKH0DFKQXPEHUD VHFWLRQ' "
- 3 $LU DWVWDJQDWLRQFRQGLWLRQV RI . DQG NBD IORZV VKURXJK D QR]]OH \$ VHFWLRQ ZKHUHWKHDUHDLV FP VKHUHLVDQRUPDOVKRFNZDYH,I WKHPDVIORZ LV NJ V HVWLPDW D VKH0DFKQXPEHUDQG E VKH VWDJQDWLRQSUHVXUH IXU GRZQVWUHDP RI VKHVKRFN
- 3 : KHQDSLWRWWXEHVXFKDVLQ)LJ LVSODFHGLQDVXSHUVRQLFIORZ DQRUPDOVKRFN ZLOOVWDQGLQIURQWRI WKHSUREH6XSSRVHWKHSURBHUHDGVS NBDDQGS NBD,I WKHVWDJQDWLRQWHPSHUDXUHLV . HVWLPDWWKH VXSHUVRQLF0DFKQXPEHDQG YHORFLWXSVWUHDP RI VKHVKRFN
- 3 \$LUIORZVIURP DODUJHVDQN ZKHUH7 . DQGS NBD WRDGHVLJQFRQGLWR ZKHUHWKHSUHVXUHLV 3D 7KHPDVIORZ LV NJ V +RZHYHU WKHUHLV QRUPDOVKRFNLQWKHH[LWSODQHMXVWDWWUWKLVFRQGLWRQLVUHFKG (VWP DW D VKRUDWDUHD DQG MXVWGRZQVWUHDP RI WKH VKRFN E WKH0DFKQXPEHU F WK WP SHUDXUH DQG G WKHSUHVXUH
- 3 \$QDWPLF H[SORVLRQSURSDJDWWVLQRXWOODLUDW CEI LQ DEVROXWDQG *f*5 7K SUHVXUHMXVWVLGHWKHVKRFNLV CEI LQ DEVROXW\$VXPLQJ N ZKDWDU WKHVSHG& RI WKHVKRFNDQGWKHYHORFLW9 MXVWLQVLGHWKHVKRFN"

&RQYHUJLQJ DQG GLYHUJLQJ QR]]OHV

- 3 6HDOHYHOVWDQGDUG DLU LV VXFNHG LQWR D YDFXXP VWDQN WKURXJK D QR]]OH DV LQ)LJ 3 \$ QRUPDOVKRFN VWDQGVZKHUHWKHQR]]OHDUHDLV FP DV VKRZQ (VWP DW D WKHSUHVXUHLQWKHVWDQNDQG E WKHPDVIORZ



- 3 $LU IURP DUHVHUYRLUDW . DQG N3D IORZ VWKURXJK DFRQYHUJLQJ±GLYHUJLQ QR]]OH 7KH WKURDWDUHD LV FP $ QRUPDO VKRFN DSSHDUV IRU ZKLFK WK GRZQVWUHDP 0 DFKQXPEHULV D : KDWLV WKHDUHDZKHUHWKHVKRFNDSSHDUV &DOFXODWH E WKHSUHVVXUHDQG F WKHWHPSHUDWXUHGRZQVWUHDP RI WKHVKRFN
- 3 $LUIORZ V WKURXJK DFRQYHUJLQJ±GLYHUJLQJ QR]]OHEHWZHHQWZR ODUJH UHVHUYRLUV D VKRZ QLQ) LJ 3 $ PHUFXU PDQRPHWHUEHWZHHQWKHWKURDWDQG WKHGRZQVWUHDP UHVHUYRLUUHDGV K FP (VWLPDWHWKHGRZQVWUHDP UHVHUYRLUSUHVVXUH ,V WKHUH QRUPDOVKRFNLQWKHIORZ" ,I VR GHVFULEHWKHSODFHLQWKHH[LWSODQHRIDWKHUXSVWUHDP"

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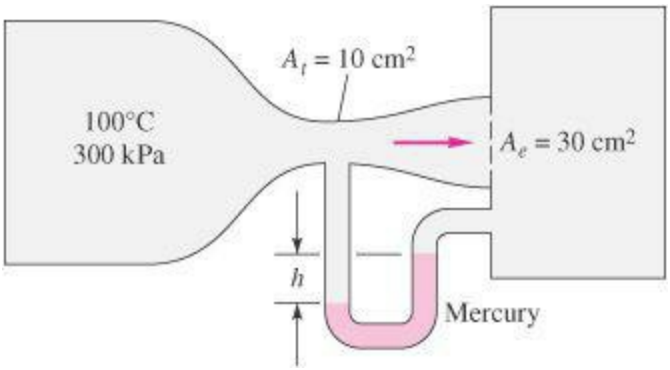

- 3 ,Q3URE 3 ZKDWZRXOGEHWKHPHUFXU PDQRPHWUHDGLQJ K LI WKHQR]]OHZHU RSHUDWLQJ H[DFWO DWXSHUQFHGLWJQFRQGLWLRQ"
- 3 $ VXSSO VDQN DW N3D DQG . IHHGV DLU WR D FRQYHUJLQJ±GLYHUJLQ QR]]O ZKRVH WKURDWDUHD LV FP 7KH H[LWDUHD LV FP 6WDWH WKH FRQGLWLRQV LQ WK QR]]OHL WKH SUHVVXUHRXWLGHWKHH[LWSODQHLV D N3D E N3D DQG F N3D G ,QHDFKFR WKHVHFDVHV ILQGWKHPDVVIORZ
- 3 $LULQDWDQN DW N3DDQG . H[KDXVWVWR WKHDWPRVSKHUHWKURXJK D FP WKURD FRQYHUJLQJ QR]]OHDWDUDWHRI NJ V : KDWLV WKHDWPRVSKHULF SUHVVXUH' : KDW WKHPD[LPXP PDVVIORZ SRVVLEOHDWRZ DWPRVSKHULF SUHVVXUH'
- 3 : LWKUHHUHQFHWR3URE 3 VKRZ WKDWWKHWKUXVWRI DURFNHWHQJLQHH[KDXVWLQJ LQ DYDFXXP LVJLYHQE\

$$F = \frac{p_0 A_e(1 + k\,\mathrm{Ma}_e^2)}{\left(1 + \dfrac{k-1}{2}\mathrm{Ma}_e^2\right)^{k/(k-1)}}$$

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  -

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FP ,WLV VXSSOLHG E\ DQ DLU WDQN DW N3D DQG . D : KDWLV WKH GHVLJ
SUHVVXUH DW WKH H[LW$ WRQH RSHUDWLQJ FRQGLWLRQ WKH H[LWSURPHUWHV DUH S$
N3D 7$. DQG9$ P V E &DQWKLV FRQGLWLRQ EH H[SODLQHG E\ DQRUPDO
VKRFN LQVLGH WKH QR]]OH" F ,I VR DW WKDW0 DFK QXPEHU GRHV WKH QRUPDO VKRF
RFFXU + LQW8 VHWKHFKDQJHLQ$ VRORFDWLRQ RFFXUUHQFH WKLV ORFDWLRQ

- 3 $ GDUJH WDQN DW . DQG N3D IHHGV DLU WR D FRQYHUJLQJ QR]]OH 7KH EDF
SUHVVXUH RXWLGH WKH QR]]OH H[LWV VHDOHYHOWDQGDUG : KDWV WKH DSSURSULDWH[L
GLDPHWHU LI WKH GHVLUHG PDVV IORZ LV NJ K"
- 3 $LUIORZ V LVHQWURSLFDOO LQ D FRQYHUJLQJ±GLYHUJLQJ QR]]OH Z LWK D WKURDWDUHD RI
FP $VVHFWLRQ WKH SUHVVXUH LV N3D WKH WHPSHUDWXUH LV . DQG WK
YHORFLWV LV P V D ,V WKH QR]]OH FKRNHG" ' HWHUPLQH E $ DQG F WKH PDV
IORZ 6XSSRVH Z LWKRXW FKDQJLQJ WDQDQWRQFRQGLWLRQV RU$ WKH IH[LEOH WKURDW
UHGXFHG VR FP $VVXPLQJ VKRFN IUHH IORZ Z LOO WKHUH EH DQ FKDQJH LQ WKH JD
SURSHUWLHV DVVRFLDWHG " ,I VR FRPSXWH Q Z S 9 DQG7 DQG H[SODLQ

- 3 8VH \RXU VWDQGDUG LGHDV IURP SDUW E RI 3URE 3 VR DFWXDOO FDUU RXW WK
FDOFXODWRQV IRUP DW IORZ RI VWDP Z LWK S$ N3D DQG7$. GLVFKDUJLQ
WKURXJK DFRQYHUJLQJ QR]]OH RI FKRNHG H[LWDUHD FP 7DNH DGYDQWDJH RI WKH Q(
((6 WKHUPRG\QDPLFIXQFWLRQ63(('6281' 6WDP S S V V

- 3 $ GRXEOH WDQN VVWP LQ) LJ 3 KDV WZR LGHQWFDO FRQYHUJLQJ QR]]OHV RI LQ
WKURDWDUHD 7DQN LV YHU ODUJH DQG WDQN LV VPDOOHQRXJK WR EH LQ VWDG IORZ
HTXLOLEULXP Z LWK WKH HQWWURP WDQN 1 R]]OH IORZ LV LVHQWURSLF EXW HQWURS\ FKDQJH
EHWZHHQ DQG GXH WR NHWWGLVLSDWLQ WDQN &RPSXWH WKH PDV IORZ ,I \R
JLYHXS 5H SS ± KDVJRRGGLVFXVLRQ

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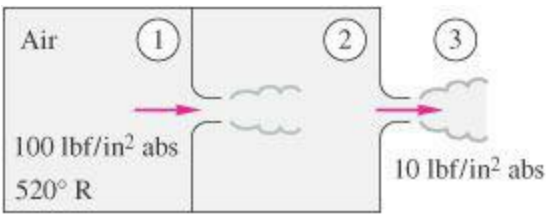
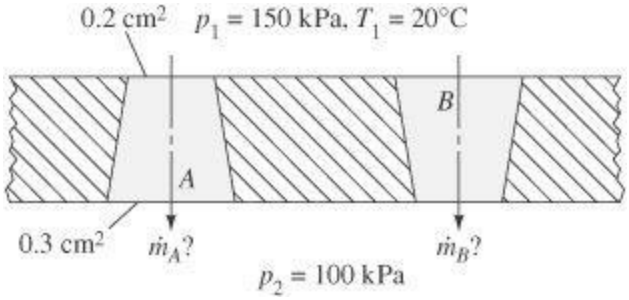
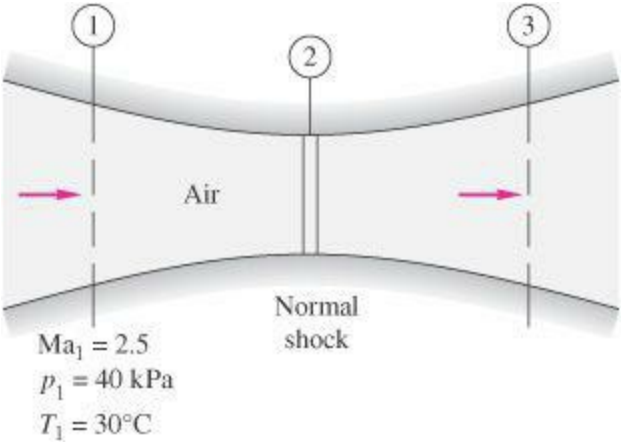


- 3 $ GDUJHUHVHUYRLUDW *& DQG N3DLVXVHGWRILOODVPDOOLQXODUGWDQNWKURXJK
FRQYHUJLQJ±GLYHUJLQJ QR]]OH Z LWK FP WKURDWDUHD DQG FP H[LWDUHD 7K
VPDOOWDQNKDV DYROXPHRI P DQGLVLQLWLDOO DW *& DQG N3D (VWXP DWWK
HODSVHGWPHZKHQ D VKRFN ZDYHVEHJLQVWR DSSHDULQWKH QR]]OH DQG E WK
PDW IORZ EHJLQVWRGURSEHORZ LWVP[LPXP YDOXH
- 3 EES $ SHUHFWJDV QRWDU H[SDQGVLVHQWURSLFDOO WKURXJK DVXSHUVRQLF QR]]OH Z LWKD
H[LWDUHD WPHVLWWKURDWDUHD 7KH H[LW0 DFK QXPEHULV : KDWV WKH VSHFLLF
KHDWUDWLR RI WKH JDV" : KDWP LJKWKLV JDV EH" ,I S N3D Z KDWLV WKH H[I
SUHVVXUHRI WKHJDV"
- 3 7KH RULHQWDWLRQ RI D KROHFDQ PDNH D GLIHUHQFH &RQVLGHUKROHV$ DQG% LQ) LJ
3 Z KLFK DUH LGHQWFDOEXWHYHUHG)RUWKHJLYHQDUSURPHUWHVRQHWKHUHG
FRPSXWWKHPDW IORZ WKURXJKHDFKKROHDQG H[SODLQZK\ WKH DUHGLIHUHQW



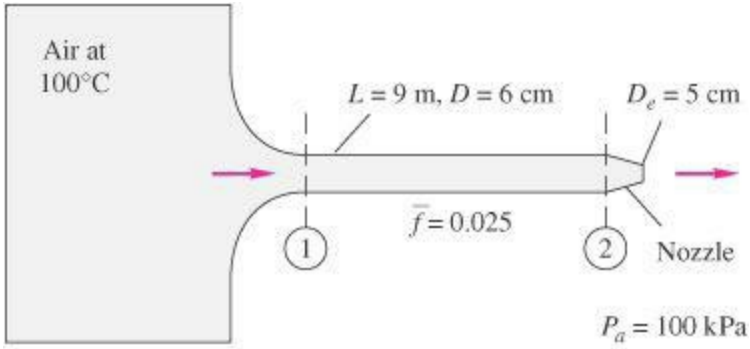
- 3 $ CJHUHVLYRLUDW . VXSSOLHVDLUIORZ WKURXJKDFRQYHUJLQJ±GLYHUJLQJ QR]C ZLWKDWKURDWDUHDRI FP $ QRUPDOVKRFNZDYHIRUPVDWDVHFWRQRI DUHD FP -XWGRZQWUHDP RI WKLVVKRFN WKHSUHVXUHLV N3D &DOFXODWH D WKHSUHVXUHL WKHWKURDW E WKHPDVIORZ DQG F WKHSUHVXUHLQWKHUHVHUYRLU
- 3 $ VHDOHYHODXWRPRELOHWUHLVLQLWLDOO DW CEI LQ JDJHSUHVXUHDQG *f*) : KHQL LVSXQFWXUHGZLWKDKROHWKDWUHVPEOVDFRQYHUJLQJ QR]CHI LWSUHVXUHGURSVW CEI LQ JDJHLQ PLQ (VWPDWWKHVL]HRI WKHKROH LQWKRXDQGWKVRI DQLQFK 7KHWUHYROXPHLV IW
- 3 $LU DWS$R CEI LQ DQG7R *f*) IORZ VLVHQWRSLFDOO WKURXJKDFRQYHUJLQJ± GLYHUJLQJ QR]CHI $WWHFWRQ ZKHUH$ LQ WKHYHORFLW9 IWV &DOFXODWH D 0D E $ F S DQG G WKHPDVIORZ LQVOXJV
- 3 $LUDW . IORZ VWKURXJKDFRQYHUJLQJ±GLYHUJLQJ QR]CHZLWKWKURDWDUHDRI FP DQG H[LWDUHDRI FP : KHQWKHPDVIORZ LV NJ K DSLWRWXEHDW SURE SODFHGLQWKH H[LWSODQHUHDGVS N3DDQGS N3D (VWPDWWKHH[I YHORFLW,VWKHUHDQRUPDOVKRFNZDYHLQWKHGXFW,I VR FRPSXWWKH0DFKQXPEH MXVWGRZQWUHDP RI WKLVVKRFN
- 3 : KHQRSHUDWLQJ DWGHVLJQFRQGLWLRQV VPRRWK H[WRV DEOHSUHVXUH DURFN HQJLQHKDV DWKUXWR PLOGRQCEI 7KHFKDPEHUSUHVXUHDQGWHPSHUDWXUHDUH CEI LQ DEVROXWDQG *f5 UHVSHFWLYHO 7KHH[KDXVWJDVHVSSUR[LPDWN ZLWKDPROHFXODUZHLJKWR (VWPDWD WKHH[LW0DFKQXPEHUDQG E WKHVKURC GLDPHWU
- 3 $LUIORZ VWKURXJKDGXFWDVLQ) LJ 3 ZKHUH$ FP $ FP DQG$ FP $ QRUPDOVKRFNVDQGVDWWHFWRQ &RPSXWH D WKHPDVIORZ E WK 0DFKQXPEHUDQG F WKHVWDJQDWLRQSUHVXUHDWWHFWRQ



- 3 $ VWSLFDOFDUERQGLR[LGHVDQNIRUDSDLQHDOJXQKRGVDERXW R] RI GTXLG &2 7KHVDQNLVILOOHGQRPRUHWKDQRQHWKLUGZLWKGTXLG ZKLFK DWURRP WHPSHUDWXUH PDLQWDLQV WKH JDVHRXV SKDVH DWDERXW SVLD D ,I D YDOYH LV RSHQHG WKH VLPXOWDQHRXV GHFRQYHUJLQJ QR]]OHZLWKDQH[WUHPHO\ E 5HSHDWWKHFDOFXODWLRQVIRUKHOXP

' XFWLOHZ ZLWK IULFWLRQ

- 3 $LUHQWHUVD FP GLDPHWHUSLSH P ORQJ DW9 P V S N3D DQG7 *f*& 7KHIULFWLRQIDFWRULV &RPSXWH9 S 7 DQGS DWWKHHQGRI WKSLSH +RZ PXFKDGGLWLRQDOSLSHOHQJWKZRXOGFDXVHWKHH[LWWRZ WREHVRQLF'
- 3 3UREOHP & JLYHVGDWDIRUDSURSRVHG$DVD&DQDGDQDXWRJDV DWXPH&+ SLSHOLQH ,I WKHGHVLJQIORZ UDWHLV NJ VDQGWKHHQWDQFHFRQGLWLRQV DUH ORE LQ DQG *f* GHWHUPLQHWKHPD[LPXP OHQJWKRI DGLDEDWLF SLSHEHRUHFKRNLQ RFFXUV
- 3 $LUIORZVDGLDEDWLFDOO WKURXJKD FP GLDPHWUSLSH &RQGLWLRQV DWWHFWLRQ DUHS N3D7 *f*& DQG9 P V 7KHDYHUDJHIULFWLRQIDFWRULV $ VHFWLRQ ZKLFKLV PHWHUVXSVWUHDP ILQG D WKHPDVVIORZ E S DQG F S
- 3 &DUERQGLR[LGHIORZVWKURXJKDQLQVXODWHG SLSH P ORQJ DQG FP LQGLDPHWU7K IULFWLRQIDFWRULV $VWHDG\VWDWHS N3DDQG7 . 7KHPDVVIORZ LV NJ V (VWLPDWHWKHSUHVVXUHGURS E\ D FRPSUHVVLEOHDQG E LQFRPSUHVVLEO 6HF IORZ WKHRU F)RUZKDWSLSHOHQJWKZLOOWKHH[LWEHFKRNHG'
- 3 $LUIORZVWKURXJKDURXJKSLSH IWORQJ DQG LQLQGLDPHWU (QWDQFHFRQGLWLRQ DUHS OEI LQ 7 *f* DQG9 IWV 7KHIORZ FKRNHVDWWKHHQGRI WKSLSH D: KDWLVWKHDYHUDJHIULFWLRQIDFWRU' E : KDWLVWKHSUHVVXUHDWWKHHQGRI WKSLSH'
- 3 $LUIORZVWKURXJKDQ IURP DWDQNWKURXJKWKHSLSHLQ) LJ 3 7KHUHLVDFRQYHUJLQ QR]]OHRQWKHHQG ,I WKHPDVVIORZ LV NJ VDQGWKHQR]]OHLVFKRNHG HWWP DW WKH0 DFKQXPEHUDWWHFWLRQ DQG E WKHSUHVVXUHLQWKHWDQN



- 3 $LUHQWLVD FP GLDPHWUSLSHDW NBD NJ P DQG P V 7KHIULFWLR IDFWRULV) LQGWKHSLSHOHQJWKIRUZKLFKWKHYHORFLWY D GURSEOHV E WUSOH DQG F TXDGUXSOHV
- 3 $LUIORZV DGLDEDWLFDOO LQ D FP GLDPHWUGXFW7KH DYHUDJH IULFWLRQ IDFWRULV ,I DWWKHHQWUDQFH 9 P VDQG7 . KRZ IDUGRZQWKHWXEHZLO D WKH0DFKQXPEHUEH RU E WKHIORZ EHFRPHG'
- 3 &RPSUHVVLEOHSLSHIORZ ZLWKIULFWLRQ6HF DWXPHVFRQVWDQWWDUQDWHQWKDOS\ DQGPDWIORZ EXWYDULDEOHPRPHQWXP 6XFKDIORZ LVRWWQFDOOHG) DQQRIORZ DQ DOOHUHSUHVHQWQJ DOOSRVVLEOHSURSHUWFKDQJHVRQDWWPSHUDXUHHQWRS\ FKDWW FDOOHG D) DQQR OQH $VXPLQJ D SHUHFWJDV ZLWKN DQG WKH GDWD RI 3URE 3 GUDZ D) DQQR FXUYHRI WKHIORZ IRUDUQJHRI YHORFLWWVIURP YHU ORZ 0 ≪ WR YHU KLJK 0 D≫ &RPPHQWRQWKH PHDQLQJ RI WKH PD[LPXP HQWRS SRLQWRQWKLVFXUYH
- 3 + HDXP 7DEOH$ HQWUJVD FP GLDPHWUSLSHDWS NBD 9 P V DQG7 *f& 7KHIULFWLRQIDFWRULV ,I WKHIORZ LVFKRNHG GHWUPLQH D WK OHQJWKRI WKHGXFWDQG E WKHH[LWSUHVXUH
- 3 0 HWKDQH &+ IORZV WKURXJK DQ LQVXODWHG FP GLDPHWUSLSHZLWKI (QWUDQFHFRQGLWLRQVDUH NBD *f& DQGDPDVVIORZ RI NJ V: KDWOHQJWKVI SLSHZLOO D FKRNH WKHIORZ E UDLVH WKHYHORFLWE\ SHUFHQWRU F GHFUHDV WKHSUHVXUHE\ SHUFHQW
- 3 %\ PDNLQJ DIHZ DJHEUDLF VXEVWLWXWLRQV VKRZ WKDW*T PD\ EHZULWWHQLQWK GHQVLWIRUP

$$\rho_1^2 = \rho_2^2 + \rho^{*2}\left(\frac{2k}{k+1}\frac{\bar{f}L}{D} + 2\ln\frac{\rho_1}{\rho_2}\right)$$

: K\ LV WKLV IRUP XOD DZ NZ DUG LI RQH LV WWLQJ WR VROYH IRU WKH PDVIORZ ZKHQ WK SUHVXUHV DUH JLYHQ DWWHFWRQV DQG "

- 3 &RPSUHVVLEOHDPLQDUIORZ I§ 5H PD\ RFFXULQFDSLOODU WXEHV &RQVLG DU DWXDOQDWRQFRQGLWRQV RI *f& DQG NBD HQWHULQJ DWXEH FP ORQJ DQ PP LQ GLDPHWU ,I WKH UHFLYHU SUHVXUH LV QHDU YDFXXP HWWP DW D WK DYHUDJH 5H\QROGV QXPEHU E WKH0DFKQXPEHUDWWKHHQWUDQFH DQG F WKHPDV IORZ LQNJ K
- 3 EES $ FRPSUHVVRUIRUHVDUWKURXJKDVPRRWKSLSH P ORQJ DQG FP LQGLDPHWU DV LQ) LJ 3 7KHDUUHDYHVDW NBD DQG *f& 7KHFRPSUHVVRUGDWIF

SUHVVXUHULVHYHUVXVPDVVIORZ DUHVKRZQLQWKHILJXUH 8VLQJWKH0RRG\FKDUW
HVWLPDWH $ FRPSXWHWKHUHYQROGVQXPEHUVIORZ

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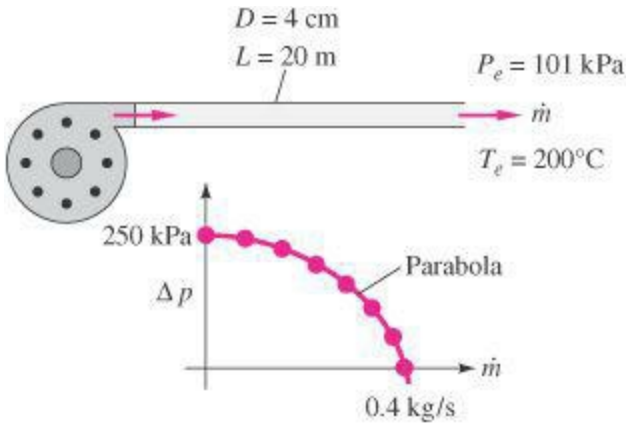

- 3 $\$ LUIORZ VDGLDEDWFDOO VKURXJKDSLSH FP LQGLDPHWUDQG P ORQJ (QWUDQF
FRQGLWLRQV DUHS NBD 7 . DQG9 P V 7KHDYHUDJHIULFWLR
IDFWRU LV D ,V WKHSLSHIORZ FKRNHG DWWKHH[LW E : KDWLV WKHH[L
SUHVVXUH" F $VVXPHDVLGHDOFHGRZQWKHSLSHLVWKHYHORFLW P V"
- 3 +RZ GRWKHFRPSUHVVLEOHSLSHIORZ IRUPXODVEHKDYHIRUVPDOOSUHVVXUHGURSV" / H
DLUDW *& HQWHUVDWEHRI GLDPHWU FP DQGHQJWK P ,I ZLWKS
NBD DQGS NBD HVWLPDWHWKHPDVVIORZ LQNJ KIRU D LVRWKHUPDOIORZ E
DGLDEDWFIORZ DQG F LQFRPSUHVVLEOHIORZ &KDS DWWKHHQWDQFHGHQVLW
- 3 $LUDW NBDDQG *& HQWHUVDVPRRWK P ORQJ SLSHDQGWHQSDWHVWKURXJK
VHFRQGVPRRWKSLSHWRD NBDUHVHUYRLUDVLQ) LJ 3 8VLQJWKH0RRG\FKDU
WRFRPSXWH HVWLPDWHWKHPDVVIORZ WKURXJKWKLVVXVWHP ,VWKHIORZ FKRNHG'

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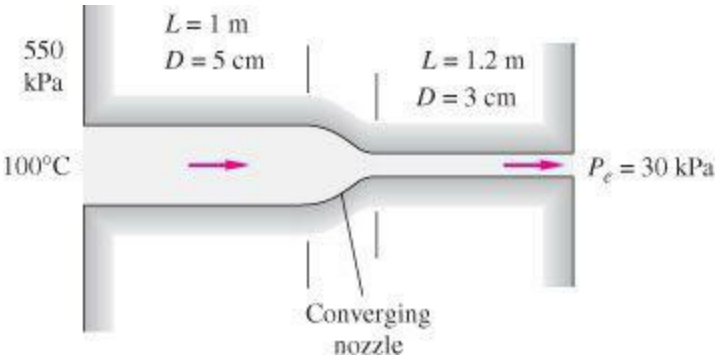

- 3 1DWXUDOJDV ZLWKN§ DQG DPROHFXODUZHJKWRI LVWREHSXPSHGWKURXJI
NP RI FP GLDPHWUSLSHDQH 7KHGRZQWHDP SUHVVXUHLV NBD ,I WKHJD
HQWHUV DW *& WKHPDVVIORZ LV NJ V DQG HVWLPDWHWKHUHTXLUHG HQWDQFH
SUHVVXUHIRU D LVRWKHUPDOIORZ DQG E DGLDEDWFIORZ
- 3 $ VDQNR RI R[\JHQ 7DEOH$ DW *& LV WR VXSSO DQ DXWRQDXWKURXJK D
XPELOLFDOWXEH P ORQJ DQG FP LQGLDPHWU 7KHH[WSUHVVXUHLQWKHWXEHLV
NBD ,I WKHGHVLUHGPDVVIORZ LV NJ K DQG ZKDWWKRXGEHWKHSUHVVXUHLQWK

VDQN'

- 3 0 RGLI\ 3URE 3 DVIROORZV 7KHSLSHOLQHZLOOQRWEHDOORZHGWRFKRNH ,WZLO KDYHSXPSLQJVWDWLRQVDERXWHYHU PLOHV D)LQGWKHHQWKRSLSHIRUZKLF WKHSUHVVXUHKDVGURSSHGWR GE LQ E : KDWLVWKHWRWDOSXPSHUDXUHDWKDWSRLQW
- 3 $LU IURP D FXELF PHWHU WDQN LQLWLDOO DW NPD DQG *& EORZV GRZ DGLDEDWLFDOO WKURXJK DVPRRWKSLSH FP LQGLDPHWUDQG P ORQJ (VWDPDWWK WKH UHTXLUHG WR UHGXFH WKH WDQN SUHVVXUH WR NPD) RU VLPSOLFLW DVVXPH FRQVWDQWNVNJ WKHSUHVVXUHDQGI §

3

) ULFWLRQOHVVIORZ ZLWK KHDWWUDQVIHU

- 3 $ IXHOEDLUPL[WXUH DVXPHGHTXLYDOHQWWR DLU HQWHUV DGXFWFRP EXWLRQFKDPEHUD 9 P VDQG7 . : KDWDP RXQWRI KHDWDGGLWLRQLQN- NJ ZLOOFDXVHWK H[LWWORZ WREHFKRNHG' : KDWZLOOEHWKH H[LW DFKQXPEHUDQG WKPSHUDXUHLL N- NJ DUHDGGHGGXULQJFRPEXWLRQ'
- 3 : KDWKDSSHQVWRWKHLQLWLDWORZ RI 3URE 3 LI WKHFRPEXWLRQ\LHOGV N- N KHDWDGGLWLRQ DQGS DQG7 UHPDLQ WKH VDPH' +RZ PXFK LV WKH PDVV IORZ UHGXFHG'
- 3 $ MHWHQJLQHDV P DOWLWXGHVDWLV NJVR DLUDQG DGGV N- NJ LQWK FRPEXWLRQFKDPEHU 7KHFKDPEHUFURVVVHFWLRQLV P DQG WKHDLUHQWHUV WK FKDPEHUDW NPDDQG *& $IWHUFRPEXWLRQWKHDLUH[SDQGVWKURXJKDQLVHQWURSL FRQYHUJLQJ QR]]OHWR H[WDWDWRSSHUF SUHVXUH (VXPDW D WKH QR]]OHWKURD GLDPHWU E WKHQR]]OHH[LWYHORFLW DQG F WKHWKUXVWSURGXFHGE\ WKHHQJLQH
- 3 &RPSUHVVLEOHSLSHIORZ ZLWKKHDWDGGLWLRQ6HF DXXPHVFRQVWDQWPRPHQWXP S !9 DQGFRQVWDQWPDVIORZ EXWYDULDEOHVWDJQDWRQHQWKDOS\ 6XFKDIORZ LV RIWHQFDOOHG5D\OHLJKIORZ DQGDQHUHPRYLQJDOOSRVLEOHSURSHUWFKDQJHVR DVPSHUDXUHHQWRPS\ FKDUWVFDOHGD5D\OHLJKQH $VXPLQJ DLUSDVLQJ WKURXJI WKHIORZ VWDWS NPD7 . 9 P V DQG$ P GUDZ I 5D\OHLJKFXUYHR WKHIORZ IRUDUDQJHR YHORFLWWVIURP YHU ORZ 0D≪ V YHU KLJK 0D≫ &RPPHQWRQWKHPHDQLQJR WKHPD[LPXP HQWRPS\ SRLQWRQWKH FXUYH
- 3 $GG WR \RXU5D\OHLJK QQH R 3URE 3 D)DQR QQH VHH3URE 3 IR VWDJQDWRQHQWKDOS\ HTXDOWR WKHYDOXHDWRFLDWG ZLWKVWDW LQ3URE 3 7K VZR FXUYHV ZLOOLQWHUVHFWDWVWDW ZKFK LV VXEVRQLF DQG DWD FHUDLQ VWDW ZKFK LVVXSHUVRQLF ,QWSUHWWKHVHWZR VWDWVYLVj YLV7DEOH%

- 3 $LUHQWUDGXFWXEVRQLFDO DWWHFWRQ DW NJV : KHQ N RI KHDWDU DGGHG WKH IORZ FKRNHV DWWKH H[LWDWS N3D DQG7 . $VXPLQ IULFWLRQOHVV KHDWDGGLWLRQ HXPDW D WKHYHORFLWDQG E WKHVWDJQDWLRQSUHVVXUHD VHFWLRQ
- 3 $LUHQWUDFRQWDQWUHDGXFWDWS N3D 9 P V DQG7 *& ,WV WKHQFRROHG ZLWKQHJOLJLEOHIULFWLRQXQWLOLWH[LWDWS N3D (VWPDW D 9 E 7 DQG F WKHVRXQGSHRXQWRI FRRODQJ LQN- NJ
- 3 7KHVFUDPMHWRI)LJ RSHUDWHVZLWKVXSHUVRQLF IORZ WKURXJKRXW$QLGHDOL]H VHHWKK LVLQ)LJ & DWWKHHQGRI WKLVFKDSWU$VXPHWKDWWKHKHDWDGGLWLRQR N- NJ EHWZHHQVHFWLRQV DQG LVIUHFWRQOHVV DQGDWFRQXDQWUHDRI P * LYHQ0 D S N3D DQG7 . $VXPHDLUIORZ DWN $WWK FRPEXVWLRQVHFWLRQH[LWLQG D 0 D E S DQG F 7
- 3 $LUHQWUDG FP GLDPHWUSLSHDW N3D NJP DQG P V $VXPI IULFWLRQOHVV IORZ ZLWKKHDWDGGLWLRQ)LQGWKHPRXQWRI KHDWDGGLWLRQIRUZKLFKWK YHORFLW D GRXEOHV E WUSOHV DQG F TXDGUXSOHV

0 DFK ZDYHV

- 3 $QREVHUYHUDWWHDOHYHOGRHVQRWKHDUDQDLUFUDIWO0LQJ DW IWXDQGDU DQDXGHXQDLWLV VWDWXM PLSDWWKHU(VWPDWWKHDLUFUDIWSHGLQIW
- 3 $ VWQ VFUDFKLQWKHVLGHRI DVXSHUVRQLF ZLQGWXQQHODFUHDWV DYHU ZHDNZDYHR DQJOH f DV VKRZQ LQ)LJ 3 DWWUZKLFK D QRUPDOVKRFN RFFXUV 7KHD VWPSHUDXUHLQUHJLRQ LV . (VWPDWWKHVWPSHUDXUHLQUHJLRQ

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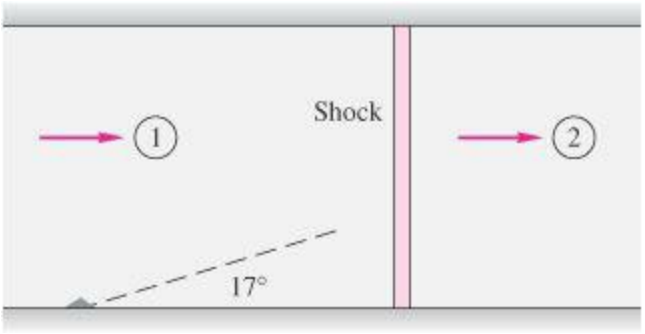

- 3 $ SDUWFOH PRYLQJ DWXQLRUP YHORFLW LQ VHDOHYHODWDQGDU DUFHDWW WKHVWZ GLWXEDQFHVSKHUHV VKRZQ LQ)LJ 3 &RPSXWWKHSDUWFOHYHORFLW DQG0 DFK QXPEHU

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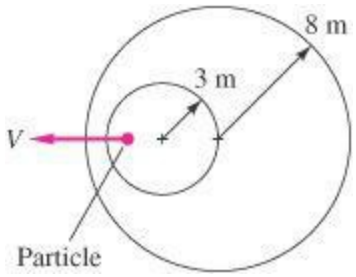

- 3 7KH SDUWLFOH LQ) LJ 3 LV PRYLQJ VXSHUPRQLFDOO LQ VHDOHYHOVDQGDUG DLU) URP VKH VZR JLYHQ GLVWXUEDQFH VSKHUHV FRPSXWH WKH SDUWLFOH 0 DFK QXPEHU YHORFLWV DQG0 DFKDQJOH

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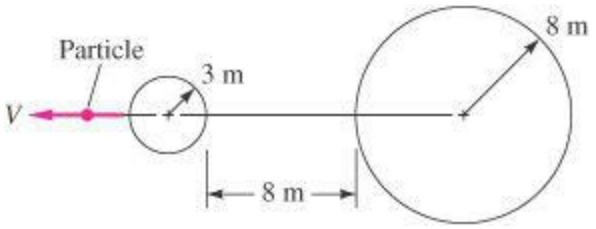

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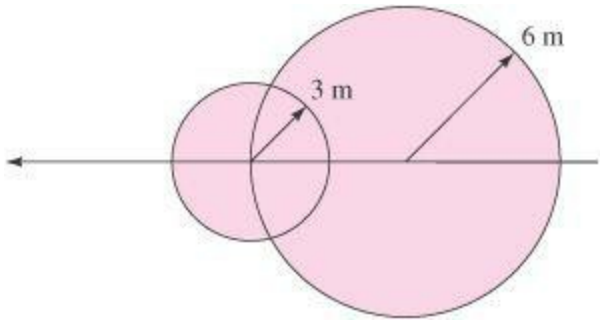

- 3 $ VKHUPLWXU SURFHVV LQ VKH VKDSH RI DQ HGGH SDUDOOHOVR WKH IORZ UHDGV D VDWXU VWP SHUDXUH RI í f& ZKHQ LQVHUWHG LQWR D VXSHUVRQLF DLUVWUHDP $ FRQFD GLVWXUEDQFHFRQHRI KDOI DQJOH f LVFUHDWHG (VWP DW D VKH0 DFK QXPEHU E VK YHORFLWV DQG F VKHVDQDQQXODUSRLQWVP SHUDXUHRI VKHVWUHDP

7KHREOTXHVKRFN Z DYH

- 3 6XSHUVRQLF DLU VWUHV D f FRPSUHVVLRQ VKLQ DV LQ) LJ 3 &RPSXWH VK GRZQVWUHDP SUHVVXUH DQG0 DFK QXPEHU DQG VKH Z DYH DQJOH DQG FRPSDUH Z LV VP DOOGLVWXUEDQFHVKHRU

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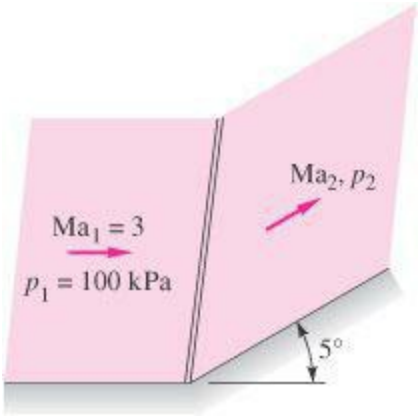

- 3 7KH ƒ GHOHFWLRQ LQ([DPSOH FDXVHG D ILQDO0 DFK QXPEHUR DQG I SUHVVXUHUDWLR R &RPSDUHWKLVZLWKWKHFDVHRI WKHIORZ SDVLQJ WKURXJKVWI ƒ GHOHFWLRQV &RPPHQWRQWKHUHVXOWDQGZK\ WKH PLJKWEHKLJKHURUORZHULQWK VHFRQGFDVH
- 3 : KHQ D VHDOHGHYHOIORZ DSSURDFKHV D UDPS R DQJOH ƒ DQREOTXHVKRFNZDY IRUPVDVLQ) LJXUH3 &DOFXODWH D 0 D E S F 7 DQG G 9

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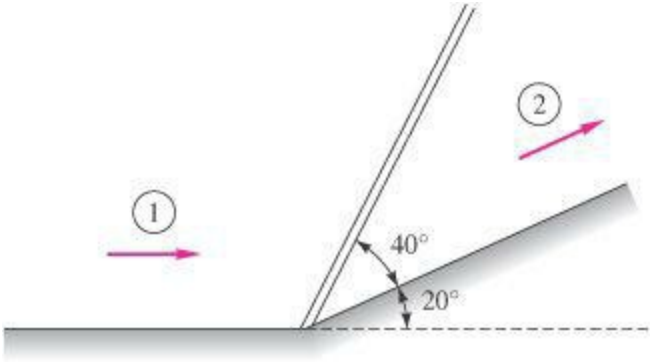

- 3 : HVDZ LQWKHWHVWVWKDWWRUN WKHPD[LPXP SRVVLEOHGHOHFWLRQFDXVHGE\ D REOTXHVKRFNZDYHRFFXUVDWQLQLWDSSURDFK0 DFKQXPEHUDQGLV PD[ƒ \$VXPLQJ DQLGHDOJDV ZKDWLV PD[IRU D DUJRQDQG E FDUERQGLR[LGH'
- 3 &RQVLGHUDLUFRZ DW0 D &DOFXODWH WKRWZR GHFLPDOSODFHV D WKHGHOHFWLRQ DQJOH IRUZKLFK WKH GRZQVWHP IORZ LV VRQLF DQG E WKH PD[LPXP GHOHFWLRQ DQJOH
- 3 ' RWKH0 DFKZDYHVXSVWUHDP RI DQREOTXHVKRFNZDYHLQWUHFWZLWKWKHVKRFN \$VXPLQJ VXSHUVRQLF GRZQVWHP IORZ GRWKHGRZQVWHP 0 DFKZDYHVLQWUHF WKHVKRFN' 6KRZ WKDWRUVPDOOGHOHFWLRQV WKHVKRFNZDYHDQJOH GHVKDOZD EHWZHHQ DQG IRUDQ\0 DFKQXPEHU
- 3 \$LUIORZVSDVWDZHGLPHQVLRQDOZHGJHGRVHGERG\ DVLQ) LJ 3 ' HWUPLQ WKHZHGJHKDOIDQJOH/ IRUZKLFK WKHKRUL]RQWDOFRPSRQHQWRI WKHVRDOSUHVVXU IRUFHRQWKHQRVHLV N1 P RI GHSWKLQRWKHSDSHU

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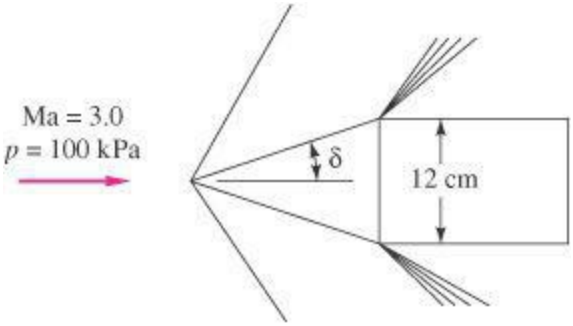

- 3 $LUIORZ V DWXSHUVRQLF VSHHG WKURXJK D FRPSUHVVLRQ UDPS DV LQ) LJ 3 $ VFUDWFK RQ WKH ZDOO DW SRLQW D FUHDWHV D ZDYHIURQW f DQJOH Z KLFH WKH HEOTXHVKRFN FUHDWHG KDV D f DQJOH : KDWLV D WKH UDPS DQJOH DQG E WKH ZDYH DQJOHİ FDXVHGE\ D VFUDWFK DWE"

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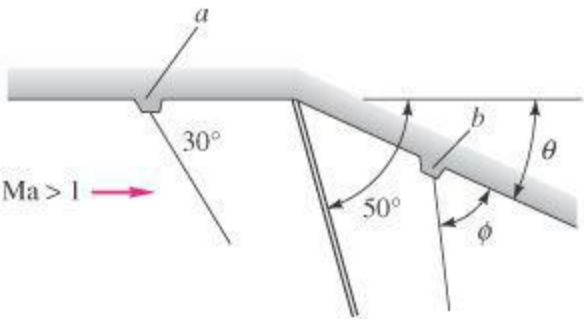

- 3 $ VXSHUVRQLF DLUIORZ DWD WKP SHUDXUH R . WWLNHV D ZHGJH DQG LV GHIOHFWH f ,I WKH UHVXOWLQJ VKRFN ZDYH LV DWWDFKHG DQG WKH WKP SHUDXUH DIWUWKH VKRFN L . D HWUP DWWKH DSSURDFK 0 DFK QXP EHUDQG Z DYH DQJOH E : K\ DUH WKH VZR VROXWRQV"
- 3 7KH IRORZLQJ IRUP XOD KDV EHHQ VXJJHVWHG DV DQ DSSUR[LPDWH VR UHDO XSVWUHP 0 DFK QXP EHUR WKH REOTXH VKRFN ZDYH DQJOH DQG WXUQLQJ DQJOH

$$\sin^2 \beta = \frac{1}{\mathrm{Ma}_1^2} + \frac{(k+1)\sin\beta\sin\theta}{2\cos(\beta-\theta)}$$

&DQ\RX SURYH RUGLVSURYH WKLV UHODWRQ" ,I QRW WU DIHZ QXP HULFDO YDOXHV DQG FRPSDU ZLWK WKH UHVXOWV IURP (T

- 3 $LUIORZ V DW0 D DQGS GEI LQ DEVROXWH WWR DQG D Z HGJH RI f DQJOH D]HUR LQFLGHQFH LQ) LJ 3 ,I WKH SRLQW G HGJH LV IRUZ DUG Z KDWZ LOO EH WKH SUHVXUH DWSRLQW$" ,I WKH EXQW HGJH LV IRUZ DUG Z KDWZ LOO EH WKH SUHVXUH DWSRLQ %'

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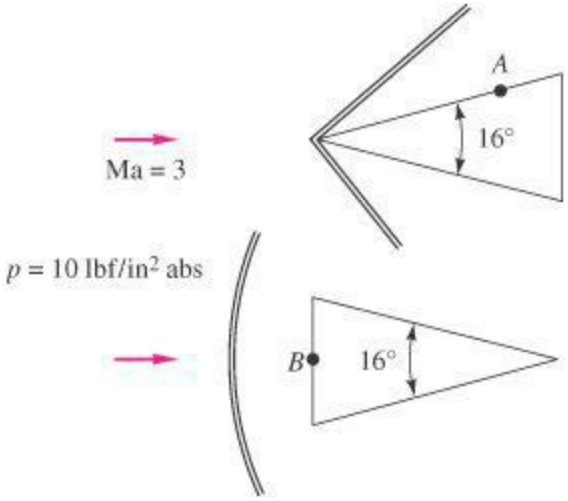

- 3 $LUIOR Z VVXSHUVRQLFDOO VWZ DUG WKH GRXEOH Z HGJH V\VWHP LQ) LJ 3 7KH [\ FRRUGLQDWHV RI WKH WSV DUH JLYHQ

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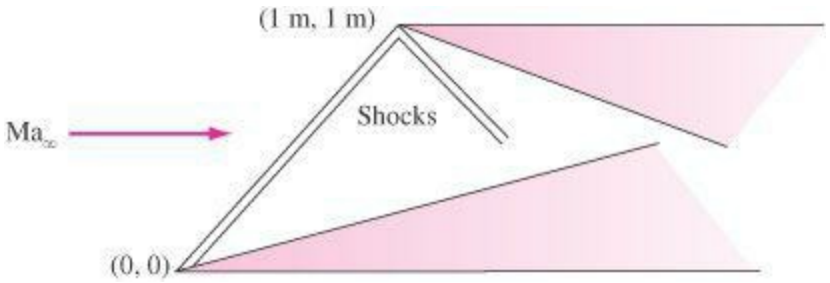

7KH VKRFN Z DYH RI WKH IRUZ DUG Z HGJH VWUNHV WKH WS RI WKH DIWZ HGJH %RWK Z HGJHV KDY ƒ GHOHFWRQ DQJOHV : KDWV WKH IUHH VWUHDP 0 DFK QXPEHU

- 3 EES : KHQ DQ REOTXH VKRFN VWUNHV D VRQG Z DOO LWUHOHFWV D VKRFN RI VXILFLHQ VWUHQJWK WR FDXVH WKH H[LWRZ 0 D WR EH SDUDOOHO WR WKH Z DOO DV) LJ 3) R DLUIOR Z LW0 D DQG S N3D FRP SXW0 D S DQG WKH DQJOHË

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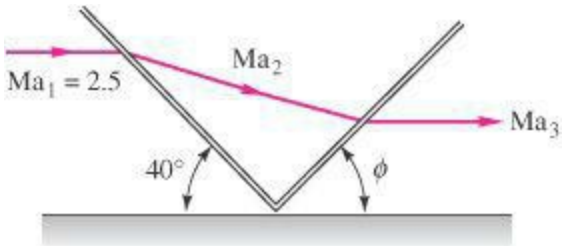

- 3 $ EHQG LQ WKH ERWWP RI D VXSHUVRQLF GXFWOR LQGXFHV D VKRFN Z DY H WKDW UHOHFW IURP WKH XSSHU Z DOO DV) LJ 3 &RP SXW WKH0 DFK QXPEHU DQG SUHVVXUH L UHJLRQ

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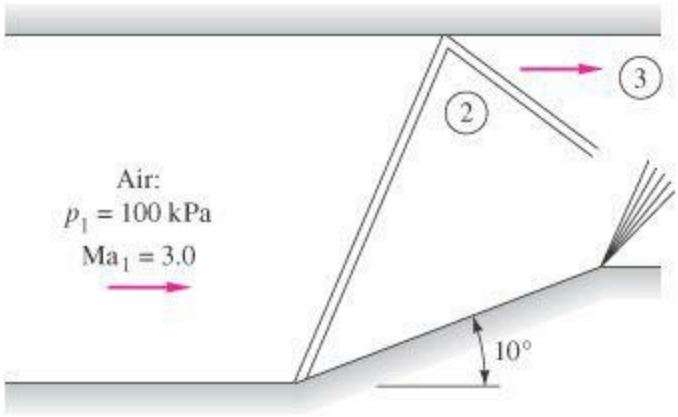

- 3) LJXUH3 LVDVSHFLDOODSSOLFDWLRQRI 3URE 3 : LWKFDUHXOGHVLJQ RQ FDQRUHQWKHEHQGRQWKHOHZ DOOVRWKDWWKHUHFWDQJXODUSODWH EHQGVWRDFKLHYHDFRQH[DFWO FDQFHOH E\ WKHUHXLQEHQG DVVKRZQ 7KLVLVDPHWKRGRI UHGXFLQJ WKH0DFKQXPEHULQ FKDQQHO D VXSHUVRQLF GLIIXVHU ,I WKH EHQG DQJOH LVË *f* ILQG D WK GRZQVWUHDP ZLGWKDQG E WKHGRZQVWUHDP 0DFKQXPEHU\$VXPHDZHDNVKRFN ZDYH

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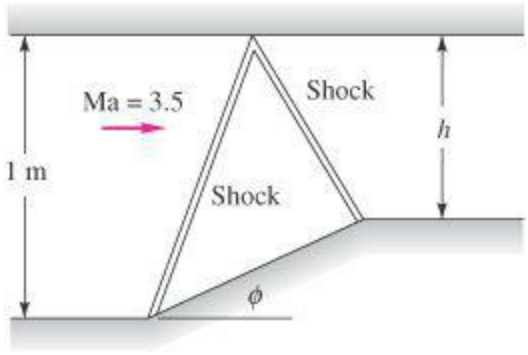
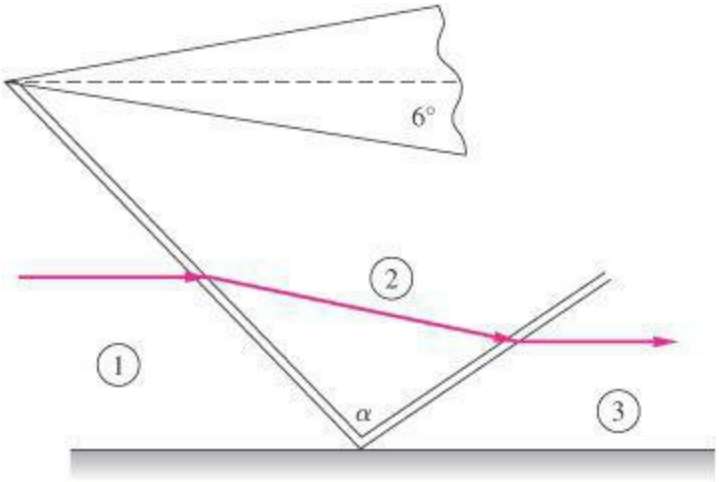


- 3 $ *f* KDO DQJOHZHGJHFUHDWVWKHUHOHFWHGVKRFNVWUHP LQ) LJ 3 ,I 0 D ILQG D 0 D DQG E WKHDQJOH.

3



- 3 7KHVXSHUVRQLF QR]]OHR) LJ 3 LVRYHUH[SDQGHG FDVH* R) LJ E ZLW

$_H$ $_W$ DQG D VXDQDQRSUHVXUH RI NBD ,I WKH MHWHGJHPDNHV D ƒ DQJ
ZLWKWKHQR]]OHFHQWHOLQH ZKDWLVWKHEDFNSUHVXUHS$ LQNBD'

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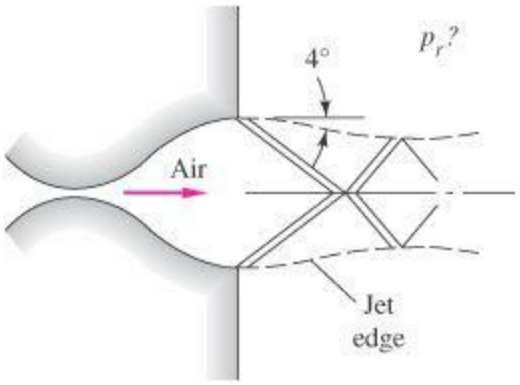

- 3 $ LUGRZ DW0 D VDNHV DFRPSUHVVLRQWXUQR ƒ DQGWKHQDQRWKHUWXUQR
DQJOH LQ) LJ 3 : KDWLVWKHPD[LPXP YDOXHRI IRUWKHVHFRQGVKRFNWRE
DWDFKHG' : LOOWKHVZRVKRFNVLQWHUVHFWRUDQ GHVWKDQ $PD["

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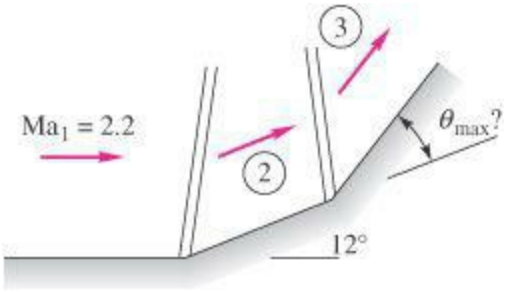

3UDQGWM0 H\HUH[SDQVLRQ ZDYHV

- 3 7KHVROXWLRQ R3URE 3 LV0 D DQGS NBD &RPSDUHWKHV
UHVXOWZLWKDQLVHQWURSLF FRPSUHVVLRQWXUQR ƒ XVLQJ 3UDQGWM0 H\HUWKHRU
- 3 6XSHUVRQLF DLUGRZ VDNHV D ƒ H[SDQVLRQ WXUQ DV LQ) LJ 3 &RPSXWH WK
GRZQVWUHDP 0DFK QXPEHU DQG SUHVVXUH DQG FRPSDUH ZLWK VPDOOGLVWXUEDQF
WKHRU

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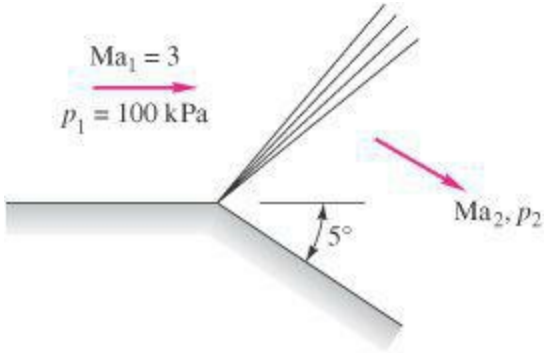

- 3 $ VXSHUVRQLF DLUIORZ DW0 D DQGS NBD XQGHUJRHV DFRPSUHVVLRQVKRFN IROORZHG E\ DQLVHQWURSLF H[SDQVLRQ WKURXJK 7KH IORZ GHOHFWLRQ LV f IRUHDFK VKRFN &RPSXWH 0 D DQGS LI D WKH VKRFN LV IROORZHG E\ WKH H[SDQVLRQ DQG E WK H[SDQVLRQ IROORZHG E\ WKH VKRFN
- 3 $LUIORZ DW0 D SDVVHV WKURXJK D f RECQTXH VKRFN GHOHFWLRQ : KL LVHQWURSLF H[SDQVLRQ WKURXJK VUHTXLUHG WR EULQJ WKH IORZ EDFN WR 0 D DQG E S"
- 3 7KH f GHOHFWLRQ LQ([DPSOH FDXVHG WKH 0 DFK QXPEHU WR GURS WR D : KDW WKLQ DQJOH ZLOO FUHDWH D 3UDQGWO H\HU DQG EULQJ WKH0 DFK QXPEHU BDFN WR " E : KDW ZLOL EH WKH ILQDO SUHVVXUH'
- 3 EES $LUDW0 D DQGS NBD XQGHUJRHV DQ LVHQWURSLF H[SDQVLRQ WR I GRZQVWUHDP SUHVVXUHR NBD : KDW VWKHGHVLUHG WKLQDQJOHILQGHJUHHV"
- 3 $LUIORZ V VXSHUVRQLFDOO RYHUD VXUIDFH WKDWFKDQJHV GLUHFWLRQ WZLFH DV LQ) LJ 3 &DOFXODW 0 D DQG E S

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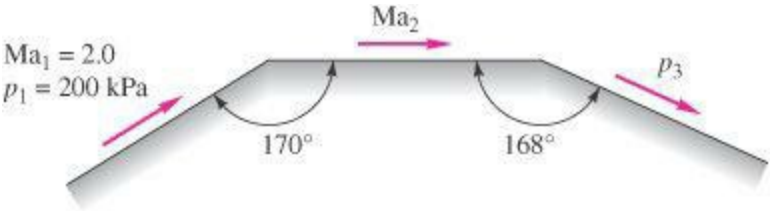
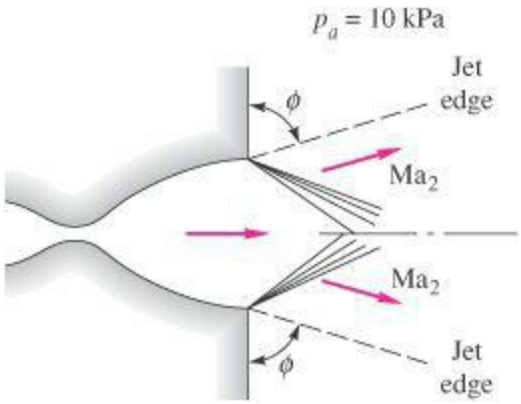


- 3 $ FRQYHUJLQJ±GLYHUJLQJ QR]]OHZ LWKD H[LWDUHDUDWR DQGS NBDRSHUDWH LQ DQ XQGHU[SDQGHG FRQGLWLRQ FDVH, RI) LJ E DV LQ) LJ 3 7K UHFHLYHU SUHVXUH LVS NBD ZKLFK LV OHVV WKDQ WKH H[LWSUHVXUH VR WKL H[SDQVLRQ ZDYHV IRUP RXWLGH WKH H[LW) RU WKH JLYHQ FRQGLWLRQV ZKDWZLOO WK 0 DFK QXPEHU0 D DQG WKH DQJOHË RI WKH HGJH RI WKH NHWWEH' $VXPHN D XVXDO



- 3 $ LUIORZ VVXSHUVRQLFDOO RYHUDFFXUDWHDVLQ) LJ 3 (VWLPDWH D VKH 0 DFK QXPEHU 0 D DQG E VKH SUHVVXUHS DV VKH IORZ GHDYHV VKH FFXUDFH VXUDFH

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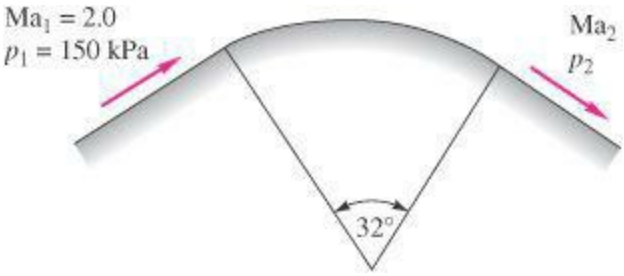

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- 3 $ LUIORZ VDW0 D SDWD GRXEO VPPHWUF GLDPRQG DLUIRLOZ KRVH IURQWDQ UHDUJLQFXGHG DQJOHV DUH ERWK f)RU]HUR DQJOH RI DWWDFN FRPSXWH WKH GUD FRHILFLHQWREWDLQHG XVLQJ VKRFN H[SDQVLRQ WKHRU DQG FRPSDUH ZLWK \$FNHU WKHRU
- 3 $ IODWSODWDLUIRLOZLWK& P LVWKDYHDQLWR N1 P ZKHQIO LQJ DW P VDQGEDG DOWXGHZLWK8, P V 8VLQJ \$FNHUHWKHRU HWLP DW D WKHDQJO RI DWWDFNDQG E WKHGUDJ IRUFHLQ1 P
- 3 \$LUIORZ VDW0 D SDWD KDO ZHGJH DLUIRLOZKRVH DQJOHV DUH f DV LQ) LJ 3 &RPSXWHWKHOWDQGGUDJ FRHILFLHQWDW HTXDOWR D f DQG E f

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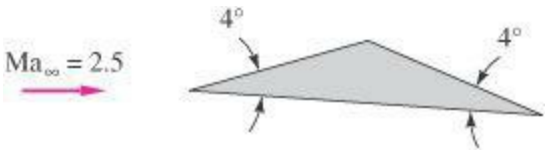

- 3 7KH ; PRGHO\$ VFUDPMHWDLUFUDWWLQ) LJ LV VP DOO: GE DQ XQP DQQHG RQO IWOBRQJ DQG IWZ LGH 7KH DHURG\QDPLFV RI D VBHQG-DLUZ KHDG VKDSHGK\SHUVRQLF YHKLFOHLVEH\RQGRXUVFRSH ,QWKDG GHWXVDXXP HL

G NJ V
H NJ V

) (

-) (,I WKH H[LVW0 DFKQXP EHULQ) LJ) (LV ZKDWLV WKH H[LWDUHD'
  D P
  E P
  F P
  G P
  H P
-) (,I WKH UHDUHQR VKRFNZDYHV DQG WKH SUHVVXUHDVRQHGXFWVFLQ) LJ) (LV NBD ZKDWLV WKH YHORFLWDVWKDWVFLQ'
  D P V
  E P V
  F P V
  G P V
  H P V
-) (,I LQ) LJ) (WKHUH LV DQRUP DOVKRFNZDYHDWD VFLQZKHUH WKH DUHDLV P ZKDWLV WKH DUGHQVLWVWXSWUHP RI WKDWVKRFN'
  D NJ P
  E NJ P
  F NJ P
  G NJ P
  H NJ P
-) (,Q3URE) (ZKDWLV WKH0 DFKQXP EHUWXWGRZQWUHP RI WKHVKRFNZDYH'
  D
  E
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&RP SUHKHQVLYH3 URE OPV

- & 7KH FRQYHUJLQJ±GLYHUJLQJ QR]]OH VNHWKHG LQ) LJ & LV GHVLJQHG WR KDYH D0 DF QXP EHUR DWWKH H[LWSODQH DVXPLQJ WKH IORZ UHPDLQV QHDU0 LVHQWURSLF 7KH IORZ WDYHOVIURP WDQNDWR WDQNE ZKHUHWDQNDLVPXFKODUJHUWKDQWDQNE D) LQGWK

DUHD DWWKH H[LWS DQG WKH EDFN SUHVVXUH S WKDW ZLOO DOORZ WKH VWXPS WR RSHUDWH D
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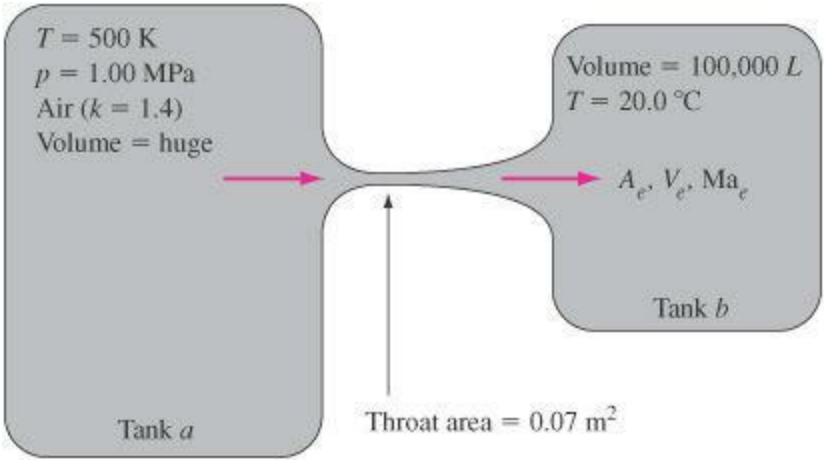
WDQN VORZV GRZQ ILOOV XS ZLWK PRUH DLU 6LQFH WDQN D LV KXJH WKH IORZ LQ WKH QR]]OH ZLO
UHPDLQ WKH VDPH KRZHYHU XQOLNH QRUPDO VKRFN ZDYH DSSHDUV DW WKH H[LW SODQH $
ZKDW EDFN SUHVVXUH ZLOO WKLV RFFXU F ,I WDQN E LV KHOG DW FRQVWDQW SHUDWXUH 7
f& HVWLPDWH KRZ ORQJ LW ZLOO WDNH IRU WKH IORZ WR JR IURP GHVLJQ FRQGLWLRQV WR WK
FRQGLWLRQ SDUW E ² WKDW V ZLWK DVKRFN ZDYH DW WKH H[LW SODQH

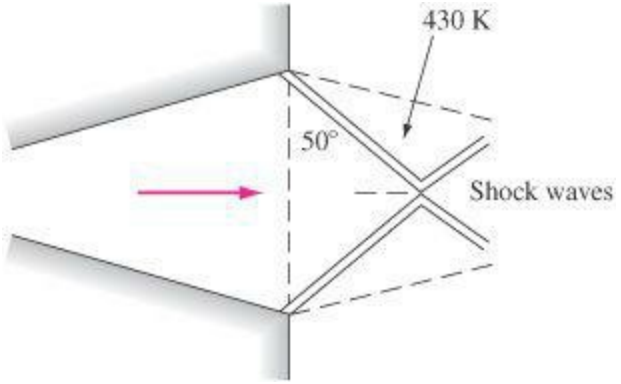
- & 7ZR ODUJH DLU WDQNV RQH DW . DQG N3D DQG WKH RWKHU DW . DQG N3D
DUH FRQQHFWHG E\ D VKDUS WKLQ WXEH P ORQJ DQG FP LQ GLDPHWHU 7KH YHUJHIUFWR
IDFWRUV $VVXPH LQ GLDEDWLF IORZ HVWLPDWH WKH PDVV IORZ WKURXJK WKH WXEH

&



- &) LJXUH & VKRZV WKH H[WHUQDO FRQYHUJLQJ±GLYHUJLQJ QR]]OH ZKHUH DQ REOTX
VKRFN SDWWHUQ LV IRUPHG ,Q WKH H[LWSODQH ZKLFK KDV DQ DUHD RI FP WKH DL
SUHVVXUHLV N3D DQG WKH WHPSHUDWXUHLV . -XVWRXWVLGH WKH H[LWVKRFN ZKLF
PDNHV DQDQJOH RI f ZLWK WKH H[LWSODQH WKH WHPSHUDWXUHLV . (VWLPDWH D WK
PDVV IORZ E WKH WKURDWDUHD F WKH WXUQLQJ DQJOH RI WKH H[LWOW DQG LQ WKH VDQ
VXSSOLQJ WKH DLU G WKH SUHVVXUHDQG H WKH WHPSHUDWXUH

&



- & 7KH SURSHUWLHV RI D GHQVH JDV KLJK SUHVVXUH DQG ORZ WHPSHUDWXUH DUH RIWH
DSSUR[LPDWHG E\ YDQGHU: DDOV¶V HTXDWLRQ RI VWDWH> @

$$p = \frac{\rho RT}{1 - b_1\rho} - a_1\rho^2$$

ZKHUHFRQVWDQWV D DQG E FDQEHIRXQGIURP WKHFULWLFDOWHPSHUDWXUHDQGSUHVVXUH

$$a_1 = \frac{27R^2T_c^2}{64p_c} = 9.0 \times 10^5 \text{ lbf} \cdot \text{ft}^4/\text{slug}^2$$

IRUDLU DQG

$$b_1 = \frac{RT_c}{8p_c} = 0.65 \text{ ft}^3/\text{slug}$$

IRUDLU)LQG DQ DQDO\WLF H[SUHVVLRQ IRU WKH VSHHG RI VRXQG RI D YDQGHU: DDOV JDV $VVXPLQJ N FRPSXWH WKH VSHHG RI VRXQG RI DLU LQ IWV DW *f* DQG DW IRU D I SHUIHFWDV DQG E DYDQGHU: DDOV JDV : KDWSHUFHQWDJHKJKHUGHQVLW\GRHVWKHYDQGH : DDOVUHDWRQSUHGLFW

- & &RQVLGHU RQH GLPHQVLRQDO VWHDG\ IORZ RI D QRQLGHDO JDV VWHDP LQ D FRQYHUJLQ QR]]OH 6WDUWLQJ DW RQ FRQGLWLRQV DUHS N3D DQG7 *f*& 7KH QR]]OH H[I GLDPHWHUV FP ,I WKH QR]]OH[LWSUHVVXUHLV N3D FDOFXODWH WKH PDVV IORZ DQG WK H[LWWHPSHUDWXUH IRUUHDOVWHDP HLWKHUIURP WKH VWHDP WDEOHVRUXVLQJ ((6 $ VDILO HVWLPDWH DW DVXPHVWHDP WR EH DQLGHDOJDV IURP 7DEOH$,V WKHIORZ FKRNHG' + RZ GRHV((6 HVWLPDWH WKH H[LW0 DFK QXPEHU' E)LQG WKH QR]]OH[LWSUHVVXUHDQGPDV IORZ IRUZKLFK WKHVWHDP IORZ LVFKRNHG XVLQJ ((6 RUWKHVWHDP WDEOHV
- & ([WHQG3URE & DVIRORRWV / HWWKH QR]]OHEH FRQYHUJLQ±GLYHUJLQ ZLWK DQH[L GLDPHWHUURI FP $VVXPHLVHQWURSLF IORZ D)LQG WKH H[LW0 DFK QXPEHU SUHVVXU DQG WHPSHUDWXUHIRUDQLGHDOJDV IURP 7DEOH$ ' RHV WKHPDVV IORZ DJUHZ LWK WK YDOXHRI NJ VLQ3URE & " E ,QHVWLDWH EUHO WKHXVHRI ((6 IRUWK SUREOHP DQG H[SODLQ ZK\ SDWD LV XQUHDOLVWLF DQG SRRUFRQYHUJHQFH RI ((6 L REWDLQHG +LQW6WXG\ WKHSUHVVXUHDQGWHPSHUDWXUHVDWSUHGLFWHG\ SDWD
- & 3URHVVRU* RUGRQ+ROZD\ DQG KLV VWXGHQW-DVRQ%HWWH RI WKH8QLYHUVLWV RI 1H %UXQZLFN REWDLQHG WKH IROORZLQJ WDEXODWHG GDWD IRUEORZ GRZQ LQWKURXJK FRQYHUJLQ±GLYHUJLQ QR]]OHVLP LODULQVKDSHVR) LJ 3 7KHVXSSO WDQNSUHVVXU DQG WHPSHUDWXUHZHUH SVLJ DQG *f UHVSHFWLYHO $WRVSKHULF SUHVVXUHZDV SVLD : DOO SUHVVXUHV DQG FHQWHUOLQH WDUJHWQXPEHUV ZHUH PHDVXUHG LQ WK H[SDQVLRQVHFWLRQ ZKLFK ZDV DIUXVWXP RI DFRQH 7KHQR]]OHWKURDWV DW 8 V WKHWDUJHWQXPEHUSUHVVXUHGDWDWRHVWLP DWH WKHORFDO DFK QXPEHU &RPSDUHWKHPHDVXUH 0 DFK QXPEHUVDQG ZDOOSUHVVXUHV ZLWKWKSUHGLFWLRQVRI RQHGLPHQVLRQDOWKHRU) R [! FP WKHWDUJHWQXPEHUSUHVVXUHGDWZDVQRWWKRXJKWE\ +RORZD\ DQG%HWWHWREH YDOLGPHDVXUHRI 0 DFK QXPEHU: KDWV\WKHSUREDEOHUHDVRQ'

x(cm)	0	1.5	3	4.5	6	7.5	9
Diameter (cm)	1.00	1.098	1.195	1.293	1.390	1.488	1.585
p_{wall} (psig)	7.7	−2.6	−4.9	−7.3	−6.5	−10.4	−7.4
$p_{stagnation}$ (psig)	29	26.5	22.5	18	16.5	14	10

- & (QJLQHUV FDOO WKH VXSHUVRQLF FRPEXVWRU LQ WKH VFUDPMHW) LJ DOP RV PLUDFXORXV ³QDNHGJKWQJ DP DWK LQDKXULFDQH´) LJXUH & LV DFXGHLGHDQ]DWR RI WKH HQJLQH $LUHQWUV EXLOV IXHOLQ WKH QDUURZ VHFWLRQ WKHQH[W DOODWXSHUVRQL VSHHGV 7KHUHDUHQR VKRFNZDYHV $VXPHDUHVRI P DWWHFWLRQV DQG DQG P DWWHFWLRQV DQG / HWWKHHQWDQFHFRQGLWLRQVEH0 D DW P WDQGDU DOWXGH $VXPHLVHQWRSLF IORZ IURP WR IUHHVWUHDPWKDWVHQWHUIURP WR ZLWK 4 N- NJ DQG LVHQWURSLF IORZ IURP WR &DOFXODWWKHH[LWFRQGLWLRQV DQG WK WKUXWSURGXFHG

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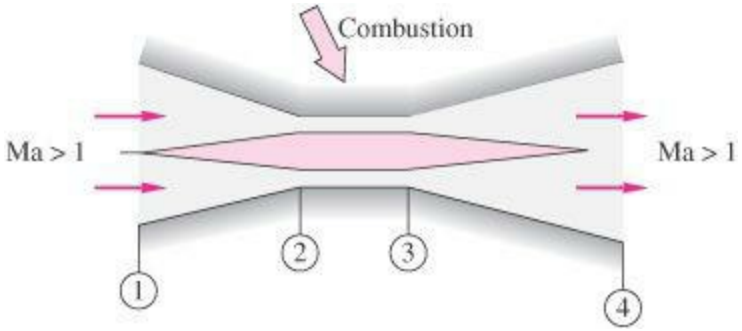
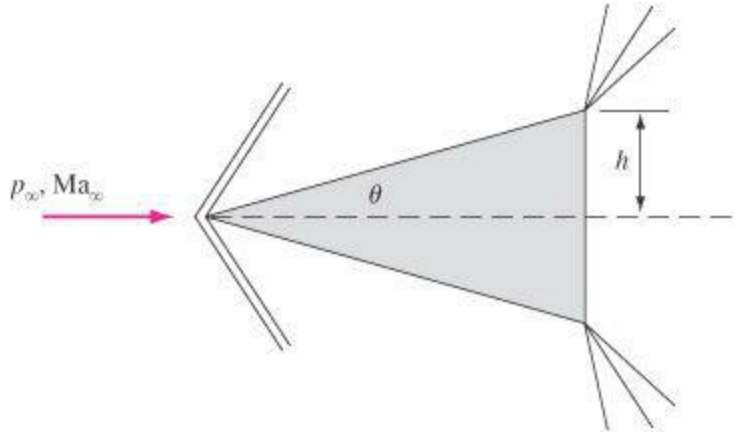


' HVLJQ3URFHVV

- ' ,WLVGHVLUHGWRVHOHFWDUHFWDQJXODUZLQJIRUDILJKWHUDLUFUDIW7KHSDQHPXWEHDEO D WR WDNH RII DQG ODQG RQ D IWGRJ VHDOHYHOUXQZD\ DQG E WR FULVH VXSHUVRQLFDOO DW0 D DW IWDOWXGH) RUVLPSOLFLW DWXPHDZLQJ ZLWK]HURVZHHSEDFN / HWWKHDLUFUDIWP D[LPXP ZHJKWTXDO Q GE ZKHUHQL WKHQXPEHURI OHWWHUV LQ\RXUVXUQDPH / HWWKHDYDLODEOHVHDOHYHOP D[LPXP WKUXWE RQHWKLUG RI WKH P D[LPXP ZHJKWGHFUHDVLQ DWDOWXGH SURSRUWRQDO WR P EHQ GHQVLW 0 DNLQ VXUHEHDWXP SWRQVDERXWKHHIHFWRI ILQWDVSHFWDWRRQZLQJ QL DQG GUDJ IRUERWK VXEVRQLF DQG VXSHUVRQLF IOLJKWVHOHFWD ZLQJ RI PLQLPXP DUH VXIILFLHQWWR SHUPLW WKHVH WDNHRII DQGLQJ DQG FUXLVH UHTXLUHPHQWV 6RPH WKRXJK VKRXOGEHJLYHQWR DQDO]LQJ WKHZLQJVDQGZLQJ URRWVLQVXSHUVRQLF IOLJKWZKHU 0 DFK FRQHVIRUP DQG WKH IORZ LV QRWWR GLPHQVLRQDO ,I QR VDWLVIDFWRU VROXWLRQ L SRVVLEOH JUDGXDOO LQFUHDVHWKHDYDLODEOHWKUXWWRFRQYHUWDQDFFHSWDEOHGHVLJQ
- ' &RQVLGHUXSHUVRQLF IORZ RI DLUDWWHD OHYHOFRQGLWLRQVSDWWD ZHGJHRI KDOI DQJOH DVVKRZQLQ) LJ ' $VXPHWKDWWKH SUHVXUH RQWKHEDFN RI WKHZHGJHHTXDOV WK IOXLG SUHVXUH DVLWH[LWVWKH 3UDQGWM0 H\HUIDQ D 6XSSRVH0 D) RUZKD DQJOH ZLOOWKHVXSHUVRQLF ZDYHGUDJ FRHILFLHQW& EDVHGRQIURQDODUHD EHH[DFW " E 6XSSRVHWKDW ƒ ,VWKHUHDIUHVWUHP 0 DFKQXPEHUIRUZKLFKWKHZDY GUDJ FRHILFLHQW& EDVHGRQIURQDODUHD ZLOOEH H[DFW " F ,QYHVWLDWWKH SHUFHQWDJH LQFUHDVH LQ& IURP D DQG E GXH WR LQFOXGLQJ ERXQGDU\ OD\HUIULFWLRQ GUDJ LQWKHFDOFXODWLRQ

'



5 HIHUHQFHV

- ($ -RKQ DQG 7 * . HLWK* DV' \QDPLFV G HG 3HDUVRQ (GXFDWLRQ 8SSHU6DGGO 5LYHU 1 -

% . +RGJH DQG . . RHQLJ &RPSUHVVLEOH) OXLG ' \QDPLFV : LWK 3HUVRQDO &RPSXWH $SSOLFDWLRQV 3HDUVRQ3UHQWLFH +DOO8SSHU6DGGOH5LYHU 1 -

5 ' =XFNHUDQG2 %LEODU] G HG) XQGDPHQWDOVRI * DV' \QDPLFV : LOH\ 1HZ <RUN

- ' $QGHUVRQ0 RGHUQ &RPSUHVVLEOH) ORZ ZLWK +LVWRULFDO 3HUVSHFWLYH G HG 0 F* UDZ +LOO1 HZ <RUN

$ + 6KDSLUR7KH' \QDPLFVDQG7KHUPRG\QDPLFVRI &RPSUHVVLEOH) OXLG) ORZ YROV : LOH\ 1HZ <RUN

& &HUFLJQDQL5DUHLHG* DV' \QDPLFV &DPEULGJH8QLYHUVLW3UHVV 1HZ <RUN

+ : /LHSPDQQDQG$ 5RVKNR (OHPHQWVRI * DV' \QDPLFV ' RYHU 1HZ <RUN

, 6WDVNXDED ,QWURGXFWRQ WR WKH 0 DWKHPDWLFDO7KHRU RI &RPSUHVVLEOH) ORZ 2[IRU 8QLYHUVLW3UHVV 1HZ <RUN

3 $ 7KRPSVRQ &RPSUHVVLEOH) OXLG' \QDPLFV 0 F* UDZ +LOO1 HZ <RUN

3 + 2RVWKX]HQ DQG : (&DUFDQRQ &RPSUHVVLEOH) OXLG) ORZ 0 F* UDZ +LOO1 H <RUN

- ' $QGHUVRQ +\SHUVRQLF DQG +LJK7HPSHUDWXUH* DV' \QDPLFV G HG $,$$ 5HVWR 9$

* $ %LUG 0 ROHFXODU* DV' \QDPLFVDQGWKH' LUHFW6LPXODWRQ RI * DV) ORZV &ODUHQGR 3UHVV 2[IRUG

' ' . QJKW(OHPHQWV RI 1XPHULFDO 0 HWKRGV IRU &RPSUHVVLEOH) ORZV &DPEULGJ 8QLYHUVLW3UHVV 1HZ <RUN

/ : 5HLVPDLHU0 DFK DQG %HRQG 7KH ,QWURGXFWLG * XLGH WR +LJK 6SHHG) OJK 0 F* UDZ +LOO

: 7 3DUU $60 (,QWHUQDWRQDO6WDQ 7DEOHV IRU ,QGXVWULDO8VH$ 60 (1HZ <RU

- + . HHQDQHWDO * DV7DEOHV ,QWHUQDWRQDO9HUVLRQ . UHJHU3XEOLVKLQJ 0 HOERXUQH) /

< $ &HQJHODQG 0 $ %RDUGV 7KHUPRG\QDPLFV $Q (QJLQHHULQJ $SSURDFK VW HG
0F* UDZ +LOO 1HZ <RUN
0 - 0RUDQ DQG + $ 6KDSLURI)XQGDPHQWDOV RI (QJLQHHULQJ 7KHUPRG\QDPLFV : LOH
1HZ <RUN
) 0 : KLWH 9LVFRXV) OXLG) ORZ GHG 0F* UDZ +LOO 1HZ <RUN
- 3DPHU . 5DPVGHQ DQG (* RRGJHU&RPSUHVVLEOH) ORZ 7DEOHV IRU (QJLQHHUV : LO
$SSURSULDWH&RPSXWHU3URJUDPV 6FKRODUV ,QQ 3RUW DVKLQJWRQ 1<
0 5 /LQGHEXUJ &RQVROLGDWHG * DV ' \QDPLFV 7DEOHV 3URHVVLRQDO 3XEOLFDWLRQV ,QF
%HOPRQW&$
$ 0 6KHNWDQ* DVG\QDPLF) XQFWLRQV RI 5HDO* DVHV 7D\ORU DQG) UDQFLV 1HZ <RUN

- + . HHQDQ DQG (3 1HXPDQQ ³0 HDVXUHPHQWV RI) UHFWLRQ LQ D 3LSH IRU 6XEVRQLF D
6XSHUVRQLF) ORZ RI $LU´ -RXUQDO RI $SSOLHG0 HFKDQLFV YRO QR S $
5 3 %HQHGLFW) XQGDPHQWDOV RI 3LSH) ORZ -RKQ: LOH 1HZ <RUN
- / 6LPV 7DEOHV IRU 6XSHUVRQLF) ORZ DURXQG 5LJKW&LUFXODU &RQHV DW=HUR $QJOH F
$WWDFN 1$6$ 63 VHHDOVR 1$6$ 63
- / 7KRPDV ³5HYRQGV 1XPEHU(IIHFWVRQ6XSHUVRQLF $VPPHWULFDO) ORZ VRYHUD&RQ-
-RXUQDORI $LUFUDIWYRO QR SS ±
: %REHUDQG5 $. HQRQ) OXLG0 HFKDQLFV : LOH 1HZ <RUN
- $FNHUHW³$LU) RUFHV RQ $LUIRLOV 0 RYLQJ) DVWHU WKDQ 6RXQG 9HDFLW´ 1$&$ 7HFK
0 HPR
: % %URZHU 7KHRU 7DEOHV DQG ' DWD IRU&RPSUHVVLEOH) ORZ 7D\ORU) UDQFLV 1HZ
<RUN
0 %HDLRUH ³7KH+\SHUVRQLF $JHV1HDU 3RSXODU6FLHQFH -DQXDU SS ±

– 7KH DXWKRU LV LQGHEWHG WR * HRUJHV $LJUHWRI &KLPD\ %HQJLXP IRU VXJJHVWLQJ WKLV XVH
IXQFWLRQ

– 7KLVLVWXHDOVRIRUPRWWDOJDHV VHH5H 6HF

– 7KLVLVWKHSULQFLSOHRI VKHVKRFNWXEHZLQGVXQQHDLQZKLFKDFRQWROOHG[SDQVLRQFUHDWHVI
EUHIORZ DVYHU KLJK0 DFKQXPEHU ZLWKGDWDVDN-HQE\ IDWWHVSRQVHLQXXP HQW 6HH HJ 5H

– 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVVRI FRQWQXLW

– 7KLVVHFWLRQPD\ EHRPLWWGZLWKRXWORVVRI FRQWQXLW

– 7KH DXWKRUVDYHVWKHVHOLWHSURJUDPV IRUIXUWKHUXVH JLYLQJ WKHP QDPHV VXFK DV3UDQGW
0 H\HU

&KDSWMU 2SHQ&KDQQHO)ORZ

$JLDQWZDMUHDVHIURP WKH;LDRDQJGLUHVHUYRLUIHQWDO&KLQD¶V+HQDQSURYLQFH7K
UHDDVHZDVKHVVLOWURP WKHULYHUEHGDQGSURYLGHVZDMUVRGURXJKWDUHDVGRZQWRDP 7KL
VSHFWDFXODURSHQFKDQQHOIORZ FDQEHDQDO]HGE\ WKHPHWKRGVRI WKHSUHVHQWFKDSWMUDOWKRXJK
LWLVVRPHZKDWEH\RQGWKHLPDJLQDWRQRI WKLVZULWMU

&RXUWMX RI WKH$VVRFLDWG3UHVV

0 RWLYDWLRQ

$QRSHQFKDQQHOIORZ GHORWV DIORZ ZLWKDIUHHVXUIDFHWRXFKLQJ DQDWRVSKHUH GNHDULYHUR
DFDQDORUDIXPH &RVHG GXFWORZ V &KDS DUHIXODRI IOXLG HLWKHUGTXLGRUJDV KDYHQRIUH
VXUIDFHZLWKLQ DQG DUHGULYHQE\ DSUHVXUHJUDGLHQWDORQJ WKHGXFWD[LV 7KHRSHQFKDQQHOIORZ
KHUHDUHGULYHQE\ JUDYLWDORQH DQGWKHSUHVXUHJUDGLHQWDWKHDWRVSKHULF LQWHUIDFHLVQHJOLGLEOH
7KHEDVLF IRUHEDDQFHLQDQRSHQFKDQQHOLVEHWZHHQJUDYLWDQGIULFWLRQ

2SHQFKDQQHOIORZV DUH DQ HVSHFLDOO LPSRUWDQWPRGH RI IOXLG PHFKDQLFV IRU FLYLO DQ
HQYLURQPHQWDOHQJLQHHUV 2QHQHGVWRSUHGLFWWKHIORZ UDWHVDQGZDWUGHSWKVWKDWWHVXOWURP DJLYH
FKDQQHOJHRPHWU ZKHWKHUDQXDORUDUWLFLDO DQG DJLYHQZHWXUHURXJKQHVV : DWULV DOP RV
DQZDVWKHUHDYDQWXLG DQGWKHFKDQQHOVL]HLVXVXDOO CDJH 7KVRSHQFKDQQHOIORZ VDUHJHQHUDO
VXUEXHQWWKUHHGLP HQVLRQDO VRPHWMP HV XQVWDG\ DQG RIWQ TXLWFRPSO[7KLV FKDSWMU SUHVHQ
VRPHVLPSOHHQJLQHULQJ WKRUHVDQGH[SHULPHQWDOFRUHODWLRQVIRUVWDG\ IORZ LQVWUDJKWFKDQQHO
RI UHJXODU JHRPHWU : HFDQ ERURZ DQG XVH VRPHFRQFHSWIURP GXFWIORZ DQDOVLV K\GUDXOL
UDGLXV IULFWLRQIDFWRU DQGKHDGORVVHV

,QWURGXFWLRQ

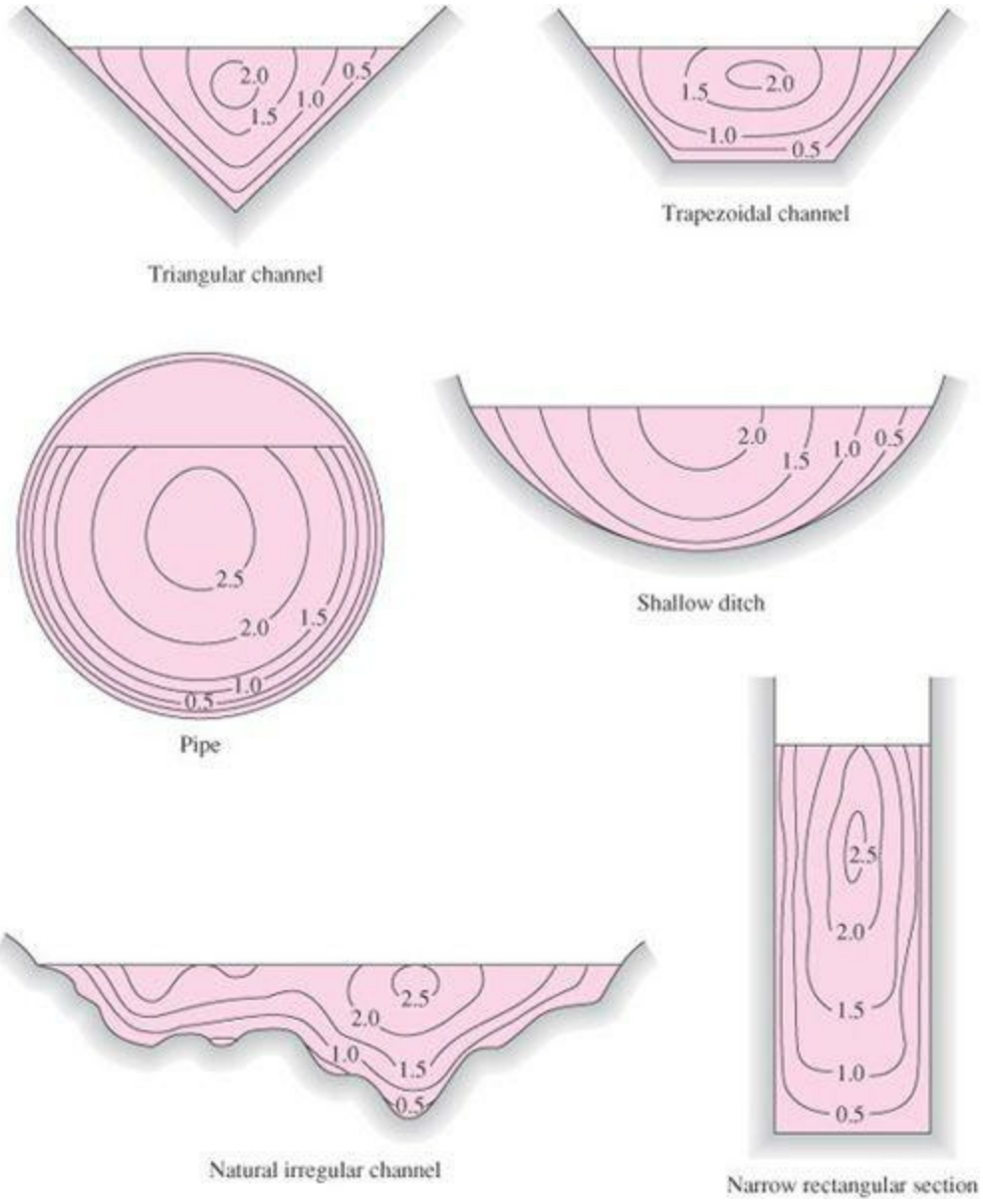
6LPSOH VWDWH RSHQFKDQQHOIGRZ LV WKH IGRZ RI DQ LQGLQDFWLYH LWHUDWLYH 7KHU
DUH PDQ SUDFWLFDO H[DPSOHV ERWK DUWLFLDO IXPHV VSLOOLQJ DV FDQDOV ZHLV GUDLQDJH GLWFKHV
FXOYHUWV DQG QDWXUDO VWUHDPV ULYHUV HVWXDULHV IGRRGSODLQV 7KLV FKDSWHU ULQWRGXFHV WKH HOHPHQWDU
DQDOVLV RI VXFK IGRZ V ZKLFK DUH GRPLQDWHG E\ WKH HIFWV RI JUDYLW

7KH SUHVHQFH RI WKH IUHH VXUIDFH ZKLFK LV HVVHQWLDOO DWDWRVSKHUF SUHVXUH ERWK KHSV DQ
KXWWKH DQDOVLV ,WKHSV EHFDXVH WKH SUHVXUHFDQEHWDNHQDV FRQVWDQWWDORQJ WKH IUHH VXUIDFH ZKLF
WKHUHRUH LV HTXLYDOHQWWR WKHK\GUDXOLF JUDGHQW +* / RI WKH IGRZ 8QIN IGRZ LQFORVHG GXFW
WKH SUHVXUH JUDGLHQWLV QRWD GLUHFWDFWRULQ RSHQFKDQQHOIGRZ ZKHUH WKH EDODQFH RI IRUFHV LV
FRQLQHG WR JUDYLWDQG IULFWLRQ %XWWKH IUHH VXUIDFH FRP SOLFDWHVWKH DQDOVLV EHFDXVHLWWVKDSHLV
SURUL XQNQRZQ 7KH GHSWK SURILOH FKDQJHV ZLWK FRQGLWLRQV DQG PXWWEH FRP SXWHG DV SDUWRI WK
SUREOP HVSHFLDOO LQXQWDG SUREOP VLQYROYLQJ ZDYHPRWLQ
%HRUH SURFHHGLQJ ZHUHPDUN DVXXDO WKDWZKROHERRNVKDYHEHQZULWWQRQSHQFKDQQ
K\GUDXOFV> WR @7KHUHDUHDOR VSHFLDOQHG WR[WGHYRWHG WR ZDYHPRWLQ> WR @DQG V
HQJLQHULQJ DSHFWRI FRDXDOIUHVXUFHIGRZV> WR @7KLVFKDSWULVRQO DQLQWRGXFWRQV
EURDGUDQG PRUHGHWDLOHG WUHDWHQW 7KH ZULWUHFRPPHQGV DV DQ RFFDVLRQDOEUHDN IURP IUH-
VXUFHIGRZ DQDOVLV 5H ZKLFK LV DQ HQFKDQWQJ DQG VSHFWFXOU JDOOHU RI RFHDQ ZDY
SKRWRJUDSKV

7KH2 QH' LP HQVLRQDO$ SSUR[LP DWLRQ

$ QRSHQ FKDQQHODZ DV KDV WZR VLGHV DQG D ERWWRP ZKHUH WKH IGRZ VDWWLHV WKH QR VOS
FRQGLWLRQ 7KHUHRUH HYHQ D VWUDJKWFKDQQHOKDV D WKUHHGLP HQVLRQDOYHORFLW GLVWULEXWLRQ 6RP
PHDVXUHPHQWV RI VWUDJKWFKDQQHOYHORFLW FRQRXV DUH VKRZ Q LQ) LJ 7KH SURILOHV DUH TXL
FRP SOH[ZLWK PD[LPXP YHORFLW\ \WSLFDOO RFFXUULQJ LQ WKH PLGSOQH DERXW SHUFHQWEHORZ WK
VXUIDFH ,QYHUV EURDG VKDOORZ FKDQQHOV WKH PD[LPXP YHORFLW\ LV Q-DUWKH VXUIDFH DQG WKH YHORFLW
SURILOH LV Q-DUO GRJDULWKPLF IURP WKH ERWWRP WR WKH IUHH VXUIDFH DV LQ (T ,Q QRQFLUFXO
FKDQQHOV WKH ERWWRP DQG VLGHZDOV PRWRQV VLP LODU WR) LJ IRUFORVHG GXFWV ,I WKH FKDQQ
FXUYHV RUP HDQGHU WKH VHFRQGDU PRWRQ LQWQVLWLHV GXH WR FHQWULXJDOH IHFW ZLWK KJK YHORFLW
RFFXUULQJ Q-DUWKH RXWUDGLXV RI WKH EHQG &XUYHG QDWXDOFKDQQHOV DUH VXEMHFWWR VWRQJ ERWWRP
HURVLRQDQG GHSRVLWLRQ HIFWV

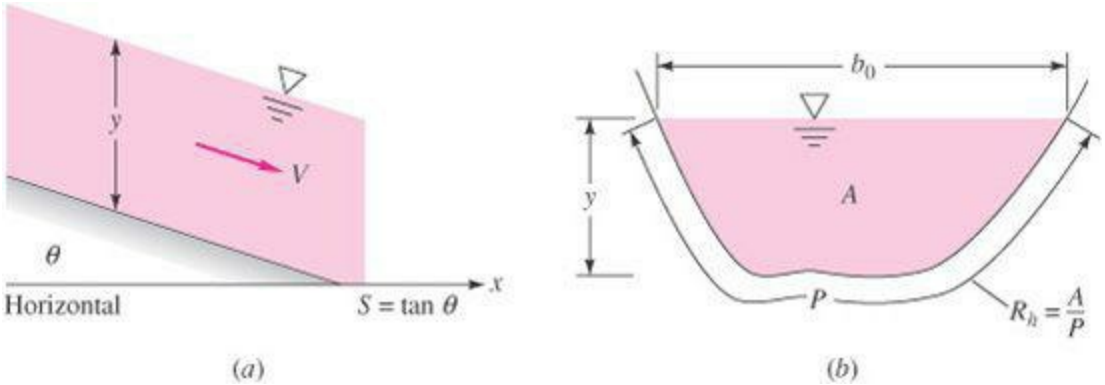
) LJ



0 HDVXUHGLVRYHOFLW FRQWRXUVLQWSLFDOWDJKWRSHQFKDQQHOIORZV

) URP 5H

) LJ



* HRP HWU DQG QRWDWLRQ IRU RSHQ FKDQQHOIORZ *a* VLGH YLHZ *b* FURVV VHFWLRQ $OOWKHU SDUDP HWHUV DUH FRQWDQWQWQXQLRUP IORZ

: LWKWKH DGYHQWRI WKH VXSHUFRPSXWHU SURYLGHV PDNHQXPHULFDOLPXODWRQV RI FRPSOH[IORZ SDWWHUQV VXFK DV LQ) LJ > @+ RZ HYHU WKH SUDFWDOHQJLQHHULQJ DSSURDFK XVHG KHUH LV WR PDNH DRQH GLPHQVLRQDO IORZ DSSUR[LPDWRQ DV LQ) LJ 6LQFH WKH GTXLG GHQVLW\ LV QHDU FRQVWDQW WKH VWDG\ IORZ FRQWLQXLW\ HTXDWRQ UHGXFHV WR FRQVWDQW YROXPHIORZ 4 DORQJ WKH FKDQQHO

$$Q = V(x)A(x) = \text{const} \qquad (10.1)$$

ZKHUH 9 LV DYHUDJH YHORFLW\ DQG $ WKH FRFDO FURVV VHFWLRQDO DUHD DV\VNHWFKHG LQ) LJ $ VHFRQG RQH GLPHQVLRQDO UHDWRQ EHWZHHQ YHORFLW\ DQG FKDQQHO JHRPHWU\ LV WKH HQHUJ\ HTXDWRQ LQFOXGLQJ IULFWLRQ ORVVHV ,I SRLQWV XSVWUHDP DQG GRZQVWUHDP DUH RQ WKH IUHH VXUIDFH S S S DQG ZH KDYH IRUWKH IORZ

$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 + h_f \qquad (10.2)$$

ZKHUH] GHQRWHV WKH HOHYDWLRQ RI WKH IUHH VXUIDFH ZKLFK LQFUHDVHV WKH ZDWHUGHSWK\ VHH) LJ D SOXV WKH KHLJKW RI WKH VORSLQJ ERWWRP 7KH IULFWLRQ KHDG ORVV K LV DQDORJRXV WR KHDG ORVV LQ GXFWORZ IURP (T

$$h_f \approx f\frac{x_2 - x_1}{D_h}\frac{V_{av}^2}{2g} \qquad D_h = \text{hydraulic diameter} = \frac{4A}{P} \qquad (10.3)$$

ZKHUH I LV WKH DYHUDJH IULFWLRQ IDFWRU) LJ EHWZHHQ VHFWLRQV DQG 6LQFH FKDQQHOV DUH LUUHJXODU LQ VKDSH WKH³ VL]H´ LV XVXDOO\ H[SUHVVHG LQ WHUPV RI WKH K\GUDXOLF UDGLXV

$$R_h = \frac{1}{4}D_h = \frac{A}{P} \qquad (10.4)$$

7KH ORFDO 5H\QROGV QXPEHU RI WKH FKDQQHO ZRXOG EH 5H 95$ ZKLFK LV XVXDOO\ KLJKO\ WXUEXOHQ ! (7KH RQO\ FRPPRQ RFF XUUHQ ODPLQDU FKDQQHO IORZ V DUH WKH WKLQ VKHHWV WKDW RUP D UDLQZDWHU GUDLQV IURP FURZ QHG VXUIHWV DQG DLUSRUWV DQG D\V

7KH ZHWWHG SHULPHWHU3 VHH) LJ E LQFOXGHV WKH VLGH V DQG ERWWRP RI WKH FKDQQHO EXW QRW WKH IUHH VXUIDFH DQG RI FRXUVH QRW WKH SDUW RI WKH VLGH V DERYH WKH ZDWHU OHYHO) RU H[DPSOH LI I UHFWDQJXODU FKDQQHO LV E ZLGH DQG K KLJK DQG FRQWDLQV ZDWHU WR D GHSWK\ LWZ HWWHG SHULPHWHU LV

3 E \

QRWE K

$ OWHUQXJK WKH 0 RRG\ FKDUW) LJ ZRXOG JLYH D JRRG HVWLPDWH RI WKH IULFWLRQ IDFWRU L FKDQQHO IORZ LQ SUDFWLFH LWV VHOGRP XVHG $Q DOWHUQDWLYH FRUUHODWLRQ GXH WR 5REHUW0 DQQLQJ GLVFXVVHG LQ6HF LV WKH IRUPXOD RI FKROLFHLQRSHQ FKDQQHOK\GUDXOLFV

) ORZ &ODVVLILFDWLRQ E\ ' HSWK 9 DULDWLRQ

7KH PRVW FRPPRQ PHWKRG RI FODVVLI\LQJ RSHQ FKDQQHO IORZ V LVE\ WKH UDWLR FKDQJH RI WKH IUH-

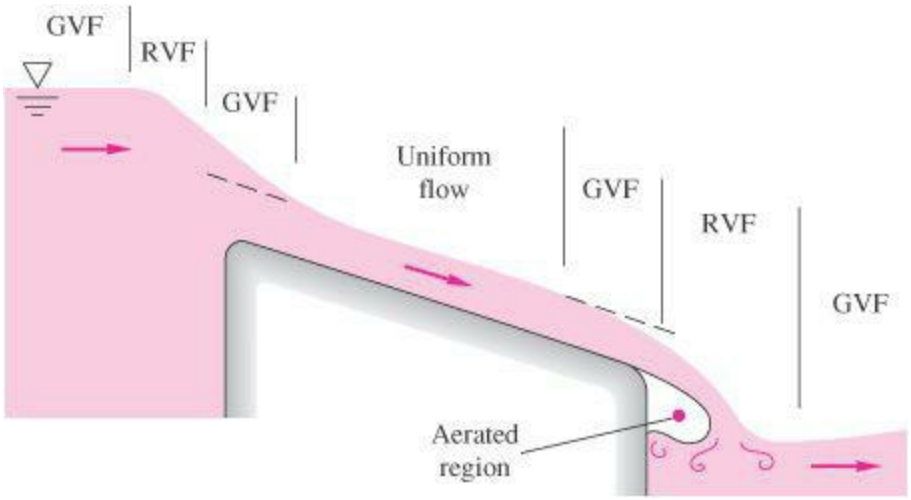
VXUIDFHGHSWK 7KHVLPSOHVWDQGPRVWZLGHO DQDO]HGFDVHLVXQLIRUP IORZ ZKHUHWKHGHSWKDQGDUH
KHQFHWKHYHORFLWLVLQVWDG\ IORZ UHPDLQFRQVWDQW8QLIRUP IORZ FRQGLWLRQV DUHDSSUR[LPDWHGE\
ORQJVWUDLJKWXQVRI FRQVWDQWVORSHDQGFRQVWDQWDUHDFKDQQHO$ FKDQQHOLQXQLIRUP IORZ LVVDLGWREH
PRYLQJ DWXQLIRUPGHSWK\Q ZKLFKLVDOLPSRUWDQWGHVLJQSDUDPHWHU

,I WKHFKDQQHOVORSHRUFURVVVHFWLRQFKDQJHVRUWKHUHLVDQREVWUXFWLRQLQWKHIORZ WKHQWKHGHSWK
FKDQJHV DQG WKH IORZ LV VDLG WR EHYDULHG 7KH IORZ LVJUDGXDOO\ YDULQJ LI WKH RQHGLPHQVLRQ
DSSUR[LPDWLRQLVYDOLGDQGUDSLGO\ YDULQJ LI QRW6RPHH[DPSOHVRI WKLVPHWKRGRI FDOFXODWLRQDU
VKRZQLQ) LJ 7KHFDOFXODWLRQEHVXPPDUL]HGVIROORZV

8QLIRUP IORZ FRQGLWLRQVHDSSURDFKDQGVORSH
9DULHG IORZ
* UDGXDOO\ YDULHG RQHGLPHQVLRQDO
5DSLGO\ YDULHG PXOWLGLPHQVLRQDO

7\SLFDO XQLIRUP IORZ LVVHSDUDWHG IURP UDSLGO\ YDULQJ IORZ E\ DUHJLRQ RI JUDGXDOO\ YDULHG
IORZ * UDGXDOO\ YDULHG IORZ FDQEHDQDO]HG E\ DILIIHUHQWLDOHTXDWLRQ 6HF EX
UDSLGO\ YDULQJ IORZ XVXDOO\ UHTXLUHV H[SHULPHQWDWLRQ RU WKHRHWLFDOPRGLILFDWLRQ RI WKHRHWLFDO FRPSXWDWLRQDO IORZ
GYQDPLFV> @

) LJ



2 SHQFKDQQHOIORZ FDWLLHGE\ UHJLRQVRI UDSLGO\ YDULQJ IORZ 59) JUDGXDOO\ YDULQ
IORZ * 9) DQGXQLIRUP IORZ GHSWKSURILOHV

) ORZ &ODVVLILFDWLRQ E\ ) URXGH1 XPEHU

$ VHFRQGDQGYHU XVHXOFODVVLILFDWLRQ RI RSHQFKDQQHOIORZ LV E\ WKHGLPHQVLRQOHVV) URXG
QXPEHU) U ZKLFK LVWKHUDWLR RI FKDQQHOYHORFLWVRWKHVSHHG RI SURSDJDWLRQ RI DVPDOOGLVWXUEDQFH
ZDYHLQWKHFKDQQHO) RUDUHFWDQJXODUURYHU ZLGHFRQVWDQWGHSWKFKDQQHOWKLVLVJLYHQE\WKHIRUP

$$\mathrm{Fr} = \frac{\text{flow velocity}}{\text{surface wave speed}} = \frac{V}{\sqrt{gy}} \qquad (10.5)$$

ZKHUH\ LVWKHZDWHUGHSWK7KHIORZ EHKDYHVGLIIHUHQWO GHSHQGLQJ RQWKHVHWKUHHIORZ UHJLPHV

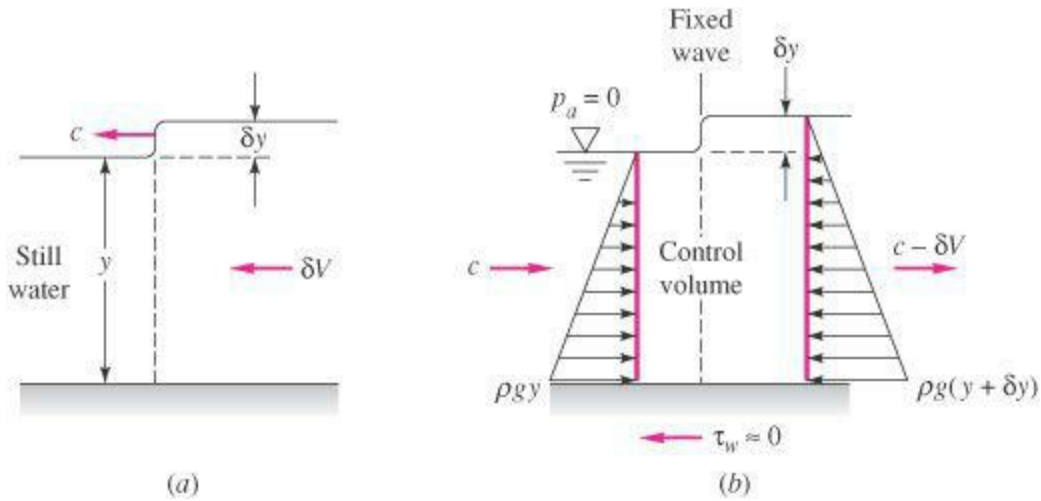
$$\begin{aligned} \mathrm{Fr} &< 1.0 \quad \text{subcritical flow} \\ \mathrm{Fr} &= 1.0 \quad \text{critical flow} \\ \mathrm{Fr} &> 1.0 \quad \text{supercritical flow} \end{aligned} \tag{10.6}$$

7KH)URXGHQXPEHUIRULUUHJXODUFKDQQHOV LV GHILQHG LQ6HF $ VPHQWRQHG LQ6HF VWKHUHLVDWWRQJ DQDORJ\ KHUHZLWKWKHWKUHHFRPSUHVVLEOHIORZ UHJLPHVRI WKH0DFKQXPEHU VXEVRQLF 0D VRQLF 0D DQG VXSHUVRQLF 0D! : HVKDOOXVHWKHDQDORJ\ LQ6HF 7KDQDORJ\ LVSXUVXHGLQ5H

6XUIDFH: DYH6SHHG

7KH)URXGHQXPEHUGHQRPLQDWRU J\ LV WKHVSHHGRI DQLQILQLWHVLPDOVKDOORZ ZDWHUVXUIDFH ZDYH : HFDQGHULYHWKLVZLWKUHIHUHQFHWR) LJ D ZKLFKVKRZVDZDYHRI KHLJKW\ SURSDJDWLQ DWVSHHGF LQWR VWLOO OTXLG 7R DFKLHYH D VWDG\ IORZ LQHUWLDOIUDPH RI UHIHUHQFH ZH IL[WK FRRUGLQDWHVRQWKHZDYHDVLQ) LJ E VRWKDWWKHVWLOOZDWUPRYHVWRWKHULJKWDWVSHHGF) LJXU LVH[DFWO DQDORJRXVWR) LJ ZKLFKDQDO]HGWKHVSHHGRI VRXQGLQDIOXLG ,WFDQEHXVHGWDQDO]HWKDOOERUHV) LJ 3 ZKLFKDUHGHVFULEHGE\ &KDQRQ> @

) LJ



$ QDOVLVRI D VPDOOVXUIDFH ZDYH SURSDJDWQJ LQWR VWLOOVKDOORZ ZDWU $ PRYLQJ ZDYH-QRQVWDG\ IUDPH $ IL[HGZDYH LQHUWDOIUDPHRI UHHUHQFH

)RUWKH FRQWROYROXPH RI) LJ E WKH RQHGLPHQVLRQDO FRQWLQXLW UHDWLRQ LV IRUFKDQQH-ZLGWK E

$$\rho c y b = \rho(c - \delta V)(y + \delta y)b$$

or

$$\delta V = c\frac{\delta y}{y + \delta y} \tag{10.7}$$

7KLVLVDQDORJRXVWR(T WKHYHORFLW FKDQJH/9 LQGXFHGE\ DVXUIDFHZDYHLV VPDOOL WK

$ VWDLJKWUHFDQXDUFKDQQHOLV IWZLGHDQG IWGHSDQGDLGRQDVRSHR f 7KHIUFWR
IDFWRUV (VWLPDWHXQIRUP IRZ UDWLQFXELFHHVSHUVHFRQG

(

6RXWLRQ

- 6\VWHP VNHWFK 7KHFKDQQHOFURVVVHFWLRQLVVKRZQLQ) LJ (
- $VXPSWLRQV 6WDGH XQIRUP FKDQQHOIRZ ZLWK f
- $SSURDFK (YDOXDWHV &Kp]\ IRUPXOD (T RU
- 3URSHUWLHV YDOXHV 3OHDVH QRWH WKDWWKHUH DUHQR IXLG SK\VLFDOSURSHUWLHVLQYROYHG LQ WK &Kp]\ IRUPXOD &DQ\RXH[SODLQWKLV"
- 6RXWLRQVWHS 6LPSOHYDOXDWHDFKWLP LQWH&Kp]\ IRUPXOD (T

- &RPPHQWV 8QIRUP IRZ HVWLPDWHV DUH VWDLGKWRZDUGLI WKHJHRPHWU LV VLPSOH 5HVXO DUHLQGHSHQGHQWRI ZDWUGHQVLW DQG YLVFRVLW EHFDXVHWKHIRZ LV IXOO URXJK DQG GULYHQ E JUDYLW 1RWWKHKLJK IRZ UDWH ODUJHUWKDQ VRPHULYHUV 7ZRGHJUHHVLV DVXEVWDQWLDOFKDQQ VOPSH

7KH0 DQQLQJ 5RXJKQHVV&RUUHODWLRQ

7KH PRVWXQGDPHQWDO VRXQG DSSURDFK WR WKH &Kp]\ IRUPXOD LV WR XVH(T ZLWK HVWLPDWHG IURP WKH 0 RRG\ IULFWLRQ IDFWRU FKDUW) LJ ,QGHG WKH RSHQFKDQQHO UHVHDUF HVWDEOLVKPHQW> @XWRQJO UHFRPPHQGV XVHR WKHIULFWLRQIDFWRULQ DOOFDOFXODWLRQV 6LQFH WWSLFD FKDQQHOV DUH ODUJH DQG URXJK ZH ZRXOG JHQHUDOO XVH WKH IXOO URXJK WXUEXOHQW RZ OPLWRI (T

$$f \approx \left(2.0 \log \frac{14.8R_h}{\epsilon}\right)^{-2} \qquad (10.15)$$

ZKHUHëLVWKHURXJKQHVVKHLJKWZLWKWVSLFDOYDOXHVOLVWHGLQ7DEOH

,QVSLWHRI WKHDXWKRUVRI WKLVIULFWLRQIDFWRUDSSURDFK PRVWHQJLQHHUV SUHIHUWRXVH L
VLPSOH GLPHQVLRQDO FRUUHODWLRQSXEOLVKHGLQ E\ 5REHUW0 DQQLQJ > @DQ,ULVK HQJLQHHU,
WWWZLWKUHDFKDQQHOV 0 DQQLQJ IRXQG WKDWWKH &Kp]\ FRHILFLHQW& LQFUHDVHG DSSUR[LPDWH DV WK
VL[WKURRWRI WKHFKDQQHOV]H + HSURSRVHGWKHVLPSOHIRUPXOD

$$C = \left(\frac{8g}{f}\right)^{1/2} \approx \alpha \frac{R_h^{1/6}}{n} \qquad (10.16)$$

ZKHUHQ LV D UROXJKQHVV SDUDPHWHU 6LQFH WKH IRUPXOD LV FOHDUO QRWGLPHQVLRQDOO FRQVLVWHQWL
UHTXLUHVDFRQYHUVLRQIDFWRU. WKDWFKDQJHVZLWKWKHV\VWHP RI XQLWVXVHG

$$\alpha = 1.0 \quad \text{SI units} \qquad \alpha = 1.486 \quad \text{BG units} \qquad (10.17)$$

5HFDOOWKDWZHZDQWHGDERXWWKLVDZNZDUGQHVVLQ([DPSOH <RXPD\ YHUL\ WKDW LVWKHFXEH
URRWRI WKHFRQYHUVLRQIDFWRUEHWZHHQWKHPHWULDQG \RXUFKRVHQ QHQJWK VFDOH ,Q %* XQLWV.
IWP

7DEOH ([SHULPHQWDO9DOXHVRI 0 DQQLQJ¶VQ) DFWRU

		Average roughness height ϵ	
	n	ft	mm
Artificial lined channels:			
Glass	0.010 ± 0.002	0.0011	0.3
Brass	0.011 ± 0.002	0.0019	0.6
Steel, smooth	0.012 ± 0.002	0.0032	1.0
Painted	0.014 ± 0.003	0.0080	2.4
Riveted	0.015 ± 0.002	0.012	3.7
Cast iron	0.013 ± 0.003	0.0051	1.6
Concrete, finished	0.012 ± 0.002	0.0032	1.0
Unfinished	0.014 ± 0.002	0.0080	2.4
Planed wood	0.012 ± 0.002	0.0032	1.0
Clay tile	0.014 ± 0.003	0.0080	2.4
Brickwork	0.015 ± 0.002	0.012	3.7
Asphalt	0.016 ± 0.003	0.018	5.4
Corrugated metal	0.022 ± 0.005	0.12	37
Rubble masonry	0.025 ± 0.005	0.26	80

Excavated earth channels:			
Clean	0.022 ± 0.004	0.12	37
Gravelly	0.025 ± 0.005	0.26	80
Weedy	0.030 ± 0.005	0.8	240
Stony, cobbles	0.035 ± 0.010	1.5	500
Natural channels:			
Clean and straight	0.030 ± 0.005	0.8	240
Sluggish, deep pools	0.040 ± 0.010	3	900
Major rivers	0.035 ± 0.010	1.5	500
Floodplains:			
Pasture, farmland	0.035 ± 0.010	1.5	500
Light brush	0.05 ± 0.02	6	2000
Heavy brush	0.075 ± 0.025	15	5000
Trees	0.15 ± 0.05	?	?

*A more complete list is given in Ref. 2, pp. 110–113.

7KH0 DQQLQJ IRUPXOD IRUXQLIRUP IORZ YHORFLWLVWKV

$$V_0\,(\text{m/s}) \approx \frac{1.0}{n}[R_h\,(\text{m})]^{2/3}S_0^{1/2}$$
$$V_0\,(\text{ft/s}) \approx \frac{1.486}{n}[R_h\,(\text{ft})]^{2/3}S_0^{1/2} \qquad (10.18)$$

7KHFKDQQHOVOPSH6 LVGLPHQVLRQOHVV DQGQLVVDNHQWRE HWKHVPHLQERWKV\VWHPV 7KHYROXPH IORZ UDWHLPSOPXOWSOHVWKLVUHVXOWE\ WKHDUHD

$$Q = V_0 A \approx \frac{\alpha}{n} A R_h^{2/3} S_0^{1/2} \qquad (10.19)$$

([SHULPHQWDOYDOXHVRI Q DQGWKHFRUUHVSRQGLQJ URXJKQHVVKHJKWDUHGLVWULEXWHGLQ7DEOH IR YDULRXVFKDQQHOVXUIDFHV 7KHUHLVDIDFWRUI YDULDWLRQIURP DVPRRWKJODVVXUIDFH Q§ WR WHQGHG IORRGSODLQ Q§ ' XHWRWKHLUHJXODULWLRI WSLFDOFKDQQHOVKDSHVDQGURXJKQHVV WK VFDWWU EDQGV LQ7DEOH VKRXOG EH WDNHQ VHULRXVO) RU URXWLQH FDOFXODWLRQV DQZ D\ XVH WK DYHUDJHURXJKQHVVLQ7DEOH

6LQFH 0 DQQLQJ¶V VL[WKURRWW]H YDULDWLRQ LV QRW[DFWUDOFKDQQHOV FDQ KDYH D YDULDEOH GHSHQGLQJ RQWKHZDWUGHSWK 7KH0 LWLWLSSL5 LYHUHQHDU0 HPSKLV 7HQQHVVHH KDVQ§ DW IWORRG GHSWK DWQRUPDO IWGHSWK DQG DW IWORRG VWDJHGHSWK 6HDVRQDOYHJHWDWLYH JURZWKDQGIDFWRUVXFK DVERWWP HURVLRQFDQDOR DIHFWWKHYDOXHRI Q (YHQQHDU LGHQWFDOP DQ PDGH FKDQQHOV FDQ YDU %UDWUHWDO> @UHSRUWWKDW8 6 %UHDX R 5HFODPDWLRQ WWW RQ0DJ FRQFUHWHOLQHGFDQDOV\LHOGHGYDOXHVRI QUDQJLQJ IURP VR

(; $0 3/ (

(QJLQHHUVILQG WKDWWKH PRWHILFLHQWUHVDQJXODUFKDQQHO PD[LPXP XQLRUP IORZ IRUD JLYH DUHD IORZ VDWGHSWKHTXDOVRKDOI WKHEHWWP ZLGWK &RQVLGHUDUHVDQJXODUEUFNZ RUNFKDQQHOOL RQDVOPSHRI : KDWLV WKHEHWWERWWP ZLGWKIRUDIORZ UDWRI IWV"

6RXWLRQ

- $VVXPSWLRQV 8QLIRUP IORZ LQDVWDLJKWFKDQQHORI FRQVWDQWVORSH6
- $SSURDFK 8VHWKH0 DQQLQJ IRUPXODLQ(QJOLVKXQLWV(T VRSUHGLFWWKHIORZ UDWH
- 3URSHUWLYDOXHV)RUEULFNZRUN IURP 7DEOH WKHURXJKQHVVIDFWRUQ§
- 6ROXWLRQ)RUERWWRP ZLGWKE VDNHWKHZDWUDSSHUVKRWEH\ E (TXDWLRQ EHFRPHV

$$A = by = b(b/2) = \frac{b^2}{2} \quad R_h = \frac{A}{P} = \frac{by}{b + 2y} = \frac{b^2/2}{b + 2(b/2)} = \frac{b}{4}$$

$$Q = \frac{\alpha}{n} A R_h^{2/3} S^{1/2} = \frac{1.486}{0.015}\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3}(0.006)^{1/2} = 100\,\frac{\text{ft}^3}{\text{s}}$$

Clean this up: $b^{8/3} = 65.7$ solve for $b \approx 4.8$ ft *Ans.*

- &RPPHQWV 7KH0 DQQLQJ DSSURDFK LV VLPSOH DQG HIHFWLYH 7KH0 RRG\ IULFWLRQ IDFWR PHWKRG(T UHTXLUHVODERURXVLWHUDWLRQDQGGHDGVWRDUHXQOHVV§ IW

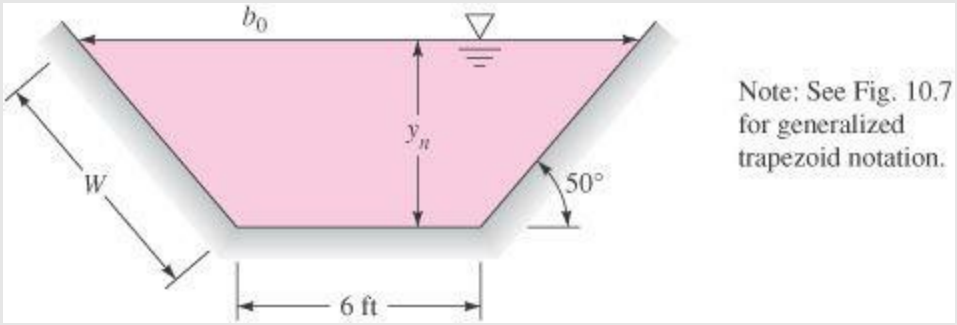
1RUPDO'HSWK(VWLPDWHV

: LWKZDWUGHSWK< NQRZQ WKHFRPSXWDWLRQRI 4 LVTXLWVWUDLJKWRUZDUG +RZHYHULI 4 LVJLYHQ WKH FRPSXWDWLRQ RI WKH QRUPDO GHSWK\Q PD\ UHTXLUH LWHUDWLRQ 6LQFH WKH QRUPDO GHSWK LV I FKDUDFWHULVWLF IORZ SDUDPHWHU WKLVLVDQLPSRUWDQWVSHFLDO SUREOHP

(; $0 3/ (

7KHDVSKDOWOLQHGWDSH]RLGDOFKDQQHOLQ)LJ(FDUULHV IWVRI ZDWXQGHUXQIRUP IORZ FRQGLWLRQVZKHQ6 : KDWLVWKHQRUPDOGHSWK\Q"

(



Note: See Fig. 10.7 for generalized trapezoid notation.

6ROXWLRQ

)URP 7DEOH IRUDVSKDOWQ§ 7KHDUHDDQGK\GUDXOLFUDGLXVDUHIXQFWLRQVRI \Q ZKLFK LVXQNQRZQ

$$b_0 = 6\text{ ft} + 2y_n \cot 50^\circ \qquad A = \tfrac{1}{2}(6 + b_0)y_n = 6y_n + y_n^2 \cot 50^\circ$$

$$P = 6 + 2W = 6 + 2y_n \csc 50^\circ$$

) URP 0 DQQLQJ¶V IRUPXOD Z LWK DNQRZ Q4 IWV Z HKDYH

$$300 = \frac{1.49}{0.016}(6y_n + y_n^2 \cot 50^\circ)\left(\frac{6y_n + y_n^2 \cot 50^\circ}{6 + 2y_n \csc 50^\circ}\right)^{2/3}(0.0015)^{1/2}$$

or
$$(6y_n + y_n^2 \cot 50^\circ)^{5/3} = 83.2(6 + 2y_n \csc 50^\circ)^{2/3} \quad (1)$$

2 QH FDQ LWHUDWH(T ODERURXV) DQG HYHQWXDOO ILQG\ Q § IW+ RZ HYHU LWLV D SHUFHF
FDQGLGDWH IRU ((6 ,QWKDG RI PDQSXODWLQJ DQG SURJUDPPLQJ WKH ILQDO IRUPXOD RQH PLJKWWLPSO
HYDOXDWH DFK VHSDUDWH SDUWV WR WKH &Kp]\ HTXDWLRQ LQ (QJOLVK XQLWV Z LWK DQJOHV LQ GHJUHHV

3 \QVLQ

$ \Q \QA WDQ

5K $ 3

$ 5KA A

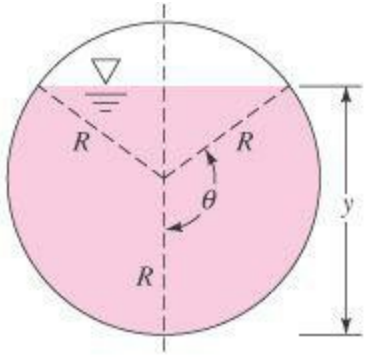
6HOHFW6RYHIURP WKHPHQXEDU DQG ((6 FRPSODLQVR I ³QHJDWLYHQXPEHUVWR DSRZHU ´ * REE
WR 9DULDEOH ,QRUPDWRQ WKH PHQXEDU DQG PDNH VXUH WKDW Q LV SRVLWLYH ((6 WKHQ LPPHGLDWH
VRQYHVIRU

P = 17.95 A = 45.04 $R_h = 2.509$ $y_n = 4.577$ ft *Ans.*

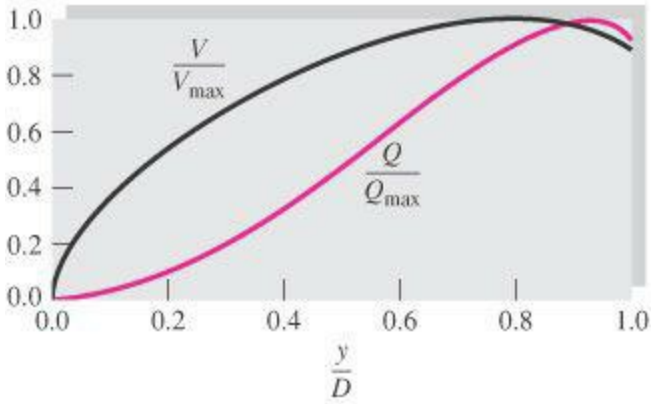
* HQHUDO ((6 LVLGHDO IRUPQFKDQQHO SUREOPVZ KHUHWKHGHSWKLV XQNQRZ Q

8 QLIRUP) ORZ LQ D 3 DUWV) XOO&LLF XODU3 LSH

) LJ



(a)



(b)

8QLIRUP IORZ LQD SDUWLDOO IXOOFLUFXODUFKDQQHO *a* JHRPHWU\ *b* YHORFLW\DQG IORZ UDWH YHUVXV GHSWK

&RQVLGHUWKH SDUWLDOO IXOOSLSH RI) LJ DLQXQLRUP IORZ 7KHPD[LPXP YHORFLW\DQG IORZ UDWHDFXDOO RFFXUEHRUHWKHSLSHLVFRPSOHWHO IXOO,QWHUPVRI WKHSLSHUDGLXV5 DQGWKHDQJOH XS VRWKHIUHHVXUIDFH WKHJHRPHWULFSURSHUWLHVDUH

$ = $R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$ $P = 2R\theta$ $R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)$

7KH0 DQQLQJ IRUPXODV SUHGLFWVDXQLRUP IORZ DVIROORZV

$$V_0 \approx \frac{\alpha}{n}\left[\frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right]^{2/3} S_0^{1/2} \qquad Q = V_0 R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \tag{10.20}$$

)RUDJLYHQQDQGVORSH6 ZHPD\ SORWWKHVHZRUHDWRQVYHUVXV\ ' LQ) LJ E 7KHUHDU VZRGLIIHUHQWPD[LPD DVIROORZV

$$V_{\max} = 0.718\frac{\alpha}{n}R^{2/3}S_0^{1/2} \quad \text{at} \quad \theta = 128.73^\circ \quad \text{and} \quad y = 0.813D$$
$$Q_{\max} = 2.129\frac{\alpha}{n}R^{8/3}S_0^{1/2} \quad \text{at} \quad \theta = 151.21^\circ \quad \text{and} \quad y = 0.938D \tag{10.21}$$

$ VVKRZ QLQ) LJ E WKHPD[LPXP YHORFLWLV SHUFHQWPRUHWKDQWKHYHORFLWZKHQUXQQLQ IXOODQGVLPLODUO WKHPD[LPXP GLVFKDUJHLV SHUFHQWPRUH6LQFHUHDOSLSHVUXQQLQJ QHDU IXOOMQ

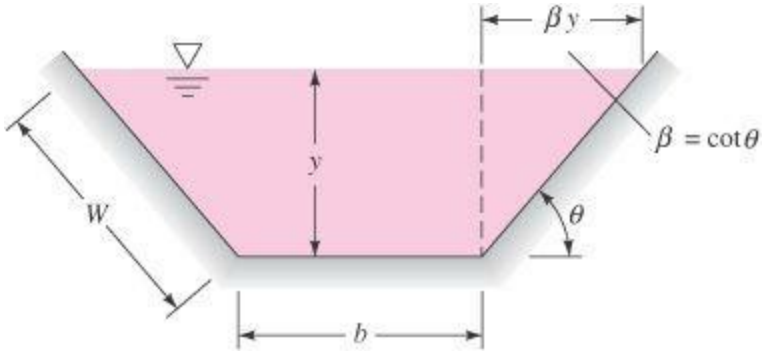
VRKDYHVRPHZKDWXQXVXDOEHIRUH VKHVHGLIIHUHQFHVDUHQRWKDWWLJQLFDQW

(IILFLHQW8 QLIRUP) ORZ &KDQQHOV

7KHHQJLQHHULQJ GHVLJQRI DQRSHQFKDQQHOKDVPDQ SDUDPHWHUV ,I WKHFKDQQHOVXUIDFHFDQHURG RUVFRXU D ORZ YHORFLW GHVLJQ PLJKWEH VRXJKW$ GLVFKDQQHOFRXOG EH SDDQG ZLWK JUDV VP PLQPL]HURVLRQ)RUQRQHURGLEOHVXUIDFHV FRQWUDFWRQDQGGDQJ FRVWPLJKWGRPLQDWH VXJJHVWLQJ D FURVV VHFWLRQ RI PLQPXP ZHWWG SHULPHWU 1RQHURGLEOHFKDQQHOV FDQEH GHVLJQHG IRUPD[LPXP ICRZ

7KH VLPSOHFLW RI 0 DQQLQJ¶V IRUPXODWRQ HQDEOHV XV WR DQDO]H FKDQQHO ICRZ V VL GHWHUPLQHWKHPRVWHILFLHQWORZ UHVLVWDQFHVHFWLRQV7KHPRVWFRPPRQSUREOHP LVWKDWRI PD[LPL]LQJ 5K IRUDJLYHQICRZ DUHDDQGGLVFKDUJH 6LQFH5K $ 3 PD[LPL]LQJ 5K IRUJLYH $ LVWKHVDPHDVPLQPL]LQJ WKHZHWWG SHULPHWU3 7KHUHVQR JHQHUDOURXWRQIRUDUEWUDU FURVVHFWRQV EXWDQDQDOVLVRI WKHWDSH]RLGVHFWRQZLOOVKRZ WKHEDVLF UHVXOWV

) LJ



* HRPHWU RI DWDSH]RLGDOFKDQQHOVHFWLRQ

&RQVLGHUWKHJHQHUDO]HG WDSH]RLG RI DQJOH LQ) LJ) RUD JLYHQ VLGH DQJOH WKHICRZ DUHDLV

$ = by + βy² β = cot θ (10.22)

7KHZHWWG SHULPHWULV

P = b + 2W = b + 2y(1 + β²)^1/2 (10.23)

(OLPLQDWLQJ E EHWZHHQ DQG JLYHV

P = A/y − βy + 2y(1 + β²)^1/2 (10.24)

7RPLQPL]H3 HYDOXDWHG3 G IRUFRQVWDQW$ DQG DQG VHWWTXDOWR]HUR 7KHUHVXOWV

A = y²[2(1 + β²)^1/2 − β] P = 4y(1 + β²)^1/2 − 2βy R_h = ½y (10.25)

7KH GDWWUHVXOWLV YHU LQWHUHVWLQJ) RUDQ DQJOH WKH PRWWH ILFLHQWFURVV VHFWLRQ IRUXQLRUP IORZ RFFXUV ZKHQWKH K\GUDXOLF UDGLXVLVKDOO WKHGHSWK

6LQFHDUHDQJOHLVDWDSH]RLGZLWK WKHPRWWH ILFLHQWDQJXODUVHFWLRQOVVXFKWKDW

$$A = 2y^2 \qquad P = 4y \qquad R_h = \tfrac{1}{2}y \qquad b = 2y \tag{10.26}$$

7R ILQG WKH FRUUHFWGHSWK\ WKHVHUHODWLRQV PXVWEH VROYHG LQFRQMXQFWLRQ ZLWK 0 DQQLQJ¶V IORZ UDWHIRUP XOD IRUWKHJLYHQGLVFKDUJH4

%HVW7 UDSH]RLG $ QJOH

(TXDWLRQ DUHYDOGIRUDQ YDOXHRI : KDWLVWKHEHVWYDOXHRI IRUDJLYHQGHSWKDQ DUHD' 7R DQVZHUWKLV TXHVWLRQ HYDOXDWG3 G IURP(T ZLWK$ DQG< KHGFRQWDQW7KH UHVXOWLV

$$2\beta = (1 + \beta^2)^{1/2} \qquad \beta = \cot\theta = \frac{1}{3^{1/2}}$$

$$\text{or} \qquad \theta = 60^\circ \tag{10.27}$$

7KXVWKHPD[LPXP IORZ WDSH]RLGVHFWLRQLVKDOO DKH[DJRQ

6LPLODUFDOFXODWLRQV ZLWKDFLUFXODUFKDQQHOVHFWLRQ UXQQLQJ SDUWDOO IXOOVKRZ EHVWHILFLHQF IRUDVHPLFLUFOH \ ½ ' ,QDFWWKHVHPLFLUFOHLV WKHEHVWRI DOOSRVVLEOHFKDQQHOVHFWLRQV PLQLPXP ZHWWG SHULPHWHUIRUDJLYHQ DUHD 7KHSHUHQWDJHLPSURYHPHQWRYHU VD\ KDOO DKH[DJRQ LV YHU VDJKWKRZ HYHU

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D : KDWDUHWKHEHVWGLPHQVLRQV\ DQGE IRUDUHFWDQJXODUEUFNFKDQQHOGHVLJQHGWRFDUU P V RI Z DWULQXQIRUP IORZ ZLWK6 " E &RPSDUHUHVXOWZ LWKDKDOO KH[DJRQDQG VHPLFLUFOH

6RXWLRQ

3DUW D

) URP(T $ \ DQG 5$ ½ \ 0 DQQLQJ¶V IRUPXOD LQ6, XQLWV JLYHV ZLWKQ 8 IURP7DEOH

$$Q = \frac{1.0}{n} AR_h^{2/3} S_0^{1/2} \qquad \text{or} \qquad 5\ \text{m}^3/\text{s} = \frac{1.0}{0.015}(2y^2)\left(\frac{1}{2}y\right)^{2/3}(0.001)^{1/2}$$

ZKLFKFDQEHVROYHGIRU

$$y^{8/3} = 1.882 \text{ m}^{8/3}$$

$$y = 1.27 \text{ m} \qquad \textit{Ans.}$$

7KHSURSHUDUHDDQGZLGWKDUH

$$A = 2y^2 = 3.21 \text{ m}^2 \qquad b = \frac{A}{y} = 2.53 \text{ m} \qquad \textit{Ans.}$$

3DUWE

,WLVFRQYHQLHQWWRVHHZKDWGRHVUDLVLQJWKHKHLJKWRIWKHVOXLFHJDWHKDYHRQ
DUHDRI P

)RUWKHKDOIKHLJKW ++ ZLWK (T SUHGLFWV

$ \ ++> @ \ ++

RU\ ++ P ZKHQFH5K ½ \ 7KHKDOIKHLJKWIORZ UDWHLVWKXV

$$Q = \frac{1.0}{0.015}(3.214)(0.681)^{2/3}(0.001)^{1/2} = 5.25 \text{ m}^3/\text{s}$$

RUDERXW SHUFHQWPRUHWKDQWKDWRUWKHUHFWDQJOH

)RUDVHPLFLUFOH$ P CE RU' P ZKHQFH3 ½ ' P DQG5K $3 P 7KHVHPLFLUFOHIORZ UDWHZLOOWKXVEH

$$Q = \frac{1.0}{0.015}(3.214)(0.715)^{2/3}(0.001)^{1/2} = 5.42 \text{ m}^3/\text{s}$$

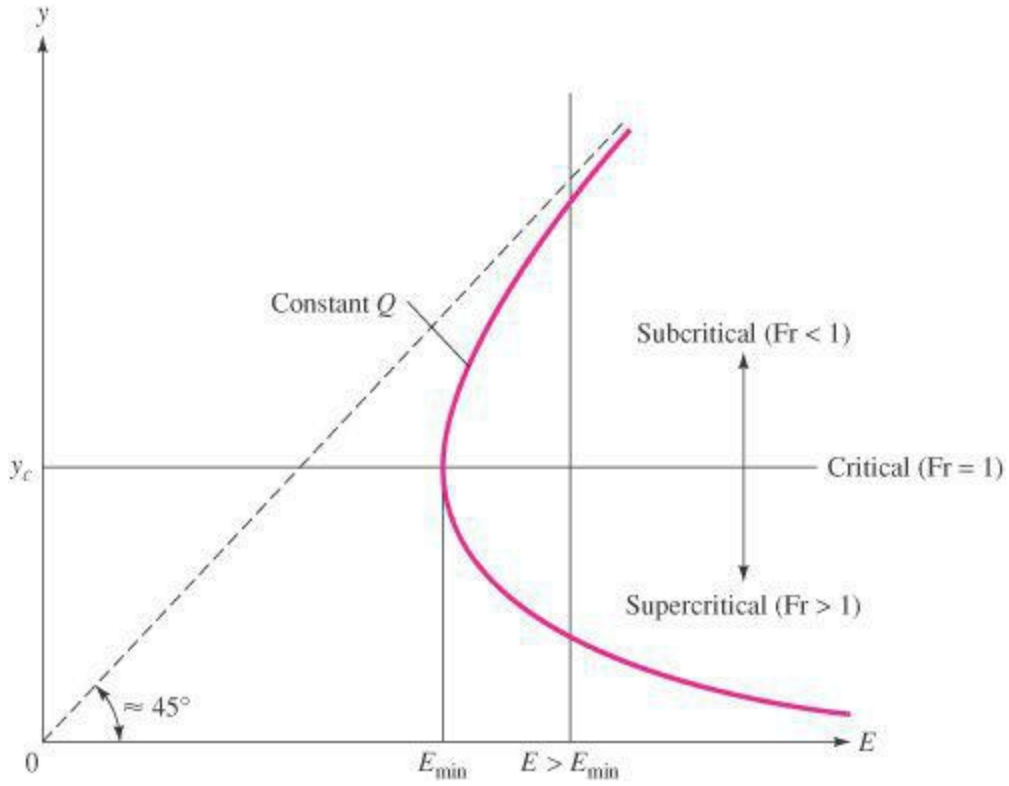
RUDERXW SHUFHQWPRUHWKDQWKDWRI WKHUHFWDQJOHDQG SHUFHQWPRUHWKDQWKDWRI WKHKDOIKHLJKW

6SHFLILF(QHUJ\ &ULWLFDO'HSWK

7KHWRWDOKHDGRI DQ LQFRPSUHVVLEOHIORZ LVWKHVXP RI LWVYHORFLW\KHDG. 9 J SUHVVXUHKHDG S DQGSRWHQWLDOKHDG])RUSHQFKDQQHOIORZ VXUIDFHSUHVVXUHLVHYHUZKHUHDWPRVSKHULF VR WKH FKDQQHOHQHUJ\ LVDEDQGHEHWZHHQYHORFLWDQGHOHYDWLRQKHDGRQO 6LQFHWKHIORZ LVWKURXJKHQWZLWK DWXPH WKDW § ² UHFDOO(T 7KHILQDOUHVXOWLV WKH TXDQWLWFDOOHGVSHFLILF HQHUJ\ (D VXJJHVWHGE\ %DNHUHWDO I > @LQ

$$E = y + \frac{V^2}{2g} \qquad (10.28)$$

ZKHUH\ LV WKH ZDWHU GHSWK ,W LV VHHQ IURP) LJ WKDW LV WKH KHLJKW RI WKH HQHUJ\ JUDGH OLQH (* / DERYH WKH FKDQQHO ERWWRP) RU D JLYHQ IORZ UDWH WKH HDUHU XVXDOO VWR WDWH V SRVVLEOH FDOF DOWHUQDWH VWDWHV IRU WKH VDPH VSHFLILF HQHUJ\ 7KHUH LV DOVR D PLQLPXP HQHUJ\ (PLQ ZKLFK FRUUHVSRQGV WR D) URXGHQXP EHUR XQLW



) LJ ,OOXVWUDWLRQ RI D VSHFLILF HQHUJ\ FXUYH 7KH FXUYH IRU HDFK IORZ UDWH KDV D PLQLPXP HQHUJ\ FRUUHVSRQGLQJ WR FULWLFDO IORZ) RUHQHUJ\ JUHDWHU WKDQ PLQLPXP WKHUH DUH WZR *alternate* IORZ VWDWHV RQH VXEFULWLFDO DQG RQH VXSHUFULWLFDO

&RQVLGHU WKH SRVVLEOH VWDWHV DVVRFLDWHG ZLWK D JLYHQ IORZ UDWH RI / HVT 4 E 9\ EHWKHGLVFKDUJH SHU XQLW ZLGWK RI DUHFWDQJXODU FKDQQHO 7KHQ ZLWK T FRQWDQW(T EHFRPHV

$$E = y + \frac{q^2}{2gy^2} \qquad q = \frac{Q}{b} \qquad (10.29)$$

) LJXUH LV D SORW RI \ YHUVXV (IRU FRQWDQW T IURP (T 7KHUH LV D PLQLPXP YDOXH RI (DW D FHUWDLQ YDOXH RI \ FDOOHG WKH FULWLFDO GHSWK %\ VHWWLQJ G(G DW FRQVWDQW T ZH ILQG WKDW (PL RFFXUV DW

$$y = y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{b^2 g}\right)^{1/3} \qquad (10.30)$$

7KH DVRFLDWHG PLQLPXP HQHUJ\ LV

$$E_{min} = E(y_c) = \tfrac{3}{2} y_c \qquad (10.31)$$

7KH GHSWK\ $F FRUUHVSRQGV WR FKDQQHO YHORFLW\ HTXDO WR WKH VKDOORZ ZDWHU SURSDJDWLRQ VSHHG& IURP (T 7RVHHWKLV UHZULWH(T DV

$$q^2 = gy_c^3 = (gy_c)y_c^2 = V_c^2 y_c^2 \qquad (10.32)$$

%\ FRPSDULVRQ LWRGBRZ VWKDWWKHFULWLFDOFKDQQHOYHORFLWLV

$$V_c = (gy_c)^{1/2} = C_0 \qquad \mathrm{Fr} = 1 \qquad (10.33)$$

) RU((PLQ QR VROXWRQH[LWWLQ) LJ DQG WKXV VXFK DIGRZ LV LPSRVVLEOH SK\VLFDOO) RU(! (PLQ WR VROXWRQV DUHSRVVLEOH ODUJHGHSWKZLWK9 $F FDOHG VXEFULWLFDO DQG VPDOOGHSWK ZLWK9 ! $F FDOHG VXSHUFULWLFDO ,Q VXEFULWLFDOIGRZ GLVXUEDQFHV FDQ SURSDJDWH XSVWUHDP EHFDXV ZDYH VSHHG& ! 9 ,Q VXSHUFULWLFDOIGRZ ZDYHV DUH VZHSWGRZQVWUHDP 8SVWUHDP LV D]RQH R VLOHQFH DQG D VPDOOREVWUXFWLRQ LQ WKHIGRZ ZLOOFUHDWH D ZHGJH VKDSHG ZDYH H[WHQGLQJ GRZQVWUHDP WKH0 DFK ZDYHVLQ) LJ F 7KHDQJOHRI WKHVHZDYHVP XUWEH

$$\mu = \sin^{-1}\frac{c_0}{V} = \sin^{-1}\frac{(gy)^{1/2}}{V} \qquad (10.34)$$

7KHZDYHDQJOHDQG WKHGHSWKFDQ WKXV EH XVHG DV D VLPSOHPHDXUHPHQWRI VXSHUFULWLFDOIGRZ YHORFLW\

1RWH IURP) LJ WKDWWKH DOOFKDQJHVLQ(QHU(PLQ FDXVHD ODUJHFKDQJHLQ WKHGHSWK\ E\ DQDOG\ ZLWKVPDOOFKDQJHVLQGXFWDUHQHUWKHVRLFSRLQWVLQ) LJ 7KXVFULWLFDOIGRZ LVQHXWDO VWDEOHDQG LV RIWQ DFFRPSDQLHG E\ ZDYHV DQG XQGXODWRQV LQ WKHIUHH VXUIDFH &KDQQHOGHVLJQHU VKRXOGDYRLGRQJUKQVRI QHDUFULWLFDOIGRZ

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$ ZLGHUHFWDQJXODUFKDQQHOWKFKDQQHOKDV DIGRZ UDWT IW V IW D : KDWLV WKHFULWLFDGHSWK' E : KDWWSHRI IGRZ H[LWWLL\ IW

6RXWLRQ

3DUWD

7KHFULWLFDOGHSWKLVIQGHSHQGHQWRI FKDQQHOURXJKQHVDQGVLP S0) IRGBRZ VIURP (T

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{50^2}{32.2}\right)^{1/3} = 4.27 \text{ ft} \qquad \textit{Ans. (a)}$$

3DUWE

,I WKH DFWXDO H SWK LV IWZ KLFK LVOHVV WKDQ \ F WKH IORZ PXVW EH VXSHUFULWLFDO $QV E

1 RQUHFWDQJXODU &KDQQHOV

,I WKH FKDQQHOZ LGWK YDULHVZ LWK< WKH VSHFLILF HQHUJ\ PXVW EH ZULWWHQ LQ WKH IRUP

$$E = y + \frac{Q^2}{2gA^2} \qquad (10.35)$$

7KH FULWLFDO SRLQW PLQLPXP HQHUJ\ RFFXUV ZKHUH G G DW RQ 4 6LQFH $ \ (T \LHOGV IRU((PLQ

$$\frac{dA}{dy} = \frac{gA^3}{Q^2} \qquad (10.36)$$

%XW G$ E G Z KHUH E LV WKH FKDQQHO ZLGWK DW WKH IUHH VXUIDFH 7KHUHIRUH(T LV HTXLYDOHQW WR

$$A_c = \left(\frac{b_0 Q^2}{g}\right)^{1/3} \qquad (10.37a)$$

$$V_c = \frac{Q}{A_c} = \left(\frac{gA_c}{b_0}\right)^{1/2} \qquad (10.37b)$$

) RU D JLYHQ FKDQQHO VKDSH $ \ DQG E \ DQG D JLYHQ4 (TV KDYH WR EH VROYHG E\ WULDO DQG HUURU RU E\ ((6 WR ILQG WKH FULWLFDO DUHD $F IURP Z KLFK 9F FDQ EH FRPSXWHG

%\ FRPSDULQJ WKH DFWXDO H SWK DQG YHORFLW\ Z LWK WKH FULWLFDO YDOXHV Z H FDQ GHWHUPLQH WKH ORFDO IORZ FRQGLWLRQ

\ ! \F 9 9F VXEFULWLFDO IORZ) U

\ \F 9 9F FULWLFDO IORZ) U

\ \F 9 ! 9F VXEFULWLFDO IORZ) U

1 RWH WKDW 9F LV HTXDO WR WKH VSHHG RI SURSDJDWLRQ F RI DVKDOORZ ZDWHU Z DYH LQ WKH FKDQQHO DQG LV GHSHQGHQW XSRQ WKH GHSWK DV LQ) LJ D) RU D UHFWDQJXODU FKDQQHO F J\

&ULWLFDO8 QLIRUP) ORZ 7KH &ULWLFDO6 ORSH

,I D FULWLFDO FKDQQHO IORZ LV DOVR PRYLQJ XQLIRUPO DW FRQVWDQW GHSWK LWP XWFRUHVSRQGV WR I FULWLFDO VORSH 6F ZLWK \Q \F 7KLV FRQGLWLRQ LV DQDO]HG E\ HTXDWLQJ (T D WR WKH &Kp]\ R 0 DQQLQJ IRUPXOD

$$Q^2 = \frac{gA_c^3}{b_0} = C^2 A_c^2 R_h S_c = \frac{\alpha^2}{n^2} A_c^2 R_h^{4/3} S_c$$

or

$$S_c = \frac{n^2 g A_c}{\alpha^2 b_0 R_{hc}^{4/3}} = \frac{n^2 V_c^2}{\alpha^2 R_{hc}^{4/3}} = \frac{n^2 g}{\alpha^2 R_{hc}^{1/3}} \frac{P}{b_0} = \frac{f}{8} \frac{P}{b_0} \qquad (10.38)$$

ZKHUH. HTXDOV IRU6, XQLWVDQG IRU%* XQLWV (TXDWLRQ LVYDOGIRUDQ FKDQQ VKDSH)RUDZLGHUHFWDQJXODUFKDQQHOE≫ \F WKHIRUPXODUHGXFHVWR

Wide rectangular channel: $$S_c \approx \frac{n^2 g}{\alpha^2 y_c^{1/3}} \approx \frac{f}{8}$$

7KLVLVDVSHFLDOFDVH DUHHUHQFHSRLQW,QPRVWFKDQQHOIORZV\Q \F)RUIXOO URXJK WXUEXOHQ IORZ WKHFULWLFDOVORSHYDULHVEHWZHHQ DQG

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7KH *f WUDQJXODUFKDQQHOLQ) LJ (KDVDIORZ UDWH4 P V &RPSXWH D \F E 9F DQ F 6F LI Q

6RXWLRQ

3DUWD

7KLVLVDQHDX\ FURVVVHFWLRQEHFDXVHDOOJHRPHWULF TXDQWLWLHVFDQEHZULWWHQGLUHFWO LQWHUPVR GHSWK\

$$P = 2y \csc 50° \qquad A = y^2 \cot 50°$$
$$R_h = \tfrac{1}{2} y \cos 50° \qquad b_0 = 2y \cot 50° \qquad (1)$$

7KHFULWLFDOIORZ FRQGLWLRQVDUHWLHV(T D

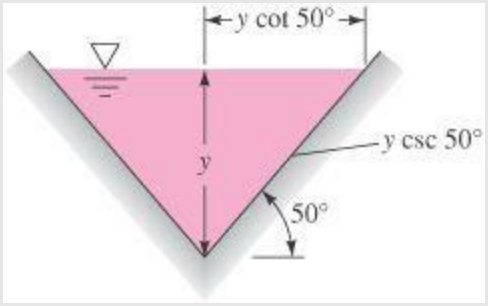
$$gA_c^3 = b_0 Q^2$$

or

$$g(y_c^2 \cot 50°)^3 = (2y_c \cot 50°) Q^2$$

$$y_c = \left(\frac{2Q^2}{g \cot^2 50°}\right)^{1/5} = \left[\frac{2(16)^2}{9.81(0.839)^2}\right]^{1/5} = 2.37 \text{ m} \qquad \textit{Ans. (a)}$$

(



3DUWE

: LWK\ NQRZQ IURP (TV ZHFRPSXWH 3 P 5KF P $ P DQG E P 7KHFULWLFDOYHORFLW\IURP (T E LV

$$V_c = \frac{Q}{A_c} = \frac{16\ \text{m}^3/\text{s}}{4.70\ \text{m}^2} = 3.41\ \text{m/s} \qquad \textit{Ans. (b)}$$

3DUWF

: LWQ ZHFRPSXWIURP (T DFULWLFDOVORSH

$$S_c = \frac{gn^2P}{\alpha^2 R_h^{1/3} b_0} = \frac{9.81(0.018)^2(6.18)}{1.0(0.760)^{1/3}(3.97)} = 0.00542 \qquad \textit{Ans. (c)}$$

) ULFWLRQOHVV) ORZ RYHUD%XPS

LV9 9 \ \ P V 7KH) URXGHQXPEHUDWWKHFUHVWLV) U) ORZ GRZQVWUHDP RI WK
EXPSLVVXEFULWLFDO7KHVHIORZ FRQGLWLRQVDUHVKRZQLQ) LJ (

(

3DUWE

) RUFULWLFDOIORZ LQDZLGHFKDQQHOZLWKT 9\ P VIURP (T

$$E_{2,\min} = E_c = \frac{3}{2} y_c = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} = \frac{3}{2}\left[\frac{(1.5\ \text{m}^2/\text{s})^2}{9.81\ \text{m/s}^2}\right]^{1/3} = 0.918\ \text{m}$$

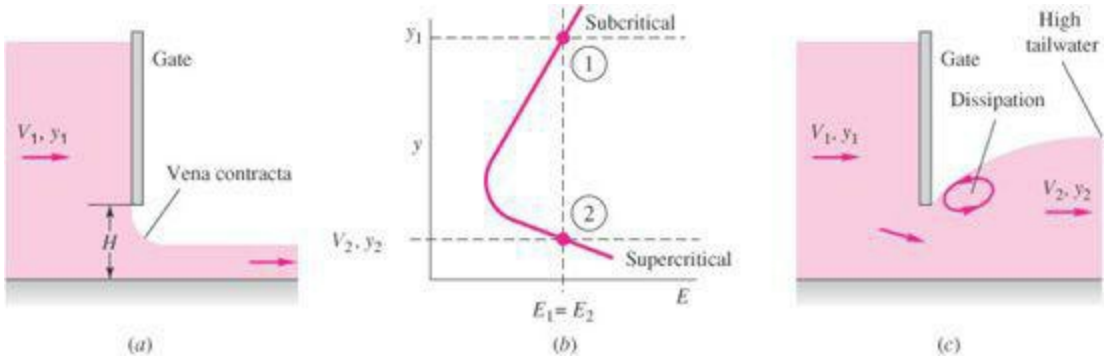
7KHUHRUHWKHPD[LPXP KHJKWRUIUHVWRQDVWHSIORZ RYHUWKLVSDUWFXDUEXPSLV

$$\Delta h_{\max} = E_1 - E_{2,\min} = 1.115 - 0.918 = 0.197\ \text{m} \qquad \textit{Ans. (b)}$$

) RUWKLVEXPS WKHVRXUFHRQRI (T LV\ \F P DQGWKH) URXGHQXPEHULVXQLWD
WKHFUHVW$WFULWLFDOIORZ WKHVXUFHOHYHOKDVGURSSHGE\ \ í \ í ûK P

) ORZ XQGHUD6OXLFH* DWH

$ VOXLFHJDWHLVDERWWRP RSHQLQJ LQDZDOODVVNHWKHGLQ) LJ D FRPPRQO XVHGLQFRQWR
RI UHVHUYRLUSDQGFKDQQHOIORZV ,I WKHIORZ LVDORZHGIUHHGLVFKDUJHWKURXJK WKHJDS DVLQ) LJ D
WKHIORZ VPRRWKO DFFHOHUDWHVIURP VXEFULWLFDO XSVWUHDP VRFULWLFDO QHDUWKHJDS VRVXSHUFULWLFDO
GRZQVWUHDP 7KHJDWHLVWKHQDQDORJRXVVRDFRQYHUJLQJ±GLYHUJLQJ QR]]OHLQJDVGLQDPLFV DVL
) LJ RSHUDWLQJ DWDWGHVLJQFRQGLWLRQ VLPLODUWRSRLQW+ LQ) LJ E



) LJ) ORZ XQGHU D VOXLFH JDWH SDWWHUQ VKRZLQJ FULWLFDO) ORZ *a* IUHH GLVFKDUJH ZLWK YHQD FRQWUDFWD *b* \SHFLILF HQHUJ\ IRU IUHH GLVFKDUJH *c* GLVWULEXWLRQ RI) ORZ XQGHU D GURZQHG JDWH

) RU IUHH GLVFKDUJH IURP D\ EH HQHUJ\ EDODQFHG DQG VLQFH WKHUH LV QR EXPS û K (T DSSOLHV ZLWK ((

$$y_2^3 - \left(\frac{V_1^2}{2g} + y_1\right)y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0 \qquad (10.40)$$

* LYHQ VXEFULWLFDO XSVWUHDP) ORZ 9 \ WKLV FXELF HTXDWLRQ KDV RQO RQH SRVLWLYH UHDO URRW VXSHUFULWLFDO) OR DWWKH VDPH VSHFLILF HQHUJ\ DV LQ) LJ E 7KH) OR UDWH YDULHV ZLWK WKH UDW \ \ ZHDN DVDSURECOP H[HFLVH WR VKRZ WKDWWKH) OR UDWLVDPD[LPXP ZKHQ \ \

7KH IUHH GLVFKDUJH) LJ D FRQWUDFWV WR D GHSWK \ DERXW SHUFHQWOHVV WKDQ WKH JDWH\JDS KHLJKWDV VKRZ Q 7KLV LV VLP LODU WR D IUHH RULILFH GLVFKDUJH DV LQ) LJ ,I + LV WKH HHLJKW RI WK JDWH JDS DQG E LV WKH JDS Z LGWK LQ R WKH SDSHU Z HFDQ DSSUR[LPDWH WKH) OR UDWH E\ RUILFH WKHRU

$$Q = C_d H b\sqrt{2gy_1} \qquad \text{where} \qquad C_d \approx \frac{0.61}{\sqrt{1 + 0.61H/y_1}} \qquad (10.41)$$

LQ WKH UDQJH+ \ 7KLV D FRQVXRXV YDULDWLRQ LQ) OR UDWH LV DFFRPSDVKHG E\ UDLVLQJ WK JDWH

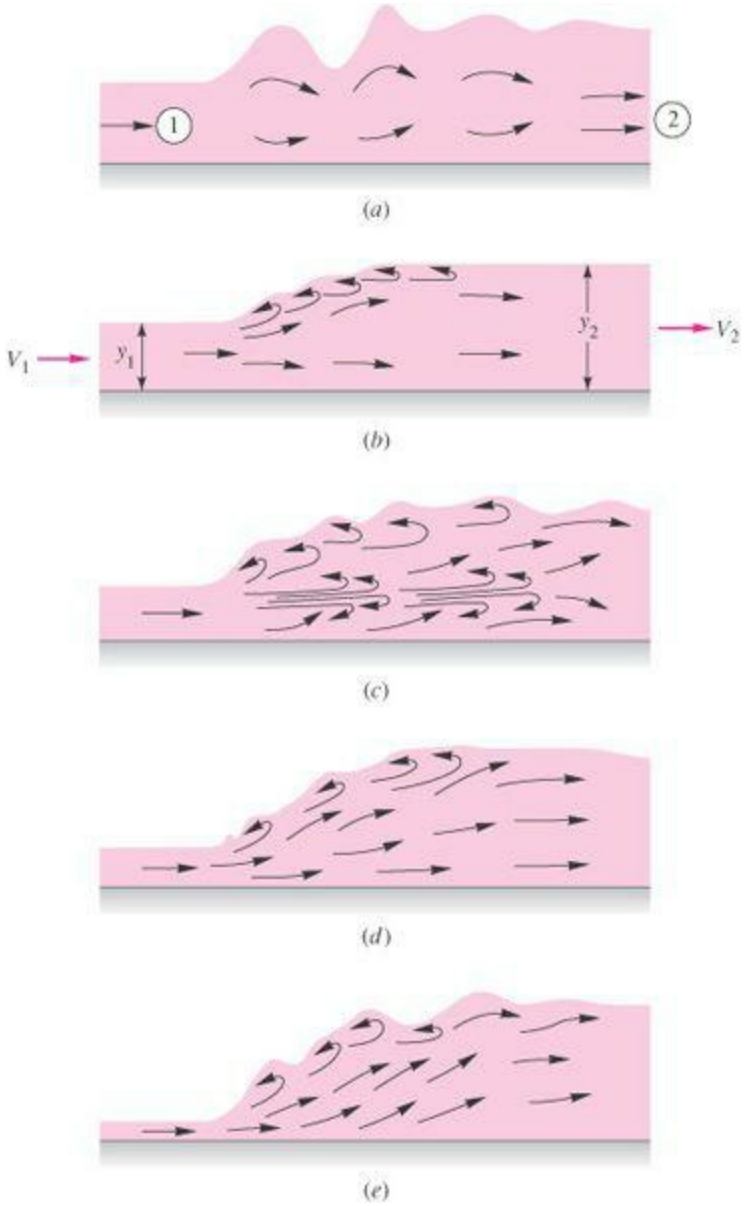
,I WKH VDLG Z DWUL VKLJK DV LQ) LJ F IUHH GLVFKDUJH LV QRV SRVVLEOH 7KH VXLFH JDWH LV VDLG V EH GURZQHG RUSDUWDOO GURZQHG 7KHUH Z LOO EH HQHUJ\ GLVVLSDWLQ WKH H[WHQW SURFEDE LQ WK IRUP RI D GURZQHG K\GUDXOLF MXP DQG WKH GRZQWUHDP) OR Z LOO UHGXFH WR VXEFULWLFDO (TXDWLRQ DQG GR QRW DSSO WR WKLV VLWXDWLRQ DQG H[SHULPHQWDO GLVFKDUJH FRHIILHQWV DU QHFHVVDU > @6HH3UREOHP 3

7KH+ \ GUDXOLF - XP S

,Q RSHQ FKDQQHO) OR D VXSHUFULWLFDO) OR FDQ FKDQJH TXLFNO EDFN WR D VXEFULWLFDO) OR E SDVLQJ WKURXJK D K\GUDXOLF MXP DV LQ) LJ 7KH XSWUHDP) OR LV IDVWDQG VKDOORZ DQG WK GRZQWUHDP) OR LV VORZ DQG GHHS DQDORJRXV WR WKH QRUPDO VKRFN Z DYH RI) LJ 8QDNH WK LQLQLWLDOP DOO WKLQ QRUPDO VKRFN WKHK\GUDXOLF MXP SLV TXLWH WKLFN UDQJLQJ LQ HQJWK IURP WR WLPH WKH GRZQWUHDP GHSWK\ > @

%HLQJ H[WUHPHO VXUEXOHQWDQG DJLWDWHG WKH KYGUDXOLF MXPS LV DYHU HIHFWLYHHQHUJ\ GLVLSDWL
DQGLVDIHDWXUH RI VWUDLQJ EDVLQDQG VSLOOZD\ DSSOLFDWLRQV> @) LJXUH VKRZV WKH MXPS IRUPH
LQDODERUDWRU RSHQFKDQQHO ,WV YHU LPSRUWDQWWKDWWXFK MXPSV EH GRFDWHG RQ VPHDOO GHVLJQH
DSURQV RWKHUZLVHWKHFKDQQHOERWWRP ZLOOEHEDGO VFRXUHG E\ WKHDJLWDWRQ -XPSV DOR PL[IOXLG
YHU HIHFWLYHO DQGKDYHDSSOLFDWLRQVWRVHZDJHDQGZDVWHDWHQWUHDWPHQWGHVLJQV

&ODVVLILFDWLRQ



) LJ &ODVVLILFDWLRQRI K\GUDXOLF MXPSV *a*) U VR XQGXODUMXPS *b*) U VW
ZHDN MXPS *c*) U VR RVFLOODWLQJ MXPS *d*) U VR VWHDG\ MXPS *e*) U!
VWURQJ MXPS

$GDSWHGIURP 5H
7KHSULQFLSDOSDUDPHWHUDIHFWLQJ K\GUDXOLF MXPS SHUIRUPDQFHLV WKHXSVWUHDP) URXGHQXPEHU) U
9 J\ 7KH5H\QROGVQXPEHUDQG FKDQQHOJHRPHWU KDYHRQD DVHFRQGDU HIHFW\$VGHVLUHG
LQ5H WKHIORZLQJUDQJHVRI RSHUDWRQFDQEHRXWOLQHG DVLOOXVWUDWHGLQ) LJ

$Fr_1 < 1.0$:	Jump impossible, violates second law of thermodynamics.
$Fr_1 = 1.0$ to 1.7:	Standing-wave or *undular jump* about $4y_2$ long; low dissipation, less than 5 percent.
$Fr_1 = 1.7$ to 2.5:	Smooth surface rise with small rollers, known as a *weak jump;* dissipation 5 to 15 percent.
$Fr_1 = 2.5$ to 4.5:	Unstable, *oscillating jump;* each irregular pulsation creates a large wave that can travel downstream for miles, damaging earth banks and other structures. Not recommended for design conditions. Dissipation 15 to 45 percent.
$Fr_1 = 4.5$ to 9.0:	Stable, well-balanced, *steady jump;* best performance and action, insensitive to downstream conditions. Best design range. Dissipation 45 to 70 percent.
$Fr_1 > 9.0$:	Rough, somewhat intermittent *strong jump,* but good performance. Dissipation 70 to 85 percent.

)XUWKHUGHWDLOVFDQEHIRXQGLQ5H DQG5H &KDS

7KHRU\ IRUD+RUL]RQWDO-XPS

$ MXPS WKDWRFFXUVRQDVWHS FKDQQHOVORSHFDQEHDIIHFWHGE\ WKHGLIIHUHQFHLQZDWHUJK FRPSRQHQWDORQJ WKHIORZ 7KHHIIHFWV VPDOONRZHYHU VRWKDWWKHFDOFXODWHG MXPSRFFXUVRQDKRUL]RQWDOERWWRP

<RX ZLOOEH SOHDVHG WR NQRZ WKDWZHKDYH DOUHDG\ DQDO]HG WKLV SUREOHP LQ6HF $ K\GUDXOLF MXPSLVH[DFWO\ HTXLYDOHQWWR WKHVWRQJ IL[HGZDYHLQ) LJ E ZKHUHWKHFKDQJHLQGHSW /\ LVQRWQHJOHFWHG ,I9 DQG\ XSWUHDP DUHNQRZQ9 DQG\ DUHFRPSXWHGE\ DSSO\LQJ FRQWLQXLW DQGPRPHQWXP DFURVV WKHZDYH DVLQ(TV DQG (TXDWLRQ LVWKHUHRUHWKHFRUUHVRXWLRQ IRUD MXPS LI ZH LQWURGXFH& DQG\ LQ) LJ E DV XSWUHDP FRQGLWLRQV9 DQG\ UHVSHFWLYHO ZLW& í /9 DQG\ /\ EHLQJ WKHGRZQWUHDP FRQGLWLRQV9 DQG\ UHVSHFWLYHO DVLQ) LJ E (TXDWLRQ EHFRPHV

$$V_1^2 = \tfrac{1}{2} g y_1 \eta(\eta + 1) \qquad (10.42)$$

ZKHUH \ \ ,QWURGXFLQJ WKH)URXGH QXPEHU)U 9 J\ DQG VROYLQJ WKLV TXDGUDWLF HTXDWLRQIRU ZHREWDLQ

$$\frac{2y_2}{y_1} = -1 + (1 + 8\,\mathrm{Fr}_1^2)^{1/2} \qquad (10.43)$$

: LWK\ WKXVNQRZQ9 IRQBRZ VIURP WKHZLGHFKDQQHOFRQWLQXLWUHODWLRQ

$$V_2 = \frac{V_1 y_1}{y_2} \qquad (10.44)$$

)LQDOO ZHFDQHYDOXDWWKHGLVWLSDWLRQKHDGORVV DFURVV WKHMXPS IURP WKH WXDG\ IORZ HQHUJ HTXDWLRQ

$$h_f = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

,QWURGXFLQJ\ DQG9 IURP (TV DQG ZH ILQG DWWU FRQVLGHUDEOH DJHEUDL PDQLSXODWRQWKDW

$$h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} \qquad (10.45)$$

(TXDWRQ VKRZV WKDWWKHGLVWLSDWRQORVWLVSRVLWLYHRQO LI \ ! \ ZKLFKLVDUHTXLUHPHQ RI WKH VHFRQG ODZ RI WKHUPRG\QDPLFV (TXDWRQ WKHQ UHTXLUHV WKDW) U ! WKDWLV WK XSVWUHDP IORZ PXVWEH VXSHUFULWLFDO) LQDOO (T VKRZV WKDW9 9 DQG WKH GRZQVWUHDP IORZ LV VXEFULWLFDO \$ OWKHVH UHVXOWV DJUHH ZLWK RXUSUHYLRXV H[SHULHQFH DQDO]LQJ WKH QRUPDO VKRFN ZDYH

7KH SUHVHQWWKRU LV IRUK\GUDXOF MXPSV LQ ZLGH RU UHFWDQJXODUK RU] RQWDO FKDQQHOV) RUWK WKH RU RI SULVP DWF RUVORSLQJ FKDQQHOV VHDGYDQFHGWR[W>IRUH[DPSOH &KDSV DQG @

(; \$0 3/ (

: DWU IORZ V LQ D ZLGH FKDQQHO DT P V P DQG\ P ,I WKH IORZ XQGHUJRHV K\GUDXOF MXPS FRPSXW D \ E 9 F) U G K, H WKH SHUFHQWDJH GLVWLSDWRQ I WKH SRZH GLVWLSDWHG SHUXQLWZ LGWK DQG J WKH WMP SHUDXU HULVHGXHWR GLVWLSDWRQ LI F_S - NJ .

6RXWLRQ

3DUW D

7KH XSVWUHDP YHORFLWLV

$$V_1 = \frac{q}{y_1} = \frac{10\ \text{m}^3/(\text{s}\cdot\text{m})}{1.25\ \text{m}} = 8.0\ \text{m/s}$$

7KH XSVWUHDP) URXGHQXP EHULV WKH UHRUH

$$\text{Fr}_1 = \frac{V_1}{(gy_1)^{1/2}} = \frac{8.0}{[9.81(1.25)]^{1/2}} = 2.285$$

) URP) LJ WKLVLVDZHDNMXPS 7KHGHSWK\ LVREWDLQHGIURP (T

$$\frac{2y_2}{y_1} = -1 + [1 + 8(2.285)^2]^{1/2} = 5.54$$

or $$y_2 = \tfrac{1}{2}y_1(5.54) = \tfrac{1}{2}(1.25)(5.54) = 3.46\ \text{m} \qquad \textit{Ans. (a)}$$

3DUWE

) URP (T VKHGRZ QXWHDP YHORFLWLV

$$V_2 = \frac{V_1 y_1}{y_2} = \frac{8.0(1.25)}{3.46} = 2.89 \text{ m/s} \qquad \textit{Ans. (b)}$$

3DUWF

7KHGRZ QXWHDP) URXGHQXP EHULV

$$\text{Fr}_2 = \frac{V_2}{(gy_2)^{1/2}} = \frac{2.89}{[9.81(3.46)]^{1/2}} = 0.496 \qquad \textit{Ans. (c)}$$

3DUWG

$VH[SHFWG) U LVVXEFULWFDO) URP (T VKHGLVWLSDWRQGRVV LV

$$h_f = \frac{(3.46 - 1.25)^3}{4(3.46)(1.25)} = 0.625 \text{ m} \qquad \textit{Ans. (d)}$$

3DUWH

7KHSHUFHQWDJHGLVWLSDWRQUHDWVK\ VRXSVWHDP HQHUJ\

$$E_1 = y_1 + \frac{V_1^2}{2g} = 1.25 + \frac{(8.0)^2}{2(9.81)} = 4.51 \text{ m}$$

Hence $$\text{Percentage loss} = (100)\frac{h_f}{E_1} = \frac{100(0.625)}{4.51} = 14 \text{ percent} \qquad \textit{Ans. (e)}$$

3DUWI

7KHSRZ HUGLVWLSDWG SHUXQLWZ LGWKLV

$$\text{Power} = \rho g q h_f = (9800\ \text{N/m}^3)[10\ \text{m}^3/(\text{s}\cdot\text{m})](0.625\ \text{m})$$
$$= 61.3\ \text{kW/m} \qquad \textit{Ans. (f)}$$

3DUWJ

)LQDOO WKHPDVVIORZ UDWHLVP !T NJP > P V P @ NJ V P DQGWK
WPSHUDWXUHULVHIURP WKHWDQGIORZ HQHJ\ HTXDWLRQLV

$$\text{Power dissipated} = \dot{m}c_p\,\Delta T$$

or $$61{,}300\ \text{W/m} = [10{,}000\ \text{kg/(s}\cdot\text{m)}][4200\ \text{J/(kg}\cdot\text{K)}]\Delta T$$

from which

$$\Delta T = 0.0015\ \text{K} \qquad \textit{Ans. (g)}$$

7KHGLVLSDWLRQLVODUJH EXWKHWPSHUDWXUHULVHLVQHJOLJLEOH

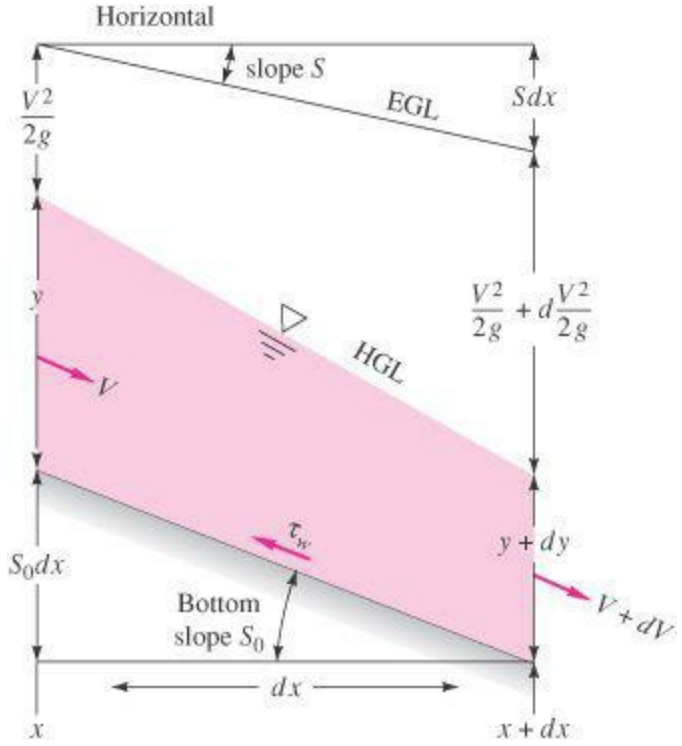
* UDGXDOO 9DULHG)ORZ

,QSUDFWLFDOFKDQQHOIORZ VERWKWHERWWRP VORSHDQGWKHZDWUGHSWKFKDQJHZLWKSRVLWLRQ DVL
)LJ $QDSSUR[LPDWHDQDOVLVLVSRVVLEOHLI WKHIORZ LVJUDGXDOO YDULHG VXFK DVLI WKHVORSH
DUHVPDOODQGFKDQJHVQRWWRR VXGGHQ 7KHEDVLF DVVXPSWLRQVDUH

6ORZ 9FKDQJLQJ ERWWRP VORSH
6ORZ 9FKDQJLQJ ZDWUGHSWK QR K\GUDXOLF MXP SV
6ORZ 9FKDQJLQJ FURVV VHFWLRQ
2QHGLPHQVLRQDOYHORFLW\GLVWULEXWLRQ
3UHVVXUHGLVWULEXWLRQDSSUR[LPDWH K\GURVWDWLF

7KHIORZ WKHQVDWLVI\HVWKHFRQWLQXLW\HTXDWLRQZLWKERWWRP IULFWLR
ORVVHVLQFOXGHG 7KHVZRXQQORZQVIRUVWDGG\ IORZ DUHYHORFLW9 [DQGZ DWUGHSWK\ [ZKHUH[
LVGLVWDQFHDORQJ WKHFKDQQHO

%DVLF' LIHUHQWLDO TXDWLRQ



) LJ (QHUJ\ EDODQFH EHWHHQ WZR VHFWLRQV LQ D JUDGXDOO\ YDULHG RSHQ FKDQQHO IORZ

&RQVLGHU WKH HQHUJ\ FKDQQHO[LOOXVWUDWHG LQ) LJ \$OJHEUDLF WUHDWPHQW WKH VWHDG\ IORZ HQHUJ\ HTXDWLRQ DUH VKRZ Q DQG WKH EDODQFH EHWHHQ [DQG[G[LV

$$\frac{V^2}{2g} + y + S_0\,dx = S\,dx + \frac{V^2}{2g} + d\left(\frac{V^2}{2g}\right) + y + dy$$

or

$$\frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) = S_0 - S \qquad (10.46)$$

Z KHUH6 LV WKH VORSH RI WKH FKDQQHO EHG P SRVLWLYH DV VKRZ Q LQ) LJ DQG6 LV WKH VORSH RI WKH(* / Z KLFK GURSV GXH WR Z DOO IULFWLRQ RQ WKH 7R HOLPLQDWH WKH YHORFLW\ GHULYDWLYH GLIHUHQWLDWH WKH FRQWLQXLW\ UHODWLRQ

$$\frac{dQ}{dx} = 0 = A\frac{dV}{dx} + V\frac{dA}{dx} \qquad (10.47)$$

%XW\$ E G\ Z KHUH E LV WKH FKDQQHO Z LGWK DW WKH VXUIDFH (OLPLQDWLQJ 9 G[EHWHHQ(T\ DQG Z HREWDLQ

$$\frac{dy}{dx}\left(1 - \frac{V^2 b_0}{gA}\right) = S_0 - S \qquad (10.48)$$

) LQDOO\ UHFDOOLQJ (T WKDW9 E J\$ LV WKH VTXDUH RI WKH) URXGH QXPEHU WKH GRFL FKDQQHO IORZ 7KHIILQDOGHVLUHGIRUP RI WKHJUDGXDOO\ YDULHG IORZ HTXDWLRQ LV

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S}{1 - \mathrm{Fr}^2}} \qquad (10.49)$$

FURVV VHFWLRQDO VKDSH SDUDPHWHUV E 3 $ DUH NQRZQ HYHU\ZKHUH DORQJ WKH FKDQQHO 7KH RQH PD
VROYH(T IRU GHSWK ZDWHU GHSWK\ [E\ DQ VDQGEDU QXPHULFDO PHWKRG 7KH DXWKRUV XVHV D
([FHOVSUHDGVKHHWV RU D SHUVRQDO FRPSXWHU 6WHS VL]HV û[PD\ EH VHOHFWHG VR WKDW WHFKFKDQJHû\ LV
QP LWHG WR QR JUHDWHU WKDQ VD\ SHUFHQW7KH VROXWLRQ FXUYHV DUH JHQHUDWHG ZHOEHKDYHG XQGHUVWKH
DUHGLVFRQWLQXRXV FKDQJHV LQ FKDQQHO SDUDPHWHUV 1RWH WKDW LQ HDSSURFKHV WKH FULWLFDO GHSWK\F WK
GHQRPLQDWRU R(T DSSURFKHV]HUR VR VP DOO VWHS VL]HV DUH UHTXLUHG ,WKHQ SV SK\VLFDOO V
NQRZ ZKDWWSH VROXWLRQ FXUYH 0 6 RUWKH OLNH \RX DUH SURFHGLQJ DORQJ EXWWKLV LV QF
PDWKHPDWLFDOO QHFHVVDU

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/ HWXVH[WHQGWKHGDWDR ([DPSOH VRFRPSXWHDSRUWRQR WKHSURFLOHVKDSH * LYHQLVDZLG
FKDQQHOZLWKQ 6 DQGT IW V IW ,I \ IWDW[KRZ IDUDORQJ WK
FKDQQHO[/ GRHV LWWDNH WKH GHSWK WR UHVHW\ IW ,V WKH IWGHSWK SRVLWLRQ XSWUHDP R
GRZ QWUHDP LQ)LJ (D'

6RXWLRQ

,Q([DPSOH ZHFRPSXWHG\F IW6LQFHRXULQLWLDOGHSWK IWLVOHVWVWKDQ\F ZHNQRZ
WKHIORZ LV VXSHUFULWLFDO/ HWXVDOVR FRPSXWWKHQRUPDOGHSWKIRUWKHJLYHQVORSH6 E\ VHWWLQJ T
IW V IWLQWKH&Kp]\ IRUPXOD ZLWK5K \Q

(D

Solve for: $y_n \approx 4.14$ ft

7KXVERWK\ IWDQG\ / IWDUHOHVVWKDQ\Q ZKLFKLVOHVVWKDQ\F VRZHPXWWEHRQDQ6

FXUYH DVLQ) LJ D) RUDZ LGHFKDQQHO(T UHGXFHVWR

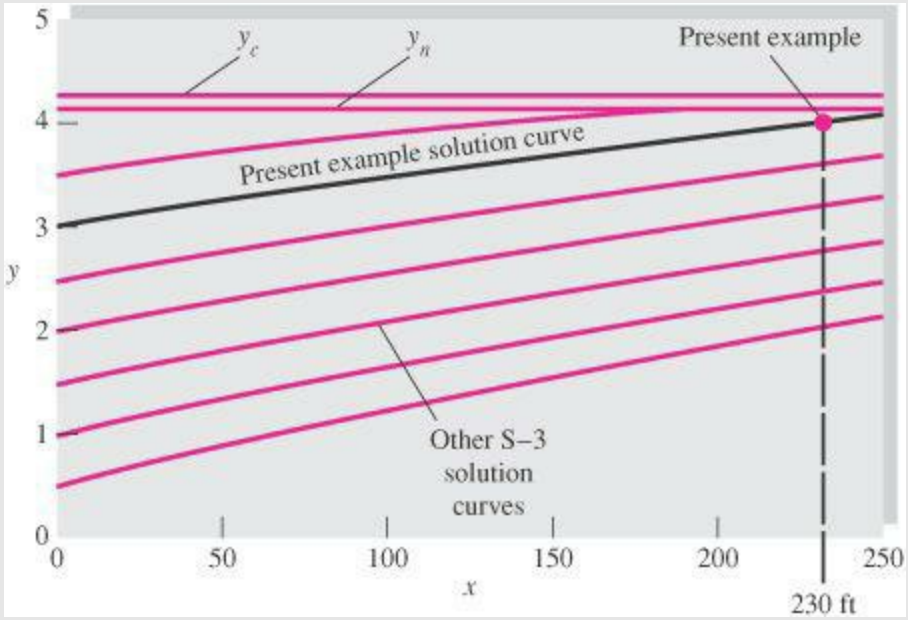
$$\frac{dy}{dx} = \frac{S_0 - n^2q^2/(\alpha^2 y^{10/3})}{1 - q^2/(gy^3)}$$

$$\approx \frac{0.0048 - (0.022)^2(50)^2/(2.208y^{10/3})}{1 - (50)^2/(32.2y^3)} \qquad \text{with } y(0) = 3 \text{ ft}$$

7KH LQLWLDO VORSH LV\ • § DQG D VXPS VL]H û[IWZ RXGG FDXVH D FKDQJH û\ §
IW§ IWOHVV WKDQ SHUFHQW: HWKHUHRUHLQWUDVQXPHULFDOO ZLWK û[IWW
GHWHUPLQHZ KHQWKHGHSWK\ IWLVDFKLHYHG 7DEXODWHVRPHYDOXHV

x, ft	0	50	100	150	200	230
y, ft	3.00	3.25	3.48	3.70	3.90	4.00

(E



7KHZ DWUGHSWK VWDUWVXSHUFULWLFDO UHDFKHV\ IWDW

$x \approx 230$ ft downstream *Ans.*

: HYHUI\ IURP) LJ D WKDWZ DWUGHSWK GRHV LQFUHDVH GRZQVWHDP RQDQ6 FXUYH 7K
VROXWRQFXUYH\ [LVVKRZQDVWKHERGGQHLQ) LJ (E
) RUDWDWHH[WDHIRUWZ HFDQLQYHVWDWWKHHQWUHIDPLO RI 6 VROXWRQFXUYHVIRUWKLVSUREOP
) LJXUH(E DQRVKRZVZKDWKDSSHQVLI WKHLQLWDOGHSWKLVYDULHGIURP WR IWLQLQFUHPHQ
RI IW$ COG6 VROXWRQ/VP RRWKO ULVHDQGDX PSWRWDFDO DSSURDFKWKHXQLIRUP IORZ FRQGLWRQ\
\Q IW

$SSUR[LPDWH6ROXWLRQ IRU,UUHJXODU&KDQQHOV

' LUHFWQXPHULFDOVROXWLRQRI (T LVDSSURSULDWHZKHQZHKDYHDQDOWDFDOIRUPXODVIRUWK
FKDQQHOYDULDWLRQV$ [6 [Q[E [DQG5K [ ) RUQDWXUDOFKDQQHOV KRZHYHU FURVVVHFWLRQ
DUH RIWHQ KLJKO\ LUUHJXODU DQG GDWD FDQ EH VSDUVH DQG XQHYHQO\ VSDFHG) RU VXFK FDVHV FLYL
HQJLQHHUV XVH DQ DSSUR[LPDWH PHWKRG WR HVWLPDWH JUDGXDOO\YDU\HG FKDQJHV : UULW(T LQ ILQLWH
GLIIHUHQFHIRUP EHWZHHQVZRGHSWKV< DQG< û\

$ YHUDJH YDOXHVRI YHORFLW VORSH DQGK\GUDXOLF UDGLXV DUHHVWLPDWHGEHWZHHQWKHVZRVHFWLRQV
) RUH[DPSOH

$ JDLQ FRPSXWDWLRQDOSURFHGHLWKHUXSVWUHDP RUGRZQVWUHDP XVLQJ VPDOOYDOXHVRI û\ ) XUWKH
GHWDLOVRI VXFK FRPSXWDWLRQV DUHJLYHQLQ&KDS RI 5H

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5HSHDW([DPSOH XVLQJ WKHDSSUR[LPDWHPHWKRGRI(T ZLWKD IRRVWHSFUHPHQW
û\ ) LQGWKHGLVWDQFHUHTXLUHGIRU\ WRULVHIURP IWWR IW

6ROXWLRQ

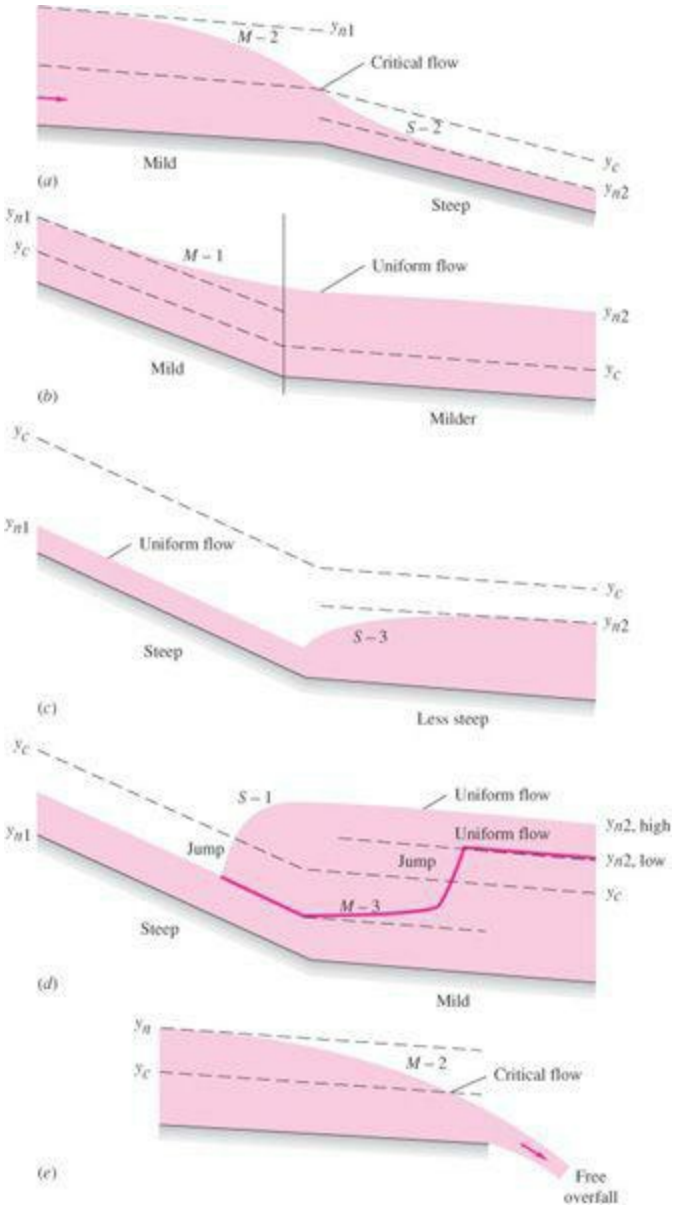
5HFDOOIURP ([DPSOH WKDWQ 6 DQGT IW VIW 1 RWHWKDW5K \ IRU
ZLGHFKDQQHO0 DNHDWDEOHZLWK\ YDULQJ IURP WR IWLQLQFUHPHQWVRI IWFRPSXWLQJ 9
T\ (\ 9 J DQG6DYJ >Q9 \ @DYJ

y, ft	V (ft/s) = 50/y	$E = y + V^2/(2g)$	S	S_{avg}	$\Delta x = \Delta E/(S_0 - S)_{avg}$	$x = \Sigma \Delta x$
3.0	16.67	7.313	0.01407	—	—	0
3.25	15.38	6.925	0.01078	0.01243	51	51
3.5	14.29	6.669	0.00842	0.00960	53	104
3.75	13.33	6.511	0.00669	0.00756	57	161
4.0 ft	12.50 ft/s	6.426 ft	0.00539	0.00604	69 ft	230 ft

&RPPHQW7KHDFFXUDF\ LVH[FHOOHQWJLYLQJ WKHVDPHUHVXOW[IWDVWKH([FHOVSUHDGVKHH
QXPHULFDOLQWHJUDWRQLQ([DPSOH 0 XFK RI WKLV DFFXUDF\ LVGXHWR WKH VPRRWK VORZ0 YDULQ
QDWXUH RI WKHSURFHVV / HVVSUHFLVRQLV H[SHFWHG ZKHQWKHFKDQQHOLV LUUHJXODUDQGJLYHQDV XQHYH
FURVVVHFWLRQV

6RPH,QWXLWLYH&RPSRVLWH) ORZ 7UDQVLWLRQV

7KH VROXWLRQ FXUYHV LQ) LJ DUH VRPHZKDW SDUDOOHO VLQFH WKH SRWXDOW FRQGLWLRQV EHWZHHQ VORSHV ,Q SUDFWLFH FKDQQHO VORSHV FDQ YDU JUHDWO 6 6 [DQG WKH VROXWLRQ FXUYHV FDQ FURV EHWZHHQ VWR UHJLPHV 2WKHUSDUDPHWULF VXFK DV$ [E [DQG Q[FDQ FDXVH LQWHUVHFW FRPSOLFDWLRQ SUREOHPV 6RPHH[DPSOHV DUH VKRZQ LQ) LJ



) LJ 6RPH H[DPSOHV RI FRPSRVLWH SURILOHV

) LJXUH DVKRZV DWDQVLWLRQ IURP DPLOG VORSH WR DVWHS VORSH LQ D FRQGXLW ZLWK FKDQQHO 7KH LQLWLDO0 FXUYH PXVW FKDQJH WR6 FXUYH ID WKHUGRZQVWHS VORSH 7KH RQO ZD\ WKI FDQ KDSSHQ SK\VLFDOO LV IRU WKH VROXWLRQ FXUYH WR SDV VP RRWKO WKURXJK WKH FULWLFDOGHSWK DV VKRZ Q 7KH FULWLFDO SRLQWV P DWK P DWLFDO VLQJXODU> 6HF @DQG WKH IGRZ QHDU WKLV SRLQWV JHQHUDO UDSLGO QRVJUDGXDOO YDULHG 7KH IGRZ SDWWHUQ DFFHOHUDWQJ IURP VXEFULWLFDO WR VXSHUFULWLFDO L VLP LODU WR D FRQYHUJLQJ±GLYHUJLQJ QR]]OH LQ JDV G\QDPLFV 2WKHU VFHQDULRV IRU) LJ D DU LP SRVVLEOH) RUH[DPSOH WKH XSWUHDP FXUYHFDQQRWEH0 IRUWKEUHDNLQVOSHZRXGFDXHDQ6 FXUYHWKDWZRXGPRYHDZ IURP XQLRUP VWHS IGRZ

) LJXUH E VKRZV DP LOG VORSHWKDWXGGHQO FKDQJHVWR DQHYHQPLOGHUVORSH 7KHDSSURDF IGRZ LVDWXP HGXQLRUP DQG WKHEUHDNLQVOSHP DNHVLWVPUHVHQFH 7KHZDWUGHSW PRYHVVP RRWKO DORQJ DQ0 FXUYHXQWLWH[DFW P HUJHV DWWKHEUHDNSRLQWZLWKDXQLRUP IGRZ D

VKHQHZ PLGGHU GHSWK\Q

) LJXUH F VKRZV D VWHS VORSH WKDW XGGHQO FKDQJHV WR D OHVV VWHS VORSH 1RWH IRU EHWWHU VORSHV WKDW Q \F %HFDXVH RI WKH VXSHUFLDO 9 ! 9F DSSURDFK IORZ WKH EUHDNLQJ VORSH FDQQRW PDNH LWV SUHVHQFH NQRZQ XSVWUHDP 7KXV QRWXQGOLQJ WKH EUHDNSRLQW GRHV DQ6 FXUYH IRUP DQG WKH WKLV SURFHGXUH SURFHHGV VP RRWKO WR XQIRUP IORZ DW WKH QHZ KLJKHU QRUP DOGHSWK

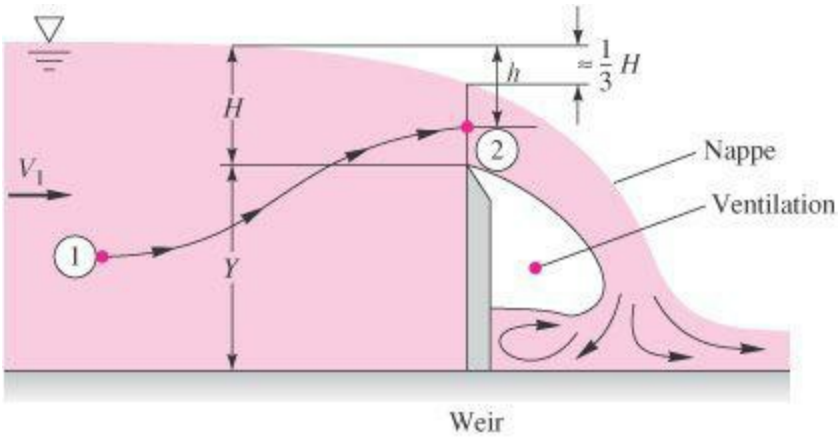
) LJXUH G VKRZV D VWHS VORSH WKDW XGGHQO FKDQJHV WR D PLGG VORSH 9DULRXV FDVHV PD RFFXU SRVVLE0 EH\RQG WKH DELOLW\ RI WKLV DXWKRUV GHVFUEH 7KH WZR FDVHV VKRZ Q GHSHQGRQ WK UHDWLY HP DJQLWXGH RI WKH PLGG VORSH ,I WKH GRZQVWUHDP GHSWK\Q LV VKDOORZ DQ0 FXUYHZ LOOWUDW DW WKH EUHDNLQJ GGHYHORSXQLOWKHORFDOVXSHUFULWFDO IORZ LV MXVWXIILFLHQWWR IRUP DK\GUDXOF MXP XS WR WKH QHZ QRUP DOGHSWK $V\Q LQFUHDVHV WKH MXP PRYHVXSWUHDP XQLO IRUWKH³KLJK FDV VKRZQ LWWRUP VRQWKHVWHS VLGH IRGBZ HGE\ DQ6 FXUYHWKDWP HUGHVLQRQ QRUP DOGHSWK\Q DW WKH EUHDNSRLQW

) LJXUH HLOOXVWUDWV DIUHRYHUDOOZ LWK D PLGG VORSH 7KLV DFWXDV DFRQWROVHFWRQ WR WKH XSWUHDP IORZ ZKLFK WKHQIRUP VDQ0 FXUYHDQG DFFHOHUDWV WR FULWFDO IORZ QHDUWKHRYHUDOO7K IDQQQJ VWUHDP ZLOOEHVXSHUFULWFDO7KHRYHUDOO³FRQWRO WKHZDWUGHSWKVXSWUHDP DQG FDQVHUY DV DQ LQLWLDO FRQGLWRQ IRU FRPSXWDWRQ R\ [7KLV LV WKH\SH RI IORZ WKDWRFFXU LQ D ZHUR ZDWUDOO6HF

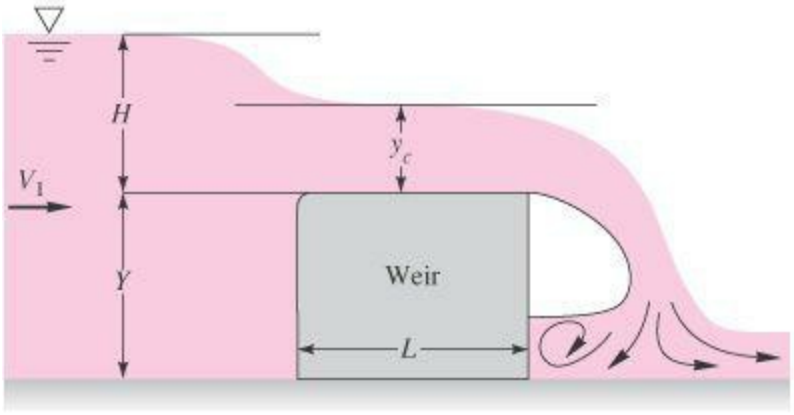
7KH H[DPSOHV LQ) LJ VKRZ WKDWFKDQJLQJ FRQGLWRQV LQ RSHQFKDQQHOIORZ FDQ UHVXOW FRPSOH[IORZ SDWWHUQV 0 DQ PRUHH[DPSOHVR FRPSOVWIORZ SURFHVVDUHJLYHQLQ5H SS :

) ORZ 0 HDVXUHP HQWDQG &RQWURO :HLUV

$ ZHLU R ZKLFK WKHRUGLQDU GDP LV DQ H[DPSOH LV DFKDQQHORBVWUFWRQRYHUZKLFK WKHIORZ PXWGHOFW) RUVLPSOH JHRPHWULHV WKH FHQQHDGLVFKDUJH4 FRUHDWV ZLWK JUDYLW DQG ZLWK WKH EORFNDJHKHJKW- WRZKLFK WKH XSWUHDP IORZ LVEDFNHGXS DERYH WKHZHLUHOHYDWRQ VHH) LJ 7KXV D ZHULV D VLPSOHEXWHIFLYH RSHQFKDQQHOIORZ PHWU: HXVHG DZHUDV DQ H[DPSOHR GLPHQVLRQDODQDOVLVQ3URE 3



(a)



(b)

) LJ) ORZ RYHUZLGH ZHOOYHQWHGZHLUV *a* VKDUS FUHVWHG *b* EURDG FUHVWHG

) LJXUH VKRZV VWRFRPPRQZHLUV VKDUS FUHVWHG DQG EURDG FUHVWHG DVXPHG WR EH YHUWLFDO ,QERWKFDVHVVWKHIORZ XSVWUHDP LV VXEFULWLFDO DFFHOHUDWHV WR FULWLFDO QHDUWKHWRS RI WKHZHLUDQG VSLOOV RYHU LQWR D VXSHUFULWLFDO QDSSH) RU ERWK ZHLUV WKH GLVFKDUJHT SHU XQLWZLGWK LVSURSRUWLRQDO WRJ + EXWZLWK VRPHZKDWGLIHUHQWFRHILFLHQWV 7KHVKRWFUHVWHG RUWKLQSODWHZHLUQDSSHVKRXOGEHYHQWLODWHGWR WKHDWPRVSKHUH WKDWLV LWWKRXOGVSULQJ FOHDURI WKHZHLUFUHVW 8QYHQWLODWHGRUGURZQHGQDSSHVDUHPRUHGLIILFXOWWRFRQWURODQGGHSHQGRQWDLOZDWHUFRQGLWLRQ 7KHVSLOOZD\ RI) LJ LVDVRUWRI XQYHQWLODWHGZHLU

) LJ +\GUDXOLF MXP S LQ D ODERUDWRU RSHQ FKDQQHO 1RWH WKH H[WHPH GLVWDQFH XSVWUHDP IORZ QWWHDP IORZ ' DWD DUHJLYHQLQ3URE 3

&RXUWHV\ RI 3URI +XEHUW&KDQVRQ 8QLYHUVLWH RI 4XHHQVODQG
$ YHU\ FRPSOHWH GLVFXVVLRQ RI ZHLU LQFOXGLQJ RWKHU GHVLJQV VXFK DV WKH SRO JRQDO³ &UHP S
ZHLUDQG YDULRXV FRQWUDFWLQJ IORPHV LVJLYHQLQWKHWH[WE\ $FN-HVHWDO> @6HH3URE 3

$QDO\VLVRI 6KDUS &UHVWHG: HLUV

,WLV SRVVLEOHWR DQDO]H ZHLUIORZ E\ LQYLVFLG SRWHQWLDOWKHRU ZLWK DQ XQNQRZQ EXWRQLYDEOH IUHH VXUIDFH DV LQ) LJ 3 +HUH KRZHYHU ZH VLPSO XVH RQH GLPHQVLRQDOIORZ WKHRU SOX GLPHQVLRQDODQDO\VLVWRGHYHORS VXLWDEOHZHLUIORZ UDWHFRUUHODWLRQV
$ YHU\ HDUO WKHRUHWFDODSSURDFK LV FUHGLWHG WR - : HLVEDFK LQ 7KH YHORFLW\ KHDG DWDQ SRLQW DERYH WKH ZHLUFUHVWVLV DVVXPHG WR HTXDOWKH WRWDOKHDG XSVWUHDP LQRWKHUZRUGV %HUQRXOOL HTXDWLRQLV XVHGZLWKQR ORVVHV

$$\frac{V_2^2}{2g} + H - h \approx \frac{V_1^2}{2g} + H \qquad \text{or} \qquad V_2(h) \approx \sqrt{2gh + V_1^2}$$

ZKHUHK LV WKH YHUWFDOGLVDQFH GRZQWR SRLQW DV VKRZQ LQ) LJ D ,I ZH DFFHSWRUWK PRPHQWZLWKRXWSURRI WKDWWKHIORZ RYHUWKHFUHVWGUDZVGRZQWRK$YH LQ WKH YROXPHIORZ T 4 E RYHUWKHFUHVWLV DSSUR[LPDWHG

$$q = \int_{\text{crest}} V_2\, dh \approx \int_{H/3}^{H} (2gh + V_1^2)^{1/2}\, dh$$
$$= \frac{2}{3}\sqrt{2g}\left[\left(H + \frac{V_1^2}{2g}\right)^{3/2} - \left(\frac{H}{3} + \frac{V_1^2}{2g}\right)^{3/2}\right]$$

1 RUP DOO WKHXSVWUHDP YHORFLW\ KHDG9 J LVQHJOHFWHG VR WKLVH[SUHVVLRQUHGXFHVWR

Sharp-crested theory: $$q \approx 0.81\left(\tfrac{2}{3}\right)(2g)^{1/2}H^{3/2} \qquad (10.53)$$

7KLVIRUPXOD LVIXQFWLRQDOO FRUUHFWEXWKHFRHILFLHQW LVWRR KLJK DQG VKRXOGEHUHSODFHGE DQH[SHULPHQWDOO GHWHUPLQHG GLVFKDUJHFRHILFLHQW

$QDO\VLVRI %URDG &UHVWHG: HLUV

7KHEURDG FUHVWHG ZHLUR) LJ E FDQEHDQDO]HGP RUHDFFXUDWHO EHFDXVHLWFUHDWVDVKRU UXQRI QHDUO RQH GLPHQVLRQDOFULWLFDOIORZ DV VKRZQ %HUQRXOOL VHTXDWLRQIURP XSVWUHDP WR WKH ZHLUFUHVWLVDOVR

$$\frac{V_1^2}{2g} + Y + H \approx \frac{V_c^2}{2g} + Y + y_c$$

,I WKH FUHVWLVYHUV ZLGHLQRRWKHSDSHU 9F J\F IURP (T 7KXVZHFDQVROYHIRU

$$y_c \approx \frac{2H}{3} + \frac{V_1^2}{3g} \approx \frac{2H}{3}$$

7KLV UHVXOWZDVXVHG ZLWKRXWSURRI WR GHULYH(T) LQDOO WKH IORZ UDWHIRORRZV IURP ZLGH FKDQQHOFUHVWHGORRZ (T

$$\text{Broad-crested theory: } q = \sqrt{gy_c^3} \approx \frac{1}{\sqrt{3}}\left(\frac{2}{3}\right)\sqrt{2g}\left(H + \frac{V_1^2}{2g}\right)^{3/2} \qquad (10.54)$$

$ JDLQZHPD\ XVXDOO QHJOHFWWKHXSWUHDP YHORFLW KHDG9 J 7KHFRHILFLHQW9 J LV DERXWLJKWEXW[SHULPHQWDOGDWDUHSUHHUHG

([SHULPHQWDO 'HWHUPLQDWLRQ RI &RHIILFLHQWV

7KHRUHWLFDO ZHLUIORZ IRUPXODV PD\ EH PRGLILHG H[SHULPHQWDOO DV IROORZV (OPLQDWH WK QXPHULFDOFRHILFLHQW DQG¥ IRUZKLFK WKHUH LV PXFK VHQVLWLYLW LQ WKH GDWDUHGXFWLRQ DQGUHGXFH WKHIRUPXODWR

$$Q_{\text{weir}} = C_d b\sqrt{g}\left(H + \frac{V_1^2}{2g}\right)^{3/2} \approx C_d b\sqrt{g}H^{3/2} \qquad (10.55)$$

ZKHUHE LV WKH FUHVWZLGWK DQG &G LV D GLPHQVLRQOHVV H[SHULPHQWDOO GHWHUPLQHG ZHLUGLVFKDUJH FRHILFLHQWZKLFK PD\ YDU ZLWK WKH ZHLUJHRPHWU 5H\QROGV QXPEHU DQG: HEHUQXPEHU 0 DQG GDWDIRUP DQ GLIIHUHQWZHLVKDYHEHHQUHSRUWHGLQWKHOLWHUDWXUH DVGHWDLOHGLQ5H $Q DFFXUDWH " SHUFHQW FRPSRVLWH FRUUHODWLRQ IRU ZLGH YHQWXUL VKDUS FUHVWH LV UHFRPPHQGHGDVIROORZV> @

$$\text{Wide sharp-crested weir: } C_d \approx 0.564 + 0.0846\frac{H}{Y} \quad \text{for} \quad \frac{H}{Y} \le 2 \qquad (10.56)$$

7KH5H\QROGVQXPEHU9 + IRUWKHVHGDWDYDU IURP (VR (EXWWKHIRUPXODVKRXOGDSSO VRKLJKHU5H VXFKDVOLQHGPVRQULYHUV

7KHEURDGFUHVWHG ZHLUR) LJ E LVFRQVLGHUDEO PRUHVHQVLWLYH WR JHRPHWULF SDUDPHWHU LQFOXGLQJ WKH VXUIDFH URXJKQHVV RI WKH FUHVW,I WKH GHDGLQJ HGJH QRVH LV URXQGHG 5 / DYDLODEOHGDWD> &KDS @P D\ EHFRUUHODWHGDVIROORZV

$$\text{Round-nosed broad-crested weir: } C_d \approx 0.544\left(1 - \frac{\delta^*/L}{H/L}\right)^{3/2} \qquad (10.57)$$

$$\text{where} \quad \frac{\delta^*}{L} \approx 0.001 + 0.2\sqrt{\epsilon/L}$$

7KH FKLH HIIHFWLV GXH WR WKH EXOHQW ERXQGDU OD\HU GLVSODFHPHQW WKLFNQHVV JURZ WK/ RQ WKH FUH DV FRP SDUHG WR XSVWUHDP KHDG+ 7KH IRUP XOD LV OM LWHG WR + / ë / " DQG 9 + ! (,I WKH QRVH LV URXQG WKHUH LV QR VLJQLILFDQW HIIHFW RI ZHLUKHJKW< DVWHDG\ + < ,I WKH EURDG FUHVWHG ZHLUKDV D VKDUS OHDGLQJ HGJH WKXV FRP PRQO FDOHG DUHFWDQJXODU ZHLUKHJKW< DQG + RZ HYHU LQ DFHUWDLQ UDQJHV RI ZHLUKHJKWDQG OHQJWK &G LVQHDU FRQVWDQW

$$\text{Sharp-nosed broad-crested weir:} \quad C_d \approx 0.462 \quad \text{for} \quad 0.08 < \frac{H}{L} < 0.33 \quad \text{and} \quad 0.22 < \frac{H}{Y} < 0.56 \qquad (10.58)$$

6XUIDFH URXJKQHVV LV QRW D VLJQLILFDQW DFWRU KHUH) RU + / VKHUH LV ODJH VFDOH WXU " SHUFHQW LQ WKH GDWD) RU + / ! DQG + < ! &G LQFUHDVHV XS WR SHUFHQW GXH WR HDF SDUDP HWU DQG FRP SOH[FKDUWDUHQHG GIRUWKHGLVFKDJHFRHILFLHQW> &KDS @

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$ ZHLUQD KRU]RQWDO FKDQQHOLV P KLJK DQG P ZLGH 7KH ZDWHU GHSWK XSWUHDP LV P (VWLP DWH WKH GLVFKDUJH LI WKH ZHLU LV D VKDUS FUHVWHG DQG E URXQG QRVHG ZLWK DQ XQLQLVKHG FRQFUHWH EURDG FUHVW P ORQJ 1HJOHFW9 J

6RO XWLRQ

3DUW D

: HDUH JLYHQ< P DQG + < § P KHQFH + § P 6LQFH + E ZH DVXPH WKDW WKH ZHLV³ ZLGH´) RU D VKDUS FUHVW(T DSSOLHV

$$C_d \approx 0.564 + 0.0846 \frac{0.6\ \text{m}}{1\ \text{m}} \approx 0.615$$

7KHQ WKH GLVFKDUJH LV JLYHQ E\ WKH EDVLF FRUUHODWLRQ (T

$$Q = C_d b \sqrt{g} H^{3/2} = (0.615)(4\ \text{m}) \sqrt{(9.81\ \text{m/s}^2)}(0.6\ \text{m})^{3/2} \approx 3.58\ \text{m}^3/\text{s} \qquad \textit{Ans. (a)}$$

: H FKHFN WKDW + < IRU(T WR EH YDOLG) URP FRQWLQXLW 9 4 E\ > @ P V JLYLQJ D5H\QROGVQXPEHU9 + § (

3DUW E

) RU D URXQG QRVHG EURDG FUHVWHG ZHU(T DSSOLHV) RUDQ XQLQLVKHG FRQFUHWH VXUIDFH UHDGë § PP IURP 7DEOH 7KH QHGLVFKDUJH FRHIILFLHQWLV

$$\frac{\delta^*}{L} \approx 0.001 + 0.2\sqrt{\epsilon/L} = 0.001 + 0.2\left(\frac{0.0024\text{ m}}{1.2\text{ m}}\right)^{1/2} \approx 0.00994$$

7KHQ(T SUHGLFWVWKHGLVFKDUJHFRHIILFLHQW

$$C_d \approx 0.544\left(1 - \frac{0.00994}{0.6\text{ m}/1.2\text{ m}}\right)^{3/2} \approx 0.528$$

7KHHVWLPDWHGIORZ UDWHLVWKXV

$$Q = C_d b\sqrt{g}H^{3/2} = 0.528(4\text{ m})\sqrt{(9.81\text{ m}^2/\text{s})}(0.6\text{ m})^{3/2} \approx 3.07\text{ m}^3/\text{s} \quad \textit{Ans. (b)}$$

&KHFNWKDW/ DVUHTXLUHG7KHDSSURDFK5H\QROGVQXPEHULV9 + § (MX
EDUHDEHORZWKHUHFRPPHQGHGOLPLWQ(T
6LQFH9 § P V9 J § P VRWKHHUURULQWKLVDOKHDGHTXDOR P LVDERXW
SHUFHQW: HFRXOGFRUUHFWWKLVIRUXSVWUHDP YHORFLWKDGLI GHUHQFH

2WKHU7KLQ3ODWH:HLU'HVLJQV

: HLV DUH RIWQ XVHG IRUIORZ PHDVXUHPHQWDQG FRQWROORI DUWLFLDOFKDQQHOV 7KH VZR PRV
FRPPRQVKDSHVDUHDUHFWDQJXODUDQGD9 QRWFK DVVKRZQLQ7DEOH $OVKRXOGEHIXOO YHQWLODWHG
DQGQRWGURZQHG

7DEOH DVKRZVDIXOOZLGWKUHFWDQJXODU ZKLFK ZLOOKDYHVOJKWHQGERXQGDU\ OD\HUHIIHFWVEX
QR HQGFRQWUDFWLRQV)RUDWKLQSODWHGHVLJQ WKHVSS LVDSSUR[LPDWHO VKDUS FUHVWHG DQG(T
VKRXOG JLYH DGHTXDWH DFFXUDF\ DVVKRZQ LQ WKH WDEOH 6LQFH WKH HQWUH FKDQQHO
DUWLFLDOYHQWLODWLRQPD\ EHQHHGHG VXFK DVKROHVLQWKHFKDQQHOZDOOV

7DEOH EVKRZVDSDUWLDOZLGWKUHFWDQJXODU E / ZKLFK ZLOOFDXVHWKHVLGHVRI WKHRYHUDOOF
FRQWUDFWLQZ DUGDQG UHGXFHWKHIORZ UDW $QDGHTXDWFRQWUDFWLRQ> @LVWR UHGXFHWK
HIIHFWLYHZHLUZLGWKE\ + DVVKRZQLQWKHWDEOH ,WWHPV KRZHYHU WKDWWKLVWSHRI ZHLULVUDWH
VHQVLWLYHWR VP DOOHIIHFW VXFK DVSODWHWKLFNQHVV DQG VLGHZDOOERXQGDU\ OD\HUJURZWK 6PDOOKDG
+ PP DQGVP DOOVORWZLGWKV E FP DUHQRWUHFRPPHQGHG6HH5HV DQG IRUIXUWKHU
GHWDLOV

7KH9 QRWFK LQ7DEOH F LVLQWQVLFDOO LQWHUWLQJ LQ WKDWWKH RYHUDOOKDV RQO RQH OHQJWV
VFDOH +² WKHUHLVQR VHSDUDWH³ZLGWK´ 7KHGLVFKDUJHZLOOWKXVEHSURSRUWLRQDOWR+ UDWHUWKDQI
SRZHU RI $SSOLFDWLRQ RI %HUQRXOOL¶V HTXDWLRQ WR WKH WDQJXODU RSHQLQJ LQ WKH VSLUWRI (T
 HDGVWR WKHIROORZLQJ LGHDOLGRZ UDWIRUD9 QRWFK

$$\text{V notch:} \qquad Q_{\text{ideal}} = \frac{8\sqrt{2}}{15}\tan\frac{\theta}{2}\,g^{1/2}H^{5/2} \qquad (10.59)$$

ZKHUH LVWKHVXSSOLFDWHGDQJOHRI WKHQRWFK 7KHDFWXDOP HDVXUHGIORZ LVDERXW SHUFHQWOHV
WKDQ WKLV GXH WR FRQWUDFWLRQ VLP LODU WR D WKLQSODWH RUILFH ,QWHUP V RI DQ H[SHUP HQWDOGLVFKDUJ

FRHILFLHQWKHUHFRPPHQGHGIRUPXODV

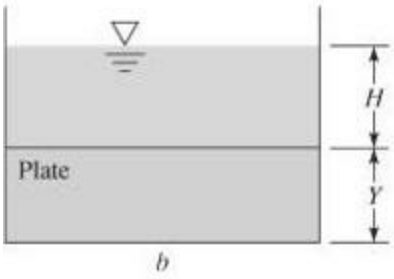
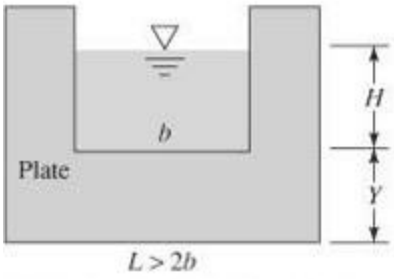
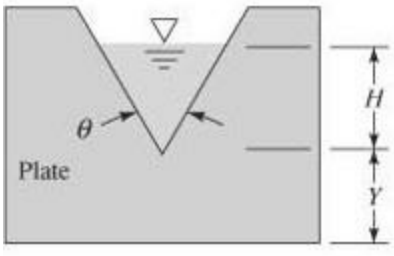
$$Q_{\text{V notch}} \approx C_d \tan\frac{\theta}{2} g^{1/2} H^{5/2} \qquad C_d \approx 0.44 \qquad \text{for} \qquad 20° < \theta < 100° \qquad (10.60)$$

IRUK HDGV + ! PP) RUVPDOOHUK HDGV EHWZ HQRQVQXPEHUDQG: HEHUQXPEHUHIHFWVPD EHLPSRUWDQWDQGDUHFRPPHQGHGFRUUHFWLRQ> @LV

$$\text{Low heads, } H < 50 \text{ mm:} \qquad C_{d,\ \text{V notch}} \approx 0.44 + \frac{0.9}{(\text{Re We})^{1/6}} \qquad (10.61)$$

ZKHUH5H ! J + DQG: H ! J+ < ZLWK< EHLQJ WKHFRHILFLHQWRI VXUIDFHWHQVLRQ / LTXLG RWKHUWKDQZDWHUPD\ EHXVHGZLWKWKLVIRUPXOD DVORQJ DV5H! WDQ DQG: H!

7DEOH 7KLQ3ODWH: HLUVIRU) RZ 0 HDVXUHPHQW

Thin-plate weir	Flow-rate correlation
 (a) Full-width rectangle.	$Q \approx \left(0.564 + 0.0846\frac{H}{Y}\right) b g^{1/2} H^{3/2}$
 (b) Rectangle with side contractions.	$Q \approx 0.581(b - 0.1H) g^{1/2} H^{3/2} \qquad H < 0.5Y$
 (c) V notch.	$Q \approx 0.44 \tan\frac{\theta}{2} g^{1/2} H^{5/2} \qquad 20° < \theta < 100°$

$ QXPEHUR RWKHUWKLQSODWHZHUGHVLJQV² VWDSH]RLGDO SDUDEROLF FLUFXODUDUF DQG8 VKDSH ² DUHGLVFXVVHGLQ5H ZKLFKDOVR FRQWDLQVFRQVLGHUDEOHGDWDRQEURDG FUHVWHGZHLV6HHDOVR 5HV DQG

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$ 9 QRWK ZHLUV WR EH GHWJQHG WR PHWDQ LUUJDWRQ FKDQQHO IORZ) RUHDVHLQUHDGLQJ WK
XSWUHDP ZDWHUJHYHOJDJH DUHDGLQJ + FP LVGHWHUPLQHGIRUWKHGHVLJQIORZ UDWHRI P K: KD
LVWKHDSSURSULDWHGHSWKIRUWKH9 QRWK'

6ROXWLRQ

- $VVXPSWLRQV 6WDGH IORZ QHJOLJLEOH: HEHUQPEHUHIHFWEHFDXVH+ ! PP
- $SSURDFK (TXDWLRQ DSSOLHVZLWK ZHKRSH DQRWK DQJOH *f* *f*
- 3URSHUWYDOXHV ,I VXUIDFHWHQVLRQLVQHJOHFWHG QRIOLG SURPHUWHVDUHQHGHG: K\"
- 6RXWLRQ $SSO(TXDWLRQ WRWKHNQRZQIORZ UDWHDQG VROYHIRU

$$Q = \frac{150\ \text{m}^3/\text{h}}{3600\ \text{s/h}} = 0.0417\frac{\text{m}^3}{\text{s}} \geq C_d \tan\left(\frac{\theta}{2}\right) g^{1/2} H^{5/2} = 0.44 \tan\left(\frac{\theta}{2}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)^{1/2} (0.3\ \text{m})^{5/2}$$

$$\text{Solve for } \tan\left(\frac{\theta}{2}\right) \leq 0.613 \quad \text{or} \quad \theta \leq 63° \qquad \textit{Ans.}$$

- &RPPHQWV $QDQJOHRI *f* ZLOOFUHDWHDQXSWUHDP KHDGRI FP $Q DQJOHOWVWKDQWK
  ZLOOFUHDWHDQHYHQODUJHUKHDG : HUIRUPXODVGHSHQGSUPDULO RQJUDYLWDQGJHRPHWU) OXL
  SURSHUWLHVVXFK DV ! < HQWURQ DVVOJKWP RGLILFDWRQVRUDVFRUHFWRQIDFWRUV

%DFNZDUG&XUYHV

$ ZHLUDIORZ EDUULHUWKDWQRWRQO DOWHUV WKHORFDOIORZ RYHUWKHZHLUEXWDOR PRGLILHV WK
IORZ GHSWKGLVWUEXWRQIDUXSWUHDP $Q VWRQJ EDUULHULQDQRSHQFKDQQHOIORZ FUHDWHVDEDFNZDUG
FXUYH ZKLFKFDQEHFRPSXWHG E\ WKHJUDGXDOO YDULHGIORZ WKHRU RI 6HF ,I 4 LVNQRZQ WK
ZHLUFRUP XOD (T GHWHUPLQHV+ DQGKQFHWKHZDWHUGHSWKXWXSXWHP RI WKHZHLU< +
< ZKHUH< LVWKHZHLUKHLJKW: HWKHQFRPSXW< [XSWUHP RI WKHZHLUFRP (T IRORZ LQ
LQWKLVFDVHDQ0 FXUYH) LJ F 6XFK DEDUULHU ZKHUHWKHZDWHUGHSWKFRUUHODWHV ZLWKWK
IORZ UDWH LVFDOOHG DFKDQQHOFRQWURSRLQW7KHVHDUHWKHVXLWDEOH SRLQWVFRUQP HUFDODQDOVLV F
IORRGZDWUSURILOHVLQULYHUV> @

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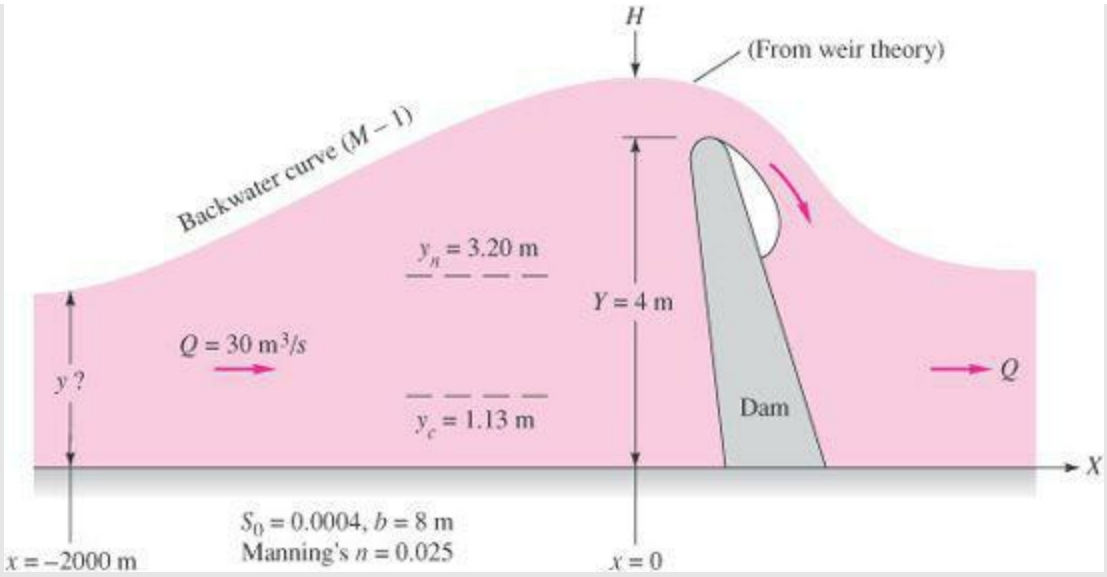
$ UHFWDQJXODUFKDQQHO P ZLGH ZLWKDIORZ UDWRI P V HQFRXQWHUV D P KLJK VKDUSHGJH
GDP DVVKRZQLQ) LJ (D ' HWHUPLQHWKHZDWHUGHSWK NP XSWUHP LI WKHFKDQQHOVORSHLV6
DQGQ

(D



6RO XWLRQ

) LUVW GHWHUPLQH WKH KHDG + SURGXFHG E\ WKH GDP XVLQJ WKH SUHVFULEHG IXOOZ LGWK Z HLUWKHRU (T

$$Q = 30\ \text{m}^3/\text{s} = C_d b g^{1/2} H^{3/2} = \left(0.564 + 0.0846\frac{H}{4\ \text{m}}\right)(8\ \text{m})(9.81\ \text{m/s}^2)^{1/2} H^{3/2}$$

6LQFH WKH WRWDO + LQ SDUHQWKHVHV LV VPDOO Z H PD\ SURFHHG E\ LWHUDWLRQ RU((6 VR WK VROXWLRQ + § P 7KHQ RXU LQLWLDO FRQGLWLRQ DV\ IXWXUH SXWHDP RI WKH GDP LV\ < + P &RPSDUH WKLV WR WKH FULWLFDO GHSWK IURP (T

$$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left[\frac{(30\ \text{m}^3/\text{s})^2}{(8\ \text{m})^2(9.81\ \text{m/s}^2)}\right]^{1/3} = 1.13\ \text{m}$$

6LQFH\ LV JUHDWHU WKDQ\F WKH IORZ XSVWHDP LV VXEFULWLFDO) LQDOO IRU UHHUHQFH SXUSRVHV HVWLP DWH WKH QRUPDO GHSWK IURP WKH&Kp]\ HTXDWLRQ

$$Q = 30\ \text{m}^3/\text{s} = \frac{\alpha}{n} b y R_h^{2/3} S_0^{1/2} = \frac{1.0}{0.025}(8\ \text{m}) y_n \left(\frac{8y_n}{8 + 2y_n}\right)^{2/3} (0.0004)^{1/2}$$

%\ WULDO DQG HUURU ((6 VROYH IRU\Q § P ,I WKH UHDUHQR FKDQJHV LQ FKDQQHO Z LGWK RU VORSH WKH Z DWHU GHSWK IDUXSWHDP R WKH GDP Z LOO DSSURDFK WKLV YDOXH $ COWKHVH UHUHHUHQFH YDOXHV\ \F DQG\Q DUH VKRZ QLQ) LJ (E

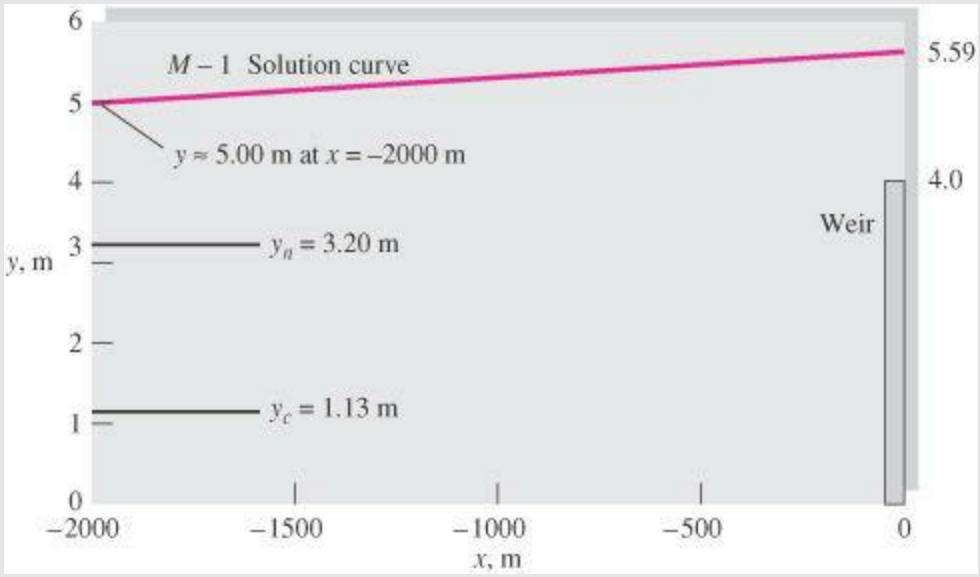
6LQFH\ ! \Q ! \F WKH VROXWLRQ Z LOO EH DQ 0 FXUYH DV FRP SXWHG IURP JUDGXDOO YDULHG WKHRU (T IRU DUHFWDQJXODU FKDQQHO Z LWK WKH JLYHQ SXWEDWV

$$\frac{dy}{dx} \approx \frac{S_0 - n^2 Q^2/(\alpha^2 A^2 R_h^{4/3})}{1 - Q^2 b_0/(g A^3)} \qquad \alpha = 1.0 \quad A = 8y \quad n = 0.025 \quad R_h = \frac{8y}{8 + 2y} \quad b_0 = 8$$

%HJLQQLQJ ZLWK\ P DW/ ZHLQMUDWEDFNZDUGVR[í P)RUWKH5XQJH.XW
PHWKRG IRXUILJXUHDFFXUDF\ LVDFKLHYHGIRUû[í P 7KHFRPSDUDWLYHFRQYHUVHQFH
)LJ (E 7KHGHWLHGVRQXQRQYDOXHLV

At $x = -2000$ m: $y \approx 5.00$ m *Ans.*

(E



7KXV HYHQ NP XSVWUHDP WKHGDP KDVSURGXFHGD³EDFNZDWÚ WKDWLV P DERYHWKHQRUPDO GHSWKWKDWZRXOGRFFXUZLWKRXWDGDP)RUWKLVH[DPSOH DQHDUQRUPDOGHSWKRI VD\ FP JUHDWHUWKDQ\Q RU\ § P ZRXOGQRWEHDFKLHYHGXQGHU[í P %DFNZDWUFXUYHVDUHTXLWHIDUUHDFKLQJXSWUHDP HVSHFLDOO\ LQIORRGVWDJHV

6XPPDU\

7KLVFKDSWUKDVLQWURGXFHGRSHQFKDQQHOIORZ DQDOVLV GPLWGWRWUDG\RQHGLPHQVLRQDOIORZ FRQGLWLRQV 7KHEDVLF DQDOVLVFRPELQHVWKHFRQWLQXLW\HTXDWLRQZLWKWKHH[WQGHG%HUQRXOOLHTXDWLR LQFOXGLQJIULFWLRQORVVHV

2SHQFKDQQHOIORZV DUH FODVVLILHG HLWKHU E\ GHSWK YDULDWLRQ RU E\ )URXGH QXPEHU WKH ODWWH EHLQJ DQDORJRXVWRWKH0DFKQXPEHULQFRPSUHVVLEOHGXFWORZ &KDS)ORZ DWFRQWUDQWWRSHDQ GHSWKLV FDOOHG XQLIRUP IORZ DQG VDLWLHVWKH FRQWLFDO&Kp]\ HTXDWLRQ 6WDJJKWSULVP DU FKDQQHOV FDQ EH RSWLP L]HG WR ILQG WKH FURVV VHFWLRQ WKDWJLYHV P D[LPXP IORZ UDWH Z LWK PLQLPXP IULFWLRQORVVHV $VWKHVORSH DQG IORZ YHORFLW LQFUHDVH WKHFKDQQHOUHFKVDFULWLFDOFRQGLWLRQ R)URXGH QXPEHUXQLW Z KHUH YHORFLW HTXDOV WKH VSHHG RI D VP DOODP SOLWXGH VXUIDFH Z DYH LQ WK FKDQQHO (YHU FKDQQHOKDV DFULWFDOVORSHWKDWYDULHV Z LWKWKHIORZ UDWH DQG URXJKQHVV ,I WKHIORZ EHFRPHV VXSHUFULWFDO)U! LWPD\ XQGHUJR D K\GUDXOLF MXPS WR D JUHDWHU GHSWK DQG IORZ H

VXEFULWLFDO YHORFLW DQDORJRXV VR DQRWKHU QDPH VKRFN ZDYH

7KH DQDORJ\ JUDGXDOO\ YDULHG IORZ OHDGV VR DGLIIHUHQWLDO HTXDWLRQ VKDSHDQEHVROYH E\ QXPHULFDOPHWKRGV 7KHFKDSWHUHQGVZLWKDGLVFXVVLRQRI VKHIORZ RYHUDGDP RUZHLUZKHUHVW VRPDOIORZ UDWHFDQEHFRUUHODWHGZLWKXSWUHDP ZDWHUGHSWK

SURECOPV TXHVWLRQV3URECOPV

0RVWRI VKHSURECOPVKHUHQDUHIDLU VWDLJKWRUZDUG 0RUHGLIILFXOWURSHQHGGHDVLJQPHQ DUHODEHOHGZLWKDQDVWHULVN 3URECOPVODEHOHGZLWKQ((6LFRQ ZLOOEHQHILWURP VKHXVHRI VW (QJLQHHULQJ(TXDWLRQ6ROYHU ((6 ZKLFKSURECOPVODEHOHGZLWKDFRPSXWHULFRQ PD\ UHTXLUH VKHXVHRI DFRPSXWHU7KHVWDQGDUGHQGRI FKDSWUSURECOPV3 VR3 FDWDORJ]HGLQVK SURECOPOLVWKHUH DUHIROORZHGE\ ZRUGSURECOPV: VR: IXQGDPHQWDOVRI HQJLQHHULQ H[DP SURECOPV)(VR)(FRPSUHHQVLYH SURECOPV& VR& DQG GHVLJQ SURMHF ' DQG'

3URECOP ' LVWULEXWLRQ

Problem Distribution

Section	Topic	Problems
10.1	Introduction: Froude number, wave speed	P10.1–P10.10
10.2	Uniform flow: the Chézy formula	P10.11–P10.36
10.3	Efficient uniform-flow channels	P10.37–P10.47
10.4	Specific energy: critical depth	P10.48–P10.58
10.4	Flow over a bump	P10.59–P10.68
10.4	Sluice gate flow	P10.69–P10.78
10.5	The hydraulic jump	P10.79–P10.96
10.6	Gradually varied flow	P10.97–P10.112
10.7	Weirs and flumes	P10.113–P10.123
10.7	Backwater curves	P10.124–P10.128

,QWURGXFWRQ) URXGHQXP EHU ZDYHVSHHG

- 3 7KHIRUPXODIRUVKDOORZ ZDWHUZDYHSURSDJDWLRQVSHHG(T RU LV LQGHSHQGHQWRI WKH SK\VLFDO SURSHUWLHV RI WKH OTXLG ONH GHQVLW\ YLVFRVLW\ R VXUIDFHWHQVLRQ ' RHVWKLV PHDQWKDWZDYHVSURSDJDWHDWWKHVDPHVSHHGLQZDWH PHUFXU JDVROLQH DQGJO\FHULQ'([SODLQ
- 3 : DWHUDW *& IORZVLQD FP ZLGHUHFWDQJXODUFKDQQHODWDGHSWKRI FP DQGI IORZ UDWHRI FP V (VWLPDWH D WKH)URXGHQXPEHUDQG E WKH5HQROG QXPEHU
- 3 1DUURZJDXJHWW%D\ LVDSSUR[LPDWHO WDXWW PLORQJ DQGKDVDQDYHUDJHGHSWKF IW7LGDOFKDUWIRUWKHDUHDLQGLFDWHVWKDWWKHGHD WKHPRXWKRI WKHED\ 1HZSRUW5KRGH,VODQG DQGLWKHDG 3URYLGHQFH 5KRG ,VODQG ,VWKLVGHD\ FRUUHODWHGZLWKWKHSURSDJDWLRQRI DVKDOORZ ZDWHUWZDOOFUH ZDYHWKURXJKWKHED\"([SODLQ
- 3 7KHZDWUFKDQQHOIORZ LQ)LJ 3 KDV DIUHH VXUIDFHLQWKUHHSDFH ' RHVL TXDOLW\ DVQRSHQFKDQQHOIORZ"([SODLQ: KDWGRHVWKHGDVKHGOQHUHSUHVHQW

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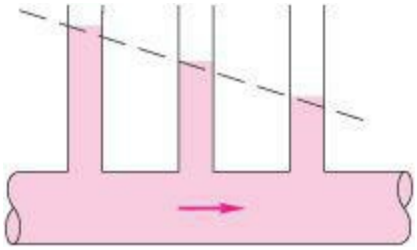
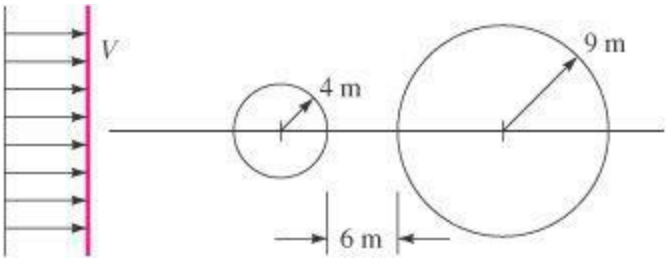


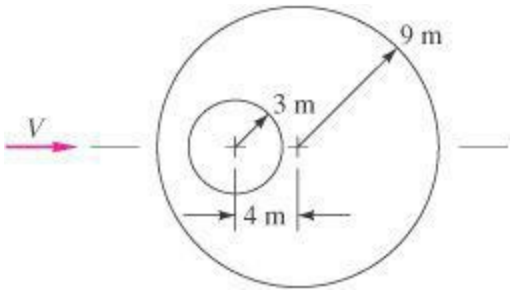
- 3 : DWUIORZ VORZ QDUHDQJXODUFKDQQHODWLV IWZLGHDQG IWGHHS 7KHIORZ UDL LV JDOPLQ (VXPDWKH) URXGHQXPEHURI WKHIORZ
- 3 3HEEOHV GURSSHG VXFFHVVLYHO DWWKHVDPHSRLQWLQWR DZ DWUFKDQQHOIORZ RI GHSV FP FUHDWHVZDYHV RFLUFXODUULSSOHV DVLQ) LJ 3) URP WKLVLQIRUPDWRQHVWLPDL D WKH) URXGHQXPEHUDQG E WKHVWHDP YHORFLW

3



- 3 3HEEOHV GURSSHG VXFFHVVLYHO DWWKHVDPHSRLQWLQWR DZ DWUFKDQQHOIORZ RI GHSV FP FUHDWHVZDYHV RFLUFXODUULSSOHV DVLQ) LJ 3) URP WKLVLQIRUPDWRQHVWLPDL D WKH) URXGHQXPEHUDQG E WKHVWHDP YHORFLW

3



- 3 $Q HDUWKTXDNH QHDU WKH . HQDL 3HQLQVXOD $ODVND FUHDWHV D VLQJOH³WVXQDPL ZDY FDOOHG DWXQDPL WKDWSURSDJDWHV VRXWKZDUG DFURVV WKH 3DFLILF 2FHDQ ,I WK DYHUDJHRFHDQGHSWKLV NP DQG VHDZDWHUGHQVLWLV NJ P HVWLPDWWKHWPI RI DULLYDORI WKLVWXQDPLLQ+LOR +DZDL
- 3 (TXDWRQ LV IRUD VLQJOH GLVWXUEDQFH ZDYH) RUSHURGLF VP DOODP SOLWG VXUIDFH ZDYHV RI ZDYHOHQJWK DQG SHULRG7 LQYLVFLG WKHRU > WR @SUHGLFWWI ZDYHSURSDJDWRQVSHHG

$$c_0^2 = \frac{g\lambda}{2\pi}\tanh\frac{2\pi y}{\lambda}$$

ZKHUH\ LV WKH ZDYHVSHHG VXUIDFH WHQVLRQ LV QHJOHFWHG D ' HWHUPLQH LI WKL
H[SUHVVLRQ LV DIIHFWHG E\ WKH 5H\QROGV QXPEHU) URXGHQXPEHU: HEHUQXPEHU' HULY
WKH OPLWLQJ YDOXHV RI WKLV H[SUHVVLRQ IRU E \≪ DQG F \≫ G) RUZKDWDDR\ L
WKH ZDYHVSHHG ZLWKLQ SHUFHQW RI OPLWF"

- 3 ,I VXUIDFH WHQVLRQ< LV LQFOXGHG LQ WKH DQDOVLV RI 3URE 3 WKH UHVXOWLQJ ZDY
VSHHGLV> WR @

$$c_0^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi Y}{\rho\lambda}\right)\tanh\frac{2\pi y}{\lambda}$$

D ' HWHUPLQH LI WKLV H[SUHVVLRQ LV DIIHFWHG E\ WKH 5H\QROGV QXPEHU) URXGHQXPEHU
: HEHUQXPEHU' HULYH WKH OPLWLQJ YDOXHV RI WKLV H[SUHVVLRQ IRU E \≪ DQG F \≫ G
) LQDOO GHWHUPLQH WKH ZDYHOHQJWK $FW JRUDPLQP XP YDOXHRI F DVXPLQJ WKDW≫

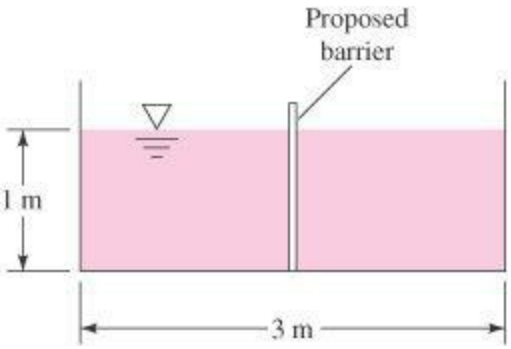
8QLIRUP IORZ WKH&Kp]\ IRUPXOD

- 3 $ UHFWDQJXODU FKDQQHOLV P ZLGH DQG FRQWDLQV ZDWHU P GHHS ,I WKH VORSH LV
f DQG WKH DQQLQJ LV FRUUXJDWHG PHDVXUHPDWWKH GLVFKDUJHIRUXQIRUP IORZ
- 3 D) RUOPLQDUGUDLQLQJ RI DZLGHWKLQ WKH ZDWHUURQSDYHPHQWVORSHGDWDQJO
DVLQ) LJ 3 VKRZ WKDWWKH IORZ UDWHLVJLYHQE\

$$Q = \frac{\rho g b h^3 \sin\theta}{3\mu}$$

ZKHUHE LV WKH VKHHWZLGWK DQG K LV GHSWK E %\ VRPHZKDWDEHORZ FRPSDUHV
ZLWK(T VKRZ WKDWWKLV H[SUHVVLRQ LV FRPSDUEOHZLWK DIUFWLRQDFWRUI 5H
ZKHUH5H 9DYK

- 3 $ GUDJH SRQG GUDLQV GRZQ DQ DSKDOWUHFDQJXODU FKDQQHOWKDWLV IWZLGH 7K
FKDQQHOVORSHLV GHJUHHV ,I WKH IORZ LV XQIRUP DWDGHSWKRI LQ HWWP DWWK
WPHWRGUDLQ DFUHIRRWWR ZDWU
- 3 7KH&Kp]\ IRUPXOD LVLQGHSHQGHQWRI IXLGGHQVLW\ ' RHVWKL
PHDQWKDWZDWU PHUFXU DOFRKRODQG6\(RLOZLOODOOIORZ GRZQDJLYHQRSH
FKDQQHODWWKHVDPHUDWH ([SODLQ
- 3 7KHSDLQWHG VWHHOFKDQQHORI) LJ 3 LVGHVLJQHG ZLWRXWWKHEDUULHUIRUDIORZ
UDWRI P VDWD QRUPDOGHSWKRI P ' HWHUPLQH D WKHGHVLJQVORSHRI WK
FKDQQHODQG E WKHUHGXFWLRQLQWRWDOIORZ UDWLI WKHSURSRVHGSDLQWHG VWHHOFHQWH
EDUULHULVLQVWDOOHG



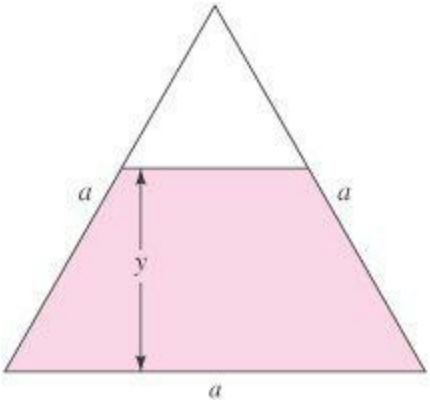
- 3 $ EUDFNZDWHUUHVHUYRLU FRQWDLQHGE\DFRQFUHWH GDP ZLGHDQGGHHS P SHUP D)LQGWKHFRUUHVSRQGLQJZLGWKRIWKHZDWHUVXUIDFH FP E)RUWKHVDPH VSHFLILHGWKHZDWHUGHSWKWKDWZLOOEHREWDLQHGDW &RPPHQW
- 3 7KHWDSHUHGGDPFRQQHFWLQJ) LJ 3 LVPDGHRI EUDFNZRUNDQGVSHFLILHV ' HWHUPLQHWKHIORZ UDWHLI WKHFRUUHVSRQGLQJ FP

3

- 3 $ 9 VKDSHG SDLQWHG VWHHOFKDQQHO VLP LODU WR) LJ (KDV DQ LQFOXGHG DQJOH R GHJUHHV ,I WKH VSHHG LQ XQLIRUP IORZ LV P SHUP HVWLP DWH D WKH IORZ UDWH LQP VDQG E WKH DYHUDJHZDOOVKHDUVWUHVV 7DNH\ P
- 3 0RGLI\ 3URE 3 WKH *9HHFKDQQHO WR OHWWKH VXUIDFHEHFOHDQHDUWK ZKHUHVLI WKHDYHUDJHYHORFLW\H[FHHGV IWV) LQGWKHPD[LPXP GHSWKWKDWDYRLG HURVLRQ 7KHVSHFLILFVWLO P SHUP
- 3 $ FLUFXODUFRUUXJDWHGPHWDOVWRUP GUDLQLVIORZLQJ KDOI IXOORYHUDVSHF IWP L (VWLPDWHWKHQRUPDOGLVFKDUJHLI WKHGUDLQGLDPHWULV IW
- 3 $QHQJLQHHUPDNHVFDUHXOPHDVXUHPHQWZLWKDZHLU VHH6HF WKDWPRQVWUL DUHFDQJXODUXQLQVKHGFRQFUHWHFKDQQHOLGRQDVSHF *6KHILQGV SHUKDS ZLWKVXSSULHV WKDWZKHQWKHZDWHUGRXEOHVIURP IW LQFKHVVR IW LQFKHV WKHQRUPDOIORZ UDWHPRUHWKDQGRXEOHVIURP VR IWV D ,VWKLVSODXVLEOH E ,I VR HVWLPDWHWKHFKDQQHOZLGWK
- 3 $ WDSHUHGGODTXHGXFW) LJ KDVE P DQG *DQGFDUULHVDQRUPDO IORZ RI P V RI ZDWHU ZKHQ\ P)RUFOD\ WKHVXUIDFHV HVWLPDWHWKH UHTXLUHGHOHYDWLRQGURSLQP NP
- 3 ,WLVGHVLUHGWRH[FDYDWHFHDQHDUWKFKDQQHOVDVDWDSHUHGGDOFURVVVHFWLRQZLWK *VHH) LJ 7KHH[SHFWHGIORZ UDWHLV IWV DQGWKHVOSHLV IWSH PLOH 7KHXQLIRUP IORZ GHSWKLV SODQQHG IRUHILFLHQWSHUIRUPDQFH VXFKWKDWWKH IORZ FURVVVHFWLRQLV KDOI DKH[DJRQ: KDWLVWKHDSSURSULDWHERWWRP ZLGWKRI WKH FKDQQHO"
- 3 $ UHFWDQJXODUFKDQQHOODLGRXWRQD *VORSH GHOLYHUVDIORZ UDWHRI JDOPLQ LQXQLIRUP IORZ ZKHQWKHGHSWKLV IWDQGWKHZLGWKLV IWD (VWLPDWHWKHYDOXHRI 0 DQQLQJ¶VIDFWRUQ E : KDWZDWHUGHSWKZLOOWULSHWKHIORZ UDWH"

- 3 7KH HTXLOLEULXP DQDO\VLV FKDQJHV) LJ 3 KDV FRQVLGHUDEOH6R DQG FRQVLG
0 DQQLQJ IDFWRUQ ,I \ D ILQGDQDO\VLV H[SUHVVLRQ IRU WKH IORZ UDWH4

3



- 3 EES ,Q WKH VSUHDG) LJ E DQDO]H D UHFWDQJXODU FKDQQHOLQ XQIRUP IORZ ZL
FRQVLGHUDEOH D$ E\ FRQVLGHULQJ EXWYDULQJ ZLGWK E DQG GHSWK\ 3ORWWK
UHVXOWLQJ IORZ UDWH4 QRUPDOL]HG E\ LWV PD[LPXP YDOXH4PD[LQWKHUDQJH E\
DQG FRPPHQWRQ WKHUHLWLV FUXFLDO IRUGLVFKDUJH HILFLHQF\ WR KDYH WK
FKDQQHOIORZ DWDGHSWK H[FHVV HTXDOWR KDOI WKHFKDQQHOZLGWK
- 3 $ FLUFXODUFRUUXJDWHGPHWDOZDWUFKDQQHOKDV DVORSHFR DQGDGLPHWUR
IW D (VWDPDWWKHQRUPDOGLVFKDUJH LQJDOPLQ WKHZDWUDGHSWKLV IW E
) RUWKLVFRQGLWLRQ FDOFXODWWKHDYHUDJHZDOVKHDUVWUHVV
- 3 6KRZ WKDWIRUDQ VWDLJKWSULPDWF FKDQQHOLQ XQIRUP IORZ WKH DYHUDJH ZD
VKHDUVWUHVVLVJLYHQE\

$$\tau_{av} \approx \rho g R_h S_0$$

,I \RX KDSSHQWR VSRWKLVUHVXOWHDUO \RX FDQXVHLWQVROLQJ 3URE 3
- 3 6XSSRVH WKDWWKH WDSH]RLGDOFKDQQHORI) LJ 3 FRQVLVWVDQG DQG VLOWWKD
ZHZLVK QRWWR HURGH $FFRUGLQJ WR DQHPSLULFDOFRUUHODWLRQE\ $ 6KLHOGVLQ
WKHDYHUDJHZDOOVKHDUVWUHVV2FULWJUHTXLUHG WR HURGHVDQG SDUWFOHVRI GLDPHWUG LV
DSSUR[LPDWHGE\

$$\frac{\tau_{crit}}{(\rho_s - \rho) g\, d_p} \approx 0.5$$

ZKHUH!V§ NJ P LVWKHGHQVLWRI VDQG ,I WKHVORSHRI WKHFKDQQHOLQ) LJ 3
LV DQGQ§ GHWHUPLQHWKHPD[LPXP ZDWUGHSWKWR NHHSIURP HURGLQJ SDUWFOH
RI PP GLDPHWU
- 3 $ FOD\ VWHH9 VKDSHGFKDQQHO ZLWK DQ LQFOXGHG DQJOHRI LV NP ORQJ DQG LV
OLG RXWRQ D VORSH : KHQ UXQQLQJ DWD GHSWRI P WKH XSVWUHDP HQG LV
VXGGHQO FORVHG ZKLOH WKH ORZHU HQG FRQQHFWV WR GUDLQ $VVXPLQJ TXDL VWDG
QRUPDOGLVFKDUJH ILQGWKHWP HIRUWKHFKDQQHOGHSWKWRGURS WR FP

- 3 $ Q XQLQVKHG FRQFUHWH IWGLP HWU VHZHU SLSH IORZV KDOI IXOO: KDWLV WK DSSURSULDWH VORSH WR GHOLYHU JDOPLQ RI ZDWHU LQ IRUP IORZ"
- 3 ' RHV KDOI D 9 VKDSHG FKDQQHO SHUIRUP DV ZHOO DV IXOO 9 VKDSHG FKDQQHO" 7KDQN ZHU WR 3URE LV4 P V ' R QRWH Y HDOWLV WR \RXU IULHQGV WKZ ZRUNLQJ RQ3) RU WKH SDLQWHG WHHOKDOI 9 LQ) LJ 3 DWWKH VDPH VORSH RI ILQG WKH IORZ DUHDWKDWJLYHV WKH VDPH4 DQG FRPSDUHZLWK3

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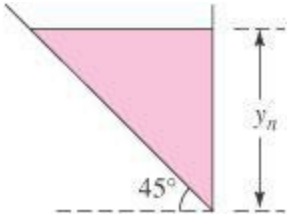

- 3) LYHVHZHUSLSHV HDFKD P GLDP HWUFOD\ WDHSLSHUXQQLQJ KDOI IXOORQD VORSHR f HPSWLQR DVLQJHDSKDOWSLSH DOR OLG RXWDW f ,I WKHODUJHSLSHL' DOR WRUXQKDOI IXOO ZKDWWKRXGEHLWGLDP HWU'
- 3 $ EUFNUHFWDQJXODUFKDQQHOZLWK6 LVGHVLJQHGWRFDUU IWVRI ZDWLQ XQIRUP IORZ 7KHUHLVDQDUPHQWRYHUZKHWKHUWKHFKDQQHOZLGWKVKRXGEH RU IW. KLFKGHVLJQQHGVIHZHUEUFNV" %\ ZKDWSHUFHQWDJH'
- 3 ,QIORRGVWDJHDQDXDOFKDQQHORIWQFRQLWWR DGHSPDLQFKDQQHOSXVWZI IORRGSODLQV DVLQ) LJ 3 7KHIORRGSODLQVDUHRIWQVKDOORZ DQGURXJK ,I WKFKDQQHOKDVWKHVDPHVORSHHYHUZKHUH KRZ ZRXGG\RX DQDO]HWKLVVLWXDWLRQIR WKHGLVFKDUJH' 6XSSRVHWKDW IW\ IWE IWE IWQ DQGQ ZLWKDVORSHRI (WUP DWWKHGLVFKDUJHLQIWV

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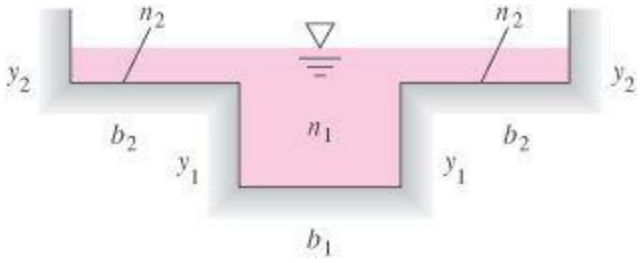
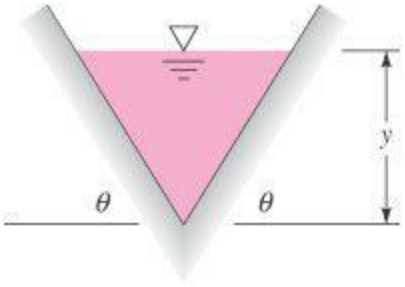


- 3 EES 7KH%DFNWDWHU5LYHULQRUWKHUQ5RGH,VODQGQRUPDOO IORZV DWDERXW P V DQG UHVPEOHV) LJ 3 ZLWKDFHDQHDWKFHQWUHFKDQQHOE § P DQG\ § P 7KHEHGVORSHLV DERXW IWPL 7KHVLGHVDUH KHDY\ EUKK ZLWKE § P ' XULQJ KXUULFDQH &DUROLQ DUHFRUGIORZ UDWHR P V ZDVHVWLPDWHG 8VHWKLVLQRUPDWLRQWRHVWLPDWHWKHPD[LPXP IORRGGHSWK\ GXULQJWKLVHYHQW

(IILFLHQWXQLRUP IORZ FKDQQHOV

- 3 $ VWDQGXDUFKDQQHO VHH) LJ (LV WR EH FRQVWUXFWHG RI FRUUXJDWHG PHWDO
ZLOOFDUU P VRQDVORSHRI 7KHVXSSO RI VKHHWPHWDOLVOLPLWHG VR WK
HQJLQHHUV ZDQWWR PLQLPL]H WKH FKDQQHOVXUFH : KDWDUH D WKH EHVWOLQFXGH
DQJOH IRUWKH FKDQQHO E WKH QRUPDOGHSWK IRUSDUW D DQG F WKH ZHWWH
SHUPHWHUIRUSDUWE "
- 3) RUWKHKDO 9HHFKDQQHOLQ) LJ 3 GHWWKHLQWHUIRUDQJOHRI WKH9HHEH) RU
JLYHQ YDOXHRI DUHD VOSH DQGQ ILQG WKH YDOXHRI IRUZKLFK WKHIORZ UDWHLVI
PD[LPXP 7RDYRLGFXPEHUVRPHDQGEUD VLPSO SOPV4 YHUXV IRUFRQWDQW$
- 3 $ WSH]RLGDOFKDQQHOKDVQ DQG6 DQGLVPDGHILQWKHVKDSHRI I
KDO KH[DJRQ IRUPD[LPXP HILFLHQF\ : KDWWKRXGG WKH OHQJWK RI WKH VLGH RI WK
KH[DJRQEHLI WKHFKDQQHOLV WR FDUU IWV RI ZDWU : KDWLV WKHGLVFKDUJHRI I
VHPLFLUFXODUFKDQQHORI WKHVDPHFURVV VHFWLRQDODUHDDQG WKHVDPH6 DQGQ'
- 3 8VLQJ WKHJHRPHWU RI) LJ D SURYHWKDWWKHPRVWHILFLHQWFLUFXODUSHQFKDQQ
PD[LPXP KLGUDXOF UDGLXVIRUDJLYHQIORZ DUHD LVDVHPLFLUFOH
- 3 ' HWHUPLQHWKHPRVWHILFLHQWYDOXHRI IRUWKH9 VKDSHGFKDQQHORI) LJ 3

3



- 3 ,WLV GHVLUHG WR GHOLYHU JDOP LQ RI ZDWU LQ D EULFN ZRUN FKDQQHOODLG RQ
VORSH RI : KLFK ZRXOG UHTXLUH IHZHU EULFNV LQ XQLRUP IORZ D D 9
FKDQQHOZ LWK *f* DVLQ) LJ 3 RU E DQHILFLHQWUHFWDQJXODUFKDQQHOZ LW
E \"
- 3 : KDWDUH WKH PRVWHILFLHQWGLPHQVLRQV IRUD ULYHWHG WURXJKFUHWHDQJXODUFKDQQHOV
FDUU P VDWD VORSHRI "
- 3 : KDWDUH WKHPRVWHILFLHQWGLPHQVLRQV IRUDKDO KH[DJRQFDVWLURQFKDQQHOWRFDUU
JDOP LQRQDVORSHRI *f*"
- 3 &DOFXOXV VWXGHQW XV WKDWWKH PRVWHILFLHQWZ DOODQJOHIRUD 9 VKDSHG FKDQQHO) LJ
3 LV *f* ,WLHOGV WKHKLJKHVWQRP DCIORZ UDWIRUDJLYHQDUHD %XWLV WKDW
D VKDUS RUDIODWP D[LPXP") RUDIORZ DUHD RI P DQG DQXQLQLVKHG FRQFUHWH
FKDQQHOZ LWKDVORSHRI SOPWKHQRP DCIORZ UDW4 LQP V YHUXV DQJOHIR
WKHUDQJH *f" " *f DQG FRPPHQW
- 3 ,WLV VXJJHVWHG WKDWD FKDQQHOWKDWUHGXFHV HURVLRQ KDV DSDUDEROF VKDSH DV) LJ 3) RUPXODV IRUDUHD DQG SHULPHWHUIRU WKHSDUDEROF FURVV VHFWLRQ DUHD
IRORZV> S @

$$A = \frac{2}{3}bh_0;\ P = \frac{b}{2}\left[\sqrt{1+\alpha^2} + \frac{1}{\alpha}\ln(\alpha + \sqrt{1+\alpha^2})\right]$$

$$\text{where}\quad \alpha = \frac{4\,h_0}{b}$$

) RU XQLRUP IORZ FRQGLWLRQV GHWHUPLQH WKH PRXWHILFLHQWUDWR E IRUWKLV FKDQQ-
PLQP XP SHUP HWUIRUDJLYHQFRQXDQWDUHD

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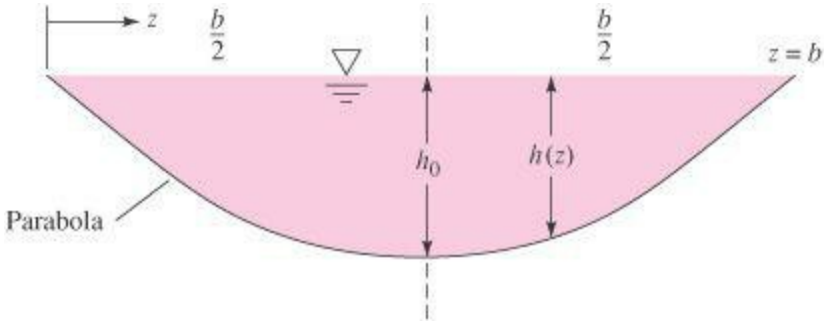

- 3 &DOFXOXV WKDW XV WKDWWKH PRXWHILFLHQWZ DWUGHSWKIRUDUHFWDQJXODUFKDQQHO VXF
DV) LJ (LV\ E ,WWLHOGVWKHKJKHVWQRP DOIORZ UDWIRUDJLYHQDUH
%XWLV WKLV D VKDUS RUDIODWP D[LPXP")RUDIORZ DUHD R P DQG DFDO WK
FKDQQHOZ LWKDVORSHR SORWWKHQRP DOIORZ UDW4 LQP V YHUXV\ E IR
WKHUDQJH ”\ E” DQGFRPPHQW

6SHFLILF HQHUJ\ FULWLFDOGHSWK

- 3 $ ZLGH FKDQHDWKULYHUKDV DIORZ UDWT IW V IW : KDWLV WKH FULWLFD
GHSWK' ,I WKHDFWXDOGHSWKLV IWZ KDWLV WKH) URXGHQXPEHUR WKHULYHU' &RP SX\
WKHFULWLFDOVORSHE\ D 0 DQQLQJ¶VIRUPXODDQG E WKH0 RRG\ FKDUW
- 3) LQG WKHFULWLFDOGHSWKR WKHEUFNFKDQQHOLQ3UREOHP 3 IRUERWWKH DQG
ZLGWKV \$UHWKHQRP DOIORZ VXEFULWLFDORUXSHUFULWLFDO"
- 3 $ SHQFLO SRLQWSLHFLQJ WKH VXUDFH R D UHFWDQJXODU FKDQQHOIORZ FUHDWV I
ZHGJHDNH ƒ KDO DQJHZDYH DVLQ) LJ 3 ,I WKHFKDQQHOVXUDFHLVSDLQWH
WWHODQG WKHGHSWKLV FP GHWHUPLQH D WKH) URXGHQXPEHU E WKHFULWLFD
GHSWK DQG F WKHFULWLFDOVORSHIRUXQLRUP IORZ

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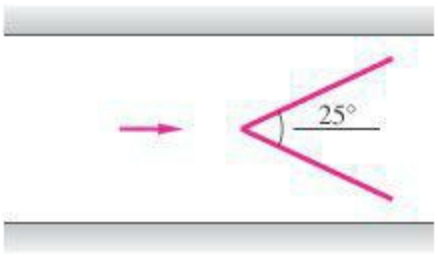

- 3 $QDVSKDOWFLUFXODUFKDQQHORI GLDPHWU FP LVIORZLQJ KDOIXOODWDQDYHUDJ YHORFLWVRI P V(VWDWDW D VKHYROXPHIORZ UDWH E WKH)URXGHQXPEHUDQ F VKHFULWLFDOVORSH
- 3 : DWUIORZVIXOOLQDQDSKDOWKDOIKH[DJRQFKDQQHORI EHWWRP ZLGWK: 7KHIORZ UDWHLV P V(VWDW: LI WKH)URXGHQXPEHULVH[DFWO
- 3)RUWKHULYHUIORZ RI 3URE 3 ILQG WKHGHSWK WKDWKDV WKHVDPH VSHFLILF HQHUJ\ DV WKHJLYHQGHSWK\ IW7KHVHDUHFDOOHGFRQMXJDWHGHSWKV : KDWL)U"
- 3 $FOD\ WDQH9 VKDSHGFKDQQHOKDV DQLQFOLQHGDQJOHRI ƒ DQGFDUULHV P V &RPSXWH D WKHFULWLFDOGHSWK E WKHFULWLFDOYHORFLW DQG F WKHFULWLFDOVORSHIR XQLIRUP IORZ
- 3 $ WUDSH]RLGDOFKDQQHOUHVPEOHV) LJ ZLWKE P DQG ƒ 7KHZDWH GHSWKLV P DQGWKHIORZ UDWHLV P V,I \RXWDNH\RXUILQJHUDQGLQWKHVXUFH DVLQ) LJ 3 ZKDWKDO DQJOHZDYHPLJKWSSHDU
- 3 $ IWGLDPHWUILQLVKHGFRQFUHWHVHZHUSLSHIORZVIXOOR ZDWU D ,QWKHVSLU RI) LJ D HXMP DW WKH VSHHG RI SURSDJDWRQ RI D VPDOOPSOXGH ZDV SURSDJDWQJ DORQJ WKH FKDQQHO E ,I WKH ZDWU LV IORZLQJ DW JDOPL FDOFXODWWKH) URXGHQXPEHU
- 3 &RQVLGHUWKH9 VKDSHGFKDQQHORI DUELWDU DQJOHLQ) LJ 3 ,I WKHGHSWKLV D ILQG DQDQDO WF H[SUHVVLRQIRUWKHSURSDJDWQVSHGF RI DVPDOOGLVXUEDQFH ZDYHDORQJ WKLVFKDQQHO>+LQW(OP LQDWWIORZ UDWIURP WKHDQDO VHVLQ6HF ,I ƒ DQG WKHGHSWKLV P GHWUPLQH E WKHSURSDJDWQVSHGDQG F WK IORZ UDWLI WKHFKDQQHOLVUXQQLQJ DWD) URXGHQXPEHURI
- 3 7R IRORZ XS WKH KDOIXOOFLUFXODUFKDQQHORI 3URE 3 FDOFXODWH D WK FULWLFDOVORSHDQG E WKHDFWXDOVORSHLI WKHXQLIRUP IORZ UDWLV JDOPLQ

) OORZ RYHUDEXPS

- 3 8QIRUP ZDWUIORZ LQDZLGHEUFNFKDQQHORI VORSH ƒ PRYHVRYHUD FF EXPS DV LQ) LJ 3 $ VOJKWGHSUHVVLRQ LQWKHZDWU VXUIDFH UHVXOWV ,I WK PLQLPXP ZDWUGHSWKRYHUWKHEXPS LV FP FRPSXWH D WKHYHORFLWRYHUWK EXPSDQG E WKHIORZ UDWSHUPHWURI ZLGWK

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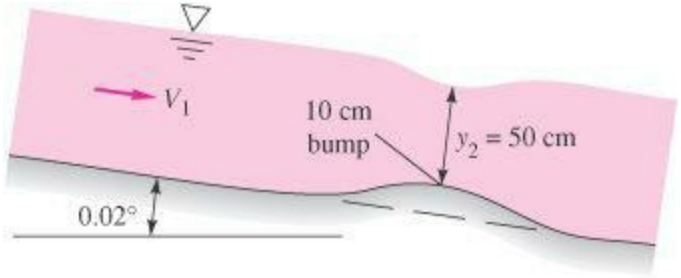

- 3 : DWUIORZLQJ LQDUHFWDQJXODUFKDQQHO P ZLGH HQFRXQWHUV DERWWRP EXPS FF KLJK 7KHDSSURDFK GHSWKLV FP DQGWKHIORZ UDWH P V' HWUPLQH D WK

Z DWUGHSWK E YHORFLW DQG F)URXGHQXPEHUDERYHWKHEXPS +LQW7KHFKDQJH LQZ DWUGHSWKLVUDWHUVOLJKWRQO DERXW FP

- 3 0 RGLI\ 3URE 3 DV IROORZV $JDLQ DVVXPLQJ XQLIRUP VXEFULWLFDO DSSURDF IORZ 9 \ ILQG D WKHIORZ UDWHDQG E \ IRUZKLFKWKHIORZ DVWKHFUHVWRI WK EXPSLVH[DFWO FULWLFDO)U

- 3 &RQVLGHU WKHIORZ LQ D ZLGHFKDQQHORYHUD EXPS DV LQ)LJ 3 2QHFD HVLPDWWKHZDWUGHSWKFKDQJHRUWDQVLWRQZLWKIUFWLRQOHVVIORZ 8VHFRQWLQXL DQGWKH%HUQRXOOLHTXDWLRQWRVKRZ WKDW

$$\frac{dy}{dx} = -\frac{dh/dx}{1 - V^2/(gy)}$$

,V WKHGUDZGRZQRI WKHZDWUVXUIDFHUHDOWF LQ)LJ 3 " ([SODLQXQGHUZKI FRQGLWLRQVWKHVXUIDFHPLJKWULVHDERYHLWXSVWUHDP SRVLWLRQ\

- 3 ,Q)LJ 3 GHW9 P VDQG\ P ,I WKHPD[LPXP EXPSKHLJKWLV FP HVWLPDW D WKH)URXGHQXPEHUDYHUWKHWRS RI WKHEXPS DQG E WKH PD[LPXP GHSUHVLRQLQWKHZDWUVXUIDFH

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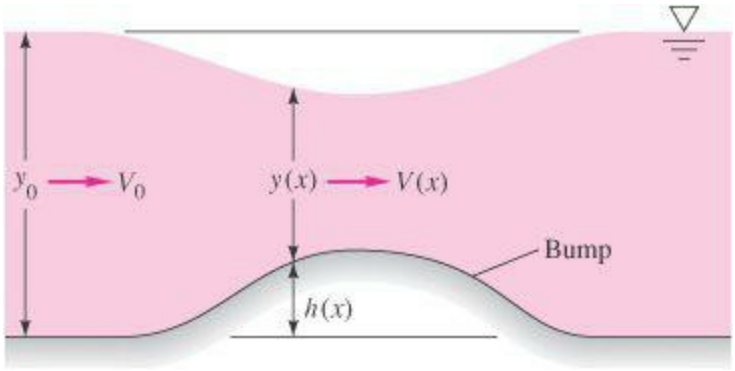

- 3)RUWKHUHFWDQJXODUFKDQQHOLQ3URE 3 WKH)URXGHQXPEHUDYHUWKHEXPS I DERXW ZKLFKLV SHUFHQWOHVVWKDQWKHDSSURDFKYDOXH)RUWKHVDPHHQWUDQF FRQGLWLRQV ILQGWKHEXPSKHLJKW?K WKDWFDXVHVWKHEXPS)URXGHQXPEHUWREH

- 3 3URJUDP DQG VROYH WKHGLIIHUHQWLDOHTXDWLRQ RI ³IUFWLRQOHVV IORZ RYHUD EXPS IURP 3URE 3 IRUHQWUDQFHFRQGLWLRQV9 P VDQG\ P /HWWKHEXPS KDYH WKH FRQYHQLHQWVKDSHK K$PD[> í FRV CE[/ @ ZKLFK VLPXODWV)LJ 3 /HW P DQGJHQHUDWDQXPHULFDOVROXWLRQIRU\ [LQWKHEXPSUHJLRQ [/ ,I \RX KDYHWPHIORURQO RQHFDVH XVHK$PD[FP 3URE 3 IR ZKLFK WKH PD[LPXP)URXGHQXPEHUV ,I PRUH WPHLV DYDLODEOH LWL LQWUXFWLYHWRH[PLQHDFRPSOHWIDPLO RI VXUIDFHSURILOHVIRUK$PD[§ FP XS V FP ZKLFKLVWKHVROXWLRQRI 3URE 3

- 3 ,Q)LJ 3 GHW9 P VDQG\ P ,I WKHPD[LPXP EXPSKHLJKWLV FP HVWLPDW D WKH)URXGHQXPEHUDYHUWKHWRS RI WKHEXPS DQG E WKH PD[LPXP LQFUHDVHLQWKHZDWUVXUIDFHOHYHO

- 3 0 RGLI\ 3URE 3 VR WKDWWKH FP FKDQJHLQERWWRP OHYHOLVDGHSUHVLRQ QR

DEXPS (VWDWH) URXGHQXPEHUDERYHWKHGHSUHVVLRQDQG E WKHPD[LPXP FKDQJHLQZDWUGHSWK

- 3 0 RGLI\ 3URE 3 WRKDYHDVXSHUFULWFDOSSURDFKFRQGLWLRQ9 P VDQ \ P ,I \RXKDYHWPHIRURQH RQHFDVH XVHK$ FP 3URE 3 IR ZKLFK WKH PD[LPXP) URXGH QXPEHU LV ,I PRUH WPH LV DYDLODEOH LWL LQWUDFWLYHWRH[DPLQHDFRPSOHWHIDPLO RI VXUIDFHSURILOHVIRU FP K$ FP ZKLFKLVWKHVROXWLRQWR3URE 3

6ROLFH-GAWMIOBRZ

- 3 * LYHQ LV WKH IOBRZ RI DFKDQQHORI ODUJH ZLGWKE XQGHUD VOLFH-GAW DVLQ) LJ 3 $VVXPLQJ IULFWLRQOHVV VWDG\ IOBRZ ZLWK QHJOLJLEOH XSVWUHDP NLQHWL HQHUJ\ GHULYH D IRUPXOD IRU WKH GLPHQVLRQOHVV IOBRZ UDWR4 \ EJ DV I IXQFWLRQ RI WKH UDWR\ \ 6KRZ E\ GLIHUHQWDWLQJ WKDWWKH PD[LPXP IOBRZ UDW RFFXUVDW \

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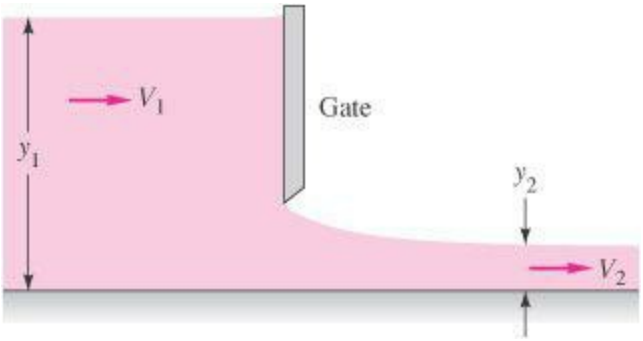
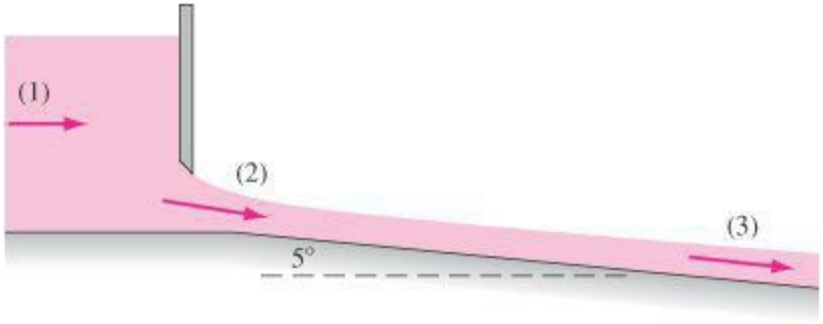


- 3 7KH VSHFLFXOD ZDWUHDVH LQ WKH FKDSWURSHQHUSKRVIOBRZ V WKURXJK D JLDO VOLFH-GAW $VVXPH WKDWWKH-GAW LV P ZLGH DQG LWV RSHQLQJ LV P KLJK 7K ZDWUGHSWK IDUXSWUHDP LV P $VVXPLQJ IUHHGLVFKDUJH HVWLPDWH WKH YROXPH IOBRZ UDWWKURXJK WKH-GAW
- 3 ,Q) LJ 3 OHW FP DQG\ FP (VWLPDWH WKH IOBRZ UDWPHUXQLWZLGWK LI WKH XSVWUHDP NLQHWF HQHUJ\ LV D QHJOHFWGDQG E LQFOXGHG
- 3 : DWUDSSURDFKHV WKH ZLGH VOLFH-GAW RI) LJ 3 DV9 P VDQG\ P $FFRXQWLQJ IRUXSWUHDP NLQHWF HQHUJ\ HVWLPDWH DWWKH RXOHWVHFWLRQ WK D GHSWK E YHORFLW DQG F) URXGHQXPEHU

3



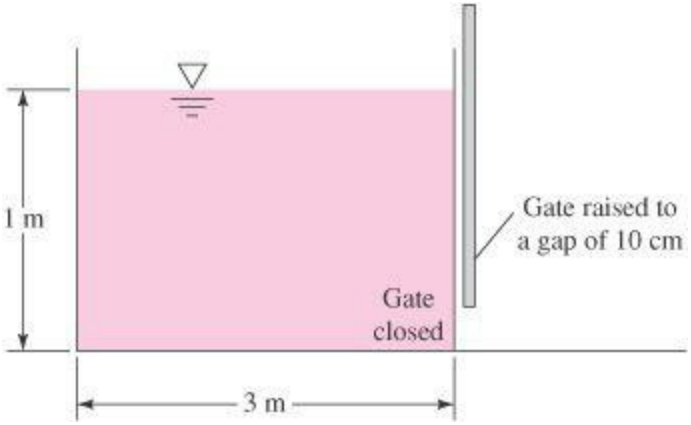
- 3 ,Q) LJ 3 OHW IWDQGVWKHJDWHZLGWKE IW) LQGWKHJDWHRSHQLQJ + WKH ZRXOGDOORZ DIUHGLVFKDUJHIORZ RI JDOPLQXQGHUWKHJDWH
- 3 : LWKUHVSHFWWR) LJ 3 VKRZ WKDWRUIUFWLRQOHVVIORZ WKHXSWUHDP YHORFLW PD\ EHUHDGLOHVWRWKHZDWUDFHOHYHOE\

$$V_1 = \sqrt{\frac{2g(y_1 - y_2)}{K^2 - 1}}$$

ZKHUH. \ \

- 3 $ VDQNRI ZDWU P GHHS P ORQJ DQG P ZLGHLQRWKHSDSHUKDVFORVHGVOLF JDWHRQWKHULJKWVLGH DVLQ) LJ 3 $VW WKHJDWHLVRSHQHGWRDJDS RI FP $VXPLQJ TXDLVWDWLF VOLFHJDWWKHRU HWUPDWWKHWPH UHTXLUHGIRUWK ZDWUOHYHOWRGURSWR FP $VXPHIUHRXWORZ

3



- 3) LJXUH3 VKRZVDKRUL]RQWDOORZ RI ZDWUWKURXJKDVOLFHJDWH DK\GUDXOL MXPS DQGRYHUD IWWKDLSFUHVWHGZHLU&KDQQHOJDWH MXPS DQGZHLUDUHDOO I ZLGHXQLQVKHGFRQFUHWH ' HWHUPLQH D WKHIORZ UDWLQIWV DQG E WKHORVVH GHSWK

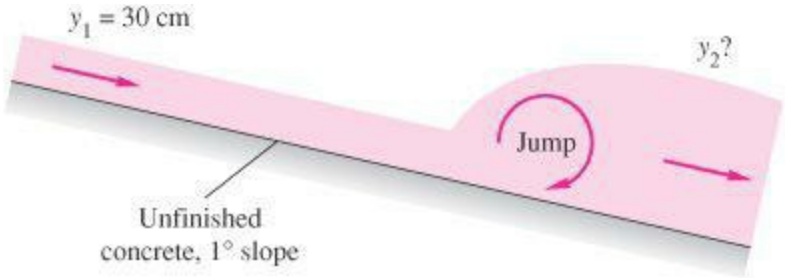
3

7LGDOEERUHRQWKH6pH6pCQHULYHUFKDQQHOLQQRUWKZHVWUDQFH

&RXUWV RI 3UR +XEHUW&KDQVRQ 8QLYHUVLWV RI 4XHHQVODQG

- 3 $ WLGDOEERUHPD\ RFFXUZKHQWKHRFHDQWLGHIHQMUVDQHWXDU DJDLQWDQRQFRPLQ
  ULYHUGLVFKDUJH VXFK DV RQWKH6HYHQ5LYHULQ(QJODQG 6XSSRVHWKDWWKHWLGDOERU
  LV IWGHHS DQG SURSDJDMV DW PLK XSWUHDP LQWR D ULYHUWKDWLV IWGHHS
  (VWLP DMWWKHULYHUFXUHQWLQQ
- 3)RUWKH VLWXDWLRQ) LJ 3 VXSSRVHWKDWDWWHFWRQ WKHGHSWK LV P DQG WK
)URXGHQXPEHUV (VWLP DMW D WKHIORZ UDMSHUHMWURI ZLGWK E \F F \
  G WKHSHUFHQWDJHRI GLVLSDMRQLQWKHMXP S DQG H WKHJDS KHLJKW RI WKHJDMW
- 3 : DMU FP GHSLV LQXQIRUP IORZ GRZ QD ƒ XQLQLVKHG FRQFUHMWVORSHZKHQI
  K\GUDXOLF MXP S RFFXUV DV LQ) LJ 3 ,I WKHFKDQQHOLV YHU ZLGH HWMP DMW WK
  ZDMUGHSWK\ GRZ QWUHDP RI WKHMXP S

3



- 3)RU WKH JDMMXP S ZHU V\VWMP VNHWKHG LQ) LJ 3 WKH IORZ UDMW ZD
  GHMWUPLQHG WREH IWV ' HMWUPLQH D WKHZDMUGHSWK\ DQG\ DQG E WK
)URXGHQXPEHUV) U DQG) U EHRUHDQGDMWUWKHK\GUDXOLF MXP S
- 3 1R GRXEWXRX XVHG WKH KRUL]RQWDO MXP S IRUPXOD WR VROYH 3UREL 3
  ZKLFK LV UHDVRQDEOH WKH VORSH LV VR VP DOO + RZ HYHU &KRZ > S
  SRLQWV RXWWKDWWK\GUDXOLF MXP SV DUHKJKHUQ VORSHGFKDQQHOV GXHWR³WKHZHJK
  RI WKHIOXLGLQWKHMXP S´ 0 DNHDFRQWROYROXP HVNHWFK RI DVOPSLQJ MXP S WR VKRZ

ZK\ WKLV LV VR 7KH VORSHG MXPS FKDUWJLYHQ LQ &KRZ¶V) LJ PD\ E
DSSUR[LPDWHGE\ WKHIROORZLQJ FXUYHILW

$$\frac{2y_2}{y_1} \approx [(1 + 8\,\mathrm{Fr}_1^2)^{1/2} - 1]e^{3.5S_0}$$

ZKHUH 6 DUH WKH FKDQQHO VORSHV IRU ZKLFK GDWD DUH DYDLODEOH 8VH WKL
FRUUHODWLRQ WR PRGLI\ \RXUVROXWLRQ WR 3UREOHP 3 ,I WKH SHUPLWWHG PDNHDJUDSK RI\ \
” YHUWLFDO) U ” IRUYDULRXV6 ”

- 3 $ IDPLOLDU VLJKWLVWKHFLUFXODUK\GUDXOLF MXPSIRUPHGE\ DIDXFHWWHWWDQQJRQWKI
IODWWLQN VXUIDFH DV LQ) LJ 3 %HFDXVH RI WKH VKDOORZ GHSWKV WKLV MXPS L
VWURQJO\ GHSHQGHQWRQERWWRP IULFWLRQ YLVFRVLW\ DQG VXUIDFHWHQVLRQ> @,WV DOW
XQWUDEOH DQG FDQ IRUP UHPDUNDEOH QRQFLUFXODU VKDSHV DV VKRZQ LQ WKH ZHEVL
KWWS ZHEPLWHGXMHIDZZZ MXPSKWP!

3

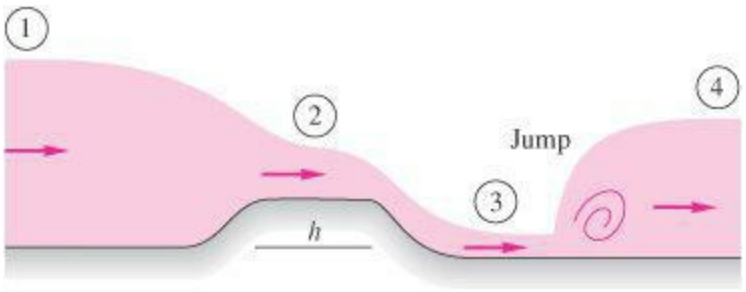
$ FLUFXODUK\GUDXOLF MXPSLQDNLWFKHQVLQN

&RXUWHV\ RI 3UR +XEHUW&KDQRQ 8QLYHUVLWL RI 4XHHQVODQG
) RUWKLV SUREOHP DVXPH WKDWWZR GLPHQVLRQDO MXPS WKHRU LV YDOG ,I WKH ZDWHUGHSW
RXWLGHWKHMXPSLV PP WKHUDGLXVDWZKLFKWKHMXPSDSSHDUVLV5 FP DQGWKHIDXFHWWORZ
UDWLV FP V ILQGWKHFRQGLWLRQVIXWXSWXUHP RI WKHMXPS

- 3 : DWULQDKRUL]RQWDOFKDQQHOLFH0HUDWVPRRWKO RYHUDEXPSDQGWKHQXQGHUJRH
DK\GUDXOLF MXPS DVLQ) LJ 3 ,I\ P DQG\ FP HVWLPDWH D 9 E

9 F \ DQG G WKHEXPSKHLJKWK

3



- 3 ,Q) LJ WKHXSWUHDP IORZ LVRQ FP GHHS 7KHFKDQQHOLV FP ZLGH DQGWKHIORZ UDWHLV P V' HWHUPLQH D WKHXSWUHDP)URXGHQXPEHU E WK GRZQWUHDP YHORFLW F WKHGRZQWUHDP GHSWK DQG G WKHSHFLFVSDWRQ
- 3 $ FP KLJK EXPS LQD ZLGHKRUL]RQWDOZDWUFKDQQHOFUHDWHV DK\GUDXOLF MXPS MXWXSWUHDP DQGWKHIORZ SDWWHUQ LQ) LJ 3 1HJOHFWQJ ORVVHV[FHSWLQWK MXPS IRUWKHFHVH\ FP HWUPDWD 9 E \ F 9 DQG G \

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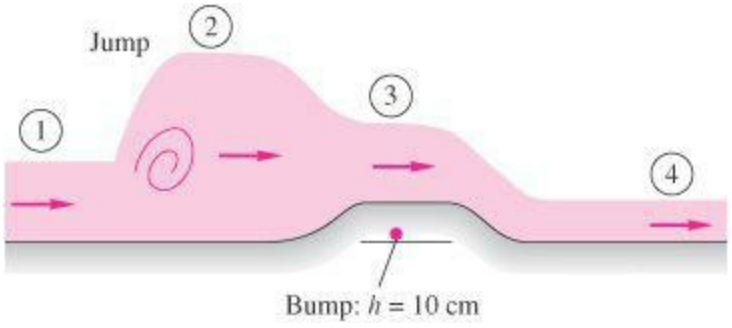

- 3)RUWKHFLUFXODUK\GUDXOLF MXPS LQ) LJ 3 WKHZDWUGHSWKVEHRUHDQGDWWUWK MXPS DUH PP DQG PP UHVSHFWLYHO $VXPHWKDWWZR GLPHQVLRQDOMXPS WKRU LVYDOG ,I WKHIDFHWWORZ UDWHLV FP V HWUPDWWKHUDGLXV5 DWZKLFKWKHMXPS ZLOOSSHDU

* UDGXDOO YDULHG IORZ

- 3 $ EUFNZRUNUHFWDQJXODUFKDQQHO P ZLGHLVIORZLQJ DW P VRQDVORSHRI *f* ,VWKLVDPLGG FULWFDORUWWHSVORSH": KDWWSHRI JUDGXDOO YDULHGVRXUQFXUY DUHZHRQLI WKHORFDOZDWUGHSWKLV D P E P DQG F P"
- 3 $ JUDYHOO HDWKZ LGHFKDQQHOLVIORZLQJ DW P VHUPHWUR ZLGWKRQDVORS RI *f* ,VWKLVDPLGG FULWFDORUWWHSVORSH": KDWWSHRI JUDGXDOO YDULH VRXUQFXUYHDUHZHRQLI WKHORFDOZDWUGHSWKLV D P E P RU F P"
- 3 $ FOD\ WDH9 VKDSHGFKDQQHORI LQFXGHGDQJOH *f* LVIORZLQJ DW P VRQ VORSHRI *f* ,VWKLVDPLGG FULWFDORUWWHSVORSH": KDWWSHRI JUDGXDOO YDULH VRXUQFXUYHDUHZHRQLI WKHORFDOZDWUGHSWKLV D P E P RU F P"

- 3 ,I EROWWRP IUDFWLRQLV LQFOXGHG LQWKH VOLFH JDAMICRZ RI 3URE 3 WKHGHSW
  \ \ \ ZLOO YDU ZLWK 6NHWK WKH WSH DQG VKDSH RI JUDGXDOO YDULH
  VRXWRQ FXUYH LQ HDFK UHJLRQ DQG VKRZ WKH UHJLRQV RI UDSLG YDULH
  ICRZ
- 3 &RQVLGHUWKH JUDGXDOFKDQJH IURP WKH SURILOH EHJLQQLQJ DWSRLQWD LQ) LJ 3
  RQDPLOGVORSH6 WRDPLOGEXWWHSHUVORSH6 GRZQWUHP 6NHWK DQGGDE-
  WKHFXUYH\ [H[SHFWHG

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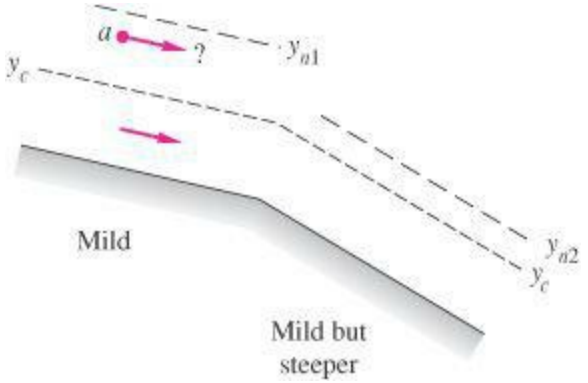
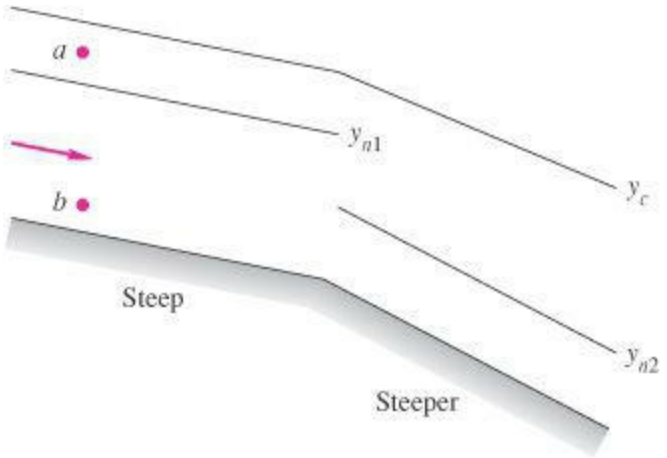
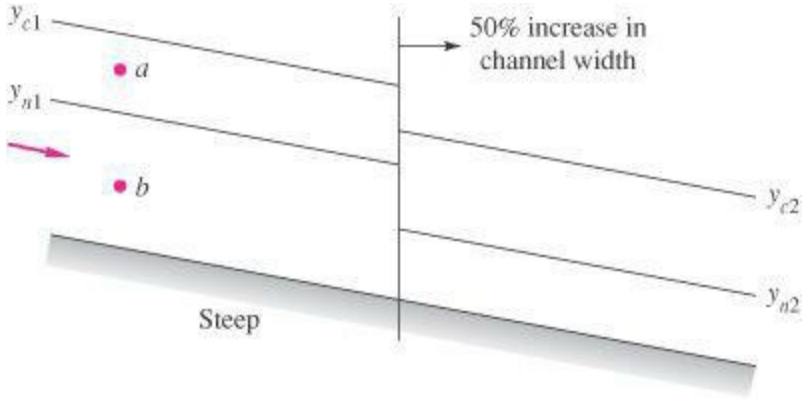


- 3 7KHZLGHFKDQQHOICRZ LQ) LJ 3 FKDQJHVIURP DWWHSVORSHWRRQHHYH
  WWHSHU %HJLQQLQJ DWSRLQWD DQGE WKHWK DQGGDE-DWKHZDWUXUDFHSURILOH
  H[SHFWHGIRUJUDGXDOO YDULHGICRZ

3



- 3 $ FLUFXODUSDLQWHG WWHOFKDQQHO RI UDGLXV FP LV UKQLQJ KDO IXOODW P
  RQD VORSH RI P NP ' HWHUPLQH D ZKHWKHU WKH VORSHLV PLOG RU WWHSDQG E
  ZKDWWWSH RI JUDGXDOO YDULHG VRXWRQ DSSOLHV DWWKLV SRLQW F 8VH WK
  DSSUR[LPDWPHWKRG RI(T DQG DVLQJHGHSWKLQFUHPHQWû\ FP W
  FDOFXODWWKHHWXP DWGû[IRUWKLVQHZ \
- 3 7KH UHFWDQJXODUFKDQQHOICRZ LQ) LJ 3 H[SDQGV WR D FURVV VHFWRQ
  SHUFHQWZLGHU %HJLQQLQJ DWSRLQWD DQGE WKHWK DQGGDE-DWKHZDWUXUDF
  SURILOHVH[SHFWHGIRUJUDGXDOO YDULHGICRZ



- 3 ,Q3URE 3 VKHIULFWLRQOHVVVROXWLRQLV\ IWZKLFKZHGHQRWHDV[MXVWGRZQVWUHDP RI VKH JDWH ,I VKH FKDQQHOLV KRUL]RQWDOZLWKQ DQ VKHUHLVQR K\GUDXOLF MXPS FRPSXWHIURP JUDGXDOO YDULHGVKHRU VKHGRZQVWUHDF GLVWDQFHZKHUH\ IW
- 3 $ UHFWDQJXODUFKDQQHOZLWKQ DQGDFRQVWDQWVORSHRI LQFUHDVH LW ZLGWK LQFUHDVHV IURP E WR E RYHU D GLVWDQFH/ DV LQ) LJ 3 D ' HWHUPLQHWKHYDULDWLRQ\ [DORQJ WKHFKDQQHOLE P / P WKHLQLWL GHSWKLV\ P DQGWKHIORZ UDWHLV P V E 7KHQ LI \RXUFRPSXWH SURJUDP LVUXQQLQJ ZHOO GHWHUPLQHWKHLQLWLDOGHSWK\ IRUZKLFKWKHH[LWGRZ ZLOOEHH[DFWO FULWLFDO

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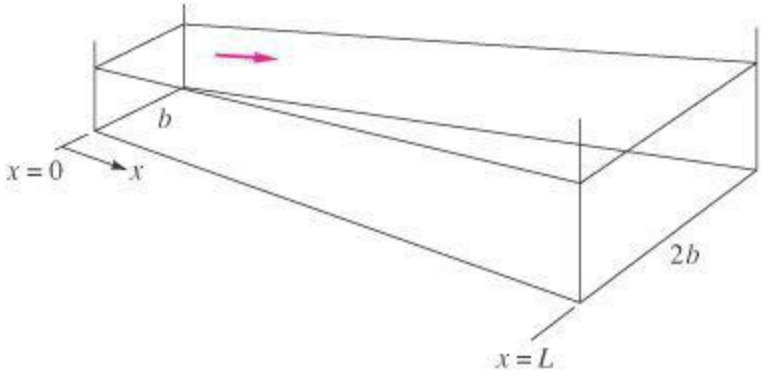

- 3 $ FRQHDUWKZLGHFKDQQHOIORZ LV FOP EHLQJ DQ DGYHUVH VORSH ZLWK6 í ,I WKHIORZ UDWHLVT P V P XVH JUDGXDOO YDULHGWKHRU VF FRPSXWHWKHGLVWDQFHIRUWKHGHSWKWRGURSIURP WR P
- 3 : DWHUIORZVDW P VDORQJ DVWUDLJKWULYHWGWUDHO ƒ 9 VKDSHGFKDQQHO VH) LJ ƒ $VWHFWLRQ WKHZDWUGHSWKLV P D $VZHSURFHHG GRZQVWUHDP ZLOOWKHZDWUGHSWKULVHRUIDOO ([SODLQ E ' HSHQGLQJ XSRQ\RX DQVZHUWR SDUWD FDOFXODWH LQRQH QPHULFDOVZRRS IURP JUDGXDOO YDULH WKHRU WKHGLVWDQFHGRZQVWUHDP IRUZKLFKWKHGHSWKULVHV RUIDOOV P
- 3) LJXUH 3 LOOXVWUDWHV D IUHH RYHUIDOO RUGURSGRZQ IORZ SDWWLQ ZKHUH I FKDQQHOIORZ DFFHOHUDWHVGRZQDVORSHDQGIDOOV IUHH) RYHUDQ DEUXSWHGJH $ VKRZQ WKHIORZ UHDFKHVFULWLFDOXWWEHRUHWKHRYHUIDOO %HZHHQ\ DQGWKHGJI

VKHIORZ LVUDSLGO YDULHGDQG GRHVQRWDLW\ JUDGXDOO YDULHGWKHRU 6XSSRV
VKDWKHIORZ UDWLVT P V P DQGWKHVXUFHLVXQLQLVKHGFRQFUHWH 8V
(T VRHWLPDWWKHZDWUGHSWK P XSVWUHDP DVKRZQ

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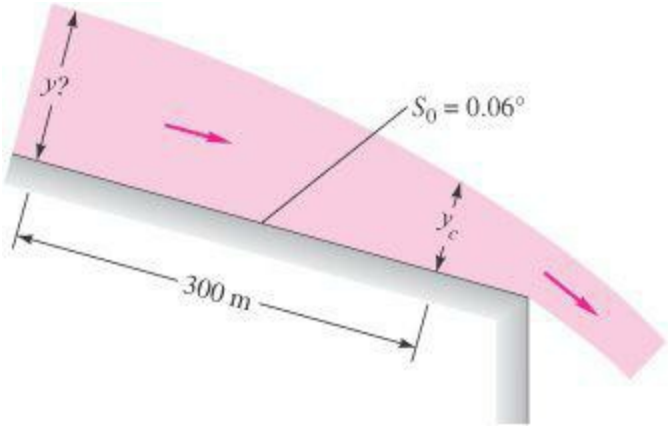

- 3 : HDVXPHGIUFWLRQDWVIORZ LQVRLOLQJ WKHEXPS FDVH 3URE 3 IRUZKLF
9 P VDQG\ P RYHUWKHFUHWZKHQK$PD FP 9 P V
DQG\ P +RZHYHULI WKHEXPSLVORQJDQGURXJK IUFWLRQPD EHLPSRUWDQ
5HSHDW3URE 3 IRUWKHVDPHEXPSVKDSHK K$PD> í FRV CE/ @V
FRPSXWHFRQGLWLRQV D DWWKHFUHWDQG E DWWKHHQGRI WKHEXPS [/ /HWK$PD
FP DQG/ P DQGDXXPHDFOHDQHDUWKVXUFH
- 3 6RLVH 3URE 3 D KRUL]RQWDO YDULDWLRQ DORQJ DQ + FXUYH E\ WK
DSSUR[LPDWHPHWKRGRI(T EHJLQQLQJ DW[\ IWDQGXVLQJ I
GHSWKLQFUHPHQWVû\ IW7KHILQDOLQFUHPHQWWKRXOGEHû\ IWWRHULQ
XVH[DFWO WR\ IW
- 3 7KHFOHDQHDUWKFKDQQHOLQ)LJ 3 LV P ZLGHDQGVORSHVDW ƒ : DW
IORZVDW P VLQWKHFKDQQHODQGHQWHUVDUHVHUYRLUVR WKDWWKHFKDQQHOGHSWKL
P MXVWEHRUHWKHHQW $VVXPLQJ JUDGXDOO YDULHGIORZ KRZ IDULVWKHGLVWDQFH
/ WR D SRLQWLQ WKH FKDQQHOZKHUH\ P" : KDWWSH RI FXUYH LV WKH Z DW
VXUFH'

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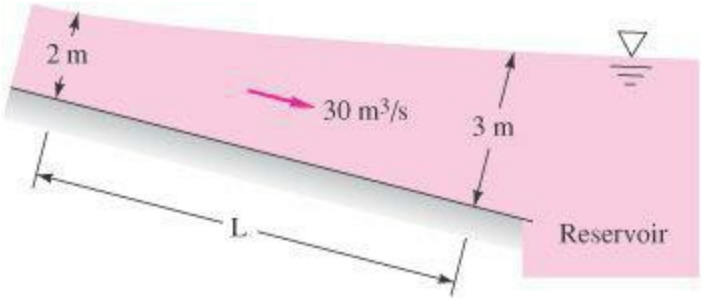


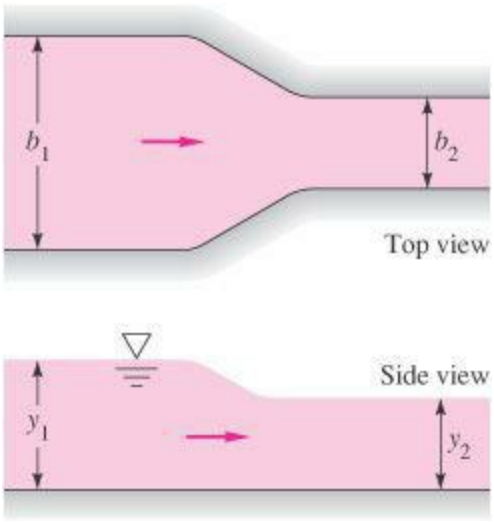
: HLUVDQGICXPHV

- 3) LJXUH3 VKRZV DFKDQQHOFRQWUDFWRQVHFWRQRIWKHFDQDGDYHQWXUL ICXPI
  > S @EHFDXVHPHDVXUHPHQWVRI \ DQG\ FDQEHXVHGWRPHWUWKHIORZ
  UDWH 6KRZ WKDWLI QRVVHV DUH QHJOHFWHG DQG WKH IORZ LV RQH GLPHQVLRQDO DQ
  VXEFULWLFDO WKHIORZ UDWHLVJLYHQE\

$$Q = \left[\frac{2g(y_1 - y_2)}{1/(b_2^2 y_2^2) - 1/(b_1^2 y_1^2)}\right]^{1/2}$$

$ SSO\ WKLVWRWKHVSHFLDOFDVHE P E P DQG\ P D) LQGWKHIORZ UDW
LI \ P E $OVRILQGWKHGHSWK\ IRUZKLFKWKHIORZ EHFRPHVFULWLFDOLQWKHWKURDW

3

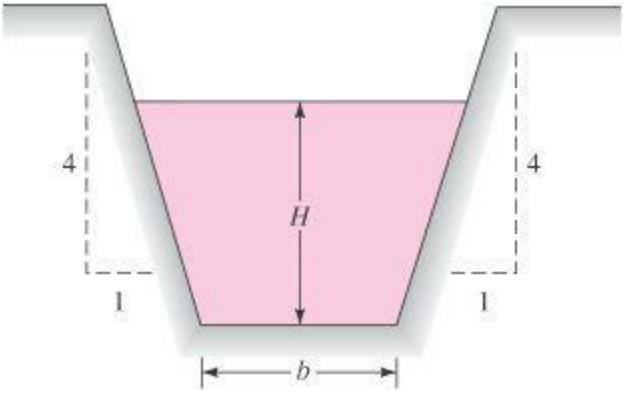


- 3) RU WKH JDWHMXPS ZHUV\VWHP VNHWFKHG LQ) LJ 3 WKH IORZ UDWH ZD
  GHWHUPLQHGWREH IWV ' HWHUPLQHWKHZDWHUGHSWK\ MXVWXSVWUHDP RI WK
  ZHLU
- 3 * UDGXDOO YDULHGWKHRU(T QHJOHFWWKHHIHFWRI ZLGWK FKDQJHV GE G[
  DVXPLQJWKDWWKH\ DUHVPDOO%XWWKH\ DUHQRWVPDOOIRUDVKRUWWKDUSFRQWUDFWR
  VXFK DV WKHYHQXUL ICXPHLQ) LJ 3 6KRZ WKDWIRUDUHFWDQJXODUVHFWR
  ZLWKE E[(T VKRXOGEHPRGLILHGDVIROORZV

$$\frac{dy}{dx} \approx \frac{S_0 - S + [V^2/(gb)](db/dx)}{1 - \mathrm{Fr}^2}$$

,QYHVWLJDWHDFULWLFDOIORZUHGXFLQJWKLVUHDWR(T

- 3 $ &LSRODHWWZHU SRSXODULQLUULJDWRQV\VWHPV LV WDSH]RLGDO Z LWK VLGHV VORSHG DW KRUL]RQWDO WR YHUWLFDO DV LQ) LJ 3 7KH IRUOZLQJ DUHIORZ UDWHV YDOXHV IURP WKH8 6 ' HSWRI $JULFXOWXUH IRUDIHZ GLIIHUHQW\ VXPS SDUDPHWHUV

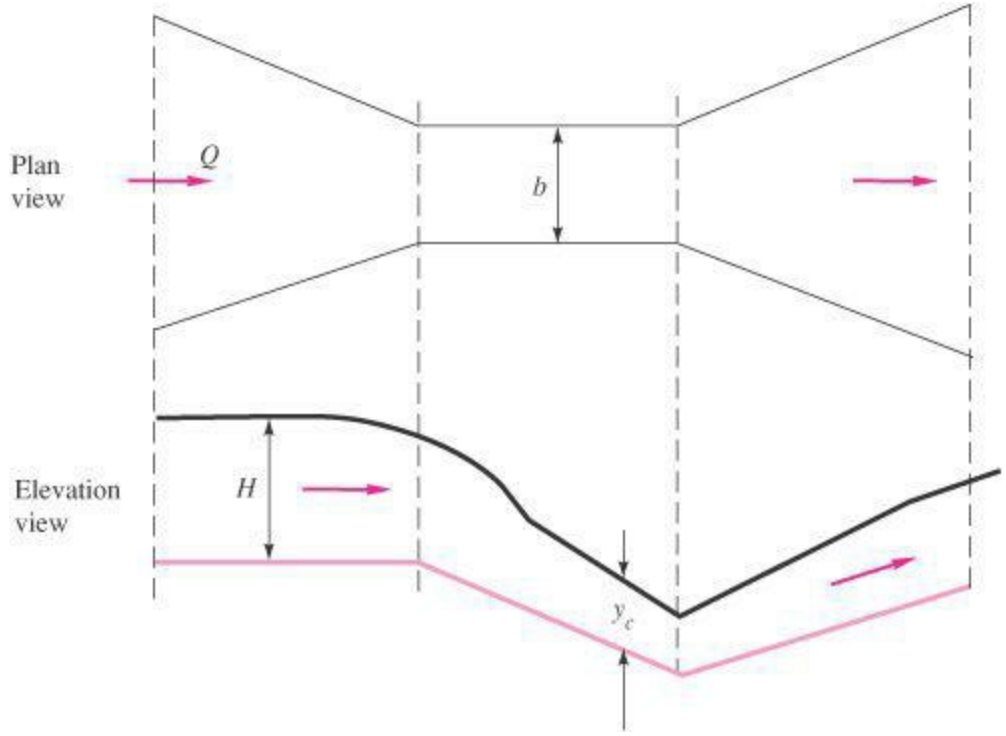


8VH WKLV GDWD WR FRUUHODWH D &LSROOHWWL Z HLUIRUP XOD Z LWK D UHDVRQDEO FRQVWDQW Z H FRHILFLHQW

H, ft	0.8	1.0	1.35	1.5
b, ft	1.5	2.0	2.5	3.5
Q, gal/min	1620	3030	5920	9740

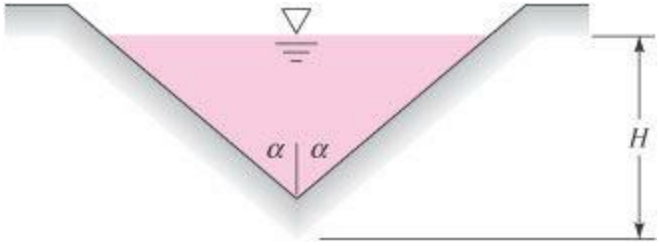
- 3 $ SRSXODU IORZ PHDVXUHPHQW GHYLFH LQDJULFXOWXUHLVWKH3DUVKDOOIXPH> @) LJ 3 QDPHG DIWHU LWV LQYHQWRU 5DOSK / 3DUVKDOO Z KR GHYHORSHG LWLQ IRUWKH8 6 %XUHDX RI 5HFODPDWLRQ 7KH VXEFULWLFDOSSURDFK IORZ LVGULYHQ E\ I VWHSFRQWUDFWLRQ WRJRFULWLFDO\ \F DQGWKHQVXSHUFULWLFDO,WJLYHVDFRQWDC UHDGLQJ + IRUDZLGHUDQJHRI VDLZDWHUV ' HULYHDIRUPXODIRUHVWLPDWLQJ4 IURP PHDVXUHPHQWVRI + DQGNQRZOHGJHRI FRQWUDFWLRQ Z LGWKE 1HJOHFWWKHHQDQFH YHORFLWKHDG
- 3 8VLQJ D%HUQRXOOLVSHDQDOVLVPLOODUW) LJ D VKRZ WKDWWKH WKHRUHWLFD GLVFKDUJHRI WKH9 VKDSHGZHLUQ) LJ 3 LVJLYHQE\

$$\frac{dy}{dx} \approx \frac{S_0 - S + [V^2/(gb)](db/dx)}{1 - \mathrm{Fr}^2}$$



3 7KH3DUVKDOOIOXPH

3

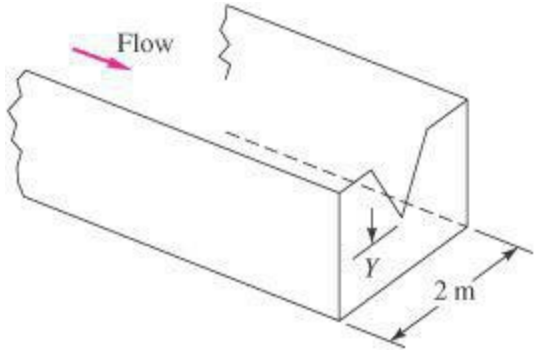


- 3 ' DWD E\ $ 7 / HQ IRU Z DWU DW *f*& UHSRUWHG LQ 5H VKRZ D VLJQLILFD LQFUHDVHR GLVFKDUJHFRHILFLHQWR 9 QRWK Z HLV) LJ 3 DWGRZ KHDG) RU. *f* VRPHPHDVXUHG YDOXHV DUH DVIROORZV ' HWUPLQHLI WKHVHGDWDFD EHFRUUHDWHGZLWKWKH5HQROGV DQG: HEHUQXPEHUVYLV j YLV(T ,I QR VXJJHVWDQRWKHUFRUUHDWRQ

H, ft	0.2	0.4	0.6	0.8	1.0
C_d	0.499	0.470	0.461	0.456	0.452

- 3 7KHUHFDQJXODUFKDQQHOLQ) LJ 3 FRQWDLQV D 9 QRWK Z HLUDV VKRZQ 7KH LQWQWLVWRPHWUIORZ UDWHVEHWZHHQ DQG P VZLWKDQXSWUHDP KRRNJDJH VHWWR PHDVXUH Z DWU GHSWKV EHWZHHQ DQG P : KDWDUH WKH PRV DSSURSULDWHYDOXHVIRUWKHQRWK KHLJKWV DQGWKHQRWK KDO DQJOH. "

3



- 3 : DAUIOR LQDUHFDQJXODUFKDQQHOVWREHPHDVXUHGE\ DWKLQSODWHZHLUWKVLG FRQWUDFWLRQV DVLQ7DEOH E ZLWK/ IWDQG< IW,WVVGHVLUHGWRPHDVXU IOR UDWHVEHWZHHQ DQG JDOPLQZLWKRQO D LQFKDQJHLQXSVWUHDP ZDWHUGHSWK: KDWLVWKHPRVWDSSURSULDWHQJWKIRUWKHZHLUWLGWKE'
- 3 ,Q (6 &UXPS GHYHORSHGWKHWDQXODUZHLUVKDSHVKRZQLQ 3 > &KDS @7KHIURQWVORSHV WRDYRLGVHGLPHQWGHSRVLWLRQ DQGWKHUH VORSHV WRPDLQWDLQDVXEHWWDLZDUOR 7KHEHDXWRI WKHGHVLJQLVWKDW KDVDXQLTXHGLVFKDUJHFRUUHVSRQGLQJ FRQGLWLRQV + + ”

$$Q = C_d b g^{1/2}\left(H_1 + \frac{V_1^2}{2g} - k_h\right)^{3/2}$$

7KHWHUP N LVDORZ KHDGORVVIDFWRU6XSSRVHWKDWKHZHLUV P ZLGHDQGKDVDFUH KHLJKW FP ,I WKHZDWUGHSWXSVWUHDP LV FP HVWLPDWHWKHIORZ UDWHLQJDOPLQ

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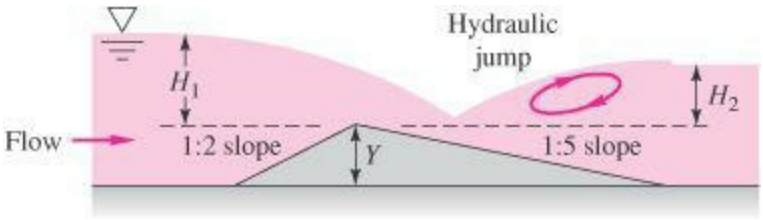

7KH&UXPSZHLU> &KDS @

- 3 7KH&UXPSZHLUFDQEHUDWHQLQ3URE 3 LVIRUPRGXODUIORZ ZKLFKRFFXU ZKHQ WKHIORZ UDWH LV LQGHSHQGHQWRI GRZQVWUHDP WDLOZDWU : KHQ WKHZHL EHFRPHVGURZQHG WKHIORZ UDWHGHFUHDVHVE\ WKHIRORZLQJ IDFWRU

$$Q = Q_{\text{mod}} f$$

where $$f \approx 1.035\left[0.817 - \left(\frac{H_2^*}{H_1^*}\right)^4\right]^{0.0647}$$

IRU ”+ + ” ZKHUH+ GHQRWV+ 9 J í N IRUEUHYLW 7KHZHLU WKHQGRXEOHJDJHGWRPHDVXUHERWK+ DQG+ 6XSSRVHWKDWKHZHLUFUHWWV P KLJK DQ P ZLGH ,I WKHPHDVXUHG XSVWUHDP DQG GRZQVWUHDP ZDWUGHSWKV DUH DQG P

UHVSHFWLYHO HVWLPDWH WKH IORZ UDWH LQ &RPPHQW RQ WKH SRVVLEOH XQFHUWDLQWLHV RI \RX HVWLPDWH

%DFNZDWHU FXUYHV

- 3 : DWHU IORZV DW IWV LQ DUHFWDQJXODU FKDQQHO IWZLGHZLWK§ DQG LV VORSHG ƒ $ GDP LQFUHDVHV WKH GHSWK WR IWDV LQ) LJ 3 8VLQ JUDGXDOO\ YDULHG WKHRU HVWLPDWH WKH GLVWDQFH/ XSVWUHDP DWZKLFK WKH ZDWH GHSWKZLOOEH IW: KDWWSHR VRXWRQFXUYHDUHZHRQ' : KDWWKRXGEHWK ZDWUGHSWKDX\PSWRWLFDOO IDUXSWUHDP"

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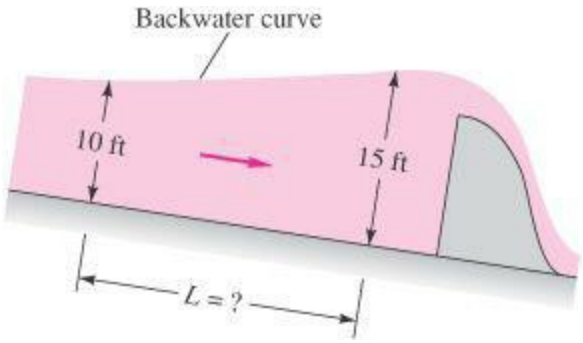
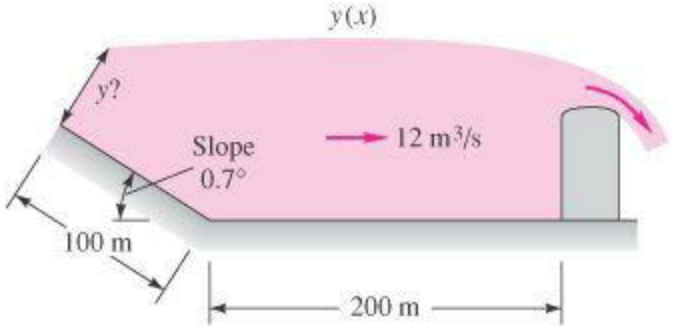


- 3 7KH7XSSHUZDUHGDP RQWKH%DFNWRQH5LYHULV IWKLJK IWZLGH DQ VKDUSHGJHG ,WFUHDWV D EDFNZDWUVLPLODUWR) LJ 3 $VXPH WKDWWKH ULYHULVDZHG HDWKUHFWDQJXODUFKDQQHO IWZLGHZLWKDIORZ UDWRI IWV (VWLPDWHWKHZDWUGHSWK PLXSWUHDP RI WKHGDP LI 6
- 3 6XSSRVHWKDWWKHUHFWDQJXODUFKDQQHORI) LJ 3 LVPDGHRI ULYHWHG VWHDQGFDUULHVDIORZ RI P VRQDVORSHRI ƒ ,I WKH9 QRWFKZHLUKDV. DQG< FP HVWLPDWH IURP JUDGXDOO\ YDULHG WKHRU WKHZDWUGHSWK P XSWUHDP
- 3 $ FHQHDWKULYHULV IWZLGHDQG DYHUDJHV IWV ,WFRQWLQVDGDP WKDW LQFUHDVHV WKHZDWUGHSWKWR IWWR SURYLGHKHDGIRUDK\GURSRZHUSODQW7K EHGVORSHLV D : KDWLVWKHORUPDOGHSWRI WKLVULYHU' E (QJLQHHU SURSRVH SXWWQJ IODVKERDUGV RQ WKH GDP WR UDLVH WKH ZDWUOHYHODWR I\ 5HVLGHQWV DKDO PLOHXSWUHDP DUH ZRUULHG DERXWORRGLQJ DERYH WKHLUSURSHUW ZDWUGHSWKRI DERXW IW8VLQJ(T LQRQHELJKDO PLOHVWHS HVWLPDWH WKHQHZ ZDWUGHSWKXSWUHDP
- 3 $ UHFWDQJXODUFKDQQHO P ZLGHLVEORFNHGE\ DEURDG FUHVWHG ZHLU P KLJK DVLQ) LJ 3 7KHFKDQQHOLVKRUL]RQWDOFRU P XSWUHDP DQGWKHQVORSH DW ƒ DVVKRZQ 7KHIORZ UDWLV P V DQGQ &RPSXWHWKHZDWUGHSWK\ DW P XSWUHDP IURP JUDGXDOO\ YDULHGWKHRU



SURE OPPVTXHVWLRQV: RUG3URE OPPV

- :)UHHVXUIDFHSURE OPVDUHGULYHQE\ JUDYLW: K\ GRVRPDQ RI WKHIRUPXODVLQWKL FKDSWUFRQWDLQWKHVTXDUHURRWRI WKHDFFHOHUDWLRQRI JUDYLW"
- : ([SODLQZK\ WKHIORZ XQGHUDVOXLFHJDW)LJ HLWKHULVRULVQRWDQDORJRXVW FRPSUHVVLEOHJDVIORZ WKURXJKDFRQYHUJLQJ±GLYHUJLQJ QR]]OH)LJ
- : ,QXQLRUP RSHQFKDQQHOIORZ ZKDWLVWKHEDQFH RI IRUFHV" &DQ\RX XVHVXFK IRUFHEDODQFHWRGHULYHWKH&Kp]\ HTXDWLRQ "
- : $ VKDOORZ ZDWHU ZDYH SURSDJDWHV DWWKH VSHHGF § J\ : KDWPDNHV L SURSDJDWH 7KDWLV ZKDWLVWKHEDQFH RI IRUFHVLQVXFK ZDYHPRWLRQ' ,QZKLF GLUHFWLRQGRHVVXFKDZDYHSURSDJDWH
- : : K\ LV WKH0DQQLQJ IULFWLRQFRUUHDWLRQ(T XVHGDOPRVWXQLYHUVDOO E\ K\GUDXOLFVHQJLQHHUV LQWKHGRI WKH0RRG\ IULFWLRQIDFWRU
- : ' XULQJ KRUL]RQWDO FKDQQHOIORZ RYHU D EXPS LV WKH VSHFLILF HQHUJ\ FRQWDQ ([SODLQ
- : &LWHVRPHVLPLODULWLHV DQGSHUKDSVVRPHGLVLPLODULWLHV EHWZHHQDK\GUDXOF MXP DQGDJDVG\QDPLFQRUPDOVKRFNZDYH
- : * LYHWKUHH[DPSOHVRI UDSLGO YDULHGIORZ)RUHDFKFDVH FLWUHDVRQVZK LWGR QRWWDNH\ RQHRUPRUHRI WKHILYHEDVLF DVXPSWLRQVRI JUDGXDOO YDULHGIORZ WKHRU
- : ,VDIUHHRYHUDOO)LJ HVLPLODUWRDZHLU &RXOGLWEHFDOLEUDWHGYHUVXVIORZ UDWHLQWKHVDPHPDQQHUDVDZHLU ([SODLQ
- : &LWHVRPHVLPLODULWLHV DQGSHUKDSVVRPHGLVLPLODULWLHV EHWZHHQDZHLUDQG %HUQRXOOLREVWUXFWLRQIORZPHWHUIURP6HF
- : ,VDEXPS)LJ D VLPLODUWRDZHLU ,IQRWZKHQGRHVDEXPSEHFRPHGUL HQRXJK RUVKDUSHQRXJK WRHBDZHLU
- : $IWUGRLQJ VRPHUHDGLQJ DQGRUWKLQNLQJ H[SODLQWKHGHVLJQDQGRSHUDWLQJ RI IORZPHWHUV : KDWDUH LV DGYDQWDJHVFRPSDUHGWRWKHYHQXULIORPHWHU3URE 3 "

SURE OPPVTXHVWLRQV)XQGDPHQWDOVRI (QJLQHHULQJ ([DP 3URE OPPV

7KH)(([DP LVIDLUO OQJWRQRSHQFKDQQHOSURE OPVLQWKHJHQHUDO PRUQLQJ VHVVLRQ EX WKLVVXEMHFWSDOVDEJSDUWLQWKHVSHFLDO]HGFLYLOHQJLQHHULQJ DWWUQRRQ H[DP

-)(&RQVLGHUDUHFDQXUFKDQQHO P ZLGHOLGRQD ƒ VORSH ,I WKHZDWUGHSWKLV P WKHK\GUDXOLFUDGLXVLV
  P
  P
  P
  P
  P
-)( )RUWKHFKDQQHORI 3URE)(WKHPRWWHILFLHQWZDWUGHSWK EHWWRZ IRU JLYHQVORSHDQGUHVLVWDQFH LV D P E P F P G P H P
-)(,I WKHFKDQQHORI 3URE)(LVEXLOWRI UXEEOHFHPHQW0 DQQLQJ¶VQ§ ZKDWLVWKHXQIRUP IORZ UDWHZKHQWKHZDWUGHSWKLV P"
  P V
  P V
  P V
  P V
  P V
-)( )RUWKHFKDQQHORI 3URE)(LI WKHZDWUGHSWKLV P DQGWKHXQIRUP IORZ UD LV P V ZKDWLVWKHSSUR[LPDWHYDOXHRI 0 DQQLQJ¶VURXJKQHVVFRHIILFLHQW'

-)( )RUWKHFKDQQHORI 3URE)(LI 0 DQQLQJ¶VURXJKQHVVFRHIILFLHQW§ DQG4 §
  P V ZKDWLVWKHORUPDOGHSWK\Q"
  P
  P
  P
  P
  P
-)( )RUWKHFKDQQHORI 3URE)(LI4 § P V ZKDWLVWKHFULWLFDOGHSWK\F"
  P
  P
  P
  P
  P
-)( )RUWKHFKDQQHORI 3URE)(LI4 § P VDQGWKHGHSWKLV P ZKDWLVWK)URXGHQXPEHURI WKHIORZ"

SURE OPPVTXHVWLRQV&RPSUHKHQVLYH3URE OPV

- &)HEUXDU VDZ WKHIDLOXUHRI WKHHDUWKHQGDP LPSRXQGLQJ &DOLRUQLD-LP¶V3RQ
  LQVRXWKHUQ5KRGH,VODQG 7KHUHVXOWLQJ IORRGUDLVHG WKPSRUDU\ KDYRF LQWKHQHDUE
  YLOODJHRI 3HDFH' DOH 7KHSRQGLV DFUHVLQDUHDDQG IWGHHS DQGZDVIX
  IURP KHDY\ UDLQV 7KHEUHDFKLQWKHGDP ZDV IWZLGHDQG IWGHHS (VWLPDWWK
  WKHUHTXLUHGIRUWKHSRQGWRGUDLQWRDGHSWKRI IW
- & $ FLUFXODUXQLQVKHGFRQFUHWHGUDLQSLSHLVODLGRQDVORSHRI DQGLVSODQHG
  WRFDUU IURP WR IWVRI UKRI ZDWU ' HVLJQFRQWUDLQWVDUH WKDW WK
  ZDWUGHSWKVKRXOGEHQR PRUHWKDQWKUHHIRXUWKVRI WKHGLDPHWUDQG WKHIORZ
  VKRXOGDZD\VEHVXEFULWLFDO: KDWLVWKHDSSURSULDWHSLSHGLDPHWUWR VDWLV WKHP
  UHTXLUHPHQW' ,I QR FRPPHUFLDOSLSHLVH[DFWO WKLVFDOFXODWHGVL]H VKRXOG\RX EX
  WKHQH[WPDOOHUWRUWKHQ[WDJHWWSLSH'
- & ([WQG3URE 3 ZKRVHVROXWLRQZDV9 § P V D 8VHJUDGXDO YDUH
  WKHRU WR HWWP DWWKH ZDWU GHSWK P GRZQWUHDP DWWHFWLRQ IRU WKH
  XQLQVKHGFRQFUHWHVORSHVKRZQLQ) LJ 3 E 5HSHDWRXUFDOFXODWRQIRUD
  XSZDUG DGYHUVH VORSHRI ƒ F : KHQ\RX ILQG WKDWSDUWE LVLP SRVWLEOHZLW
  JUDGXDOO YDUHG WKRU H[SODLQZK\ DQGUHSHDWRUDQDGYHUVHVORSHRI ƒ
- & ,WLV GHVLUHG WR PHWU DQ DSKDOWUHFDQJXODUFKDQQHORI ZLGWK P ZKLFK L
  GHVLJQHGIRUXQLRUP IORZ DWD GHSWRI FP DQGDVORSHRI 7KHYHUWFL
  VLGHVRI WKHFKDQQHODUH P KLJK &RQVLGHUXVLQJ DWKLQSODWUHFDQJXODUZHL
  HLWKHUIXOORUSDUWDOZLGWK 7DEOH D E IRUWKLV SXUSRVH 6WUP > S
  UHFRPPHQGVIRUDFFXUDWHFRUHDWRQ WKDWWXFKDZHLUKDYH<≡ FP DQG+ <”
  ' HWHUPLQHWKHIHDVLELOLWRI LQWDOOLQJ VXFKDZHLUWKDWZLOOEHDFFXUDWDQG\HWQ
  FDXVHWKHZDWUWRRYHUIORZ WKHVLGHVRI WKHFKDQQHO
- &)LJXUH& VKRZVDK\GUDXOF PRGHORI DFRPSRXQGZHLU RQHWKDWFRPELQHVWZ
  GLIIHUHQWWKDSHV D 2WKHUWKDQ PHDVXUHPHQWIRUZKLFK LWP LJKWEH SRRU ZKD
  FRXOGEH WKHHQJLQHHULQJ UHDVRQIRUVXFKDZHLU' E)RUWKHSURWRWSHULYHUDVXPI
  WKDWWERWKVHFWLRQVKDYH VLGHVDWD ƒ DQJOHWRWKHYHUWFDO ZLWKWKHERWWRP VHFWLR
  KDYLQJ DEDVHZLGWKRI P DQG WKHXSSHUVHFWLRQKDYLQJ DEDVHZLGWKRI P
  LQFOXGLQJ WKHFXWRXWSRUWRQ 7KHKLJKWRI ORZHUDQGXSSHUKRUL]RQWDOVHFWLRQV/DU
  P DQG P UHVSHFWLYHO 8VHHQJLQHHULQJ HWWP DWWSDQG PDNHDSORWRI XSVWUHDP
  ZDWUGHSWKYHUVXV3HDXP D5LYHUIORZ UDWHLQWKHUDQJH WR P V F)RUZKD
  ULYHUIORZ UDWZLOOWKHZDWURYHUIORZ WKHVPSRI WKHGDP"
- &)LJXUH& VKRZVDKRUL]RQWDOORZ RI ZDWUWKURXJK DVXLFHJDWH DK\GUDXOL
  MXPS DQGRYHUD IWWKDUSFUHVWZHLU&KDQQHOJDWH MXPS DQGZHLUDUHDOO I
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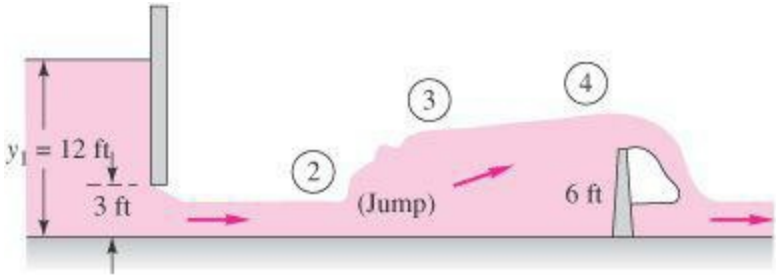
  G \ DQG H \

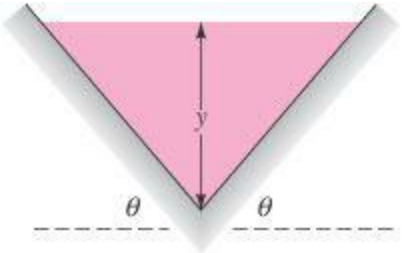


&

&RXUWHV\ RI WKH8 6 $UP\ &RUSVRI (QJLQHHUV: DWHUZD\V([SHULPHQWDWLRQ

- & &RQVLGHU WKH 9 VKDSHG FKDQQHOLQ) LJ & ZLWK DQ DUELWUDU\ DQJOH 0 DNH FRQVLGXLW DQG PRPHQWP DQDOVLV RI D VP DOOGLVWXUEDQFH/\ ⩽ \ DV LQ) LJ 6KRZ WKDWWKHZ DYHSURSDJDWRQVSHHGLQWKLV FKDQQHOLV LQGHSHQGHQWRI DQGGRH QRWHTXDOWKHZ LGH FKDQQHUHXXOWF J\

&



SURE OPP VTXHVWLRQV' HVLJQ3URMHFW

- ' $ VWDLJKWZHHG HDUWKFKDQQHOKDVWKHWDSHRIGDOVKDSHR) LJ ZLWKE F DQG ƒ 7KHFKDQQHOKDVDFRQWDQWERWWRP VORSHRI 7KHIORZ UDWHYDULH VHDVRQDOO IURP XSWR P V,WLVGHVLUHGWRSODFHDVKDSHGZHLUDFURVVWK FKDQQHOVR WKDWWKH ZDWUGHSWK NP XSWUHDP UHPDLQV DW P " SHUFHQ WKURXJKRXWWKH \HDU ,QYHVWLDWWKH SRVLELOLWV RI DFFRPSOLVKLQJ WKLV ZLWK DIXOO ZLGWK ZHLU LI VXFFHVVIXO GHWUPLQHWKH SURSHUZHLUKHLJKW ,I XQVXFFHVVIXO WU RWKHUDOWHUQDWLYHV VXFK DV D DIXOOZLGWK EURDGFUHVWHG ZHLURU E DZHLUZLV FRQVLGHUHGV RU F D 9 QRWFK ZHLU : KDWYHU \RXU ILQDO GHVLJQ FLWW VHDVRQDOYDULDWLRQ RI QRUPDOGHSWKV DQG FULWFDOGHSWKV IRUFRPSDULVRQ ZLWKW GHVLUHG\HDUURXQGGHSWKRI P

- ' 7KH &DURVHOO' DP RQ WKH 3DZFDWFN 5LYHULV IWKLJK IWZLGH DQG VKDO HGJHG 7KH&RDNOH\ &RPSDQ XVHVWKLVKHDGWRJHQHUDWHK\GURSRZHUHOFWULFLW DQ ZDQWVPRUH KHDG 7KH\ DVN WKH WZQIRUSHUPLWVRQ WR UDLVHWKH GDP KJKHU 7K ULYHUDERYH WKH GDP PD\ EH DSSUR[LPDWG DV UHFWDQJXODU IWZLGH VORSLQ XSWUHDP DW IWSHUWDWXWPLGH DQGZLWKD VRQ FREEOHGEHG 7KHDYHUDJHIORZ UDWHV ZLWKD \HDUSUHGLFWHGIORRGUDWHV IWV 7KHULYHUVLGH DUH VWHS XQLO PL XSWUHDP ZKHUH WKHUH DUH OLQJ UHVLGHQFHV 7KH VRZ FRXQFLODJUHHVWKHGDP PD\ EHKHJKWHQHG LI WKHQHZ ULYHUOHYHDQHDUWKHVHKRXVH GXULQJ WKH \HDUIORRG LV QR PRUH WKDQ WKH SUHVHQWOHYHDQGJH IORZ FRQGLWRQV <RX DV SURMHFWHQJLQHU KDYH WR SUHGLFWKRZ KJK WK GDP FUHWWDQEHUDLVHGDQGWUDOOPHHWWKLVUHTXLUHPHQW

5 HIHUHQFHV

% $ %DNKPHWHI +\GUDXOLFVRI 2SHQ&KDQQHOV 0 F* UDZ +LOO1HZ <RUN

9 7 &KRZ 2SHQ&KDQQHO+\GUDXOLFV 3UHQWLFH+DOO8SSHU6DGGOH5LYHU1-

0 + &KDXGKUL 2SHQ&KDQQHO)ORZ GHG 6SULQJHU1HZ <RUN

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+ &KDQJ 7KH+\GUDXOLFVRI 2SHQ&KDQQHO)ORZ GHG (OVHYLHU1HZ <RUN

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- 3HGHRVN : DYHV LQ WKH 2FHDQ DQG $WPRVSKHUH ,QWURGXFWLRQ WR : DYH ' \QDPLFV 6SULQJHU1HZ <RUN

/ + +RORXLWHQ: DYHV LQ 2FHDQF DQG &RDVWDO: DWHUV &DPEULGJH 8QLYHUVLW 3UHV 1HZ <RUN

0 . 2FKL2FHDQ: DYHV 7KH6WRFKDVWF$SSURDFK &DPEULGJH8QLYHUVLW3UHVV / RQGR

$ 7 ,SSHQ (VWXDU DQG&RDVWDOH+\GURG\QDPLFV 0 F* UDZ +LOO1HZ <RUN

0 % $ EERWWDQG: $ 3UHH &RDVWDO (VWXDULDO DQG+DUERU(QJLQHHULQ5HHUHQFH%RRN 7D\ORU) UDQFLV 1HZ <RUN

3 ' . RPDU %HDFK3URFHVVHVDQG6HGLPHQWDWLRQ GHG 3HDUVRQ(GXFDWLRQ 8SSHU6DGGO 5LYHU1-

: <XH & / / LQ DQG 9 & 3DWD³/ DUJH (GG 6LPXODWRQ RI 7XUEXOHQW 2SHQ &KDQQ
) ORZ ZLWK) UHH6XUIDFH6LPXODWGE\ / HYHO6HW0 HWRG´3K\VLFVRI) OXLGV YRO QR) HE
SS ±
- 0 5REHUWRQDQG+ 5RXVH³7KH) RXU5HJLPHVRI 2SHQ&KDQQHO) ORZ´&LY (QJ YR
QR 0 DUFK SS ±
5 : 3RZHOO³5HVLVWDQFHWR) ORZ LQ5RXJK &KDQQHOV´7UDQV $P * HRSK\V 8QLRQ YR
QR $XJXVW SS ±
5 0 DQQLQJ ³2QWKH) ORZ RI : DWHU LQ2SHQ&KDQQHOV DQG 3LSHV´7UDQV , & (,UHDQ
YRO SS ±
³) UHFWRQ) DFWRUVLQ2SHQ&KDQQHOV 5HSRUWRI WKH&RPPLWWHRQ+\GURPHFKDQLFV´$6&(-
+\GUDXO' LY 0 DUFK SS ±
( ) %UDWW + : . LQJ - (/ LQGHOO DQG & < : HL + DQGERRN RI +\GUDXOLFV WKHG
0 F* UDZ +LOO1 HZ <RUN
8 6 %XUHDX RI 5HFODPDWRQ³5HVHDUFK 6WXGLHVRQ6WUDQGHG %DLQV (QHUJ\ ' LVLSDWRUV DI
$VWRFLDWHG$SSXUWHQDQFHV´ +\GUDXOF / DE 5HS +\G -XQH
3 $ 7KRPSVRQ &RPSUHVVLEOH) OXLG' \QDPLFV 0 F* UDZ +LOO1 HZ <RUN
5 0 2OVRQDQG6 - : UJKW(VWHQVLRQ (QJLQHHULQJ) OXLG0 HFKDQLFV WKHG +DUSH
5RZ 1HZ <RUN
3 $FNHUVHWWO : HLVDQG) OXPHVIRU) ORZ 0 HDVXUHPHQW: LOH\ 1HZ <RUN
0 * %RV - $ 5HSGRJOH DQG$ - &OPPHQV) ORZ 0 HDVXULQJ) OXPHVIRU2SHQ&KDQQ
6\VWPV $PHULFDQ6RF $JULFXOWXUDODQG%LRORJLFDO(QJLQHHUV 6W-RVHSK 0 ,
0 * %RV/ RQJ 7KURDWG3UHVVDQG%URDG &UHVWG: HLV 6SULQJHU9HUODJ 1HZ <RU

' + +RJGQ&RPSXWU$VVLVWG) ORRGSODLQ +\GURJ\ DQG +\GUDXOLFV G HG
0 F* UDZ +LOO1 HZ <RUN
: 5RGL 7XUEXOHQFH0 RGHOVDQG7KHLU$SSOLFDWLRQLQ+\GUDXOLFV$ $ %DOHPD / HLGHQ
7KH1 HWKHUODQGV
3 ' ' XUELQ % $ 3HWWHUVRQ DQG% $ 5HI 7KHRU DQG0 RGHOQJ RI 7XUEXOHQW) ORZ\
: LOH\ 1HZ <RUN
5 %DEDQ ' HVLJQRI ' LYHUVLRQ: HLV 6PDOO6FDOH,UULJDWLRQLQ+RW&OPDWV: LOH\ 1HZ
<RUN
+ &KDQJRQ+\GUDXOF ' HVLJQ RI 6WHSSHG &DVFDGHV &KDQQHOV : HLV DQG 6SLOOZD\
3HUJDPRQ3UHVV 1HZ <RUN
' . DPSLRQDQG$ %UHZHU7KH%RRN RI : DYHV) RUP DQG%HDXWRQWKH2FHDQ GHG
5RZ PDQDQG/ LWWHLHOG / DQKDP 0 '
/ 0 D\V : DWHU5HVRXUFHV(QJLQHHULQJ : LOH\ 1HZ <RUN
' . : DOKRZLDN HG ,VFR 2SHQ &KDQQHO) ORZ 0 HDVXUHPHQW+ DQGERRN WK HG
7HODG\QH,VFR ,QF / LQFROQ1 (
+ &KDQJRQ³3KRWRJUDSKLF 2EVHUYDWRQVRI 7LGDO%RUHV 0 DVFDUHWV LQ) UDQFH´ +\GUDX
0 RGHO5HSRUW&+ 7KH8QLYHUVLWRI 4XHHQVDQG SDJHV
(- : DWRQ³7KH6SUHDG RI D/ LTXLG -HWRYHUD +RUL]RQWDO3ODQH´ -) OXLG0 HFKDQLFV
YRO SS ±

– 6XUIDFHWHQVLRQLVUDUHO LPSRUWDQWEHFDXVHRSHQFKDQQHOVDUHQRUP DOO TXLWODUJHDQGKDYH
YHU ODUJH: HEHUQXPEHU6XUIDFHWHQVLRQDIIHFWVP DOOP RGHOVRI ODUJHFKDQQHOV

– $ PRUHFRPSOHWHDVVHVVPHQWRI5H SS ±

– $QLQWHUHVWLQJGLVFXVVLRQRI WKHKLVWRU\ DQG³GLPHQVLRQDOLW\ RI 0 DQQLQJ¶V IRUPXODLVJLYHQL
5H SS ±

– 7KLV LV WKH EDVLV RI WKH ZDWHUFKDQQHODQDO\ IRUVXSHUFJDVG\QDPLFVH[SHULPHQWDWR
> &KDS @

– 7KLVVHFWLRQPD\ EHRPLWWHGZLWKRXWORVVRI FRQWLQXLW\

– 7KHDXWKRULVLQGHEWHGWR3URI %UXFH/ DURFNIRUFDUHIXOUHDGLQJRI WKHVHVWDQVLWLRQSURFHHV

)LJ 6FKHPDWLF GHVLJQ RI SRVLWLYH GLVSODFHPHQWSXPSV *a* UHFLSURFDWLQJ SLVWRQ R
SOXQJHU *b* H[WHUQDOJHDUSXPS *c* GRXEOHVFUHZ SXPS *d* VOLGLQJ YDQH *e* WKUHHOREH SXPS
f GRXEOHFLUFXPIHUHQWLDOSLVWRQ *g* ICH[LEOHVXEHVTXHHJHH

6LQFH3' 3VFRPSUHVVPHFKDQLFDO DJDLQVWDFDYLWLWHGZLWKOLTXLG DFRPPRQIDXUHLVWK
VKH GHYHORS LPPHQVH SUHVXUHV LI VKH RXWHWLV VKXWGRZQ IRUDQ UHDVRQ 6WXG FRQWUDFWRQ L
UHTXLUHG DQG FRPSOHWHVKXWRI ZRXOGFDXVHGDPDJHLI SUHVXUHUHOHYDOYHVZHUHQRWXVHG
' \QDPLF SXPSVVLPSO DGGPRPHQWP WKWHIXLGE\ PHDQVRI IDWPRYLQJ EODGHVRUYDQHVR
FHUWDLQVSHFLDOGHVJQV 7KHUHLVQR FORVHG YROXPH 7KHIXLGLQFUHDVHVPRPHQWP ZKLOHPRYLQ
WKURXJKRSHQSDVDJHVDQGWKHQFRQYHUWVWKJK YHORFLWVWRD SUHVXUHLQFUHDVHE\ H[LWQJ LQWRI
GLIIXVHUVHFWRQ ' \QDPLF SXPSVFDQEHFODVVLILHGDVIROORZV

5RWDU\
&HQWULXJDORUUDGLDOH[WURZ
$[LDOIORZ
0 L[HGIORZ EHWZHHQUDGLDODQGD[LDO
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(OHFWURP DJQHWLF SXP SV IRU OLTXLG PHDOV
) OXLG DFWXDWHG JDVOLWRUK\GUDXOLF UDP

: HVKDOOFRQFHQWUDWHRQWKLVFKDSWHURQWKHURWDU\GHVLJQV VRPHWLPHVFDOOHGURWRG\QDPLF SXP SV
2WKHUGHVLJQV RI ERWK 3' 3 DQG G\QDPLF SXP SV DUHGLVFXVVHG LQVSHFLDO]HGWH[WVRIH[DPSOH
@
' \QDPLF SXP SV JHQHUDOO SURYLGHDKLJKHUIORZ UDWHWKDQ3' 3V DQG DPXFK VPDOOHUGLVFKDUJH
EXWDUHLQHIIHFWLYHLQKDQGOLQJ KLJK YLVFRVLWLTXLGV ' \QDPLF SXP SVDOR JHQHUDOO QHGSUIPLQJ L
WKH DUHILOOHG ZLWK JDV WKH FDQQRWXFN XS DTXLG IURP EHORZ LQRWKHULQGHW7KH3' 3 RQWK
RWKHUKDQG LVVHO SUIPLQJ IRUPRWDSSOFDWRQV $ G\QDPLF SXPS FDQ SURYLGHYHU KJK IORZ UDWH
XS WR JDOPLQ EXWXVXDOO ZLWKPRGHUDWSUHVXUHULVH DIHZ DWRVSKHUHV ,QFRQWDVW
3' 3 FDQ RSHUDWH XS WR YHU KJK SUHVXUHV DWV EXWWSLFDOO SURGXFHVORZ IORZ UDWHV
JDOPLQ
7KHUHDLYHSHUURUP DQFH ûS YHUXV4 LVTXLWGLIHUHQWRUWKHWZRWSHVRI SXPS DVKRZ
LQ) LJ $WFRQVWDQWWKDWWURDWRQ VSHHG WKH3' 3 SURGXFHV QHDUO FRQVWDQWIORZ UDWH DQ
YLUWXDOO XQOLPLWHG SUHVXUH UHVH ZLWKOLWWOH HIHFWRI YLVFRVLW 7KHIORZ UDWHRI D3' 3 FDQQRWE
YDULHG H[FHSWE\ FKDQJLQJ WKHGLVSODFHPHQWRUWKHVSHG 7KHUHDEEHFRQVDQWSHGGLVFKDUJHIURP
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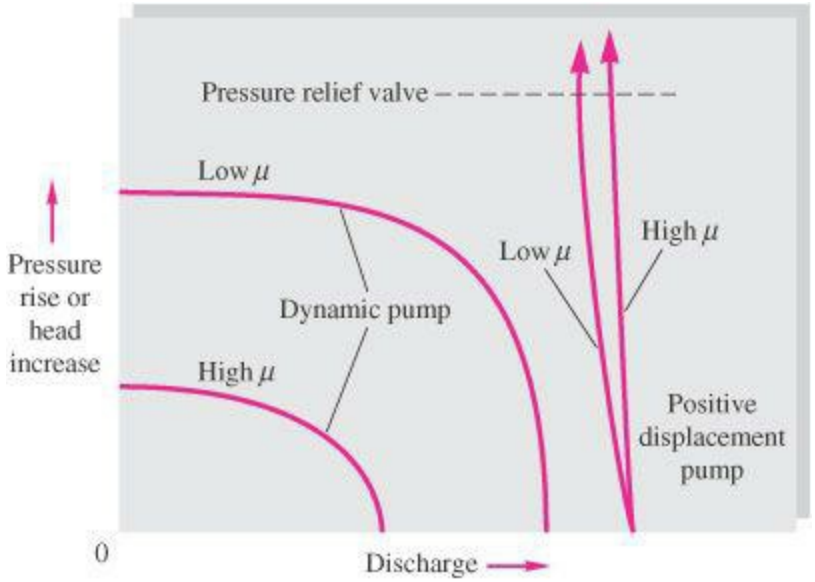

) LJ &RPSDULVRQ RI SHUIRUPDQFH FXUYHV RI WSLFDOG\QDPLF DQG SRVLWLYH GLVSODFH P HQ
SXPSVDWFRQVWDQWSHHG

7KHG\QDPLF SXPS E\ FRQWDWLQ) LJ SURYLGHVD FRQWLQXRXVFRQVWDQWSHHGYDULDWRQ R
SHUIRUPDQFH IURP QHDUPD[LPXP ûS DW]HUR IORZ VKXWRI FRQGLWLRQV WR]HUR ûS DWP D[LPXP IORZ
UDWH+LGK YLVFRVLWIOXLGVVKDUSO GHJUDGHWKHSHUIRUPDQFHRI DG\QDPLF SXPS
\$V XVXDO DQG IRU WKH OWWLP H LQ WKLV WH[W ZH UHPLQG WKH UHDGHU WKDWWKLV LV P HUHO D
LQWRGXFWRU\ FKDSWU 0DQ ERRNV DUHGHYRWHG VROHO WR WKLERP DFKLQHV JHQHUDO]HG WHDWHQW> W
@WH[WSHFLDO]LQJ LQSXPSV> WR @IDQV> WR @FRPSUHVVRUV> WR @JDVWXELQH
> WR @K\GURSRZHU> @DQG3' 3V> WR @7KHUHDUHVHYHUDOXH XCKDQGERRNV> \
@ DQG DWOHDWWZR XQGHUJUDGXDW WH[WERRNV > @KDYH D FRP SUHHQVLYH GLVFXVLRQ R
WKLERP DFKLQHV 7KHUHDGHULVUHHUHGWRWKHVHVRUFHVIRUIXUWKHUGHWDLOV

7KH&HQWULIXJDO3XPS

/HWXV EHJLQ RXU EULHI GRRN DWURRG QDPLF PDFKLQHV E\ H[DPLQLQJ WKH FKDUDFWULVWLFV RI WK
FHQWULIXJDOSXPS $VVXPHWKHGLQ) LJ WKLVSXPS FRQVLVWVRI DQLPSHOOHUURWDWLQJ ZLWKLQDFDVLQJ
) OXLGHQWHUVD[LDOO\ WKURXJKWKHH\HRI WKHFDVLQJ LVFDXJKWXSLQWKHLPSHOOHUEODGHV DQGLVZKLUO
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LPSHOOHU 7KH GRXJKQXWWKDSHG GLIIXVHU RUVFURRO VHFWLRQ RI WKH FDVLQJ GHFHOHUDWHV WKH IOXLG DQ
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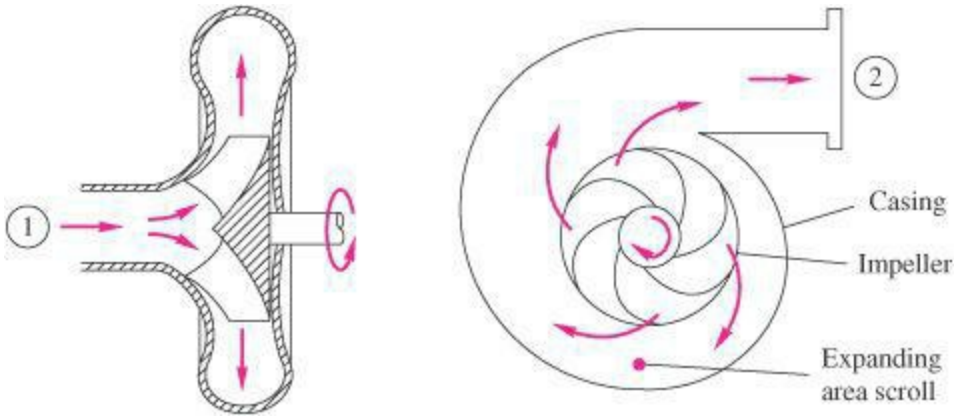

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ERWKVLGHV E\ DQLPSHOOHUZDOO 7KHGLIIXVHUPD\ EHYDQHOHVV DVLQ) LJ RUILWWHG ZLWKILQ H
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WKH H\H DQG SRLQW WKH H[LW) URP (T QHJOHFWLQJ YLVFRXV ZRUN DQG KHDWWUDQVIHU WKLV
FKDQJHLVGHQRWHGE\ +

$$H = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_2 - \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_1 = h_s - h_f \qquad (11.1)$$

ZKHUHK, LV WKH SXPS KHDG VXSSOLHG DQGK, WKH ORVVHV 7KH QHWKHDG+ LV D SUPDU\ RXWSX
SDUDPHWHUIRUDQ\ WKUERPDFKLQH 6LQFH(T LVIRULQFRPSUHVVLEOHIOXLG LWP XVWEHPRGLILHGIR
JDVFRPSUHVVRUV ZLWKODUJHGHQVLWFKDQJHV

8VXDOO\ 9 DQG9 DUHDERXWWKHVDPH] í] LVQR PRUHWKDQDPHWURVR DQGWKHQHWSXP
KHDGLVHVWHQWLDOO\ HTXDOWRWKHFKDQJHLQSUHVVXUHKHDG

$$H \approx \frac{p_2 - p_1}{\rho g} = \frac{\Delta p}{\rho g} \qquad (11.2)$$

7KH SRZHU GHOLYHUHG WR WKH IOXLG VLPSO HTXDOV WKH VSHFLILF ZHJKW WLPHV WKH GLVFKDUJH WLPHV WK
QHWKHDG FKDQJH

$$P_w = \rho g Q H \tag{11.3}$$

7KLV LV WDGLWLRQDOO FDOOHG WKH ZDWHUKRUVHSRZHU 7KH SRZHU UHTXLUHG WR GULYH WKH SXPS LV WK
EUDNHKRUVHSRZHU

$$\text{bhp} = \omega T \tag{11.4}$$

ZKHUH& LV WKH VKDIWDQJXODU YHORFLW DQG7 WKH VKDIWWRUTXH ,I WKHUH ZHUH QR ORVVHV 3$ DQG EUDN
KRUVHSRZHU ZRXOG EH HTXDO EXWR FRXUH3$ LV DFWXDOOO OHVV DQG WKH HILFLHQF\ RI WKH SXPS L
GHILQHG DV

$$\eta = \frac{P_w}{\text{bhp}} = \frac{\rho g Q H}{\omega T} \tag{11.5}$$

7KHFKLH DLP RI WKHSXPS GHVLJQHULV WR PDNH DVKLJK DVSRVVLEOHRYHUDVEURDG DUDQJHR
GLVFKDUJH4 DVSRVVLEOH

7KHHILFLHQF\ LVEDVLFDOO FRPSRVHG RI WKUHHSDUW YROXPHWULF K\GUDXOLF DQG PHFKDQLFDO 7K
YROXPHWULF HILFLHQF\ LV

$$\eta_v = \frac{Q}{Q + Q_L} \tag{11.6}$$

ZKHUH4/ LV WKH ORVV RI IOXLG GXH WR OHDNDJH LQ WKH LPSHOOHU FDVLQJ FOHDUDQFHV 7KHK\GUDXOL
HILFLHQF\ LV

$$\eta_h = 1 - \frac{h_f}{h_s} \tag{11.7}$$

ZKHUHK, KDV WKUHHSDUW VKRFN ORVV DVWKHH\HGXHV WR LPSHUFWP DWK EHWZHQLQDWWRZ DQ
WKH EODGH HQWDQFHV IULFWRQ ORVVHV LQ WKH EODGH SDVVDJHV DQG FLUFXODWRQ ORVV GXH V
LPSHUFWP DWK DVWKH H[LVWLG HR WKHEODGHV
)LQDOO WKHPHFKDQLFDOHILFLHQF\ LV

$$\eta_m = 1 - \frac{P_f}{\text{bhp}} \tag{11.8}$$

ZKHUH3, LV WKHSRZHUORVV GXHVWR PHFKDQLFDOIULFWRQ LQ WKHEHDULQJV SDFNLQJ JODQGV DQG RVKH
FRQDFWVSRLQWV LQ WKHPDFKLQH
%\ GHILQLWRQ WKHVRDOOHILFLHQF\ LVVLPSO WKHSURGXFWRI LWVWKUHHSDUW

$$\eta \equiv \eta_v \eta_h \eta_m \tag{11.9}$$

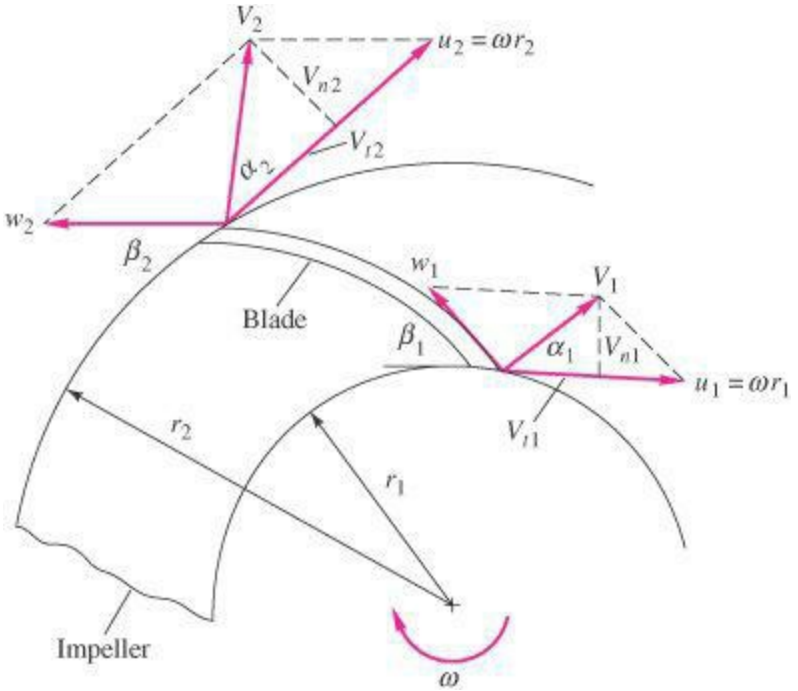
7KHGHVLJQHUKDVWRZRUNLQDOOWKUHHDUHDVWRLPSURYHWKHSXPS

(OHPHQWDU\ 3XPS 7KHRU\

<RXPD\ KDYHWKRXJKWWKDW(TV VR ZHUHIRUPXODWIURP SXPSWKHRU\ 1RWWR WKH DUHPHUH GHILQLWLRQV RI SHUIRUPDQFHSDUDPHWHUV DQG FDQQRWEHXVHGLQDQ SUHGLFWLYH PRGH 7R DFWXDOO SUHGLFWWKHKHDG SRZHUHILFLHQF\ DQGIORZ UDWH RI DSXPS VZRWKRUHWFDOSSURDFKHVDUH SRVVLEOH VLPSOHRQHGLPHQVLRQDOIORZ IRUPXODV DQG FRPSOH[FRPSXWHUPRGHOVWKDWFRXQ IRUYLVFRVLWLHVDQGWKUHHGLPHQVLRQDOW 0 DQ RI WKHEHWWHUGHVLJQLPSURYHPHQWVWXOFRPHIURP WHVWLQ DQG H[SHULHQFH DQGSXPS UHVHDUFK UHPDLQV DYHU DFWLYHILHOG > @7KHODVW \HDUVKDYHVHH FRQVLGHUDEOHDGYDQFHVLQFRPSXWDWRQDOIOXLGG\QDPLFV &)' PRGHOLQJ RI IORZ LQWKUERPDFKLQ> @DQGDVWHDWWHJKWFRPPHUFLDOXVHQWORZ WKUHHGLPHQVLRQDO&)' FRGHVDUHQRZ DYDLODEOH

7RFRQWUDFWDQHPHQW WKRU RI SXPS SHUIRUPDQFH ZHDVXPHQHGLPHQVLRQDOIORZ DQ FRPELQHGDQDO\WLFDOH[SHULPHQWDO WKURXJK WKHLPSOHPHQWDWLRQRIPRQWH &DUORWHFKQLTXHV DFRQWUROYROXPH(T

7KH HGDQ YHORFLW GLDJUDPV DUH VKRZQ LQ) LJ 7KH IOXLG LV DVXPHG WR HQWHU WK LPSHOHUDW U ZLWKYHORFLWFRPSRQHQWZ VDQJHQWWRWKHEODGHDQGH SUHVVLUHPRYHQ X &U PDWKLQJ WKHWS VSHHGRI WKHLPSHOHU,WDEVRUEVHQHUJHY HORFLWLVWKXVWKHYHFWRU R Z DQGX VKRZQDV9 6LPLODUO WKHIORZ H[LWVDW U ZLWKFRPSRQHQWZ SDUDOOHOWR WKHEODGH DQJH SUHVVWS VSHHGX &U ZLWKUHVXOWDQWYHORFLW9



) LJ ,QBWDQGH[LWYHORFLW GLDJUDPVIRUDQLGHDO[HGSXPSLPSHOHU

: HDSSOHGWKHDQJXODUPRPHQWXP WKHRUHP WR DWXUERPDFKLQHLQ([DPSOH) LJ DQG DULLYHGDWDUHVXOWRUWKHDSSOHGWRUTXH7

$$T = \rho Q(r_2 V_{t2} - r_1 V_{t1}) \qquad (11.10)$$

ZKHUH9$_W$ DQG9$_W$ DUHWKHDEVROXWHFLUFXPIHUHQWLDOYHORFLW FRPSRQHQWVRI WKHIORZ 7KHSRZH

IWPLQIRUJDVHV 7KHGHSHQGHQWYDULDEOHV RU³RXWSXW DUHVDNHQWREHKHDG+ SUHVVXUHULVHûSIR
JDVHV EUDNHKRUVHSRZHU EKS DQGHILFLHQF\

) LJXUH VKRZV WSLFDO SHUFRUPDQFH FXUYHV IRU D FHQWULXJDO SXPS 7KH KHDG L
DSSUR[LPDWH FRQVWDQWVDWORZ GLVFKDUJH DQG WKHQ GURSV WR]HUR DW4 4PD[$WWKLV VSHHG DQ
LPSHOOHUVL]H WKHSXPS FDQQRWGHOLYHUDQ PRUHIOXLGWKDQ4PD[7KHSRWLQWHVOSHSDWWR WKHKHD
LV VKRZQGDVKHG DV PHQWRQHG HDUOLHU WKLV UHJLRQ FDQ EH XQVWDEOH DQG FDQ FDXVH KXQWLQJ IRUWK
RSHUDWQJ SRLQW

) LJ 7\SLFDOFHQWULXJDOSXPS SHUFRUPDQFH FXUYHV DWFRQVWDQWLPSHOOHUURWDWQVSHHG
7KHXQLWVDUHDUELWUDU

7KHHILFLHQF\ LVDOZD\V]HUR DWQR IORZ DQGDW4PD[DQGLWUHDFKHVDPD[LPXP SHUKDSV V
SHUFHQWDWDERXW 4PD[7KLV LV WKHGHVLJQIORZ UDWH4 RUEHWWHILFLHQF\ SRLQW %(3
PD[7KH KHDG DQG KRUVHSRZHU DW%(3 ZLOOEH WUHGG+ DQG3 RUEKS UHVSHFWLYHO ,WV
GHVLUDEOHWKDWWKHHILFLHQF\ FXUYHEHIODWQHDU PD[VR WKDWD ZLGHUDQJHRI HILFLHQWRSHUDWQJ L
DFKLHYHG +RZHYHU VRPHGHVLJQV VLPSO GR QRWDFKLHYHIODWHILFLHQF\ FXUYHV 1RWWKDW LV QR
LQGHSHQGHQWRI + DQG3 EXWDWKHULVFDOFXODWGIURP WKHUHDWRQLQ(T !J4 + 3
$VVKRZQLQ) LJ WKHKRUVHSRZHUUHTXLUHGWRGULYHWKHSXPSWSLFDOO ULVHVPRQRWRQFDOO
ZLWKWKHIORZ UDWH 6RPHWLPHVWKHUHLV DOJHSRZHUULVHEH\RQGWKH%(3 HVSHFLDOO IRUUDGLDO
VSSHGDQGIRUZDUGFXUYHGEODGHV 7KLVFRQVLGHUHGXQGHVLUDEOHEHFDXVHDPXFKODUJHUPRWRUL
WKHQHHGHGWRSURYLGHKJKIORZ UDWVW %DFNZDUGFXUYHGEODGHVWSLFDOO KDYHWKHLUKRUVHSRZH
OHYHORIDERYH%(3 ³QRQRYHUORDGLQJ´WSHRI FXUYH

0 HDVXUHG3HUIRUPDQFH&XUYHV

³ 6KRFN GRZV GXH WR PLVPDWFK EHWZHHQ WKH EODGH DQJOHV DQG WKH LQGXFHU GLUHFWRU HVSHFLDOO VLJQLILFDQWDWKLJK IORZ UDWHV

7KHVH DUH FRPSOLFDWHG WKUHH GLPHQVLRQDO IORZ HIHFWV DQG KHQFH DUH GLIILFXOW WR SUHGLFW $OWKRXJK DV PHQWLRQHG QXPHULFDO &)' WHFKQLTXHV DUH EHFRPLQJ PRUH LPSRUWDQW> @PRGHUQ SHUIRUPDQFH SUHGLFWLRQ LV VWLOO D EODQG RI H[SHULHQFH HPSLULFDO FRUUHODWLRQV LGHDOL]HG WKHRU DQG &)' PRGLILFDWLRQV> @

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7KH LQSXPS SRI)LJ DLVWRSXPS JDOPLQRI ZDWUDW UPLQIURP DUHVHUYRLU ZKRVH VXUIDFH LV DW EHLQ DEVROXWH ,I KHDG GRZ IURP UHVHUYRLU WR SXPS LQGHWLV IWZKHU VKRXOG WKH SXPS LQGHWE HSODFHG WR DYRLG FDYLWDWRQ IRUZDWUDWD f) S# EHI LQ DEVROXW 6* DQG E f) S# EHI LQ DEVROXW 6* "

6ROXWLRQ

3DUWD

)RUHLWKHUFDVH UHDGIURP)LJ DDW JDOPLQWKDWWKH UHTXLUHG1 36+ LV IW)RUWKH FDVH! J EHI IW)URP(T LWLVQHFHVVDU WKDW

7KHSXPS PXWEHSODFHGDWOHDW IWEHORZ WKHUHVHUYRLUVXUIDFHWRDYRLGFDYLWDWRQ

3DUWE

)RUWKLVFDVH! J EHI IW(TXDWRQ DSSOLHVDJDLQZLWKWKHKLJKHUS%

$$40 \text{ ft} \le \frac{(14.7 - 11.52 \text{ lbf/in}^2)(144 \text{ in}^2/\text{ft}^2)}{60.1 \text{ lbf/ft}^3} - Z_i - 6.0$$

or $$Z_i \le 1.6 - 40 = -38.4 \text{ ft}$$ *Ans.* (*b*)

7KHSXPS PXWQRZ EHSODFHGDWOHDW IWEHORZ WKHUHVHUYRLUVXUIDFH 7KHVHDUHXQXVXDOO VWULQJHQWFRQGLWLRQVEHFDXVHDOODUJH KLJK GLVFKDUJHSXPS UHTXLUHVDODUJH1 36+

' LP HQVLRQOHVV3 XP S 3 HUIRUP DQFH

) RUDJLYHQSXPS GHVLJQ WKHRXSXWYDULDEOHV+ DQG EUDNHKRUVHSRZHUVKRXOG EH GHWHUPLQHGIRUDJLYHQ GLVFKDUJH4 LPSHOOHUGLDPHWUG DQG VKDIWVSHHGQ DWGHVLJ2 WKHUSRWLEOHSDUDPHWHUVDUH WKHIOXLG GHQVLW ! YLVFRVLW DQG VXUIDFH URXJKQHVV Å 7KXV WKH SHUFRUPDQFH FXUYHV LQ) LJ DU HTXLYDOHQWWR WKHIRORZLQJ DWXPHGIXQFWLRQDOUHODWLRQV

$$gH = f_1(Q, D, n, \rho, \mu, \epsilon) \qquad \text{bhp} = f_2(Q, D, n, \rho, \mu, \epsilon) \tag{11.21}$$

7KLVLVDVWDLJKWIRUZDUGDSSOLFDWLRQRI GLPHQVLRQDODQDO VLVSULQFLSOHVIURP &KDS $VDPDWW RI IDFWLWZ DV JLYHQ DV DQ H[HUFLVH ([ ) RUHDFK IXQFWLRQ LQ(T WKHUH DUH VHYH YDULDEOHV DQG WKUHH SULPDU GLPHQVLRQV 0 / DQG7 KHQHZ H[SHFW í GLPHQVLRQOHVS JURXSV DQG WKDWWZ KDWZ HJHW<RX FDQ YHUI\ DV DQH[HUFLVHWKDWDSSURSULDWHGLPHQVLRQOHVIRUP IRU(TV DUH

$$\frac{gH}{n^2D^2} = g_1\left(\frac{Q}{nD^3}, \frac{\rho nD^2}{\mu}, \frac{\epsilon}{D}\right)$$
$$\frac{\text{bhp}}{\rho n^3 D^5} = g_2\left(\frac{Q}{nD^3}, \frac{\rho nD^2}{\mu}, \frac{\epsilon}{D}\right) \tag{11.22}$$

7KH TXDQWLWLHV! Q DQGë' DUH UHFRJQL]HG DV WKH 5H\QROGV QXPEHU DQG URXJKQHVV UDWLR UHVSHFWLYHO 7KUHHQHZ SXPS SDUDPHWHUVKDYHDULVHQ

$$\text{Capacity coefficient } C_Q = \frac{Q}{nD^3}$$
$$\text{Head coefficient } C_H = \frac{gH}{n^2D^2} \tag{11.23}$$
$$\text{Power coefficient } C_P = \frac{\text{bhp}}{\rho n^3 D^5}$$

1 RWH WKDWRQO WKH SRZHU FRHIILFLHQWFRQWDLQV IOXLG GHQVLW WKH SDUDPHWHUV&4 DQG&+ EHLQ NLQHPDWF WWSHV

) LJXUH JLYHVQR Z DUQLQJ RI YLVFRXVRUURXJKQHVV HIHFW 7KH5H\QROGV QXPEHUVDUHIURP VR î RUIXOO WXUEXOHQWRZ LQDOOSDWJHVSUREEO 7KHURXJKQHVV LV QRWJLYHQ DQ YDUHVJUHDW DP RQJ FRPPHUFLDOSXPSV %XWDWXFK KJK5H\QROGVQXPEHUVZHH[SHFWPRUHORVV WKHVPH SHUFHQDJH HIHFWRQ DOWKHVH SXPPS 7KHUHOWLV FRPPRQWR DWXPHWKDWWKH5H\QROGVQXPEHUDQG WKHURXJKQHVVUDWR KDYHDFRQWDQWHIHFWWR WKDW(TV UHGXFHWR DSSUR[LPDWO

$$C_H \approx C_H(C_Q) \qquad C_P \approx C_P(C_Q) \tag{11.24}$$

) RUJHRPHWFDO VLPLODUSXPPS ZHH[SHFWKHDG DQG SRZHUFRHIILFLHQWVWR EH QHDU XQLTX IXQFWLRQVRI WKHFDSDFLWFRHIILFLHQW. HKDYHWRZDWKRXWKDWWKHSXPPSVDUHJHRPHWFDO VLPLODUR QHDU VR EHFDXVH PDQXIDFWXUHUVSXWGLIHUHQWL]HGLPSHOHUVLQWKH VDPHFDVLQJ WKXV YLRODWLQ

JHRPHWULF VLPLODULW DQG ODUJHSXPSVKDYHVPDOOUUDWRVRI URXJKQHVDQGFOHDUDQFHVWRLPSHOO
GLPHQVLRQDO VPDOOSXPSV ,QDGGLWRQ WKH PRUH YLVFRXV OLTXLGV ZLOOKDYH VLJQLILFDQW5H\QROG
QXPEHUHIHFWV IRUH[DPSOH DIDFWRURI RUPRUH YLVFRVLW\LQFUHDVHFDXVHVDFOHDUDQ YLVLEOHHIHF
RQ&+ DQG&3

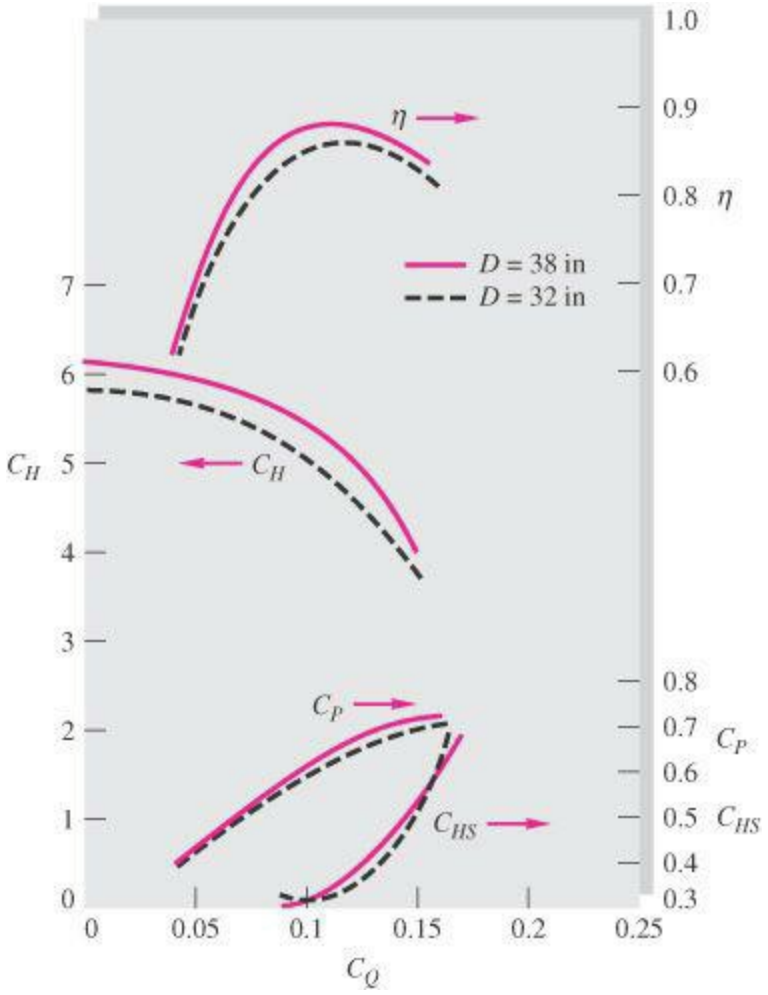
7KH HILFLHQF\ LV DOVHDG GLPHQVLRQOHVV DQG LV XQLTXHO UHDWHG WR WKH RWKHUWKUHH ,WYDULH
ZLWK&4 DOVR

$$\eta \equiv \frac{C_H C_Q}{C_P} = \eta(C_Q) \qquad (11.25)$$

: HFDQWWW(TV DQG IURP WKHGDWDRI)LJ 7KHLPSHOOHUGLPHWULVRI DQ
LQDUHDSSURSULDWH SHUFHQWGLIIHUHQWLQVL]H DQGVRWKHUUDWRRI LPSHOOHUWRFDVLQJ VL]HLVWK
VDPH 7KHSDUDPHWULV&4 &+ DQG&3 DUHFRPSXWHGZLWKQLQUV 4 LQIWV JDOPLQî î í
+ DQG' LQIWJ IWV DQGEUDNHKRUVHSRZHULQKRUVHSRZHUWPHV IW OEI V KS 7K
QRQGLPHQVLRQDOGDWDUHWKHQSORWWHGLQ) LJ $ GLPHQVLRQDWVXFWLRQKHDGFRHILFLHQWLVDOVR
GHILQHG

$$C_{HS} = \frac{g(\mathrm{NPSH})}{n^2 D^2} = C_{HS}(C_Q) \qquad (11.26)$$

7KHFRHILFLHQW&3 DQG&+6 DUHVHQVLWLYHFRUUHODWHGWRSHUFHQW LQWRDVLQJOHIXQFWLRQRI &4
ZKLOH DQG&+ GDWDGHYLDWHE\ DIHZ SHUFHQW7KHODWWZRSDUDPHWULVDUHPRUHVHQVLWLYHWRVOJK
GLVFUHSDQFLHVLQPRGHOLPLODULW VLQFHWKHODJHUSXPS KDVVPDOOHUURXJKQHVDQGFOHDUDQFHUDWR
DQGD SHUFHQWDJHU5H\QROGVQXPEHUWHGHYHORSVVOJKWO PRUHKHDGDQGLVPRUHHILFLHQW7K
RYHUDOOHIIHFWVDUHVRXQGLQJ YLFWRU IRUGLPHQVLRQOOQDOVLV



) LJ 1RQGLPHQVLRQDOSORWVRI WKHSXPS SHUIRUPDQFHGDWDIURP) LJ 7KHVHQXPEHUV DUHQRWUHSUHVHQWDWLYHRI RWKHUSXPS GHVLJQV

7KHEHVWHILFLHQF\ SRLQWLQ) LJ LVDSSUR[LPDWHG

$$C_{Q^*} \approx 0.115 \qquad C_{P^*} \approx 0.65$$
$$\eta_{\max} \approx 0.88 \qquad (11.27)$$
$$C_{H^*} \approx 5.0 \qquad C_{HS^*} \approx 0.37$$

7KHVHYDOXHVFDQEHXVHGWRHVWLPDWHWKH%(3 SHUIRUPDQFHRI DQ VL]HSXPSLQWKLVJHRPHWULFDO VLPLODUIDPLO\ ,QOLNHPDQQHUWKHVKXWRII KHDGLV&+ § DQGE\ H[WDSRODWLRQWKHVKXWRII SRZHU LV&3 § DQGWKHPD[LPXP GLVFKDUJHLV&4 PD[§ 1RWHKRZHYHUWKDW) LJ JLYHVQ UHODWLRQLQRUPDWRQ DERXWVD WKH RU LQLPSHOOHUV LQ) LJ ZKLFK KDYH D GLIIHUHQ LPSHOOHUWR FDVLQJ VL]HUDWR DQGWKXVPXWEHFRUUHODWHGVHSDUDWHO

%\ FRPSDULQJ YDOXHVRI Q' Q DQGQ' IRUWZRSXPSVLQ) LJ ZHFDQVHHUHDGLO ZKWKHODUJHSXPSKDGWKHVPHGLVFKDUJHEXWDSRZHUDQGKHDG

' LVFKDUJHJRHVDVQ ZKLFKLVDERXWWKHVPHIRUERWSXPSV +HDGJRHVDVQ' DQGSRZHUD Q' IRUWKHVPH! ZDWU DQGWKHVHDUHDERXWKDOIDVPXFKIRUWKHODUJHUSXPS 7KH136+ JRHVD Q' DQGLVDORKDOIDVPXFKIRUWKH LQSXPS

	D, ft	n, r/s	Discharge nD^3, ft^3/s	Head n^2D^2/g, ft	Power $\rho n^3 D^5/550$, hp
Fig. 11.7*a*	32/12	1170/60	370	84	3527
Fig. 11.7*b*	38/12	710/60	376	44	1861
Ratio	—	—	1.02	0.52	0.53

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$ SXPS IURP VKHIDPLO RI) LJ KDV' LQDQGQ UPLQ (VWLPDWH D GLVFKDUJH E KHDG F SUHVVXUHULVH DQG G EUDNHKRUVHSRZHUR VKLSXPS IRUZDWUDW f) DQGEHWWHILFLHQF\

6RXWLRQ

3DUWD

,Q %* XQLW VDNH IWDQGQ UV $W f) ! RI ZDWULV VOXJVIW 7KH%(3 SDUDPHWHUVDUHNQRZ QIURP) LJ RU(TV 7KH%(3 GLVFKDUJHLVVKXV

$$Q^* = C_{Q^*} n D^3 = 0.115(25 \text{ r/s})(1.75 \text{ ft})^3 = (15.4 \text{ ft}^3/\text{s})\left(448.8 \frac{\text{gal/min}}{\text{ft}^3/\text{s}}\right) = 6900 \text{ gal/min}$$

Ans. (*a*)

3DUWE

6LPLODUO VKH%(3 KHDGLV

$$H^* = \frac{C_{H^*} n^2 D^2}{g} = \frac{5.0(25)^2(1.75)^2}{32.2} = 300\text{-ft water} \qquad \textit{Ans.}\ (b)$$

3DUWF

6LQFHZ HDUHQRWJLYHQHOHYDWLRQRUYHORFLW\KHDGFKDQJHVDFURVVWKHSXPS ZHQHJOHFWWKHP DQG HXLOLP DWH

$$\Delta p \approx \rho g H = 1.94(32.2)(300) = 18{,}600 \text{ lbf/ft}^2 = 129 \text{ lbf/in}^2 \qquad \textit{Ans.}\ (c)$$

3DUWG

)LQDOO VKH%(3 SRZHUV

$$P^* = C_{P*}\rho n^3 D^5 = 0.65(1.94)(25)^3(1.75)^5$$
$$= \frac{323{,}000 \text{ ft} \cdot \text{lbf/s}}{550} = 590 \text{ hp} \qquad \textit{Ans. (d)}$$

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: HZDQWRXOGDSSURPSIURP VKHIDPLOLRI)LJ ZKLFKGHOLYHUV JDOPLQZDWUDW UPLQDWEHWWHILFLHQF\ (VWLPDWH D VKHLPSHOOHUGLDPHWU E VKH PD[LPXP GLVFKDUJH F VK VKXWRIIKHDG DQG G VKH136+ DWEHWWHILFLHQF\

6RXWLRQ

3DUWD

JDOPLQ IWVÑ Ñ DQGÑ Ñ UPLQ UV$W%(3 ZHKDYH

$$Q^* = C_{Q*}nD^3 = 6.68 \text{ ft}^3/\text{s} = (0.115)(20)D^3$$
$$D = \left[\frac{6.68}{0.115(20)}\right]^{1/3} = 1.43 \text{ ft} = 17.1 \text{ in} \qquad \textit{Ans. (a)}$$

3DUWE

7KHPD[LPXP 4 LVUHDFKHGWR4 E\ DUDLVRRI FDSDFLWVFRHILFLHQW

$$Q_{\max} = \frac{Q^* C_{Q,\max}}{C_{Q^*}} \approx \frac{3000(0.23)}{0.115} = 6000 \text{ gal/min} \qquad \textit{Ans. (b)}$$

3DUWF

)URP)LJ ZHHVWLPDWGWKHVKXWRIIKHDGFRHILFLHQWREH 7KXV

$$H(0) \approx \frac{C_H(0)n^2D^2}{g} = \frac{6.0(20)^2(1.43)^2}{32.2} = 152 \text{ ft} \qquad Ans.\ (c)$$

3DUWG

)LQDOO IURP (T VWH136+ DW%(3LVDSSUR[LPDWHO

$$\text{NPSH*} = \frac{C_{HS*}n^2D^2}{g} = \frac{0.37(20)^2(1.43)^2}{32.2} = 9.4 \text{ ft} \qquad Ans.\ (d)$$

6LQFHWKLVLVDVPDOOSXPS LWZLOOEHGHVHILFLHQWWKDQWKHSXPSVLQ) LJ SURGEO DERX SHUFHQWP D[LPXP

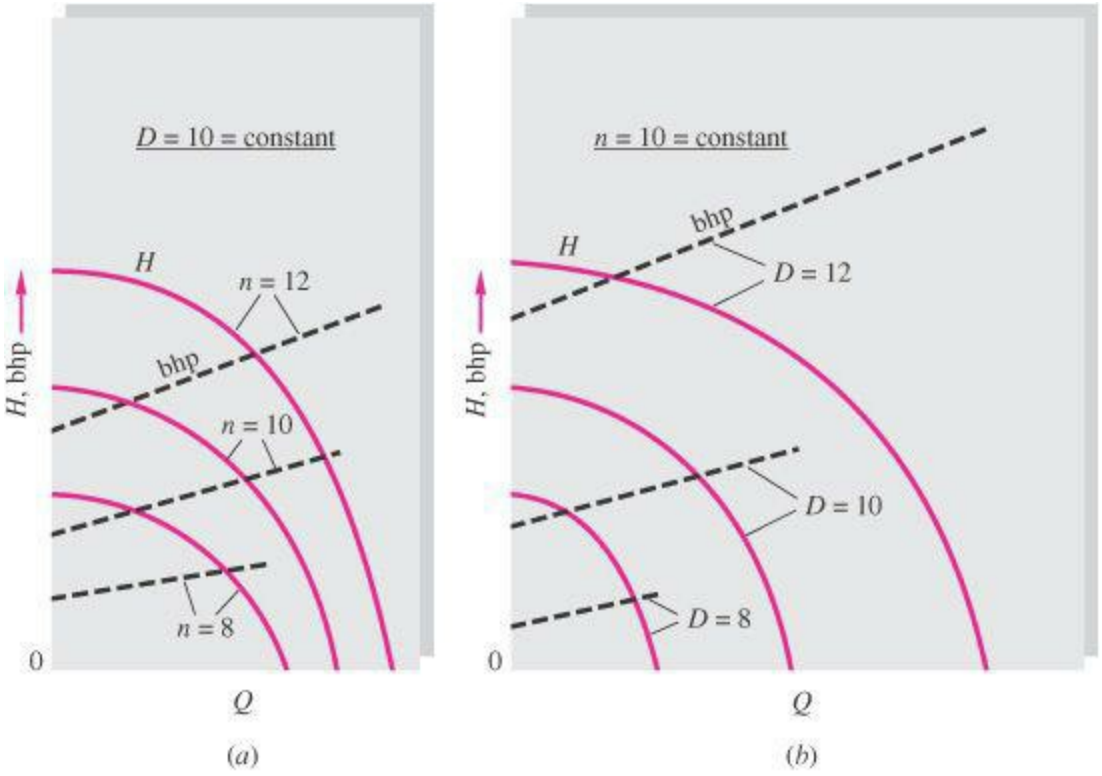
6LPLODULW5XOHV

7KH VXFFHVV RI) LJ LQ FRUUHODWLQJ SXPS GDWD OHDGV WR VLPSOH UXOHV IRU FRPSDULQJ SXPS SHUIRUPDQFH ,I SXPS DQG SXPS DUH IURP WKH VDPH JHRPHWULF IDPLO DQG DUH RSHUDWLQJ D KRPRORJRXV SRLQW WKH VDPH GLPHQVLRQOHVV FKDUWXFK DV) LJ WKHLIICRZ UDWH KHDG DQG SRZHU ZLOOEHUHDWHGDVIROORZV

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3 \qquad \frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{D_2}{D_1}\right)^2$$

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{n_2}{n_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 \qquad (11.28)$$

7KHVH DUH WKHVLPLODULW UXOHV ZKLFK FDQ EH XVHG WR HVWLP DWH WKH HIIHFWV RI FKDQJLQJ WKH IOXLG VSHHG RU VL]H RQ DQ G\QDPLF WKUERPDFKLQH SXPS RU WKUELQH ZLWKLQ D JHRPHWULFDOO VLPLODU IDPLO \$ JUDSKLF GLVSOD\ RI WKHVH UXOHV LV JLYHQ LQ) LJ VKRZLQJ WKH HIIHFWV RI VSHHG DQG GLDPHWUFKDQJHVRQSXPS SHUIRUPDQFH ,Q) LJ DWKHVL]HLVKHGFRQWDQWDQGWKHVSHHGLVYDULH SHUFHQWZKLOH) LJ E VKRZ VD SHUFHQWVL]HFKDQJHDWFRQWDQWVSHHG 7KHFXUYHVDUHSORWWH WR VFDOHEHWZLWKDUELWUDU XQLW 7KHVSHHGHIIHFW) LJ D LV VXEWDQWLDO EXWWKHVL]HHIIHFW) LJ E LV HYHQ PRUH GUDP DWF HVSHFLDOO IRU SRZHU ZKLFK YDULHV DV' * HQHUDO ZHVHHWKDW JLYHQSXPS IDPLO FDQEHDGMXVWHGLQVL]HDQGVSHHGWRILWDYDULHWRI VVWHP FKDUDFWHULVWLFV



) LJ (IIHFWRI FKDQJHV LQ VL]H DQG VSHHG RQ KRP RGRXV SXPS SHUIRUPDQFH *a* SHUIHQWFKDQJHLQVSHHGDWFRQVWDQWVL]H *b* SHUIHQWFKDQJHLQVL]HDWFRQVWDQWVSHHG

6WUDWW VSHDNLQJ ZH ZRXOG H[SHFWRU SHUFWLPLODULW WKDW EXWZH KDYH VHQWKL ODUJHUSXPSVDUHPRUHHILFLHQWKDYLQJ DKJKHU5H\QROGVQXPEHUDQGORZHUURXJKQHVDQGFHDUDQF UDWRV 7Z R HPSLULFDOFRUUHODWRQV DUH UHFRPPHQGHG IRUPD[LPXP HILFLHQF\ 2QH GHYHORSHG E\ 0 RRG\ > @IRUWXUELQHVEXWDOVR XVHG IRUSXPSV LVDVL]HHIHFW7KHRWKHU VXJJHVWHG E\ \$QGHUVR > @URP WKRXDQGVRI SXPS SWXGLVDIORZ UDWHIHFW

Size changes [43]:
$$\frac{1-\eta_2}{1-\eta_1} \approx \left(\frac{D_1}{D_2}\right)^{1/4} \qquad (11.29a)$$

Flow rate changes [44]:
$$\frac{0.94-\eta_2}{0.94-\eta_1} \approx \left(\frac{Q_1}{Q_2}\right)^{0.32} \qquad (11.29b)$$

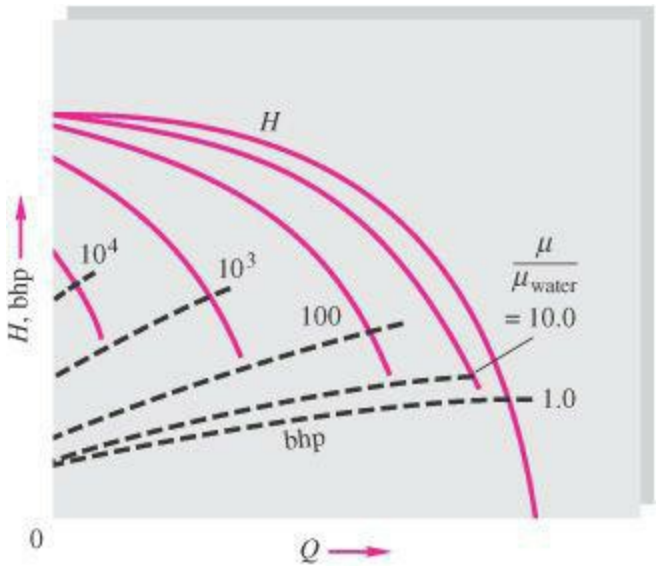
\$QGHUVRQ¶V IRUPXOD E PDNHVWKH SUDFWDOREVHUYDWRQWKDWHYHQDQLQLQWHUQ ODUJHSXP ZLOOKDYH ORWHV +HWKXV SURSRVHVD PD[LPXP SRVWLEOHHILFLHQF\ RI SHUFHQWUDWKHUWKDQ SHUFHQW\$QGHUVRQ UHFRPPHQGV WKDWWKH VDPH IRUPXOD EH XVHG IRUWXUELQHV LI WKH FRQWDQW L UHSODFHG E\ 7KHIRUPXODVLQ(T DWXPHWKH VDPH YDOXHRI VXUIDFHURXJKQHVIRUERW PDFKLQHV² RQHFRXOGPLFURSRODVKDVPDOOSXPSDQGDFKLHYHWKHHILFLHQF\ RI DODUJHUPDFKLQH

(IIHFWRI 9 LVFRVLW\

&HQWULXJDOSXPSV DUH RIWQ XVHG WR SXPS RLOV DQG RWKHU YLVFRXV OLTXLGV XS WR WPHV WK YLVFRVLW\ RI ZDWU %XWWKH 5H\QROGVQXPEHUV EHFRPH VXEVWDQWLDOO\ ORZHU WKDQ WKRVH HIHFWRQ SHUIRUPDQFH) LJXUH VKRZV WSLFDOWWWFXUYHV RI KHDGDQG EUDNHKRUVHSRZHUYHUX GLVFKDUJH + LJK YLVFRVLW FDXVHV D GUDPDWF GURS LQ KHDGDQG GLVFKDUJH DQG LQFUHDVHV LQ SRZH

UHTXLUHPHQWV 7KHHILFLHQF\ DQG RSHUDWLQJ UDQJH RI D FHQWULIXJDO SXPS GHSHQG RQ WKH GHVLJQ RI WKH LPSHOOHU DQG YROXWH DV ZHOO DV RQ WKH VSHHG DQG GLDPHWHU %HFDXVH ZDWHU KDV D NQRZQ YLVFRVLW\ WKH GHVLJQHU FDQ SUHGLFW WKH SHUIRUPDQFH RI D SXPS LQ YLVFRXV OLTXLGV

μ/μ_{water}	1.0	10.0	100	1000
η_{max}, %	85	76	52	11



)LJ (IIHFWV RI YLVFRVLW RQ FHQWULXJDO SXPS SHUIRUPDQFH

0 L[HG DQG $ [LDO) ORZ 3XPSV 7KH 6SHFLILF 6SHHG

: HKDYH VHHQ IURP WKH SUHYLRXV VHFWLRQ WKDW WKH PRGHUQ FHQWULXJDO SXPS LV DIRUPLGDEOH GHYLFH DEOH WR GHOLYHU KLJK KHDGV DQG UHDVRQDEOH IORZ UDWHV ZLWK [FHOOHQW HILFLHQF\ ,WV DQP DWK PDQ VXVWDP UHTXLUHPHQWV %XW HDFK FDO WKH FHQWULXJDO SXPS LV DKLJK KHDG ORZ IORZ PDFKLQH ZKHUHDV WKH UHDUHP DQ DSSOLFDWLRQV UHTXLUQJ ORZ KHDG DQG KLJK GLVFKDUJH 7R VHHWKDWWKH FHQWULXJDOGHVLJ LVQRWWRQHQHQWRUXFKVXVWDP V FRQVLGHUWKHIORZLQH[SOH

(; $0 3/ (

: HZ DQWR XVH D FHQWULXJDO SXPS IURP WKH IDPLO RI)LJ WR GHOLYHU JDOOLQR ZDWU DW *) ZLWK D KHDG RI IW: KDWWKRXOG EH D WKH SXPS VL]H DQG VSHHG DQG E EUDN KRUVHSRZ HU DWXP LQJ RSHUDWLQJ DQD WEHWWHILFLHQF\"

6RXWLRQ

3DUWD

(QWUWKH NQRZ QKHDG DQG GLVFKDUJH LQWR WKH %(3 SDUDPHWHUV IURP (T

$$H^* = 25 \text{ ft} = \frac{C_{H^*} n^2 D^2}{g} = \frac{5.0 n^2 D^2}{32.2}$$

$$Q^* = 100{,}000 \text{ gal/min} = 222.8 \text{ ft}^3/\text{s} = C_{Q^*} n D^3 = 0.115 n D^3$$

7KHVZRXQNQRZQVDUHQDQG' 6RQYHVLPXOWDQHRXVOLIRU

$$D = 12.4 \text{ ft} \qquad n = 1.03 \text{ r/s} = 62 \text{ r/min} \qquad \textit{Ans. (a)}$$

,I\RXZLVKWRDYRLGDQJHEUDFPDQSXOOLQJVLPSOLSURJUDPWKHVZRVLPXOWDQHRXVHTXDWLRQVIURP 3DUWD LQ((6 XVLQJ(QJOLVKXQLWV

$$Q = A$$

$$Q' = A$$

6SHFLI\LQ9DULDEOH,QIRUPDWLRQWKDWQDQG' DUHSRVLWLYH DQG((6 SURPSWV UHWXUQVWKHFRUUHVRXWLQ' IWDQGQ UV

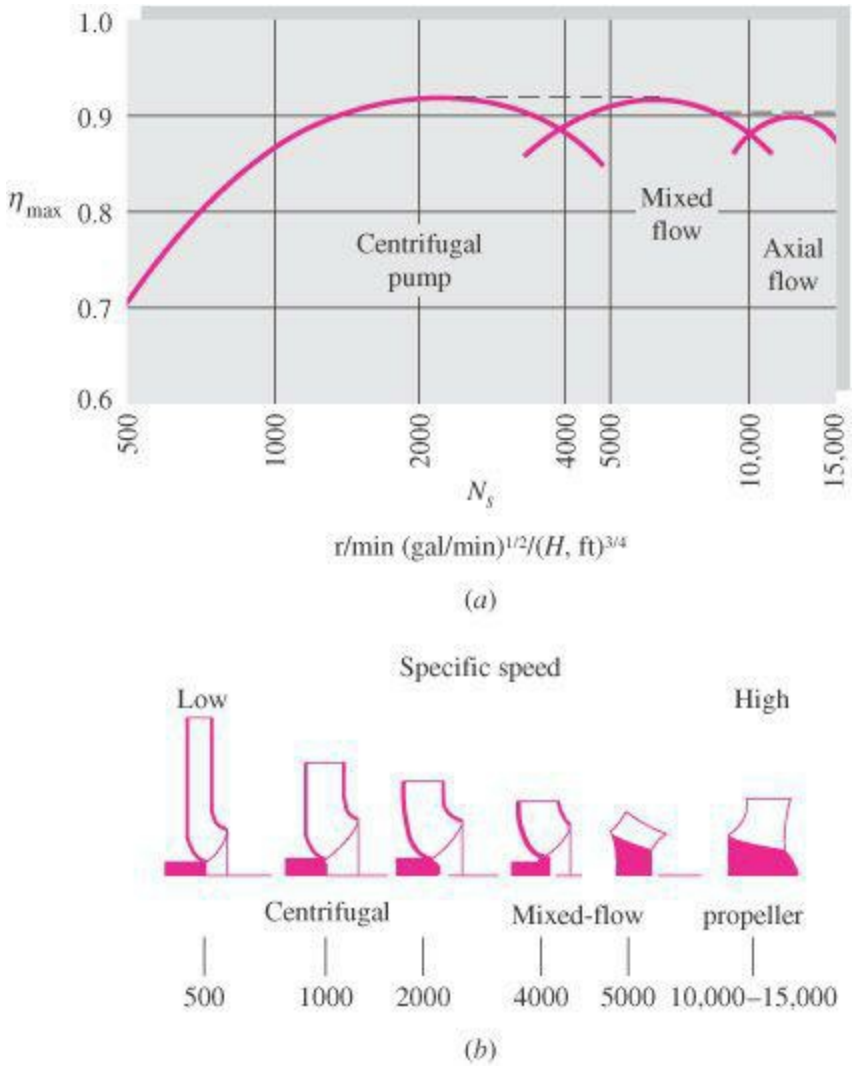
3DUWE

7KHPRWHILFLHQWKRUVHSRZHUVWKHQIURP (T

$$\text{bhp}^* \approx C_{P^*} \rho n^3 D^5 = \frac{0.65(1.94)(1.03)^3(12.4)^5}{550} = 720 \text{ hp} \qquad \textit{Ans. (b)}$$

7KH VROXWLRQ WR([DPSOH LV PDWKHPDWFDOO FRUUHFWEXWUHVXOWLQ D JURWWTXH SXPS DW LPSHOHUPRUHVHDQ IWLQGLDPHWU URWDWQJ VR VORZ0 RQHFDQ YLVXDOL]HR[HQZDOLQJ LQD FLUFG VXUQJWKHVKDW

2WKHUG\QDPLF SXPS GHVLJQVSURYLGHORZ KHDGDQGKLJK GLVFKDUJH)RU[DPSOH WKHUHLVDWS RI LQ UPLQSXPS ZLWKWKHVDPHLQSXWSDUDPHWUVV) LJ E ZKLFK ZLOOGHOYHUWKH I KHDGDQG JDOPLQIORZ UDWHFDOHGIRULQ([DPSOH 7KLVLVGRQHE\ DORZLQJ WKHIORZ V SDVVWKURXJKWKHLPSHOHUZLWKDQD[LDOIORZ FRPSRQHQWVFHQWULXJDOFRPSRQHQW7KHSDVDJH FDQEHRSHQHGXS WRWKHLQFUHDVHGIORZ UDWZLWKYHU OLWWOHVL]HLQFUHDVH EXWKHGURSLQUDGLDORXWO YHORFLW GHFUHDVHVWKHKHDG SURGXFHG 7KHHDUWKPL[HGIORZ SDWWDGLDO SDWWD[LDO DQG D[LDO IORZ SURPHOHUWSH IDPLOLHV RI GQDPLF SXPS 6RPH YDQHGHVLJQV DUH VHWWKHG LQ) LJ ZKLFK LQWRGXFHVDQLQWHUHVWLQJ QHZ ³GHVLJQ SDUDPHWUWKHVSHFLILF VSHHG1 V RU N_s'



) LJ *a* 2 SMP XP HI LFL HQF\ DQG *b* YDQH GHMJQ RI G\QDPLF SXPS IDPLOLHV DV I IXQFWLRQ RI VSHFLILF VSHHG

7KH6SHFLILF6SHHG

0 RVWSXPS DSSOLFDWLRQV LQYROYH DNQRZ QKHDG DQG GLVFKDUJH IRUWKH SDUWFXODUV\VWHP SOXV I VSHHG UDQJH GLFWDWHG E\ HDFWLF PRWRUVSHHGV RUFDYLWDWLQ UHTXLUHPHQWV 7KHGHMJQHUWKHQVHOHFW WKHEHWWL]HDQG VKDSH FHQWUIXJDO PL[HG DIORZ IRUWKH SXPS 7RKHOS WKLV VHOHFWLRQ ZHQHG GLPHQVLRQOHVV SDUDPHWUILQJ VSHHG GLVFKDUJH DQG KHDGEXWQRWVL]H 7KLVLVDFFRPSOLVKHGE HDPLQDWQJ WKHGLPHWUEHWZHHQ&4 DQG&+ DSSOLQJ WKHUHXOWRQD WR WKH%(3 7KLVUDWR LVFDOOH WKHVSHFLILF VSHHGDQG KDVERWKDGLPHQVLRQOHVVIRUP DQGDVRPHZKDWD\ SUDFWLFDOIRUP

Rigorous form:

$$N_s' = \frac{C_{Q^*}^{1/2}}{C_{H^*}^{3/4}} = \frac{n(Q^*)^{1/2}}{(gH^*)^{3/4}} \qquad (11.30a)$$

Lazy but common:

$$N_s = \frac{(\text{r/min})(\text{gal/min})^{1/2}}{[H\,(\text{ft})]^{3/4}} \qquad (11.30b)$$

,QRWKHUZRUGV SUDFWLFDOLQHQJLQHHUVGRQRWERWKHUWRFKDQJHQWRUHYROXWLRQVSHUVHFRQG4 W
FXELFIHHWSHUVHFRQGRUWRLQFUHDVHJUDYLWZLWKKHDG DQKRXJKWKHODWWHUZRXOGEHQHFHVVDU IRU VD
DSXPSRQWKHPRRQ 7KHFRQYHUVLRQIDFWRUV

$$N_s = 17{,}182N'_s$$

1RWHWKDW1V LVDSSOLHGRQO WR%(3 WKXVDVLQJOHQXPEHUFKDUDFWHUL]HVDQHQWLUHIDPLO RI SXPS\
) RUH[DPSOH WKHIDPLO RI) LJ KDV1V• § 1V UHJDUGOHVV R
VL]HRUSHHG

,WWKLQVRXWKDWWKHVSHFLILF VSHHGLVGLUHFWO UHODWHGWRWKHPRWWHILFLHQWSXPSGHVLJQ DVVKRZ
LQ) LJ / RZ 1V PHDQVORZ 4 DQG KLJK+ KHQFHDFHQWULXJDOSXPS DQGDUJH1V LPSOLHVDI
DILDOSXPS 7KHFHQWULXJDOSXPS LVEHWWHUR1V EHWWHQ DQG WKHPL[HGIORZ SXPS IRU1V
EHWWHQ DQG DQG WKHDILDOIORZ SXPS IRU1V DERYH 1RWWKHFKDQJHV LQ
LPSHOOHUVKDSHDV1V LQFUHDVHV

6XFWLRQ6SHFLILF6SHHG

,I ZHXVH136+ UDWKHUWKDQ+ LQ(T WKHUHVXOWLVFDOOHGVXFWLRQVSHFLILF VSHHG

Rigorous:

$$N'_{ss} = \frac{nQ^{1/2}}{(g\ \text{NPSH})^{3/4}} \qquad (11.31a)$$

Lazy:

$$N_{ss} = \frac{(\text{r/min})(\text{gal/min})^{1/2}}{[\text{NPSH (ft)}]^{3/4}} \qquad (11.31b)$$

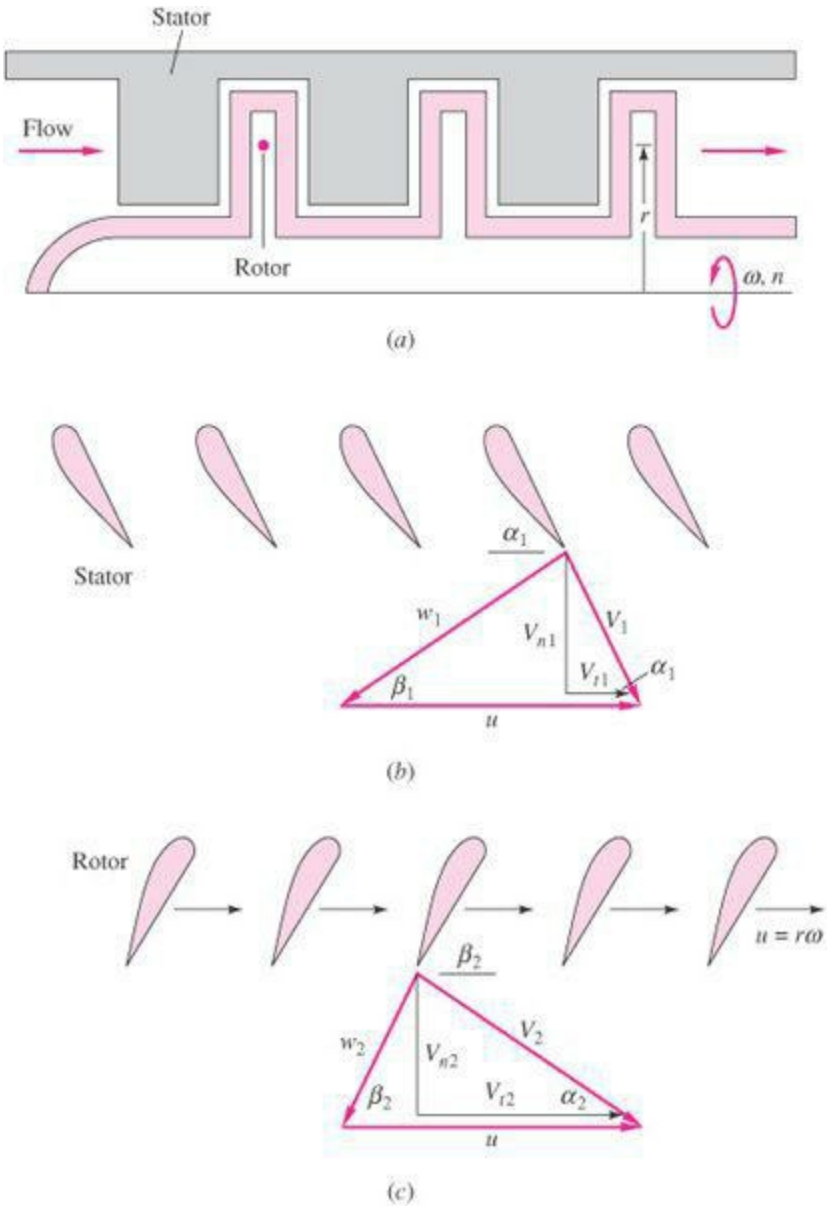
ZKHUH136+ GHQRWHVWKHDYDLODEOHVXFWLRQKHDGRI WKHVXVWHP ' DDVIURP : LVOEHQXV> @WKRV
WKDWDJLYHQSXPSLVLQGDQJHURI LQGHWFDYLWDWLQJ

$$N'_{ss} \geq 0.47 \qquad N_{ss} \geq 8100$$

,QWKHDEVHQFHRI WHVWGDWD WKLV UHDVRQFDQEHXVHG JLYHQQ DQG4 WR HVWLPDWHWKHPLQLPXP
UHTXLUHG136+

$[LDO)ORZ3XPS7KHRU

$ PXOWLVWDJHD[LDOIORZ JHRPHWU LVVKRZQLQ) LJ D 7KHIOXLGHVWHQWHUV DOPRV
D[LDOO WKURXJK DQQXODUDUURZV RI IL[HGVWDWRUEODGHV DQG PRYLQJ URWRUEODGHV 7KHLQFRP SUHVVLEO
IORZ DVXPSWLRQ LV IUHTXHQWO XVHG HYHQ IRU JDVHV EHFDXVH WKH SUHVXUH ULVH SHU VWDJH LV XVXDOO
VPDOO



) LJ $ QDO VLV RI DQ D[LDO IORZ SXPS *a* EODGH JHRPHWUL *b* VWDWRU EODGHV DQG H[LW YHORFLW GLDJUDP *c* URWRU EODGHV DQG H[LW YHORFLW GLDJUDP

7KH VLPSOLILHG YHFWRU GLDJUDP DQDO VLV DVVXPHV WKDW WKH IORZ LV RQH GLPHQVLRQDO DQG HDFK EODGH URZ DWD UHGXFHG UDGLXV HYDOXDWHV SDUDPHWHUV DFURVV WKH H[LW EODGH DQG) LJXUH E VKRZV VWDWRU EODGHV DQG WKHLU H[LW YHORFLW GLDJUDP 6LQFH WKH VWDWRU LV ¿[HG LGHDO WKH DEVROXWH YHORFLW 9 LV SDUDOOHO WR WKH WDQJHQW RI WKH EODGH $IWHU HYHQWXDO VXEVDFLQJ WKH URWRU DQG HQWHULQJ WKH YHORFLW 9 ZHREWDLQ WKH YHORFLW ZUHDWLYH WR WKH URWRU ZKLFK LGHDO VKRXOG EH SDUDOOHO WR WKH URWRU EODGLQJ HGJH

) LJXUH F VKRZV VWKH URWRUBEODGHV DQG WKHLU H[LW YHORFLW GLDJUDP +HUH WKH UHODWLYH YHORFLW Z LV SDUDOOHO WR WKH EODGH WDLQJ HGJH ZKLOH WKH DEVROXWH YHORFLW 9 VKRXOG EH GHVLJQHG V VP RRWKO HQWHUWKH QH[W URZ RI VWDWRU EODGHV

7KH WKHRUHWLFDO SRZHU DQG KHDG DUH JLYHQ E\ (XOHUV WXUELQH HTXDWLRQ 6LQFH WKH UHVLGXDO KHDG LQ WKH H[LW YHORFLW LV QHJOLJLEOH DQG RQH GLPHQVLRQDO FRQGLWLRQ LV DVVXPHG WKH H[LW YHORFLW FRPSRQHQW PLQFRQWDLQV

$$V_{n1} = V_{n2} = V_n = \frac{Q}{A} = \text{const}$$

) URP VKH JHRPHWU RI VKH YHORFLWG LDJUDPV VKH QRUPDO YHORFLW RUYRXPHIORZ FDQEH GLUHFW UHDWG VR VKE EODGH URDWRQDO VSHHGX

$$u = \omega r_{av} = V_{n1}(\cot \alpha_1 + \cot \beta_1) = V_{n2}(\cot \alpha_2 + \cot \beta_2) \qquad (11.32)$$

7KXV VKH IORZ UDWH FDQ EH SUHGLFWG IURP VKH URDWRQDO VSHHG DQG VKE EODGH DQJOHV 0 HDQZKLG VLQFH 9$W 9Q FRV DQG 9W Xí 9Q FRV (XOHUÀV UHDWRQ IRUVKHSXPS KDG EHFRPHV

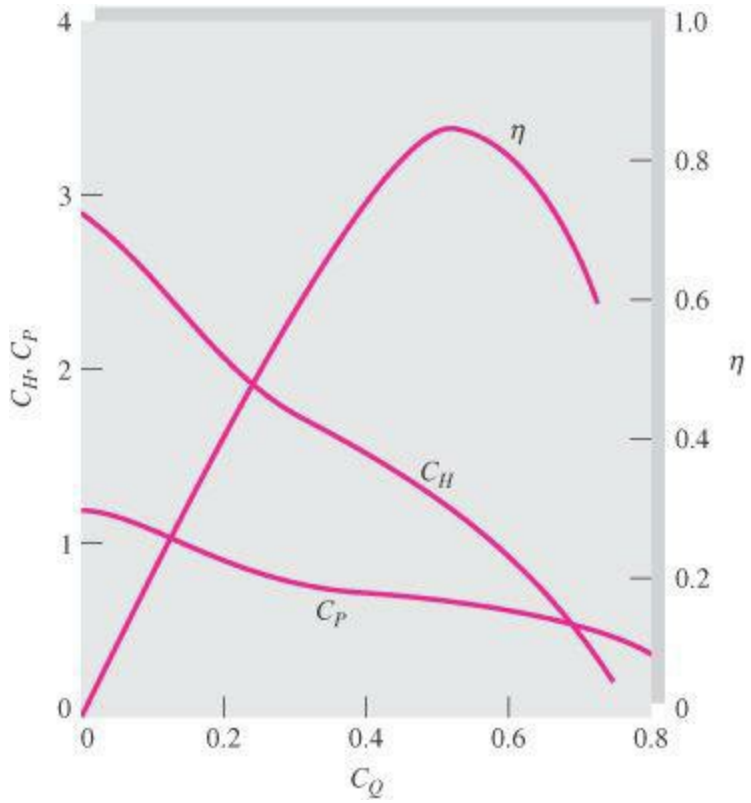
$$\begin{aligned} gH &= uV_n(\cot \alpha_2 - \cot \alpha_1) \\ &= u^2 - uV_n(\cot \alpha_1 + \cot \beta_2) \end{aligned} \qquad (11.33)$$

VKH SUHUHGIRUP EHFDXVHLVWHDWV VR VKE EODGH DQJOHV. DQG 7KHVKXWI RUQR IORZ KHDGL VHHQVR EH+ X J MXVWDVLQ(T IRUD FHQWULXJDOSXPS 7KH EODGH DQJOHSDUDPHWUFRV FRV FDQEHGHVLJQHG VR EHQHJDWLYH]HUR RUSRVLWLYH FRUUHVSRQGLQJ VR DULVLQ IDWRUIDQQL KHDGFXUYH DVLQ) LJ

6WUFWV VSHDNLQJ (T DSSOLHVRQO VR D VLQJOH VWHDPVWEHR UDGLXVU EXWLWLV DJRRG DSSUR[LPDWRQIRUYHU VKRUWEODGHVLI UGHQRWVVKHDYHUDJHUDGLXV)RUORQJ EODGHVLWLV FXVWRP DU VXP (T LQ UDGLDOVWUSVRYHUVKE EODGH DUHD 6XFK FRPSOH[LWV PD\ QRWEH Z DUUDQWG VLQF VKHRU EHLQJ LGHDOL]HG QHJOHFWGRWHVDQG XVXDOO SUHGLFWVVKH KHDGDQG SRZ HUDUJHUWKDQWRVH LQ DFWXDOSXPS SHURUP DQFH

3HUIRUP DQFHRI DQ$[LDO) ORZ 3XPS

$ WKLJK VSHFLILF VSHHGV VKH PRWH ILFLHQWFKRLFH LV DQ D[LDOIORZ RUSURSHOOHU SXPS Z KLFH GHYHORSVKLJK IORZ UDWH DQG ORZ KHDG $ WVSLFDOGLPHQVLRQOHVFKDUWRUD SURSHOOHUSXPS LV VKRZ LQ) LJ 1RWH DV H[SHFWG VKH KLJKHU&4 DQG ORZ HU&+ FRPSDUHG Z LWK) LJ 7KH KHDG FXUYH GURSV VKDUSO Z LWK GLVFKDUJH VR VKDWD ODUJH V\VWP KHDG FKDQJH Z LOO FDXVH D PLQG ORZ FKDQJH 7KH SRZ HUFXUYH GURSV Z LWK KHDG DOVR Z KLFK PHDQV D SRWLEOHRYHUORDGLQJ FRQGLWRQLI VK V\VWP GLVFKDUJH VKRXOG VXGGHQO GHFUHDVH) LQDOO VKH HILFLHQF\ FXUYH LV UDWKHU QDUURZ DQ VWDQXODU DSSSRVHG WR VKE EURDG SDUDERLF VKDSHG FHQWULXJDOSXPS HILFLHQF\ ) LJ

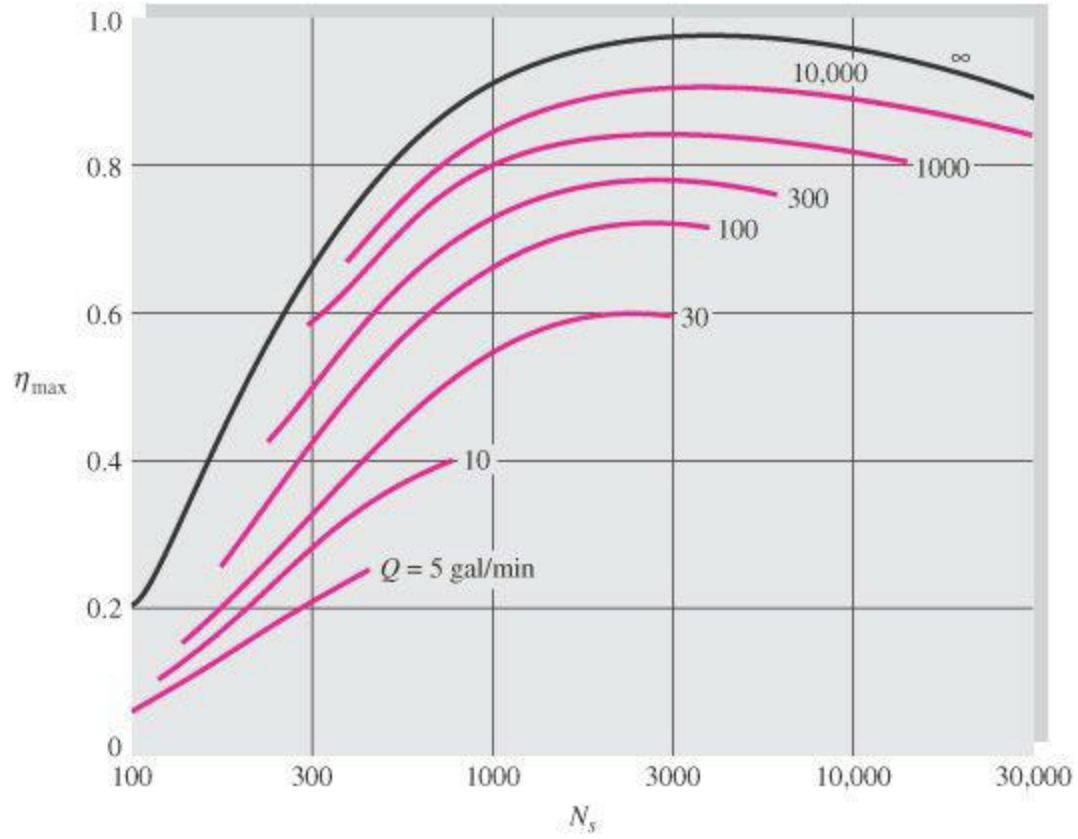


) LJ ' LPHQVLRQOHVV SHUIRUPDQFH FXUYHV IRU D WZLFDOD[LDOI ORZ SXPS N_s
&RQWLQXHFWMG IURP GDWD JLYHQ E\ 6WHSDQRII > @RUD LQSXPS DW UPLQ

%\ LQVSHFWLRQ RI) LJ &4 § &+ § &3 § DQG PD[§) URP WKLV Z
FRPSXWH1V • § 1V 7KHUHDIWHU GRZ HIILFLHQF\ LV GHWHUPLQ
SXPS VL]H G LQ Q UPLQ 4 JDOPLQ
$ UHSHWLWLRQ RI ([DPSOH XVLQJ) LJ ZRXOG VKRZ WKDWWKLV SURSHOOHUSXPS IDPLO FD
SURYLGHD IWKHGDQG JDOPLQGLVFKDUJHLL ' LQDQGQ UPLQ ZLWKEKS
WKLV LV D PXFK PRUH UHDVRQDEOH GHVLJQ VROXWLRQ ZLWK LPSURYHPHQW VWLOO SRVVLEOH DWODUJHU1
FRQGLWLRQV

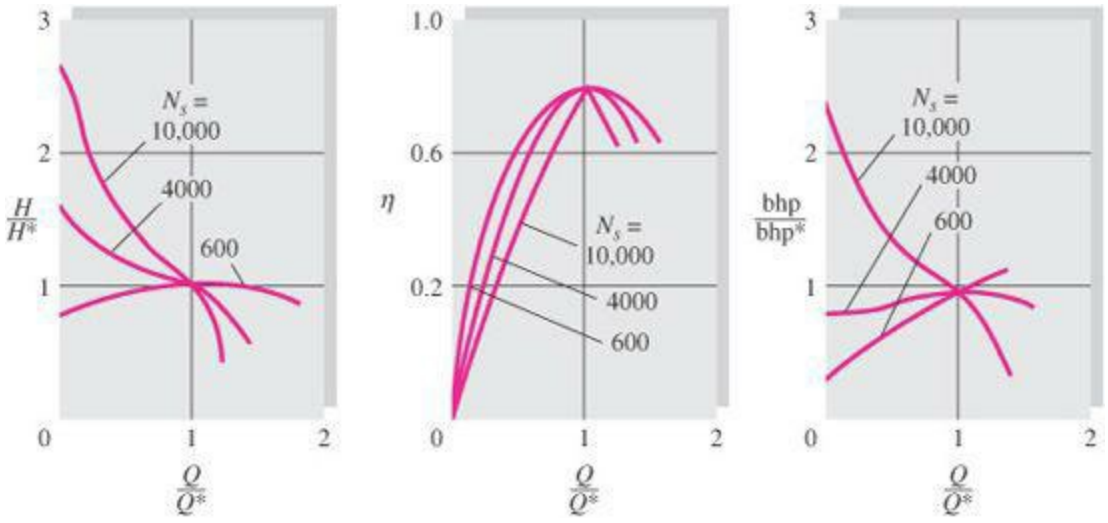
3XP S 3HUIRUP DQFH YHUVXV 6SHFLILF 6SHHG

6SHFLILF VSHHG LV VXFK DQ HIHFWLYH SDUDPHWHU WKDWWLV XVHG DV DQ LQGLFDWRU RI EHWK SHUIRUP DQF
DQG HILFLHQF\ ) LJXUH VKRZ VDFRUUHODWLRQ RI WKHRSWLPXP HILFLHQF\ RI DSXPS DVDIXQFWLRQ R
WKHVSHFLILF VSHHG DQG FDSDFLW\ %HFDXVHWKHGLPHQVLRQOHVVSDUDPHWU4 LV DURXJK PHDVXUH RI EWK VL]I
DQG 5H\QROGV QXPEHU LQFUHDVHV ZLWK4 : KHQ WKLV WSH RI FRUUHODWLRQ ZDV ILUWVXEOLVKHG E
: LVOLFHQXV> @LQ LWEHFDPHNQRZ QDVWKHSXPS FXUYH DFKDQHQJHWRDOOP DQXIDFWXUHU : HFD
FKHFNWKDWWKHSXPSVRI) LJV DQG ILWWKHFRUUHODWLRQYHU ZHOO



) LJ 2 SMP XP HILFLHQF\ RI SXP SV YHUVXV FDSDFLW DQG VSHFLLF VSHHG*(Adapted from Refs. 4 and 31.)*

) LJXUH VKRZV VWKH HIHFWRI VSHFLLF VSHHG RQ WKH VKDSH RI WKH SXP S SHUFRUP DQFH FXUYH QRUP DOL]HG Z LWK UHVSHFWWR WKH %(3 SRLQW 7KH QXP HULFDO YDOXHV VKRZ Q DUH UHSUHVHQWDWLYH EX VRP HZ KDWXDOXDWLYH 7KHKLJK VSHFLLF VSHHGSXP SV 1V§ KDYHKHDG DQG SRZ HUFXUYHVWKD GURS VKDUSO Z LWK GLVFKDUJH LP SOLQJ RYHUDOD RUWDUWKS SUREOPV DWORZ IORZ 7KHUHIILFLHQF FXUYHLVYHU QDUURZ



) LJ (IIHFWRI VSHFLLF VSHHGRQSXP S SHUFRUP DQFHFXUYHV

\$ ORZ VSHFLLF VSHHGSXP S 1V KDVDEURDGHILFLHQF\ FXUYH DULQJ SRZHUFXUYH DQ DKHDGFXUYHWKDW?GURSV' DWKXWRI LP SOLQJ SRVLEOHVXJHRUKXQWQJ SUREOPV

7KH) UHH3 URSHOOHU

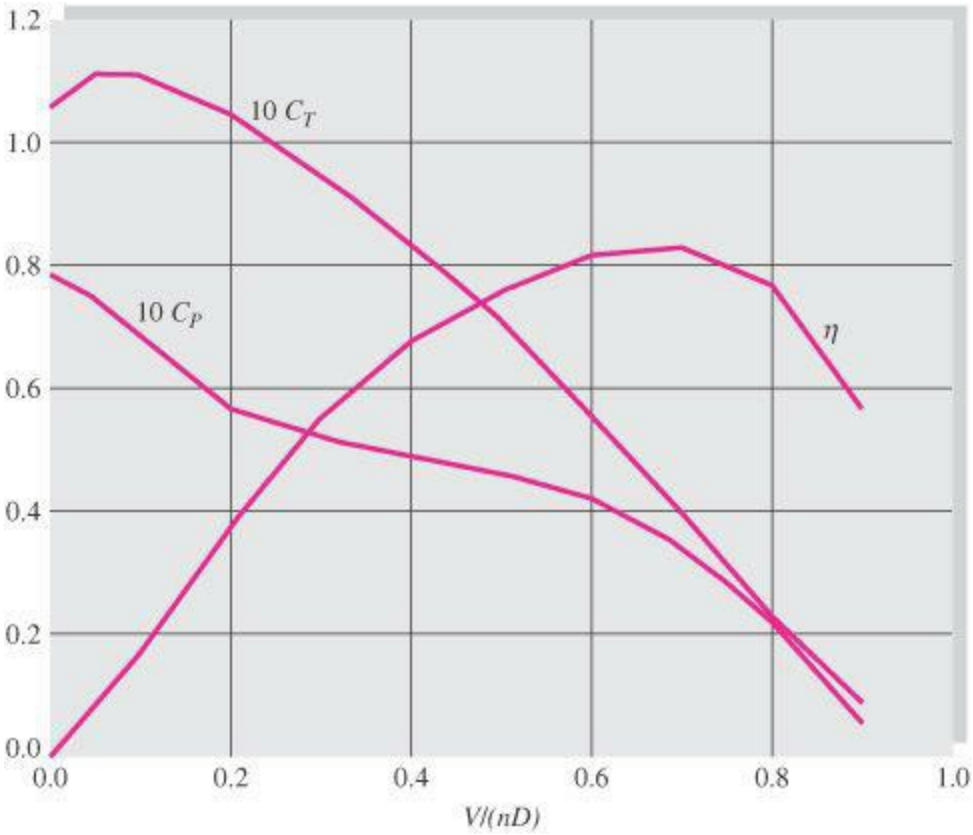
7KH SURSHOOHU VWXGLHG LQ) LJ LV HQFORVHG LQ D GXFWHG FDSXUHV DOWKH DSSURDFK IORZ ,Q FRQWUDVW WKH IUHH SURSHOOHU IRUHVLWKHUL FUDIW RUPDULQH DSSOLFDWLRQV DFWLQ DQ XQERXQGHG IOXLG DQG WKXV LV PXFK OHVV HIFLHQW 7KH DQDORJ RI SURSHOOHU SXPS SUHVXUH ULVH LV WKH IUHH SURSHOOHU VKURXGSHUXQWDUHD CE VZHSWRXWE\ WKHEODGHV ,QDFXUDWH GLPHQVLRQDODQDOVLV WKURXW DQG SRZHU UHTXLUHG3 DUHIXQFWLRQV RI IOXLG GHQVLW! URWDWLRQUDWH Q UHY V IRUZDUG YHORFLW9 DQG SURSHOOHUGLDPHWUG 9LVFRVLW HIFLHQF\ DUH VPDOODQGQHJOHFWG <RXPLJKWHQR DQDO]LQJ WKLV DV D &KDS DVLJQPHQW7KH 1\$&\$ QRZ WKH 1\$6\$ FKRVH ! Q G DV UHSHDWQJ YDULDEOHV DQG WK UHVXOWDUHWKHDFFHSWG SDUDPHWUV

$$C_T = \text{thrust coefficient} = \frac{T}{\rho n^2 D^4} = \text{fcn}(J),\ J = \text{advance ratio} = \frac{V}{nD}$$

$$C_P = \text{power coefficient} = \frac{P}{\rho n^3 D^5} = \text{fcn}(J),\ \eta = \text{efficiency} = \frac{VT}{P} = \frac{JC_T}{C_P} \quad (11.34)$$

7KH DGYDQFH UDWLR - ZKLFK FRPSDUHV IRUZDUG YHORFLW VR D PHDVXUH SURSRUWLRQDO WR EODGH WS VSHHG KDVDVWURQJ HIHFWXSRQWKXWDQG SRZHU

) LJXUH VKRZV SHUIRUPDQFHGDWDIRUDSURSHOOHUXVHGRQWKH&HVQD DLUFUDW7KHWKUX DQG SRZHUFRHILFLHQWV DUH VPDOO RI 2 DQG DUH PXOWSOLHG E\ IRUSORWWQJ FRQYHQLHQF 0 D[LPXP HILFLHQF\ LV SHUFHQWDW ZKHUH&7 § DQG&3 §



) LJ 3HUIRUPDQFHGDWDIRUDIUHHSURSHOOHUXVHGRQWKH&HVQD DLUFUDW&RPSDU WR) LJ IRUD GXFWHG SURSHOOHUSXPS 7KHWKUXWDQGSRZHUFRHILFLHQWVDUHPXFK VPDOOHU IRUWKHIUHHSURSHOOHU

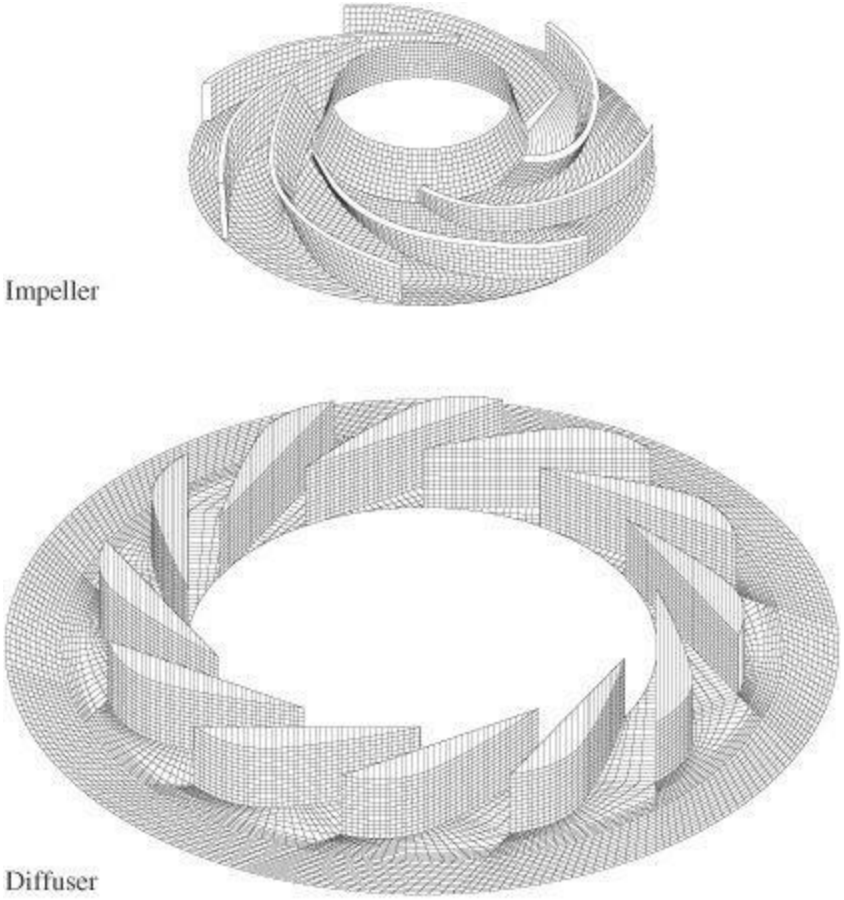
7KHUHDUHVHYHUDOHQJLQHHULQJ PHWKRGVIRUGHVLJQLQJ SURSHOOHUV 7KHVHWKRUHVDUHGHVFULEHGL
VSHFLDOL]HGWR[WV ERWKIRUP DULQH> @DQGDLUFUDIW> @SURSHOOHUV

&RP SXWDWLRQDO) OXLG ' \QDP LFV

7KHGHVLJQRI VXUERP DFKLQHU KDVWDGLWLRQDOO EHHQKJKO H[SHULP HQWDO ZLWKVLP SOHWKRUHV
VXFK DVLQ6HF RQO DEOHWR SUHGLFWWHQGV ' LPHQVLRQOHVFRUHDWLRQV VXFK DV) LJ DU
XVHXOEXWUHLH[WHQVLYH H[SHULP HQWDWLRQ &RQVLGHUWKDWGR LQ D SXP S LV WKUHHGLP HQVLRQDO
XQWDG ERWKSHULRGLF DQG WXUEXOHQW DQG LQYROYHV IORZ VHSDUDWLRQ UHFLUFXODWLRQ LQWKHLP SHOOH
XQWDG EODGH ZDNHV SDWLQJ WKURXJK WKH GLIIXVHU DQG EODGH URRWV WSV DQG FHDUDFHV ,WLV QI
ZRQGHUWKDWRQH GLP HQVLRQDOWKRU FDQQRWJLYH HILLP TXDQWLWDWLYH SUHGLFWLRQV

0 RGHUQ FRP SXWHU DQDOVLV FDQ JLYH UHDOLVWF UHVXOWV DQG LV EHFRP LQJ D XVHXO WRRO IR
WXUERP DFKLQHU GHVLJQHUV $ JRRG H[DP SOH LV 5H UHSRUWQJ FRP ELQHG H[SHULP HQWDO DQ
FRP SXWDWLRQDO UHVXOW IRU D FHQWULXJDO SXP S GLIIXVHU $ SKRWJUDSK RI WKH GHYLFH LV VKRZ QLQ) LJ
D ,WLV P DGH RI FHDU 3HUSH[VR WKDWODVHU P HDVXUHP HQW RI SDUWFOH WDFNLQJ YHORFLP HW
/ 379 DQG GRSSOHU DQHP RP HWU / ' $ FRXOG EH VDNHQ WKURXJKRXWWKH VWXPP 7KH GDWZH
FRP SDUHG ZLWK D &) ' VLP XODWLRQ RI WKHLP SHOHUDQG GLIIXVHU XVLQJ WKH JULGV VKRZ QLQ) LJ E
7KH FRP SXWDWLRQV XVHG D WXUEXOHQFH IRUP XODWLRQ FDOOHG WKHNë PRGHO SRSXODU LQ FRP P HUFLDO &) '
FRGHV VHH6HF 5HVXOWZ HUH JRRG EXWQRW H[FHOOHQW7KH &) ' PRGHOSUHGLFWHG YHORFLW DQ
SUHVVXUHGDWDDGHTXDWHO XS XQWO IORZ VHSDUDWLRQ DWWUZKFK LWZ DV RQO TXDOLWDWLYH &OHDU &) '
LVGHYHORSLQJ DVJQLFDQWWRROLQWXERP DFKLQHU GHVLJQ> @

(*a*)



(b)

) LJ 7XUERPDFKLQHU\ GHVLJQ QRZ LQYROYHV ERWK H[SHULPHQWDWLRQ DQG FRPSXWDWLRQ IOXLG G\QDPLFV &)' *a* DFHQWULXJDOLPSHOOHUDQGGLIIXVHU*(courtesy of K. Eisele et al., "Flow Analysis in a Pump Diffuser: Part 1, Measurements: Part 2, CFD," Journal of Fluids Eng. Vo 119, December 1997, pp. 968–984/American Society of Mechanical Engineers)* b DVWUHH GLPHQVLRQDO&)' PRGHOJULG IRU WKLV VWXG*(from Ref. 56 by permission of the American Society of Mechanical Engineers)*

0 DWFKLQJ 3XPS SVWR6\ VWHP &KDUDFWHULVWLFV

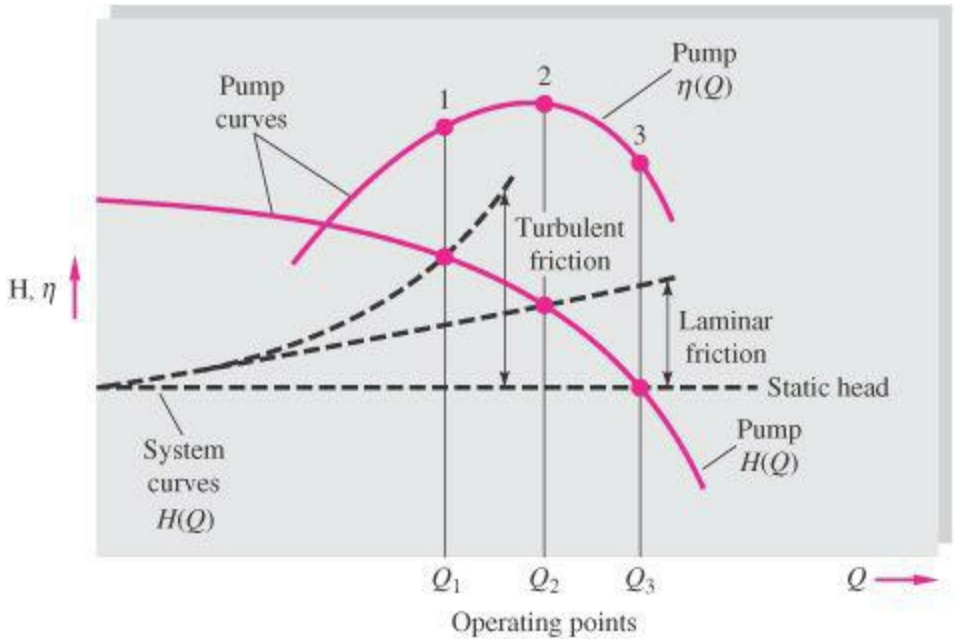
7KH XQLP DW WKH RSHUDWLQJ V\VWHP FKDUDFWHULVWLFV 3K\VLFDOO WK V\VWHP KHDG PXVW PDWFK WKH KHDG SURGXFHG E\ WKH SXPS DQG WKLV LQWHUVHFWLRQ VKRXOG RFFXU LQ WK UHJLRQ RI EHVW HILFLHQF\

7KH V\VWHP KHDG ZLOO SURFEDE0 FRQVLQ DVWDWF HOHYDWLRQ FKDQJH] í] SOXV IULFWLRQ ORVVHV L SLSHV DQG ILWWLQJV

$$H_{sys} = (z_2 - z_1) + \frac{V^2}{2g}\left(\sum \frac{fL}{D} + \sum K\right)$$

ZKHUH . GHQRWHV PLQRU ORVVHV DQG9 LV WKH IORZ YHORFLW LQ WKH SULQFLSDO SLSH 6LQFH9 LV SURSRUWLRQDO WR WKH SXPS GLVFKDUJH4 WKH HTXDWLRQ UHSUHVHQWV D V\VWHP KHDG FXUYH+V 4 7KH

H[DPSOHVDUHVKRZQLQ) LJ DVWDWF KHDG+V D VWDWF KHDGSXVODPLQDUIULFWLRQ+V D E4
DQG VWDWF KHDG SOXV WXUEXOHQWIULFWLRQ+V D F4 7KH LQWHUVHFWLRQ RI WKH VXVWHP FXUYH ZLWK WK
SXPS SHUFRUPDQFHFXUYH+ 4 GHILQHVWKHRSHUDWLQJSRLQW,Q) LJ WKHODPLQDUIULFWLRQRSHUDWLQ
SRLQWLV DWP D[LPXP HILFLHQF\ ZKLOH WKH WXUEXOHQWDQG VWDWF FXUYHV DUH RI GHVLJQ 7KLV PD\ E
XQDYRLGDEOH LI VXVWHP YDULDEOHV FKDQJH EXWWKH SXPS VKRXOG EH FKDQJHG LQ VL]H RU VSHHG LI LV
RSHUDWLQJ SRLQWLV FRQVLGHUHG RI GHVLJQ 2I FRXUVH D SHUFHWP DWK PD\ QRWEH SRVVLEOH EHFDX
FRP P HUFLDOSXPSSVKDYHRQO FHUWDLQGLVFUHWVL]HV DQG VSHHGV / HWXVLOOXVWDWWKHVHFRQFHSWZLW
DQH[DPSOH



) LJ ,OOXVWDWRQRI SXPS RSHUDWLQJ SRLQWVIRUWKUHHWSHVRI VXVWHP KHDGFXUYHV

(; $0 3/ (

: HZDQWRXVHWKH LQSXPS RI) LJ DDW UPLQWRSXPS ZDWUDW *f* IURP RQ
UHVHUYRLUWRDQRWKHU IWKLJKHUWKURXJK IWRI LQ,' SLSHZLWKIULFWLRQIDFWRU D
: KDWZLOOWKHRSHUDWLQJ SRLQWDQGHILFLHQF\ EH' E 7R ZKDWSHGVKRXOGWKHSXPS EHFKDQJHGW
RSHUDWDWWKH%(3"

6RROXWLRQ

3DUWD

) RUUHVHUYRLUVWKHLQLWLDODQGILQDOYHORFLWLHVDUH]HUR WKXVWKHV\VWHP KHDGLV

$$H_s = z_2 - z_1 + \frac{V^2}{2g}\frac{fL}{D} = 120\ \text{ft} + \frac{V^2}{2g}\frac{0.030(1500\ \text{ft})}{\frac{16}{12}\ \text{ft}}$$

) URP FRQWLQXLWLQWKHSLSH $V = Q/A = Q/[\frac{1}{4}\pi(\frac{16}{12}\text{ ft})^2]$ DQGVR ZHVXEVWLWXWHIRU9 WRJHW

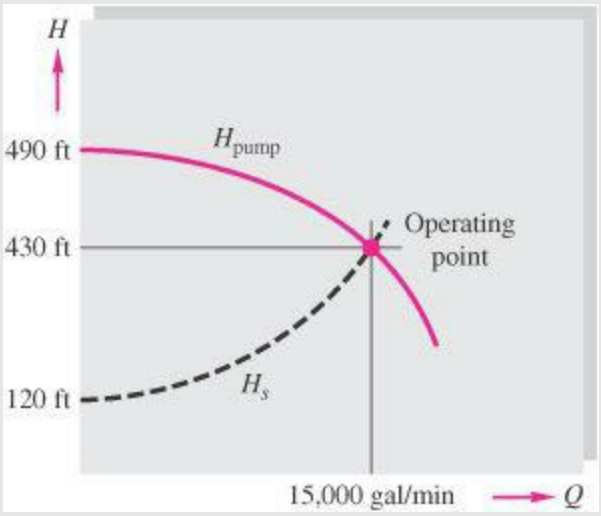
$$H_s = 120 + 0.269Q^2 \qquad Q \text{ in ft}^3/\text{s} \tag{1}$$

6LQFH) LJ DXVHVWKRXVDQGVRI JDOORQVSHUPLQXWHIRUWKHDEVFLVVD ZHFRQYHUW4 LQ(T WR
WKLVXQLW

$$H_s = 120 + 1.335Q^2 \qquad Q \text{ in } 10^3 \text{ gal/min} \tag{2}$$

: HFDQSORW(T RQ) LJ DQGVHHZKHUHLWLQWHUVHFWVWKH LQSXPS KHDGFXUYH DV L
) LJ ($ JUDSKLFDOVROXWLRQJLYHVDSSUR[LPDWD

+ § IW4 § JDOPLQ



(

7KHHILFLHQF\ LVDERXW SHUFHQWWRJKWW RI GHVLJQ
$ QDQDO\WF VROXWLRQ LV SRVVLEOHLI ZH ILWWKH SXPS KHDG FXUYH WR D SDUDERODZKLFK LV YHU
DFFXUDWH

$$H_{pump} \approx 490 - 0.26Q^2 \qquad Q \text{ in } 10^3 \text{ gal/min} \tag{3}$$

(TXDWLRQV DQG PXWP DWFK DWWKHRSHUDWLQJ SRLQW

$$490 - 0.26Q^2 = 120 + 1.335Q^2$$

or

$$Q^2 = \frac{490 - 120}{0.26 + 1.335} = 232$$

$$Q = 15.2 \times 10^3 \text{ gal/min} = 15{,}200 \text{ gal/min} \qquad \textit{Ans. (a)}$$

$$H = 490 - 0.26(15.2)^2 = 430 \text{ ft} \qquad \textit{Ans. (a)}$$

3DUWE

7RPRYHWKHRSHUDWLQJSRLQWRI%(3 ZHFKDQJHQ ZKLFKFKDQJHVERWK4 ' QDQG+ ' Q) UR
) LJ D DW%(3 + § IWWKHXVIRUDQ Q + Q $OR UHDG4 § î JDOPLQ
WKHXVIRUDQ Q 4 Q 0 DWK+ WRWKHVXVWHP FKDUDFWHULVWLFV (T

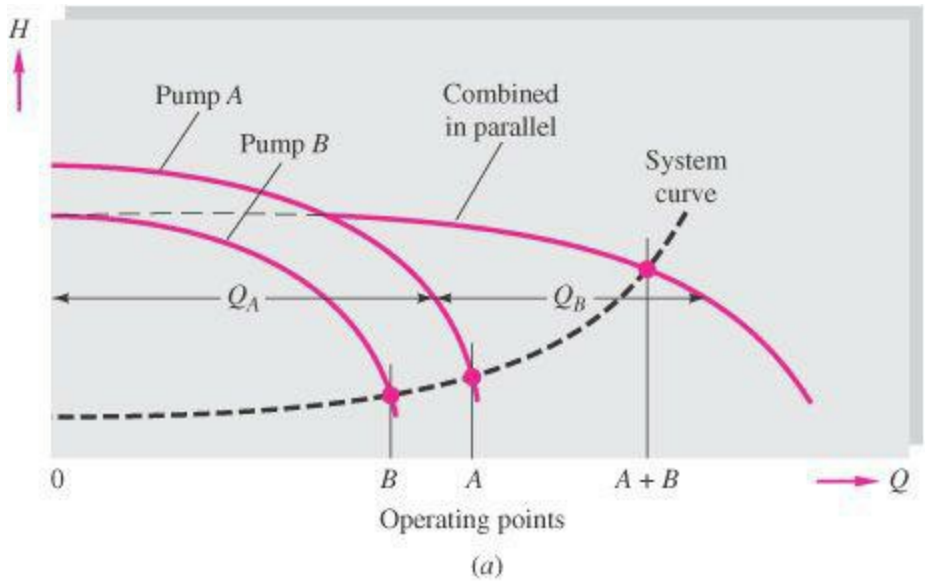
$$H^* = 386\left(\frac{n}{1170}\right)^2 \approx 120 + 1.335\left(20\frac{n}{1170}\right)^2 \qquad \textit{Ans. (b)}$$

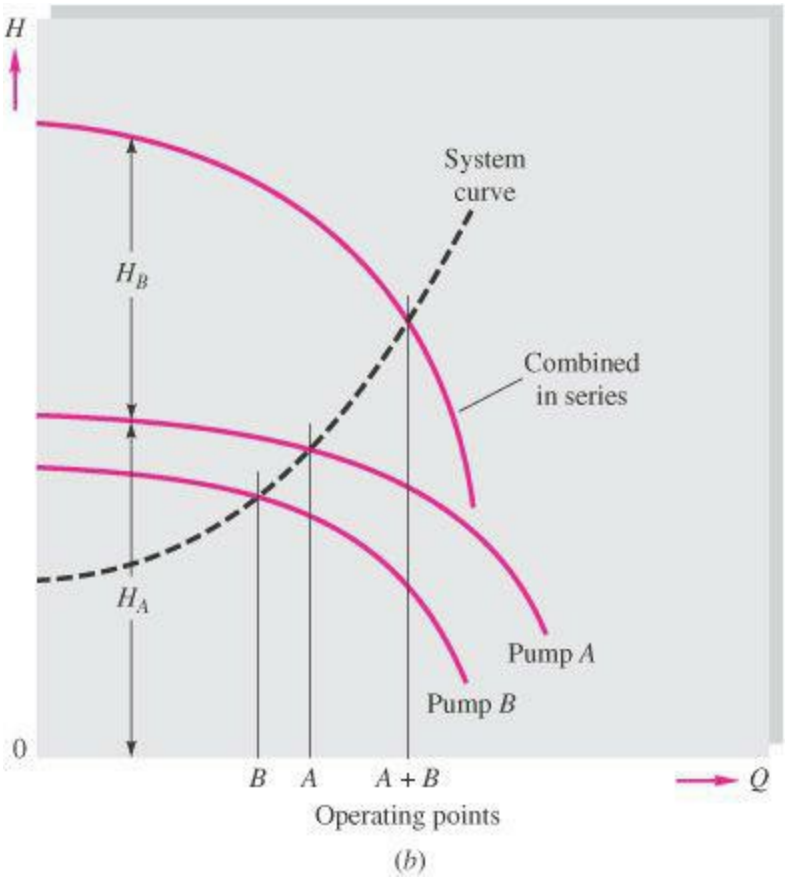
ZKLFK JLYHVQ 7KXVLWLVLPSRVVLEOHWRRSHUDWHDWPD[LPXP HILFLHQF\ ZLWKWKLVSDUWFXOD
V\VWHP DQGSXPS

3XPSV&RPELQHGLQ3DUDOOHO

,I DSXPS SURYLGHVWKHULJKWHDGEXWWRR OWWHGLVFKDUJH DSRVLEOHUHPHG\ LVWRFRPELQHWZ
VLPLODUSXPPSVLQSDUDOOHO VKDULQJ WKHVDPHVXFWLRQ DQG LQOHWFRQGLWLRQV $ SDUDOOHODUUDQJHPHQWV
DOORXHGLI GHOYHU GHP DQG YDULHV VR WKDWRQHSXPS LVXHG DWORZ IORZ DQG WKHVHFRQGSXPS LV
WDUWHGXS IRUKLJKHUGLVFKDUJHV %RWKSXPPSVKRXOGKDYHFKHFNYDOYHVWRDYRLGEDFNORZ ZKHQRQ
LVVKXWGRZQ

7KHWZR SXPPSVLQSDUDOOHOQHGQRWEHLGHQWLFDO 3K\VLFDOO WKHUIORZ UDWHVZLOOVXP IRUWK
VDPHKHDG DVLOOXVWUDWHGLQ) LJ D ,I SXPPS$ KDVPRUHHDGWKDQSXPS% SXPPS%FDQQRWEH
DGGHGLQXQLOWKHRSHUDWQJ KHDGLVEHORZ WKHVKXWRII KHDGRI SXPPS% 6LQFHWKHVXVWHP FXUYHUL
ZLWK4 WKHFRPELQHGGHOYHU 4$ %ZLOOEHOWVWKDQWKHVHSDUDWHRSHUDWQJ GLVFKDUJHV4$ 4% EX
FHHGLQJ JUHDWHUWKDQHLWKHURQH)RUDYHU IODWVXDFH FXUYHVZR VLPLODUSXPPSVLQSDUDOOHZL
GHOYHUQHDUO VWLFHWKHIORZ 7KHFRPELQHGEULNHKRUVHSRZHULVIRXQGE\ DGGLQJEULNHKRUVHSRZH
IRUHDFKRI SXPPSV$ DQG%DWWKHVDPHKHDGDVWKHRSHUDWQJ SRLQW7KHFRPELQHGHILFLHQF\ HTXDO
! J 4$ % +$ % EKS$ %





) LJ 3HUFHQWDJH GLIIHUHQFH RI VWHDG\ VWDWH SXPS RSHUDWLQJ VLQJOH DQG *a* LQ SDUDOOHORU *b* LQ VHULHV

,I SXPSV$ DQG%DUHQRWLGHQWLFDO) LJ D SXPS%VKRXOGQRWEHUXQDQGFDQQRWHYH EHVXUHWKXS LI WKHRSHUDWLQJ SRLQWVDERYHWKXWKI KHDG

3XPSV&RPELQHG LQ6HULHV

,I D SXPS SURYLGHV WKH ULJKWGLVFKDUJH EXWWRR OWWH KHDG FRQVLGHUDGLQ D VLPLODU SXPS L VHULHV ZLWK WKH RXSXWRI SXPS%IHGGLUHFW LQWR WKH VXFWLRQ VLGH RI SXPS$ $V VNHWFKHG LQ) LJ

E WKH SK\VLFDOSULQFLSOHIRUVXPPLQJ LQVHULHVLVWKDWWKHWZR KHDGVDGGDWWKHVDPHIORZ UDWHV JLYHWKHFRPELQHG SHURUPDQFHFXUYH 7KHVZR QHGQRWEHLGHQWLFDODWDOOVLQFHWKH PHUHO KDQGL WKHVDPHGLVFKDUJH WKH PDY HYHQKDYHGLIIHUHQWSHGV DOWKRXJK QRUPDOO EHWWDUHGULYHQE\ WK VDPHVKDIW

7KHQHGIRUDVHULHVDUUDQJHPHQWPSOHVWKDWWKHV\VWHP FXUYHLVVWHSWKDWV LWHTXLUHVKLJKHDG WKDQHLWKHUSXPS\$ RU%FDQ SURYLGH 7KHFRPELQHG RSHUDWQJ SRLQWKHDG ZLOOEH PRUH WKDQHLWKHU\$ RU%VHSDUDWHO EXWQRWDV JUHDWDV WKHLUVXP 7KHFRPELQHG SRZHULV WKH VXP RI EUDNKRUVHSRZHUFRU\$ DQG%DWWKHRSHUDWQJ SRLQWRZ UDW 7KHFRPELQHGHILFLHQF\ LV

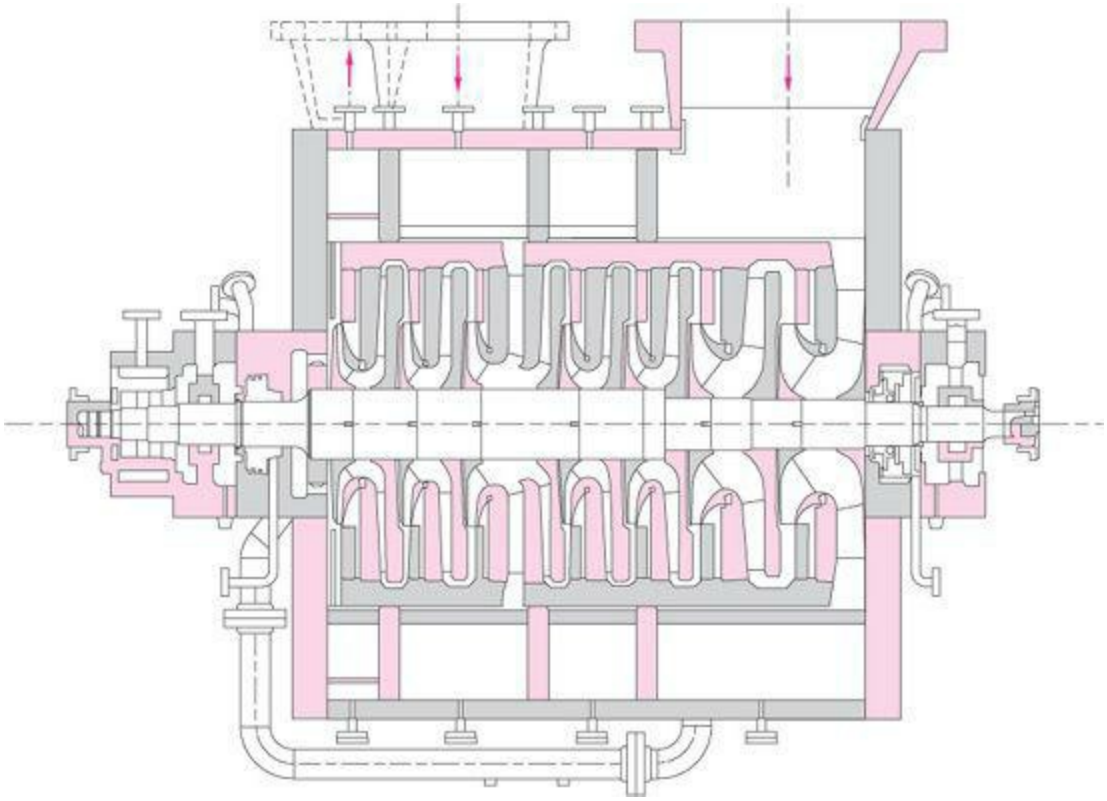
$$\frac{\rho g(Q_{A+B})(H_{A+B})}{550\ \text{bhp}_{A+B}}$$

VLPLODUWRSDUDOOHOSXPSV

: KHWKHUSXPSV DUHXVHGLQVHULHVRULQSDUDOOHO WKH DUUDQJHPHQWZ LOOEH XQHFRQRPLFDOXQOHV ERWSXPPSV DUHRSHUDWQJ QHDUWKHLUEHVWHILFLHQF\

0 XOWLSXPSH3XPSV

)RUYHU KJKKHDGVLQFRQGLWLRQVRSHUDWQJ WKHVRXWRQLVDPXOWLSXPS ZLWKWHH[LWR RQ LPSHOOHUIHHGLQJ GLUHFWO LQWR WKHQH[W&HQWULXJDO PL[HG IORZ DQGD[LDOIORZ SXPSVKDY DOOEHHQJURXSHGLQDVPDQ DV VWDJHV ZLWKKHDGVXS WR IWR ZDWUDQG SUHVXUHULVHXS W GBI LQ DEVROXWH) LJXUH VKRZ VDVHFWLRQ RI DVHYHQVWDJHFHQWULXJDOSURPSDQHFRPSUHVVR WKDWGHYHORSV GBI LQ ULVHDW IWPLQDQG EKS



) LJ &URVV VHFWLRQ RI D VHYHQVWDJH FHQWULXJDO SURPSDQH FRPSUHVVRU WKDWGHOLYHU IWPLQDW EKS DQG D SUHVXUH ULVH RI GBI LQ 1RWH WKH VHFRQGLQGHWDWWDJH

DQG WKH YDULQJ LPSHOODUGHVLJQV*(Courtesy of DeLaval-Stork V.O.F., Centrifugal Compressor Division.)*

&RPSUHVVRUV

0 RVWRI WKHGLVFXVVLRQLQWKLVFKDSWHUFRQFHQWUDWHVRQFHQWULIXJDOFRPSUHVVLEOHIORZ² WKDWV QHJOLJLEOHFKDQJHLQ IOXLG GHQVLW (YHQ WKH SXPS RI) LJ ZKLFK FDQ SURGXFH IWRI KHDG DW UPLQ ZLQFUHDVHVDQGDUDPSUHVVXUHRQO E\ GEI IW DERXWD SHUFHQWFKDQJHLQGHQVLW 7KH SLFWXUFKDQJHVDWKLJKHUVSHHGV ûS ’ Q DQG PXOWLSOHVWDJHV ZKHUH YHU ODUJHFKDQJHVLQSUHVVXUHDQGGHQVLW DUHDFKLHYHG 6XFK GHYLFHVDUHFDOOGFRPSUHVVRUV DVLQ) LJ 7KHFRQFHSWRI VWDWKHDG + ûS!J EHFRPHVLQDSSURSULDWH VLQFH! YDULHV &RPSUHVVRUSHUIRUPDQFHLVPHDVXUHGE\ WKHSUHVXUHUDWLR DFURVV WKHVWDJHS S DQG WKHFKDQJHLQVWDJQDWLRQHQWKDOS\ K í K ZKHU $h_0 = h + \frac{1}{2}V^2$ VHH6HF &RPELQLQJP VWDJHVLQVHULHVUHVXOWLQS$ η_{comp} S S P $\V GHQVLW LQFUHDVHV GHV DUHD LV QHGG QRWWKHG FUHDVHLQLPSHOOHUVL]HIURP ULJKWWRGHVLQ) LJ &RPSUHVVRUVPD\ EHHLWKHURI WKHFHQWULXJDORUD[LDOIORZ WSH> WR @ &RPSUHVVRUHILFLHQF\ IURP LQGHWHUPLQDWHRQGLWLRQ WR ILQDORXWOHW LV GHILQHG E\ WKHFKDQJHLQJD HQWKDOS\ DVXPLQJ DQDGLDEDWF SURFHVV

$$\eta_{\text{comp}} = \frac{h_f - h_{01}}{h_{0f} - h_{01}} \approx \frac{T_f - T_{01}}{T_{0f} - T_{01}}$$

&RPSUHVVRUHILFLHQFLHVDUHVLPLODUWRK\GUDXOFPDFKLQHV $P[$ § WR SHUFHQWEXWWKHPDV IORZ UDQJHLVPRUHOLPLWHG RQWKHORZ VLGHE\ FRPSUHVVRUVXUJH ZKHUHEDGHVWDOOQG YLEUDWRU RFFXUDQGRQWKHKJK VLGHE\ FKRNLQJ 6HF ZKHUHWKH0DFKQXPEHUHDFKHV VRPHZKHUH WKHVXVWHP &RPSUHVVRUPDWIORZ LVQRUPDOO SORWWHGXVLQJ WKHVDPHWSHRI GLPHQVLRQOHVVIXQFWRR IRUPXODVGLQ(T À 57 ' S ZKLFK ZLOOUHDFK DPD[LPXP ZKHQFKRNLQJ RFFXUV)R IXUWKHUGHWDLOV VHH5HV WR

(; $0 3/ (

,QVWUXFWDWH[WQGLQJ ([DPSOH E\ XVLQJ WZR LQSXPSVLQSDUDOOHOWRGHOYHUPRUHIORZ ,W WKLVHILFLHQW

6RXWLRQ

6LQFHWKHSXPSVDUHLGHQWFDOHDFKGHOYHU $WKHVDPH UPLQVSHG 7KHVVWHP FXUYHLV WKHVDPH DQGWKHEDODQFH RI KHDGUHDWRQEHFRPHV

$$H = 490 - 0.26(\tfrac{1}{2}Q)^2 = 120 + 1.335Q^2$$

or $$Q^2 = \frac{490 - 120}{1.335 + 0.065} \qquad Q = 16{,}300 \text{ gal/min} \qquad \textit{Ans.}$$

7KLVLVRQO SHUFHQWPRUHWKDQDVLQJOHSXPS (DFKSXPSGHOLYHUV $Q = 8130$ JDOPLQ IRUZKLFK
WKHHILFLHQF\ LVRQO SHUFHQW7KHWRWDOEUDNHKRUVHSRZHUUHTXLUHG LV ZKHUHDV DVLQJO
SXPSXVHGRQO EKS 7KLVLVDSRRUGHVLJQ

(;$03/(

6XSSRVHWKHHOHYDWRQFKDQJHLQ([DPSOH LVUDLVHGIURP WR IWJUHDWHUWKDQDVLQJO
LQSXPSFDQVXSSO ,QYHVWLJDWHXVLQJWZR LQSXPSVLQVHULHVDW UPLQ

6RXWLRQ

6LQFHWKHSXPSVDUHLGHQWLFDOWKHWRWDONHDGLVWZLFHDVPXFKDQGWKHFRQWDQW LQWKHV\VWHP
KHDGFXUYHLVUHSODFHGE\ 7KHEDODQFHRI KHDGVEHFRPHV

$$H = 2(490 - 0.26Q^2) = 500 + 1.335Q^2$$

or $$Q^2 = \frac{980 - 500}{1.335 + 0.52} \qquad Q = 16.1 \times 10^3 \text{ gal/min} \qquad \textit{Ans.}$$

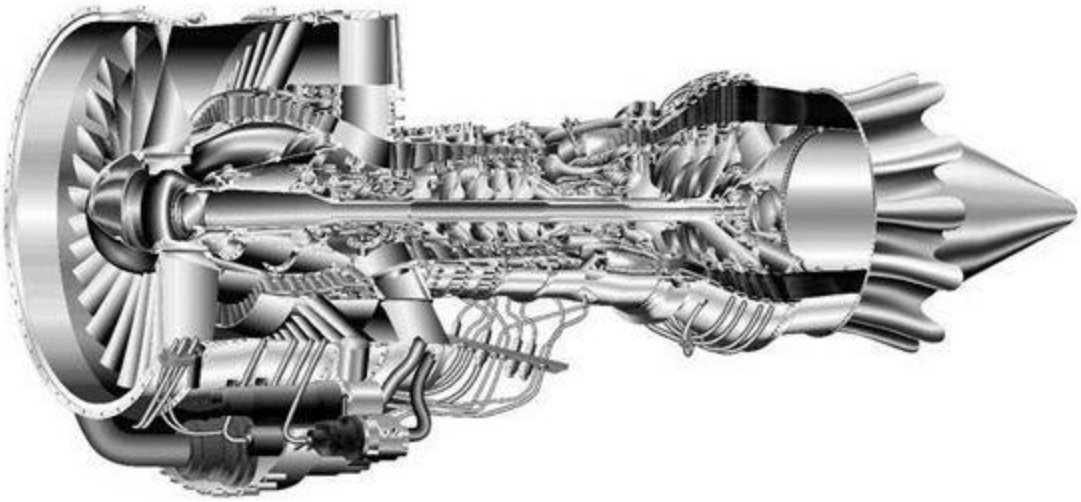
7KHRSHUDWQJKHDGLV IWRU SHUFHQWPRUHWKDQWKDWRUDVLQJOHSXPS
LQ([DPSOH (DFKSXPSLVRSHUDWQJDW î JDOPLQ ZKLFKIURP) LJ DLV SHUFH
HILFLHQWDSRLWW JRRGPDWK WR WKHV\VWHP 7RSXPSDWWKLVRSHUDWQJSRLQWUHTXLUHV EKS R
DERXW EKSIRUHDFKSXPS

* DV7XUELQHV

6RPHPRGHUQGHYLFHVFRQVLQERWKSXPSVDQGWXUELQHV $ FDVWLF FDVHLVWKHJDVWXUELQH Z KLF
FRPELQHVDFRPSUHVVRUDFRPEXVWRQFKDPEHUDWXUELQH DQG RIWQ DIDQ * DVWXUELQHVDUHXVHGW
GULYHDLUFUDIWKHOLFRSWHUV $UP\ WDQNV DQGVPDOOHOHFWULF SRZHUSODQWV 7KH\ KDYHDKJKHUSRZH
WR ZHJKWUDWR WKDQ UHFLSURFDWQJ HQJLQHV EXWWKH\ VSLQ DWYHU KJK VSHHGV DQG UHTXLUH KJI
WHPSHUDWXUHPDWHULDOVDQGWKXVDUHFRVWO 7KHFRPSUHVVRUDLVWKHLQWHUPHGLDWHSUHVXUHDVPXFKD
WR WPHVKJKHUEHRUHHQWULQJ WKHFRPEXVWRQFKDPEHU7KHKDNGDLUWKHQSDVHVKURXJKD
WXUELQH Z KLFK GULYHVWKHFRPSUHVVRU7KHDLUOHZ WKHQH[WWRSURYLGHWKHWKUXWDQGLVJHQHUDOO I
VXSHUVRQLFIORZ

7KHH[DPSOHLOOXWUDWHV LQ) LJ LVD3UDWW : KLWQH WXUERIDQDLUFUDIWHQJLQH 7K
OLJHHQWDQFH IDQ JUHDWO LQFUHDVHV WKHDLUIORZ LQWR WKHHQJLQH VRPHRI Z KLFK E\SDVVHV WK

FRP SUHVVRUV 7KH FHQWULIXJDO IORZ HQJLQHV DORZ SUHVVXUH / 3& DQG DKLJK SUHVVXUH +3& FRP SUHVV
DQG WKHQ HLWKHU WKH FRPEXVWLRQ \$IWHU FRP EXVWLRQ WKH KRWKLJK YHORFLWV JDVHV SDVV WKURXJK D KLJK
SUHVVXUH WXUELQH +37 ZKLFK GULYHV WKH+3& DQG DORZ SUHVVXUH WXUELQH / 37 ZKLFK VHSDUDWH
GULYHV ERWK WKH / 3& DQG WKH IDQ 7KH H[KDXVWJDVHV WKHQ FUHDWH WKH WKUXVW LQ WKH XVXDOP RP HQXP
H[FKDQJH P DQQHU 7KH HQJLQH VKRZ Q GHVLJQHG IRU VKRUWHU DLUOLQH IOLJKW KDV D PD[LPXP WKUXVWR
GE



) LJ &XWDZD\ YLHZ RI D 3UDWW : KLWQH\ VXUERIDQ DLUFUDIW HQJLQH*(Copyright United Technologies Corporation 2008. Used with permission.)*

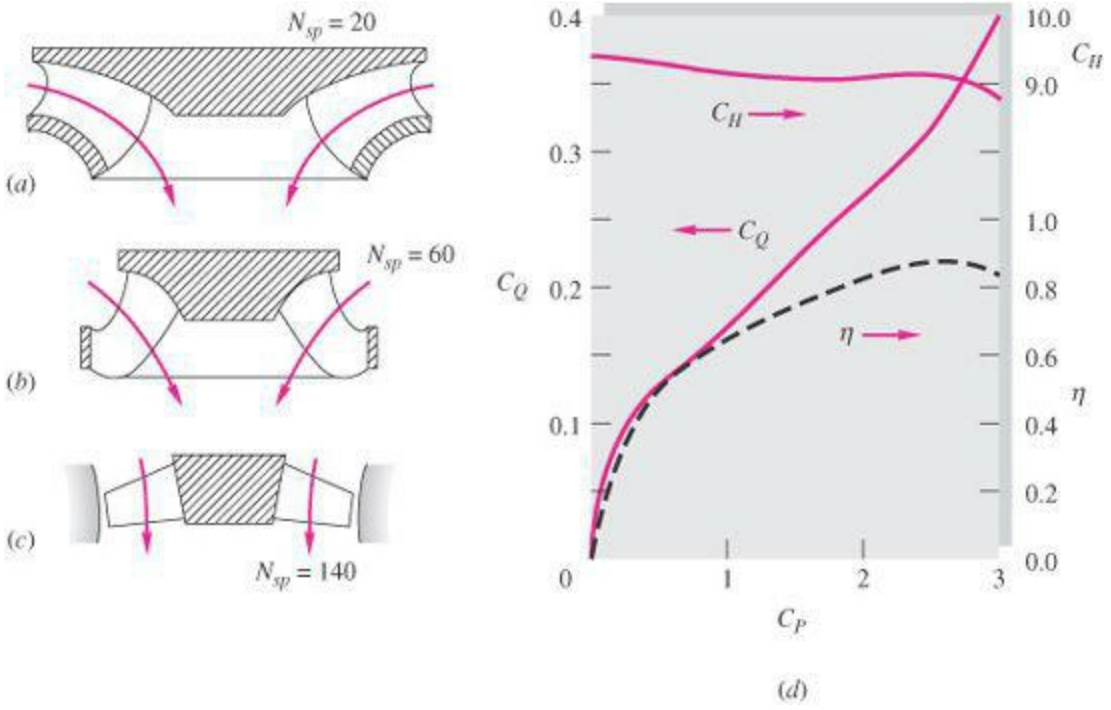
7XUELQHV

\$ VXUELQH H[WDFWHQHU\ IURP DIXLG WKDW SRVHVVHV KLJK KHDG EXWWV IDUR XV WR D WXUELQH DSXP SUXQEDFNZDUG %DVLFDOO WKH UHDUZR\WSHV UHDFWRQ DQG LPSXOH WKHGLIIHUHQFH OLQJ LQWR PDQQHUR KHDG FRQYHUWRQ ,QWHUDFWRQ WXUELQH WKH IOXG ILOOV WKH EODGH SDVDJHV DQG WKH KHDG FKDQJHRUSUHVVXUHGURS RFFXUZ LWKLQ WKH LPSOBU 5HDFWRQGHVLJQV DUH R WKH UDGLDOIORZ PL[HG IORZ DQG D[LDOIORZ \WSHV DQG DUH HVHQWLDOO GQDPLF GHYLFHV GHVLJQHG WR DGP LWWKH KLJK HQHUJ\ IOXLG DQG H[WDFWWV P RP HQXP \$QLPSXOH WXUELQH ILUWFRQYHUWV WKH KLJK KHDG WKURXJK D QR]]OHLQ D KLJK YHORFLW MHWZ KLFK WKHQ WXUNV WKH EODGH DWRQH SRVLWRQ DV WKH SDW E\ 7KH LPSOB SDWDJHV DUH QRWIXLG ILOOHG DQG WKH MHWZORZ SDWWKH EODGHV LV HVHQWLDOO DWFRQWDQWSUHVXUH 5HDFWRQWXUELQHV DUH VP DOOHUEHFDXVHIOXG ILOOV DOOWKH EODGHV DWRQHWP H

5HDFWRQ 7XUELQHV

5HDFWRQWXUELQHV DUHORZ KHDG KLJK IORZ GHYLFHV 7KHIORZ LVRSSRVLWWKDWLQ DPP HQWLQ DWWKH OLJHUGLDP HWUVFWRQ DQG GLVFKDUJLQJ WKURXJK WKH H[HDWWJLYLQJ XS PRVWRI LWHQHUJ\ WR WKH LPSOBU (DUO GHVLJQV ZHUH YHU LQHILFLHQWEHFDXVH WKH ODFNHG VDWRQDU JXLGH YDQHV DWWKH HQWDQFH WR GLUHFWWKH IORZ VP RRWKO LQR WKH LPSOBU SDWDJHV 7KH ILUWH ILFLHQWLQZ DUG IORZ WXUELQHZ DVEXLOWQ E\ -DPHV%) UDQFLV D8 6 HQJLQHU DQG DOOUDGLDO RUP L[HG IORZ GHVLJ

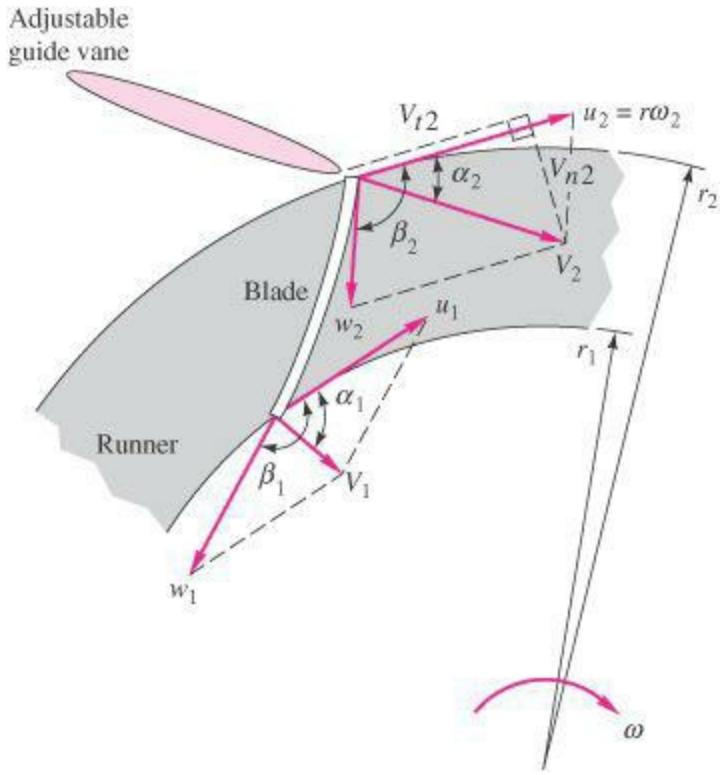
DUH QRZ FDOOHG) UDQFLV WXUELQHV $ WXUELQH FDQ EH GHVLJQHG PRUH FRPSDFW
ZLWK SXUHO D[LDO IORZ DQG LV WUHGDSURSHOOHU WXUELQH > @7KH SURSHOOHU PD\ EH HLWKHU ILL[HG
EODGHRUDGXVWDEOH . DSODQ\WSH WKHOWWUEHLQJ FRPSOHDMGPHFKDQLFDO EXWPXFKPRUHHILFLHQ
DWORZ SRZHU VHWWQJV) LJXUH VKRZV WNHWKHV RI UKQHUGHVLJQV IRU) UDQFLV UDGLDO) UDQF
PL[HG IORZ DQG SURSHOOHU WSHWXUELQHV



) LJ 5HDFWRQ WXUELQHV *a*) UDQFLV UDGLDOWSH *b*) UDQFLV PL[HG IORZ *c* SURSHOOHU D[LDO IORZ *d* SHUFRUPDQFH FXUYHV IRUD) UDQFLV WXUELQH *n* UPLQ*D* IWN$_{sp}$

,GHDOL]HG5 DGLDO7 XUELQH7 KHRU\

7KH(XOHUWXUERPDFKLQHIRUPXOD DOORSSOV WRHQHUJ\ H[WUDFWLQJ PDFKLQHVLI Z HUHYHUV
WKHIORZ GLUHFWLRQDQG UHVKDSHWKHEODGHV) LJXUH VKRZV DUDGLDOWXUELQHUXQQHU$JDLQDWXPH
RQHGLPHQVLRQDOIUFWLRQOHVIORZ WKURXJK WKHEODGHV $GXWDEOHLQOHWJXLGHYDQHV DUHDEVROXWH
QHFHVDU IRUJRRGHILFLHQF\ 7KH\ EULQJ WKHLQOHWIORZ WRWKHEODGHVDWDQJH. DQGDEVROXWHYHORFLW
9 IRUPLQLPXP³VKRFN\ RUGLUHFWLRQDOPLVPDWFKORVV $IWUYHFWRUDO DGGLQJ LQWKHUXQQHUUSVSHH
X &U WKHRXWUEODGHDQJOHVKRXOGEHVHWDWDQJOH WRDFFRPPRGDWWKHUHODWLYHYHORFLWZ D
VKRZQLQWKHILJXUH 6HH) LJ IRUWKHDQDORJRXVUDGLDOSXPS YHORFLWGLDJUDPV



) LJ ,QDGGLWLRQWRWKHYHORFLW GLDJUDPV IRUDQ LGHDO]HG UDGLDOIORZ UHDFWLRQ WXUELQH UXQQHU

\$SSOLFDWLRQRI WKH DQJXODUPRPHQWXP FRQWROYROXPH WKHRUHP (T VR) LJ VH ([DPSOH IRUDVLPLODUFDVH \LHOGVDQLGHDOHIILFLHQF\ IRUPXODIRUWKHSRZHU3 H[WUDFWHGE\ WKHUXQQHU

$$P = \omega T = \rho\omega Q(r_2 V_{t2} - r_1 V_{t1}) = \rho Q(u_2 V_2 \cos\alpha_2 - u_1 V_1 \cos\alpha_1) \quad (11.35)$$

Z KHUH9$W DQG9$W DUH WKH DEVROXWH LQHWDQG RXWHWFLUFXPIHUHQWLDOYHORFLWLHV FRPSRQHQWV RI WK IORZ 1RWWKDW(T LVLGHQWLFDOWR(T IRUDUDGLDOSXPS H[FHSWWKDWWKHEODGHVKDSH DUHGLIIHUHQW

7KHDEVROXWHQRUPDOYHORFLW9$ 9 VLQ. LVSURSRUWLRQDOWR WKHIORZ UDWH4 ,I WKHIORZ UDWHFKDQJHVDQG WKHUXQQHUVSHHGX LVFRQVWDQWWKHYDQHVPXVWEHDGMXVWHGDQHZ DQJOH. VR WKH Z VWDOOIRORZ VWKHEODGHVXUIDFH 7KXVDGMXVWDEOHLQHWYDQHVDUHYHU LPSRUWDQWWR DYRLGVKRFNORVV

3RZHU6SHFLILF6SHHG

7XUELQHSDUDPHWHUVDUHVLPLODUWRWKRVHRI DSXPS EXWKHGHSHQGHQWYDULDEOHLVWKHRXWSXWEUDN KRUVHSRZHU ZKLFK GHSHQGV RQ WKH LQHWIORZ UDWH4 DYDLODEOH KHDG+ LPSHOOHU VSHHGQ DQ GLDPHWHUG 7KHHILFLHQF\ LVWKHRXWSXWEUDNHKRUVHSRZHUGLYLGHGE\ WKHDYDLODEOHZDWHUKRUVHSRZHU ! J4+ 7KHGLPHQVLRQOHVVFRQVWDQWV &4 &+ DQG&3 GHILQHGMXVWDVIRUDSXPS (TV ,I Z QHJOHFWV6H\QROGV1XPEHUDQGURXJKQHVVHIIHFWV WKHIXQFWLRQDOUHODWLRQVKLSVDUHZULWWHQ &3 DV WKH LQGHSHQGHQWYDULDEOH

$$C_H = \frac{gH}{n^2D^2} = C_H(C_P) \qquad C_Q = \frac{Q}{nD^3} = C_Q(C_P) \qquad \eta = \frac{\text{bhp}}{\rho g Q H} = \eta(C_P) \quad (11.36)$$

where $$C_P = \frac{\text{bhp}}{\rho n^3 D^5}$$

) LJXUH GVKRZ VWSLFDOSHUFRUP DQFHFXUYHVIRUDVPDOO) UDQFLVUDGLDOXUELQH 7KHPD[LPXP HILFLHQF\ SRLQWVFDOHGWKHQRUPDOSRZHU DQGWKHYDOXHVIRUWKLVSDUWFXODUXUELQHDUH

PD[	&3	&4	&+

$ SDUDPHWULFWRPSDUHVWKHRXSXWSRZHUZLWKWHDYDLODEOHKHDG LQGHSHQGHQWRI VL]H LVIRXQ E\ HOLPLQDWLQJ WKHGLPHWUHHQ&+ DQG&3 ,WLVFDOOHGWKHSRZHUVSHFLILF VSHHG

Rigorous form: $$N'_{sp} = \frac{C_P^{1/2}}{C_H^{5/4}} = \frac{n(\text{bhp})^{1/2}}{\rho^{1/2}(gH)^{5/4}} \quad (11.37a)$$

Lazy but common: $$N_{sp} = \frac{(\text{r/min})(\text{bhp})^{1/2}}{[H\ (\text{ft})]^{5/4}} \quad (11.37b)$$

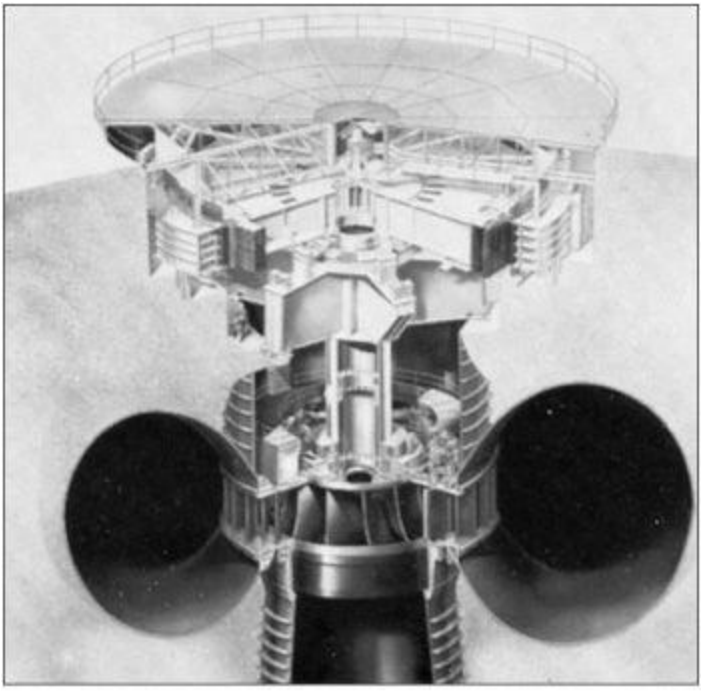
) RU Z DWU! VXJVIW DQG $N_{sp} = 273.3N'_{sp}$ 7KH YDURXV WXUELQH GHVLJQV GLYLGH XS QLFH DFFRUGLQJ WR WKHUDQJHRI SRZHUVSHFLILF VSHHG DVIROORZV

1 RWH WKDW1 $_{VS}$ QNH1 $_{V}$ IRUSXPSV LVGHILQHGRQO Z LWK UHVSHFWWR WKH %(3 DQG KDVDVLQJOH YDOX IRUDJLYHQWXUELQHIDPLO ,Q) LJ G 1 $_{VS}$ UHJDUGOHVVRI VL]H

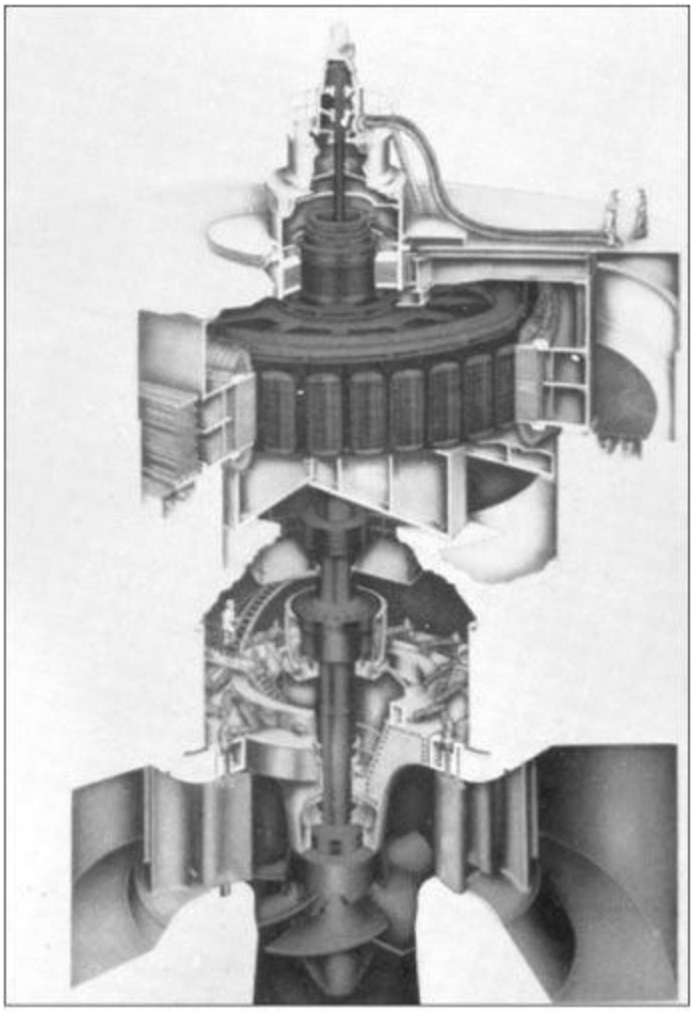
Turbine type	N_{sp} range	C_H range
Impulse	1–10	15–50
Francis	10–110	5–25
Propeller:		
Water	100–250	1–4
Gas, steam	25–300	10–80

/ LNHSXPSV WXUELQHVRI ODUJHVL]HDUHJHQHUDOO PRUHHILFLHQWDQG(TV FDQEHXVHGD DQHXWP DWWZKHQGDWDYDUHODFNLQJ

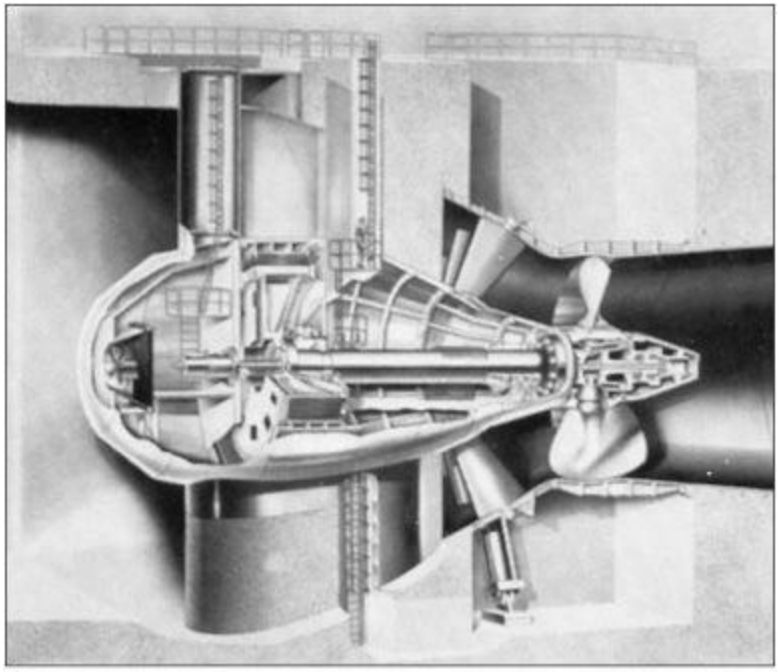
7KH GHVLJQRI DFRPSOHWHODUJH VFDOH SRZHUJHQHUDWLQJ WXUELQHV\VWHPV LV DPDWWHU RI QHFHVVLW SURVHFXWLQJ LQGHWDLOGRXEWVDQG XQFHUWDLQWLHV ZLFHWDWV VSLUDOFDVHV JHQHUDWRU ZLWK FRROLQJ FRLOV EHDULQJV DQG WDQPLWWRQ JHDUV UXQQHUEODGHV GUDIWXEHV DQG DXXPDWLQ FRQWROV 6RPHWSLFDOODUJH VFDOHUHDWRQ WXUELQHGHVLJQVDUH VKRZQLQ) LJ 7KHUHYHUVLEO SXPS DQG WXUELQH GHVLJQ RI) LJ G UHTXLUHV VSHFLDO FDUH IRU DGXXWDEOH JXLGH YDQHV WR EH HILFLHQWRUIORZ LQHLWKHUGLUHFWLRQ



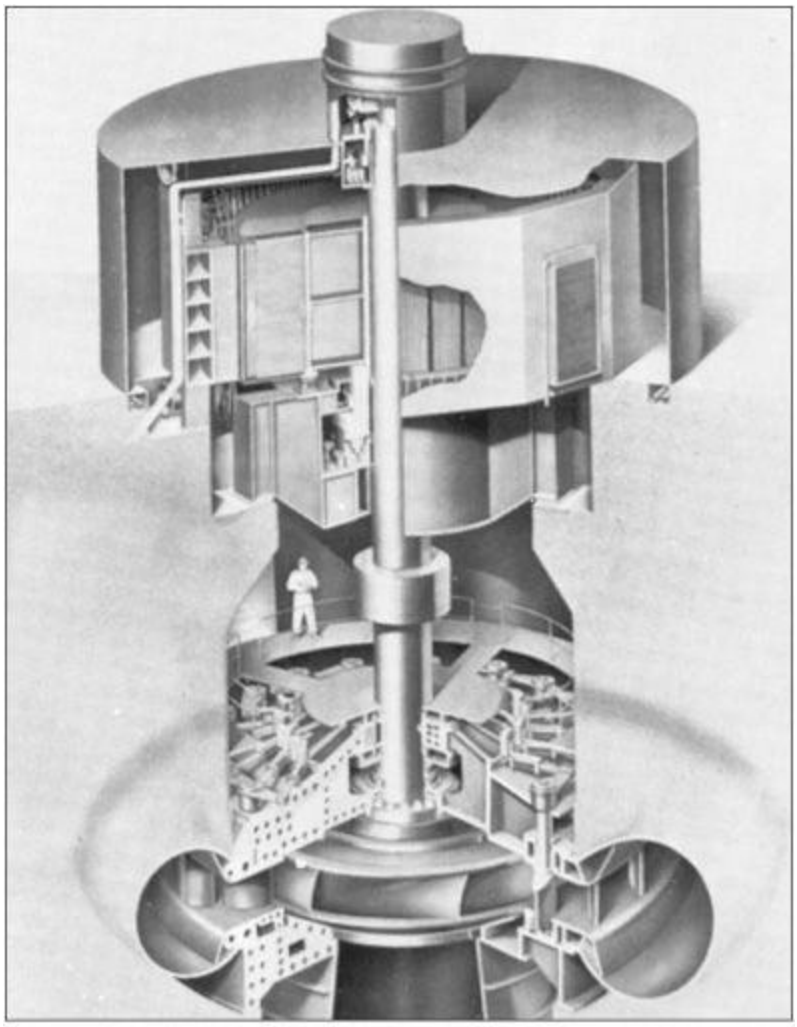
(*a*)



(*b*)



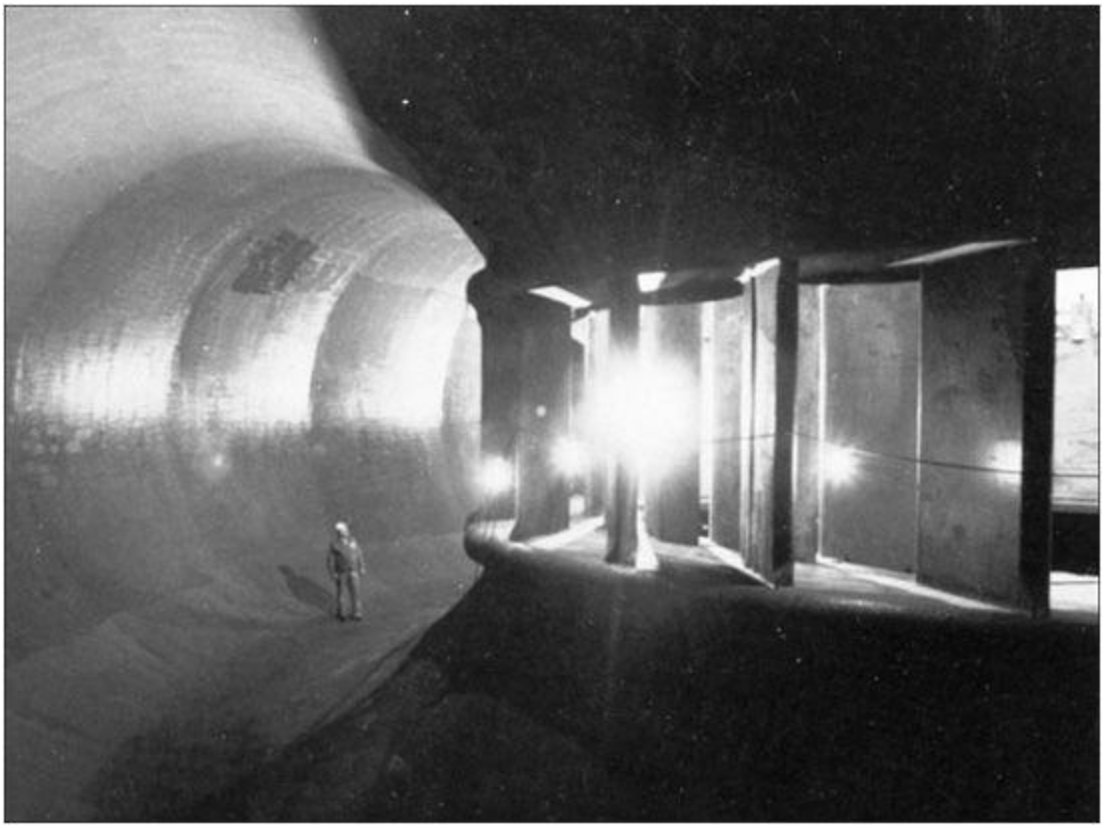
(*c*)



(*d*)

) LJ / DUJH VFDOH VWUXFWXUHV GHVLJQV GHSHQG RQ DYDLODELOLWH KHDG DQG IORZ UDWH DQG RSHUDWLQ FRQGLWLRQV *a*) UDQFLV UDGLDO *b* . DSODQ SURSHOOHU *c* EXOE P RXQWQJ ZLWK SURSHOHU UXQQHU *d* UHYHUVLEOH SXP S VWUXFWXUH ZLWK UDGLDOXXQQHU*(Courtesy of Voith Siemens Hydro Power.)*

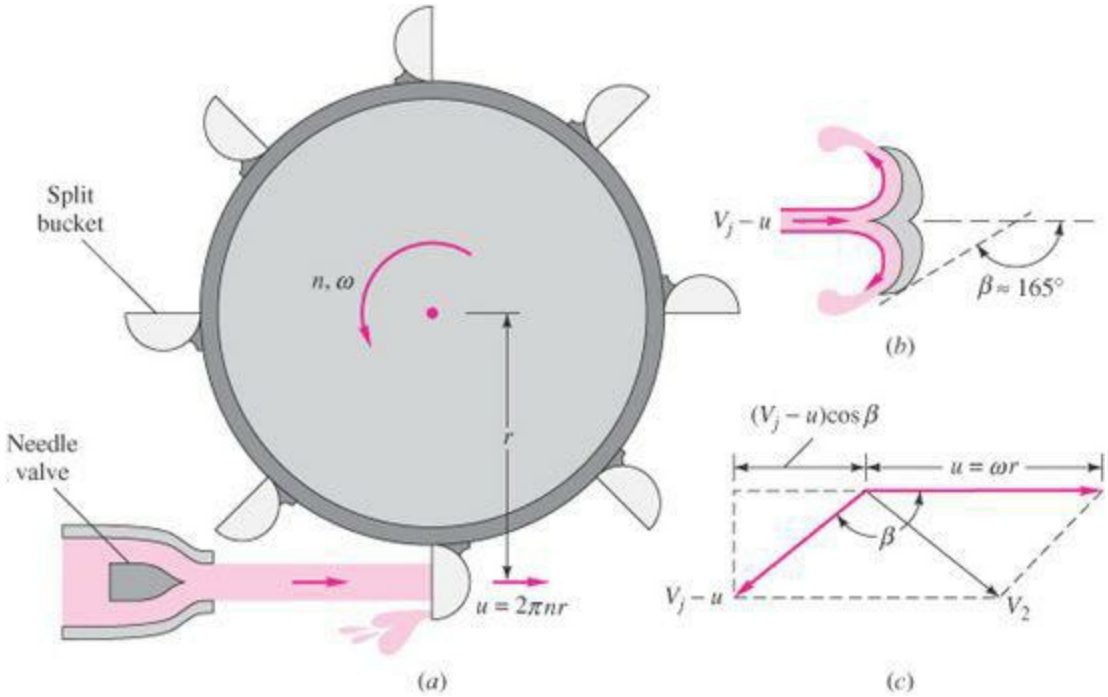
7KH ODUJHVW 0: KGURSRZHUGHVLJQVDUHDZHOOSRZHQYLHZHGRQDKXPDQVFDOH D
VKRZQLQ) LJ 7KHHFRQRPLF DGYDQWDJHVRI VPDOOVFDOHPRGHDWWULQJ DUHHYLGHQWURP WKL
SKRWRJUDSKRI WKH) UDQFLVXELQHXQLWDW UDQG&RXOHH' DP



) LJ ,QWHULRUYLHZ RI WKH PLGRQKS 0: VXELQHXQLWRQWKH* UDQG&RXOH
' DP RI WKH&RXPELD5LYHUVKRZLQJ WKHVSLUDOFDVH WKHRXWIIL[HGYDQHV ³VWD UQJ´ DQGWKH
LQQHUDGVXVWDEOHYDQHV ³ZLFNHWJDWH´ *(Courtesy of Voith Siemens Hydro Power.)*

,PSXOVH7XUELQHV

)RUKJK KHDG DQG UHDWLYH0 GRZ SRZHU WKDWLV GRZ 1$VS QRWRQD ZRXGD UHDFWRQ WXELQ
UHTXLUH WRR KJK D VSHHG EXWDOR WKH KJK SUHVXUH LQ WKH UXQQHU ZRXG UHTXLUH D PDWLYH FDVLQ
WKLFNQHVV 7KHLPSXOVHWXELQHRI) LJ LVLGHDOIRUWKLVVLWXDWLRQ 6LQFH1$VLVGRZ QZLOOEHGRZ
DQG WKH KJK SUHVXUH LV FRQLQHG WR WKH VPDOOQR]]OH ZKLFK FRQYHUWV WKH HDG WR DQ DFFRSSHUI
SUHVXUHHMHFWRU KJKYHORFLW9M 7KHMHWVWUNHVWKHEXFNHWDQGLPSDUWVDPRPHQWXP FKDQJHVLPLODUV
WKDWLQRXUFRQWROOYROXPHDQDOVLVIRUDPRYLQJ YDQHLQ([DPSOH RU3URE 3 7KHEXFNH
KDVHDQHQGSWFDOVSOWFXS VKDSH DVLQ) LJ E 7KH DUHQDPHG3HOWRQZKHOV DWWU/ HWWU$
3HOWRQ ± ZKRSURGXFHGWKHILUVWHILFLHQWGHVLJQ



) LJ , PSXOVH WXUELQH a VLGH YLHZ RI ZKHHO DQG QR]]OH b WRS YLHZ RI EXFNHW c YHORFLW GLDJUDP

, Q([DPSOH ZH IRXQG WKDW WKH IRUFH SHUXQLWP DVVIORZ RQ D VLQJOH PRYLQJ YDQH RULQWHG FDVH D VLQJOH 3HOWRQ EXFNHW ZDV 9M í X í FRV ZKHUH X LV WKH YDQH YHORFLW DQG LV WKH H[L DQJOH RI WKH NHW) RU D VLQJOH YDQH D VLQ([DPSOH WKH PDVVIORZ ZRXOG EH! $M9Mí X EXWRUI 3HOWRQ ZKHHO ZKHUH EXFNHWV NHSHQWULQJ WKH NHWDQG FDSWXUHG DOWKH IORZ WKH PDVVIORZ ZRXOG EH ! 4 ! $M9M $QDOWHUQDWLYH DQDO VLV XVHV WKH (XOHU WXUERPDFKLQH HTXDWRQ DQG WKH YHORFLW GLDJUDP RI) LJ F 1RWLQJ WKDW X X ZH VXEVWLWXWH WKH DEVROXWH H[LWDQJOH LQWHWDQJHQWLDO YHORFLWHV LQRR WKH WXUELQH SRZ HUUHDWRQ

$$P = \rho Q(u_1 V_{t1} - u_2 V_{t2}) = \rho Q\{uV_j - u[u + (V_j - u)\cos\beta]\}$$

or

$$P = \rho Q u(V_j - u)(1 - \cos\beta) \qquad (11.38)$$

ZKHUH QEU LV WKH EXFNHWDQH DUHORFLW DQG U LV WKH SLWFK UDGLXV RU GLVWDQFH WR WKH NH FHQWULDQH $ EXFNHWDQJOH f JLYHV PD[LPXP SRZ HUEXWLSK\VLFDO LPSUDFWLFDO ,QSUDFWL § f RU í FRV § RUROO SHUFHQWDWWKDQP D[LPXP SRZ HU) URP (T WKHWKHRUHWLFDOSRZ HUR DQLPSXOHWXUELQHLVSDUDERODF LQEXFNHWSHDGX DQ LVPD[LPXP ZKHQG3 GX RU

$$u^* = 2\pi n^* r = \tfrac{1}{2} V_j \qquad (11.39)$$

) RUD SHUFHFWQR]]OH WKHHQWUHDYDLODEOHKHDG ZRXOG EHFRQYHUWHG WR MHWYHORFLW9M J+ $FWXDOO VLQFHWKHUHDUH WR SHUFHQWQR]]OHORVVHV DYHORFLWFRHILFLHQW&% LVXVHG

$$V_j = C_v(2gH)^{1/2} \qquad 0.92 \le C_v \le 0.98 \qquad (11.40)$$

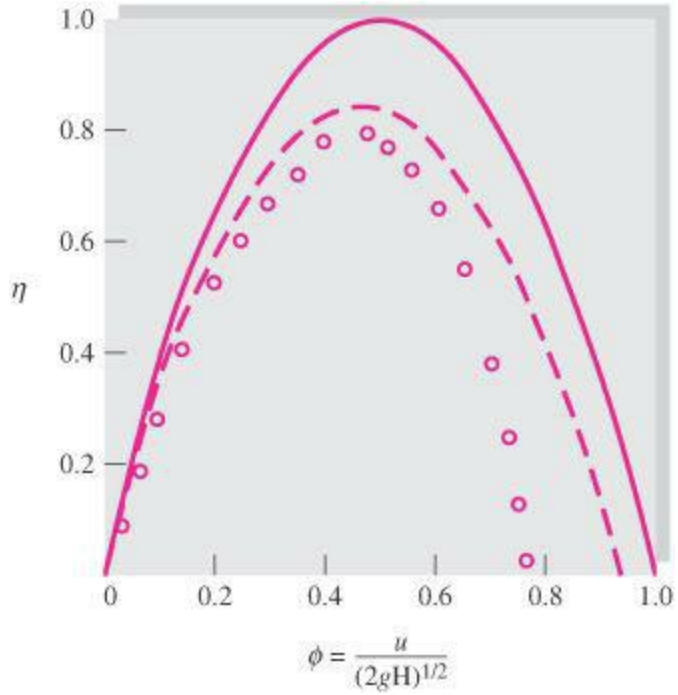
%\ FRPELQLQJ (TV DQG WKHWKHRUHWLFDOLPSXOHWXUELQHHILFLHQF\ EHFRPHV

$$\eta = 2(1 - \cos\beta)\phi(C_v - \phi) \qquad (11.41)$$

where $$\phi = \frac{u}{(2gH)^{1/2}} = \text{peripheral velocity factor}$$

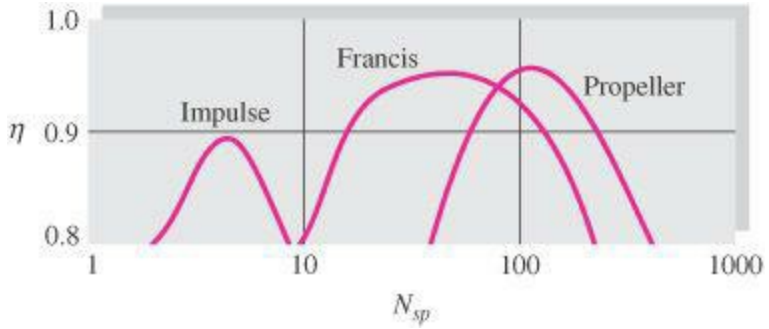
0 D[LPXP HILFLHQF\ RFFXUVDW $\phi = \frac{1}{2}C_v \approx 0.47$

) LJXUH VKRZV (T SORWWHG IRU DQ LGHDO WXUELQH f &Y DQG IRU WSLFDO ZRUNLQJ FRQGLWLRQV f &Y 7KH DWWUDFWLYH SURGLFWLRQ PD[SHUFHQWDWÏ EH VHHQ DFWXDOO\ DW IRUD LQ3HORQ ZKHOWWDUHV RPHZKDW OHVV HILFLHQW GXH WR ZLQGDJH PHFKDQLFDO ORVVHV EDFNVODVKLQJ DQG QRQXQLRUP EXFNHWWRZ) RUWKLV WWW PD[SHUFHQWDQG JHQHUDO VSHDNLQJ DQ LPSXOVH WXUELQH LV QRWTXLWH DV HILFLHQWDV WKH) UDQFLV RU SURPHOOHU WXUELQHV DWWKH %(3V



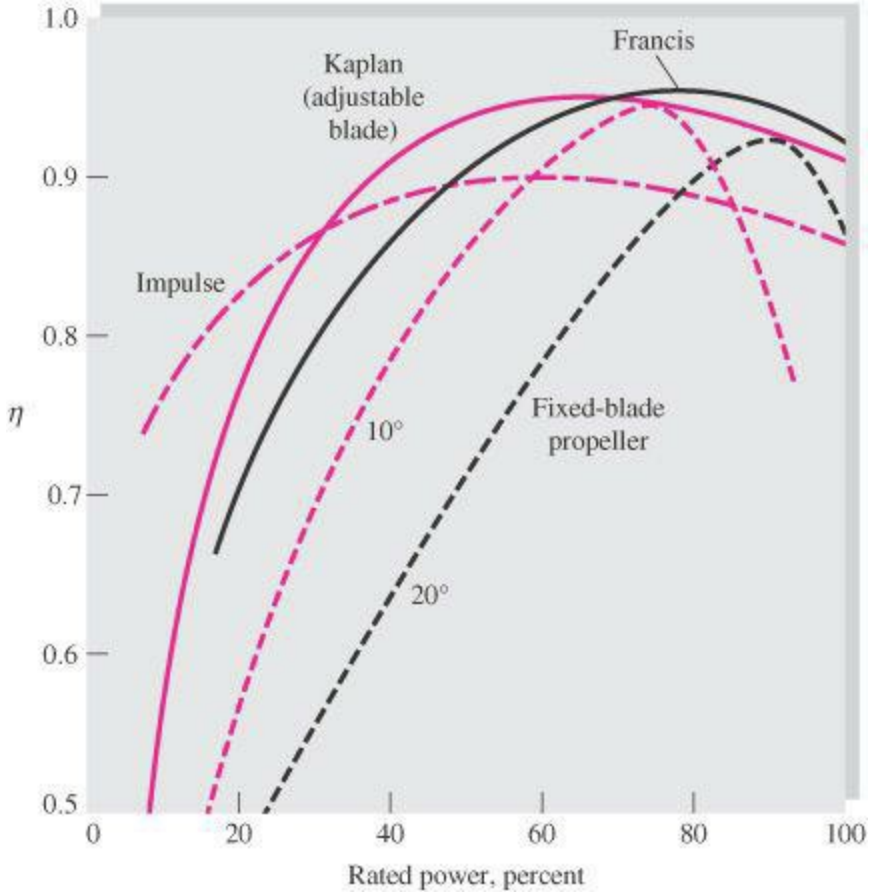
) LJ (IILFLHQF\ RI DQ LPSXOVH WXUELQH FDOFXODWHG IURP (T VRQG FXUYH LGHDOβ $f C_v$ GDVKHG FXUYH DFWXDOβ $f C_v$ RSHQ FLUFOHV GDWD 3HORQ ZKHHO GDPHWMU IW

) LJXUH VKRZV WKH RSWLPXP HILFLHQF\ RI WKH WXUELQH WSHV DQG WKH LPSRUWDQFH RI WKH SRZHUSHFLILF VSHHG1VS DV D VHOHFWLRQ FULWHULRQ WKH GHVLJQHU 7KHVH HILFLHQFLHV DUH RSWLPXP DQG DU REWDLQHG LQFDUHXOGHVLJQRI GUJHP DFKLQHV



) LJ 2 SWLP XP HILFLHQF\ RI WXUELQH GHVLJQV

7KH ZDWHU SRZHU DYDLODEOH WR D WXUELQH PD\ YDU GXH WR HLWKHU GRZQVWUHDP FKDQJHV EHWZHHQ ZKLFK DUH FRPPRQLQIILFDQW DQG VHDVRQDO YDULDWLRQV 7KH GHPDQG IRU WXUELQH SRZHU DOVR YDULHV IURP QLJKW WR GD\ DQG WKH RSHUDWLQJ UHVSRQVHV DFKDQJH LQ WKH IORZ UDWH E\ DGXVWPHQWV RI DJDWH YDOYHV RU QHGOH YDOYHV) LJ D \$V VKRZQ LQ) LJ DOO WXUELQH WSHV DFKLHYH HIILFLHQF\ DV IXQFWLRQV RI WKHLU VSHFLILF VSHHGV DQG HIILFLHQF\ YHUVXV SRZHU FXUYHV DUH XVHG E\ SURSHOOHU WXUELQH PDQXIDFWXUHUV DV SSRVHG WR QRUPDOSRZHU ZKLFK LV GHYHORSHG DW[LPXP HILFLHQF\ DQG SURSHOOHU 7KH WUDQG SRZHU LQ) LJ LV WKH OXJHVWSRZHU GHYHORSHG E\ WKH PDQXIDFWXUHU DV SSRVHG WR QRUPDOSRZHU ZKLFK LV GHYHORSHG DW[LPXP HILFLHQF\



) LJ (IILFLHQF\ YHUVXV SRZHU GHVLJQ IRU YDULRXV WXUELQH GHVLJQV DW FRQWDQW SHHG DQG KHDG

) RU IXUWKHU GHWDLOV RI GHVLJQ DQG RSHUDWLQJ FKDUDFWHULVWLFV RI WXUERPDFKLQHU WKH UHDGHU DQG LQWHUHVWHG 5HV DQG 5HV DQG

(; $0 3/ (

,QYHVWLJDWH WKH SRVVLELOLWLHV RI XVLQJ D D3HOWRQ Z KHHOV LP LODU WR) LJ RU E WKH) UDQFL VKUELQHIDPLO RI) LJ GWR GHOYHU EKS IURP DQ HWKHDGRI IW

6RXWLRQ

3DUWD

) URP) LJ WKHP RVWHILFLHQW 3HOWRQ Z KHHO RFFXUV DWDERXW

$$N_{sp} \approx 4.5 = \frac{(\text{r/min})(30{,}000 \text{ bhp})^{1/2}}{(1200 \text{ ft})^{1.25}}$$

or $$n = 183 \text{ r/min} = 3.06 \text{ r/s}$$

) URP) LJ WKHEHWWRSHUDWLQJ SRLQWLV

$$\phi \approx 0.47 = \frac{\pi D(3.06 \text{ r/s})}{[2(32.2)(1200)]^{1/2}}$$

or $$D = 13.6 \text{ ft} \qquad \textit{Ans. (a)}$$

7KLV 3HOWRQ Z KHHOLV SHUKDSV D OLWWOH VORZ DQG DWWLOHOUJH <RX FRXOG UHGXFH' DQG LQFUHDVHG E\ LQFUHDVLQJ 1_{VS} WR VD\ RU DQG DFFHSWLQJ WKH VOJKWUHGXFWLRQ LQ HILFLHQF\ 2U\RX FRXOG XVH GRXEOH KXQJ VWR Z KHHOFRQLJXUDWLRQ HDFK GHOYHULQJ EKS Z KLFK FKDQJHV' DQGQ E\ WK IDFWRU

Double wheel: $n = (183)2^{1/2} = 260 \text{ r/min}$ $D = \frac{13.6}{2^{1/2}} = 9.6 \text{ ft}$ *Ans. (a)*

3DUWE

7KH) UDQFLVZ KHHOR) LJ GPXWKDYH

$$N_{sp} = 29 = \frac{(\text{r/min})(30{,}000 \text{ bhp})^{1/2}}{(1200 \text{ ft})^{1.25}}$$

or $$n = 1183 \text{ r/min} = 19.7 \text{ r/s}$$

7KHQWKHRSWPXP SRZ HUFRH ILFLHQWV

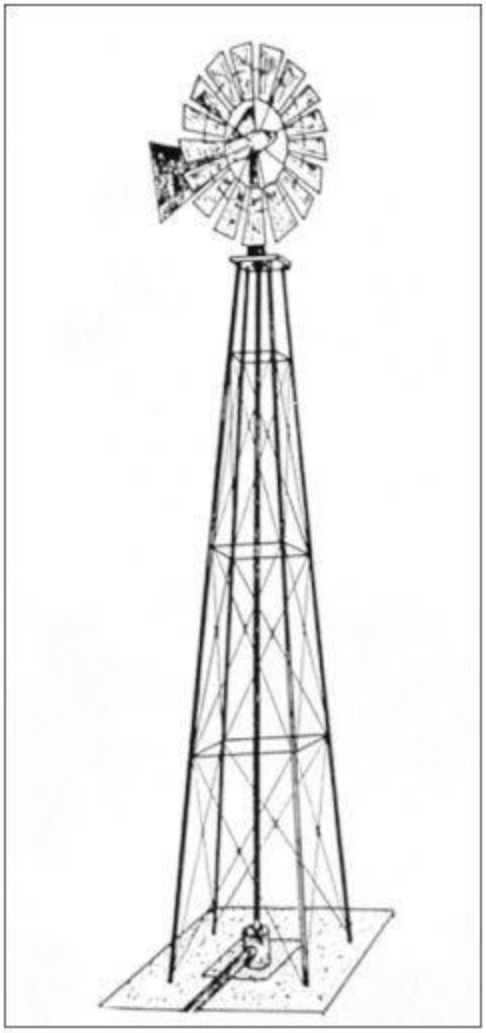
$$C_{P*} = 2.70 = \frac{P}{\rho n^3 D^5} = \frac{30{,}000(550)}{(1.94)(19.7)^3 D^5}$$

or $D^5 = 412 \qquad D = 3.33 \text{ ft} = 40 \text{ in}$ *Ans.* (*b*)

7KLV LV D IDVWHU VSHHG WKDQ QRUPDO SUDFWLFH DQG WKH FDVLQJ ZRXOG KDYH WR ZLWKVWDQG IWR
ZDWHU RU DERXW OEI LQ LQWHUQDO SUHVVXUH EXWWKH LQ VL]H LV H[WUHPHO DWWDFWLYH) UDQFL
VXUELQHV DUH QRZ EHLQJ RSHUDWHG DWKHDGVXS WR IW

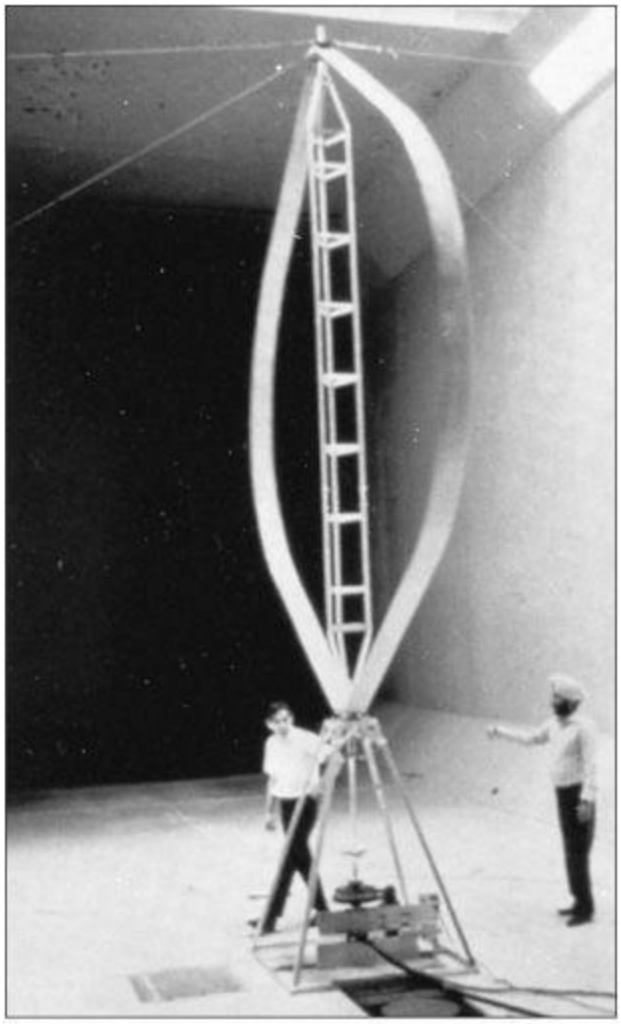
: LQG 7XUELQHV

: LQG HQHUJ\ KDV ORQJ EHHQ XVHG DV D VRXUFH RI PHFKDQLFDO SRZHU 7KH IDPLODU IRXUEODGH
ZLQGPLOOV RI +RODQG (QJODQG DQG WKH* UHHN LVODQGV KDYH EHHQ XVHG IRU FHQWXULHV WR SXPS ZDW
JULQG JUDLQ DQG VDZ ZRRG 0 RGHUQ UHVHDUFK FRQFHQWUDWHV RQ WKH DELOLW\ RI ZLQG WXUELQHV WR JHQHUD
HOHFWULF SRZHU . RHSSO> @ WXUHVHV WKH SRWHQWLDO IRU SURPOHU\WSH PDFKLQHV 6SHUD > @JLYHV I
GHWDLOHG GLVFXVVLRQ RI WKH WHFKQLFDO DQG HFRQRPLF IHDVLELOLW\ RI ODUJH VFDOH HOHFWULF SRZHU JHQHUDWR
E\ ZLQG 6HHDOR5HV DQG
6RPH H[DPSOHV RI ZLQG WXUELQH GHVLJQV DUH VKRZQ LQ) LJ 7KH IDPLODU $PHULFD
PXOWLEODGHIDUP ZLQGPLOO) LJ D LV RI ORZ HILFLHQF\ EXWKRXVDQGV DUH LQXVHDV UXJJHG
UHOLDEOH DQG LQH[SHQVLYH ZD\ VR SXPS ZDWU $ PRUH HILFLHQWGHVLJQ LV WKH SURPHOOHUPLOOLQ) LJ
E VLPLODU WR WKH SLRQHHULQJ 6PLWK3XQDP N WZR EODGHG V\VWHP WKDW SRSHUDWHG R
* UDPSDV. QRE PL ZHVWRI 5XWODQG 9HUPRQWIURP WR 7KH 6PLWK3XQDP GHVLJ
EURNHEHFDXVHRI LQDGHTXDWHEODGHVWUHQJWK EXWLWZ LWKVWRRG ZLQGVXS WR PLK DQGLWV HILFLHQF\
ZDVPS0 GHPRQVWUDWHG> @



(*a*)

(*b*)



(*c*)

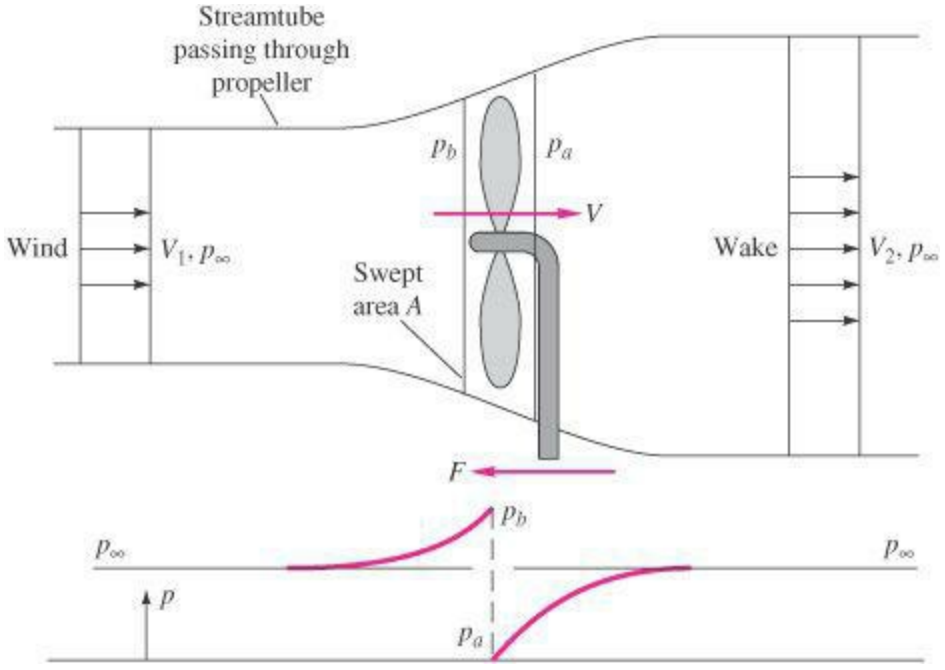
(*d*)

) LJ : LQG VXUELQH GHVLJQV *a* VKH \$PHULFDQ PXOWLEODGH IDUP + \$: 7 *b* D PRGHUQ VKURHEODGH N: + \$: 7 RQDZLQGIDUP LQ3 (RXDU] HO) UDQFH*(courtesy of Hubert Chanson)* *c* 7KH ' DUULHXV 9\$: 7*(courtesy of National Research Council Canada)* *d* D IRXUEODGH ' XWFK WLSHZLQGPLOOEXLOWLQ LQ- DPHVWRZQ 5KRGH, VODQG*(courtesy of F. M. White)*

7KH ' XFW $PHULFDQ PXOWLEODGH DQG SURSHOOHU PLOOV DUH H[DPSOHV RI KRUL]RQWDOO[LV ZLQ
VXUELQHV +$: 7V ZKLFK DUH HILFLHQWEXWRPHZKDWZ NZ DUGLWKDWWKH UHTXLUH[WQVLYHEDFLQ
DQGJHDUV\VWPVZKHQFRPELQHGZLWKDQHDFWLFJHQHUDWRU7KXVDFRPSHWQJ IDPLO RI YHUWFDOD[LV
ZLQG WXUELQHV 9$: 7V KDV EHHQ SURSRVHG WR VLPSOI\ JHDULQJ DQG WXQHQWWKUHTXLUHPHQW) LJXU
F VKRZ V\WH³HJEHDWJ 9$: 7 LQYHQWGE\ * - 0 ' DUULHXVLQ 7RPLQLPL]HFHQWULXJD
VWUHVV WKH WZLVWLQG EODGHV RI WKH ' DUULHXV WXUELQH IRORRZ DWRSRVNLQ FXUYH IRUPHG E\ D FKDL
DQFKRUHG DWZR SRLQWV RQ D VSLQQLQJ YHUWFDOURG 7KH ' DUULHXV GHVLJQ KDV WKH DGYDQWDJH WKDWWK
JHQHUDWRUDQGJHDUER[PD\ EH SODFHGRQWKHJURXQGIRUHDV\ DFFHVV ,WV QRWDV HILFLHQWWKRXJK DV
+$: 7 DQG IXUWKHUPRUH LWV QRVWHO VDUQJ 7KHOLJHWW DUULHXV GHYLFHNQRZ QWR WKHZ UWLVW
0: VXUELQH PLQGLDPHWU DW&DS&KDW4XpEHF &DQDGD
7KHIRXUDUP ' XFWK WSHZLQGPLOOLQ) LJ GLVIRXQG WKURXJKRXW(XURSHDQG WKH0 LGGO
(DVWDQGGDWVEDFNWR WKH WKFHQWXU ,QWKH1HWKHUODQGV WKH DUHSULPDULO XVHGWRGULQQRZ ODQG
7KHPLOLQWKHSKRWZDVEXLOWQ WRJULGFRQ ,WV QRZ D5HGH,VODQGWRXULVWDWWUDFWLRQ

,GHDOL]HG: LQG7XUELQH7KHRU\

7KHLGHDOIUFWRQOHVVHILFLHQF\ RI DSURSHOOHUZLQGPLOOZDVSUHGLFWHGE\ $ %HW]LQ XVLQ
WKHVLPXODWRQVKRZQLQ) LJ 7KHSURSHOOHULVUHSUHVHQWHG DQDFWXDDWUGLVN ZKLFK FUHDWH
DFURVV WKH SURSHOOHUSODQH D SUHVVXUH GLVFRQWLQXLW RI DUHD$ DQG GRFDO YHORFLW9 7KH ZLQG L
UHSUHVHQWHG DVXQLIRUPVWHDP WKEHIRUH DSSURDFKYHORFLW9 DQGDVVXPHGWR DNHYHORFLW9 7K
SUHVVXUHULVHVVRS E IXWKEHRUHWKHGLVNDQGGURSVVRS D IXWDWUHXQLQJ WRIUHHVWUHDP SUHVVXUHL
WKHIDZDNH 7RKRGWKHSURPHOOHUULJ ZKHQLWVH[WDFWQJHQHUJ\ IURP WKHZLQG WKHUHPXWEHI
OHVZDUGIRUFHI RQLWVXSSRUWDVVKRZQ



) LJ ,GHDO]HGDFWXDDWUGLVN DQG VWUHDP WXEHDQDO\VLVRI IORZ WKURXJK DZLQGPLOO

$ FRQWROYROXP H±KRUL]RQWDOP RP HQWXP UHDWRQDSSOHGEHWZHHQVHFWLRQV DQG JLYHV

) [ ) æ9 9

$ VLPLODU UHDVRQLQJ FRQFOXGHV WKDW

$$F_x = F_E - F_D = \dot{m}(V_D - V_E)$$

(TXDWLQJ WKHVH WZR \LHOGV WKH SURSHOOHU IRUFH

$$F = (p_b - p_a)A = \dot{m}(V_1 - V_2) \tag{11.42}$$

$V\VXPLQJ LGHDO IORZ WKH SUHVVXUHV FDQ EH IRXQG E\ DSSOLQJ WKH LQFRPSUHVVLEOH %HUQRXOOL UHDVRQLQJ WR SVR WKH GLVN

From 1 to b: $p_\infty + \frac{1}{2}\rho V_1^2 = p_b + \frac{1}{2}\rho V^2$

From a to 2: $p_a + \frac{1}{2}\rho V^2 = p_\infty + \frac{1}{2}\rho V_2^2$

6XEWUDFWLQJ WKHVH DQG QRWLQJ WKDW ! $9 WKURXJK WKH SURSHOOHU ZH FDQ VXEVWLWXWH IRU $p_E - p_D$ L

(T VR REWDLQ

$$p_b - p_a = \tfrac{1}{2}\rho(V_1^2 - V_2^2) = \rho V(V_1 - V_2)$$

or

$$V = \tfrac{1}{2}(V_1 + V_2) \tag{11.43}$$

&RQWLQXLW DQG PRPHQWXP WKXV UHTXLUH WKDW WKH YHORFLW9 WKURXJK WKH GLVN HTXDOV WKH DYHUDJH RI WK ZLQG DQG IDUZDNHV SHHGV

) LQDOO WKH SRZHU [WDFWHG E\ WKH GLVN FDQ EH ZULWWHQ LQ WHUPV RI 9 DQG9 E\ FRPELQLQJ (TV DQG

$$P = FV = \rho A V^2(V_1 - V_2) = \tfrac{1}{4}\rho A(V_1^2 - V_2^2)(V_1 + V_2) \tag{11.44}$$

) RUD JLYHQ ZLQG VSHHG9 ZH FDQ ILQG WKH PD[LPXP SRVVLEOH SRZHU E\ GLIIHUHQWDWLQJ 3 ZLW UHVSHFW WR9 DQG VHWWLQJ HTXDO WR]HUR 7KHUHVXOWV

$$P = P_{\max} = \tfrac{8}{27}\rho A V_1^3 \qquad \text{at } V_2 = \tfrac{1}{3}V_1 \tag{11.45}$$

ZKLFK FRUUHVSRQGV WR 9 9 WKURXJK WKH GLVN

7KH PD[LPXP DYDLODEOH SRZHU WR WKH SURSHOOHU LV WKH PDVV IORZ WKURXJK WKH SURSHOOHU WLPHV WKH VRXQG NLQHWLF HQHUJ\ RI WKH ZLQG

$$P_{\text{avail}} = \tfrac{1}{2}\dot{m}V_1^2 = \tfrac{1}{2}\rho A V_1^3$$

7KXV WKH PD[LPXP SRVVLEOH HILFLHQF\ RI DQ LGHDO IUFWLRQOHVV ZLQG WXUELQH LV XVXDOO VWDWHG L WHUPV RI WKH SRZHU FRHILFLHQW

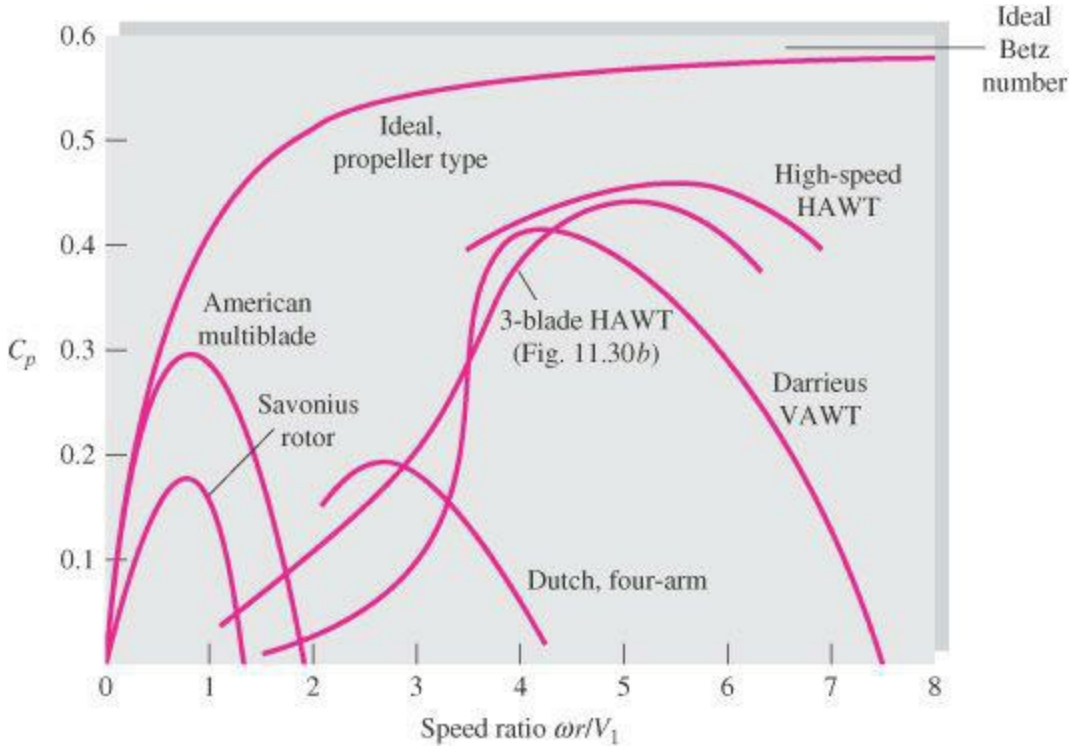
$$C_P = \frac{P}{\frac{1}{2}\rho A V_1^3} \tag{11.46}$$

(TXDWLRQ VWDWH VWKDW WKH WKHRUHWLFDO SRZHU FRHIILFLHQW LV

$$C_{p,\max} = \tfrac{16}{27} = 0.593 \qquad (11.47)$$

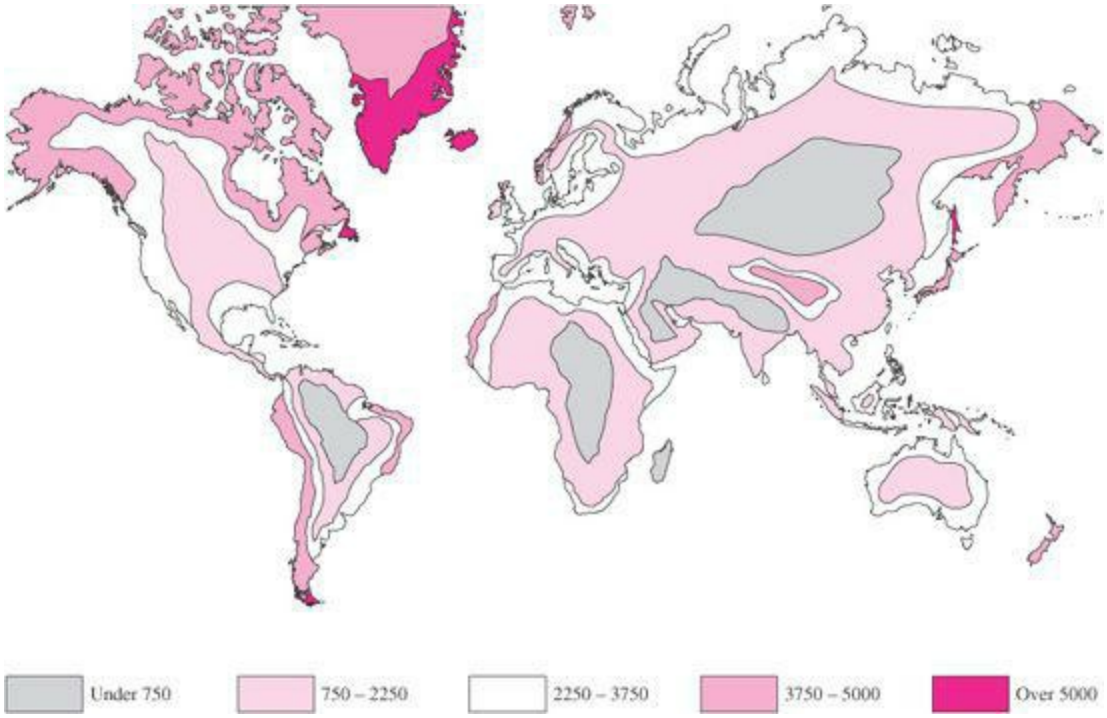
7KLV LV FDOOHG WKH %HWV QXPEHU DQG VHUYHV DV DQ LGHDO ZLWK ZKLFK WR FRPSDUH WKH DFWXDO SHUIRUPDQFH RI UHDO ZLQGPLOOV

) LJXUH VKRZV WKH PHDVXUHG SRZHU FRHIILFLHQWV RI YDULRXV ZLQG WXUELQH GHVLJQV 7KH LQGHSHQGHQW YDULDEOH LV QRW WKH DFWXDO URWDWLRQDO VSHHG EXW WKH UDWLR RI EODGH WLS VSHHG &UWR ZLQG VSHHG 1RWH WKDW WKH WLS FDQ PRYH PXFK IDVWHU WKDQ WKH ZLQG DIF GLVWXUELQJ WR WKH DLU EXW DP LQDU WR HQJLQHHUV LQ WKH EHKDYLRUR LFHERDW DQG VDLOLQJ YHVVHOV 7K ' DUULHXV KDV WKH PDQ DGYDQWDJHV RI D YHUWLFDO D[LV EXWKDV GLWWH WRUTXH DW ORZ VSHHGV VHH) LJ DQG DOVR URWDWHV P RUH VORZ O DWP D[LP XP SRZHU WKDQ DSURSHOOHU WKXV UHTXLULQJ DKJKHUJHD UDWLR IRU WKH JHQHUDWRU 7KH 6DYRQLXV URWRU) LJ E KDV EHHQ VXJJHVWHG DV D 9$: 7 GHVLJ EHFDXVH LWSURGXFHV SRZHU DWYHU ORZ ZLQG VSHHGV EXWLWV LQHILFLHQWDQG VXVFHSWLEOH WR VWRUP GDP DJHEHFDXVH LWFDQQRWEH IHDWKHUHG LQKJK ZLQGV



) LJ (VWLP DWHG SHUIRUP DQFH RI YDULRXV ZLQG WXUELQH GHVLJQV DV D IXQFWLRQ RI EODGH WLS VSHHG UDWLR *(From Ref. 53.)*

$ V VKRZ QLQ) LJ WKHUH DUH P DQ DUHDV RI WKH ZRUOG ZKHUH ZLQG HQHUJ\ LV DQ DWWUDFWLYH DOWHUQDWLYH VXFK DV ,UHODQG * UHDW %ULWDLQ ,FHODQG $UJHQWLQD &KLOH 1HZ =HDODQG DQG 1HZ IRXQGODQ 5RELQVRQ> @SRLQWV RXW WKDW $XVWUDOLD ZLWK RQO PRGHUDWH ZLQGV FRXOG JHQHUDWH KDOI LWV HOHFWULFL ZLWK ZLQG WXUELQHV ,Q[KDXVWEOH DQG DYDLODEOH WKH ZLQGV FRXSOHG ZLWK ORZ FRVW WXUELQH GHVLJQ SURPLVHD EULJKW IXWXUH IRU WKLV DOWHUQDWLYH



) LJ : RUOG DYDLODELOLW RI OQG EDVHG ZLQG HQHUJ\ HVWLPDWHG DQQXDO HQHUJ\ SURGXFWLR
N: KN: RI DZLQG WXUELQH DVGDW P V PLK *(From Ref. 54.)*

: LWK IRVVLO IXHO OLPLWDWLQG DQG LQFUHDVLQJ ZRUU DERXW JOREDO ZDUPLQJ WKH SURVSHFW IRU ZLQ
SRZHU DUH DWWUDFWLYH * OREDO ZLQG VSHHG GDWD E\ $UFKHUDQG-DFREVHQ> @VKRZ WKDWKDUQHVVLQJ
PRGHUWUDWLRQ RI DYDLODEOH ZLQG HQHUJ\ FRXOG VXSSO\ DOO RI WKH HDUWK¶V HOHFWULFLW\ QHHGV 1 RQ
H[SHFWWKDWRKDSSHQ EXWKH$PHULFDQ: LQG (QHUJ\ $VVRFLDWLRQ> @SURMHFWWKDW SHUFHQW
$PHULFD¶V SRZHUFRXOG EH ZLQG JHQHUDWHG E\ WKH\HDU $IWUDVORZ WDUWKH8 6 LQ
EHFDPHWKHFRXQWZ LWKWKHPRWZ LQG SRZHUJHQHUDWRQ
7KHJURZWKLQZLQG HQHUJ\ JHQHUDWRQKDVEHHQ[FHOOHQWbut *: LVWLOORQ SHUFHQWV
WRWDOZRUOG HOHFWULFLW\ 0RWWR WKHIXWXUH JURZWKZLOOSUREDEO EHODQGEDVHG EHFDXVHRI WK
KLJK FRWWRI FRQWUKFWRQ WDQVPLWWLRQ DQG PDLQWHQDQFH RIIVKRUH ZLQG WXUELQHV ,Q WKH8QW
6WDWHV HPSKDVLVKDV VKLWHG WR RFFDQ ZLQG WXUELQHV GXH WR WKHKLJK FRWWRI WDQVPLWWLRQ DQG
LVRODWHG ZLQG IDUPV)RU[DPSOH WKUHHVHSDUDWHJH 0: WXUELQHV DUHQRZ EHLQJ SODQQHG IR
WKHZULWLQJ VVPDOOVHDLGHYLGDJH ,QJHQHUDO FRWWZLOOEHKLJK DQG WKHSRODUHDOGHDWIUFWRX
EXWKHIXWXUHRI ZLQGHQHUJ\ VHHPVEULJKW

Country	Capacity in 2008, GW	Percent of world total
United States	25.2	20.8
Germany	23.9	19.8
Spain	16.8	13.9
China	12.2	10.1
India	9.6	8.0
Rest of the world	33.1	27.4
TOTALS	**120.8**	**100.0**

6XP P DU\

7XUERPDFKLQHU GHVLJQLVSHUKDSVWKHPRVWSUDFWDODQGPRVWDFWLYHDSSOLFDWLRQRI WKHSULQFLSOH
RI IOXLGPHFKDQLFV 7KHUHDUHELGDRQVRI SXPSVDQGWXUELQHVLQXVHLQWKHZRUOG DQGWKRXVDQGVR
FRPSDQLHVDUHVHHNLQJ LPSURYHPHQW 7KLVFKDSWHUKDVGLVFXVVHGERWKSRVLWLYHGLVSODFHPHQWG\QD
DQG PRUHH[WHQVLYHO URWRG\QDPLF PDFKLQHV : LWKWKHFHQWUXJDOSXPS DV DQH[DPSOH WKHEDVL
FRQFHSWV RI WRUTXH SRZHU KHDG IORZ UDWH DQG HILFLHQF\ ZHUH GHYHORSHG IRU D WKUERPDFKLQH
1RQGLPHQVLRQDO DQDO\VLV OHDGV WR WKH SXPS VLPLODULW\ UXOHV DQG VRPH WWSLFDO GLPHQVLRQOHV
SHUFRUPDQFHFXUYHVIRUD[LDOODQGFHQWULXJDOPDFKLQHV 7KHVLQJOHPRWXVHXOSXPS SDUDPHWULVWK
VSHFLILF VSHHG ZKLFK GHDO DW WKH WWSHRI GHVLJQ HGG $QLQWHUHVWLQJ GHVLJQ DSSOLFDWLRQ LV WK
WKHRU RI SXPSVFRPELQHGLQVHULHVDQGLQSDUDOOHO
7XUELQHV H[WDFWHQHUJ\ IURP IOXLGV DQG DUH RI VZR WWSHV LPSXOVH WXUELQHV ZKLF
FRQVWWKHPRPHQWXP RI DKLJK VSHHG VWHDP DQGUHDFWLRQWXUELQHV ZKHUH WKH SUHVXUH GURS RFFXU
ZLWKLQ WKH EODGH SDVVDJHV LQ DQ LQWHUQDOO IORZ %\ DQDORJ\ ZLWK SXPSV WKH SRZHU VSHFLILF VSHHG L
LPSRUWDQWRUWXUELQHV DQG LVXVHG WR FDWXL\ WKH LQIRUPDWLRQ) UDQFLV DQG SURPHOHU WWSHGHVLJQ
$ VPHFLDO FDVH RI UHDFWLRQ WXUELQH ZLWK XQFRQLQHG IORZ LV WKH ZLQG WXUELQH 6HYHUDO WWSHV R
ZLQGPLOOV ZHUHGLVFXVVHGDQGWKHUHODWLYHSHUFRUPDQFHVFRPSDUHG

3UREOHPV

0 RVWRI WKHSUREOHPVKHUHQDUHIDLUO VWUDLJKWRUZ DUG 0 RUHGLIILFXOWURSHQHQGHGDVLJQPHQ
DUHODEHOHGZLWKDQDVWHULVN 3UREOHPVODEHOHGZLWKDQ((6 LFRQ ZLOOEHQHILWURP WKHXVHRI WK
(QJLQHHULQJ (TXDWLRQ6ROYHU ((6 ZKLHSUREOHPVODEHOHGZLWKDFRPSXWHULFRQ PD\ UHTXLU
WKHXVHRI DFRPSXWHU 7KHVWDQGDUGHQGRI FKDSWHUSURBLEMS VR3 FDWDORJXHGLQWK
SUREOHP OLWWKHUH DUHIRORZHG E\ ZRUGSROBLEMS VR: FRPSUHHQVLYHPROBLEMS
& VR& DQGGHVLJQSURMHFWV

3UREOHP ' LVWULEXWLRQ

Section	Topic	Problems
11.1	Introduction and classification	P11.1–P11.14
11.2	Centrifugal pump theory	P11.15–P11.21
11.3	Pump performance and similarity rules	P11.22–P11.41
11.3	Net positive-suction head	P11.42–P11.44
11.4	Specific speed: mixed- and axial-flow pumps	P11.45–P11.62
11.5	Matching pumps to system characteristics	P11.63–P11.73
11.5	Pumps in parallel or series	P11.74–P11.81
11.5	Pump instability	P11.82–P11.83
11.6	Reaction and impulse turbines	P11.84–P11.99
11.6	Wind turbines	P11.100–P11.107

,QWURGXFWRQ DQG FODVVLILFDWRQ

- 3 ' HVFULEHWKHJHRPHWU DQGRSHUDWRQRI DKXP DQSHUWDOWF 3' 3 WKDWVFKHUWKHGE HYHU URP DQWF SHUWRQRQHDWK + RZ GRWKHVZR YHQWLFDVGLIIHU'
- 3 : KDWZRXGEHWKHVWFKQLFDOFDVLILFDWRQ RI WKHIORZLQJ WXUERPDFKLQHV D I KRXVHKROGIDQ E DZLQGPLOO F DQDLUFUDIWSURSHOOHU G DIXHOSXPSLQDFU H DQHGXFWRU I DIOXLG FRXSOLQJ WDQVPLWWRQ DQG J DSRZHUSODQWWMP WXUELQH'
- 3 $ 3' 3 FDQGHOYHUDOPRWDQ IOXLG EXWKHUHVDOZD\VDOPLWQJ YHU KJKYLVFRVLW

IRUZ KLFK SHUFHQWDJH ZLOO GHFUHDVH &DQ\RX H[SODLQ WKH SUREDEOH UHDVRQ"

- 3 $QLQWHUHVWLQJ VLGH ERPDFKLQH LV WKH WRUTXH FRQYHUWHU ZKLFK FRPELQHV ERWK DSP DQG D VLGH WR FKDQJH WRUTXH EHWZHHQ WZR VKDIWV ' R VRPH UHVHDUFK RQ WKH FRQFHSWDQGGHVFUEHLWZ LWKDUHSRUWVNHWKHV DQGSHUFDQGHGDW WRWKHFDW

- 3 : KDWWSHRI SXPSLVVKRZQLQ) LJ 3 " +RZ GRHVLWSHUDW

3

- 3) LJXUH3 VKRZ VWZR SRLQWDKDO SHUGDSDUWLQWKHRSHUDWRQRI DSXPS : KD WWSHRI SXPSLVWKLV> @+RZ GRHVLWZRUN' 6NHWK\RXEHWWXHWVRI IORZ UDW YHUXVWLPHIRUDIHZ F\FOHV

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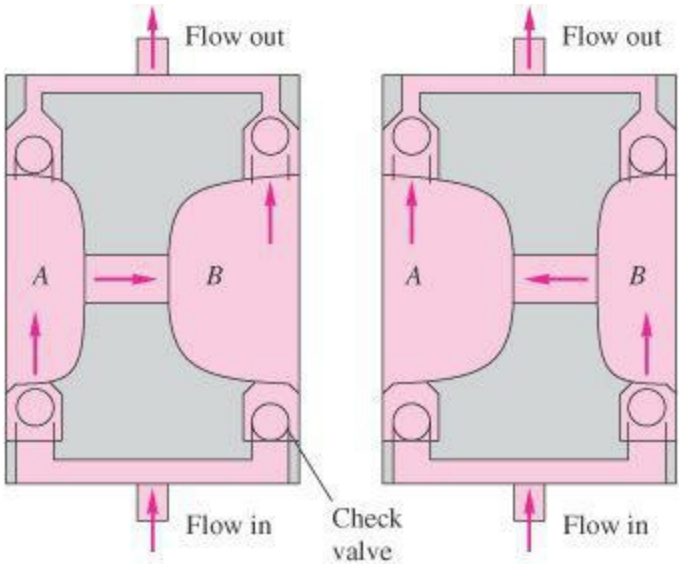


- 3 $ SLVWRQ3' 3 KDVD LQGLDPHWUDQGD LQWXRNHDQGRSHUDWVDW UPLQZLWK SHUFHQWYROXPHWULF HILFLHQF\ D : KDWLVLWGHOLYHU LQJDOPLQ' E ,I WKHSXPS GHOLYHUV6$(: RLODW *& DJDLQWDKHDGRI IWZKDWKRUVHSRZHULVUHTXLU ZKHQWKHRYHUDOOHILFLHQF\ LV SHUFHQW

- 3 $ FHQWULXJDO SXPS GHOLYHUV JDOPLQ RI ZDWUDW *& ZKHQ WKH EUDN KRUVHSRZHULV DQG WKH HILFLHQF\ LV SHUFHQW D (VWLPDWWKHKHDGULVHIQI DQG WKH SUHVVXUHULVHQGE LQ E $OR HVWLPDWWKHKHDGULVHDQGKRUVHSRZHUL LQWKDGWKHGHOLYHU LV JDOPLQRI JDVROLQHDW *&

- 3) LJXUH3 VKRZ VWKHPHDVXUHGSHUPDQFHRI WKH9LFNHUVPRGHO394 SLWWV SXPS ZKHQGHOLYHULQJ6$(: RLODW *) ! § NJ P 0 DNHVRPHJHQHUL

REVHUYDWRQV DERXWWKHVH GDWD YLVj YLV) LJ DQG \RXU LQWHUSRQ DERXWWK
EHKDYLRUR SLWRQSXPSV

- 3 6XSSRVHWKDWWKHSXPS R) LJ 3 LVUXQDW UPLQDJDLQWDSUHVVXUHULVHR
EDU D 8VLQJ WKHPHDVXUHGGLVSODFHPHQWPDWWKHWKRUHWLFDOGHOYHU L
JDOPLQ)URP WKH FKDUWHWWDW E WKH DFWXDO GHOYHU DQG F WKH RYHUDO
HIILFLHQF\

- 3 $ SXPS GHOYHUV / PLQR ZDWUDW *& DJDLQWDSUHVVXUHULVHR N3D
. LQHWF DQGSRWQWLDOHQHUJ\ FKDQJHVDUHQHJOLJLEOH ,I WKHGULYLQJ PRWRUXSSOHV
N ZKDWLVWKHRYHUDOOHIILFLHQF\"

- 3 EES ,Q DWWWR WKHFHQWULXJDOSXPS VKRZQLQ) LJ 3 WKHIROORZLQJ GDWDU
VDNHQ S PP+J YDFXXP DQGS PP+J JDJH 7KHSLSHGLDPHWU
DUH' FP DQG' FP 7KHIORZ UDWLV JDO PLQR OJKWRLO 6*
(VWLPDWH D WKHKHDGGHYHORSHG LQPHWUV DQG E WKHLQSXWSRZHUUHTXLUH
DW SHUFHQWHILFLHQF\

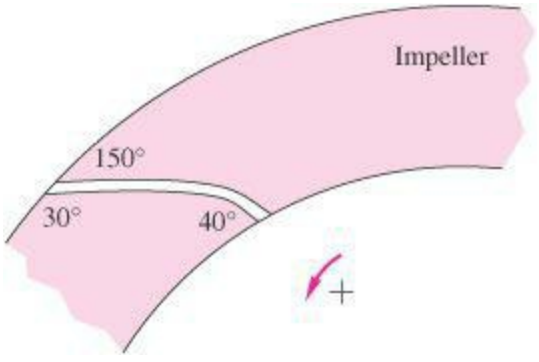
- 3 $ KSSXPS GHOYHUV OEI RI HWKDQHQHJOFRODW *& LQ VHFRQGV DJDLQWK
KHDGRI IW&DOFXODWWKHHILFLHQF\ RI WKHSXPS

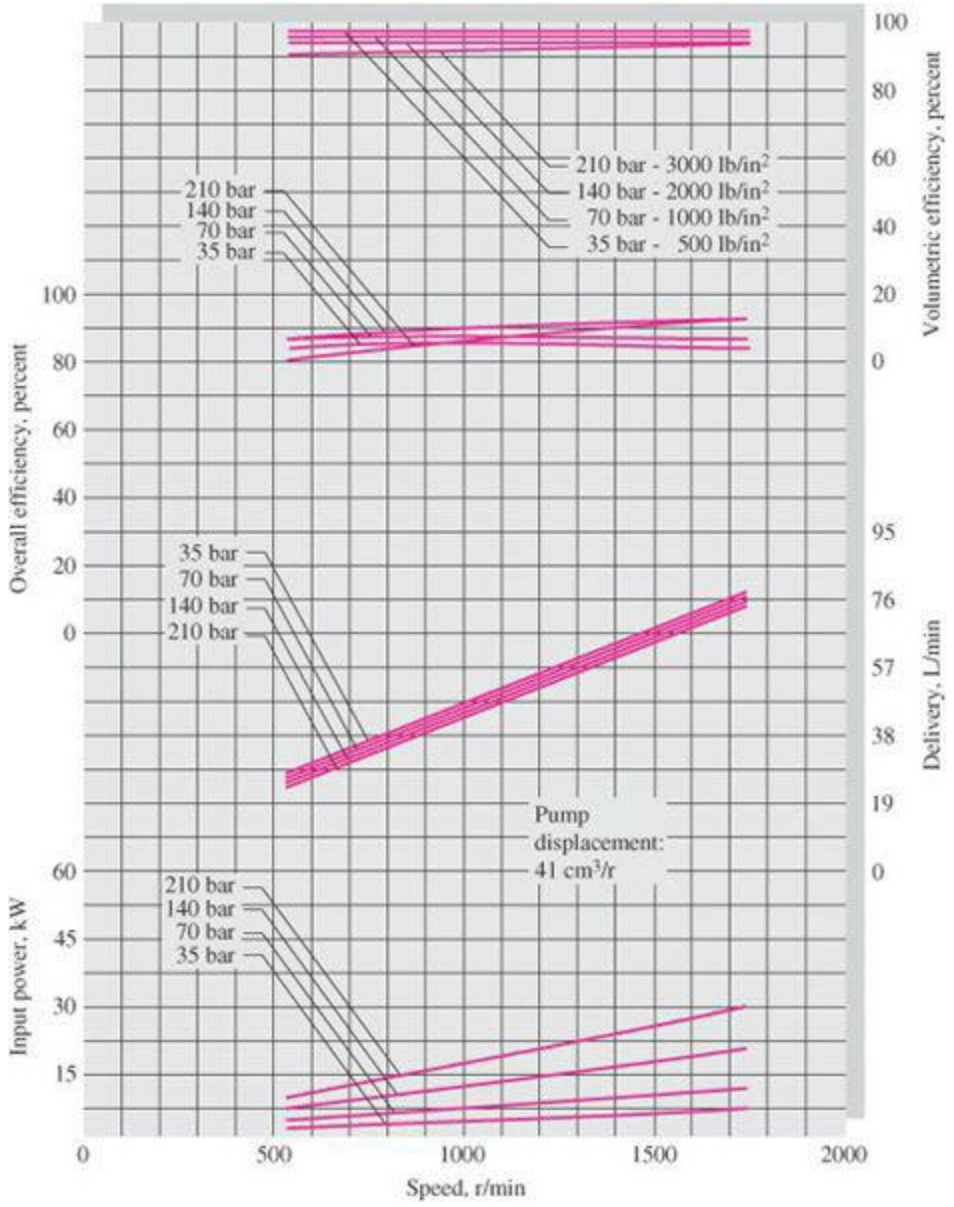
- 3 $ SXPS GHOYHUVJDVROLQHDW *& DQG P K $WWKHLQOHWS N3D] P
DQG9 P V $WWKH H[LWS N3D] P DQG9 P V +RZ PXF
SRZHULVUHTXLUHGLI WKHPRWRUHILFLHQF\ LV SHUFHQW

&HQWULIXJDOSXPS WKHRU\

- 3 $ GZ QVSULQNOHUFDQEHXVHGDVDVLPSOHWXEELQH $VVKRZQLQ) LJ 3 IORZ
HQWHUVQRUPDOWRWKHSDSHULQWKHFHQWUDQGVSOLWVHYHQO LQWR4 DQG9 $WKH
HDFKQR]]OH 7KHDUP VURWDWHDWDQJXODUYHORFLW& DQGGRZRUNRQDVKDW UDWH WK
YHORFLWGLDJUDP IRUWKLV WXEELQH 1HJOHFWQJ IULFWLRQ ILQG DQ H[SUHVVLRQ IRUWK
SRZHU GHOYHUHG WR WKH VKDW) LQG WKH URWDWLRQ UDWH IRU ZKLFK WKH SRZHU LV
PD[LPXP

- 3 7KHFHQWULXJDOSXPS LQ) LJ 3 KDVU FP U FP E E FP DQ
URWDWHVFRXQWHUFORFNZLVHDW UPLQ $VDPSOHEODGHLVVKRZQ $VVXPH. *
(VWLPDWH WKH WKHRUHWLFDO IORZ UDWH DQG KHDG SURGXFHG IRUZDWU DW *& DQ
FRPPHQW





3 3HUIRUPDQFHRI WKHPRGHO394 SLVWRQSXPS GHOLYHULQJ6$(: RLODW *f*

(Courtesy of Vickers Inc., PDN/PACE Division.)

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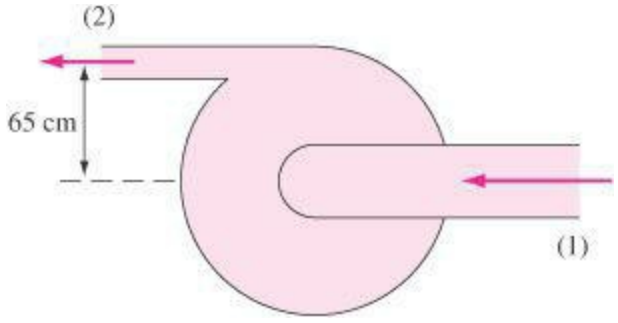

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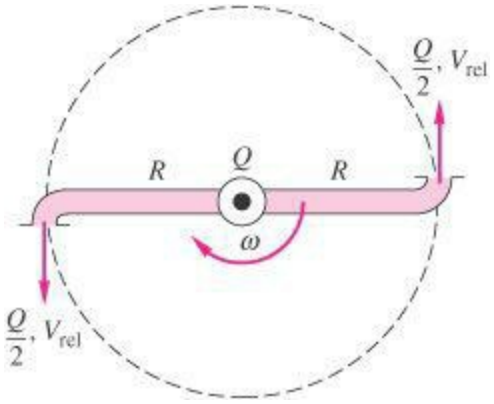
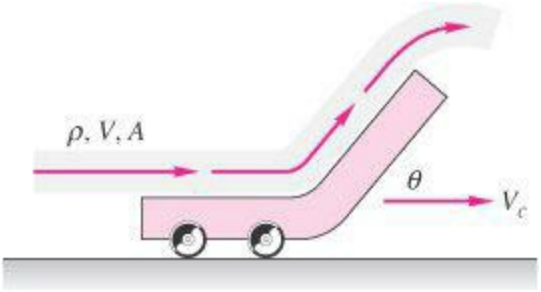


- 3 $ FHQWUIXJDOSXPS KDVG LQG LQE LQE LQ *f* DQG *f* DQGURWDWVDW UPLQ ,I WKHIOXLGLVJDVRQQHDW *f*& DQGWKHIORZ HQWHUVWKH EODGHVUDGLDOO HWDQJHQWLDOWKHWKRUHWLFDO D IORZ UDWHLQJDOPLQ E KRUVHSRZHU DQG F KHDGLQIW
- 3 $ MHWRI YHORFLW9 VWULNHVD YDQHWKDWPRYHVWKHULJKWDWVSHHG9$ DVLQ) LJ 3 7KH YDQH KDV D VWUDLJKW DQJOH ' HULYH DQ H[SUHVVLRQ IRU WKH SRZHU GHOLYHUHGWRWKHYDQHE\ WKHMHW) RUZKDWYDQHVSHHGLVWKHSRZHUPD[LPXP"

3



- 3 $ FHQWUIXJDOSXPSKDVU LQE LQDQG *f* DQGURWDWVDW UPLQ ,I LWJHQHUDWVDKHDGRI IWGHWHUPLQHWKHWKRUHWLFDO D IORZ UDWHLQJDOPLQDQ E KRUVHSRZHU $VXPHQHDUUDLDOIORZ IORZ
- 3 6XSSRVHWKDW3URE 3 LVUHYHUVHGLQRUDWDWPHQWRI WKHWKRUHWLFDOSRZHU3$ §KS &DQ\RXWKHQFRPSXWHWKHWKRUHWLFDO D IORZ UDWHDQG E KHDG' ([SODLQ DQGUHVROYHWKHGLIILFXOWVWKDWDULVH
- 3 7KH FHQWUIXJDOSXPS RI) LJ 3 GHYHORSV D IORZ UDWH RI JDOPLQ R JDVROLQH DW *f*& ZLWK QHDUUDLDOEODGHVRXWLQIORZ (VWLPDWHWKH WKRUHWLFDO D KRUVHSRZHU E KHDGULVH DQG F DSSURSULDWHEODGHQJOHDWWKHLQQHUDGLXV

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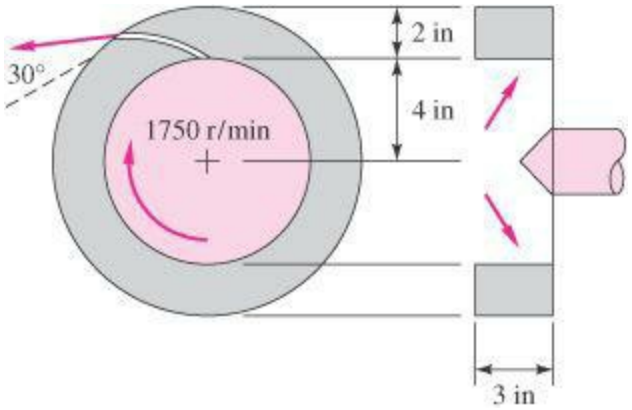


3XP S SHUIRUP DQFH DQG VLP LODULW UXOHV

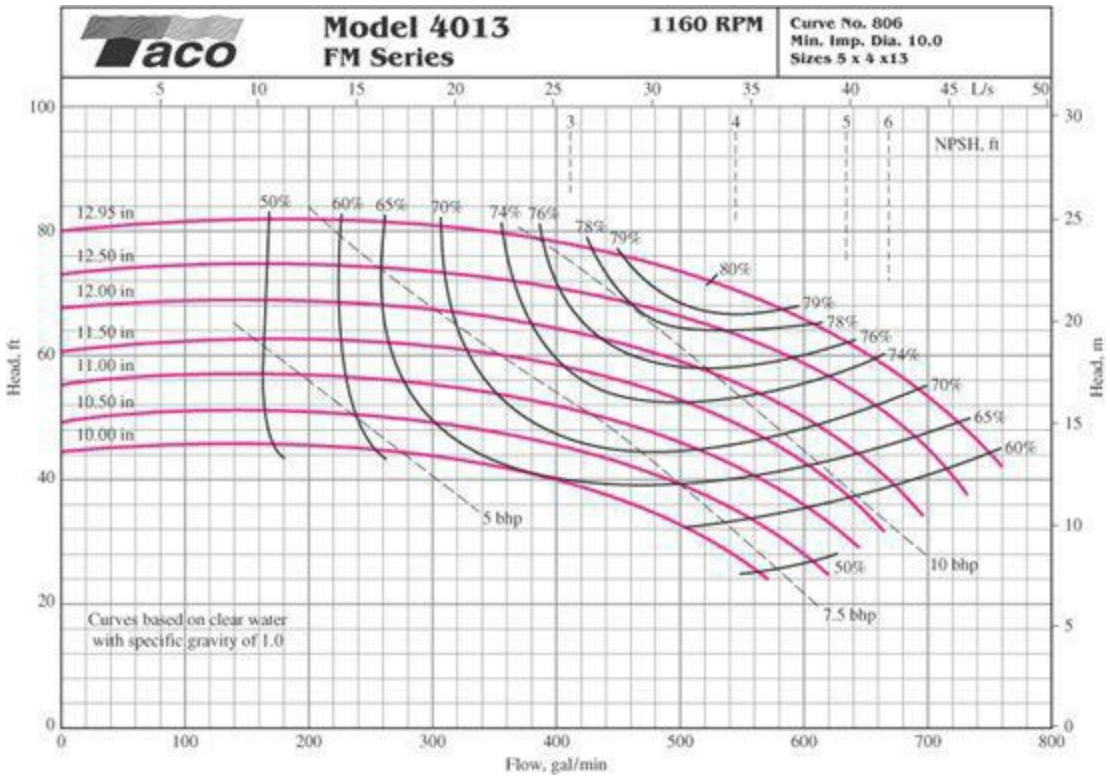
- 3 $ FP GLDP HWU FHQWULXJDO SXP S UXQQLQJ DW UPLQ Z LWK Z DWHU DW *& SURGXFHV WKH IROORZ LQJ SHUIRUP DQFH GDWD
  D ' HWHUPLQH WKH EHVW HILFLHQF\ SRLQW E 3ORW &+ YHUVXV &4 F ,I Z H GHVLUH WR XV WKLV VDP H SXP S IDP LO\ WR GHOLYHU JDOOLQRI N-HURVHQH DW *& DW DQ LQSXW SRZ HU N Z KDW SXP S VSHHG LQ UPLQ DQG LPSHOOHU]H LQFP DUH QHHGHG' : KDW KHDG Z LO EH GHYHORSHG'

Q, m³/s	0.0	0.05	0.10	0.15	0.20	0.25	0.30
H, m	105	104	102	100	95	85	67
P, kW	100	115	135	171	202	228	249

- 3 $ WZ KDWSHHG Z KHQ SXP SLQJ Z DWU VKRXOG WKH LQSXP S RI) LJ E EH UX VR UHDQ]H D DKHDG RI IW) RU WKLV FRQGLWLRQ Z KDW DUH WKH UHVXOWLQJ E IORZ UDWH DQG F EUDNHKRUVHSRZ HU'
- 3) LJXUH 3 VKRZV SHUIRUP DQFH GDWD IRU WKH 7DFR ,QF PRGHO SXP S &RPSXWH WKH UDWRV RI PHDVXUHG VKXWRII KHDG WR WKH LGHDOYDOXH8 J IRU DOO VHYH LPSHOOHU VL]HV ' HWHUPLQH WKH DYHUDJH DQG VWDQGDUG GHYLDWLRQ RI WKLV UDWLR DQ FRPSDUH LWV WR WKH DYHUDJHIRU WKH VL[LPSHOOHUVLQ) LJ
- 3 $ WZ KDWSHHG LQ UPLQ VKRXOG WKH LQGLDP HWU SXP S RI) LJ E EH UXQ V SURGXFH DKHDG RI IWDWDGLVFKDUJHRI JDOOLQ' : KDWEUDNHKRUVHSRZ H Z LOOEH UHTXLUHG' +LQW) LW 4 VR DIRUP XOD
- 3 : RXOG WKH VPDOOHVW WRUWKHOUJHVW WR WKH VHYHQ7DFR,QF SXP SVLQ) LJ 3 EH EHWWU D IRUSURGXFLQJ QHDUEHVWHILFLHQF\ DZ DWU IORZ UDWRI JDOOLQDQGI KHDGRI IW E $ WZ KDWSHHG LQUPLQ VKRXOGWKLVSXP S EHUXQ' F : KDWLQSX SRZ HULVUHTXLUHG'
- 3 EES 7KH LQSXP S RI) LJ 3 LVWR EHVFDOHGXS LQ VL]H WR SURYLGH DKHDG IWDQG D IORZ UDWH RI JDOOLQ DW%(3 ' HWHUPLQH WKH FRUUHFWD LPSHOOH GLDP HWU E VSHHGLQUPLQ DQG F KRUVHSRZ HUUHTXLUHG
- 3 7HVWVE\ WKH %URQ-DFNVRQ &R RI D LQGLDP HWU FHQWULXJDOZ DWU SXP S D UPLQ\ LHOGVWKH IROORZ LQJ GDWD
  : KDWLV WKH %(3" : KDWLV WKH VSHFLILF VSHHG' (VWLP DWH WKH PD[LPXP GLVFKDUJ SRVVLEOH

Q, ft^3/s	0	2	4	6	8	10
H, ft	340	340	340	330	300	220
bhp	135	160	205	255	330	330

- 3 ,I WKH VFDOQJ GDZ V DUH DSSOLHG WR WKH SXPS RI 3URE 3 IRU WKH VDPH LPSHOOHU GLDPHWHU D WKH VSHHG IRU ZKLFK WKH VKXWRII KHDG ZLOO EH IW E WKH VSHHG IRU ZKLFK WKH %(3 IORZ UDWH ZLOO EH F WKH VSHHG IRU ZKLFK WKH %(3 FRQGLWLRQV ZLOO UHTXLUH KS
- 3 $ SXPS JHRPHWULFDOO VLPLODU WR WKH LQGLFDWHG LQ GLDPHWHU UHSUHVHQWV D VHULHV RI GHVLJQV LQ ZKLFK DOO GLPHQVLRQV DUH LQ WKH VDPH UDWLR ' HWHUPLQH D WKH DSSURSULDWH VSHHG LQ USM E WKH %(3 KHDG DQG F WKH %(3 IORZ UDWH



3 3HUFHQWDJH GDWD IRU D FHQWULIXJDO SXPS *(Courtesy of Taco, Inc., Cranston, Rhode Island.)*

- 3 $ FHQWULIXJDO SXPS ZLWK EDFNZDUG FXUYHG EODGHV KDV WKH IROORZLQJ PHDVXUHG SHUFHQWDJH ZKHQ WKH ZDWHU DW *f*& D (VWLPDWH WKH EHVW HILFLHQF\ SRLQW DQG WKH PD[LPXP HILFLHQF\ E (VWLPDWH WKH PRV HILFLHQW ORZ UDWH DQG WKH UHVXOWLQJ KHDG DQG EUDNH KRUVHSRZHU LI WKH GLDPHWHU LV GRXEOHG DQG WKH URWDWLRQDO VSHHG LQFUHDVHG E\ SHUFHQW

Q, gal/min	0	400	800	1200	1600	2000	2400
H, ft	123	115	108	101	93	81	62
P, hp	30	36	40	44	47	48	46

- 3 7KH GDWD IRU 3URE 3 FRUUHVSRQG WR D SXPS VSHHG RI UPLQ : HUH\ RX DEOH WR VROYH 3URE 3 ZLWKRXW WKLV NQRZOHGJH' D (VWLPDWH WKH GLDPHWUL

VKHLPSHOOHU +LQW6HH3URE 3 IRUDFXH
E 8VLQJ\RXUHVWLPDWHIURP SDUWD FDOFXODWHWKH%(3SDUDPHWHUV C_Q^*, C_H^*, and C_P^* DQ
FRPSDUHZLWK(TV F)RUZKDWVSHHGRI WKLVSXPSZRXOGWKH%(3KHDGEH
IW
- 3 ,Q3URE 3 WKHSXPS%(3ICRZ UDWLV JDOPLQWKHLPSHOOHUGLDPHWUL
LQDQGWKHVSHHGLV UPLQ6FDOHWKLVSXPSZLWKWKHVLPLODULWXOHVWRILQ
D WKHGLDPHWUDQG E WKHVSHHGWKDWZLOOGHOLYHUD%(3ZDWUICRZ UDWRI
JDOPLQDQGDKHDGRI IW F :KDWEUDNHKRUVHSRZHUZLOOEHUHTXLUHGIRUWKL
QHZ FRQGLWLRQ'
- 3 <RXDUHDVNHGWRFRQVLGHUDSXPSJHRPHWUFDOO VLPLODUWRWKH LQGLDPHWU7DFH
SXPS RI)LJ 3 WR GHOLYHU JDOPLQ DW UPLQ ' HWHUPLQH WK
DSSURSULDWH LPSHOOHUGLDPHWU E %(3KRUVHSRZHU F VKXWRII KHDG DQG G
PD[LPXP HILFLHQF\ 7KHIOXLGLVNHURVHQH QRWZDWU

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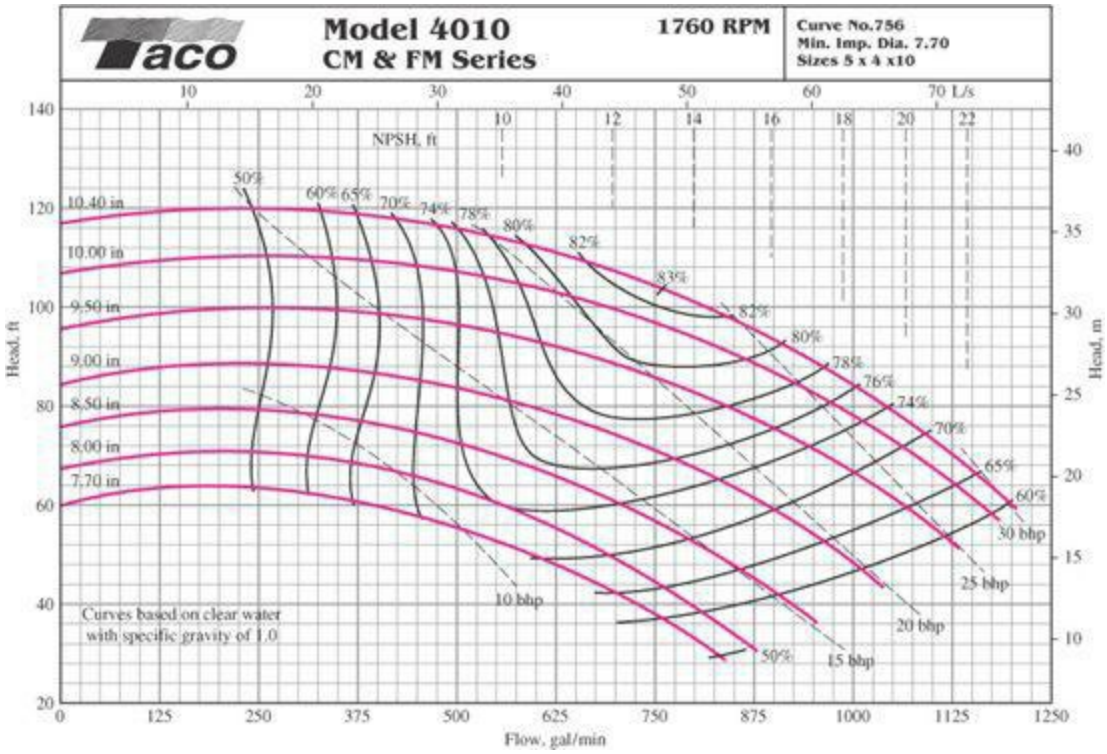


3HUFHQWDJHGDWDIRUDIDPLO RI FHQWULXJDOSXPSLPSHOHUV*(Courtesy of Taco, Inc., Cranston, Rhode Island.)*

- 3 $Q LQGLDPHWUFHQWULXJDOSXPS UKQQLQJ DW UPLQZLWK ZDWU DW *f*&
JHQHUDWHVWKHIROORZLQJSHUIRUPDQFHGDWD
' HWHUPLQH D WKH%(3 E WKHPD[LPXP HILFLHQF\ DQG F WKHVSHFLILF VSHG G 3OI
WKHUHTXLUHGLQSXWSRZHUYHUXVWKHICRZ UDWH

Q, gal/min	0.0	2000	4000	6000	8000	10,000
H, ft	92	89	84	78	68	50
P, hp	100	112	130	143	156	163

- 3 7KHSXPS RI3URE 3 KDVDPD[LPXP HILFLHQF\ RI SHUFHQWDW JDOPLQ

D &DQZHXVHWKLVSXPS DWWKHVDPHGLDPHWHUEXWDGLIIHUHQWVSHHG WRJHQHUDWH %(3 KHDG RI IWDQG D %(3 IORZ UDWH RI JDOPLQ' E ,I QRWZKDW GLDPHWHULVDSSURSULDWH'

- 3 &RQVLGHUWKHVWRSXPSVRI 3UREOHPV3 DQG3 ,I WKHGLDPHWHUVDUHQRW FKDQJHG ZKLFKLVEHWWHUIRUGHOLYHULQJ ZDWHUDW JDOPLQDQGDKHDGRI IW : KDWLVWKHDSSURSULDWHURWDWLRQVSHHGIRUWKHEHWWHUSXPS'
- 3 EES $ LQSXPS UXQQLQJ DW UPLQ KDVWKHIROORZLQJ PHDVXUHGSHUIRUPDQFH IRUZDWHUDW *&

D (VWLPDWHWKHKRUVHSRZHUDW%(3 ,I WKLVSXPS LV UHFDOHGLQZDWHUWR SURYLGH EKS DW UPLQ GHWHUPLQHWKHDSSURSULDWH E LPSHOOHUGLDPHWHU F IORZ UDWH DQG G HILFLHQF\ IRUWKLVQHZ FRQGLWLRQ

Q, gal/min	50	100	150	200	250	300	350	400	450
H, ft	201	200	198	194	189	181	169	156	139
η, %	29	50	64	72	77	80	81	79	74

- 3 7KH$GDV&KDOPHUV' /5 FHQWULXJDOFRPSUHVVRUGHOLYHUV IWPLQRI 62 ZLWKDSUHVVXUHFKDQJHIURP VR GBE LQ DEVROXWHXQLWV DQ KSPRWRUD UPLQ : KDWLVWKHRYHUDOOHILFLHQF\" : KDWZLOOWKHIORZ UDWHDQGûS EH D UPLQ' (VWLPDWHWKHGLDPHWHURI WKHLPSHOOHU
- 3 7KHVSHFLILFVSHHG1V DVGHILQHGE\ (TV GRHVQRWFRQWDLQWKHLPSHOOHU GLDPHWHU+RZ WKHQVKRXOGZHVL]HWKHSXPSIRUDJLYHQ1V" / RJDQ> @XJJHVWVI SDUDPHWHUFDOHGWKHVSHFLILFGLDPHWHU'V ZKLFKLVDGLPHQVLRQOHVVFRQELQDWLRQR 4 J+ DQG' D ,I 'V LVSURSRUWLRQDOWR' GHWHUPLQHLWVIRUP E : KDWLVWK UHODWLRQVKLS LI DQ RI 'V WR&4 &+ DQG&3 " F (VWLPDWH'V IRUWKHVZRSXPS RI) LJV DQG
- 3 ,WLV GHVLUHG WR EXLOG D FHQWULXJDOSXPS JHRPHWUFDOO VLPLODU WR WKDWRI 3URE 3 WR GHOLYHU JDOPLQ RI JDVROLQH DW *& DW UPLQ (VWLPDWHWK UHVXOWLQJ D LPSHOOHUGLDPHWHU E KHDG F EUDNHKRUVHSRZHUDQG G PD[LPXP HILFLHQF\

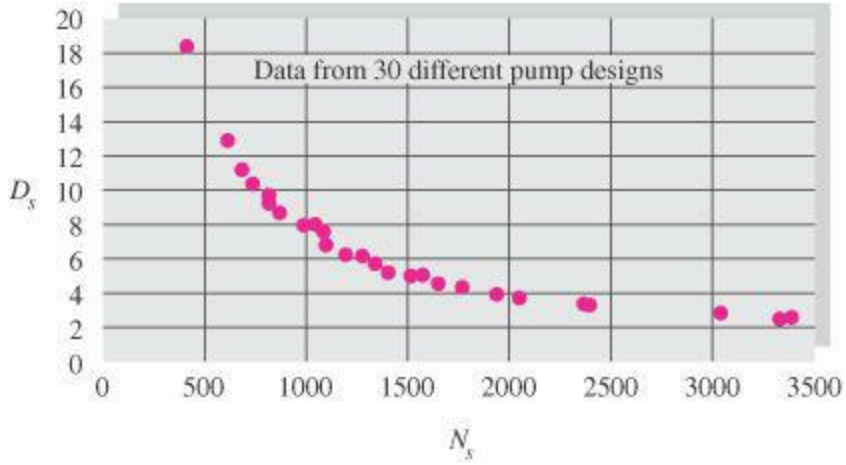
1HWSRVLWLYHVXFWLRQKHDG

- 3 $Q LQPRGHOSXPS GHOLYHULQJ *) ZDWHUDW JDOPLQDQG UPLQEHJLQ WR FDYLWDWHZKHQWKHLQOHWSUHVVXUHDQGYHORFLW DUH GBE LQ DEVROXWHDQG IWV UHVSHFWLYHO) LQGWKHUHTXLUHG136+ RI DSURWRWSHWKDWV WPHVODJHUDQGUXQ DW UPLQ
- 3 7KH LQGLDPHWHUSXPSLQ) LJ DDW UPLQLVXVHGWRSXPSZDWHUDW *& WKURXJK D SLSLQJ V\VWHP DW JDOPLQ D ' HWHUPLQHWKH UHTXLUHG EUDNKRUVHSRZHU 7KHDYHUDJHIULFWLRQIDFWRULV E ,I WKHUHLV IWRI LQ GLDPHWHUSLSHXSWUHDP RI WKHSXPS KRZ IDUEHORZ WKHVXUIDFHVKRXOGWKHSXPS LQGHWEHSODFHGWRDYRLGFDYLWDWLRQ'
- 3 7KHSXPS RI 3URE 3 LVVFDOHGXS WR DQ LQGLDPHWHUSRHUDWLQJ LQZDWHUD EHWWHHILFLHQF\ DW UPLQ 7KHPHDVXUHG136+ LV IWDQGWKHIULFWLRQORV

EHZHHQWKHLQDWDQGWKHSXPSLV IW: LOOLWEHVXILFLHQWRDYRLGFDYLWDWRQL
WKHSXPSLQDWLVSODFHG IWEHORZ WKHVXUIDFHRI DVHDOHYHOUHVHUYRLU'

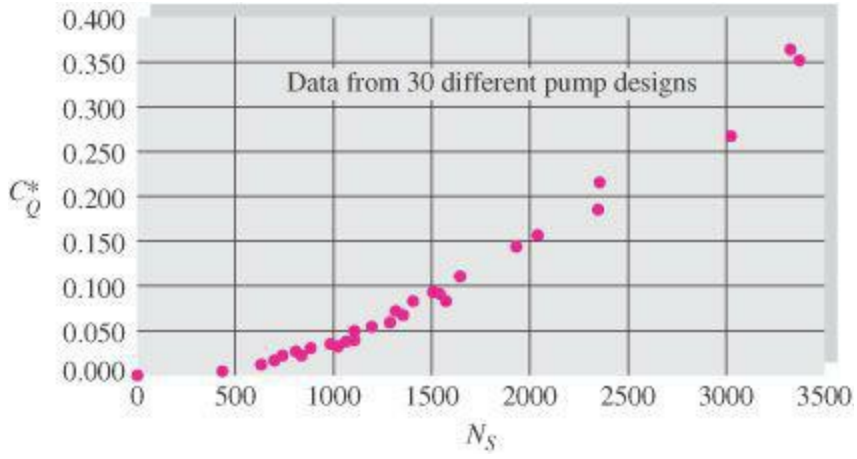
6SHFLILFVSHHG PL[HG DQGD[LDOIORZ SXPSV

- 3 ' HWHUPLQH WKH VSHFLILF VSHHGV RI WKH VHYHQ7DFR ,QF SXPS LPSHOOHUV LQ) LJ 3 $UH WKH\ DSSURSULDWHIRUFHQWULXJDOGHVLJQV" $UH WKH\ DSSUR[LPDWD HTXD ZLWKLQH[SHULPHQWDOXQFHUWDLQW\" ,I QRWZK\ QRW
- 3 7KHDQZHUWR3URE 3 LVWKDWWKHGLPHQVLRQOHVV³VSHFLILFGLPHWJ VDNHVWK IRUP ' V ' J+ 4 HYDOXDWHGDWWKH%(3 ' DWFRQGHFWGE\ WKHDXWKRUIR GLIIHUHQWSXPSVLQGLFDWH LQ) LJ 3 WKDW V FRUUHODWHZHOZLWKVSHFLI VSHHG1 V 8VHWKLVILJXUHWRHVWLPDWWKHDSSURSULDWLPSHOOHUGLPHWUIRUDSXP WKDWGHOYHUV JDOPLQRI ZDWUDQG DKHDGRI IWZKHQUXQQLQJ DW UPLQ 6XJJHVWDFXUYHILWWGIRUPXODWRWKHGDW + LQW8VHDK\SHUEROF IRUPXOD



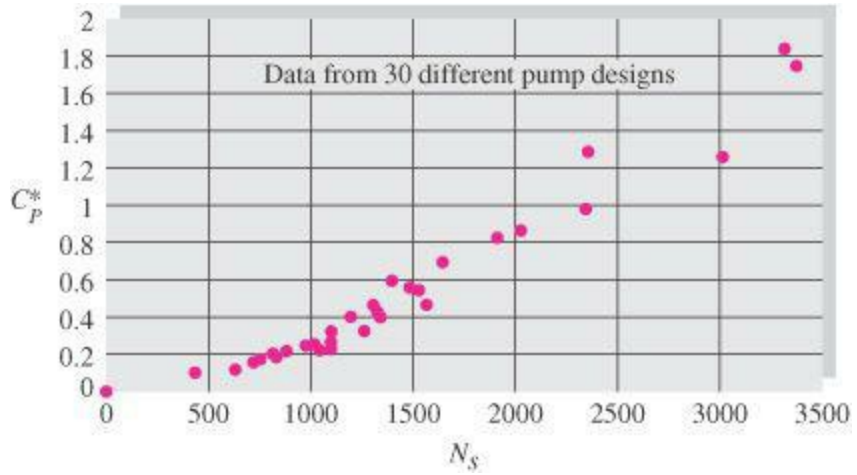
3 6SHFLILFGLDPHWUDW%(3 IRU FRPPHUFLDOSXPSV

- 3 $ SXPS PXVWEHGHVLJQHGWRGHOLYHU P VRI ZDWUDJDLQWDKHDGRI P 7K VSHFLILHGVKDWWSHHGLV UV: KDWWSHRI SXPSGR\RXUHFRPPHQG'
- 3 $ FRPPHUFLDOSXPS UKQVDW UPLQDQGGHOLYHU QHU%(3 DIORZ RI JDOPLQDQGDKHDGRI P D: KDWWSHRI SXPSLVWKLV" E (VWLPDWWKHLPSHOOH GLPHWUXVLQJ WKHGDWDRI3URE 3 F (VWLPDW$C*D DQGDGGDQRWKHUGDWSRLQ WR) LJ 3
- 3 ' DWFRQGHFWGE\ WKHDXWKRUIRUIORZ FRHILFLHQWDW%(3 IRU GLIIHUHQWSXPSVDL SORWWGYHUXV VSHFLILF VSHGLQ) LJ 3 ' HWHUPLQHL WKHYDOXHVRI C*D IRUWK WKUHHSXPSVLQ3UREV 3 3 DQG3 DOVRILWWRQWKLVFRUUHODWLRQ,I VF VXJJHVWDFXUYHILWWGIRUPXODIRUWKHGDW



3) GRZ FRHILFLHQWDW%(3 IRU FRPPHUFLDOSXPSV

- 3 ' DWDFRQILUPHGE\ WKHDXWKRUIRUSRZHUFRHILFLHQWDW%(3 IRU GLIHUHQWSXPS DUHSORWWHGYHUVXVSHFLILFVSHHGLQ) LJ 3 ' HWHUPLQHLWKHYDOXHVRI$ IR WKHWKUHSXPSVLQ3URE 3 DSSRILWRQWKLVFRUUHODWLRQ ,I VR VXJJHVWDFXUYHILWWHGIRUPXODIRUWKHGDWD



3 3RZHUFRHILFLHQWDW%(3 IRU FRPPHUFLDOSXPSV

- 3 $QDLOOIGRZ EGRZHUGHOYHUV IWVRI DLUWKDWHQWHUV DW f& DQG DW 7K IGRZ SDWDJHKDVD LQRXWHUUDGLXVDQGDQ LQLQQHUUDGLXV %DGHDQJOHVDUH. f DQG f DQGWKHURWRUUXQVDW UPLQ)RUWKHILUWWXDJHFRPSXWH D WKHKHDGULVHDQG E WKHSRZHUUHTXLUHG
- 3 $QDLOOIGRZ IDQRSHUDWV LQ VHDOHYHOODLU DW UPLQ DQG KDV D EODGH WLS GLDPHWURI P DQGDURRWGLDPHWURI FP 7KHLQHWDQJOHVDUH. f DQG f ZKLOHDWWKHRXWOHW f (VXPDWWKHWKRUHWFDOYDOXHVRI WKH D IGRZ UDW E KRUVHSRZHUDQG F RXWHWDQJOH.
- 3) LJXUH3 LVDQH[DPSOHRI DFHQWULXJDOSXPS FRUUHODWLRQ ZKHUH' V LVGHILQHG LQWKHSUREOHP / RJDQDQG5R\ > @VXJJHVWWKHIROORZLQJ FRUUHODWLRQIRUD[LDOIGRZ SXPSVDQGIDQV

$$D_s \approx \frac{130}{N_s^{0.485}} \qquad \text{for } N_s > 8000$$

ZKHUH1$V LV WKH GLPHQVLRQDOVSHFLILF VSHHG (T E 8VH WKLV FRUUHODWLRQ WR ILQ
WKH DSSURSULDWH VL]H IRU D IDQ WKDW GHOLYHUV IWPLQ RI DLU DW WKH GHVLJQ
ZKHQ UXQQLQJ DW UPLQ ZLWK D SUHVVXUH ULVH RI LQFKHV RI ZDWHU + LQW([SUHVV WKH ID
KHDGLQH HWHU DLU QRWHHWHU ZDWHU

- 3 ,WLV GHVLUHG WR SXPS IWV RI ZDWHU DW D SHHG RI UV DJDLQVW D KHDG RI I
D : KDW WSHR SXPS ZRXOG \RX UHFRPPHQG' (VWLPDWH E WKH UHTXLUHG LPSHOOH
GLDPHWHU DQG F WKH EUDNH KRUVHSRZHU

- 3 6XSSRVH WKDW WKH [LDO IORZ SXPS RI) LJ ZLWK' LQ UXQV DW UPL
D &RXOG LW HILFLHQWO SXPS JDOPLQ RI ZDWHU' E ,I VR ZKDW HDG ZRXO
UHVXOW F ,I DKHDG RI IWV GHVLUHG ZKDW YDOXHV RI ' DQG QZRXOG EHEHWWU'

- 3 $ SXPS LV QHHGHG WR GHOLYHU JDOPLQ RI JDVROLQH DW *f*& DJDLQVW D KHDG R
IW) LQG WKH LPSHOOHU VL]H VSHHG DQG EUDNH KRUVHSRZHU QHHGHG WR XVH WKH SXP
IDPLOLHV RI D) LJ DQG E) LJ : KLFK LV WKH EHWWU GHVLJQ'

- 3 3HUIRUPDQFH GDWD IRU D LQGLDQ HWHU UXQQLQJ DW UPLQ DUH D
IROORZV

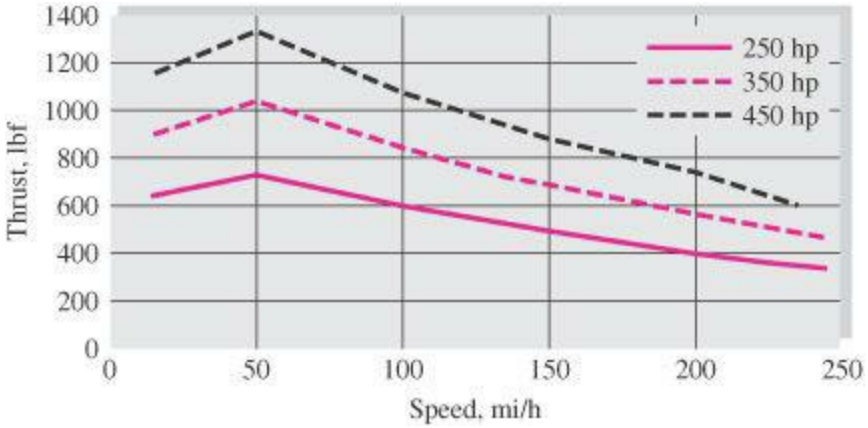
1RWH WKH ILFWLWRXV[SUHVVLRQ SUHVXUHULVH LQ WKH SURVH ZDWHU UDWKHU WKDQ DLU : KDW
WKH VSHFLILF VSHHG' +RZ GRHV WKH SHUIRUPDQFH FRPSDUH ZLW) LJ " : KDW DU
C_Q^*, C_H^*, and C_P^*?

Δp, in H_2O	29	30	28	21	10
Q, ft^3/min	500	1000	2000	3000	4000
bhp	6	8	12	18	25

- 3 $ LIFUDVSURSHOOHUVSHFLDOLVW FODLP WKDW GLPHQVLRQOHVV SURSHOOHU GDWD ZKHQ SORWWH
DV &7 - YHUVXV &3 - IRUP D QHDUO VWUDLJKW OLQH \ P[E D 7HVWLQJ
K\SRWKHVLV IRU WKH GDWD RI) LJ LQ WKH KLJK HILFLHQF\ UDQJH- 9 Q HTXD
VR DQG E ,I VXFFHVVIXO WHVWLQJ WLV VWUDLJKWOLQHV RU SUHGLFWWKHURDWRQUDW
LQ UPLQ IRU D SURSHOOHU ZLWK' IW3 KS 7 GEI DQG9 PLK IR
VHDOHYHOVDQGGUDIRQGLWRQV &RPPHQW

- 3 6XSSRVH HLWLV GHVLUHG WR GHOLYHU IWPLQ RI SURSDQH JDV PRDFXODU ZHLGKW
DW DWP DQG *f*& ZLWK DVLQJOH VWDJH SUHVVXUHULVH RI LQ+ 2 ' HWHUPLQ
WKH DSSURSULDWH VL]H DQG VSHHG IRU XVLQJ WKH SXPS IDPLOLHV RI D 3URE 3 DQ
E) LJ : KLFK LV WKH EHWWU GHVLJQ'

- 3 3HUIRUPDQFH FXUYHV IRU D FHUWDLQ IUHH SURSHOOHU FRPSDUDEOH WR) LJ FDQ EH
SORWWHG DV VKRZQ LQ) LJ 3 IRU WKUXVW7 YHUVXV VSHHG9 IRU FRQVWDQW SRZHU3
D : KDW V VXLWDEOH DWD DW WKH ZULWHU DERXW WKH WHFKQLYHV" E &DQ \RX GHGXF
WKLV EHKDYLRUE\ UHDUUDQJLQJ RUUHSORWWLQJ WKH GDWD RI) LJ "



- 3 $ PLQHYHQWDWRQIDQ UXQQLQJ DW UPLQ GHOLYHUV P VRI VHD OHYHODLUZLV DSUHVVXUH ULVH RI 3D ,V WKLV IDQ D[LDO FHQWULXJDO RU PL[HG' (VWLPDWH WKH GLDPHWULQIW,I WKH IORZ UDWH LV LQFUHDVHG SHUFHQWRUWKH VPHGLDPHWU E\ ZKDWSHUFHQWDJHZLOOWKHSUHVVXUHULVHFKDQJH'
- 3 7KHDFWXDOPLQHYHQWDWRQIDQGLVFXVVHGLQ3URE 3 KDGDGLDPHWURI >5H S @: KDWZRXOGEHWKHSURSHUGLDPHWUIRUWKHSXPSIPLO RI) LJ VR SURYLGH P VDW UPLQDQG %(3" : KDWZRXOGEHWKHUHVXOWQ SUHVVXUHULVHQ3D'

0 DWFKLQJ SXPSVWRVVWMP FKDUDFWMULVWLFV

- 3 $ JRRGFXUYHILWWRWKHKHDGYV IORZ IRUWKH LQSXPSLQ) LJ DLV

  + LQIW§ (4 4 LQJDOPLQ

  $VXPHWKHVDPHURWDWQUDW UPLQ DQGHVLPDWWKHIORZ UDWWKLVSXPS ZLO SURYLGHWRGHOLYHUZDWUIURP DUHVHUYRLUWKURXJK IWR LQSLSH WRDSRLQW I DERYHWKHUHVHUYRLUVXUIDFH $VXPHDIULFWLRQIDFWRU
- 3 $ FHDI EORZHULVHVHQWDOO DFHQWULXJDOLPSHOOHU[LWQJ WRDWEH 6XSSRVHWKDWWK VWEHLV VPRRWK39& SLSH IWORQJ ZLWK DGLDPHWUR LQ 7KHGHVLUHG H[YHORFLW LV PLK LQVHOHFWHDVXDQGDU ,I ZHXVHWKHSXPSIPLO R(TV VR GULYHWKHEORZHU ZKDWSSUR[LPDW D GLDPHWUDQG E URWDWQSSHH DUHDSSURSULDW' F ,VWKLVDJRRGGHVLJQ'
- 3 $Q LQGLDPHWUFHQWULXJDOSXPS UXQQLQJ DW UHY PLQ GHOLYHUV JDOPLQDQGDK HDGR IWDWEHWWHILFLHQF\ SHUFHQW D &QWKLVSXP RSHUDW HILFLHQ ZKHQ GHOLYHULQ ZDWUDW *& WKURXJK P R FP GLDPHWU VPRRWK SLSH' 1HJOHFWPLQRUORVVHV E ,I \RXDQZHU WR D L QHJDWLYH FDQWKHVSHGQEHFKDQJHGWRRSHUDWHILFLHQ" F ,I \RXDQZHUV E LVDOWR QHJDWLYH FDQWKHLPSHOOHUGLDPHUEHFKDQJHGWRRSHUDWHILFLHQ DQGVWLOOXQDW UHY PLQ'
- 3 ,WLVSURSRVHGWRUXQWKHSXPSR 3URE 3 DW UPLQWRSXPSZDWUDW *& WKURXJK WKHV\VWHP LQ) LJ 3 7KHSLSHLV FP GLDPHWUFRPPHUFLDOWWH: KDWORZ UDWLQIWPLQZLOOUHVXOW,VWKLVDQHILFLHQWDSSOLFDWRQ'

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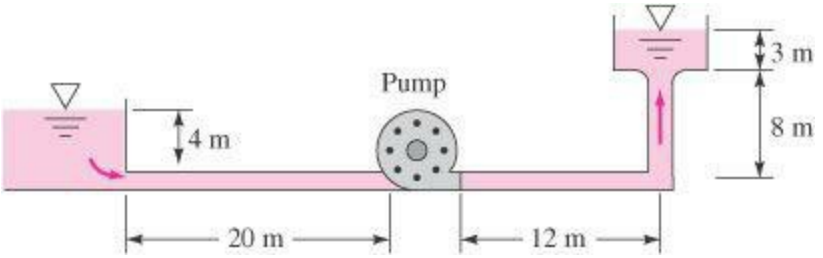

- 3 EES 7KH SXPS RI 3URE 3 UKQQLQJ DW UPLQ LV WR SXPS ZDWHU DW *f*& WKURXJK P RI KRUL]RQWDO JDOYDQL]HG LURQ SLSH $OO RWKHU V\VWHP GHWDLOV DUH QHJOHFWHG ' HWHUPLQH WKH HOERZ UDWH DQG LQSXW SRZHU IRU D SLSHGLDPHWHU FF DQG E WKH SLSHGLDPHWHU IRXQG WR \LHOG PD[LPXP SXPS HILFLHQF\
- 3 6XSSRVH WKDW ZH XVH WKH D[LDO IORZ SXPS RI) LJ WR GULYH WKH HDI EORZHUV 3URE 3 : KDW DSSURSULDWH GLDPHWHU DQG E URWDWLRQ VSHHG DUH DSSURSULDWH' F ,V WKLV DJUHHG HVLJQ'
- 3 7KH SXPS RI 3URE 3 UKQQLQJ DW UPLQ LV XVHG WR GHOLYHU ZDWHU DW *f*& WKURXJK IWRI FDVWLURQ SLSH WR DQ HOHYDWLRQ IWKLJKHU ' HWHUPLQH D WKH SURPHUSLSHGLDPHWHU RU%(3 RSHUDWLRQDO GE VKHIORZ UDWH WKDW WHVXOWL WKH SLSH GLDPHWHUV LQ
- 3 7KH SXPS RI 3URE 3 RSHUDWLQJ DW UPLQ LV XVHG ZLWK *f& ZDWHULQ WKH V\VWHP RI) LJ 3 D ,I LWV RSHUDWLQJ DW%(3 ZKDW V WKH SURPHUHOHYDWLRQ] ' E ,I] IWZ KDW V WKH IORZ UDWH L G LQ"

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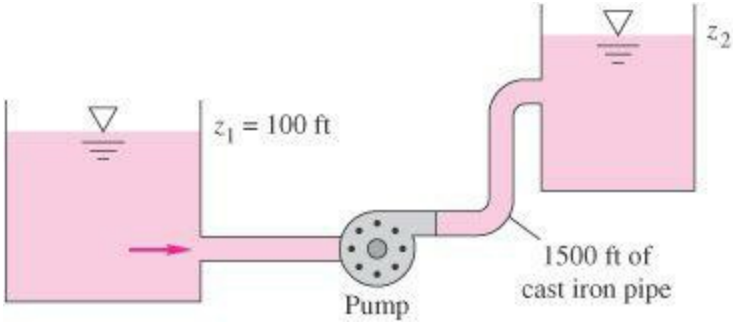

- 3 7KH SXPS RI 3URE 3 UKQQLQJ DW UPLQ GHOLYHUV ZDWHU DW *f*& WKURXJ IWRI KRUL]RQWDO LQGLDPHWHU FRPPHUFLDO VWHHO SLSH 7KHUH DUH D VKDUS HQJLQHHUV VKDUS H[LWV RU ORVVHV DQG DJDLQVW YDOYH (VWLPDWH D WKH IORZ UDWH L WKH YDOYH LV ZLGH RSHQ DQG E WKH YDOYH FORVLQJ SHUFHQWDJH WKDW FDXVHV WKH SXPS V RSHUDWLQJ DW%(3 F ,I WKH GXW\ FRQGLWLRQ KROGV FRQWLQXRXVO IRU \HDU HVWLPDWH WKH HQHUJ\ FRVWDW „ N K
- 3 3HUIRUP DQDOHGDWD IRU D VLPLODU FRPPHUFLDO SXPS DUHDV IROORZV

7KLV SXPS VXSSOLHV *f& ZDWHU WR D KRUL]RQWDO $ LQGLDPHWHU JDUGHQ KRVH ë§ LQ WKDW IWRQ (VWLPDWH D WKH IORZ UDWH DQG E WKH KRVH GLDPHWHU WKDW ZRXOG FDXVH WKH SXPS WR RSHUDWH DW%(3

Q, gal/min	0	10	20	30	40	50	60	70
H, ft	75	75	74	72	68	62	47	24

- 3 7KH&HVWQD DLUFUDIWKDVDZLQJDUHDRI IWDQDVSHFWUDWLRI DQGI EDVLF GUDJ FRHILFLHQW&, ,WSURSHOOHU ZKRVHGDWDLVVKRZQLQ) LJ KDVDGLDPHWURI IW,I WKHSODQHZHLJKV OE DQGIOLHVDW IW DW P VWDQGDUG DOWLWXGH HVWLPDWH D WKH DSSURSULDWH SURPHOOHU VSHHG L UHYPLQ DQG E WKH SRZHU UHTXLUHG ,V WKH SURPHOOHU HILFLHQW+LQW 7KH HILFLHQF\ LVJRRG EXWQRWEHWW

3XPSVLQSDUDOOHORUVHULHV

- 3 7KH LQSXPSLQ) LJ DLVXVHGDW UPLQLQDVWXGHQWODE ZKRVHKHDGFXUYHLV +V IW 4 ZLWK4 LQWKRXVDQGVRI JDOORQVRI ZDWUSHUPLQXW) LQGWK GLVFKDUJH DQG EUDNH KRUVHSRZHU UHTXLUHG IRU D RQH SXPS E WZR SXPSV L SDUDOOHODQG F WZR SXPSVLQVHULHV : KLFKFRQLJXUDWRQLVEHWW
- 3 7ZR LQSXPSVIURP) LJ E DUHLQWDOOHGLQSDUDOOHOIRUWKHVWXWP RI) LJ 3 1HJOHFWPLQRUORVVHV)RUZDWUDW *& HVWLPDWWKHIORZ UDWDQGSRZH UHTXLUHGL D EHWWSXPSVDUHUXQQLQJ DQG E RQHSXPSLVVKXWRI DQGLVRODWHG

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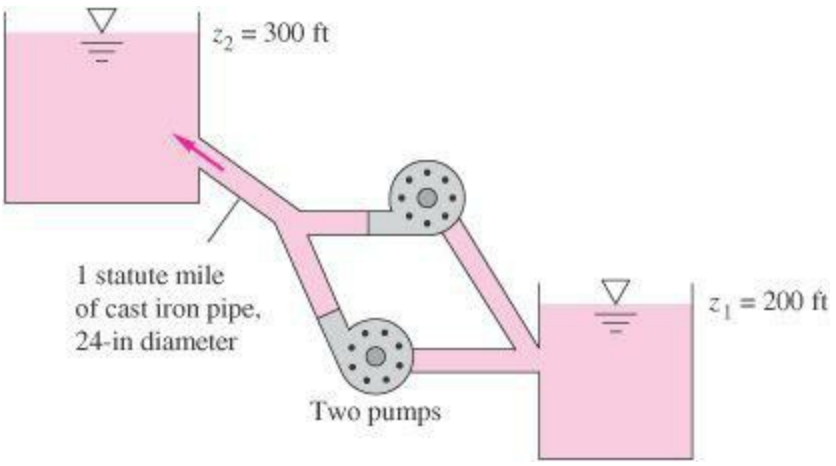


- 3 7ZR LQSXPSVIURP) LJ DDUHFRPELQHGLQSDUDOOHOWRGHOLYHUZDWUDW *) WKURXJK IWRI KRUL]RQWDOSLSH ,I ZKDWSLSHGLDPHWUZLOOHQVXUHI IORZ UDWRI JDOPLQIRUQ UPLQ'
- 3 7ZRSXPSVRI WKHWSHWWWWGLQ3URE 3 DUHWREHXVHGDW UPLQWRSXPS ZDWUDW *& YHUWFDOO XSZDUGWKURXJK P RI FRPPHUFLDOWHOSLSH 6KRXO WKH EHLQVHULHVRULQSDUDOOHO: KDWLVWKHSURSHUSLSHGLDPHWUIRUPRWHILFLHQ RSHUDWQ'
- 3 &RQVLGHUWKHDIOIORZ SXPSRI) LJ UKQQLQJDW UPLQ ZLWKDQLPSHOOH GLDPHWURI LQ 7KHHIXLG LV SURSDQHJDV PROHFXDUZHLJKW D +RZ PDQ SXPSVLQVHULHVDUHQHHGHGWRLQFUHDVHWKHJDVSUHVXUHIURP DW WR DW' E (VWLPDWWKHPDVVIORZ RI JDV
- 3 7ZR LQSXPSVIURP) LJ D DUHWREHXVHGLQVHULHVDW UPLQWRGL

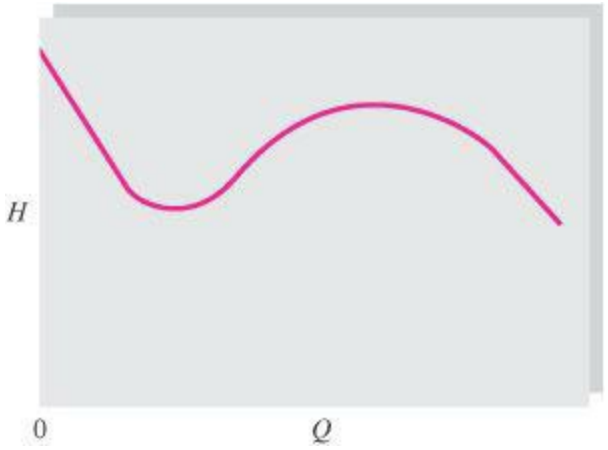
ZDWUKURXJK IWRYHUWFDOFDWURQSLSH: KDWKRXGWKHSLSHGLDPHWUEHIR
PRVWHILFLHQWRSHUDWRQ'1HJOHFWPLQRUORVVHV

- 3 ' HWHUPLQHLHWHUD WKHVPDOOHVWRU E WKHGLJHVWRI WKHVHYHQ7DFR SXPSVLQ)LJ 3 UXQQLQJ LQVHUHVDW UPLQ FDQHILFLHQF\ SXPS ZDWUDW *& WKURXJK NP RI KRUL]RQWDO FP GLDPHWUFRPPHUFLDOWXEHSLSH
- 3 5HFRQVLGHUWKHVVWHP RI)LJ 3 8VHWKH%\UQ-DFNVRQ SXPS RI3URE 3 UXQQLQJDW UPLQ QRVFDOQJ WRGULYHWKHIORZ ' HWHUPLQHWKHUHVXOWLQ IORZ UDWHEHWZHHQWKHUHVHUYRLUV: KDWLVWKHSXPSHILFLHQF\"

3XPSLQWDELODW

- 3 7KH6VKDSHGKHDGYHUVXVIORZ FXUYHLQ)LJ 3 RFFXUVLQVRPHDJLDOIORZ SXPSV ([SODLQKRZ DIDLOHG IODWVXVWHP ORVFXUYHPLJKWFDXVHLQXDELODWHVLQWK RSHUDWRQRI WKHSXPS +RZ PLJKWZHDYRLGLQXDELODW"

3



- 3 7KHORZ VKXWRII KHDGYHUVXVIORZ FXUYHLQ)LJ 3 RFFXUVLQVRPHFHQWULXJD SXPSV ([SODLQKRZ DIDLOHG IODWVXVWHP ORVFXUYHPLJKWFDXVHLQXDELODWHVLQWK RSHUDWRQRI WKHSXPS : KDWDGGLWLRQDOYH[DWRQRFFXUZKHQWZR RI WKHVHSXPS DUHLQSDUDOOHO +RZ PLJKWZHDYRLGLQXDELODW"

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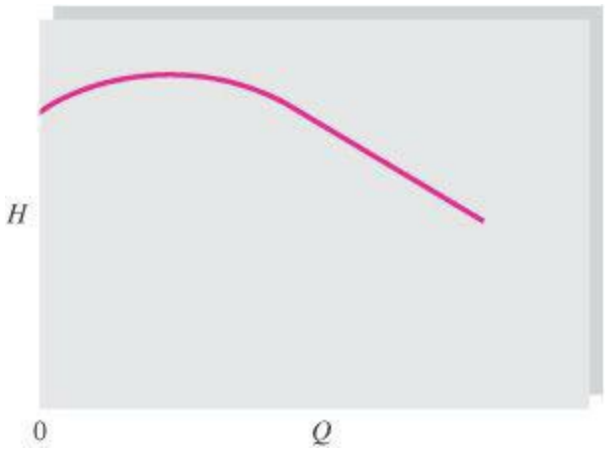

5 HDFWRQ DQG LP SXOVH WXUELQHV

- 3 7XUELQHV DUH WR EH LQVWDOOHG ZKHUH WKH QHWW KHDGLV IWDQG WKH IORZ UDWH JDOP LQ ' LVFXVV WKH W\SH QXPEHU DQG VL]H RI WXUELQH WKDW PLJKW EH VHOHFWHG LI WK JHQHUDWRUV HOHFWHG LV D SRO H FRQH Q UPLQ DQG E SRO H Q UPLQ : K\ DUH DVWHULVNV WR EH DVVXPHG WR RXUELQHV GHVLUDEOH IURP DSDQQLQJ SRLQWR YLHZ"
- 3) RU D KLJK IORZ VLWH ZLWK D KHDG RI IWLVLV GHVLUHG WR GHVLJQ D VLQJOH I\ GLDPHWHU WXUELQH WKDW GHYHORSV EKS DW D SHHG RI UPLQ DQG SHUFHQ HIILFLHQF\ ,WV GHFLGHG ILUVW WR WU\ D JHRPHWULFDOO\ VLPLODU PRGHO RI GLDPHWHU I\ UXQQLQJ DW UPLQ D : KDW LV WKH VSHHG RI WXUELQH LV LQ WKH SURPRWSH' : KDW LV WKH DSSURSULDWH E KHDG DQG F IORZ UDWH IRU WKH PRGHO WXUELQH G (VWLPDWH WKH SRZHU H[SHFWHG WR EH GHOLYHUHG E\ WKH PRGHO WXUELQH
- 3 7KH 7XSSHUZ DUH K\GURHOHFWULF SODQW RQ WKH %ODFNVWRQH 5LYHU KDV IRXU LG GLDPHWHU WXUELQHV HDFK SURYLGLQJ N DW UPLQ DQG IWV IRU D KHDG RI IW: KDW WSHR RI WXUELQHV DUH WKHVH' +RZ GRHV WKLV SHUFRUPDQFH FRPSDUH Z LV) LJ "
- 3 $QLGHDOL]HG UDGLDO WXUELQH LV VKRZ QLQ) LJ 3 7KH DEVROXWH IORZ HQWHUV DQG OHDYHV UDGLDOO LQZDUG 7KH IORZ UDWH LV P VRI Z DWUDW *& 7KH EODG WKLFNQHVV LV FRQVWDQW DW FP &RPSXWH WKH WKHRUHWLFDO SRZHU GHYHORSHG

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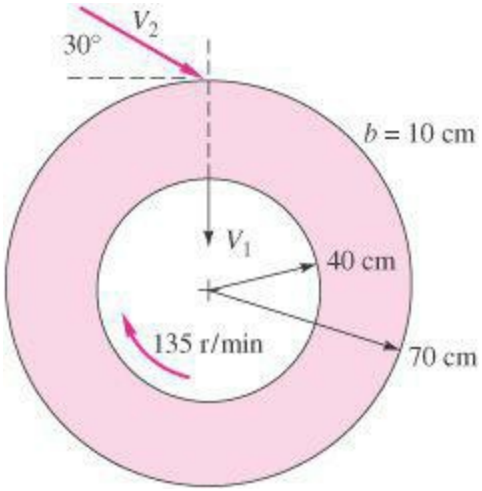

- 3 3HUIRUP DQDO\VHV IRU D YHU VPDOO ' FP PRGHO Z DWU WXUELQH RSHUDWQ Z LWK DQ DYDLODEOH HKHDG RI IWDUHDV IROORZV

D : KDW WSHR RI WXUELQH LV WKLV QHZ VR EH' E : KDW LV VR GLIIHUHQW DERXW WKH VHGD FRPSDUHG WR WKH GLPHQVLRQOHVV SHUUP DQH SORW) LJ G' 6XSSRVH WLWV GHVLUHG XVH D JHRPHWULFDOO\ VLPLODU WXUELQH WR VHUYH Z KHUH WKH DYDLODEOH HKHDG DQG IORZ DUH I DQG IWV UHVSHFWLYHO (VWLPDWH WKH PRWH HILFLHQF\ F WXUELQH GLDPHWHU G UHVXOWLQ VSHHG DQG H KRUVHSRZHU

Q, m³/h	18.7	18.7	18.5	18.3	17.6	16.7	15.1	11.5
RPM	0	500	1000	1500	2000	2500	3000	3500
η	0	14%	27%	38%	50%	65%	61%	11%

- 3 $ 3HOWRQ ZKHHORI IWSLWFK GLDPHWUR SHUDWV XQGHU D QHWKHDG RI I\ (VWLPDWH WKH VSHHG SRZHUHG SRZ HURXSXWDQG IORZ UDWH IRUEHWWHU HILFLHQF\ LI WKHQR]]OH[L GLDPHWULV LQ
- 3 $QLGHDOL]HGUDGLDOWXUELQHLVVKRZQLQ) LJ 3 7KHDEVROXWHIORZ HQWHUVDW ZLWKWKHEODGHDQJOHVDVVKRZQ7KHIORZ UDWHLV P VRI ZDWHUDW *& 7KHEODG WKLFNQHVVLVFRQVWDQWDW FP &RPSXWHWKHWRUTXHDQGSRZ HUGHYHORSHG

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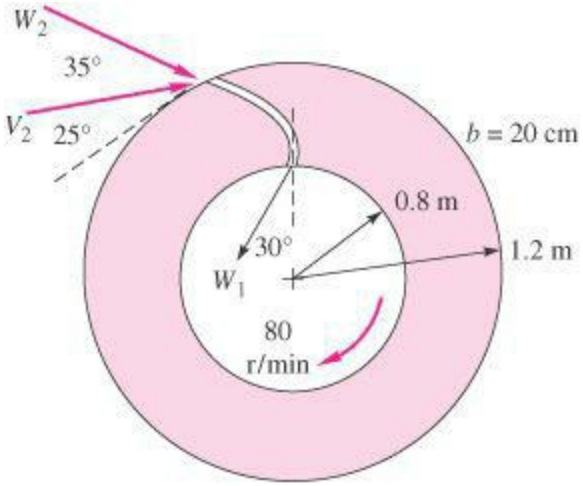

- 3 7KHIORZ WKURXJK DQ DQLDOIORZ WXUELQH FDQ EH LGHDOL]HG E\ PRGLI\LQJ WKH VDPH URWRU GLDJUDPV RI) LJ IRU HQHUJ\ DEVRUSWLRQ 6NHWFK D VXLWDEOH EODGH DQG IORZ DUUDQJHPHQWDQG WKHDVRFLDWHGYHORFLW\YHFWRUGLDJUDPV
- 3 $ GDP RQDULYHUVEHLQJ VLWHGIRUDK\GURHOHFWULFWXUELQH 7KHIORZ UDWHLV P K WKHDYDLODEOHKHDGLV P DQG WKHWXUELQHVSHHGLV WR EH UPLQ ' LVFXVV WK HVXLWDEOHWXUELQHV]HDQGIHDWXUHVIRU D D)UDQFLV WXUELQHDQG E D 3HOWR ZKHHO
- 3) LJXUH 3 VKRZV D FXUZD\ RI DFURVVIORZ RU³%DQL´ WXUELQH> @ZKLF UHVHPEOHV D VTXLUUHOFDJH ZLWK VORWWHG FXUYHG EODGHV 7KHIORZ HQWHUV DWDERXW RTFORFN DQG SDVVHV WKURXJK WKHFHQWUDQG WKHQDJDLQWKURXJK WKHEODGHV GHDYLQJ DERXW RTFORFN 5HSRUWWR WKHFDVRQWKHRSHUDWRQDQGDGYDQWDJHVRI WKLVGHVLJQ LQFOXGLQJHDQ\YHORFLW\YHFWRUGLDJUDPV
- 3 $ VLPSOH FURVVIORZ WXUELQH) LJ 3 ZDV FRQWUXFWHG DQG WHVWHG DWWKH 8QLYHUVLWRI 5KRGH,VODQG 7KHEODHVZHUHPDGHRI 39& SLSHFXWORQJWKZLVHLQ WKUHH f DUF SLHFHV : KHQLWZDVWHVWHGLQZDWHUDWDKHDGRI IWDQGDIORZ UDWH RI JDOOLQ WKH PHDVXUHG SRZHUSXWZDV KS (VWLPDW D WK HILFLHQF\ DQG E WKHSRZHUVSHFLILFVSHHGLQ UHY PLQ

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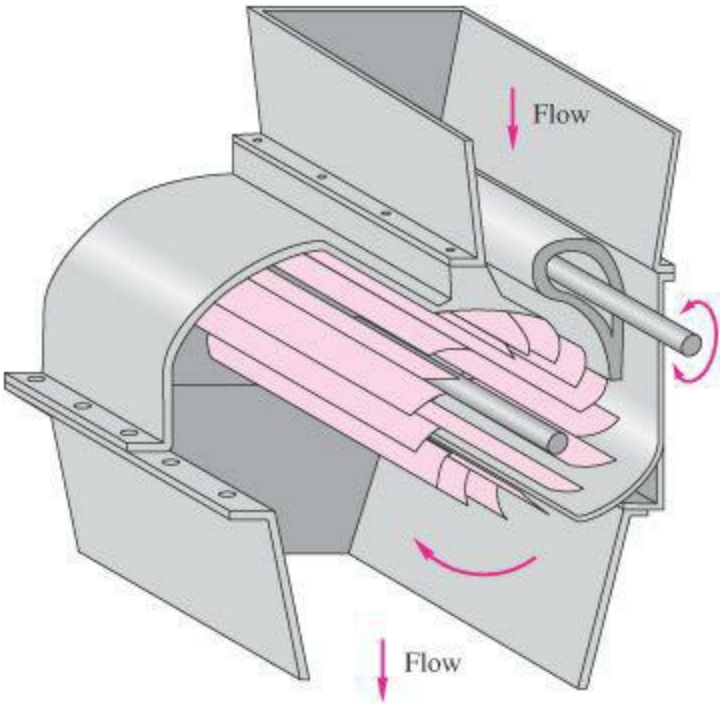

- 3 2QH FDQ PDNH D WKHRUHWLFDO HVWLPDWH RI WKH SURSHUGLDPHWHU RI D SHQWRFNLQ D LPSXOVH WXUELQH LQVWDOODWLRQ DVLQ) LJ 3 / HW/ DQG+ EH NQRZQ DQG GHWHUPLQH WKH WXUELQH SHUIRUPDQFH EH LGHDOL]HG E\ (TV DQG $ FFRXQW IRU FRQVWDQW IULFWLRQ ORVVHV LQ WKH SHQWRFN EXW QHJOHFW PLQRU ORVVHV 6KRZ WKDW WKH PD[LPXP SRZHU JHQHUDWHG ZKHQK + E WKH RSWLPXP MHWVSHHFLW LV J+ DQG F WKE EHWWQR]]OHGLDPHWHUV' M > I/ @ ZKHUH I LV WKH SLSHIULFWLRQIDFWRU

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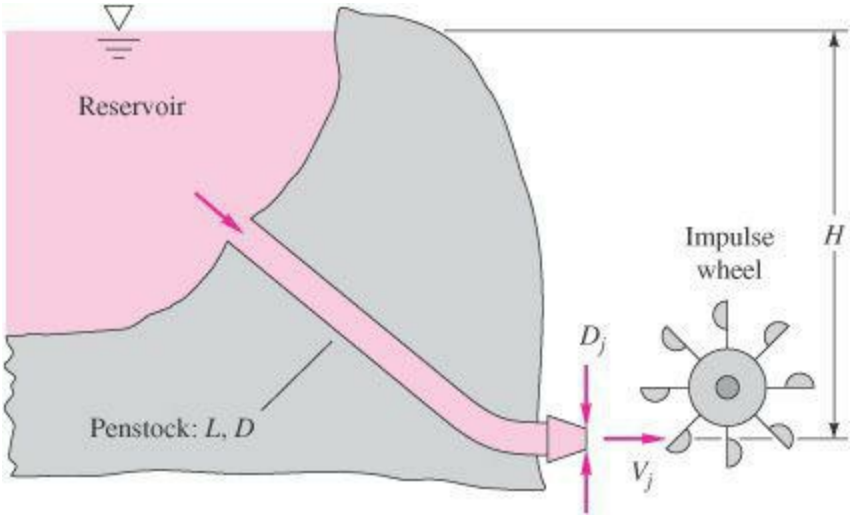
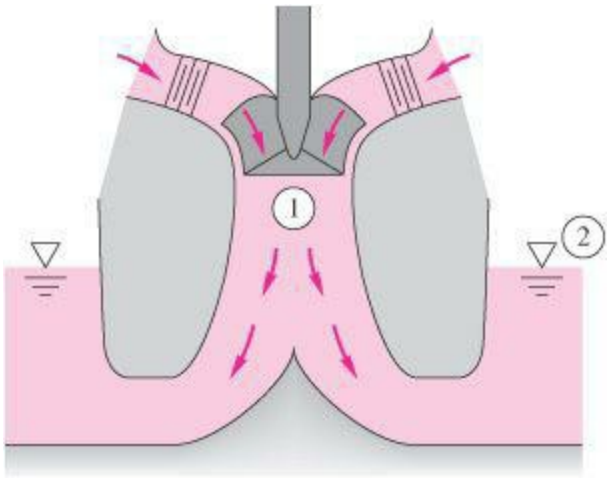


- 3 $SSO WKHUHVXOWV RI 3UREOHP 3 WR GHWHUPLQH WKHRSWLPXP D SHQWRFNGLDPHWHU DQG E QR]]OHGLDPHWHUIRU D KHDG RI IWDQG DIORZ UDWH RI JDOPLQZLWK DFRPPHUFLDOO\DYDLODEOHQRPLQDOSLSHVL]HV IW
- 3 EES &RQVLGHU WKH IORZ LQ QRQRSWLPXP YHUVLRQ RI 3UREOHP 3 + P / NP ' P ' M FP 7KHSHQWRFNLVFRQFUHWë PP 7KHLPSXOVHZKHHOGLDPHWHULV P (VWLPDWH D WKH SRZHUJHQHUDWHG E\ WKHZKHHODW SHUFHQ

HILFLHQF\ DQG E WKHEHWWSHGRI WKHZKHOLQUPLQ 1HJOHFWPLQRUORVVHV

- 3)UDQFLV DQG . DSODQ WXUELQHV DUH RIWHQ SURYLGHG ZLWK GUDIW WXEHV ZKLFK GHGHFUHDVH WKH H[LWIORZ LQWR WKH WDLOZDWHU UHJLRQ DV LQ) LJ 3 ([SODLQ DWWHDWWWZ DGYDQWDJHV LQXLQJ GUDIWWXEH

3



- 3 7XUELQHV FDQ DOVR FDYLWDWH ZKHQ WKH SUHVVXUH DWSRLQW LQ) LJ 3 GURSV WR OHVV WKDQ WKH 136+ GHILQHG E\ (T WKH HPSLULFDO FULWHULRQ JLYHQ E\ : LVOLFHQXV> @RUFDYLWDWRQLV

$$N_{ss} = \frac{(\text{r/min})(\text{gal/min})^{1/2}}{[\text{NPSH (ft)}]^{3/4}} \geq 11{,}000$$

8VHWKLVFULWHULRQWRFRPSXWHKRZ KLJK] í] WKHLPSHOOHU HLQ) LJ 3 FDQEH SODFHGIRUD)UDQFLVWXUELQHZLWKDKHDGRI IW1 $_{VS}$ DQG$_{D}$ GEI LQ DEVROXWH EHRUHFDYLWDWRQRFFXUVLQ *f*) ZDWHU

: LQG WXUELQHV

- 3 &RQVLGHUWKHGDJHZLQG WXUELQHLQWKHFKDSWHURSHQHUSKRWR 3UHWHQG WKDWYRX GRWKQRW WKDW0 PHDQVILYHPHJDZDWW 8VHWKHPHDJHUSKRWRFDSWRQGDDSOX \RXUNQRZ OHGJH RI ZLQG WXUELQH WKHRU DQG H[SHULPHQWDWLRQ DW HXUPDWWKH SRZH GHOLYHUHGE\ WKHWXUELQHIRUDUDQJHZLQGVSHHGRI P VDQGDURXQGWRQUDWR UHY PLQ
- 3 $ ' DUULHXV9$: 7 LQRSHUDWRQLQ/ XPVGHQ 6DNDWKHZDQ WKDWV IWKLJK DQ IWLQGLDPHWHUWZHSVRXWDQDUHDRI IW(VXUPDW D WKHPDLPXP SRZH DQG E WKHURWRUVSHGLI LWWVRSHUDWQJ LQ PLKZLQGV
- 3 QPHULFDQ IWGLDPHWHUPXOWLEODGH+$: 7 LVXVHGWRSXPS ZDWHUWR DKLJKWR IWWKURXJK LQGLDPHWHUFDWWLRQSLSH ,I WKHZLQGVDUH PLK HWXP DWWK UDWHRI ZDWHUIORZ LQJDOPLQ
- 3 7KHQHZ WKUHEODGH+$: 7 PXQLFSDOZLQGWXUELQHLQ3RUWPRXWK 5, KDVDWZH IWKLJK ZLWKDWXUELQHGLDPHWURI IW7KHQHZ VSDSHUUDWHV WKHWXUELQHD

0 : D : KDVZLQGYHORFLWDQGWXUELQHURWDWRQVSHHG ZRXOGJHQHUDWHWKH SRZHU E (WXPDWHWKHDQQXDOSRZHUSURGXFWLRQIRUDWXUELQHZLQGVSHHGRI PLK

- 3 7KHFRQYHUVLRQ&DSH&RG: LQG3URMHFWSURSRVHV GUDJHZLQGWXUELQHVL 1DQWXFNHW6RXQG LQMQGHGWRSURYLGH SHUFHQWRI WKHHOHFWULF SRZHUQHGVR &DSH&RGDQGWKH,VODQGV 7KHWXUELQHGLDPHWULV IW)RUDQDYHUDJHZLQ YHORFLW RI PLK ZKDWDUH WKHEHVWURWDWRQ UDWHDQGWRWDO SRZHURXWSX HVWLPDWHVIRU D D+$: 7DQG E D9$: 7"
- 3 ,Q DZLQGSRZHUHGYHKLFOHFRQWHVWKHGLQ1RUWK+RODQG> @ZDVZR ZLWK D GHVLJQ E\ VWXGHQWV DWWKH 8QLYHUVLW RI 6WXWWDUW$ VFKHPDWF RI WK ZLQGLQJ WKUHZKHHOHUV VKRZQLQ) LJ 3 ,WV SRZHUHG E\ DVKURXGH ZLQGWXUELQH QRWD SURPHOOHU DQG XQDNHDVDLGERDWFDQPRYHGLUHFW LQWR WK ZLQG D +RZ GRHVLWZRUN' E : KDWL WKHZLQGLVRI WRWKHVLGH' F &LV VRPHGHVLJQTXHVWLRQV\RXPLJKWDVN
- 3 $QDO]H WKH ZLQGSRZHUHGYHKLFOHRI) LJ 3 ZLWK WKHIROORZLQJ GDWD WXUELQHGLDPHWU' IWSRZHUFRHILFLHQW) LJ YHKLFOH& $ IW DQGWXUELQHURWDWRQ UPLQ 7KHYHKLFOHPRYHVGLUHFWO LQR DKHDG ZLQG: PLK 7KHZLQGEDFNZDUGWKUXVWRQWKHWXUELQHVDSSUR[LPDWHO 7 § &7 ! 9UHO$WXUELQH ZKHUH9UHOLVWKHDLUYHORFLWUHODWLYHWRWKHWXUELQH DQG&7 § (LJKWSHUFHQWRI WKHWXUELQHSRZHULVGHOLYHUHGE\ JHDUVWRWKHZKHHOV WR SURSHOWKHYHKLFOH(VWLPDWHWKHVHDOHYHOYHKLFOHYHORFLW9 LQPLK

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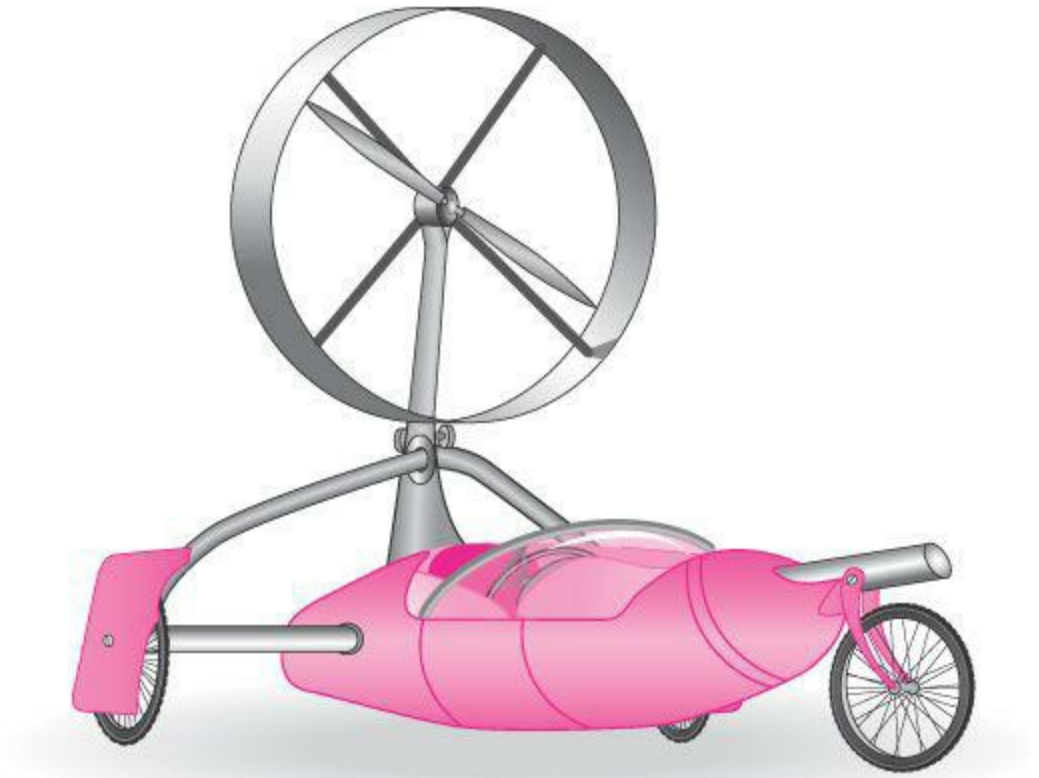

- 3) LJXUH VKRZHGWKHWSLFDOSRZHUSHUFRPDQFHRI DZLQGWXUELQH 7KHZLQ DQR FDXVHV DWKUXWWRFH WKDWPXWWEH UHVLVWHG E\ WKH WUXFWXUH 7KH WKUX\

FRHILFLHQW&7 RI DZ LQGWXUELQHPD\ EHGHLQHGDVIROORZV

$$C_T = \frac{Thrust\ force}{(\rho/2)\,AV^2} = \frac{T}{(\rho/2)\,[(\pi/4)D^2]\,V^2}$$

9DOXHVRI &7 IRUDWWSLFDOKRUL]RQWDOD[LVZLQGWXUELQHDUHVKRZQLQ) LJ 3 7K
DEVFLVVDLVWKHVDPHDVLQ) LJ &RQVLGHUWKHWXUELQHRI3URE 3 ,I WKHZLQGL
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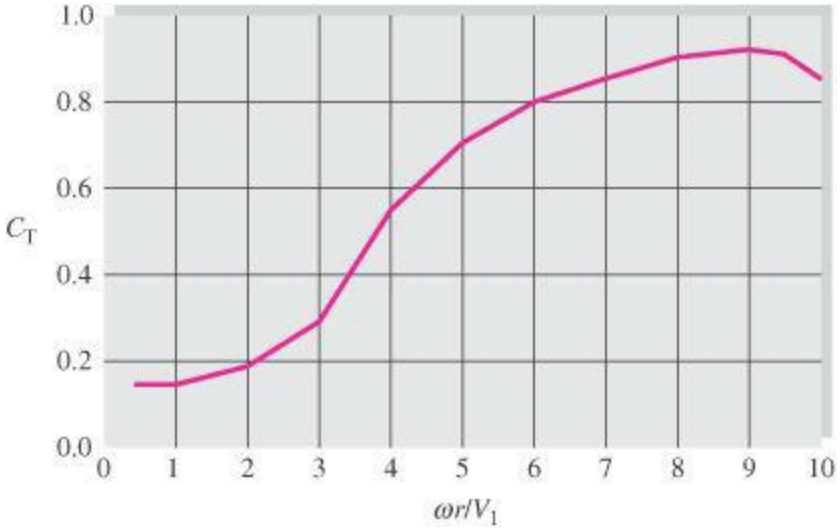


3 7KHXWFRHILFLHQWRUDWSLFDO+$: 7

: RUG3 URECOPV

- : : HNQRZ WKDWDQHQFRVHGURWDWQJ EODGHG LPSHOOHUZ LOOLPSDUWHQHJ\ WR DIOXLG XVXDOO LQWKHIRUP RI D SUHVVXUH ULVH EXWKRZ GRHVWLDFWXDOO KDSSHQ' LVFXVV ZLWKVNHWFKHV WKHSK\VLFDOPHFKDQLVPVWKURXJKZKLFKDQLPSHOOHUDFWXDOO WUDQVIHUV HQHUJ\ WR DIOXLG
- : ' \QDPLF SXPSV DVRSSRVHGWR3' 3V KDYHGLIILFXOWPRYLQJKJKO YLVFRXVIOXLG / REDORI DQG5RW> @VXJJHVWWKHIROORZLQJ UKHR WKXPE ' LQ ! ZDWL ZKHUH' LV WKH GLDPHWUR WKH GLVFKDUJH SLSH) RUH[DPSOH 6$(: RLO ZDWU VKRXOGUHTXLUHDWOHDVW LQRXOHW&DQ\RX H[SODLQVRPHUHDVRQVIR WKLVOLPLWDWRQ'
- : 7KH FRQFHSWRI 1 36+ GLVFXVV WKDWOTXLG G\QDPLF SXPSV VKRXOG JHQHUDOO E LPPHUVHGEHORZ WKHVXUFH &DQ\RX H[SODLQWKLV" : KDWLV WKHHIHFWRI LQFUHDVLQ WKHOTXLG\WPSHUDXUH'
- :) RUORQGLPHQVLRQDOIDQSHURUPDQFH : DOOV> @VXJJHVWWKDWWKHKHDGFRHILFLHQ VKRXOGEHUHSODFHGE\ )73 !Q' ZKHUH) 73 LV WKHIDQVWDOSUHVXUH FKDQJH ([SODLQWKHXVHXOHVRI WKLVPRGLILFDWRQ
- : 3HUORUPDQFHGDWDIRUFHQWULXJDOSXPSV HYHQLI ZHOOFDOHGJHRPHWUFDOO VKRZ I GHFUHDVH LQ HILFLHQF\ ZLWK GHFUHDVLQJ LPSHOOHU VL]H ' LVFXV VRPH SK\VLFO

UHDVRQVZK\ WKLVLVVR

- : &RQVLGHU D GLPHQVLRQDWV SXPS SHURUPDQFH FKDUWVXFK DV) LJ : KD DGGLWLRQDOGLPHQVLRQDWV SDUDPHWHUV PLJKWPRGLI\ RUHYHQGHWHUPLQH WKH VLPLODULW LQGLFDWHGLQVXFKGDWD'
- : 2QHSDUDPHWHUQRWGLVFXVVHGLQWKLVWH[WLV WKHQXPEHURI EODGHVRQDQLPSHOOHU' R VRPH UHDGLQJ RQ WKLV VXEMHFWDQG UHSRUWWR WKH FODVV DERXWLW HIHFWRQ SXPS SHURUPDQFH
- : ([SODLQ ZK\ VRPH SXPS SHURUPDQFH FXUYHV PD\ OHDG WR XQXVXDOHRSHUDWQ FRQGLWLRQV
- : : K\ DUH)UDQFLV DQG . DSODQ WXUELQHV JHQHUDOO FRQVLGHUHG XQXVXDOEOH IF K\GURSRZHUWWZKHUHWKHDYDLODEOHHKHDG[FHHGV IW
- : ' R VRPHUDGLQJ RQWKHSHURUPDQFHRI WKHIUHSURSHOOHUWKDWLV XVHG RQVPDC ORZ VSHHGDLFUDW: KDWGLPHQVLRQDWSDUDPHWWVDUH UHSRUWHG IRUWK GDWD' +RZ GR WKHSHURUPDQFHDQGHILFLHQF\ FRPSDUHZLWKWKRVHIRUWKHDQDO IORZ SXPS'

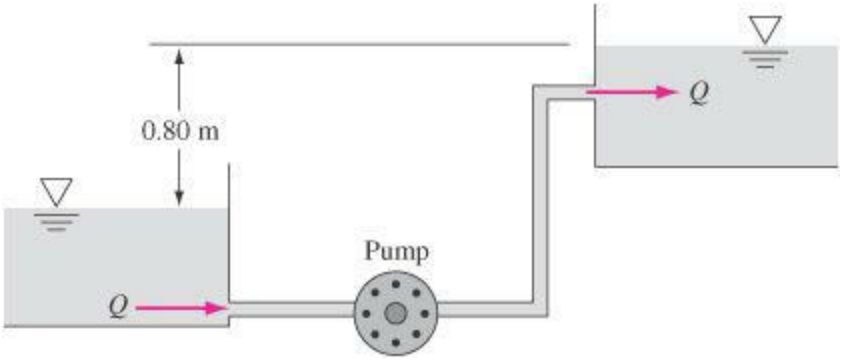
&RP SUHKHQVLYH3 URE OHP V

- & 7KHQHWKHDGRI DOXWHDTXDULXP SXPS LVJLYHQE\ WKHPDQXIDFWXUHUDVDIXQFWLRQRF YROXPHIORZ UDWHDVQXWGEHORZ

: KDWLV WKHPDQLPXP DFKLHYDEOHIORZ UDWLL \RX XVHWKLVSXPS WRPRYHZDWUIURP WK ORZ HUUHVHUYRLUWR WKHXSSHUHVHUYRLUDVVKRZQLQ) LJ & " 1RWH 7KHWXEQJ LV VPRRW ZLWK DQ LQQHUGLDPHWUR PP DQG D WRWDOOHQJWK RI P 7KHZDWUL DWURRP WHPSHUDWXUHDQG SUHVXUH 0 LQRUGHUWRVLQWKHVVWHP FDQEHQHJOHFWHG

Q, m^3/s	H, mH_2O
0	1.10
1.0 E-6	1.00
2.0 E-6	0.80
3.0 E-6	0.60
4.0 E-6	0.35
5.0 E-6	0.0

- & 5HFRQVLGHU3URE 3 DVDQH[HUFLVHLQSXPS VHOHFWLRQ 6HOHFWDQLPSHOOHUVL]H DQG URWDWLRQDO VSHHG IURP WKH %\URQ-DFNVRQ SXPS IDPLO RI3URE 3 W GHOYHUDIORZ UDWRI IWV VR WKHVVWHP RI)LJ 3 DWP LQLPXP LQSXWSRZH &DOFXODWWKHKRUVHSRZHUHTXLUHG



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- & 7KH VVWHP RI)LJ & LVGHVLJQHGWRGHOYHUZDWUDW *& IURP DVHDOHY UHVHUYRLUWRDQRWKHUURXJKQHZ FDWWLRQSLSHRI GLDPHWU FP 0LQRUORVVHVDU . EHRUHWKHSXPSHQWDQFHDQG . DWWUWKHSXPSH[LW D 6HDFW SXPSIURP HLWKHU)LJ DRU E UXQQLQJDWWKHJLYHQVSHGV WKDWFDQSHUFU WKLVVDWNDWP D[LPXP HILFLHQF\ ' HWHUPLQH E WKHUHXQLQJIORZ UDW F WKHEDN KRUVHSRZHUDQG G ZKHWKHUWKHSXPSDVSUHVHQW VLXDWGLVDHIURP FDYLWDWRQ

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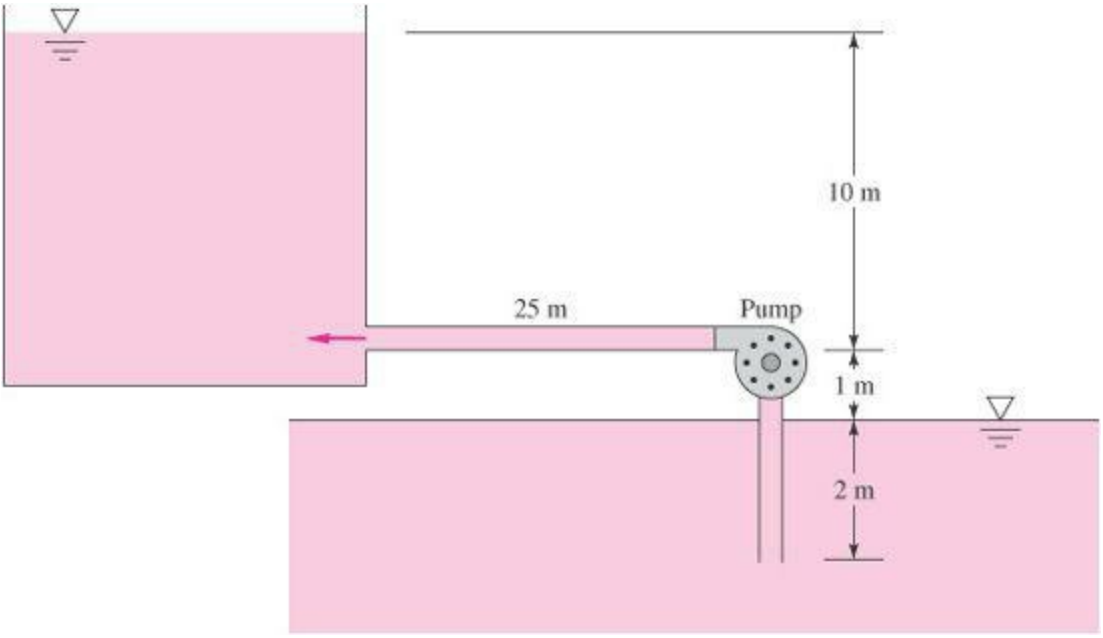

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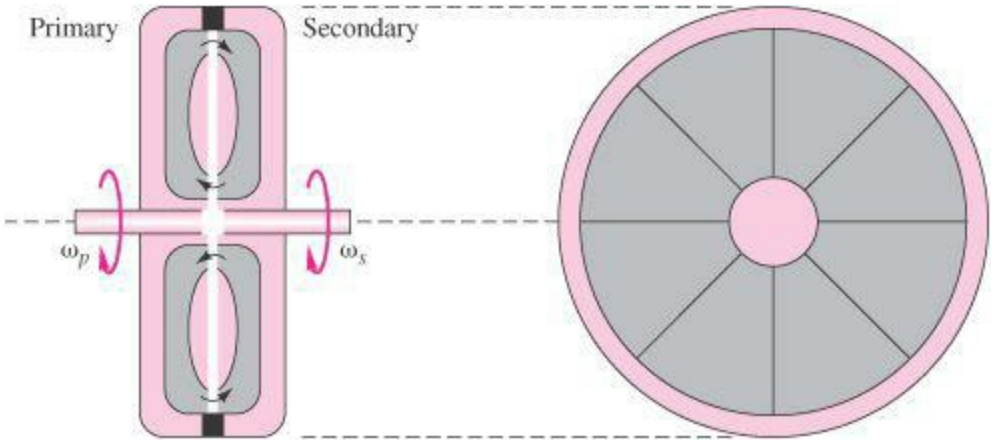


- & ,Q3URE 3 HVWLPDWHWKHHILFLHQF\ RI WKHSXPSLQWZRZD\V D 5HDGLWGLUHFW
IURP)LJ E IRUWKHG\QDPLFDOO VLPLODUZDWUSXPS DQG E FDOFXODWHWKH
(T IRU WKH DFWXDONHURVHQH IORZ &RPSDUH \RXU UHVXOWV DQG GLVFXVV DQ
GLVFUHSDQFLHV

- & $Q LQWHUHVWLQJ WXUERPDFKLQH > @LV WKHIOXLG FRXSOLQJ RI)LJ & ZKLF
FLUFXODWVIOXLGIURP DSULPDU\ SXPS URWRUDQGWKXVWXUQVDVHFRQGDU\WXUELQHRQ
VHSDUDWHWKDW%RWKURWRUVKDYHUDGLDOEODGHV &RXSOLQJVDUHFRPPRQLQDOOWSHVF
YHKLFOHDQGPDFKLQHWRROVDQGGULYHV 7KHVOS RI WKHFRXSOLQJ VGHILQHGD
WKH GLPHQVLRQOHVV GLIIHUHQFH EHWZHHQ VKDWURWDWLRQ UDWHV V í &V &S) RUI
JLYHQYROXPHRI IOXLG WKHWRUTXH7 WDQVPLWWHGLVDIXQFWLRQRI V ! &S DQGLPSOH
GLDPHWHU' D 1RQGLPHQVLRQDOL]H WKLV IXQFWLRQ LQWR WZR SL JURXSV Z LWK RQH S
SURSRUWLRQDO WR7 7HVW RQ D IWGLDPHWHU FRXSOLQJ DW UPLQ ILOOHG Z LW
K\GUDXOLF IOXLGRI GHQVLW\ OEP IW\LHOGWKHIROOWLQJ WRUTXHYHUVXVVOSGDWD
E ,I WKLV FRXSOLQJ LV UXQDW UPLQ DWZKDWVOS YDOXHZLOOLWWDQVPLWD WRUTXHR
IWOEI" F : KDWLV WKHSURSHUGLDPHWUIRUDJHRPHWFDOO VLPLODUFRXSOLQJ WR UXQD
UPLQDQG SHUFHQWVOS DQGWDQVPLW IWOEI RI WRUTXH'

Slip, s	0%	5%	10%	15%	20%	25%
Torque T, ft-lbf	0	90	275	440	580	680

- & 5HSRUW WR WKH FODVV RQ WKH&RUGLHU PHWKRG > @IRU RSWPDO GHVLJQ R
WXUERPDFKLQHU 7KHPHWKRGLVUHDGLO\ WR DQGJUHDW H[SDQGHG IURP 3URE 3
DQGXVHVERWKVRIWZDUHDQGFKDUWVRGHYHORSDQHIILFLHQWGHVLJQIRUDQ JLYHQSXPS
RUFRPSUHVVRUDSSOLFDWLRQ
- & $ SXPS WXUELQHLV DUHYHUVLEOHGHYLFHWKDWXVHVDUHYROXWLRJHQHUDWSRZHULQWK
GD\ WPHDQG WKHQSXPSVZDWUEDFNXS WR WKHUHVHUYRLUDWQJKW/ HWXVUHVH3URE
3 DVDSXPS WXUELQH 5HFDOOWKDW] IWDQG WKHZDWUIORZ VWKURXJK
IWRI LQGLDPHWUFDWWLRQSLSH)RUVLPSOLFLW DVXPHWKDWWKHSXPS RSHUDWVD
%(3 ZLWK+ S IWDQG WKHWXUELQHRSHUDWVDW%(3 ZLWK+ W
IW1HJOHFWPLQRUORVVHV (VWLPDW D WKHLQSXWSRZHU LQZDWW UHTXLUHG E\ WK
SXPS DQG E WKHSRZHU LQZDWW JHQHUDWG E\ WKHWXUELQH)RUIXUWKHUWFKQLFD
UHDGLQJ FRQVXOWWKH85/ ZZZ XVEUJRY SPWK\GUDXOFVBDDE SXEV(0 (0 SG

' HVLJQ3URMHFW

- ' 7R VDYH RQ HDFK WUDFN WKH VWDUWLQJ DWRZ QZ DWUXSSO VW XVHV JUDYLW GULYHQ IORZ IURP ILYH DJHV WKURXJH WDQN VGXULQJ WKH GD\ DQG WKH QHLJKERU WKH HDVW 30 WR $0 DWDFKHDSHUQJKWDWRI „ N K 7KH WRDOUHXSSO QHHGHG HDFK QJK YDULHV IURP (WR (JDO ZLWK QR PRUH WKDQ (JDOBRQ WR DQ RQH WDQN 7DQ HDHYDWRQ VYDU IURP WR IW$ VLQJOH FRQWDQWSHG SXPS GUZ LQJ IURP D OUJH JURXQGZDWUDTXLHUDQG YDOYHG LQR ILYH GLIHUHQWFDWLRQ WDQNV XSSO QQH GRHV WKLV MRE ' LWDQFHV IURP WKH SXPS WR WKH ILYH WDQNV YDU PRUH RQHW HY HQ IURP WR PL (DFK QQH DY HUDJHV RQH HDBRZ HY HU IWDQG KDV IRXU EXWWUC YDOYHV WKDWFDQ EH FRQWROOHG DWDQ GHVLUDEOH DQJOH 6HDFWD VXLWDEOH SXPS IDPLC IURP RQH RI WKH VL[GDWD VHW LQ WKLV FKDSWU) LJV 3 DQG 3 SOX 3UREV 3 3 DQG 3 $VXPH LGHDOL PLQDUW QR 5H\QROGV QXPEH RUSXPS URXJKQHVV HIFW 7KH JRDOLV WR GHWUPLQH SXPS DQG SLSHDQH VL]HV WKC DFKLHYH PLQP XP WRWDO FRVWRY HUD \HDUSHUIRG 6RPHVXJJHVWWG FRVWGDWDUH 3XPS DQG PRWRU SOX SHUFHQW RI SLSHVL]H 9DOYHV SOX SHUFHQW RI SLSHVL]H 3LSHDQHV „ SHUFHQW GLDPHWUSHUIRRWRQJWK 6LQFH WKH IBRZ DQG HDYDWRQ SDUDPHWUV YDU FRQVLGHUDEO DUDQGRP GDL0 YDUDWR ZLWKLQ WKH SHFLILHG UDQJHV PLJKWLYHDUHDQWF DSSURDFK

5HIHUHQFHV

' * : LOVRQ ³7XUERPDFKLQHU²)URP 3DGGOH: KHHOVWR7XUERMHWV´ 0HFK (QJ YRO 2FW SS ±

' -DSLNVHDQG1 & %DLQHV,QWURGXFWLRQWR7XUERPDFKLQH &RQFHSWV(7, ,QF +DQRYH1+

(6 /RJDQDQG5 5R\ (GV +DQGERRNRI7XUERPDFKLQH GHG 0DUFHO' HNNHU1HZ <RUN

*) : LVOHHQXV)OLG0HFKDQLFVRI7XUERPDFKLQH GHG 0F* UDZ +LOO1HZ <RUN

6 / ' L[RQ)OLG0HFKDQLFVDQG7KHUPRG\QDPLFVRI7XUERPDFKLQH VWHG (OVHYLHU 1HZ <RUN

- $ 6FKHIVDQG$ ( )XKV+DQGERRNRI)OLG'\QDPLFVDQG)OLG0DFKLQHU : LOH\ 1HZ <RUN

(6 /RJDQ -U7XUERPDFKLQH %DVLF7KHRU DQG$SSOLFDWLRQV GHG 0DUFHO' HNNH 1HZ <RUN

$ - 6WHSDQRII &HQWULXJDODQG$[LDO)ORZ 3XPSV GHG : LOH\ 1HZ <RUN

- 7X]VRQ &HQWULXJDO3XPS' HVLJQ : LOH\ 1HZ <RUN

3 * LGKDUDQG2 0RQ] 3UDFWLFDO&HQWULXJDO3XPSV (OVHYLHU 1HZ <RUN

/ 1HON &HQWULXJDODQG5RWDU 3XPPV &5& 3UHVV %RFD5DWRQ)/

' -DSLNVH 5)XUVDQG : ' 0DUFKHU&HQWULXJDO3XPS' HVLJQ DQG 3HUIRUPDQFH &RQFHSWV(7, ,QF +DQRYHU1+

5 . 7XUWRQ5RWRG\QDPLF 3XPS' HVLJQ &DPEULGJH 8QLYHUVLW\3UHVV &DPEULGJH 8.

, - . DUDVLN DQG7 0F* XLUH&HQWULXJDO3XPSV GHG 6SULQJHU9HUODJ 1HZ <RUN

9 / /REDQRII DQG5 5 5RV&HQWULXJDO3XPSV ' HVLJQ DQG $SSOLFDWLRQ GHG (OVHYLHU 1HZ <RUN

+ / 6WHZDUW3XPPV VWHG 0F*UDZ+LOO1HZ <RUN

$ % 0F. HQ]LH$[LDO) ORZ) DQV DQG &RPSUHVVRUV $HURG\QDPLF ' HVLJQ DQG 3HUIRUPDQFH $VKJDWH3XEOLVKLQJ %URRNILHOG 97

$ - : HQQHUWURP ' HVLJQRI +LJK0 / RDGHG$[LDO) ORZ) DQVDQG&RPSUHVVRUV&RQFHSWV(7, ,QF +DQRYHU1+

) 3 %OHLHU) DQ+DQGERRN 6HOHFWLRQ $SSOLFDWLRQ DQG' HVLJQ 0F* UDZ +LOO1HZ <RUN

5 $: DOOV $[LDO) ORZ) DQVDQG' XFWV : LOH\ 1HZ <RUN

+ 3 %RFK $ 3UDFWLFDO* XLGHWR&RPSUHVVRU7HFKQRORJ\ GHG 0F* UDZ +LOO1HZ <RUN

5 1 %URZQ&RPSUHVVRUV 6HOHFWLRQ DQG6L]LQJ GHG * XO 3URFHVVRQDO3XEOLVKLQ %XOODQWRQ 0$

5RQDOG + $XQJLHU$[LDO) ORZ &RPSUHVVRUV $ 6WDWH\ IRU $HURG\QDPLF ' HVLJQ DQ $QDO\VLV $60(3UHVV 1HZ <RUN

+ , + 6DUDYDQDPXWWR + &RKHQ DQG*) & 5RJHUV* DV7XUELQH 7KHRU 3UHQWI

+DQG8SSHU6DGGOH5LYHU1-

3 3 : DQK DQG3)OHWFKHU * DV7XUELQH3HUIRUPDQFH $60 (3URF 1HZ <RUN

0 3 %R\FH * DV7XUELQH (QJLQHHULQJ +DQGERRN G HG * XO3URHVWRQDO3XEOLVKLQ
%XWWHQJWRQ0 $

)OLG0DFKLQHU * URXS ,QWHUQDWLRQ R 0HFKDQLFDO (QJLQHHUV+\GURSRZHU : LOH\ 1HZ
<RUN

-HUHP\ 7KDNH7KH0LFUR+\GUR3RZHU7XUELQH0DQXDO,QWHUPHGLDWH7HFKQRORJ\ 3XE
&RQFKHWWH(VVH[8.

3 & +DQGRQ HG &RPSUHVVRU+DQGERRN0F* UDZ +LOO1HZ <RUN

+\GUDXOLF ,QWHUQDW +\GUDXOLF ,QWHUQDW 3XPS 6WDQGDUGV &RPSOHW VW HG 1HZ <RU

3 &RRSHU - 0HWLOD & +HDG DQG , - . DUDWLN HG 3XPS +DQGERRN VW HG
0F* UDZ +LOO1HZ <RUN

- 6 * XOOLYHUDQG5 ($ $UQGW+\GURSRZHU(QJLQHHULQJ +DQGERRN0F* UDZ +LOO1H
<RUN

5 / ' DXJKHUW - %)UDQNLQ DQG (-)LQQHPRUH)OLG0HFKDQLFV DQG (QJLQHHULQ
$SSOLFDWLRQV VW HG 0F* UDZ +LOO1HZ <RUN

5 + 6DEHUVN\ (0 * DWW $ - $FRVWD DQG (* +DXSWDQQ)OLG)RZ $)LU
&RXUVHIO)OLG0HFKDQLFV VW HG 3HDUVRQ(GXFDWLRQ 8SSHU6DGGOH5LYHU1-

- 3 3R\QWRQ 0HWHULQJ 3XPSV 0DUFHO HNNHU1HZ <RUN

+\GUDXOLF,QWHUQDW 5HFLSURFDWLQJ 3XPS 7HVW6WDQGDUG 1HZ <RUN

7 / +HQVKDZ 5HFLSURFDWLQJ 3XPSV : LOH\ 1HZ <RUN

- (0 LOOHU 7KH5HFLSURFDWLQJ 3XPS 7KHRU ' HVLJQDQG8VH : LOH\ 1HZ<RUN

' * : LOVRQDQG7 . RUDNDQWW 7KH' HVLJQRI +LJK(IILFLHQF\ 7XUERPDFKLQHU DQG* D
7XUELQHV GHG 3HDUVRQ(GXFDWLRQ 8SSHU6DGGOH5LYHU1-

6 2 . UDXV HWDO ³3HUIRUPLF 9HORFLW\0HDVXUHPHQWVLQ D : LGH DQG / DUJH 5DGLXV 5DW
$ XWRPRWLYH7RUTXH&RQYHUWHUDWWKH3XPS 7XUELQH,QWHUIDFH´ -)OLGV(QJLQHHULQJ YRO
QR SS ±

(0 * UHLJHU³7KH6WDELOLW\RI 3XPSLQJ 6\VWHPV 7KH)UHHPDQ6FKRODU/ HFWXUH´-
)OLGV(QJ YRO -XQH SS ±

5 (OHUHWDO HGV $GYDQFHVRI &)' LQ)OLG0DFKLQHU ' HVLJQ: LOH\ 1HZ <RUN

/) 0RRG\ ³7KH3URSHOOHU7\SH7XUELQH´ $6&(7UDQV YRO S

+ + $QGHUVRQ ³3UHGLFWLQJ4XDQWLW DQG (IILFLHQF\ LQ3XPSV² 7KH$UD 5DW
3UHFLSLWDWLQ3HUIRUPDQFH3UHGLFWLRQ RI &HQWULIXJDO3XPSV DQG &RPSUHVVRUV YRO
$60 (6\PS 1HZ <RUN SS ±

0 6FKREHUW 7XUERPDFKLQHU)ORZ 3K\VLFV DQG' \QDPLF 3HUIRUPDQFH 6SULQJHU 1HZ
<RUN

' - 0DKRQH HG 3URFHHGLQJV RI WKH ,QWHUQDWLRQDO&RQIHUHQFH RQ +\GURSRZHU
$6&(5HWURQ 9$

* : . RHSSO3XWQDP¶V3RZHUIURP WKH: LQG GHG 9DQ1RVWDQG5HLQKROG 1HZ <RUN

3 * LSH : LQG (QHUJ\ %DVLFV $ * XLGHWR6PDOODQG0LFUR: LQG6\VWHPV &KHOVHD* UHH
3XEOLVKLQJ : KLWH5LYHU-XQFWLRQ 97

' $ 6SHUD: LQG 7XUELQH 7HFKQRORJ\ )XQGDPHQWDO &RQFHSWV RI : LQG 7XUELQ

(QJLQHHULQJ $60 (3UHVV 1HZ <RUN

(+DX : LQG 7XUELQHV) XQGDPHQWDOV 7HFKQRORJLHV $SSOLFDWLRQ (FRQRPLFV G HG 6SULQJHU 9HUODJ 1HZ <RUN

5 +DUULVRQ (+DX DQG + 6QHO/ DUJH: LQG 7XUELQHV : LOH\ 1HZ <RUN

5 + $XQJLHU 7XUELQH $HURG\QDPLFV $[LDO) ORZ DQG 5DGLDO) ORZ 7XUELQH' HVLJQ DQ $QDO\VLV $60 (3UHVV 1HZ <RUN

0 / 5REHUWV ³7KH' DUUHXV: LQG 7XUELQH IRU(OHFWULFDO3RZHU* HQHUDWLRQ´$HURQDX - -XQH SS ±

') : DUQHDQG3 * &DQDQ ³* HQHUDWLRQ RI (OHFWULFLW\ IURP WKH: LQG´ ,((5HY YRO QR 5 1RYHPEHU SS ±

/ $ +DLPHU ³7KH &RVW OHZ 7XUELQH´ : DWHUSRZHU -DQXDU\ SS ± VHHDOVR $60 (6\PS 6PDOO+\GURSRZHU) OXLG0 DFK YRO DQG YRO

. (LVHOHHWDO ³) OHZ $QDO\VLV LQD3XPS' LIIXVHU 3DUW 0 HDVXUHPHQW 3DUW &)' ´-) OXLGV(QJ YRO ' HFHPEHU SS ±

' -DSLNVHDQG1 & %DLQHV7XUERPDFKLQHU ' LIIXVHU' HVLJQ7HFKQRORJ\ &RQFHSW(7 ,QF +DQRYHU1+

% 0 DVH\ DQG- : DUG6PLWK0 HFKDQLFV RI) OXLGV WKHG 1HOVRQ7KRUQHV 3XEOLVKLQ &KHOWHQKDP 8.

$PHULFDQ : LQG (QHUJ\ $VVRFLDWLRN ³* OREDO : LQG (QHUJ\ 0 DUNHW5HSRUW´ 85/ KWWS ZZZ DZHDRUJ SXEVGRFXPHQWJORBEDOPDUNHW SG!

- &DUORQ0 DUNH3URPHWKHXVDQG3URPXOLQ GHG %XWWZRUWK+HLQHPDQQ1HZ <RUN

0 +RGPDQQ0 RGHUQ3URPHWHXVDQG' XFW HVLJQ $LUFUDIW HVLJQV ,QF 0 RQWUH\ &$

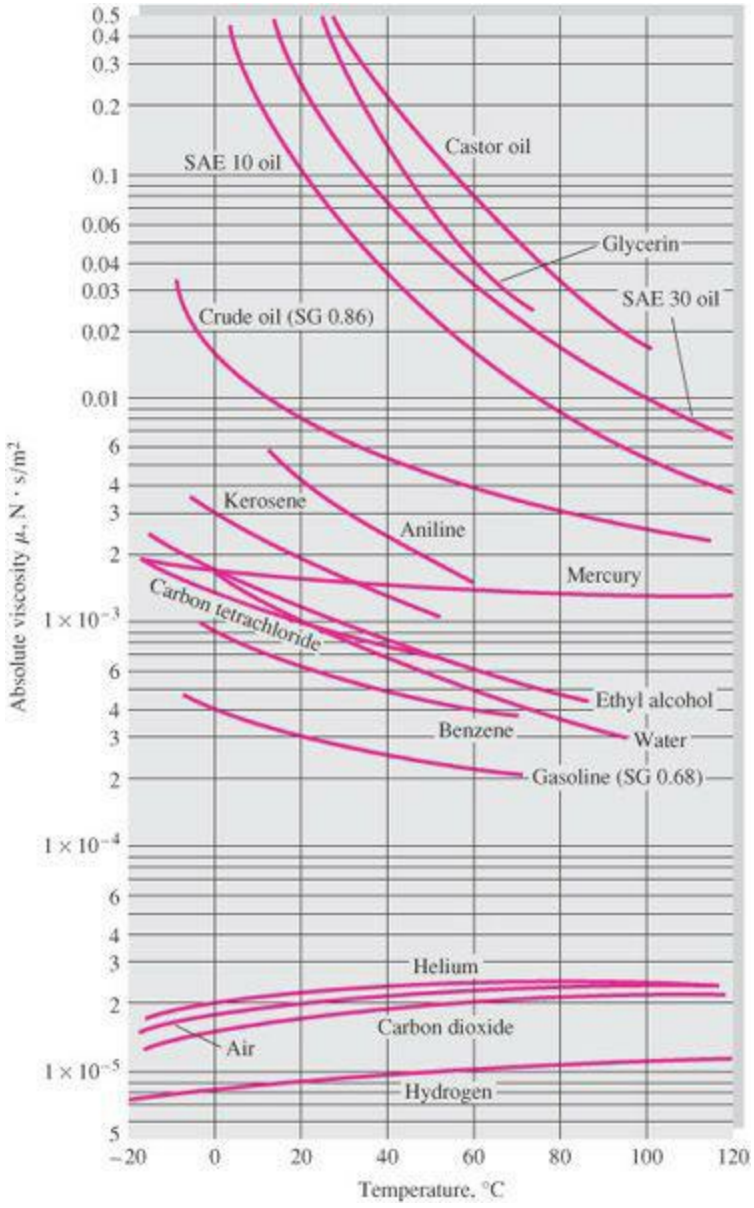
& / $UFKHUDQG0 = -DFREVRQ ³(YDOXDWLRQRI * OREDO: LQG3RZHU- * HRSK\V 5HV $VW YRO GRL -'

0)DUQDVDQG$ * DURQ ³$SSOLFDWLRQRI ' 2(IRU2SWLPDO7XUERPDFKLQHU ' HVLJQ´ 3DS $,$$ $,$$)OXLG' \QDPLFV&RQHUHQFH 3RUWODQG 25 -XQH

& &UDQ ³5XQQLQJ $JDLQVWWKH: LQG´ 3RSXODU6FLHQFH 0 DUFK SS ±

– &RQYHUVLRQIDFWRUV PD\ EHQHHGHG KS IW GEI V :

– : HDGRSWJ+ DVDYDULEOHWADGR + IRUGLPHQVLRQDOUHDVRQV



)LJ $ $EVROXWHYLVFRVLWLHV RI FRPPRQ IOXLGV DW DWP

7DEOH$ 9LVFRVLWLHVDQG' HQVLWLHVRI $LUDW DWP

6XJJHVWHGFXUYHILWWIRUDLU

$$\rho = \frac{p}{RT} \qquad R_{\text{air}} \approx 287 \text{ J/(kg} \cdot \text{K)}$$

Power law: $$\frac{\mu}{\mu_0} \approx \left(\frac{T}{T_0}\right)^{0.7}$$

Sutherland law: $$\frac{\mu}{\mu_0} \approx \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0 + S}{T + S}\right) \qquad S_{\text{air}} \approx 110.4 \text{ K}$$

with $T_0 = 273$ K, $\mu_0 = 1.71$ E$-$5 kg/(m $\cdot$ s), and T in kelvins.

7DEOH$ 3URSHUWLHVRI &RPPRQ/ LTXLGVDW DWP DQG *f*& *f*)

Liquid	ρ, kg/m^3	μ, kg/(m · s)	Y, N/m*	p_v, N/m^2	Bulk modulus K, N/m^2	Viscosity parameter C†
Ammonia	608	2.20 E−4	2.13 E−2	9.10 E+5	1.82 E+9	1.05
Benzene	881	6.51 E−4	2.88 E−2	1.01 E+4	1.47 E+9	4.34
Carbon tetrachloride	1590	9.67 E−4	2.70 E−2	1.20 E+4	1.32 E+9	4.45
Ethanol	789	1.20 E−3	2.28 E−2	5.7 E+3	1.09 E+9	5.72
Ethylene glycol	1117	2.14 E−2	4.84 E−2	1.2 E+1	3.05 E+9	11.7
Freon 12	1327	2.62 E−4	—	—	7.95 E+8	1.76
Gasoline	680	2.92 E−4	2.16 E−2	5.51 E+4	1.3 E+9	3.68
Glycerin	1260	1.49	6.33 E−2	1.4 E−2	4.35 E+9	28.0
Kerosene	804	1.92 E−3	2.8 E−2	3.11 E+3	1.41 E+9	5.56
Mercury	13,550	1.56 E−3	4.84 E−1	1.1 E−3	2.85 E+10	1.07
Methanol	791	5.98 E−4	2.25 E−2	1.34 E+4	1.03 E+9	4.63
SAE 10W oil	870	1.04 E−1‡	3.6 E−2	—	1.31 E+9	15.7
SAE 10W30 oil	876	1.7 E−1‡	—	—	—	14.0
SAE 30W oil	891	2.9 E−1‡	3.5 E−2	—	1.38 E+9	18.3
SAE 50W oil	902	8.6 E−1‡	—	—	—	20.2
Water	998	1.00 E−3	7.28 E−2	2.34 E+3	2.19 E+9	Table A.1
Seawater (30‰)	1025	1.07 E−3	7.28 E−2	2.34 E+3	2.33 E+9	7.28

*In contact with air.
†The viscosity–temperature variation of these liquids may be fitted to the empirical expression

$$\frac{\mu}{\mu_{20^\circ C}} \approx \exp\left[C\left(\frac{293\ \text{K}}{T\ \text{K}} - 1\right)\right]$$

ZLWKDFFXUDF\ RI “ SHUFHQWLQWKHUDQJH ” 7 ” *f*&

Á5HSUHVHQWDWLYH YDOXHV 7KH 6$(RLOFODVVLILFDWLRQV DQGRZ D YLVFRVLW\ YDULDWLRQ RI XS WR “ SHUFHQWHVSHFLDOO DWWRZ HUWHPSHUDWXUHV

7DEOH$ 3URSHUWLHVRI &RPPRQ* DVHVDW DW DQG *f*& *f)*

Gas	Molecular weight	R, m^2/(s^2 · K)	ρg, N/m^3	μ, N · s/m^2	Specific-heat ratio	Power-law exponent n*
H_2	2.016	4124	0.822	9.05 E−6	1.41	0.68
He	4.003	2077	1.63	1.97 E−5	1.66	0.67
H_2O	18.02	461	7.35	1.02 E−5	1.33	1.15
Ar	39.944	208	16.3	2.24 E−5	1.67	0.72
Dry air	28.96	287	11.8	1.80 E−5	1.40	0.67
CO_2	44.01	189	17.9	1.48 E−5	1.30	0.79
CO	28.01	297	11.4	1.82 E−5	1.40	0.71
N_2	28.02	297	11.4	1.76 E−5	1.40	0.67
O_2	32.00	260	13.1	2.00 E−5	1.40	0.69
NO	30.01	277	12.1	1.90 E−5	1.40	0.78
N_2O	44.02	189	17.9	1.45 E−5	1.31	0.89
Cl_2	70.91	117	28.9	1.03 E−5	1.34	1.00
CH_4	16.04	518	6.54	1.34 E−5	1.32	0.87

*The power-law curve fit, Eq. (1.27), $\mu/\mu_{293K} \approx (T/293)^n$, fits these gases to within ±4 percent in the range $250 \le T \le 1000$ K. The temperature must be in kelvins.

7DEOH$ 6XUIDFH7HQVLRQ 9DSRU3UHVVXUH DQG6RXQG6SHHGRI : DWU

T, °C	Y, N/m	p_v, kPa	a, m/s
0	0.0756	0.611	1402
10	0.0742	1.227	1447
20	0.0728	2.337	1482
30	0.0712	4.242	1509
40	0.0696	7.375	1529
50	0.0679	12.34	1542
60	0.0662	19.92	1551
70	0.0644	31.16	1553
80	0.0626	47.35	1554
90	0.0608	70.11	1550
100	0.0589	101.3	1543
120	0.0550	198.5	1518
140	0.0509	361.3	1483
160	0.0466	617.8	1440
180	0.0422	1002	1389
200	0.0377	1554	1334
220	0.0331	2318	1268
240	0.0284	3344	1192
260	0.0237	4688	1110
280	0.0190	6412	1022
300	0.0144	8581	920
320	0.0099	11,274	800
340	0.0056	14,586	630
360	0.0019	18,651	370
374*	0.0*	22,090*	0*

*Critical point.

7DEOH$ 3URSHUWLHV RI WKH6WDQGDUG$WPRVSKHUH

7 DEOH% ,VHQWURSLF) ORZ RI D3HUIHFW DV N

7 DEOH% 1 RUP DO6KRFN5 HODWLRQV IRU D3HUIHFW DV N

Ma_{n1}	Ma_{n2}	p_2/p_1	$V_1/V_2 = \rho_2/\rho_1$	T_2/T_1	p_{02}/p_{01}	A_2^*/A_1^*
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.10	0.9118	1.2450	1.1691	1.0649	0.9989	1.0011
1.20	0.8422	1.5133	1.3416	1.1280	0.9928	1.0073
1.30	0.7860	1.8050	1.5157	1.1909	0.9794	1.0211
1.40	0.7397	2.1200	1.6897	1.2547	0.9582	1.0436
1.50	0.7011	2.4583	1.8621	1.3202	0.9298	1.0755
1.60	0.6684	2.8200	2.0317	1.3880	0.8952	1.1171
1.70	0.6405	3.2050	2.1977	1.4583	0.8557	1.1686
1.80	0.6165	3.6133	2.3592	1.5316	0.8127	1.2305
1.90	0.5956	4.0450	2.5157	1.6079	0.7674	1.3032
2.00	0.5774	4.5000	2.6667	1.6875	0.7209	1.3872
2.10	0.5613	4.9783	2.8119	1.7705	0.6742	1.4832
2.20	0.5471	5.4800	2.9512	1.8569	0.6281	1.5920
2.30	0.5344	6.0050	3.0845	1.9468	0.5833	1.7144
2.40	0.5231	6.5533	3.2119	2.0403	0.5401	1.8514
2.50	0.5130	7.1250	3.3333	2.1375	0.4990	2.0039
2.60	0.5039	7.7200	3.4490	2.2383	0.4601	2.1733
2.70	0.4956	8.3383	3.5590	2.3429	0.4236	2.3608
2.80	0.4882	8.9800	3.6636	2.4512	0.3895	2.5676
2.90	0.4814	9.6450	3.7629	2.5632	0.3577	2.7954
3.00	0.4752	10.3333	3.8571	2.6790	0.3283	3.0456
3.10	0.4695	11.0450	3.9466	2.7986	0.3012	3.3199

7 DEOH% $ GLDEDWLF) ULFWLRQDO) ORZ LQD&RQVWDQW$ UHD' XFWRUN

Ma	$\bar{f}L/D$	p/p^*	T/T^*	$\rho^*/\rho = V/V^*$	p_0/p_0^*
0.00	∞	∞	1.2000	0.0000	∞
0.10	66.9216	10.9435	1.1976	0.1094	5.8218
0.20	14.5333	5.4554	1.1905	0.2182	2.9635
0.30	5.2993	3.6191	1.1788	0.3257	2.0351
0.40	2.3085	2.6958	1.1628	0.4313	1.5901
0.50	1.0691	2.1381	1.1429	0.5345	1.3398
0.60	0.4908	1.7634	1.1194	0.6348	1.1882
0.70	0.2081	1.4935	1.0929	0.7318	1.0944
0.80	0.0723	1.2893	1.0638	0.8251	1.0382
0.90	0.0145	1.1291	1.0327	0.9146	1.0089
1.00	0.0000	1.0000	1.0000	1.0000	1.0000
1.10	0.0099	0.8936	0.9662	1.0812	1.0079
1.20	0.0336	0.8044	0.9317	1.1583	1.0304
1.30	0.0648	0.7285	0.8969	1.2311	1.0663
1.40	0.0997	0.6632	0.8621	1.2999	1.1149
1.50	0.1361	0.6065	0.8276	1.3646	1.1762
1.60	0.1724	0.5568	0.7937	1.4254	1.2502
1.70	0.2078	0.5130	0.7605	1.4825	1.3376
1.80	0.2419	0.4741	0.7282	1.5360	1.4390
1.90	0.2743	0.4394	0.6969	1.5861	1.5553

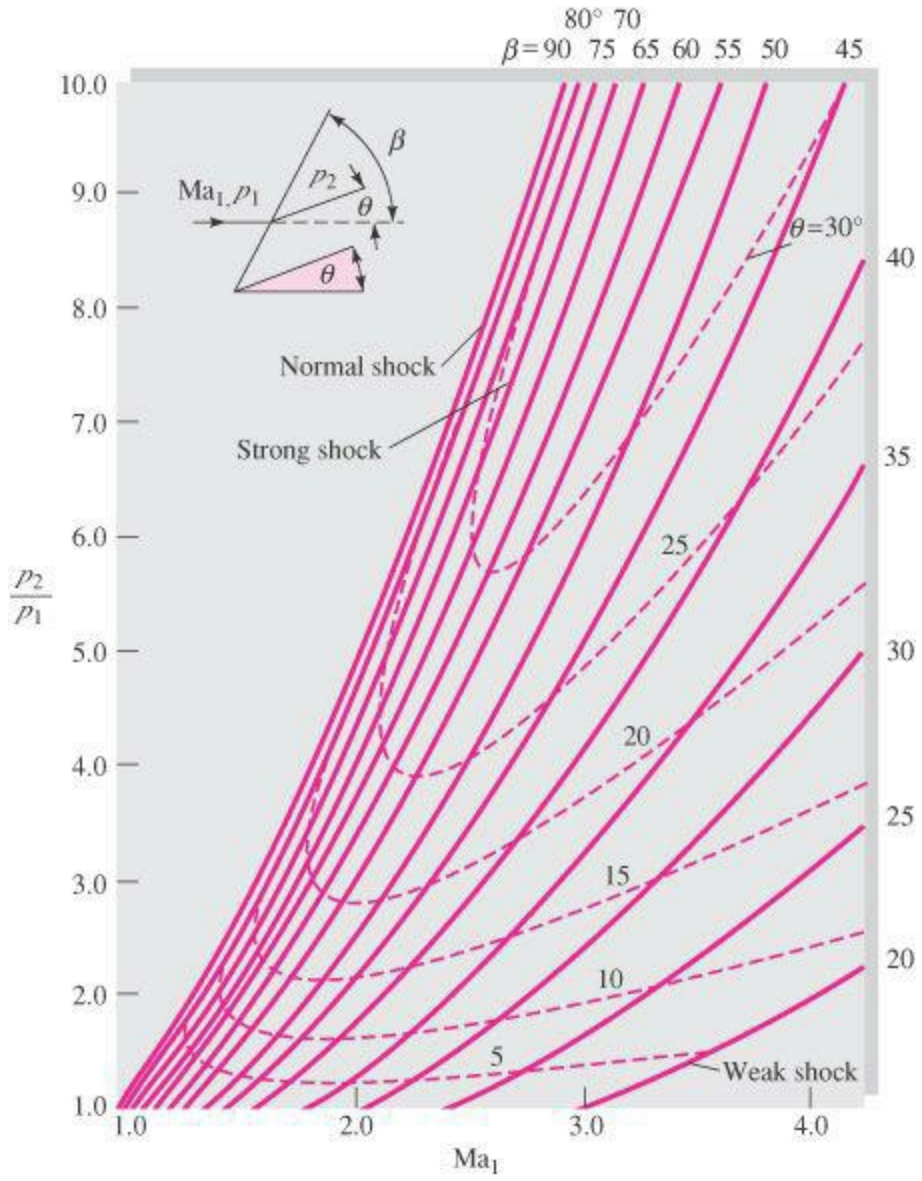
7DEOH%)UDFWRQDVV' XFWOHVV RZ LWK+ HDW7UDQVIHUFRUN

Ma	T_0/T_0^*	p/p^*	T/T^*	$\rho^*/\rho = V/V^*$	p_0/p_0^*
0.00	0.0000	2.4000	0.0000	0.0000	1.2679
0.10	0.0468	2.3669	0.0560	0.0237	1.2591
0.20	0.1736	2.2727	0.2066	0.0909	1.2346
0.30	0.3469	2.1314	0.4089	0.1918	1.1985
0.40	0.5290	1.9608	0.6151	0.3137	1.1566
0.50	0.6914	1.7778	0.7901	0.4444	1.1141
0.60	0.8189	1.5957	0.9167	0.5745	1.0753
0.70	0.9085	1.4235	0.9929	0.6975	1.0431
0.80	0.9639	1.2658	1.0255	0.8101	1.0193
0.90	0.9921	1.1246	1.0245	0.9110	1.0049
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.10	0.9939	0.8909	0.9603	1.0780	1.0049
1.20	0.9787	0.7958	0.9118	1.1459	1.0194
1.30	0.9580	0.7130	0.8592	1.2050	1.0437
1.40	0.9343	0.6410	0.8054	1.2564	1.0777
1.50	0.9093	0.5783	0.7525	1.3012	1.1215
1.60	0.8842	0.5236	0.7017	1.3403	1.1756
1.70	0.8597	0.4756	0.6538	1.3746	1.2402
1.80	0.8363	0.4335	0.6089	1.4046	1.3159
1.90	0.8141	0.3964	0.5673	1.4311	1.4033
2.00	0.7934	0.3636	0.5289	1.4545	1.5031
2.10	0.7741	0.3345	0.4936	1.4753	1.6162
2.20	0.7561	0.3086	0.4611	1.4938	1.7434

7DEOH% 3UDQGWO0 H\HU6XSHUVRQLF ([SDQVLRQ) XQFWLRQIRUN

Ma	ω, deg	Ma	ω, deg	Ma	ω, deg	Ma	ω, deg
1.00	0.00						
1.10	1.34	3.10	51.65	5.10	77.84	7.10	91.49
1.20	3.56	3.20	53.47	5.20	78.73	7.20	92.00
1.30	6.17	3.30	55.22	5.30	79.60	7.30	92.49
1.40	8.99	3.40	56.91	5.40	80.43	7.40	92.97

) LJ % 0 DFKQXP EHUGRZ QWWHDP RI DQREQTXHVKRFNIRUN



) LJ % 3UHVVXUHUDWLR GRZQVWHDP RI DQREOTXHVKRFN IRUN

$SSHQGL[& &RQYHUVLRQ)DFWRUV

' XULQJWKLVSHULRGRIWDQVLWLRQWKHUHLVDFRQVLGHUDEOHFRQIXVLRQVEHWZHHQ%* DQG6, XQLWV VHH7DEOH 6RPHDGGLWLRQDOFRQYHUVLRQVDUHJLYHQKHUH &RQYHUVLRQIDFWRUVDUHJLYHQLQVLGHWKH IURQWFRYHU

Length	Volume
1 ft = 12 in = 0.3048 m 1 mi = 5280 ft = 1609.344 m 1 nautical mile (nmi) = 6076 ft = 1852 m 1 yd = 3 ft = 0.9144 m 1 angstrom (Å) = 1.0 E−10 m	1 ft^3 = 0.028317 m^3 1 U.S. gal = 231 in^3 = 0.0037854 m^3 1 L = 0.001 m^3 = 0.035315 ft^3 1 U.S. fluid ounce = 2.9574 E−5 m^3 1 U.S. quart (qt) = 9.4635 E−4 m^3
Mass	**Area**
1 slug = 32.174 lbm = 14.594 kg 1 lbm = 0.4536 kg 1 short ton = 2000 lbm = 907.185 kg 1 tonne = 1000 kg	1 ft^2 = 0.092903 m^2 1 mi^2 = 2.78784 E7 ft^2 = 2.59 E6 m^2 1 acre = 43,560 ft^2 = 4046.9 m^2 1 hectare (ha) = 10,000 m^2
Velocity	**Acceleration**
1 ft/s = 0.3048 m/s 1 mi/h = 1.466666 ft/s = 0.44704 m/s 1 kn = 1 nmi/h = 1.6878 ft/s = 0.5144 m/s	1 ft/s^2 = 0.3048 m/s^2
Mass flow	**Volume flow**
1 slug/s = 14.594 kg/s 1 lbm/s = 0.4536 kg/s	1 gal/min = 0.002228 ft^3/s = 0.06309 L/s 1 × 10^6 gal/day = 1.5472 ft^3/s = 0.04381 m^3/s
Pressure	**Force**
1 lbf/ft^2 = 47.88 Pa 1 lbf/in^2 = 144 lbf/ft^2 = 6895 Pa 1 atm = 2116.2 lbf/ft^2 = 14.696 lbf/in^2 = 101,325 Pa 1 inHg (at 20°C) = 3375 Pa 1 bar = 1.0 E5 Pa	1 lbf = 4.448222 N = 16 oz 1 kgf = 2.2046 lbf = 9.80665 N 1 U.S. (short) ton = 2000 lbf 1 dyne = 1.0 E−5 N 1 ounce (avoirdupois) (oz) = 0.27801 N
Energy	**Power**
1 ft · lbf = 1.35582 J 1 Btu = 252 cal = 1055.056 J = 778.17 ft · lbf 1 kilowatt hour (kWh) = 3.6 E6 J	1 hp = 550 ft · lbf/s = 745.7 W 1 ft · lbf/s = 1.3558 W
Specific weight	**Density**
1 lbf/ft^3 = 157.09 N/m^3	1 $slug/ft^3$ = 515.38 kg/m^3 1 lbm/ft^3 = 16.0185 kg/m^3 1 g/cm^3 = 1000 kg/m^3
Viscosity	**Kinematic viscosity**
1 slug/(ft · s) = 47.88 kg/(m · s) 1 poise (P) = 1 g/(cm · s) = 0.1 kg/(m · s)	1 ft^2/h = 0.000025806 m^2/s 1 stokes (St) = 1 cm^2/s = 0.0001 m^2/s

Temperature scale readings

$T_F = \frac{9}{5}T_C + 32$ $T_C = \frac{5}{9}(T_F - 32)$ $T_R = T_F + 459.69$ $T_K = T_C + 273.16$

where subscripts F, C, K, and R refer to readings on the Fahrenheit, Celsius, Kelvin, and Rankine scales, respectively.

Specific heat or gas constant*	Thermal conductivity*
1 ft · lbf/(slug · °R) = 0.16723 N · m/(kg · K) 1 Btu/(lbm · °R) = 4186.8 J/(kg · K)	1 Btu/(h · ft · °R) = 1.7307 W/(m · K)

*Although the absolute (Kelvin) and Celsius temperature scales have different starting points, the intervals are the same size: 1 kelvin = 1 Celsius degree. The same holds true for the nonmetric absolute (Rankine) and Fahrenheit scales: 1 Rankine degree = 1 Fahrenheit degree. It is customary to express temperature differences in absolute temperature units.

$SSHQGL[' (TXDWLRQVRI0RWLRQLQ&\OLQGULFDO&RRUGLQDWHV

7KHHTXDWLRQVRI PRWLRQRI DQLQFRPSUHVVLEOHQHZWRQLDQIOXLGZLWKFRQVWDQW N DQGF$S DUHJLYH KHUHLQF\OLQGULFDOFRRUGLQDWHV U] ZKLFKDUHUHODWHGWRFDUWHVLDQFRRUGLQDWHV [\ ] DVLQ) LJ

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z \tag{D.1}$$

7KHYHORFLWYFRPSRQHQWDUHY$U Y DQGY] +HUHDUHWKHHTXDWLRQV &RQWLQXLW

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0 \tag{D.2}$$

&RQYHFWLYHWPHGHULYDWLYH

$$\mathbf{V}\cdot\nabla = v_r\frac{\partial}{\partial r} + \frac{1}{r}v_\theta\frac{\partial}{\partial\theta} + v_z\frac{\partial}{\partial z} \tag{D.3}$$

/DSODFLDQRSHUDWRU

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2} \tag{D.4}$$

7KHUPRPHQWXP HTXDWLRQ

$$\frac{\partial v_r}{\partial t} + (\mathbf{V}\cdot\nabla)v_r - \frac{1}{r}v_\theta^2 = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g_r + \nu\left(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right) \tag{D.5}$$

7KH PRPHQWXP HTXDWLRQ

$$\frac{\partial v_\theta}{\partial t} + (\mathbf{V}\cdot\nabla)v_\theta + \frac{1}{r}v_r v_\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + g_\theta + \nu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right) \tag{D.6}$$

7KH] PRPHQWXP HTXDWLRQ

$$\frac{\partial v_z}{\partial t} + (\mathbf{V}\cdot\nabla)v_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z + \nu\nabla^2 v_z \tag{D.7}$$

7KHHQHUJ\ HTXDWLRQ

$$\rho c_p\left[\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)T\right] = k\nabla^2 T + \mu[2(\epsilon_{rr}^2 + \epsilon_{\theta\theta}^2 + \epsilon_{zz}^2) + \epsilon_{\theta z}^2 + \epsilon_{rz}^2 + \epsilon_{r\theta}^2] \tag{D.8}$$

ZKHUH

$$\epsilon_{rr} = \frac{\partial v_r}{\partial r} \qquad \epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)$$

$$\epsilon_{zz} = \frac{\partial v_z}{\partial z} \qquad \epsilon_{\theta z} = \frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \tag{D.9}$$

$$\epsilon_{rz} = \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \qquad \epsilon_{r\theta} = \frac{1}{r}\left(\frac{\partial v_r}{\partial \theta} - v_\theta\right) + \frac{\partial v_\theta}{\partial r}$$

9 LVFRXVVWUHVVFRPSRQHQWV

$$\tau_{rr} = 2\mu\epsilon_{rr} \qquad \tau_{\theta\theta} = 2\mu\epsilon_{\theta\theta} \qquad \tau_{zz} = 2\mu\epsilon_{zz}$$

$$\tau_{r\theta} = \mu\epsilon_{r\theta} \qquad \tau_{\theta z} = \mu\epsilon_{\theta z} \qquad \tau_{rz} = \mu\epsilon_{rz} \tag{D.10}$$

$ QJXODUYHORFLWFRPSRQHQWV

$$2\omega_r = \frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}$$

$$2\omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \tag{D.11}$$

$$2\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta}$$

$QVZHUVWR6HOHFWHG3UREOHPV

&KDSWHU

```
3      (    NJ    (    PRGHFXOHV
3     D 7KHIRUPXODLVLQFRQVLVWHQW
3    1§      0\ ,
3     <HV DOOWHUPVDUH^0 / 7 `
3     ^%   ^/í `
3     4   &RQW%J  +
3     $OOWHUPVDUH^0 /í 7í `
3     9   9 Hí PW
3      E
3      D    N3D  E      NJ P
3     : DU      GEI
3     !ZHW      NJ P  !GU      NJ P
3     :        IW GEI
3      D    N1  E    N1
3      D !       NJ P   E !       NJ P  LGHQWLWLHV
3      E     (  3D
3     2       3D 5H
3     $         NJ  P   V  %       .
3           . §  7 .      .
3         :
3      G     P V  H     P V  I     P V
3     ) §    K      K  $9
3      D <HV  E   §     NJ  P V
3     3 §    :
3     0 § GE95  K
3          0 VLQ    GE95
3              NJ  P   V  GDWW SRLQWVDUHVXUEXGHQWWRZ
3            3D
3            3D
3        PP
3     )          1
3     K   < ! J    FRV
3     K   < FRV   ! J:
3     ] §       P
3        NP
3      D     P V  E     P V
3      D    f&  E   f&
```

```
3     0 D
3     \  [ VDQ    FRQVDQW
3     $ SSUR[LPDWH    SHUFHQW
3      D      NJ  P  V   E     SHUFHQW
3
```

&KDSWHU

```
3    1[\  í    CE IW 2$$  í    CE IW
3    $ SSUR[LPDWH    GHJUHHV
3     D     IW E     LQ  F      P  G       PP
3     D     N3D  E “   P
3           3D
3        FP
3     K       FP  K       FP
3     D       CE IW  E       CE IW
3
3        CE
3          FP
3     S VDOHV  N3D P H[DFW     (    NJ
3     D      IW E      IW
3     D      N3D  E .
3          FP
3     ûS  ûK> ZDWU    G '   í  RO  í G '   @
3       f
3     S%       N3D
3     / HVVHJ GURSV     FP  ULJKVHJ ULVHV   FP  YHUWFDO
3     S$ í S%   !  í  !  JK
3     D      CE IW  E      CE IW PDQRPHWUHDGV/IULFWRQORVV
3
3     D      N3D  E      P
3     D      IW E          CE
3     D        CE  E      IWURP $
3          IW
3          P
3         1
3          IW
3          P
3     )       (  1  0 &       (  1   P FRXQWHUFORFNZLVH QR\WSSLQJ
3            1
3          P
3     K§       P
3     +   5>CE    ^ CE          `   @
```

```
3     D     NL  F     NL  P
3     E )$%       1 )&'       1
3      !     f
3    )+       0 1 )9       0 1
3     D )9       1 )+       1
3    3     NL
3    )+       NL )9       NL \HV
3    )9         1 )+         1
3    )RQHERW§       1
3    )RUFHVRQHDFKSDQHODUHHTXDO
3    )+       NL )9       NL
3    )+        GE )9        GE
3     )+     )9       NL
3      D     NL  E     NL
3        1
3     ]§       P
3      D         P  E       NJP
3      D       1  6*
3      D    1  E
3
3      D <HV  E <HV  F       LQ
3          IW
3         f
3     DE§
3           P
3           3D YDFXXP
3     <HV VWDEOHLL 6!
3     6OUKVW XQWDEOH 0 *   í       P
3     6WDEOHLL 5 K!
3      D XQWDEOH E VWDEOH
3     0 *   /    GL5 í  5   GE!   LL / !   5
3         LQGHHS YROXPH       IOXLGRXQFHV
3     D[    D í     P V  GHFHOHUDWRQ  E í     P V  GHFHOHUDWRQ
3      D     FP  E     1
3      D D[ §     P V  E QRHIHFWS$   S%
3     / HDQVWRWKHULJKWDW     f
3      D EDFNZDUG  E IRUZDUG
3        FP QQHDUVFDOH2.
3      D     UPLQ  E     UPLQ
3         UPLQ
3         UPLQ
3           UPLQ
```

&KDSWHU

```
3   U SRVLWRQYHFWRUIURP SRLQW2
3       LQ
3   4    E    J   >K  /    í  Kí /   @
3    D     P V E     P V F     P V
3     D   P V E    P V F   FP VRXW
3    ûW    V
3    GKGW  4    4 í 4    GG
3    4WRS    8 E/
3     E 4     EKXPD[
3     D    P/ V E      FP V
3     D     NJ V E      P V F
3    K  >. WG   WDQ  @
3    4    8 EK
3     D      NJ V E     N3D
3       PLQXWHV
3    9KRXU      P V
3    9          IWW
3    8          P V
3    9  9 U  K
3        1 WRWKHOHW
3    )    S í S$ $ í ! $ 9  >'  '    í  @
3    )   !8 / E
3    .        FRV
3    9 §      P V
3        N1
3    )   !: K9  >    í VLQ  í  @WRWKHOHW
3        1
3     D      1 WROHW E     1 XS
3       1
3         1
3         1
3        1
3         1
3       GE
3    ' UDJ§       1
3    )[      )\  í   1 )]        1
3     D       1 P  E     FP  F     FP
3    )    !   JE K  í K   í ! K E9   K K í
3       P V
```

```
3        1
3         NBD
3     9      >      9M@      !4   N
3     G9 GW J
3     G9 GW JK /    K
3     9MW       IWW
3     G= GW    J= /
3       D      P VDQG      P  E       NP
3      K K   í`   `   `   `>     9    JK  @
3      9    í 9H 5  CQ   í  P ÂW0
3      )     CE   ! '  9
3       D 9    9        &9 W0   &    ! EK   í FRV
3      ) ERW          CBE
3       D       NJ V  E       NBD
3       D 9          P V  E K       FP
3      ;      >K + í  K @
3       D         IWV  E        IWV
3      KPD[          FP
3       D         IWV  E         IWV
3          NBD JDJH
3      4          FP  V
3       D     P   E     PP
3           NJ V
3           P  VXEFULWFDO          P  VXSHUFULWFDO
3      W  >  . í    K J@  .     '  G
3          1
3      7    í       N1  P
3      7%       IWCBE
3           NJ V
3      3   ! 4 & 5 9Q FRW     FRW
3      7% í          N1  P
3          KS PD[ SRZHUDW    UPLQ
3      7    ! 4 9Z  5   /     N
3      4          P  V 7           f&
3      û 7 §     f)
3       D       N+ NJ  E      0 :
3         P
3       D       KS  E       KS
3         KS
3          N
3           KS
```

3 *f*& V

3 P VDQG P V

&KDSWHU

3 D GXGW 9 / [/

3 E D 8 / [/ D 8 / \ /

3 E D [D \

3 D 9 / E DWW / 8

3 D Y 8 FRV 9 VLQ E <HV

3 ,I % %Ë %U Ú IFQ Ë

3 % IFQU RQQD

3 D <HV FRQWQXLWLVVDWLHG

3 ! ! / / í 9W

3 % % FRQWW^. ` ^/ 7` ^D` ^/í `

3 Y í %U I] U

3 E % Y K

3 E S S í ! [\

3 D E <HV FRQWQXLWDQG1 DYLU6WHVDUHVDWLHG

3 I & U I & U

3 & ! J VLQ

3 7 7 8 N í \ K

3 $ 8 UVLQ í 9 UFRV FRQW

3 D <HV $ H[LWW

3 $ í 4 CE

3 4 8/E

3 ,UUDWLRQDO]& + í & 5 J

3 $ 9\ K FRQW

3 $ í . VLQ U

3 D <HV DYHUDFLWSRWHQWLDO H[LWW

3 9 § P V . § í *f*

3 D $ í E Ë í QQU

3 $ %UVLQ %/ QQU FRQW

3 <HV $ H[LWW

3 \ $ U FRV Q FRQW

3 D Z ! J /[í [

3 2EVHUYHUHVXOW% 95 U

3 % ! JE QQ UD í ! J U í D

3 4 P V P

```
3    %    8 QQ UE >QQ DE@
3     D    NJ K  E   PP
3    K  K H[S>íŒE !JW     /$  @
3    Y   95  U
3     D     3D  E     :
```

&KDSWHU

```
3    3URJUDPPH9        PLK
3    9        P V )       1
3     D     PLK  E      GH
3    0 R  J    !<
3      D ^0 /í  7í  `   E ^0 / 7í  `
3     6W  8  ! J'
3     2QHSRWLEOHJURXSLVK/  N
3     6WDQGDUGQXPEHU  K ! 9F S
3     4   >ûS/  E@ FRQWW
3     2QHSRWLEOHJURXSLV9'  8
3     9'  9   IFQ1  + /
3     )  ! 9 /      IFQ.  ! 9/     / '   9 D
3      D ,QGHWHUPLQDWH E 7         V
3     / /   IFQ>/  '   ! 9'     (   ! 9   @
3     P" 57 R     S R'      IFQF S 5
3     4  EJ  +       FRQWW
3     N\ GURJHQ§       :   P   .
3      D 4 GW5  $û7    FRQWDQW
3     G'    IFQ! 8'     ! 8  '  <
3     + DOYLQJ P LQFUHDVHVI E\ DERXW  SHUFHQW
3      D ^1`   ^/ `
3     ) §      1   GRXEOLQJ 8 TXDGUXSOHV)
3     $ SSUR[ LP DWH0       GH RQH DWK
3      D       V  E
3     3RZ HU§   KS
3     ) DU§    1  P
3     9 §     P V
3      E        1
3      D <HV  E          NJ V
3     &DOPLQP          +]
3      D 9       P V  E ]       P
3      E $ SSUR[ LP DWH0        1
3     )       GH  H[ WDSRODWH G
```

```
3    $ERXW  N1  H[WDSRODWH
3    3URRWSHPRPHQW   N1  P
3    ' UDJ          GH
3    : HEHUQR §    LI / P / S
3     D     P V  E        F
3        N1
3    9P     FP V 7P     V +P      P
3    $W   :  '        P
```

&KDSWHU

```
3   $ERXW    EDUUHOVSHUGD\
3    D    P  K  E    P  K
3    D K\GURJHQ [     P
3    D í     3DP  E í      3DP
3     D ) URP $ WR% E K     P
3           NJ P V
3    4          P  KLI +     FP
3      PP
3              NJ P V OLPLQDUIGRZ
3         FF K
3    )      1
3     D       P  K OLPLQDU E G      PP
3     D '         FP
3    ûS     3D
3     D      P  K  E IGRZ LVXS
3     D IGRZ LVXS  E      P  K
3     D       GH IW  E    IWV
3         P V
3         PP
3    K      P  ûS     0 3D
3          JDOPLQ
3    U5    í Hí
3     D í      3DP  E    3D  F    SHUFHQW
3    S       0 3D
3    WGDQ  > : < CE  @ K    IDY/ '  J@
3     D       GH LQ  E       KS
3       P  K
3     D 1 RWLGHQWLFDWR + DOOQG
3        KS
3    4          P  K OLPLQDU 5H
3     D    N3D  E    P  K  F  X      P VDWU    FP
```

```
3     4       P  K
3     '  §     FP
3     /       P
3     4       P  KRU    P  K
3     4       P  K VRVWHOHW
3     4         P  V
3      D       P  K  E       P  K
3     '  §       P
3      D     P V  E        P P   F        3DP
3     $ERXW SDVWDJHV
3     +       LQ
3      D       P  K  E     3D YDFXXP
3     D       FP
3      E         3D
3     $SSUR[LPDWH VTXDUHV
3      $ PD[LPXP RI  HQGRZV
3       D       KS  E       KSZLWK ƒFRQH
3       D       3D  E  KG§
3       4           IWV
3       D . §       E  4  §       IWV
3          :
3       4           IWV
3       6KRUWGXFW4          IWV
3       4           P  V
3       ûS        GE LQ
3       4           P  V 4          P  V 4          P  V
3       ,QFUHDVHGë GDQG/ GDUHWHFDXVHV
3       4   í       IWV 4          IWV 4          IWV
3        RSHQLQJ    ƒ
3       4$%       4%&       4%       4&'       4$&       IWV DQG
3       4$%       4%&       4%       4&'       4$&       IWV DQG
3            ƒ ' H      P  SH       NBD
3             ƒ : H      IWSH       GE IW
3       D       P V  E       P  V  F       3D
3          P V
3       ûS        NBD
3       4         JDOPLQ GUHGXFHU       FP
3        D K       FP
3        D          P  V  E       NBD
3        D       P  E       P
3       ûS        NBD
```

```
3      '          FP
3           JDOPLQ
3      4          IWV
3       D    P  K  E       3D
```

&KDSWHU

```
3    7KLVLVSUREDEO\KHDXP
3     D    P  E   P
3    +        YHUVXV      IRU%DVLXV
3    $SSUR[LPDWHO\       1 SHUPHWUR ZLGWK
3     ' RHVQRWDDW\ %X%       DW
3     &   ! %      FRQWW    ZDOOVXFFHVVRQ
3      D )        1  E      1
3      D       P V  E          3D
3     [ §      P
3     $     [8   I
3     K        PP  K        PP
3     )D        )  )E        )
3      D )GUDJ          1  ! /   8   D
3     3UHGLFWHGWKLFNQHVVLVDERXW  SHUFHQWKLJKHU
3     )          !   /  8    /
3        SHUFHQW
3         P V      NQ
3      D     P V  E     P V
3     /          P  E        P
3      D     1 P
3     $FFXUDWHWRDERXW"  SHUFHQW
3     §   PP  8          P V      NQ
3     6HSDUDWHRQDW/              SHUFHQWHUURU
3     6HSDUDWHRQDW §     GHJUHHV
3      D 5H          E  D      PP
3     ]    7  >%Q   @Q  J  5% í
3      D     1   E FURWZLQGFUHDWHVDYHU ODJHVLGHIRUFH
3     7RZ SRZHU       KS
3     6TXDUHVLGHOHQJWK§       P
3     ûW  ±    P          V
3        P V
3        IW
3      D /        P  E       P
3     $ERXW   PLK
3      D      KS
```

3 KS
3 *f*
3 D V
3 9 P V
3 $SSUR[LPDWHO P E\ P
3 D KS E KS
3 $9RYHUWXUQ§ IWV PLK
3 7RUTXH§ &9 ' 5 PD[UPLQ
3 $9DYJ 8 '
3 E K§ P
3 D PLK E PLK
3 NQRWV
3 P
3 û[EDOO§ P
3 û\ § IW
3 $9GRZQ§ P PLQ 9XS§ P PLQ
3 D PLK E KS
3 D P V E P
3 / ' PD[. *f*
3 D P V E P V NQ
3 FUXGHWKHRU § UV
3 $SSUR[LPDWHO IW

&KDSWHU

3 + CE 5 í 5
3 1R ULVQRWDSURSHUWZRGLPHQVLRQDOSRWQWLDO
3 % %U VLQ
3 + %
3 D FP
3 *
3 ,URDWRQDORXWU URDWRQDOLQHU PLQPXP S S í !& 5 DWU
3 D P WRWKHOHIWR $
3) URP DIDU DVLQJHVRXUFH P
3 9RUW[QHDUDZDOO VHH) LJ E
3 6WDJQDWRQIORZ WRZDUGDEXPS
3 &S í^ [D > [D @ &S PLQ í DW[D
3 D P E P RQHDFKVLGH
3 &UHDWHVDVRXUFHLQDVTXDUHFRQHU
3 U P
3 P VWRWKHULJKW
3 7ZRVDGQDWRSRLQW D $x = \pm a/\sqrt{3}$

```
3    8,        P V  /      FP  9PD[        P V
3         P
3         N3D
3    .         P  V  D      N3D  E      N3DXSSHU    N3DORZHU
3    ) ERRO        1
3    K    D   8PD[     8
3    9ERRO         IWWZ LWKZ LQGDW  f
3    )SDUDOOHO          OBE )QRUPDO         OBE SRZHU§      KS YHUVDSSURSLPDWH
3    7KLVLV) LJ ____ D IORZ LQD  f FRUQHU
3    6WDJQDWRQIORZ QHDUD³EXPS´
3              P   P      LI 8   &[P
3    )ORZ SDWWD5DQNLQHRYDO
3    $SSOLHGWRZLQGVKQQHO³EORFNDJH´
3    $GYHUVHJUDGLHQWRU[ ! D
3    V%WRWDO     . L    . M    D
3    1HHGDQLQLQWHUDFWLRQ RI LPDJHV
3     D      P  V  E          F        KS
3     D          E     f
3     D     P   E         F         1
3    . §    f
3     D       P   E  9        P  VDWU                f  DQG          f
3    <HV WKH\ DUHRUWKRJRQDO
3     D       8,  D
3    <HV DFORVHGWDUGURSVKDSHDSSHDUV
3     9          P  V S$          N3D
3      D         IW E          IWFUXGHO
```

&KDSWHU

```
3    D 9          P V ûV      - NJ .    E 9          P V ûV      - NJ .
3    D       - NJ .
3    FS        - NJ .   FY       - NJ .
3         .
3    0 D
3     D       ( 3DDQG       P  V  E       ( 3DDQG       P  V  F        P V
3    $SSURSLPDWHO        P V
3    0 D§
3     D     N3D  E       P  V  F
3     D       P  V  E        P V
3     E DW0 D§
3     D        NJ V  E
3    ' HYLDWRQHVWKDQ SHUFHQWDW0 D
```

3 D N3D E N3D F
3 D .
3 D NJ V
3 D NJ V
3 D 0 D 7 . 9 P V
3 9H IW/ SH SVLD 7H f5 WKLKW GE
3 D P
3 D P V E P V F NJ V
3 D NJ V E NJ V
3 D FP E N3D
3 0 D
3 $ WDERXW$ § FP
3 D P V E .
3 8SWWHDP 0 D 9 P V
3 & IW/ 9LQLGH IW/
3 D FP E N3D
3 K P
3 SDW N3D PD[IORZ NJ V
3 N3D
3 ' § PP
3 NJ V
3 ûW VKRFNV § V ûW FKRNQJ WRSV § V
3 &DVH$ NJ V % NJ V
3 $ (IWRU' KRGH LQ
3 9H P V 0 DH \HV
3 D NJ V E F N3D
3 9 P V S N3D 7 . S N3D
3 D NJ V
3 D E GE LQ
3 D P
3 D P E P F P
3 D E F NJ K
3 E N3D F P
3) ORZ LVFKRN-GDW NJ V
3 SWQN N3D
3 DERXW V
3 0 DVIORZ GURSVE\ DERXW SHUFHQW
3 D P V E N3D
3 D E N3D F .
3 9SDQH § IW/
3 9 P V 0 D
3 3 LV P DK-DGRI WKHVP DOOFLUFH 0 D 7WWU .

```
3              ƒ 0 D        S        N3D
3       D        E      N3D  F      .    G       P V
3       D      ƒ  E      ƒ
3      /ZHGJH§      ƒ
3       D S$          SVLD  E S%          SVLD
3      0 D          S        N3D Ë       ƒ
3       D K          P   E 0 D
3      SU          N3D
3      0 D          S        N3D
3       D 0 D            S          N3D  E 0 D            S          N3D
3       D         GHJUHHV
3       D          E      N3D
3       D          E      N3D
3       D .        ƒ  E GUDJ          1 P
3      $SSSUR[LPDWLRQ    GEH
```

&KDSWHU

```
3       D )U
3      7KHVHDUHSLHFHZLVHPXOWLSOLFDWLRQV
3       D )U         E 9FXUHQW       P V
3      ûW          K
3        FUW    CE< ! J
3       )GRZ PXVWEHIXOO URXJKWXUEXOHQWKLJK5H IRU&Kp]\ WRE HYDOG
3       D         P  V
3       D       P  V  E DERXW  3D
3       4       IWV
3       6              RU      P NP
3       D Q§           E       IW
3       ûW§     PLQ
3       $          P          GDJHU
3       ,I E     IW\          IW3          IWLI E     IW\          IW3          IW
3       \          P
3       0 D[LPXP IGRZ DW       ƒ
3       7KHVHZRDUHHTXDOO HILFLHQW
3       +H[DJRQVLGHOHQJWK E          IW
3       K E§
3       D              E
3       D          E        P  F
3       :          P
3       D          P  E       P V  F
```

3 D IWW E
3 D E
3 \ P 9 P V
3 ûK FP
3 D E \ P
3 P V
3 D P E P V F
3 D IWV
3 + § IW
3 D P E P V P
3 D IW E IWW F G H IW
3 \ IW\ IW SHUFHQW
3 D IW E IWW
3 D P VP E P F P G H P
3 D \ IW\ IW
3 \ PP 9 P V
3 D E P V F FP
3 5 § FP
3 D WWHS 6 E 6 F 6
3 1RHQWU GHSWKDG/WRFULWLFDOIORZ
3 $SSUR[LPDWH P
3 D \FUHWH§ P E \ / § P
3 0 FXUYH ZLWK\ P DW/ § P
3 IW
3 < P . *f*
3 JDOPLQ
3 0 FXUYH \ IWDW[í IW
3 $W[í P \ P
3 $W P XSWUHDP \ P

&KDSWHU

3 7KLVLVDGLDSKUDJP SXPS
3 D + IWDQGûS GE LQ E + IWRI JDVRQQH 3 KS
3 D JDOPLQ E JDOPLQ F
3 D P E :
3 :
3 4 § JDOPLQ + § P
3 9YDQH 9WJRUPD[SRZHU
3 D URRW 4 DQG IWV E URRW + IWDQG IW
3 D %(3 SHUFHQWDW4 P V
3 %RWKDUHILQH WKHOJHWWLVPRUHHILFLHQW

3 %(3 DWDERXW IWV 1$V§ 4PDL§ IWV
3 D UPLQ E IW F JDOPLQ
3 D ' § LQ F Q§ UPLQ
3 D LQ E KS F IW G SHUFHQW
3 D 1R E LQDW UPLQ
3 D KS E LQ F JDOPLQ G SHUFHQW
3 D '$V ' J+ 4
3 136+SURW§ IW
3 1RFDYLWDWRQ UHTXLUHGGHSWKLVRQO IW
3 '$V§ &1$V & “ SHUFHQW
3 E DERXW LQ
3 D P V E N F ƒ
3 D LQ E EKS
3 D' IWQ UPLQ3 KS E' IWQ UPLQ3 KS
3 E $SSUR[LPDWH UPLQ
3 E 1R
3 ' IWûS 3D
3 D LQ E UPLQ
3 IWPLQ QRQ%(3 HILFLHQF\ SHUFHQW
3 D LQ E UPLQ
3 D IW E IWV
3 D JDOPLQ E LQ
3 D E F NJDOPLQ DOO
3 '$SH§ IW
3 $SSUR[LPDWH VWDJHV
3 %RWKSXPSVZRUNZLWKWKHHDFKLQVHULHV WKHODUJHWEHLQJ PRUHHILFLHQW
3 7ZRWXUELQHV D ' § IW E ' § IW
3 1$V§ KHQFH)UDQFLVWXUELQHV
3 D)UDQFLV F LQ G UPLQ H KS
3 3§ N
3 D SHUFHQW E 1$V§
3 D IW E IW
3 $ERXW 0:
3 4§ JDOPLQ
3 D 0:
3 $SSUR[LPDWH PLK

,QGH[

$

DEVROXWHSUHVVXUH___
DEVROXWHVWHSHUDWXUHVFDOHV___
DEVROXWHVWUPV___
DFFHOHUDWLQJFRRUGLQDWHVVWHP ___±___
DFFHOHUDWLRQILHOG___ ___±___
DFFHOHUDWLRQRIJUDYLW___±___
$FNHUHW-DFRE___
$FNHUHVGUDJVKHRU ___ ___
$FNHUHVQDQHDU]HGVKHRU ___
DFRXVWLFV___
DFWLYHIORZ FRQWURO___
DFWXDWRUGLVN___
DGGHGPDVV___
DGLDEDWLF DQGLVHQWURSLF VWDGH IORZ
DERXW___±___
DLU XVHXCQXP EHUVIRU___±___
FULWLFDOYDXHVDWWKHVRQLF SRLQW___±___
LVHQWURSLF SUHVVXUHDQGGHQVLWUHDWRV___±___
0 DFK QXP EHUUHDWRV___
UHDWRQKLS VRW%HUQRXOOL¶VHTXDWRQ___
DGLDEDWF IORZ___±___
DGLDEDWF IUFWRQDOIORZ___
DGLDEDWF SURFHVV___
DGLDEDWF UHDWRQV___
DGYHUVHSUHVVXUHJUDGLHQW___ ___ ___
$HURFDU FDUSDQH ___
DHURG\QDPLF IRUFHVRQURDGYHKLFOHV___±___
DHURG\QDPLFV___
DHURQDXWF VWHQGVLQ___±___
DLU XVHXCQXP EHUV___±___
DLUFUDVWANDLQQJ YRUWF___±___
DLURLOV
GHVLJQ___
IRUFHVRQ___
QLWH ILFLHQF\ ___
VKDSH___
DLURLOVKHRU
DLUFUDVWANDLQQJ YRUWF___±___

ICDVSODVWDLURLOYRUWH[VKHHWWKHRU ±
. XWDWFRQGLWLRQ ±
SRWHQWLDOWKHRU IRUWKLFNFDPEHUHGDLURLOV ±
VXSHUYRQLF ±
Z LQJVRI ILQLWHVSDQ ±
DLUPLFUREXEEOHV
DQWLQDLWVWDWHV
DQWLQDLYHLPSODFWDSSURDFK ±
$QDO\WLFDO7KHRU RI +HDW) RXUHU
$QGHUVRQ¶V IRUPXOD
$QGUDGH¶V HTXDWLRQ
DQJOHRI DWWDFN
DQJOHV
DQJOHYDOYH
DQJXODUPRPHQWXP UHDWLRQ
DQJXODUPRPHQWXP WKHRUHP ±
DQQXODUSRI Z DWU
DQQXOXV KGURGHF GLPHWHU
$QVHOPH) DELHQ
DSSDUHQWYLVFRVLW
DUELWUDU IL[HG FRQWROYROXPH ±
DUELWUDU IORZ SDWWHUQ
DUELWUDU PRYLQJ DQG GHRUPDEOH FRQWROYROXPH ±
$UFKLPHGHV
$UFKLPHGHV¶ ODZ V
DUHDUDWLR
DVSHFWDWLR $5
DWPRVSKHULF ICXLG SUHVXUH
DYDLODEOHKHDG
DYHUDJHYHORFLW
D[LDOIORZ SXPS SHURUPDQFH
D[LDOIORZ SXPSV
D[LDOIORZ SXPS VKHRU ±
D[LVPPHWULF ICRZ
D[LVPPHWULF SRWHQWLDOICRZ
VKHFRQHVWDWK KGURG\QDPLF PDW ±
SRLQWGRXEOHW
SRLQWVRXUFHRUVLQN
VSKHULFDOSRODUFRRUGLQDW ±
XQLRUP VWHDP SOXVDSRLQWVRXUFH ±

%

EDFNSUHVVXUH

EDFNZDUGFXUYHGLPSHGDUEODGHV
EDFNZDMUFXUYHV ±
%DQNLVXUEHQHV
EDVLFSODQHIORZ
%DJLQ+HQULePLOH
EHORZZDMUDUHD
%HUQRXOOL'DQLHO
%HUQRXOOLFRQVWDQWV
%HUQRXOLHTXDWLRQ
DERXW
DGLDEDWLFDQGLVHQWURSLFVWDGH IORZ
GLIXVHUSHUPDQHQW
DVHQHJ\HTXDWLRQ
IUHHVXUIDFH
IULFWLRQDOGHWV IORZ
LQIULFWLRQDOGHWV XQXVXDG IORZ
IRULQFRPSUHVVLEOHIORZ
RULJLQVRI ±
UHVXOWFLWRQV ±
LQURDMQJFRRUGLQDWHV
IRUVXDG IULFWLRQDOGHWVLQFRPSUHVVLEOHIORZ
XQXVXDG LURQDOQDO
YHORFLWVSRMQDO
%HUQRXOOREVXUYDWLRQVWKHRU
%HUQRXOUHDWLRQ
EHWWHILFLHQFSRLQWV%(3
%HV $
%HVQXP EHU
%LQJKDP SODWH
ELRORJLFDOGUDJUHGXFWLRQ ±
%ODVLXV+
%ODVLXVHTXDWLRQ
EODWZDYHQXP EHU
EORFNDJHIDFWRU
EORZHUV GHILQHG
EXQQERG IORZ
ERG GUDJ DVKLJKPDFKQXP EHUV ±
ERG IRUFHV
%RUGD &
ERXQGDULHV
ERXQGDU FRQGLWLRQV
ERXQGDU FRQGLWLRQVIRUVKHEDVLF HTXDWLRQV
DERXW ±
LQFRPSUHVVLEOHIORZ ZLWKFRQVWDQWSURSHUWLHV ±
LQYLVFLG IORZ DSSUR[LPDWLRQV ±

VLP SOLLHGIUHHVXUDFHFRQGLWLRQV
ERXQGDU\ HOHPHQWVPHWKRG %(0
ERXQGDU\ CDVHU
ERXQGDU\ CDVHUDQDOVLV
ERXQGDU\ CDVHUHTXDWLRQV
ERXQGDU\ CDVHUIORZV
ERXQGDU\ CDVHUSDWFKLQJ
ERXQGDU\ CDVHUVSDUDWLRQRQDKDOO ERG
ERXQGDU\ CDVHUVZLWKSUHVVXUHJUDGLHQW
ERXQGDU\ CDVHUWHRU
ERXQGDU\ CDVHU[PRPHQWXP HTXDWLRQ
ERXUGRQWXEH
%RXWLQHVT 7 9
ERZ VKRFN
ERZ VKRFNZDYH
EUDNHKRUVHSRZHU
EUHDNSRLQWVHFWLRQ
%UIGJPDQ 3 :
%UIQHCKDUGQHVV
%UIQNPDQQXPEHU
%UWWXK JUDYLWDWLRQDO %* XQLW
%XFNLQJKDP (
%XFNLQJKDP 3L7KHRUHP
EXGERXVERZ
EXGJHLQDGXFW
EXONPRGXOXV
EXR\DQF\
FHQWURI
QHXWDO
VSHFLILF ZHJKWRI IOXLGDQG
DQGVWDELOLW
EXR\DQWSDUWFHV
EXR\DQWLQJ OJKWVSKHUHV
EXWWHUIO YDOYH

&

FDPEHUGDUURLO
&DQQRQ8EEHOBKGHYLVFRPHWU
FDSDFLWYHVHQVRUV
FDSLOODU YLVFRPHWU
FDUV GUDJ FRHILFLHQWRI
&DUWHVLDQYHFWRUIRUP RI YHORFLWYILHOG
&DXFK\ 5LHPDQQHTXDWRQV

FDYLWDWRQ ____
FDYLWDWRQQXPEHU__
FHQWURLGXRVDQF\ __
FHQWURLSUHVVXUH &3 ___± ____ ____
FHQWULXJDOSXPSKHDG____
FHQWULXJDOSXPSV____
DERXW___
EDVLFRXWSXWSDUDPHWHUV____±___
HIIHFWRIEDGHDQJOHRQSXPSKHDG____±___
HOHPHQWDU SXPSSYKHRU ____±___
FHQWUSHWDODFFHOHUDWRQ____
FHQWULGDOPRPHQWRILQHUWLD__
&* 6\VWHP ___
FKDQQHOFRQWURORSRLQW___
FKDQQHOV
FRQWDQWWORZ DURXQG____
JHRPHWU DQG YHORFLW___
5H\QROGV QXPEHURI ____
FKDQQHOVORSH____
&KDSO JLQ 6 ____
FKDUDFWULVWLFDUHD____
&Kp]\ $QDOLQH_ ____
&Kp]\ FRHILFLHQW___
FKLPQH\ IORZ ____
FKRNLQJ
DERXW___±___
GHILQHG____
GXHWRFULFWLRQ____±___
HIIHFWVGXHWRVLPSOHKDQQJ____±___
PD[LPXP PDVIORZ ____
\RQF H[LW___
\RQF IORZ ____
FKRNLQJ SRLQW___
FLUFXODU\ OQGHU LQYLVFLG SUHVVXUHGLVWULEXWLRQ___
FLUFXODWRQ___±___
FLUFXODWRQIORZ ____
FLUFXODWRQORVV___
FODVVLILFDWRQRI SXPSV___±___
FORVHGEODGHV___
&RDQGDHIIHFW___
FRHILFLHQWRI VXUIDFHWHQVLRQ__
&ROREURRN&) ____
FRQQHFWRUHV__
FRPELQDWRQFDUDQGDLUSODQH___±___
FRPPHUFLDO&)' FRGHV___±___

FRP SRXQGZ HLU
FRP SUHVWLELOLWHIHFW
FRP SUHVWLELOLWSDUDPHWHUV ±
FRP SUHVWLEOHGXFWORZ ZLWKIUDFWLRQ
DERXW ±
DGLDEDWLF IORZ ±
FKRNLQJ GXHWRIUDFWLRQ ±
LVRWKHUPDO IORZ ZLWKIUDFWLRQ ±
ORQJ SLSHOLQHV ±
PDVV IORZ IRUDJLYHQSUHVVXUHGURS ±
PLQRUORVVHVLQFRPSUHVVLEOHIORZ
FRP SUHVWLEOHIORZ
DGLDEDWLF DQGLVHQWURSLF VWDGH IORZ ±
FRP SUHVWLEOHGXFWORZ ZLWKIUDFWLRQ ±
FRQYHUJLQJ DQGGLYHUJLQJ QR]]OHVRSHUDWRQ ±
GHILQHG
IUDFWLRQDWVGXFWORZ ZLWKKHDWWDQVIHU ±
LVHQWURSLF IORZ ZLWDUHDFKDQJHV ±
LVHQWURSLF SURFHVV ±
QRUPDOVKRFNZDYH ±
3UDQGWOO H\HU[SDQVLRQZDYHV ±
VKHUPRGQDPLFVUHYLHZ ±
VKLQDUROOKHRU ±
VKUHGLPHQVLRQDOVXSHUVRQLF ±
VZR GLPHQVLRQDOVXSHUVRQLF ±
FRP SUHVWLEHJDVIORZ FRUUHFWLRQIDFWRU
FRP SUHVWLEHHDPLQDUIORZ
FRP SUHVWRUV ±
FRP SXUDWLRQDOOLGG\QDPLFV &)'
FRQH[LWFDQQXOV
FRQH[SVWR K\GURG\QDPLF PDVV ±
FRQH SODWHYLVFRPHWHU
FRQRUP DOP DSSLQJ
FRQFDOGLIXVHU
FRQXJDWHGHSWKV
FRQVHUYDWRQR PDVV ±
FRQLVWHQWXQLW
FRQWDQWZDUHGXFWORZ
FRQWDQWJDVGHQLW
FRQDFWZDQJH
FRQWXQLWHTXDWRQ
FRQWXQLWUHDWRQ 6HHFRQVHUYDWRQR PDVV
FRQWXXP
FRQWDFWRQIORZ
FRQWDFWRQV

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FRQWURONH\VWURNH
FRQWURONH\VWURNHDQDORJV
FRQWURONH\VWURNHPHWKRG
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FRQYHUVLRQHUFRHUFLYHUDWLR
FRQYHUJLQJDQGGLYHUJLQJQR]]OHV
FRQYHUJLQJGLYHUJLQJQR]]OHV
FRQYHUJLQJQR]]OHV
FRQYHUVLRQIDFWRUV
&RUURGHUHFRYHUDELOLW\
&RXSRQVHIIHFWV
&RXSRQVPDWFKPHWKRG
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&RXSOHUZRZ
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&RYHUWRZ
&RYHUWRZVRXUFH
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&RXSOLQJVOS
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FUHHSLQJPRGH
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FULWLFDOGHSWK
DERXW
FULWLFDOVORSH
FULWLFDOXQLRUP RZ
IUHFWLRQDOWVRZ RYHUDEXPS
QRQUHFWDQJXODUFKDQQHOV
RSHQFKDQQHORZ
UHFWDQJXODUFKDQQHOV
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FULWLFDORZ
FULWLFDOSRLQWV
FULWLFDOSURSHUWLHV
FULWLFDOVORSH
FULWLFDOVRQFHSWKURDW
FULWLFDOYDOXHVDWKHVRQFSRLQW
FULWLFDOYHORFLW\
FURVVIORZ VWUHDPOLQHV

&UXPSZHLU
FXSDQHPRPHWUPHWULV
FXUDXQFWRQ
FXUYHGVKRFNZDYH
FYOQGHUGUDJ
URWDWLQJ
FYOQGULFDOSRDUFRRUGLQDWHV ±

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GT$OPEHUW-HDQ
GT$OPEHUWVSDUDGR[
' DUN +HQU
' DUN)RUVKHKHLPHUGDZ
' DUN IUHQWRQIDFWRU
' DUN ¶VIRUPXOD
' DUN ¶VODZ RI SRURXVIORZ
' DUUHXV * - 0
' DUUHXVURWRU
GD9LQFL/HRQDUGR
GHDGZDWUHJLRQ
GHRUPDEOHFRQWROYROXPH
GHRUPLQJFRQWROYROXPH
GHQVLW ± ±
GHQVLWJUDGLHQW
GHSHQGHQWYDULDEOHV
GHULYDWRQIRUZR GLPHQVLRQDOCIORZ ±
GHVLJQFRQGLWLRQ
GHVLJQIORZ UDWH
GHVLJQSUHVXUH
GHYLDWRQVIURP LGHDOSXPS\VWHRU ±
GLDPRQGIRLO
GLDSKUDJP
GLDX\VWROF SUHVXUH
GLIHUHQWLDODQDO\VLV
GLIHUHQWLDOHTXDWRQVRI DQJXODUPRPHQWXP ±
GLIHUHQWLDOHTXDWRQVRI HQHUJ\ ±
GLIHUHQWLDOHTXDWRQVRI IOXLGPRWRQ
GLIHUHQWLDOHTXDWRQVRI PDVVPRPHQWXP ±
(XOHU¶VHTXDWRQ
1DYLHU6WRNHVHTXDWRQV ±
GLIHUHQWLDOHTXDWRQVRI PDVVFRQVHUYDWRQ ±
FYOQGULFDOSRDUFRRUGLQDWHV ±
LQFRPSUHVVLEOHIORZ ±

VWDWH FRPSUHVVLEOHIORZ ____
GLIIHUHQWDOPDQRPHWU ____
GLIIHUHQWDOWHDWRQVIRUIOXLGIORZ
DERXW ____
DFFHOHUDWRQILHOGRIDIOXLG ____ ±
ERXQGDU FRQGLWRQVIRUWKHEDVLF HTXDWRQV ____ ±
&RXHWWIORZ EHWZHHQDIL[HGDQGDPRYLQJSODWH ____ ±
GLIIHUHQWDOHTXDWRQRI DQJXODUPRPHQWXP ____ ±
GLIIHUHQWDOHTXDWRQRI HQHUJ\ ____ ±
GLIIHUHQWDOHTXDWRQRI OQHDUPRPHQWXP ____ ±
GLIIHUHQWDOHTXDWRQRI PDVVFRQVHUYDWLRQ ____ ±
IORZ EHWZHHQFRQFHQWULF F\OLQGHUV ____ ±
IORZ GXHWRSUHVVXUHJUDGLHQWEHWZHHQWZR IL[HGSODWHV ____ ±
IUHFWLRQDOWLUERGWRQDOOIORZ ____ ±
IXOO GHYHORSHGPLQDUSLSHIORZ ____ ±
LQFRPSUHVVLEOHYLVFRXVIORZ H[DPSOHV ____ ±
LQWURGXFWLRQ URWDWQJ LQQHUF\OQGHUIORZ ____ ±
VWHDP IXQFWLRQ ____ ±
YRUWFLWVDQGLUURWDWRQDOW ____ ±
GLIXVHUDQJH ____
GLIXVHUIORZ SDWWHUQV ____
GLIXVHUSDUDPHWHUV ____
GLIXVHUSHUPDQHQ ____ ±
GLIXVHUWDOO ____
GLDWDQW ____
GLPHQVLRQDOFDQDO ____
GLPHQVLRQDOFDQDOVLV ____
GLPHQVLRQDOFDQDOVLVDQGVLPLODULW ____
DERXW ____ ±
GLPHQVLRQDOKRPRJHQHLW SULQFLSOH ____ ±
PRGHOQJ DQGLWSSLWDOV ____ ±
QRQGLPHQVLRQDO[DWRQ VKHEDVLF HTXDWRQV ____ ±
3LWKHRUHP ____ ±
SULQFLSOHR GLPHQVLRQDOKRPRJHQHLW ____ ±
GLPHQVLRQDOFRQVWDQW ____
GLPHQVLRQDOKRPRJHQHLW SULQFLSOH ____ ±
GLPHQVLRQDOOLQHDUWKHRU ____
GLPHQVLRQDOOLQHDUWHTXDWRQV YV KRPRJHQHRXVHTXDWRQV ____
GLPHQVLRQDOPDW[____
GLPHQVLRQDOOYDULDEOHV ____
GLPHQVLRQVGUDJ YV GLPHQVLRQVYLVFRVLW ____
GLPHQVLRQVIRUFHFRHILFLHQW ____
GLPHQVLRQVIRUFHYV GLPHQVLRQVYLVFRVLW ____
GLPHQVLRQVIRUP ____
GLPHQVLRQVJURXSVLQIXLGPHFKDQLFV ____
GLPHQVLRQVORVVFRHILFLHQW ____

GLP HQVLRQDWVSDUDPHWHUV ± ±
GLP HQVLRQDWVSUHVXUHFKDQJHYV GLP HQVLRQDWVYLVFRVLWV
GLP HQVLRQDWVSUHVXUHFRHILFLHQW
GLP HQVLRQDWVSXPS SHUURUP DQFH ±
GLP HQVLRQDWVYDULDEOHV
GLP HQVLRQDWVYLVFRVLWV
YV GLP HQVLRQDWVGUDJ
YV GLP HQVLRQDWVIRUFH
YV GLP HQVLRQDWVSUHVXUHFKDQJH
GLP HQVLRQV
GLP HQVLRQVRI IOXLG P HFKDQLFVSURSHUWLHV
GLUHFWQXP HULFDOVLP XODWRQ ' 1 6
GLVFKDUJHFRHILFLHQW&G
GLVFRQWLQXRXVERXQGDU\ FRQGLWLRQV
GLVNYDOYH
GLVSODFHP HQWWKLFNQHVV ±
GLYHUJHQFHDQJOH
' RSSOHUDQHP RP HQW /' $
' RSSOHUVKLIW
GRXEOHWV ±
GRZ QWLP SUHVXUH
GRZ QZ DUG SUHVXUH ±
GUDJWKEHV
GUDJ
DQG GIWFRHILFLHQWVRQURWDWLQJ VSKHUH
RI VXSHUVRQLF DLUFRLOV
RI VZR GLP HQVLRQDOERGLHV
GUDJ FRHILFLHQWV
EXR\DQWLLQJ GJKWVSKHUHV
RI FDLV
FKDUDFWHULVWLF DUHD
GHILQHG
LQGXFHG
ODP LQDUERXQGDU\ OD\ HU
RI VKUHH GLP HQVLRQDOERGLHV ±
VXUEXOHQWERXQGDU\ OD\ HU
GUDJ IRUFH
GUDJ IRUFHYV YHORFLW\
GUDJ IXQFWLRQ
GUDJ RI LPP HUVHGERGLHV ±
GUDJ RI VXUIDFHVKLSV ±
GUDJ UHGXFWLRQP HWKRGV
GURSGRZ QIORZ SDWWHUQ
GURZ QHGQDSSHV
GURZ QHGRXWORZ

GURZ QHG VOXLFHJDWH
GXFWG SURSHOOHUPHWUV
GXFWORZ
GXFW0 DFKQXP EHU
' XWFK WSHZ LQGP LOO
G\QDP LF SUHVXUH
G\QDP LF SXP SV
G\QDP LF VLP LODULW ±

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(FN-HUWQXP EHU
HGG\ YLVFRVLW
HGG\ YLVFRVLWP RGHO
HIHFWLYHGLDP HWU
HILFLHQF\
DLUFRLOOLW
EHWWHILFLHQF\ SRLQW%(3
K\GUDXOLF
PHFKDQLFDO
SXP S
VRLDO
RI VXUEQHGHVLJQ/ RSWP XP
YROXP HWUF
HILFLHQWXQIRUP IORZ FKDQQHOV ±
HJKWHQQ/ FHQXU\ PDWHP DWFLDQV
(NP DQQXP EHU
HODWLF GHRUP DWRQLQXWP HQW LQXWP HQDWRQ
HOFWUF RXSXWLQXWP HQW LQXWP HQDWRQ
HOFWUF RXSXWV HQVRUV
HOFWURP DJQHWF P RWRU
HOP HQDU\ SODQHIORZ VROXWRQV
FLUFXODWRQ ±
QQHLUR\DWRQDOYRUW[±
QQHVRXUFHRUVLQNDWWHRULJLQ
5DQNLQHKDO EROG ±
VLQSXV DYRUW[DWWHRULJLQ
VRXUFHSXV DQHTXDOLQN ±
VXSHUSRVLWRQ ±
XQLRUP VWHP DWDQQH
XQLRUP VWHP LQWH[GLUHFWRQ ±
XQLRUP VWHP SXV DVLQDWHRULJLQ ±
HQHJ\ HTXDWRQ
DERXW ±

IUFWRQDQGVWDWH ZRUNLQGRZ VSHHGIGRZ ±
NLQHWF HQHUJ\ FRUUHFWRQIDFWRU ±
RQHGLPHQVLRQDOHQHUJ\ IOX[WUPV ±
VWDWG\ IGRZ HQHUJ\ HTXDWRQ
HQHUJ\ H[WUDFWRQ
HQHUJ\ JUDGHQGQH (* /
HQJLQHHULQJ HTXDWRQV SHFXODU ±
(QJLQHHULQJ (TXDWRQ6RGYHU ((6 ± ±
HQWKDOS\ IOX[
HQWDQFHGRVWHV
HQWDQFHUHJLRQ
HQWRS\ JUDGLHQW
HTXDWRQRI FRQWLQXLW\ 6HHFRQVHUYDWRQRI PDVV
HTXDWRQRI PRWRQ
HTXLOLEULXP RI DIXLG ±
JDJHSUHVVXUH
YDFXXP SUHVVXUH
HTXLYDOHQWYURXGHQXPEHU
HTXLYDOHQW6H\QROGV QXPEHU
(XOHU/HRQKDUG
(XOHUDQDQG/DJUDQJLDQGHVFULSWRQV ±
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(XOHUQXPEHU
(XOHU¶VHTXDWRQ
(XOHU¶VVXUEELQHIRUPXOD
(XOHU¶VVXUEELQHUHDWWDFK
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H[LW RQHGLPHQVLRQDO
H[SDQVLRQIDFWRU
H[SHULPHQWDOGDWD
H[SHULPHQWDOGXFWWRZ V
GLIXVHUSHUPDQHQH ±
ORFDOYHORFLWPHDVXUHPHQW ±
YROXPHIORZ PHDVXUHPHQW ±
H[SHULPHQWDOH[WUDQDOV
DHURG\QDPLF IRUFHVRQURDGYHKLFOHV ±
ELRORJLFDOGUDJ UHGXFWLRQ ±
ERG\ GUDJ DWKLJKPDFK QXPEHUV ±
EXR\DQWLQLQJ OJKWWSKHUHV
FKDUDFWULVWLFWDUHD
FRPELQDWRQFDUDQGDLSODQH ±
FUHSLQJ IORZ ±
GUDJ RI LPPHUVHGERGLHV ±
GUDJ RI VXUDFHVKLSV ±

GUDJ UHGXFWRQPHWKRGV
IRUFHVRQODWQJERGLHV ±
IUHFWRQGUDJ DQGSUHVXUHGUDJ ±
. ODQH)RJHOPDQDUURLO
VKUHGLPHQVLRQDOERGLHV ±
VWR GLPHQVLRQDOERGLHV
ZLQJLQVSLUHGE\ WKHKXPSEDFNZKDOH
H[SHULPHQWDOP
H[SODFLWYLVFRXVIORZ PRGHO
H[WHQGHG VWDWHV 6HHDOVR H[SHULPHQWDOH[WHQGHG VWDWHV
)
)DQQHU6NDQIORZ V
)DQQR * LQR* ±
)DQQRIORZ ±
IDQV
IDYRUDEOHSUHVVXUHJUDGLHQW
ILQLWHDVSHFWDWLR
ILQLWHGLIIHUHQFHPHWKRG)' 0 ±
ILQLWHGLIIHUHQFHV
ILQLWHHOHPHQWPHWKRG)(0
ILQLWHUHJLRQ
ILQLWHVSDQDUURLOV
ILUVWODZ RI WKHUPRG\QDPLFV
IL[HGFRQWROYROXPH
IODWSODWHDLUIRLOYRUWH[VKHGWKHRU ±
IODWSODWHERXQGDU\ OD\HU
ODPLQDUIORZ ±
WDQJHQWLDOVORSHEXEEOHFH ±
WXEXOHQWIORZ ±
IODWSODWHERXQGDU\ OD\HUIORZ
IODWSODWIORZ IRUPXOD
IODZ DOJHEUDLIIXVHU
)OHWFKHUUHYHVVKLS
IORDWV
IORZ DORQJ DVWHDP OLQH
IORZ DURXQG DFRQHURI DUELWDU DQJOH ±
IORZ EHWZHHQORQJ FRQFHQWULF F\OLQGHUV ±
IORZ EHWZHHQSDUDOOHOSODWHV
IORZ FRHILFLHQW
IORZ GXHWRSUHVXUHJUDGLHQWEHWZHHQWZRIL[HGSODWHV ±
IORZ LQORQFLUFXODUGXFWV
DERXW

IGRZ EHWZHHQSDUDGHCSODWV ____
IGRZ VKURXJKDFRQFHQWUDWLRQ ____ ±
KYGUDXOLFGLDPHWHU ____ ±
GPLQDUIGRZ VRXUFH ____ ±
RWKHUQRQFLFXODUFURVVVHFWLRQV ____ ±
VXUEXOHQWGRZ VRXUFH ____ ±
IGRZ QRUPDOWRDIODVSODWH ____ ±
IGRZ QR]]OH ____
IGRZ SDWWDFLFXODUFYOLQGHUZLWKFLFXODWRQ ____ ±
IGRZ SDWWDYRUWH[____ ±
IGRZ SDWWPPHUVHGERGLHV
ERXQGDU\ OD\HUHTXDWRQV ____ ±
ERXQGDU\ OD\HUVZLWKSUHVXUHJUDGLHQW ____ ±
H[SHULPHQWDOGDWDIGRZV ____ ±
IGWSODWHERXQGDU\ OD\HU ____ ±
PRPHQWXP LQWHJUDOHTXDWLRQ ____ ±
5H\QROGVQXPEHUDQGJHRPHWU HIIHFW ____ ±
IGRZ SURSHUWLHV ____
IGRZ UDWHFKDQJHV ____
IGRZ UDWHHIIHFWURP VL]HFKDQJHV ____
IGRZ UDWSUREOPV ____ ±
IGRZ VHSDUDWRQ ____ ____ 6HHDOVR VWDOOV
IGRZ VXDWHV ____
IGRZ VKURXJKDFRQFHQWUDWLRQ ____ ±
IGRZ YHORFLWLQRSHQFKDQQHOIGRZ ____
IGRZ YLVXDOL]DWRQ ____ ±
IXFWXDWLQJ YHORFLWLHV FRUUHDWRQV ____
IXFWXDWRQ ____
IXLGFLUFXODWRQ ____
IXLGFRXSOLQJ VOS ____
IXLGGHQVLW _
IXLGQQH ____
IXLGPHFKDQLFV
DERXW ±
GLPHQVLRQVDQGXQLW ±
IXLGDVDFRQFHSW ±
IXLGDVDFRQWLQXXP ±
IXLGVYV VRQGV ±
KLVWRU RI ±
SUREOP VRYLQJ WFKQLTXHV _
VHFRQGDU GLPHQVLRQV ____
WKHUPRG\QDPLF SURSHUWLHV ____ ±
YHORFLWLWILHGSURSHUWLHV ____ ±
YLVFRVLW DQGRWKHUVHFRQGDU SURSHUWLHV ____ ±
IXLG PHFKDQLFVSURSHUWLHVGLPHQVLRQV ____

IOLG SUHVVXUH
DAVRVSKHULF
IOLGV
SUHVVXUHDQGGHQVLW ±
YV VRDGV ±
IOLGVXDDFV
)RJHDP))
IRUHVRQDWQJERGLHV ±
)RVFKHKHLPHU3 +
)RXULHU-RVHSK
)RXULHUJ¶VDZ ±
)UDQFLV-DPHV% _
)UDQFLVVXUEHQH
IUHHERG FRQHSW
IUHHGLVFKDUJH
IUHHRYHUDOO
IUHHSURSHOOHUPHWULV
IUHHSURSHOOHUWKUXVWSHUXQLWDUHD
IUHHVXUHDP
IUHHVXUHDP DQJOHRI DWWDFNDQGDLUFRLOVKDSH
IUHHVXUHDPGHQVRXQ
IUHHVXUJH
IUHHYRUWH[
IULFWRQDOHIIHFW
IULFWRQDQGGVXI ZRUNLQGRZ VSHGGIGRZ ±
IULFWRQGUDJ
IULFWRQGUDJ DQGSUHVVXUHGUDJ ±
IULFWRQIDFWRUV
IULFWRQDWVGXFWWGRZ ZLWKKHDWWDQVHU
DERXW ±
FKRNLQJ HIIHFWVGXHWRVLPSOHKHDWQJ ±
0 DFK QXP EHUUHDWRQV ±
UHDWRQVKLS WRQRUP DOVKRFNZ DYH ±
IULFWRQDWVGXFWGZ V
IULFWRQDWVLUURXQGQDQDGZ
DERXW
JHQHUDWRQR URXQGQDQDGV ±
RUWKRJRQDOVRI VXSHUPQVDQGSRWQWDOOQHV ±
YHORFLWVSRWQWDO
IULFWRQDWVPRPHQXP HTXDWRQ
IULFWRQDWVXQLWDG IGRZ
IULFWRQGRVWHV
IULFWRQGRVVPHWUV
IULFWRQYHORFLW
IURQWDODUHD

) URXGH 5REHUW
) URXGH : LOOLDP _
) URXGHQXPEHU
FUHDWHDFFRQGLWRQVIRU
) URXGHQXPEHUGHQRPLQDWRU
) URXGHVFDOLQJNLQHPDWLFUHDWRQV
IXQG GHYHORSHGFRQGLWRQV
IXQG GHYHORSHGIORZ
IXQG GHYHORSHGODPLQDUSLSHIORZ ±
IXQG URXJKIORZ ±
IXQG VKUEXOHQWIORZ
IXQG YLVFRXVFRPSXWDWRQDOIOXLGG\QDPLFV &)'
) XQGDPHQWDOVRI (QJLQHHULQJ)(([DPLQDWRQ
IXHGTXDQWVIRUFHEDODQFHGERXQGRQWEH

*

JDJHSUHVVXUH
JDJHSUHVVXUHIRUPXODV ±
JDVEHKDYLRULQWKHUPRGWLQWKHRQDWRQ
JDVFRQWDQW
JDVG\QDPLFV
JDVHV
K\GURVWDWLFSUHVVXUHRQ ±
QQHDUIRUPXODDGHTXDF\ IRU ±
VWDWHUHODWRQV
JDVPROHFXDUZHLJKW
JDVVXUIDFHV ±
JDWHYDOYH
JHQHUDWRQRI URWDWRQDODW ±
JHRPHWULFLQIOXHQFHRI VWHDP QR]]/HIORZ ±
JHRPHWULFSDUDPHWHUV
JHRPHWULVLPLODULW ±
JHRWKHUPDOGDWDVSKHUH
JOREDOSURSHUWLHV
* | UWKU +
JUDGLHQW
JUDGXDO[SDQVLRQ ±
JUDGXDOO YDULHG WKHRU
JUDGXDOO YDULQJ ORZ * 9)
JUDSKLFDOPHWKRGRI VXSHUSRVLWRQ
* UDYLWDWRQ QXPEHU
JUDYLWVEDVHGLQWKHRQWLQWKHRQDWRQ

+

+DDDDQG 6 (
+DJHQ* RWKLO+HQUFK/XGZLJ_
+DJHQ3RLVHXLOOHIIOHZ
KDOOERG\ RI UHYROXWRQ
KDOOERG\VKDSH
+DQQD. HLVH
+DJHQ*
+DJHQ: LOOLDPVK\GUDXJKWVIRUPXOD
KHDG
KHDGGRVV IUHFWRQIDFWRU ±
KHDGGRVVFKDLW
KHDWLQJLQGXFWLRQVRXUFHVRI
KHDWRQGXFWRQHTXDWRQ
KHDWWDQJHU
KHDWWDQJHUFRHILFLHQW
KHDWWDQJHU0 DFKQXPEHU
+HOH6KDZ DSSDUDWXV ±
+HOPKROW]V+HUPDQQYRQ
+HOPKROW]. LUFKRII VWHRU
+HURRI $OH[DQGULD
+HUVFKHO&OPHQV
KLJKPROHFXODUZHJKWSROPHUDGGLWLYH
KLJKVSHFLILFVSHHGSXPSV
KLJKVSHFLILFVSHHGV
KLVWRULFDORXWOLQH ±
KRGJUDSK
+RGRZD\ *RUGRQ
KRPRJHQHRXVHTXDWLRQVYV GLPHQVLRQDOO LQFRQVLVWHQWHTXDWLRQV
KRPRGRJRXVSRLQW
KRUL]RQWDO[LVZLQGVXEELQH +$: 7
KRWLOP DQHPRPHWHU
KRWZLUHDQHPRPHWHU
+XJRQLRW$
KXGVKDSH
+XQVDFNHU-
K\GUDXOFDOO VPRRWKZDOO
K\GUDXOF DQGHQHJ\ JUDGHQWV ±
K\GUDXOF GLPHWHU ± ___
K\GUDXOF HILFLHQF\
K\GUDXOF JUDGHQW +*/ ___ ___
K\GUDXOF MXPSV
DSSUR[LPDWVROXWLRQIRULUUHJXODUFKDQQHOV ±

EDFNZDWUFXUYHV ±
EDVLFGLIHUHQWDCHTXDWRQ ±
EURDGFUHVWGZHLV/DQDOVLV
FODVVLILFDWRQRI ±
FODVVLILFDWRQRI VRXUFHV ±
FRPSRVLWHGZ VWDQGLQRQV ±
H[SHULPHQWDZHLUGLVFKDUJHFRHILFLHQW ±
IORZ XQGHUDVXUIDFHJDW ±
QRUPDOVKRFNZDYH
QXPHULFDOVROXWRQ ±
VKDUS FUHVWGZHLV/DQDOVLV ±
VKHRU IRU ±
VKHRU IRUKRU]RQWDO ±
VKLQSODWHZHLUGHVLJQV ±
K\GURXOLF PRGHOV ±
K\GURXOLF UDGLXV
K\GURXOLF UDGLXVDQGYHORFLW
K\GURXOLFV
K\GURGQDPLF PDW
K\GURGQDPLFV
K\GURVWDWF FRQGLWRQ
K\GURVWDWF IRUFHVRQFXUYHGVXUIDFHV ±
K\GURVWDWF IRUFHVRQCHUHGVXUIDFHV ±
K\GURVWDWF IRUFHVRQSODQHVXUIDFHV ±
K\GURVWDWF SDUDGR[
K\GURVWDWF SUHVVXUH
GLVWULEXWRQ ±
LQJDVHV ±
QHDUDSSUR[LPDWRQ ±
LQCOTXLGV ±
VWDQGDUGDWPRVSKHUH ±
YDULDEOHJUDYLWHIHFW ±
K\GURVWDWF SUHVVXUHGLIHUHQFH
K\SHUVRQLF DLUSODQHV
K\SHUVRQLF IORZ

,

LGHDOCIUFWRQOHVVPDWIOX
LGHDO]HGUDGLDOXELQHWKRU
LGHDO]HGZLQGXELQHWKRU ±
LPDJHV ±
LPPHUVHGERG\ IORZV
LPSHOOHUHFLFXODWRQORWV

LPSXOHVXUELQHV ±
LQFRPSUHVVLEOHDIILX\PPHWULF ±
LQFRPSUHVVLEOH%HUQRXOOLHTXDWLRQ
LQFRPSUHVVLEOHIORZ
DQDO\VLVIRU
DIILX\PPHWULF ±
%HUQRXOOLHTXDWLRQIRU
FRQVHUYDWLRQRI PDVV ±
ZLWKFRQYDQWLSURSHUWLHV ±
HTXDWLRQRI FRQWLQXLW
LQYLVFLG
QDYLHUHTXDWLRQVIRU ±
QHDU LQFRPSUHVVLEOHIORZ
QHJDWLEHGHQVLW ±
SODQH
VWDG\
VWRGLPHQVLRQDO
QHDU LQFRPSUHVVLEOHIORZ FRQVLQXLWHTXDWLRQZLWKFRQYDQWLYLVFRVLW
QHDU LQFRPSUHVVLEOHIORZ ZLWKFRQYDQWSURSHUWLHV ±
LQFRPSUHVVLEOHSODQHIORZ LQSRURXVFRUUGLQDWHV
LQFRPSUHVVLEOHYLVFRXVIORZ H[DPSOHV
&RXHWWIORZ EHWZHHQDIL[HGDQGDPRYLQJSODWH ±
IORZ EHWZHHQORQJ FRQFHQWULF\OLQGHUV ±
IORZ GXHWRSUHVVXUHJUDGLHQWEHWZHHQWZRIL[HGSODWHV ±
IXOO GHYHORSHGPLQDUSLSHIORZ ±
LQWDELOLW\RI URWDWLQJ LQQHUF\OLQGHUIORZ ±
LQGXFHGGUDJ FRHIILFLHQW
LQHUWLDOFRRUGLQDWHV\VWHP
LQLQWHUURZ RI YRUWLFHV ±
LQLQWHUPLPDUHJLRQ
LQLQWHUPLPDO\VWHP DQDO\VLV
LQWDOFRQGLWLRQV
LQWHVWDOYLDWLRQ
LQWHVWORZ
VXEVRQLF
VXSHUVRQLF
LQWHV0 DFK QXPEHU
LQWHV6 H\QROGV QXPEHU
LQWHVW
LQWHVW RQH GLPHQVLRQDO
LQWHUZDOOOD
LQWDELOLW\RI URWDWLQJ LQQHUF\OLQGHUIORZ ±
LQWXPHQWLQWXPHQWDWLRQ
LQYUDODQDO\VLV
LQYUDOYRUWPH

LQWURGXFWRU\ SURSHUWLHV ____
LQWURGXFWLRQ YLGHR ____
LQWUXGHUV ____
LQWUXGHU\ ____
LQWUXGHU UHSRUW ____
LQWUXGHU YHUVXV [WUXGHU YLFWRU V ____ ±
,QWUXGHU DQG 6\VWHP 6, XQLW ____
LQYLVLEOH UHV
DQDO\VLV ____
DSSUR[LPDWRQV ____ ±
(XOHU¶V HTXDWLRQ ____
LQYLVLEOH LQFRPSUHVVLEOH IORZ ____
LQYLVLEOH SUHVVXUH GLVWULEXWLRQ IURP FLUFXODU\ CQGHU ____
LQYLVLEOH VKHUV ____ ____ ____
LQZDUG SUHVVXUH IRUFH ____
,PSHQDOWUDQDWWXWS E\ VWHPHWRG ____ ±
LUURWDWRQDO IORZ ____ ____ ____
LUURWDWRQDO IUHWWRQ UHJLRQ ____
LUURWDWRQDO IXQFWRQ ____
LVHQWURSLF EXOPPRGXOXV ____
LVHQWURSLF IORZ ____ ____
LVHQWURSLF IORZ ZLWK DUHDFKDQJHV
DERXW ____ ±
FKRNLQJ ____ ±
IRFDOPDVV IXQFWRQ ____ ±
SHUHFWDVDUHDFKDQJH ____ ±
LVHQWURSLF SHUHFWDV SRZHUODZ UHODWRQVKLS ____
LVHQWURSLF SUHVVXUHDQGGHQVLWUHDWRV ____ ±
LVHQWURSLF SURFHVV ____ ±
LVHQWURSLF UDWR IRUPXODYV 0 DFKQXPEHUIRUPXOD ____
LVRWKHUPDOIORZ ZLWKIUFWRQ ____ ±

-

MHWK[LVSUHVVXUHFRQGLWRQV ____ ±
MHWK[LVSUHVVXUHHTXDOVDWRVSKHULFSUHVVXUH ____
MHWWORZ ____
-RXNRZVNL 1 (____ ____ ____
MXPS 6HHK\GUDXOLFMXPSV

.

. DSODQVXUEOLQH ____

. iUPiQ 7KHRGRUHYRQ
. iUPiQ¶VDQDOVLVRI VKHIDVSODVP ±
. iUPiQ¶VPRPHQWXP LQWURGXFHRUV
. iUPiQYRUWH[VXUHWV
. HHQDQ - +
. HOYLQRYDO
NLQHPDWF EORXQGDU\ FRQGLWLRQV
NLQHPDWF VLPLODULWV ±
NLQHPDWF YHORFLWV
NLQHWF HQHUJ\
NLQHWF HQHUJ\ FRUUHFWLRQIDFWRU ±
NLQHWF HQHUJ\ IOX[
. LQJ / 9
. LQJ¶VODZ
. LUFKKRII * XWDY 5
. OQH 5)
. OQH)RJHPDQQLURLO
. QXGVHQXPEHU
. XWDY : 0
. XWDVFRQGLWLRQ ±
. XWDV-RXNRZVNLOIVKHRUHP ±

/

/ DJUDQJH -RVHSK/ RXLV
OJUDQJLDQILHOGGHVFULSWLRQ
OPLQDUEORQGDU OD\HUGUDJFRHILFLHQW
OPLQDUIORZ ± ±
OPLQDUIORZ HOHPHQW
OPLQDUIORZ VFDOQJ OPZ
OPLQDUIORZ VRXUFH ±
OPLQDUIXOO GHYHORSHGSLSHIORZ ±
OPLQDULQWUDOHKRU ±
OPLQDUSODWHIORZ VRXUFH
OPLQDUVHSDUDWRU
OPLQDUVKHDU
OPLQDUVKHRUHV
/ DSODFH 3LHUUH6LPRQ
/ DSODFHHTXDWRQV ±
OPSVHUDW
OUJHHGG\ EUHDNXSGHYLFHV / (%8V
OUJHHGG\ VLPXODWRQ / (6
OVHU' RSSOHUDQHPRPHWU / ' $
OVHUSDUWFOHWDFNLQJ YHORFLPHWU / 379

QDVHUHGIOXLG /)
3 QHDVWWTXDUHV HUURU
QLW
DQGGUDJRI URWDWQJFA\QQGHUV ±
LQYLVFLGWKHRU IRU
RI VXSHUVRQLF DLUFRLOV
QLWFRHILFLHQW
QLWGUDJSRODUSORW
QLWHILFLHQF\ RI DLUFLO
/ LQHQWKDO* XVDY
/ LQHQWKDO2 WWR
QQHDUGLIHUHQWDOHTXDWRQV
QQHDUIRUP XODDGHTXDF\ ±
QQHDUL]HV
QQHDUP RPHQWXP HTXDWRQ
DERXW
%HUQRXOOLDVHQHUJ\ HTXDWRQ
%HUQRXOOHTXDWRQUHVWULFWLRQV ±
K\GUDXOLF DQGHQHUJ\ JUDGHQWV ±
MWWW[LVSUHVVXUHHTXDOVDWRSSKHUH SUHVVXUH
QQHDUP RPHQWXP \SV
P RPHQWXP IOX[FRUUHFWLRQIDFWRU
QHWSUHVXUHIRUFHRQDFRVHGFRQWROVXUIDFH ±
QRQLQHUWDOUHHUQHIIHFW ±
RQHGLPHQVLRQDOP RPHQWXP IOX[
SUHVXUHFRQGLWLRQV/DWWW[LW ±
VXUIQDWLRQ VXUIDFDQGG\QDP LF SUHVXUH ±
VWDG\ LQFRP SUHVVLEOHIORZ
VXUFHYHORFLWVFRQGLWLRQIRUDOJHVQN ±
QQHDUP RPHQWXP \SV
QQHGRXEOHW
QQHLUUHYHUVLEOHQDOYRUWH[±
QQHVLQN
QQHVRXUFH
QQHVRXUFHDWDSRLQW
QQHVRXUFHRUVLQNDWWKHRULJLQ
QQHYRUWH[DWDSRLQW
QTXLGV ±
ORFDOCFFHOHUDWLRQ
ORFDOCORVVHV
ORFDOPDVVIORZ IXQFWLRQ ±
ORFDOSURSHUWLHV
ORFDO5H\QROGVQXPEHU
ORFDOVHORFLWVPHDVXUHPHQW ±
ORJDULWKPLF GDZ

OHJDUWKPLF OD PRGLILHGE\ URXJKQHVV
OHJDUWKPLF RYHUDSOD ±
OHJDUWKPLF RYHUDSOD\HU
OHQJSLSHOLQHV ±
OHQJWKURDWGZHLU
OHVFRHILFLHQW
OHVHV
OHZ VSHHGIOHZV
OHZ VSHHGIOHZVDQGURXJKQHVVUDWLR
OXPLQHVFHQWFRDWLQJVLQWXPHQWLQWXPHQDWLRQ

0
0 DFK (UQW ____ ____
0 DFK FRQH ____
0 DFK QXP EHU ____ ____ ____ ± ____ ____ ____
0 DFK QXP EHU IRUP XOD YV LVHQWRSLF UDWR IRUP XOD ____
0 DFK QXP EHU UHDWRQV ____ ____ ± ____ ± ____
0 DFK Z DYHV ____ ±
0 DJQXV * XVWDY ____
0 DJQXV + HLQULFK ____
0 DJQXV HIHFW ____
0 DJQXV 5 REHUW ____
0 DQQLQJ 5 REHUW ____ ____
0 DQQLQJ¶V IRUP XOD ____ ____
0 DQQLQJ¶V IRUP XOD IRU XQLIRUP IORZ YHORFLW ____
0 DQQLQJ¶V RSHQ FKDQQHO IRUP XOD ____
0 DQQLQJ¶V URXJKQHVV FRUUHODWRQ ____ ____
P DQRPHWHU IRUP XOD ____
P DQRPHWHU DSSOLFDWRQV
GRZ QZ DUG SUHVVXUH ____ ±
SUHVVXUH GLVWULEXWRQ ____ ±
VLP SOH P DQRPHWHU DSSOLFDWRQV ____ ±
P DSSLQJV ____
0 DUR W (GP H ____
P DV VRXUFHV RU VLQNV RI ____
P DV IORZ ____
P DV IORZ IRU DJLY HQSUHVVXUHGURS ____ ±
P DV IORZ IXQFWRQ ____
P DV IORZ P HWHU ____
P DV P HDVXUHP HQW ____
P DWHULDO GHULYDWLYH ____ ±
P DWUL[UDQN ____
P D[LP XP P DV IORZ ____
0 D[Z HOO - DP HV & OHUN ____
P HDQ IUHHSDWK ____
P HDQ YHORFLW JUDGLHQW DQG VXUEXOHQW VKHDU ____
P HDVXUHG SHUP DQHHFXUYHV ____ ±
P HFKDQLFDO HILFLHQF ____
P HQLVFXV ____
P HUFXU EDURP HWHU ____ ±
P HWDFHQWHU ____
P HWDFHQWULF KHLJKW ____
P HWHUV ____
P LFURE XEEOHV ____

0 LQDQFH & %
PLQRUGXFWRRVFRHILFLHQW
PLQRUDVVHW
PLQRUDVVHWVLQFRPSUHKHQVLEOHORZ
PLQRURUDFFRXQWVLQSLSHOLQHV ±
JUDGXDWH[SDQVLRQ² WKHGLIIXVH ±
PL[HG DQG DJLDOIORZ SXPSV
DJLDOIORZ SXPS WKHRU ±
FRPSXWDWRQDOIOXLGG\QDPLFV
SHUFRUP DQHHR DQDJLDOIORZ SXPS
SXPS SHUFRUP DQFHYHUXV VSHFLILF VSHHG ±
VSHFLILF VSHHG
VXFWRQ VSHFLILF VSHHG ±
VXUERP DFKLQHU ±
PL[HGIORZ SXPSV
PL[LQJ HQJWKFRQXDQW
PL[LQJ HQJWKHGG\ YHORFLW
PRGHODQDOVLV
PRGHQJ DQGLWSLWDOV ±
G\QDPLF VLPLODULW ±
JHRPHWULF VLPLODULW ±
NLQHPDWF VLPLODULW ±
Z DWUDQGDLUWXQJ GLVFUHSDQFLHV ±
PRGHQJ FRQHSW DGYDQFHG ±
PRGXODUIORZ
PROHFXODUG\QDPLF PRGHO
PRPHQXP DORQJ Z DOHTXDWRQ
PRPHQXP GHILFLW
PRPHQXP HTXDWRQ
PRPHQXP HTXDWRQZLWKFRQXDQWYLVFRVLW
PRPHQXP IOX[FRUUHFWRQIDFWRU
PRPHQXP IOX[WUP
PRPHQXP IOX[WUPV
PRPHQXP LQWUDFHOXODPDW ±
GLVSODFHPHQWWKLFNQHVV ±
. iUPiQ‖V DQDO\VLV WKHIDWSDW ±
PRPHQXP LQWUDFWKHRU
PRPHQXP UHDWRQ
PRPHQXP WKLFNQHVV
0RRG\ /)
0RRG\ FKDUW ±
0RRG\ SLSHIULFWRQIDFWRU
0RRG\ WWSHSLSHIULFWRQ
0RUWRQQXPEHU 0R
PRYLQJ QRUPDOWKRFNV ±

PRYLQJ VKRFN ___
PXOWLSKDVHIRUFHV _
PXOWLSOHSLSHV\VWHPV ___ ±
SLSHQHWZRUNV ___ ±
SLSHVLQSDUDOOHO ___ ±
SLSHVLQVHULHV ___ ±
VKUHGHGHSHQGHQFLHV ___ ±
PXOWLWKUHDGHSXPSV ___

1

1$&$ DLUFRLOV ___ ±
QDSSHV ___
QDWXUDOFRQYHFWLRQ ___
1DYLHU&ODXGH/RXLV _
1DYLHU6WRNHVHTXDWLRQV _ ___ ___ ___
QHDUHVW LQFRPSUHVVLEOHIORZ ___ ___ ___ ___
QHWFLUFXODWLRQ ___
QHWSRVLWLYHVXFWLRQKHDG ___ ±
QHWSUHVVXUHIRUFHRQDFORVHGFRQWROVXUIDFH ___ ±
QHXWURFEXR\DQF\ __
1HZWRQ,WHUF ___
1HZWRQLDQIOXLGV __ ___ ±
1HZWRQ¶VODZV __
LQFRQSUHVVLEOHWDOFRRUGLQDWH ___
VHFRQGODZ ___ ___
1LNXUDGVH - ___
QRGHV ___
QRIORZ KHDG ___
QRQFLUFXODUGXFW ___ ±
QRQGHRUPDEOHIUHWDOFRRUGLQDWH ___
QRQGLPHQVLRQDOLQLWLDOFRQGLWLRQV
DERXW ___
DSSOLFDWLRQV ___ ±
FRPSUHVVLEOHDWVSDUDPHWHUV ___ ±
GLPHQVLRQOHVVSDUDPHWHUV ___ ± ___ ±
RVFLOODWLQJ IORZV ___
QRQLQHUWLDOFRRUGLQDWH ___
QRQLQHUWLDOH IIHFW ___
QRQLQHUWLDOUHHUHQFHIUDPH ___ ±
QRQ1HZWRQLDQIOXLGV ___ ± ___
QRQZHWWLQJ OLTXLG ___
QRUPDV ___
QRUPDOGHSWK ___

QRUPDOSRZHU
QRUPDOVKRFNZDYH ±
KVGUDXCDFMXPSV
0DFKQXPEHUUHDGHUV ±
PRYLQJQRUPDOVKRFNV ±
QRVDOSEROGDUVFRQGLWLRQV
QRVDOSFRQGLWLRQV ±
QRVWPSHUDWXUHMXPSFRQGLWLRQV ±
QR]HHIORZ
QR]HVV
GHVLJQSUHVVXUH
JHRPHWU
QXPHULFDODQDOVLV
DERXW
DOWHUQDWLYHLPSOHPHQWDWLRQ ±
ERXQGDU\HOHPHQWKRG ±
FRPPHUFLDO&)' FRGHV ±
ILQLWHGLIIHUHQFHPHWKRG ±
ILQLWHHOHPHQWPHWKRG
RQHGLPHQVLRQDOXQVWHDG\ IORZ ±
VWHDG\ VWDWH GLPHQVLRQDOOHVV IORZ ±
YLVFRXV IORZ FRPSXWHUGHY
QXPHULFVPHWKV ±

2

REQTXHUHFRPSUHVVLRQVKRFN
REQTXHVKRFNSRODUKRGRJUDSK
REQTXHVKRFNUHDWRQV
REQTXHVKRFNZDYH ±
RFHDQHQJLQHHULQJ
RQHGLPHQVLRQDOSSUR[LPDWRQ
RQHGLPHQVLRQDOHQHUJ\ IOX[VWUPV ±
RQHGLPHQVLRQDOIOX[VWUP DSSUR[LPDWRQ ±
RQHGLPHQVLRQDOLQHWDQGH[LWV
RQHGLPHQVLRQDOPRPHQWXP IOX[
RQHGLPHQVLRQDOXQVWHDG\ IORZ ±
RQHVHYHQSRZHUGDZ
RQVHWSRLQW
RSHQEODGHV
RSHQFKDQQHOIORZ
DERXW ±
&Kp]\ IRUPXOD ±
FULWLFDOGHSWK ±

HILFLHQWXQLRUP IGRZ FKDQQHOV ±
IGRZ FODVVLILFDWRQE\ GHSWKYDULDWRQ ±
IGRZ FODVVLILFDWRQE\ )URXGHQXPEHU
IGRZ YHORFLWLQ
JHRPHWU DQG QRVDWRQIRU
K\GUDXOLF MXPSV ±
RQH GLPHQVLRQDOSSUR[LPDWRQ ±
\SHFLLF HQHUJ\ ±
VXUDFHZDYHVSHHG ±
XQLRUP IGRZ ±
RUILFHIGRZ ±
RUWKRJRQDOIGRZ VHW
RUWKRJRQDOWRI VXUHPDQHVDQGSRWHQWLDOIORZV ±
RVFLOODWLQJ IGRZ V
RVFLOODWLQJ MXPS
RXWUDOHU
RXWUXVWHPDQHGLVSODFHPHQW
RXWZDOOHU
RXWOHW
RXWSXWYDULDEOHV
RXWLGHSUHVVXUH
RXWZDUGQRUPDOXQLWVYHFWRU
RXWZDUGXQLWVYHFWRU
RYHUODS OD\HU

3

SDLQW
SDUDOOHOSLSHIGRZ
SDUDPHWULV
3DUKDOOIXPH
SDUWLDOO GURZQHG\OLFHJDWH
SDUWLFOHLPDJHYHORFLPHWU 3,9
3DVFDO¶VODZ
SDWKLQJERXQGDU FRQGLWLRQV
SDWKOLQHV
SD\EDFNVWPH
3HOWRQ / HWXU$
3HOWRQZKHHO
SHUHFWXOG
SHUHFWJDV ±
SHUHFWJDVDUHDFKDQJH ±
SHUHFWJDVODZ
SHURGLF VP DOODP SODWHVXUIDFHZDYHV

SHUPHDELOLW ____ ____
3KLORVRSKV + RUDWR) UHGHULFN ____
SK\VLFDOODZV ____ ± ____
VXVWDLP YV FRQWUROYROXPHV ____ ± ____
YROXPHDQGPDVVUDWLRI IORZ ____ ± ____
SLH]RHOHFWULFVWDQGXFHUV ____
SLJV ____
SLSHGLDPHWUSURERPV ____ ± ____
SLSHIORZ SURERPV VWSHVRI
DERXW ____
IORZ UDWH ____ ± ____
SLSHGLDPHWU ____ ± ____
SLSHOHQJWK ____ ± ____
SLSHIORZ WDQVLWRQ ____
SLSHIULFWRQYV 5H\QROGVQXPEHU ____
SLSHOHQJWKSURERPV ____ ± ____
SLSHQHZWRQV ____ ± ____
SLSHV ____
DYHUDJHYHORFLW ____
SLSHVLQSDUDOOHO ____ ± ____
SLSHVLQVHULHV ____ ± ____
SLVFKLQJPRPHQW ____
3LVWRQ ____ ± ____
,SVHQDOWKLQDWWXVSE\ VWSPHWKRG ____ ± ____
3LVWRW+HQULGH ____ ____
3LVWRWRUPXOD ____
SLVWRWXEDWF VWEHV ____ ____
SODQHIORZ SDWWHUVHGERG\ VKDSHV ____ ± ____
IORZ SDWWDFKFXODU\OLQGHUZLWKFLUFXODWRQ ____ ± ____
. HOYLQRYDO ____
. XWWD-RXNRZVNLOLWWKHRUHP ____ ± ____
OLWWDQGGUDJRI URWDWLQJ F\OLQGHUV ____ ± ____
SRWHQWLDOIORZ DQDORJV ____ ± ____
5DQNLQHRYDO ____ ± ____
SODQHIORZ VWSHV ____
SODQHSRODUFRRUGLQDWV ____
SODQHSRWHQWLDOIORZV
DERXW ____
IORZ DURXQGDFRUQHURI DUELWUDU DQJOH ____ ± ____
IORZ QRUPDOWKDIODWSODW ____ ± ____
OLQHVRXUFHDWSRLQW ____
OLQHYRUWH[DWSRLQW ____
XQLIRUP VWUHDP DWDQDQJOHRI DWWDFN ____
SODWXFIOXLGV ____
SODQRUP DUHD ____

3RGH / ____
3RGH¶VDQJOH____
SRLQWGRXEOHW___
SRLQWVRXUFHRUVLQN____
SRLVH___
3RLVHXLGH_
3RLVHXLGH - / 0 ___ ____
3RLVHXLGHIORZ ____ ____
3RLVHXLGHIORZ IULFWLRQIDFWRU___
3RLVHXLGHSDUDEROD____
3RLVRQ¶VUDWLR____
SRVLWLYHGLVSODFHPHQWSXPSV 3' 3V ____
SRWHQWLDOHQHUJ\ ___
SRWHQWLDOIORZ DQDORJV____ ±___
SRWHQWLDOIORZ DQGFRPSXWDWLRQDOIOXLGG\QDPLFV
DLUFRLOVKHRU____ ±___
D[LVPPHWULFSRWHQWLDOIORZ____ ±___
HOHPHQWDU SDQHIORZ VROXWLRQV____ ±___
LPDJHV____ ±___
LQWURGXFWLRQDQGGUHYLHZ____ ±___
QXPHULFDOODQDOVLV____ ±___
RVKHUSDQHSRWHQWLDOIORZ V____ ±___
SDQHIORZ SDWWFORVHGERG\ VKDSHV____ ±___
SDQHSRDUFRRUGLQDWHV___
UHYLHZ RI VWHDP IXQFWLRQFRQFHSWV____ ±___
UHYLHZ RI YHORFLW SRWHQWLDOFRQFHSWV____ ±___
VXPPDU ___
VXSHUSRVLWLRQ RI SDQHIORZ VROXWLRQV____ ±___
SRWHQWLDOOLQHV___
SRWHQWLDOVKHRU ___ ___
SRWHQWLDOVKHRU IRUVKLFNFDPEHUHGDLUIRLOV____ ±___
SRXQGRI PDW___
SRZHUFRHILFLHQW___ ___
ZLQGWXUELQH___
SRZHUODZV___ ___
UHDWLRQVKLS IRULVHQWRSLF SHUFHFWJDV___
SRZHUVSHFLLF VSHHG____ ±___
3UDQGWO/ XGZLJ_ ___ ___ ___ ___ ___ ___ ___ ___ ___
3UDQGWO0 H\HUDQJOH___
3UDQGWO0 H\HUH[SDQVLRQIXQFWLRQ___
3UDQGWO0 H\HUH[SDQVLRQZDYHV
QHZ WHQGVLQDHURQDXWFV____ ±___
3UDQGWO0 H\HUSHUHFWJDVIXQFWLRQ____ ±___
VXSHUVRQLF DLUFRLOVDSSOLFDWLRQ____ ±___
WKLQ DLUFRLOVKHRU____ ±___

VKUHHGLPHQVLRQDOVXSHUVRQFIGRZ ±
3UDQGWOO HVHUSHUHFWVDVIXQFWLRQ ±
3UDQGWOQXPEHU
SUHVXUH
SUHVXUHDQG SUHVXUHJUDGLHQW ±
SUHVXUHIRUFHVRQDIOXLGHOHPHQW ±
SUHVXUHFRHILFLHQWGLPHQVLRQOHVV
SUHVXUHFRQGLWLRQVDWWWWW[LW ±
SUHVXUHGLIIHUHQWLDO
SUHVXUHGLVWULEXWLRQ ±
HTXLOLEULXP RI DIOXLG ±
K\GURVWDWLF IRUFHVRQSDQHVXUIDFHV ±
K\GURVWDWLF SUHVXUHGLVWULEXWLRQ ±
PDQRPHWU DSSOLFDWLRQV ±
PHDQYDOXHV
SUHVXUHDQG SUHVXUHJUDGLHQW ±
SUHVXUHPHDVXUHPHQW ±
LQWULG EERG\ PRWLRQ ±
SUHVXUHGUDJ
SUHVXUHGURSOOHW
SUHVXUHHTXDWLRQ
SUHVXUHIRUFHVRQDIOXLGHOHPHQW ±
SUHVXUHJUDGLHQW
SUHVXUHJUDGLHQWWXUDFHV
SUHVXUHJUDGLHQWYHFWRU
SUHVXUHKHDG ±
SUHVXUHPHDVXUHPHQW ±
SUHVXUHUHFRYHU FRHILFLHQW
SUHVXUHVHQVRUV
SULPDU GLPHQVLRQV
SULPDU VKHUPRG QDPLF YDULDEOHV
SULPLQJ
SULQFLSDOFKRUGOQHR VKHERG ±
SULQFLSOHR GLPHQVLRQOHVVQXPEHUV 3' +
DERXW
HQJLQHHULQJ HTXDWLRQV SHFXODU ±
VFDOLQJ UHSHDWQJ YDULDEOHV VHOHFWLRQ
YDULDEOHV DQG FRQWDQW ±
YDULDEOHV DQG VFDOLQJ SDUDPHWHUV VHOHFWLRQ ±
SUREOHP VROYLQJ WHFKQLTXHV
SURSHOOHUPHWUV
SURSHOOHUWKUXVWSHUXQLWDUHD
SURSHOOHUWXUELQH
SURWRWSH
SVHXGRSODVWLFLGV

SXPS FXUYHV ___
SXPS HILFLHQF\ ___
SXPS KHDG ___
SXPS RSHUDWQJ SRLQWV ___
SXPS SDUDPHWHUV ___
SXPS SHUFHQWLOH ___
SXPS SHUFHQWLOHFXUYHVDQGVLPLODULWXGHV
DERXW ___
GHYLDWRQVIURP LGHDOSXPS SWKHRU ___ ±
GLPHQVLRQOHVSXPS SHUFHQWLOH ___ ±
HIIHFWRI YLVFRVLW ___ ±
PHDVXUHGSHUFHQWLOHFXUYHV ___ ±
QHWSRVLWLYH VXFWLRQKHDG ___ ±
VLPLODULWXGHV ___ ±
SXPS SHUFHQWYHUWXVVSHFLILF VSHHG ___ ±
SXPSV ___ 6HH DOVR FHQWULXJDOSXPSV PL[HG DQG DLDOIORZ SXPSV SXPS SHUFHQWLOH FXUYHVDQGVLPLODULWXGHV SXPSV PDWKLQJ WRV\VWHP FKDUDFWHULVWLFV
FODVVLILFDWRQ ___ ±
FRPELQHGLQSDUDOOHO ___ ±
FRPELQHGLQVHULHV ___
GHILQHG ___
GHYLDWRQVIURP LGHDOSXPS SWKHRU ___ ±
GLPHQVLRQOHVSXPS SHUFHQWLOH ___ ±
HIIHFWRI EODGHDQJOHRQKHDG ___ ±
PDWKLQJ WRV\VWHP FKDUDFWHULVWLF ___ ±
PXOWLVWDJHSXPSV ___
RSHUDWQJ VLQJOH DQGSDUDOOHO ___
SXPS SHUFHQWYHUWXVVSHFLILF VSHHG ___ ±
SXPS VWKHRU ___ ±
VSHFLILF VSHHGSXPSV ___ ±
YLVFRVLWHIIHFWRQSXPS SHUFHQWHFXUYHVDQGVLPLODULWXGHV ___ ±
SXPSV PDWKLQJ WRV\VWHP FKDUDFWHULVWLFV
DERXW ___ ±
FRPSUHVVRUV ___ ±
JDVWXUELQHV ___ ±
PXOWLVWDJHSXPSV ___
SXPSVFRPELQHGLQSDUDOOHO ___ ±
SXPSVFRPELQHGLQVHULHV ___
SXPS VXUJH ___
SXPS VWKHRU ___ ±
SXPS VWXELQH ___
SXUHFRQWDQW ___

TXDVLWDGDVXPSWRQ
TXLFNVDQG

5

UDQGRP HUURU
UDQNRI DPDZ[
5DQNLQH: - 0
5DQNLQHKDOEERG ±
5DQNLQH+XJRQRWHDWRQ
5DQNLQHRYDO ±
UDSLGDYDULQJIORZ 59)
UDUHDFWRQVKRFN
UDUHILHGJDV
UDWGSRZHU
UDWRWSHLGHDOJDVIRUPXOD
5D\OHLJK/RUG-RKQ: 6WUXWW
5D\OHLJKIORZ
5D\OHLJKQQH
UHDFWRQVXUELQHV ±
UHDOLPSHUHFWJDVIORZV
UHDOJDVHV VSHFLILFKHDWR
UHFWDQJXODUZHLU
UHGLPLQDU]DWRQUHJLRQ
UHDWRQVKLSVRQRUPDOVKRFNZDYH ±
UHDWWHWUPV
UHDWWHYHYHORFLW
UHSHDWQJYDULDEOHPHWKRGRI GLPHQVLRQDOQDO VLV
UHSHDWQJYDULDEOHV 6HHVFDOQJSDUDPHWHUV
UHVLVWDQFHFRHIILFLHQW ±
UHVRQDQFH
UHYHUVLEOHGLDEDWLFIORZ
UHYHUVLEOHSRWHQWLDOFKHRU
UHYROXWLRQV
5H\QROGV2VERUQH
5H\QROGVDQG0DFKQXPEHUHIHFW
5H\QROGVQXPEHU
RI FKDQQHO
GLPHQVLRQOHVV IRUPLPLDUIORZ ±
IRUFHFRHILFLHQW
DQGJHRPHWULHIHFW ±
KLVWRULFDOQRXQLQH ±
ORFDO
ORZ VSHHGYLVFRXVIORZVZLWKQRIUHHVXUIDFH

YV SLSHIULFWRQ
UHJLPHV ±
URXJKQHVVUDWR DQG
VRXUFHR ±
WDQVLWRQ
5H\QROGVQXPEHUUDQJHV
5H\QROGV¶WHVW HDYHUDJLQJ FRQFHSW ±
5H\QROGV WDQVSRUWWKHRUHP
DERXW
DUELWUDU IL[HG FRQWROYROXPH ±
DUELWUDU PRYLQJ DQGGHRUP DEOHFRQWROYROXPH ±
FRQWROYROXPHPRYLQJ DVFRQXDQWVYHORFLW
FRQWROYROXPHR FRQXDQWKDSHEXWYDULDEOHYHORFLW
GHILQHG
IRUGHRUP DEOHFRQWROYROXPH
DVHQHJ\ HTXDWRQ
RQHGLPHQVLRQDOCIX[WKUP DSSUR[LPDWRQ ±
UKHRORJ\
UKHRSHFWF IOXLGV
5LDERXFKLQVN\ '
5LFKDUGVRQQXPEHU
5LJKWVLGH %
ULJLGERG\ PRWRQ
SUHVVXUHGLVWULEXWRQ ±
UHDOYHPRWRQEHWZHHQSDUWFOHV ±
URWDWRQ ±
VKHDUVWUHVV
XQIRUP CQHDUFFHOHUDWRQ ±
ULJLGERG\ URWDWRQ ±
5RELQV %HQMPLQ
URFNHWG QDPLFV
URQQQJ PRPHQW
5RVVE\ QXPEHU
URWDPHWUV
URWDWQJ F\ QQGHUYLVFRPHWU
URWDWQJ GLVNYLVFRPHWU
URWDWQJ VHQVRUV
URWDWRQDCIORZ
URWRG QDPLF SXPSV
URWRUEODGHV
URXJKQHVV
URXJKQHVVUDWR
DQGORZ VSHHGIORZ V
5H\QROGVQXPEHUDQG
LQWXLWLEOHQWORZ

VXEVFULSWLRQ
YDOXHVRI FRPPHUFLDOGXFWV
ZDOO
URXJKZDOOSLSHIORZ
URXJKZDOOVHIHFW ±
5XQJH. XWLOLPHWHUG
6
6DYRQLXVURVRU
6\ERORXQLYHUVDOVHFRQGV
6\ERORYLVFRVLW\
VFDQQLQJ UHSHDWQJ YDULDEOHVVHOHFWRQ
VFDQQLQJ GDZV
VFDQQLQJ SDUDPHWHUV
6FKDFKWQJ +
VFUHHQVKRW
VFURROVHFWRQ
VHFRQGDU\ GLPHQVLRQVLQIOXLGPHFKDQLFV
VHFRQGDU\ IORZ
VHFRQGDU\ VXEVFKHPHV
VHGLPHQWDWLRQ EDVLQ
VHSDUDWRUV SLSHIORZ
VHULHVSLSHIORZ
VKDIWZRUN
VKDSHIDFWRU
VKDUS FUHVWHGZHLV ±
VKHDUIUHHFRQGLWLRQ
VKHDUDHUVKLFNHVV
VKHDUVWUDLQUDWH
VKHDUVWHVV
VKHDUVKLFNHQLQJIOXLG
VKHDUWKLQQLQJIOXLG
VKHDUZRUN
VKRFN6HHDOVR QRUPDOVKRFNZDYH
DQJOHRI
ERZ VKRFN
H[SDQVLRQVKHRU
REOLTXHUHFRPSUHVVLRQVKRFN
SRODU
UDUHFDFWLRQVKRFN
VWUDQJ VKRFN

ZHDNVKRFN ____
VKRFNOXV ____
VKRFNVWEHZLQGVWQQHO ____
VKRFNZDYHV ____
ERZVKRFNZDYH ____
FXUYHGVKRFNZDYH ____ ____
REOTXHVKRFNZDYH ____ ± ____
VKRUWFUHWWAGZHLUQDSSHV ____
VKRUWGXFW ____
VKRXOGHUYHORFLW ____
VKXWWLKHDG ____
VLGHIRUFH ____
VLPLODULW ____
VLPLODULWUXOHV ____ ± ____
VLPSOHPDQRPHWUDSSODFDWRQV ____ ± ____
VLPSOLILHGIUHHVXUDFHFRQGLWRQV ____
VLQNSOXVDYRLG[DWKHRUJLQ ____
VLSKRQLQJ ____
VL]HFKDQJHV IORZUDWHIHFWURP ____
VNLQIULFWRQFRHILFLHQW ____
VNLQIULFWRQODZ IRUWXUEXOHQWDWSODWHIORZ ____
VOHQGHUERG\ IORZ ____
VOLS IOXLGFRXSOLQJ ____
VOLSIORZ ____
VOLSIORZ LQJDVHV ____
VOLSYHORFLW ____
VOXLFHJDWHIORZ ____
6PLWK3XQGP ZLQGVKLHOG ____
VRODGVXUFH ____
VROYHUV ____
VROYHUVRIWZDUH ____ 6HHDOVR(QJLQHHULQJ(TXDWRQ6ROYHU((6
VRQLFEDUULHU ____
VRQLFERRP ____
VRQLFIORZ FKRNLQJ ____
VRQLFGHQJWK ____
VRQLFPRWRQ ____
VRQLSRLQWWUDQVYDOXHV ____ ± ____
VRQLSURSHUWLHV ____
VRQLVKURDW ____ ____
VRXQGVSHHG ____
VRXQGVSHHGUHDWRQ ____
VRXUFHSOXVDQHTXDOLQN ____ ± ____
VRXUFHVRUVLQNVRIPDW ____
VSHFLILFGLDPHWU ____
VSHFLILFHQHUJ\ ____ 6HHDOVRFULWWDOGHSWK

\SHFLILF JUDYLW
\SHFLILF KHDW
\SHFLILF KHDWDWDWR
\SHFLILF KHDWIRUUHDOJDVHV
\SHFLILF VSHHG
\SHFLILF VSHHGSXPSV ±
\SHFLILF ZHJKW
EXR\DQWRUHHDQG ±
RI FRPPRQIOLGV
GHILQHG
RI IOLGDQGEXR\DQF\
RI KXPDQEHLQJV
\SHHGRI VRXQG ±
\SKHUHGUDJIRUPXOD
\SKHULFDOSRODUFRRUGLQDWHV ± ±
\SK\JPRPDQRPHWHU
\SLQXS SUREOHP
WDEOHDW
DERXW ±
DQGEXR\DQF\ ±
DQGZDWHUHD ±
WDEOHWPDSRI GLIIXVHUIORZ SDWWHUQV
WDEEHHMXPS
WDQJHQWRQ WDWF DQGG\QDPLF SUHVVXUH ±
WDQJHQWRQGHQVLW\
WDQJHQWRQHQHUJ\
WDQJHQWRSRLQW
WDQJHQWSUHVVXUH
WDQWHDDUGDURL
WDOHSRLQW
WDOHV
WDOHVSHHG
WDQGDUGDWPRVSKHUH ±
WDUJHWQJ YRLWH[
WDWH HTXDWRQRI
WDWUHDWRQVIRUJDVHV
WDWUHDWRQVIRUJDTXLGV
WDWFDOO XQWBEHHERG
WDWF HTXLOLEULXP RI IORDWQJ EERG
WDWF KRQH
WDWF ORVVFRHILFLHQW
WDWF SUHVVXUH
WDWF V\VWHPV
WDURUEDGHV

VWDG\ FRP SUHVVLEOHIORZ ____
VWDG\ FRP SUHVVLEOHIODP LQDUIORZ ____
VWDG\ IORZ ____
VWDG\ IORZ HQHJ\ HTXDWRQ____
VWDG\ LQFRP SUHVVLEOHIORZ ____
VWDG\ MXMP S____
VWDG\ SODQHFRP SUHVVLEOHIORZ ____ ±
VWDG\ VZR GLP HQVLRQDOODP LQDUIORZ ____ ±
VWDP SURSHUWLHV__
6WDN HV * HRUJH* _ __ ____ ____
6WDN HVIORZ ____ ____
6WDN HVQXP EHU____
6WDN HV2VHQIRUP XOD__ __
VWRSSLQJ YRUWH[____
6WUP HUYLVFRP HWU__
VWUDWLFDWRQ____
VWUHDNOLQHV__
VWUHP IXQFWRQ____
DERXW ____ ±
JHRP HWULFLQIOXHQFHRQ VWUHP OLQHVRI VWHIORZ ____ ±
LQFRP SUHVVLEOHIDLX\ PP HWULFIORZ ____ ±
LQFRP SUHVVLEOHISODQHIORZ LQSRDUFRRUGLQDWHV____
VWDG\ SODQHFRP SUHVVLEOHIORZ ____ ±
VWUHP IXQFWRQFRQHVW ____ ±
VWUHP OLQHIRUP RI IULFWRQOHVVPRP HQWXP HTXDWRQ____
VWUHP OLQHRI VWHIORZ ____
VWUHP OLQHV ____ ____ ____
VWUHP OLQHVXUIDFH____
VWUHP VXEHV__
VWURQJ MXMP S____
VWURQJ VKRFN____
6WURXKDO9 ____
6WURXKDOOXMP EHU____
6WUXWW-RKQ: /RUG5D\OHLJK _ ____ ____ ____
VXEFULWLFDOIORZ ____
VXEFULWLFDOIORZ VWDWH____ ±
VXEP HUJHGH[LW____
VXEVRQLF GLIIXVHU____
VXEVRQLF IORZ ____ ____
VXEVRQLF LQOHW____
VXEVRQLF LQOHWIORZ ____
VXEVRQLF PRWRQ____
VXEVWDQWLDOGHULYDWLYH____ ±
VXFFHVVLYHRYHUHDIDWRQ 625 PHWKRG____
VXFWLRQVSHFLILF VSHHG____ ±

VXGGHQFRQWDFWRQ 6& ___
VXGGHQH[SDQVLRQ 6(___
VXSHUFULWLFDOLGRZ ___
VXSHUFULWLFDOLGRZ VWDWH ___ ±
VXSHUSRVLWLRQ ___ ±
VXSHUSRVLWLRQRI SDDQHIGRZ VROXWLRQV
ERXQGDU\ FRQGLWLRQVHSDUDWLRQRQDKDOO ERG ___ ±
GRXEOHWV ___ ±
IGRZ SDVVDYRUWH[___ ±
JUDSKLFDOPHWKRGRI VXSHUSRVLWLRQ___
LQILQLWHURZ RI YRUWLFHV ___ ±
WKHYRUWH[VKHHW ___
VXSHUVRQLF DLUFRLOV
FHQWHURI SUHVVXUHRI ___
GLVWR ___
VXSHUVRQLF DLUFRLVDSSOLFDWLRQ ___ ±
VXSHUVRQLF H[SDQVLRQDQGFRPSUHVVLRQ___
VXSHUVRQLF IGRZ ___ ___
VXSHUVRQLF LQOHW ___
VXSHUVRQLF NWV ___
VXSHUVRQLF 0 DFK QXPEHU ___
VXSHUVRQLF PRWRQ ___
VXUIDFH IRUFHV ___ ___
VXUIDFHRI DP DFKLQH ___
VXUIDFHURXJKQHVV ___
VXUIDFHVKDSH ___
VXUIDFHWHQVLRQ ___ ± ___ ___
VXUIDFHWHQVLRQFRHIILFLHQW ___
VXUIDFHYHORFLWVFRQGLWLRQIRUDODUJHVDQN ___ ±
VXUJH ___
VXUURXQGLQJV ___
6XVKHUDDQGGDZ ___ ___
VZLQJ FKHFNYDOYH ___
VWPPHW FRQGLWLRQ ___
VWVWP ___
VWVWP DWF HUURU ___
VWVWP VWDWH ___
VWVWP YV FRQWROYROXPHV ___ ±
VWVWRQF SUHVVXUH ___

7

WDJHWWGRZ PHWKU ___
7DYGRU* HRIIUH , _ ___

7D\GRU0RXQWDLQ___
7D\GRUQXPEHU___
7D\GRUYRLFHV___
\WPSHUDWXUH__
\WPSHUDWXUHVHQVRU___
\WQVRUV___
\WUPLQDOIDOOYHORFLW__
7KHRUL RI6RXQG5D\OHLJK ___
\WKHUPDOFRQGXFWLYLW___ ±___
\WKHUPRG\QDPLFODZV___
\WKHUPRG\QDPLFSURSHUWLHVRIIOXLG__
\WKHUPRG\QDPLFVUHYLHZ
DEHRXW___
LVHQWURSLFSURFHVV___ ±___
0DFKQXPEHU___
SHUHFWJDV___ ±___
\SHFLILFKHDWDWR___
\WKLFN\VKHDUOD\HUIORZV___
\WKLQDLUROOWKHRU\ ___ ±___
\WKLQSODWHRULILFH___
\WKLQSODWHZHLUQDSSHV___
\WK[RWRSLFIOXLGV__
7KRP $ _
\WUHHGLPHQVLRQDOERGLHV___ ±___
\WUHHGLPHQVLRQDOOHWWHU ___
\WUHHGLPHQVLRQDOVXSHUVRQLF ___ ±___
\WUHHUHVHUYRLUXQFHUWDLQW___ ___ ±___
\WURDV___
\WURDVWKURXJK___
\WURDVWORZ ___
\WUXVV___
\WUXVVWRUFH___
7KZDLWHV %___
7KZDLWHVMVPHWKRG ODPLQDUEHQGDU OD\HU___ ±___ ___ ___
\WGDOEHUW___
\WPHDQV__
7RUULFHOOL (YDQJHOLVWD___
\RXQJHILHOHQF\ ___
\RXQJKHDG___
\WDLQJ HGJH5H\QROGV1XPEHU___
\WDLQJ YRUWHV___
7UDQVLWRQŠ FDUSDQH ___
\WDQVLWRQDOURXJKQHVV___
\WDQVLWRQ5H\QROGV1XPEHU___
\WDQVLWRQ\RXQJ\XEXOHQFH___ ±___

WDQVRQLFIGRZ ____
WDQVSRUWDWRQ____
WDQVSRUWSURSHUWLHV__
WHHGUDJFRHILFLHQWV____
WDJRQDOFRHILFLHQWPDWUL____
WDJRQDOPDWUL DJRUWKP 7' 0$ ____
WRSRVHQFXUYH____
WRSRVSKHUH__
WXUELQHIRUPXOD____
WXUELQHPHWULV____
WXUELQHSDUDPHWHUV____
WXUELQHV
DERXW__
LGHDOLHGUDGLDOWXUELQHWKHRU ____
LGHDOLHGZLQGWXUELQHWKHRU ____±___
LPSXOVHWXUELQHV____±___
LQFRPSUHVVLEOHWXUEGG IGRZ HQHUJ\ HTXDWRQ____
RSWPXP HILFLHQF\ RI GHVLJQ____
SRZHUVSHFLILFVSHHG____±___
UHDFWRQWXUELQHV____±___
ZLQGWXUELQHV____ ____±___
WXUERPDFKLQHU 6HHDOVR WXUELQHV
DERXW__
FHQWULXJDOSXPS____±___
FODVVLILFDWRQRI SXPSV____±___
PDWKLQJ SXPSVWRVWMP FKDUDFWHULVWF____±___
PL[HG DQGDLDOIGRZ SXPSV____±___
SXPS SHURUPDQFHFXUYHVDQGVLPLODULWUKHV____±___
VSHFLILFVSHHG____±___
WXUERPDFKLQHV____
WXUEXOHQFH_
WXUEXOHQFHIRUPDWRQPRGHO___
WXUEXOHQFHPRGHOLQJ PRGHOV
DERXW____
DGYDQFHGPRGHOLQJ FRQFHSWV____±___
ORJDULWKPLF RYHUDSDZ ____±___
5HQRGM¶WPHDVHUDJLQJ FRQFHSW____±___
VRIWZDUHIRU____
WXUEXOHQFHVKHDUGPHDQYHORFLWJUDGLHQW___
WXUEXOHQWERXQGDU\ OD\HUGUDJ FRHILFLHQW___
WXUEXOHQWFRUUHFWLRQIDFWRUV____
WXUEXOHQWMODWSODWHIGRZ VNLQIULFWLRQDZ ____
WXUEXOHQWMODWSODWHVKDSHIDFWRU____
WXUEXOHQWMOZ ____ ____ ____ ____ ____±___
WXUEXOHQWMOZ SDUDOOHOSODWHVROXWLRQ____±___

VKLEXGHQWORZ VKHRU ____
VKLEXGHQWULFNOHV____
VKLEXGHQWULFNOHQIRUPXOD____
VKLEXGHQWSLSHIORZ
DERXW ± ____
HIIHFWRI URXJKZDOOV ± ____
0 RRG\ FKDLW ± ____
VKLEXGHQWVHFRQGDU IORZ ____
VKLEXGHQWSDUDPRUSKV____
VKLEXGHQWPRRVKZDOOFRQGLWRQ____
VKLEXGHQWXSHVVHV____
WZR GLPHQVLRQDOERGLHV____
WZR GLPHQVLRQDOERGLHVGUDJ____
WZR GLPHQVLRQDOOLQLG IORZ ____
WZR GLPHQVLRQDOVXSHUVRQLF IORZ
0 DFK ZDYHV ± ____
REOLTXHVKRFNZDYH ± ____
YHU ZHDNVKRFNZDYHV ± ____
WZR SKDVHIORZ ____

8

XOWDVRQLF IORZ PHWHUV____
XQFHUWDLQWLHV[SHULPHQWDOGDWD ± ____
XQGXODUUXPS____
XQLIRUP IORZ
DERXW____
0 DQQLQJ URXJKQHVVFRUUHODWLRQ ± ____
QRUPDOGHSWKHVWLPDWHV ± ____
LQSDUWV IXOOFLUFXODUSLSH ± ____
XQLIRUP IORZ YHORFLW____
XQLIRUP QQHDUFFHOHUDWRQ ± ____
XQLIRUP VWHDP
DWZQDQJOH____
DWZQDQJOHRI DWWDFN____
SOXVDSRLQWVRXUFH ± ____
SOXVDVLQNDVKHRULJLQ ± ____
LQVH[GLUHFWRQ ± ____
XQLIRUP SUHVVXUHVXEVRFWLRQ____
XQLW_
XQLYHUVDO%HUQRXOOLFRQVWDQW____
XQVWEOHPRGHO____
XQVWEOHSUHVVXUHJUDGLHQW____
XQVWHDG\ IULFWLRQDOHVV IORZ DORQJ DVWHDP QQH____

XQYHUVDODULHV
XSZLQGIORZ

9

YDFXXP SUHVVXUH
YDOYHIORZ FRHILFLHQW
YDOYHV
UHVLVWDQFHFRHILFLHQW
VWHSVRI
YDSRUSUHVVXUH ±
YDULDEOHJUDYLWHIHFW ±
YDULDEOHV
DERXW
DQGFRQWDQW ±
DQGVFDOLQJSDUDPHWHUVVHOHFWRQ ±
YDULHGRSHQFKDQQHOIORZ
9DVFN $
YHFWRUHTXDWRQ
YHFWRUUHDWRQVKLSV
YHFWRUVVKUHHFRPSRQHQW
YHJURRYHPLFURUEDW
YHORFLW
DQGFKDQQHOJHRPHWU
FULWFDO
YV GUDJ IRUFH
IULFWRQ
K\GURXOF UDGLXVDQG
LQWDO
NLQHPDWF
ORFDOPHDVXUHPHQW ±
PHDQYDOXHV
PL[LQJ OHQJWKHGG\
UHDWLYH
VKRXOGHU
VOLS
VXUIDFH ±
VWUPLQDOOD
XQLRUP IORZ
YROXPH DYHUDJH
]HUR
YHORFLWGHHFWZDZ
YHORFLWILHOG ±
YHORFLWILHOGSURSHUWLHV ±

YHOHFWKHDG ____
YHOHFWPHWKRV ____
YHOHFWRI DSSURDFKIDFWRU ____
YHOHFWSRWHQWLDO ____
YHOHFWSRWHQWLDOFRQHSWV ____ ±
YHOHFWSRWHQWLDOIXQFWLRQ ____
YHQDFRQWUDFW ____
9HQWXUL * LRYDQQL ____
YHQWXULIXPH ____
YHQWXULPHWU ____ ____
YHQWXULWHH ____
YHUWLFDO[LVZLQGWXUELQH 9$: 7 ____
YHU ZHDNVKRFNZDYHV ____ ±
YLUWXDOPDW ____
YLVFRVLW ____ ±
YLVFRVLWDQGRWKHUSHQGDU SURSHUWLHV
DERXW ____
EDVLFIORZ DQDOVLVWKQLTXHV ____ ±
IORZ EHWZHHQSODWHV ____ ±
IORZ SDWWHUQ ____ ±
IORZ YLVXDOL]DWLRQ ____ ±
QRQ1HZWRQLDQIOXLGV ____ ±
QR VOLSFRQGLWLRQ ____ ±
QR WPSHUDWXUHMXPSFRQGLWLRQ ____ ±
5H\QROGVQXPEHU ____ ±
VOLSIORZ LQJDHV ____
VSHHGRI VRXQG ____ ±
VXUIDFHWHQVLRQ ____ ±
XQFHUWDLQWLQH[SHULPHQWDOGDW ____ ±
YDSRUSUHVVXUH ____ ±
YDULDWLRQRI YLVFRVLWZLWKWPSHUDWXUH ____
YLVFRVLW ____ ±
YLVFRVLWHIIHFWRQSXPSSHURUPDQFHFXUYHVDQGVLPLODULWUXOHV ____ ±
YLVFRVLWYDULDWLRQZLWKWPSHUDWXUH ____
YLVFRXVGLVLSDWLRQIXQFWLRQ ____
YLVFRXVIORZ UHJLPHVRI ____
YLVFRXVIORZ FRPSXWDWLRQHDV ____
YLVFRXVIORZ LQGXFWV
DERXW ____
GLIXVHUSHUPDQHH ____ ±
H[SHULPHQWDOGXFWVV ____ ±
IORZ LQORQFLUFXODUGXFW ____ ±
IRXUWVSHVRI SLSHIORZ SUREOPV ____ ±
IULFWLRQIDFWRU ____ ±
KHDGORVV ____ ±

LQWHUQDWLRQDOYHUVXVH[WHUQDOYLFRXVIORZ V ±
ODPLQDUIXQG GHYHORSHGSLSHIORZ ±
PLQRUORFDOORVVHVLQSLSHV\VWHPV ±
PXOWSOHSLSHV\VWHPV ±
5HYQRGV/QXPEHUUHJLPHV ±
VXUEXOHQFHPRGHOLQJ ±
VXUEXOHQWSLSHIORZ ±
YLVFRXVIORZ PRGHOHOHPHQW
YLVFRXVWUHVV
YROXPHDQGPDVVUDWHIORZ ±
YROXPHDYHUDJHYHORFLW
YROXPHIORZ
YROXPHIORZ PHDVXUHPHQW ±
YROXPHPHDVXUHPHQW
YROXPHUDWHIORZ
YROXPHWHFHILFLHQF\
YRUWH[IORZPHWHU
YRUWH[IXQFWRQ
YRUWH[VKHGGLQJ
YRUWH[VKHHW
YRUWH[VWUHQJWK
YRUWH[
YRUWH[VWUDLJKWHQHUDWRUGHOWD ±

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ZDOOZ
ZDOOHU
ZDORVFLOODWRU
ZDOURXJKQHVVUDWR
ZDOVKHDU 6HHERXQGDU\ OD\HU
ZDOWSHUDWXUHUDWR
ZDWUDQGDLUWHWUDGHFRHI ±
ZDWUKRUVHSRZHU
ZDWOQHDUHD ±
ZDWOQHIRRWSULQW
ZDYHGUDJ
ZDYHPDNLQJGUDJ
ZDYHPRWRQ
ZDYHSURSDJDWRQVSHHG
ZDYHWRXJK
ZHDNMXPS
ZHDNVKRFN
: HEHU0 RUW]

: HEHUQXPEHU
ZHLU
ZHLUGLVFKDUJHFRHILFLHQW
ZHLUQDSSHV
ZHLV ±
: HLVEDFK-XQDXV
ZHWWGDUHD
ZHWWGSHULPHWHU
ZLGWKFKDQJHVDQGJUDGXDOO YDULHGWKHRX
ZLQGHQJLQHULQJ
ZLQGVXQQ\
ZLQGVXUELQHSRZHUFRHILFLHQW
ZLQGVXUELQHV ±
ZLQJLQVSLUHGE\ WKHKXPSEDFNZKDOH
ZLQJVRILQIWVSDQ ±
: LVDFHQV *) ±

<

\DZ
<RXQJ¶VPRGXOVRIHDWXUHV

=

]HURERG\ GUDJSDUDGR[
]HURSUHVVXUHJUDGLHQW
]HURYHORFLW\
]RQHRI DFWLRQ
]RQHRI VLOHQFH

&RQYHUVLRQ) DFWRUVIURP %* WR6, 8QLWV

Pressure	lbf/ft^2	Pa	4.7880 E + 1
	lbf/in^2	Pa	6.8948 E + 3
	atm	Pa	1.0133 E + 5
	mm Hg	Pa	1.3332 E + 2
Specific weight	lbf/ft^3	N/m^3	1.5709 E + 2
Specific heat	$ft^2/(s^2 \cdot °R)$	$m^2/(s^2 \cdot K)$	1.6723 E − 1
Surface tension	lbf/ft	N/m	1.4594 E + 1
Temperature	°F	°C	$t_C = \frac{5}{9}(t_F - 32°)$
	°R	K	0.5556
Velocity	ft/s	m/s	0.3048
	mi/h	m/s	4.4704 E − 1
	knot	m/s	5.1444 E − 1
Viscosity	$lbf \cdot s/ft^2$	$N \cdot s/m^2$	4.7880 E + 1
	$g/(cm \cdot s)$	$N \cdot s/m^2$	0.1
Volume	ft^3	m^3	2.8317 E − 2
	L	m^3	0.001
	gal (U.S.)	m^3	3.7854 E − 3
	fluid ounce (U.S.)	m^3	2.9574 E − 5
Volume flow	ft^3/s	m^3/s	2.8317 E − 2
	gal/min	m^3/s	6.3090 E − 5

(48$7,21 6+((7

Ideal-gas law: $p = \rho RT$, $R_{air} = 287$ J/kg-K	Surface tension: $\Delta p = Y(R_1^{-1} + R_2^{-1})$
Hydrostatics, constant density: $p_2 - p_1 = -\gamma(z_2 - z_1)$, $\gamma = \rho g$	Hydrostatic panel force: $F = \gamma h_{CG} A$, $y_{CP} = -I_{xx}\sin\theta/(h_{CG}A)$, $x_{CP} = -I_{xy}\sin\theta/(h_{CG}A)$
Buoyant force: $F_B = \gamma_{fluid}$(displaced volume)	CV mass: $d/dt(\int_{cv} \rho dv) + \sum(\rho AV)_{out}$ $- \sum(\rho AV)_{in} = 0$
CV momentum: $d/dt(\int_{cv} \rho \mathbf{V} dv)$ $+ \sum[(\rho AV)\mathbf{V}]_{out} - \sum[(\rho AV)\mathbf{V}]_{in} = \sum \mathbf{F}$	CV angular momentum: $d/dt(\int_{cv} \rho(\mathbf{r_0} \times \mathbf{V}) dv)$ $+ \sum \rho AV(\mathbf{r_0} \times \mathbf{V})_{out} - \sum \rho AV(\mathbf{r_0} \times \mathbf{V})_{in} = \sum \mathbf{M_0}$
Steady flow energy: $(p/\gamma + \alpha V^2/2g + z)_{in} =$ $(p/\gamma + \alpha V^2/2g + z)_{out} + h_{friction} - h_{pump} + h_{turbine}$	Acceleration: $d\mathbf{V}/dt = \partial \mathbf{V}/\partial t$ $+ u(\partial \mathbf{V}/\partial x) + v(\partial \mathbf{V}/\partial y) + w(\partial \mathbf{V}/\partial z)$
Incompressible continuity: $\nabla \cdot \mathbf{V} = 0$	Navier-Stokes: $\rho(d\mathbf{V}/dt) = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{V}$
Incompressible stream function $\psi(x,y)$: $u = \partial\psi/\partial y$; $v = -\partial\psi/\partial x$	Velocity potential $\phi(x,y,z)$: $u = \partial\phi/\partial x$; $v = \partial\phi/\partial y$; $w = \partial\phi/\partial z$
Bernoulli unsteady irrotational flow: $\partial\phi/\partial t + \int dp/\rho + V^2/2 + gz = \text{Const}$	Turbulent friction factor: $1/\sqrt{f} =$ $-2.0 \log_{10}[\varepsilon/(3.7d) + 2.51/(\text{Re}_d\sqrt{f})]$
Pipe head loss: $h_f = f(L/d)V^2/(2g)$ where f = Moody chart friction factor	Orifice, nozzle, venturi flow: $Q = C_d A_{throat}[2\Delta p/\{\rho(1-\beta^4)\}]^{1/2}$, $\beta = d/D$
Laminar flat plate flow: $\delta/x = 5.0/\text{Re}_x^{1/2}$, $c_f = 0.664/\text{Re}_x^{1/2}$, $C_D = 1.328/\text{Re}_L^{1/2}$	Turbulent flat plate flow: $\delta/x = 0.16/\text{Re}_x^{1/7}$, $c_f = 0.027/\text{Re}_x^{1/7}$, $C_D = 0.031/\text{Re}_L^{1/7}$
$C_D = \text{Drag}/(\frac{1}{2}\rho V^2 A)$; $C_L = \text{Lift}/(\frac{1}{2}\rho V^2 A)$	2-D potential flow: $\nabla^2\phi = \nabla^2\psi = 0$
Isentropic flow: $T_0/T = 1 + \{(k-1)/2\}\text{Ma}^2$, $\rho_0/\rho = (T_0/T)^{1/(k-1)}$, $p_0/p = (T_0/T)^{k/(k-1)}$	One-dimensional isentropic area change: $A/A^* = (1/\text{Ma})[1 + \{(k-1)/2\}\text{Ma}^2]^{(1/2)(k+1)/(k-1)}$
Prandtl-Meyer expansion: $K = (k+1)/(k-1)$, $\omega = K^{1/2}\tan^{-1}[(\text{Ma}^2-1)/K]^{1/2} - \tan^{-1}(\text{Ma}^2-1)^{1/2}$	Uniform flow, Manning's n, SI units: $V_0(\text{m/s}) = (1.0/n)[R_h(m)]^{2/3} S_0^{1/2}$
Gradually varied channel flow: $dy/dx = (S_0 - S)/(1 - \text{Fr}^2)$, $\text{Fr} = V/V_{crit}$	Euler turbine formula: Power $= \rho Q(u_2 V_{t2} - u_1 V_{t1})$, $u = r\omega$